BEEF-vdW+U method applied to perovskites: Thermodynamic, structural, electronic, and magnetic properties

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Abstract

The recently developed BEEF-vdW exchange-correlation method provides a reasonably reliable description of both long-range van der Waals interactions and short-range covalent bonding between molecules and surfaces. However, this method still suffers from the excessive electron delocalization that is connected with the self-interaction error and, consequently, the calculated chemical and physical properties such as formation energy and band gap deviate markedly from the experimental values, especially when strongly correlated systems are under investigation. In this contribution, BEEF-vdW+U calculations have been performed to study the thermodynamic, structural, electronic, and magnetic properties of La-based perovskites. An effective interaction parameter $U_{eff}$ and an energy adjustment $\Delta E_{\text{ME}}$ are determined simultaneously by a mixing GGA and GGA+U method, where the enthalpy or Gibbs free energy of formation of oxides containing a transition metal in different oxidation states are fitted to available experimental data. The $\Delta E_{\text{ME}}$ is found to have its origin in the fact that the GGA+U method gives rise to the offsets in the total energy that include not only the desired physical correction but also an arbitrary contribution. Calculated results indicate that the BEEF-vdW method provides a more accurate description of the bonding in the $O_2$ molecule than the PBE method and has generally smaller $U_{eff}$ values for the 3$d$-block transition metals, thereby giving rise to band gaps and magnetic moments that are in better agreement with the experimentally measured values.

Keywords: GGA+U; Perovskite; Formation energy; Band gap; Magnetic moment

Supplementary data for this article is available

1. INTRODUCTION

Density functional theory (DFT) has gained considerable ground in recent years, and has become one of the most powerful techniques for predicting the structure and property of molecules and extended solids. [1] In contrast to the Hartree-Fock approach which does not include electron correlation, DFT has its complexity and a lot of physics hidden inside the approximate exchange-correlation functional. This term works surprisingly well given its simplicity and holds the key to the success of the theory. On the other hand, the commonly used LDA/GGA suffers from the delocalization error arising from the Coulomb term that pushes electrons apart (previously known as the “self-interaction error”) and the static correlation error which is due to the failure of the description of degenerate and near-degenerate states [2]. The former is believed to be connected
with the spurious interaction of an electron with itself, which can be cancelled by an equivalent term in the exchange energy in the Hartree-Fock approach. In the DFT method, however, the exact exchange-correlation functional is still unknown and, consequently, its accuracy in determining chemical and physical properties of materials depends strongly on the degree of the cancellation of errors in energy differences.

The “self-interaction error” in the occupied states may lead to excessive electron delocalization and, consequently, to an increase in the occupied electronic energy levels [3]. This deficiency in LDA/GGA is largely responsible for the underestimation of band gap by DFT calculations, especially in strongly correlated systems that possess highly localized $d$ or $f$ electrons. When electrons are transferred between species in different chemical environments, just as in the case of oxidation of transition metal cations, the “self-interaction error” would penalize the low oxidation state over the higher one, as the latter has less localized $d$ orbitals occupied. For this reason, little error cancellation is expected and the resultant oxidation energies are invariably overestimated [4].

To give a reasonably accurate description of the electronic structures as well as energetics of transition metal oxides, several new approaches, including LDA/GGA+U [5, 6], hybrid functional [7], and random phase approximation (RPA) [8], have been developed to address the transition between localized and delocalized electronic states. Among them the LDA/GGA+U framework offers the best compromise between accuracy and computational efficiency. In this method, the essential assumption is that the strongly correlated $d$ or $f$ electrons are subject to on-site quasi-atomic interactions which can be described by a Hubbard term $U$ and a Hund’s-rule exchange term $J$ [5, 6]. A simplified (rotationally invariant) approach to LDA/GGA+U was proposed by Dudarev et al. [9], in which an effective interaction parameter $U_{\text{eff}} = U - J$, was introduced by adding a penalty function to the total energy expression in LDA/GGA. This additional Hubbard-type term localizes the electronic states to which it is applied and thus lowers the energy of the occupied states.

It is well known that the accuracy of the LDA/GGA+U method depends entirely on the $U_{\text{eff}}$ for transition metal elements. Determining an appropriate $U_{\text{eff}}$, however, can be quite tricky. Although some researchers have pointed out that purely theoretical calculations can be used in this regard [10], the method proves to be difficult to apply to all transition metal oxides, such as $\text{CeO}_2$ [11]. In practice, the $U_{\text{eff}}$ values are often obtained by fitting calculated band gap and thermodynamic quantities to available experimental data [12-14]. One problem with such a determination is the difficulty of ensuring that the fitting to one property can correctly reproduce others. Moreover, it was recently found that the $U_{\text{eff}}$ value depends upon not only the identity of transition metal elements but also upon their local environments, such as the oxidation state and surrounding ligands [15]; that is, the transferability of $U_{\text{eff}}$ across compounds containing transition metal ions is limited. Finally, the $U_{\text{eff}}$ values obtained by different calculation methods (e.g., by using different exchange-correlation functionals and pseudopotentials) are not always the same. Thus, when the calculation method is changed, it is highly required to repeat the fitting to a specific property of interest to give a reasonable $U_{\text{eff}}$ value for a particular system.

Recently, Wellendorff et al. developed a semilocal approximation with an additional nonlocal correlation term, the Bayesian error estimation functional with van der Waals correlation (BEEF-vdW) [16], using regularization and crossover point-validation methods from machine learning. By taking a linear combination of local (Perdew-Wang LDA) [17], semilocal (PBE) [18], and nonlocal correlation (vdW-DF2) [19] as the correlation model space, both a reasonably reliable description of long-range van der Waals forces and a quantitatively accurate prediction of short-range covalent bonding between molecules and surfaces can be achieved, making BEEF-vdW an excellent density functional for studies in heterogeneous catalysis and surface science. However, the self-interactions of electrons with themselves that arise from the Hartree term are still not fully compensated for in the BEEF-vdW method and, consequently, an appropriate correction to the functional (e.g., GGA+U) also needs to be made when strongly correlated systems are under investigation.

Perovskite-type oxides, which have a general formula $\text{ABO}_3$, represent an ideal system for ascertaining the relationship between chemical composition and electronic structure of materials, as their extraordinary ability allows doping and substitution for both the $A$- and the $B$-site cations while retaining the structural integrity [20]. Meanwhile, the chemical diversity makes it possible to tailor various properties of perovskites to a wide variety of applications, including cathode materials in solid oxide fuel cells (SOFCs) [21], catalysts in heterogeneous catalysis [22], oxygen sources in membrane separation [23], and high-temperature oxygen sensors [24]. Since the $B$-site cations usually come from $d$-block transition metals, it is well established that most perovskite-centered transition-metal oxides have localized $d$ electrons. Although research efforts have previously been devoted to probing the geometrical and electronic structures of perovskites using the GGA+U method [25-30], none of them have been focused on the determination of the $U_{\text{eff}}$ values for DFT investigation of vdW-type, which makes it challenging to simultaneously account for strong and weak interactions present in condensed matter physics and surface chemistry. In this contribution, BEEF-vdW+U calculations have been carried out to obtain the thermodynamic, structural, electronic, and magnetic properties of $\text{LaMO}_3$ ($M = \text{Sc}$ - $\text{Cu}$). The $O_2$ binding error is first corrected by fitting the energetics of formation of nontransition-metal oxides to the experimental values. Then, an effective interaction parameter $U_{\text{eff}}$ and an energy adjustment $\Delta E_{\text{scale}}$ are determined simultaneously by a mixing GGA and GGA+U method. Finally, we calculate other physical properties of $\text{LaMO}_3$ by using the derived $U_{\text{eff}}$ values.
and conclude by discussing the agreement between the calculated and experimental results.

2. METHODOLOGY

2.1 DFT calculations and thermodynamic corrections

All periodic DFT calculations have been carried out with the VASP software package [31], in which the wavefunction at each k-point is expanded in terms of a plane wave basis set. The projector-augmented wave (PAW) method was employed to represent the interactions between ion cores and valence electrons [32] and both the BEEF-vdW [16] and the PBE methods [18] were used to account for the exchange and correlation of the Kohn-Sham theory. The application of the PBE method was to validate the data obtained using the BEEF-vdW method and hence to evaluate the reliability of the computational methods, as PBE is by far the most widely used GGA method. By using the “hard” PAW potentials with valence configurations La (5s² 5p⁶ 6s² 5d¹), O (2s² 2p⁴), Sc (3s² 3p⁶ 4s² 3d²), Ti (3p⁶ 3d² 4s²), V (3s² 3p⁶ 3d³ 4s²), Cr (3p⁶ 3d⁵ 4s¹), Mn (3p⁶ 3d⁵ 4s²), Fe (3p⁶ 3d⁶ 4s²), Co (3d⁴ 4s²), Ni (3p⁶ 3d⁸ 4s²), Cu (3d⁹ 4s²) and a plane-wave energy cutoff of up to 600 eV was necessary to converge the total energy per atom in LaMO₃ to within 1 meV.

Sampling of the Brillouin zone was performed with the Monkhorst-Pack method [33] and electronic occupancies were determined according to the Gaussian scheme with an energy smearing of 0.1 eV. A gamma-centered 11 × 11 × 11 k-point grid was used to sample the Brillouin zone of the LaMO₃ perovskites adopting an orthorhombic structure, and sampling of the Brillouin zone of the rhombohedral structure was performed with a 7 × 7 × 5 grid. For the majority of this work, spin-polarized calculations were performed to obtain reasonably accurate structures and energetics of the transition metal-based perovskites. During geometry optimization of the bulk structures of transition metal oxides, a quasi-Newton algorithm was used, and both the atomic coordinates and lattice vectors are allowed to fully relax until the forces on each atom are converged better than 0.01 eV/Å.

The mass-weighted Hessian matrix was calculated under the finite difference approximation and diagonalized to give the vibrational frequencies of the normal modes. Then, the enthalpy and Gibbs free energy of molecular O₂ at a temperature T and a pressure P were calculated in the ideal-gas limit:

\[ \Delta H^{\text{gas}}(0 \rightarrow T) = E_{\text{tot}} + E_{\text{ZPE}} + \Delta H^{\text{gas}}(0 \rightarrow T) \]  

where \( E_{\text{tot}} \) is the total energy of molecular O₂ as determined by DFT calculations. \( E_{\text{ZPE}} \) is the zero-point energy, and \( \Delta H^{\text{gas}}(0 \rightarrow T) \) is the enthalpy change from 0 K to temperature T. As for the transition-metal oxide crystal structures, all degrees of freedom were treated in the harmonic limit and the Gibbs free energy can therefore be written as

\[ G_{\text{oxide}}(T,P) = E_{\text{oxide}} + E_{\text{ZPE}} + \Delta U^{\text{iso}}(0 \rightarrow T) - T \cdot S^{\text{iso}}(T) \]  

where \( \Delta U^{\text{iso}}(0 \rightarrow T) \) is the change in internal energy from 0 K to temperature T. A more detailed derivation of the thermodynamic properties can be found in our previous work [34].

2.2 GGA+U framework

Here the enthalpy and Gibbs free energy of formation of transition metal oxides were calculated and compared with available experimental data to determine the \( U_{\text{eff}} \) values, which were subsequently used to reproduce physical properties of La-based perovskites. As pointed out by Wang et al. [4], the error in the oxidation energy of transition-metal oxides has contributions from both the overestimation of the binding energy in the O₂ molecule by LDA/GGA and the failure in the cancellation of the “self-interaction error” in the energy difference. The O₂ binding energy can be corrected by fitting the energetics of formation of nontransition-metal oxides to experimental values, [4] and the correlation error may be removed by using a Hubbard-type term to address the on-site Coulomb interaction in the localized orbitals. It should be noted that this method is only applicable to the situation where the d orbital is atomic-like, e.g., in the oxidation of a low-valent oxide to a higher one [4]. If the formation enthalpy of transition-metal oxides is concerned, however, an additional correction needs to be made to account for the energy change accompanying the transition from the delocalized electronic states to the localized states. A methodology mixing GGA and GGA+U calculations have been proposed by Jain et al. [35], in which the overall reaction for the formation of FeAl₂O₄ was decomposed into three reactions and the total energies of the element/compound with uncorrelated and correlated d states were computed with GGA and GGA+U, respectively. By fitting the calculated formation energy of Fe₂O₃ to the experimental value, an energy adjustment that is proportional to the transition metal content in the compound was obtained. Of the three errors, the O₂ binding error and the absence of the aforementioned energy adjustment underestimate the formation enthalpy of transition-metal oxides (less negative) while the correlation error overestimates the formation enthalpy (more negative). As a consequence, the accuracy of the predication about the formation enthalpy depends strongly upon how the three contributions counteract one another.

Armed with this knowledge, we can calculate the enthalpy change (\( \Delta H_{\text{oxide}}^{\text{iso}} \)) for a transition-metal oxide formation reaction

\[ xM + \frac{y}{2} O₂ \rightarrow M_xO_y \]  

as
\[\Delta H^\text{Exp}_{f,M,O_x} = H_{M,O_x}^{\text{GGA+U}} - x \cdot \Delta E_M - x \cdot H_{M,O_x}^{\text{GGA+U}} - \frac{y}{2} \left( H_{O_2}^{\text{GGA}} + \Delta E_{O_2} \right) \]  
\[= \frac{1}{x} \left[ \Delta H_{f,M,O_x}^{\text{GGA}} + \Delta E\left(U_{\text{eff}}\right) - \Delta H_{f,M,O_x}^{\text{Exp}} \right] \]  

\[\Delta E_M = \frac{1}{x} \left[ \Delta H_{f,M,O_x}^{\text{GGA}} + \Delta E\left(U_{\text{eff}}\right) - \Delta H_{f,M,O_x}^{\text{Exp}} \right] \]  

\[\Delta H_{f,M,O_x}^{\text{GGA}} = H_{M,O_x}^{\text{GGA}} - x \cdot H_{M,O_x}^{\text{GGA+U}} - \frac{y}{2} \left( H_{O_2}^{\text{GGA}} + \Delta E_{O_2} \right) \]  

where \(H_{M,O_x}^{\text{GGA+U}}\), \(H_{M,O_x}^{\text{GGA}}\), and \(H_{O_2}^{\text{GGA}}\) are the enthalpies of \(M,O_x\), \(M\) metal, and molecular \(O_2\) as calculated by GGA+U and GGA, respectively, \(\Delta E_{O_2}\) is the energy correction to the \(O_2\) binding energy, and \(\Delta E_M\) is the energy adjustment that needs to be made to the total energies by GGA+U so that the energetics obtained from the mixing GGA and GGA+U calculations are comparable. Because the \(\Delta E_M\) has its origin in the fact that the GGA+U method gives rise to the offsets in the total energy that include not only the desired physical correction but also an arbitrary contribution [36], it can be expressed as

\[\Delta E_M = \frac{1}{x} \left[ \Delta H_{f,M,O_x}^{\text{GGA}} + \Delta E\left(U_{\text{eff}}\right) - \Delta H_{f,M,O_x}^{\text{Exp}} \right] \]  

and \(\Delta E\left(U_{\text{eff}}\right)\) represents the energy correction to the correlation error, which is defined as

\[\Delta E\left(U_{\text{eff}}\right) = H_{M,O_x}^{\text{GGA}} - H_{M,O_x}^{\text{GGA+U}} \]  

In light of this information, Eq. (5) can be rewritten as

\[\Delta H^\text{Exp}_{f,M,O_x} = \left[ H_{M,O_x}^{\text{GGA+U}} - x \cdot H_{M,O_x}^{\text{GGA+U}} - \frac{y}{2} \left( H_{O_2}^{\text{GGA}} + \Delta E_{O_2} \right) \right] + \frac{1}{x} \left[ \Delta H_{f,M,O_x}^{\text{GGA}} - \Delta H_{f,M,O_x}^{\text{Exp}} \right] \]  

where the first term on the right-hand side of the equation represents the enthalpy change for the oxidation of the transition metal in a “virtual” reduced state that has localized \(d\) electrons to an oxide where the transition metal has a positive oxidation number. Since the \(d\) electrons are strongly correlated in both the reactant and the product, the reaction enthalpy is calculated purely by the GGA+U method. The second term on the right-hand side of the equation can be envisioned as the energy required for the transition of the \(d\) electrons in the reduced metal from the delocalized state to the localized state. Upon the decomposition and rearrangement of the equation, it is clear that the \(\Delta E_M\) depends solely on the electronic structure of the transition metal and does not vary with the identity of the transition-metal oxides under investigation. For the transition-metal oxides that have no tabulated values for the formation enthalpy, the Gibbs free energy of formation (\(\Delta G_{f,M,O_x}\)) is calculated instead:

\[\Delta G_{f,M,O_x} = G_{M,O_x}^{\text{GGA+U}} - x \cdot \Delta E_M - \frac{y}{2} \left( G_{O_2}^{\text{GGA}} + \Delta E_{O_2} \right) \]  

As stated in the Introduction, for a given transition metal element, the fitted \(U_{\text{eff}}\) value may vary with the oxidation state and the property of interest. From Eqs. (5) and (10), it follows that this dependence holds true for the \(\Delta E_M\), which changes with the \(U_{\text{eff}}\) value so as to keep the calculated thermodynamic properties close to the experimental results.
### Table 1. Crystal and magnetic structures as well as thermodynamic properties of La-based perovskites and binary metal oxides.

<table>
<thead>
<tr>
<th>Oxide</th>
<th>Space group</th>
<th>ICSD</th>
<th>Magnetic structure</th>
<th>Enthalpy of formation at 298 K (eV) [37]</th>
<th>Gibbs free energy of formation at 298 K (eV) [37]</th>
</tr>
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<tr>
<td>LaScO$_3$</td>
<td>$Pbnm$</td>
<td>99538</td>
<td>NM [38]</td>
<td>-19.56 [39]</td>
<td>-</td>
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<tr>
<td>LaTiO$_3$</td>
<td>$Pbnm$</td>
<td>98414</td>
<td>GAFM [40]</td>
<td>-17.62 [41]**</td>
<td>-</td>
</tr>
<tr>
<td>LaVO$_3$</td>
<td>$Pbnm$</td>
<td>73899</td>
<td>CAFM [40]</td>
<td>-16.20 [42]</td>
<td>-15.37 [42]</td>
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<tr>
<td>LaCrO$_3$</td>
<td>$Pbnm$</td>
<td>153615</td>
<td>GAFM [40]</td>
<td>-15.92 [43]</td>
<td>-</td>
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<td>LaMnO$_3$</td>
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<td>56616</td>
<td>AAFM [40]</td>
<td>-14.90 [42]</td>
<td>-</td>
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<tr>
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<td>$Pbnm$</td>
<td>78062</td>
<td>GAFM [40]</td>
<td>-14.31 [43]</td>
<td>-</td>
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<tr>
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<td>$R3c$</td>
<td>201762</td>
<td>FM</td>
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<tr>
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<td>93919</td>
<td>FM</td>
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<tr>
<td>LaCuO$_3$</td>
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<td>73554</td>
<td>NM</td>
<td>-10.62 [44]**</td>
<td>-</td>
</tr>
<tr>
<td>Sc$_2$O$_3$</td>
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<td>-</td>
<td>NM</td>
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<td>VO</td>
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<td>AFM [45]</td>
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<tr>
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<td>AFM [49, 51]</td>
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<td>-1.33</td>
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<td>160203</td>
<td>NM</td>
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</table>

*Experimental Gibbs free energy of formation at 300 K obtained from Smithells metals reference book [53].

**Estimated enthalpy or Gibbs free energy of formation by using an empirical method.
However, despite having different possible values, the fitted $U_{\text{eff}}$ for the oxides of each transition-metal element was found to lie within a narrow range and the averaged $U_{\text{eff}}$ can often improve dramatically the description of other physical properties such as band gap and magnetic moment [4]. Hence, it is technically feasible to simultaneously determine a single value of $U_{\text{eff}}$ and $\Delta E_M$ for each transition-metal element by fitting the enthalpy or Gibbs free energy of formation of oxides containing that metal in different oxidation states. In practice, we plotted the energy difference (on a per mol metal basis) between the experimentally and theoretically determined thermodynamic properties as a function of the $U_{\text{eff}}$, and the intersection of the curves defines the $U_{\text{eff}}$ and $\Delta E_M$ that are transferable between the oxides. From this point of view, our method is quite different from the methodology used by Jain et al. [35], where the oxidation energy of a low-valent oxide to a higher one was first fitted to derive the $U_{\text{eff}}$ value and the $\Delta E_M$ was then obtained by mixing GGA and GGA+U calculations. When more than one intersection point is obtained, the $U_{\text{eff}}$ takes the value that may give rise to a closer band gap of the transition-metal oxides to the experimental data. The detailed methodology is illustrated in figure 1.

2.3 Selection of materials

Figure 2. Schematic representations of magnetic structures of antiferromagnetic substances.
In addition to the La-based perovskites LaMO₃ (M = Sc - Cu), a number of binary metal oxides that contain transition metals with a +2, +3, and +4 oxidation number were chosen to determine the U_{eff} value for each transition-metal element. LaZnO₃ was not included in the present work because 3d orbitals become more significantly contracted across the transition metal series, making it impossible to remove electrons from the 3d orbitals of Zn to form stable Zn⁺⁺ cations [54]. At room temperature the La-based perovskites may adopt either an orthorhombic or a rhombohedral structure with the Pbnm (M = Sc - Fe) or R₃̅c (M = Co - Cu) space group symmetry [55]. As for the binary oxides, most monoxides and dioxides adopt a cubic and a tetragonal structure, respectively, while the oxides containing metals in the +3 oxidation state tend to crystallize with a structure that has a primitive rhombohedral lattice (see table 1). The crystal symmetries, however, can be lowered or removed by specifying initial spin polarization on the transition-metal cations. Here the experimentally observed magnetic structures were adopted for most of the transition-metal oxides applied, as summarized in table 1. In particular, the magnetic configurations of the antiferromagnetic substances are shown schematically in figure 2.

The thermodynamic properties of the metal oxides at specified temperatures are also presented in table 1. Of the LaMO₃ perovskites, LaTiO₃ and LaCuO₃ have no experimental value available for the enthalpy and Gibbs free energy of formation. Thus, the values were estimated by using empirical methods. As for LaTiO₃, Cheng et al. [39] proposed that the reaction enthalpy for \( \frac{1}{2}La_{2}O_{3} + \frac{1}{2}M_{2}O_{3} \rightarrow LaMO_{3} \) can be approximated by an empirical expression:

\[
\Delta H_f = 630 - 720 \cdot t_{p}
\]

where \( t_{p} \) is the Goldschmidt tolerance factor of the perovskite, which is defined as

\[
t_{p} = \frac{r_{1} + r_{o}}{\sqrt{2}(r_{g} + r_{o})}
\]

where \( r_{1} \), \( r_{g} \), and \( r_{o} \) are the ionic radii of A-, B-site cation, and oxygen anion, respectively. Since the experimental formation enthalpies of LaZnO₃ and TiO₂ are already known, the formation enthalpy of LaTiO₃ can be readily obtained [41]. For LaCuO₃, however, this method is not applicable because CuO has not yet been isolated as a pure substance. Given the fact that the structural stabilities (and hence the formation energies) of the metal oxides decrease with decreasing the actual partial charges on the cations and anions [56], Sreedharan et al. [44] estimated the Gibbs free energy of formation of LaCuO₃ at a given temperature by first plotting the Gibbs free energy of formation of LaMO₃ (M = Mn, Fe, Co, and Ni) against the atomic number of the transition metal and then extrapolating linearly to LaCuO₃. In this way, an empirical relationship that relates the thermodynamic property of LaCuO₃ to temperature was developed:

\[
\Delta G_{f,LaCuO_{3}} = -1095.0 + 0.2350 \cdot T
\]

3. Results and Discussion

3.1 Correction to oxygen binding energy

As aforementioned, there exist three contributions to the error in the calculated formation energies of transition-metal oxides by LDA/GGA. Unlike the other two contributions, the overbinding in the \( O_{2} \) molecule represents a constant shift in the calculated values, making the formation energies of oxides less negative than are obtained experimentally. The overbinding cannot be cancelled because the metal-oxygen bond that results is strongly polar, with little covalent character, and it is often sufficient to assume that the interaction between them is entirely electrostatic in character. By fitting the calculated energetics of formation of nontransition-metal oxides to the experimental values, it is found that the correction to the \( O_{2} \) binding energy depends strongly on the calculation method used and the thermodynamic property of interest. For instance, if the experimental formation enthalpy is plotted against the value calculated by the BEEF-vdW and PBE methods, it is clear that the total energy of molecular \( O_{2} \) is underestimated by 0.46 and 0.84 eV respectively, as shown in figure 3a. However, in the cases where the Gibbs free energy of formation is concerned, the energy corrections are predicted to be 0.37 eV for BEEF-vdW and 0.74 eV for PBE. It can be seen that the BEEF-vdW method gives a more accurate description of the bonding in the \( O_{2} \) molecule, which is probably due to the fact that the BEEF-vdW method has been optimized by using several training data sets that include the formation energy of molecular \( O_{2} \) and the energy changes for molecular reactions involving \( O_{2} \) as a reactant.

On the other hand, even after the total energy of \( O_{2} \) has been corrected to account for the \( O_{2} \) binding error, there remains a discrepancy between the calculated and experimental \( O_{2} \) binding energy, as can be seen in table 2. Previously, Wang et al. [4] used the same method to make the least squares fit, and the formation energies of nontransition-metal oxides were calculated by using the PBE method and the standard version of the PAW potential for \( O \). An energy

\[
\Delta G_{f,LaCuO_{3}} = -1095.0 + 0.2350 \cdot T
\]
correction of 1.36 eV was obtained in their work, and the oxygen binding energy was corrected to -4.66 eV, which becomes less negative than the experimental value. They believed that this additional deviation is related to the electron transfer from the metal s and/or p states to the oxygen 2p orbital when \( O_2 \) is reduced to form \( O^2- \).

### 3.2 Determination of \( U_{\text{eff}} \) and \( \Delta E_M \)

#### 3.2.1 Lanthanum

La-based perovskites \( LaMO_3 \) (\( M = Sc - Cu \) ) constitute one of the best known classes of ternary transition-metal oxides. Apart from the period 3 elements, another transition metal, La, is also involved in the compounds. It was reported that although the \( La \) 4f electrons show strong intra-atomic Coulomb correlations in the \( LaMO_3 \) perovskites [58], the \( La \) 5d electrons are only weakly correlated because there are a large number of electron holes in the 5d orbitals of \( La^{3+} \), which gives rise to a strong hybridization between the \( La \) 5d electrons and the \( O \) 2p electrons [59]. Since the 4f states are not involved in the valence configurations of the PAW potential for \( La \) and the 5d states in \( La^{3+} \) are weakly localized, the electronic structures of \( La \)-based metal oxides are calculated without applying any \( U_{\text{eff}} \) value to the electronic states in the \( La \) cation. To demonstrate that this treatment is reasonably accurate, it is useful to make a first test by calculating the enthalpy change for the reaction,

\[
La + \frac{1}{2} M_2O_3 + \frac{3}{4} O_2 \rightarrow LaMO_3
\]  

Figure 4. Differences between the calculated and experimental enthalpies of formation of \( Sc \)-based oxides as a function of \( U_{\text{eff}} \).

#### 3.2.2 Scandium oxide

As noted earlier, the \( U_{\text{eff}} \) and \( \Delta E_M \) values for each transition metal in period 3 can be obtained by first plotting

\[
\left( \Delta H_{\text{Exp}}^{\text{forming}}(LaM,O_2) - \Delta H_{\text{Exp}}^{\text{forming}}(LaM,O_2) \right) / X \]

or

\[
\left( \Delta G_{\text{Exp}}^{\text{forming}}(LaM,O_2) - \Delta G_{\text{Exp}}^{\text{forming}}(LaM,O_2) \right) / X
\]

against the \( U_{\text{eff}} \) value for the oxides containing the metal in different oxidation states and then identifying the intersection points of the curves. However, since \( Sc \) occurs only in the +3 oxidation state, where the 3+ ion with [Ar] configuration is particularly stable, it is technically infeasible to obtain the \( U_{\text{eff}} \) value by calculating the enthalpy change for oxidation reactions. Indeed, if we plot the energy difference as a function of \( U_{\text{eff}} \) value for \( Sc \) and \( LaScO_3 \), two parallel curves are observed, without any intersection point (see Figure 4).

On the other hand, it was reported that the degree of localization of 3d electrons in the transition-metal oxides can be traced to the competition between Coulomb repulsion which makes 3d electrons more localized and the hybridization of metal 3d states with \( O \) 2p states that delocalizes 3d orbitals [60]. Because \( Sc \) is positioned at the beginning of the first transition series, it is often sufficient to assume that the metal-ligand interaction in its compounds is entirely electrostatic in character. As a result, although the calculated formation enthalpy of \( LaScO_3 \) by BEEF-vdW and PBE is 0.12 and 0.23 eV, respectively, more negative than the experimental data, introduction of a penalty function to address excessive electron delocalization that arises from the SIE seems not necessary.

#### 3.2.3 Titanium oxide

### Table 2. Experimental and calculated oxygen binding energies by the BEEF-vdW and PBE methods.

<table>
<thead>
<tr>
<th>Experimental ( O_2 ) binding energy (eV)</th>
<th>Calculated ( O_2 ) binding energy by BEEF-vdW (eV)</th>
<th>Calculated ( O_2 ) binding energy by PBE (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncorrected</td>
<td>Corrected</td>
</tr>
<tr>
<td>-5.23 [18]</td>
<td>-5.68</td>
<td>-5.22</td>
</tr>
<tr>
<td>-5.12 [57]</td>
<td>-5.22</td>
<td>-5.31</td>
</tr>
</tbody>
</table>

*Experimental \( O_2 \) binding energy with zero-point energy removed.
The thermodynamic property of TiO\textsubscript{2} and LaTiO\textsubscript{3} is used to determine the $U_{\text{eff}}$ value for Ti because Ti$_2$O$_3$ has strongly fluctuating orbitals [61] that cannot be accurately described by GGA+U [62]. Although LaTiO\textsubscript{3} has only a band gap of 0.1 - 0.2 eV [59, 63], it is an anti-ferromagnetic Mott-Hubbard insulator due to strong on-site Coulomb repulsion [64], and the weakly fluctuating orbitals make GGA+U an appropriate approach for this material. As expected, the resulting $U_{\text{eff}}$ for Ti is rather small (0.6 eV for BEEF-vdW+U and 1.3 eV for PBE+U in figure 5a), indicating that the effective hybridization strength is comparable to the Coulomb interaction between 3d electrons [40]. It should be noted that Aykol et al. [15] obtained a $U_{\text{eff}}$ value of 4.35 eV on the basis of the thermodynamic quantities of Ti$_2$O$_3$ and TiO$_2$. However, applying such a high $U_{\text{eff}}$ value gives a band gap that is ~1.5 eV higher than the experimental value for Ti$_2$O$_3$. Closer examination of their calculated dependence of the energy difference on the $U_{\text{eff}}$ indicates that the abnormally high slope of the curve referring to Ti$_2$O$_3$ could be responsible for the deviation.

### 3.2.4 Vanadium oxide

Figure 5b shows the energy difference of VO\textsubscript{3}, V$_2$O$_5$, and LaVO$_3$ as a function of $U_{\text{eff}}$, where the $U_{\text{eff}}$ ranges from 0 eV to 7 eV. Given the diversity in the oxidation state of V in its oxides, it is not surprising to learn that there exist several distinctly different intersection points of the curves. Furthermore, the calculated $U_{\text{eff}}$ spans a wide range of values, varying from 2.5 eV to 4.3 eV for BEEF-vdW and from 3.0 eV to 5.8 eV for PBE, in accord with the PBE results obtained by Aykol et al. [15] and by Lutfalla et al. [65]. Thus, one problem with the determination of the $U_{\text{eff}}$ value for V is the difficulty of obtaining the correct band structures of different metal oxides by applying a single $U_{\text{eff}}$. For this reason, unlike the other 3d-block transition metals, the $U_{\text{eff}}$ value for V in LaVO\textsubscript{3} is determined by further comparing the calculated band gaps given by all the possible $U_{\text{eff}}$ values to available experimental data. It is eventually found that the intersection point of the curves referring to VO\textsubscript{3} and LaVO\textsubscript{3} defines the appropriate $U_{\text{eff}}$ values (2.5 eV for BEEF-vdW+U and 3.0 eV for PBE+U) that provide a reasonably accurate estimate of the band gap of LaVO\textsubscript{3}.

### 3.2.5 Chromium, manganese, iron, cobalt, nickel, and copper oxides

The data for the Cr\textsubscript{2}, Mn\textsubscript{2}, Fe\textsubscript{2}, Co\textsubscript{2}, Ni\textsubscript{2}, and Cu-based oxides are shown in figure 5c-h. It can be seen from the figure 5c-h that the energy difference at $U_{\text{eff}} = 0$ becomes more positive as the transition metal has a lower oxidation number, because the SIE penalizes the low oxidation state over the higher one. Furthermore, the curves for the metal oxides containing the metal in the same formal oxidation state are almost coincident with each other, which can be explained by their similar octahedral coordination environments. For instance, the dependence of the energy differences for Cr$_2$O\textsubscript{3} and LaCrO\textsubscript{3} on the $U_{\text{eff}}$ value follows nearly the same trend. As another example, the curve for Fe$_2$O\textsubscript{3} closely resembles that for LaFeO\textsubscript{3}, regardless of the exchange-correlation functional used. The only exception occurs for Mn\textsubscript{2}O\textsubscript{3} and LaMnO\textsubscript{3}. The $U_{\text{eff}}$ values that result when these two compounds are concerned would differ by 0.22 eV for BEEF-vdW+U and 0.21 eV for PBE+U. The discrepancy is probably due to the fact that the magnetic structure of Mn\textsubscript{2}O\textsubscript{3} has not yet been completely solved, especially when temperature is varied and, consequently, the collinear model proposed by Regulski et al. [50] was used in the present work, which is only a simple approximation based on powder neutron diffraction data measured at 10 K.

As for LaCoO\textsubscript{3}, it may exhibit three different magnetic states, namely, low-spin (LS), immediate-spin (IS), and high-spin (HS) states [66]. The transition between the spin states of Co\textsuperscript{3+} at temperatures ranging from 90 to 600 K is still controversial [67]. Our calculations show that the IS state is the most stable magnetic state at room temperature, in good agreement with the experimental observations by Goodenough et al. [68-71], and the reason is probably due to the particularly strong d-p hybridization when Co\textsuperscript{3+} is in the IS state [72]. In addition, it is interesting to find that the $U_{\text{eff}}$ value obtained increases from the LS to the IS, which can be explained by the requirement of additional energy to reduce the doubly occupied orbitals [73].

### 3.2.6 $U_{\text{eff}}$ and $\Delta E_M$ values for 3d-block transition metals

#### Table 3. Calculated $U_{\text{eff}}$ and $\Delta E_M$ values for 3d-block transition metals.

<table>
<thead>
<tr>
<th>Transition metal</th>
<th>BEEF-vdW+U</th>
<th>PBE+U</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_{\text{eff}}$</td>
<td>$\Delta E_M$</td>
</tr>
<tr>
<td>Sc</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ti</td>
<td>0.6</td>
<td>0.07</td>
</tr>
<tr>
<td>V</td>
<td>2.5</td>
<td>1.40</td>
</tr>
<tr>
<td>Cr</td>
<td>2.5</td>
<td>1.38</td>
</tr>
<tr>
<td>Mn</td>
<td>3.5</td>
<td>1.27</td>
</tr>
<tr>
<td>Fe</td>
<td>2.9</td>
<td>1.46</td>
</tr>
<tr>
<td>Co</td>
<td>3.1</td>
<td>1.49</td>
</tr>
<tr>
<td>Ni</td>
<td>5.6</td>
<td>2.37</td>
</tr>
<tr>
<td>Cu</td>
<td>2.8</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The calculated $U_{\text{eff}}$ and $\Delta E_M$ values for the 3d-block transition metals are summarized in the Table 3. It can be seen from the table that the dependence of the $U_{\text{eff}}$ on atomic number exhibits a non-monotonic behavior and follows nearly the same trend as that proposed by Wang et al. [4], regardless...
of the exchange-correlation functional used. The non-monotonic relationship has its origin in the non-monotonic behavior of the multiplet splitting as a function of atomic number. [74] Saitoh et al. calculated the $U_{\text{eff}}$ values for LaMO$_3$ ($M = \text{Ti}, \text{Cr}, \text{Mn}, \text{Fe}, \text{Co}$) by using a cluster model and suggested that the $U_{\text{eff}}$ value for LaFeO$_3$ is largest because of the strongest Hund’s rule-coupling stabilization in the $d^5$ configuration [74]. In addition, the $U_{\text{eff}}$ values determined by using the BEEF-vdW method are generally smaller than those by PBE; that is, BEEF-vdW is capable of reducing the error in the calculated thermodynamic properties of transition-metal oxides, as compared to PBE. Interestingly, the training data sets used to optimize the BEEF-vdW method do not include any properties of strongly correlated systems. The improvement in the description of the electronic structures and energetics of transition-metal oxides lies in the fact that BEEF-vdW contains the vdW-DF2 nonlocal correlation functional as an essential component, which pairs

Figure 5. Differences between the calculated and experimental enthalpies or Gibbs free energies of formation of Ti-, V-, Cr-, Mn-, Fe-, Co-, Ni-, and Cu-based oxides as a function of $U_{\text{eff}}$. 

References:
nonlocal correlation with LDA correlation and therefore includes semilocal correlation. [16]

It should be noted that the GGA+U method, although not entirely accurate in all details, provides an adequate and useful first description of the electronic structure of transition-metal oxides. By using the same $U_{\text{eff}}$ values as determined in this work, Zheng et al. [75] and Li et al. [76] calculated the oxygen vacancy formation energies of $LaMO_3$ ($M = Sc - Cu$), which were found to agree well with available thermogravimetric data [77-79], even better than was obtained from the HSE06 hybrid functional [80]. On the other hand, within the GGA+U framework, all 3$d$ electrons are driven to be more localized by adding a penalty function, where the strong hybridization between the $e_g$ orbital and the $O_2p$ states makes the $e_g$ band less sensitive to the strength of the on-site Coulomb interaction than the $t_2g$ orbital [58]. Hence, it is expected that excessive electron localization may persist in the $e_g$ orbital. To ensure that the $U_{\text{eff}}$ values determined by fitting the thermodynamic quantities can correctly reproduce other properties, the GGA+U method was used to calculate the structural, electronic, and magnetic properties of $LaMO_3$, which are then compared with available experimental data.

### 3.3 Properties of Perovskites

#### 3.3.1 Lattice distortion

Unlike the ideal cubic perovskites, $La$-based perovskites $LaMO_3$ may adopt either an orthorhombic ($Sc - Fe$) or a rhombohedral ($Co - Cu$) structure at low temperatures, which results from the cooperative tiltings of $MO_6$ octahedra and the internal distortion due to the Jahn-Teller effect. It was reported that the octahedral rotation (GFO distortion) has a major effect on the ferromagnetic stability [81], band gap [82], and Mott transition [62] of materials. At room temperature, the $MO_6$ octahedra tilt in response to the size mismatch between the $La$ and $M$ cation, giving rise to low-symmetry lattice structures [83-86], with the $M-O-M$ bond angle (\(\theta_{M-O-M}\)) decreasing from 180 to 145 - 170°. Therefore, the $M-O-M$ bond angle can be used to some extent as a measure of the GFO distortion.

Figure 6a depicts the variation of the average $M-O-M$ bond angle in $LaMO_3$ as the atomic number of the constituent transition metals is changed, and the corresponding experimental data as well as the Goldschmidt tolerance factor are also presented for comparison. It can be seen that, although our calculated average $M-O-M$ bond angles differ somewhat from the experimental data, they apparently follow the same trend. Interestingly, the Goldschmidt tolerance factor varies in much the way as well. The reason behind the resemblance is that both the $M-O-M$ bond angle and the Goldschmidt tolerance factor of $LaMO_3$ depend solely on the effective ionic radius of $M$ at a function of the atomic number of the constituent transition metals in $LaMO_3$; (c). illustration of $Q_1$, $Q_2$, and $Q_3$ modes of the Jahn-Teller distortions in a $MO_6$ octahedron; (d). calculated and experimental Jahn-Teller distortion in $LaMO_3$.

As for the Jahn-Teller distortion, it has been found to have a pronounced effect on the metallicity and orbital ordering of perovskites by removing the degeneracy of 3$d$ electrons and causing splitting of the 3$d$ band [88]. The three basic modes of the Jahn-Teller distortion, namely, the breathing mode $Q_1$, the basal-plane distortion mode $Q_2$, and the octahedral stretching
mode \( Q_1 \), are illustrated in figure 6c. Since the \( Q_1 \) mode simply changes the volume of the MO₆ octahedron while retaining the symmetry of the materials, the \( Q_2 \) and \( Q_3 \) modes are often used to measure the degree of the Jahn-Teller distortion, which can be expressed in terms of the M-O bond lengths [87]

\[
Q_2 = \sqrt{2} (l - s)
\]

\[
Q_3 = \sqrt{6} (2m - l - s)
\]

where \( s \) and \( l \) are the short and long M-O bond lengths, respectively, pointing along the [100] and [010] axes, and \( m \) is the M-O bond length along the [001] axis, which is always less than or equal to \( l \). The calculated and experimental Jahn-Teller distortions are shown in figure 6d. It is important to note that LaCoO₃, LaNiO₃, and LaCuO₃ adopt the highly symmetric rhombohedral structure with the six M-O bond lengths taking the same value and do not undergo the Jahn-Teller distortion. From Figure 6d, one can see that although DFT and DFT+U are known to deal with the structures and energies at absolute zero, our calculations based on the crystal and magnetic structures at room temperature may give results that are in remarkably good agreement with the experimental data.

3.3.2 Band structure

According to the difference between the on-site Coulomb repulsion energy which separates lower and upper Hubbard bands and the charge-transfer energy required to transfer an electron from the anion valence band to the \( d \) orbitals at the transition-metal site, \( La \)-based perovskites can be classified into three categories: \( LaMO₃ \) (\( M = Sc, Cr, Fe, \) and \( LS \) \( Co \)) are charge-transfer (CT) semiconductors, whose band gap is a \( p-d \) type gap between the anion \( 2p \) filled band and the \( M 3d \) upper Hubbard band; \( LaMO₃ \) (\( M = Ti, Mn, \) and \( F \)) are Mott-Hubbard insulators, whose band gap is a \( d-d \) type gap where the anion \( 2p \) band does not play a major role [40]; \( LaNiO₃ \) and \( LaCuO₃ \) are metallic conductors, with the filled lower Hubbard band overlapping the empty upper Hubbard band. The experimentally observed electrical conductivity and measured band gap of \( LaMO₃ \) are summarized in Table 4. The general decrease in the charge-transfer gap of the CT semiconductor from \( LaScO₃ \) to \( LaCoO₃ \) arises from the increased electronegativity of the metal cation [89].

To explore whether the \( U_{\text{eff}} \) values obtained can give a reasonably accurate description of the electrical conductivity of the perovskite materials, the band structure and density of states (DOS) of \( LaMO₃ \) are calculated by using the BEEF-vdW+U method, as shown in Figs 7 and 8, respectively. The calculated band gaps are listed in Table 4 and the corresponding data calculated by using the PBE method are given in the Supplementary Data. From figure 7, it can be seen that \( LaScO₃ \) has a direct band gap, in agreement with the previous calculations [90] where the calculated band gap by BEEF-vdW is over 2 eV smaller than the experimental value. The significant error in the calculated band gap is derived from the underestimation of the \( Sc 3d \) unoccupied band energy and can only be corrected when a large and unreasonable \( U_{\text{eff}} \) value is used, because the \( U_{\text{eff}} \) has a negligible effect on the band gap determined by the unoccupied \( d-p \) state [91]. As for \( LaTiO₃ \) which has a \( d-d \) type gap (see figure 8 for details), PBE+U overestimates the band gap by ~0.2 eV. By contrast, although the \( U_{\text{eff}} \) value seems too small to split the \( Ti 3d \) bands in the BEEF-vdW+U method, which results in the \( Ti 3d \) unoccupied \( 3d \) band crossing the Fermi level, BEEF-vdW+U gives a more accurate energy gap (0.05 eV) between \( Ti 3d \) bands.

<table>
<thead>
<tr>
<th>Perovskite</th>
<th>Electrical conductivity</th>
<th>Band gap (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>Calculated</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BEEF-vdW+U</td>
</tr>
<tr>
<td>LaScO₃</td>
<td>CT semiconductor</td>
<td>5.7 [92], 6.0 [59]</td>
</tr>
<tr>
<td>LaTiO₃</td>
<td>MH insulator</td>
<td>0.1 [59], 0.2 [63]</td>
</tr>
<tr>
<td>LaVO₃</td>
<td>MH insulator</td>
<td>1.1 [94]</td>
</tr>
<tr>
<td>LaCrO₃</td>
<td>CT semiconductor</td>
<td>3.4 [59]</td>
</tr>
<tr>
<td>LaMnO₃</td>
<td>MH insulator</td>
<td>1.1 [59], 1.7 [96], 1.9 [97], 2.0 [98]</td>
</tr>
<tr>
<td>LaFeO₃</td>
<td>CT semiconductor</td>
<td>2.1 [59], 2.4 [101]</td>
</tr>
<tr>
<td>LaCoO₃</td>
<td>CT semiconductor (LS)</td>
<td>0.3 [59] (LS)</td>
</tr>
<tr>
<td>LaNiO₃</td>
<td>metal</td>
<td>0 [59]</td>
</tr>
<tr>
<td>LaCuO₃</td>
<td>metal</td>
<td>0 [59]</td>
</tr>
</tbody>
</table>
The band gap of \( \text{LaVO}_3 \) is estimated by applying the \( U_{\text{eff}} \) values determined from the thermodynamic properties of \( \text{VO}_2 \) and \( \text{LaVO}_3 \) (2.5 eV for BEEF-vdW and 3.0 eV for PBE) as well as those of \( \text{VO} \) and \( \text{LaVO}_3 \) (4.2 eV for BEEF-vdW and 5.8 eV for PBE). The calculated results indicate that the lower \( U_{\text{eff}} \) values may give the band gap energies (1.34 eV for BEEF-vdW and 1.47 eV for PBE) that are comparable to the experimental value (1.1 eV). Like \( \text{LaTiO}_3 \), \( \text{LaVO}_3 \) has a d-d type band gap, which is also overestimated by over 0.37 eV by PBE+U. By comparison, BEEF-vdW+U that has a smaller \( U_{\text{eff}} \) value can reduce the error by 0.13 eV. From figure 7 and 8, it is clear that \( \text{LaCrO}_3 \) has a calculated d-d type band gap of 2.69 and 2.80 eV by BEEF-vdW+U and PBE+U, respectively. However, it was experimentally reported that \( \text{LaCrO}_3 \) is a CT insulator with a p-d gap of 3.4 eV [59]. The discrepancy is probably due to the fact that the \( U_{\text{eff}} \) value is not large enough for the O 2p band to lie above the Cr 3d sub-band and hence at the top of the valence band [102].

As can be seen from the density of spin-up and spin-down states of \( \text{LaCoO}_3 \) and \( \text{LaNiO}_3 \) in figure 8, they both behave as

![Figure 7](image-url)
half-metals, in accord with previous calculations [72, 103, 104]. In experiment, however, LaCoO$_3$ and LaNiO$_3$ are predicted as a CT semiconductor and a metallic conductor, respectively, which can be explained by the LS Co cations in the experimentally used LaCoO$_3$ sample [67] and the excessive exchange splitting of the Ni d states by GGA [87]. For LaMnO$_3$, LaFeO$_3$, and LaCuO$_3$, both BEEF-vdW+U and PBE+U can give reasonably accurate d-d and p-d type band gaps or correctly reproduce the Cu 3d and O 2p bands crossing the Fermi level. Comparison between our calculated band gaps with previously reported results by HSE indicates the BEEF-vdW+U method shows a reasonable accuracy (see table 4). Although an improvement in the calculated band gap of LaScO$_3$ can only be achieved by using the hybrid functional (5.73 eV) with a mixing parameter [87], HSE gives larger errors in the calculated band gaps of LaTiO$_3$, LaVO$_3$, and LaFeO$_3$ than BEEF-vdW+U.

To further examine whether the fitted $U_{\text{eff}}$ can be extrapolated to other oxides, the band gaps of most of binary 3d-block transition-metal oxides were also calculated compared with available experimental data, as summarized in Table S1. Since the $U_{\text{eff}}$ values determined in the present work are based on the thermodynamic quantities of both binary and ternary oxides, it is reasonable to expect that the $U_{\text{eff}}$ values may give a reasonably accurate description of the physical properties of the binary oxides. Indeed, one can see from the table that the calculated results agree reasonably with the experimental data, except for Sc$_2$O$_3$, TiO$_2$, and VO. The reason the band gaps of Sc$_2$O$_3$ and TiO$_2$ are much smaller than the experimental values is that introduction of a penalty function has a negligible effect on the energies of the occupied O 2p states and the unoccupied metal 3d states. As for VO, because the $U_{\text{eff}}$ value for V in LaVO$_3$ is determined by the thermodynamic and electronic properties of LaVO$_3$ and VO$_2$, it is not surprising to learn that the band gap of VO which has more localized d electrons is overestimated. To summarize, the transferability of $U_{\text{eff}}$ across various binary oxides, especially to those containing the late transition metals, is satisfactory.

3.3.3 Magnetic moment

The variation in the magnetic moments of LaMO$_3$ can be readily understood in terms of the progressive filling of the $t_{2g}$ and $e_g$ bands. The magnetic configurations, the experimental and calculated magnetic moments of the perovskites are listed in Table 5, and the corresponding values of binary metal oxides are given in Table S1. It can be seen that the magnetic moment of LaMO$_3$ ($M =$ Sc – Fe) increases

![Figure 8. Calculated DOSs of LaMO$_3$ ($M =$ Sc – Cu) by BEEF-vdW+U.](image-url)
Table 5. Magnetic configuration and magnetic moment of LaMO$_3$ ($M = $ Sc – Cu)

<table>
<thead>
<tr>
<th>Perovskite</th>
<th>Magnetic configuration of $M^{3+}$</th>
<th>Experimental</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>BEEF-vdW+U</td>
</tr>
<tr>
<td>LaScO$_3$</td>
<td>$d^0$</td>
<td>0 [38]</td>
<td>0</td>
</tr>
<tr>
<td>LaTiO$_3$</td>
<td>$t_{2g}^1$</td>
<td>0.45 [105], 0.57 [106]</td>
<td>0.62</td>
</tr>
<tr>
<td>LaVO$_3$</td>
<td>$t_{2g}^{1+}$</td>
<td>1.3 [59]</td>
<td>1.81</td>
</tr>
<tr>
<td>LaCrO$_3$</td>
<td>$t_{2g}^{11+1}$</td>
<td>2.45 [107], 2.49 [108], 2.8 [109]</td>
<td>2.83</td>
</tr>
<tr>
<td>LaMnO$_3$</td>
<td>$t_{2g}^{1+1+1+1}$</td>
<td>3.42 [110], 3.7 [111], 3.87 [112]</td>
<td>3.9</td>
</tr>
<tr>
<td>LaFeO$_3$</td>
<td>$t_{2g}^{11+1+1}$ (LS)</td>
<td>3.9 [113]</td>
<td>4.05</td>
</tr>
<tr>
<td>LaCoO$_3$</td>
<td>$t_{2g}^{11+1+1}$ (LS)</td>
<td>0 [115] (LS)</td>
<td>2.11 (IS)</td>
</tr>
<tr>
<td>LaNiO$_3$</td>
<td>$t_{2g}^{1+1+1+1}$</td>
<td>1 [117, 118]</td>
<td>1.41</td>
</tr>
<tr>
<td>LaCuO$_3$</td>
<td>$t_{2g}^{1+1+1}$</td>
<td>0 [119]</td>
<td>0</td>
</tr>
</tbody>
</table>

as the transition-metal ion has more unpaired electrons, and the calculated values by BEEF-vdW+U and PBE+U agree well with the experiment data as well as those by HSE (see Tables 5 and S1). The only exception occurs for LaCoO$_3$. Although several DFT calculations [28, 115] have demonstrated that the IS LaCoO$_3$ is lower in energy than the LS LaCoO$_3$, the magnetic moment was measured under experimental conditions for which LaCoO$_3$ is in the nonmagnetic state [115]. It is important to note that the magnetic moments determined by BEEF-vdW+U compare more closely to the experimental data than those by the PBE+U method, which can be attributed to the smaller $U_{off}$ values used by the former, in the sense that the larger $U_{off}$ values may increase more dramatically the calculated magnetic moment by increasing the occupancy of spin-up ($t_{2g}^1$ and $e_g^1$) bands and by reducing the occupancy of spin-down electrons in the occupied states [102].

4. CONCLUSION

In this study, periodic DFT+U calculations have been carried out to study the thermodynamic, structural, electronic, and magnetic properties of La-based perovskites, with the use of the recently developed BEEF-vdW method and the most widely used PBE method. The “self-interaction error” in the occupied states of these strongly correlated systems is removed by introducing an effective interaction parameter $U_{off}$, which is determined together with an energy adjustment $\Delta E_M$ by a mixing GGA and GGA+U method. In this method, the enthalpy or Gibbs free energy of formation of oxides containing a transition metal in different oxidation states is fitted to available experimental data, and the $\Delta E_M$ is found to have its origin in the fact that the GGA+U method gives rise to the offsets in the total energy that include not only the transition metal and does not vary with the identity of transition-metal oxides.

To ensure that the derived $U_{off}$ values can correctly reproduce other physical properties of LaMO$_3$, lattice distortion, band structure, and magnetic moment are calculated by using the derived $U_{off}$ values and compared with experimentally reported data. Calculated results indicate that the BEEF-vdW method provides a more accurate description of the bonding in the $O_2$ molecule than the PBE method and has generally smaller $U_{off}$ values for the 3$d$-block transition metals, thereby leading to band gaps and magnetic moments that are in better agreement with the experimentally measured values.

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References

[27] Dar S A, Srivastava V and Sakalle U K 2017 A combined DFT and post-DFT investigation on cubic XMnO3 (X = Sr, Ba) perovskite oxides Mater. Res. Express 4 12
[31] Kresse G and Furthmüller J 1996 Efficiency of ab-initio total energy calculations for metals and semiconductors using a plane-wave basis set Comp. Mater. Sci. 6 15-50
[37] Haynes W M 2013 CRC handbook of chemistry and physics: CRC press


[116] Zhang X-b, Gang F and Wan H-l 2014 Density Functional Theory Study on Spin States of LaCoO\textsubscript{3} at Room Temperature \textit{Chinese Journal of Chemical Physics} \textbf{27} 274-8

