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Original Article

A possible pitfall in textbook examples of the two-team league with equal-proportion gate revenue sharing

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Abstract
Problem statement and purpose: For pedagogical purposes, textbooks and papers often use a simple two-team league to analyse the consequences on competitive balance and wage level formation from equal-proportion gate revenue sharing. In a profit-maximizing regime with a closed labour market, it is often assumed that this situation produces identical negative parallel shifts in the linear marginal revenue curves of the teams. This paper aims to analyse the validity of the abovementioned statement. **Approach:** The analysis isperformedby mathematical approaches. **Results:** This paper shows that the negative shifts are parallel and identical only when the slope coefficients are equal. **Conclusions:** In an educational context, confusions occur because of inconsistencies between theoretical textbook explanations and numerical student tasks. This paper aims to explain the negative shift theoretically and numerically in a mannerthat avoids this confusion. Furthermore, this paper shows mathematically that the parallel and identical shifts of the marginal revenue functions are only a special case.
Keywords: two-team league, equal-proportion gate revenue sharing, marginal revenue functions, pedagogics.

Introduction

Previous studies in sports economics have analysedthe effects of introducing an equal-proportion gate revenue sharing system in a theoretical profit-maximizing two-team league with a closed labour market.It is often claimedthat the linear marginal revenue (MR) curves of the two teams will have equal parallel shiftsafter the introduction ofthis gaterevenue sharing system. For example, this is statedin a textbook by Fort (2011, p. 176), who claims that: “The marginal value of winning must shift down parallel and equally for both owners in order to yield the same winning percent combination.”The main objective of this paper is to analyse the validity of the abovementioned statement.

This paper shows that the shifts are only parallel and identical when the slopes of the marginal revenue curves for the two teams are equal. More importantly, this paper shows the circumstances for which they are not. In addition, calculations confirm that Rottenberg’s (1956) invariance propositionremains unaffected regardless of whether the shifts are parallel or not. Therefore, the motivation for this paper is purely pedagogical and related to the inconsistencies between textbook statements and student tasks with numerical classroom examples. The next section of this paper presents a model of linear marginal revenue functions for equal-proportion gate revenue sharing to explain whether the shifts are parallel and identical or not. Next, based on the theoretical models, numerical examplesare presented. The last section presents the conclusions.

The model

The statement about the parallel and identical shifts in the linear marginal revenue curves when introducing an equal-proportion gate revenue sharing system originates from the traditional two-team model in a closed labour market with profit-maximizing teams.[1] This model is frequently employed in sports economics (e.g.,Dobson & Goddard, 2011; Fort, 2011; Fort & Quirk, 1995; Késenne, 2007; Quirk &Fort, 1992;Sandy, Sloane, &Rosentraub, 2004; Szymanski, 2004;Vrooman, 1995)[2] and consists of one large and one small market team. The linear marginal revenue function for the large market team (MR_L) has an intercept term (a) that is larger than the intercept term (c)for the small market team (MR_S), and the slope coefficient isequal or higher for the small market team ($b \leq d$). In general terms, equation 1 is written as:[3]

$MR_L = a - bW_L$ and $MR_S = c - dW_S$, (1)

[1] Ownership objective in professional team sports will not be discussed in this paper. The analyses here are basedonly on profit maximisation.
[2]Evaluation of this model is beyond the scope of this paper.
[3]See, for example, Dobson and Goddard (2011) for thedescription of this type of model. See also Kringstad (2015).

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whereW_L is the winning percentage forteam L, and $W_S = 1 - W_L$ is the winning percentage for team S.The intersection point is at $W_L^* = \frac{a-c+d}{b+d}$, and the resulting equilibrium price is equal to $P^* = \frac{ad+bc-bd}{b+d}$. When introducing the equal-proportion gate revenue sharing system in this model, the share of the home team is $0.5 < \propto < 1$ (Quirk& ElHodiri, 1974).According to Quirk and Fort (1992), the marginal revenue functions for the large market team (MR_L^{RS}) and small market team (MR_S^{RS})are represented by equation 2:

$MR_L^{RS} = \propto MR_L - (1-\propto)MR_S$ and $MR_S^{RS} = \propto MR_S - (1-\propto)MR_L$ (2)

By inserting the coefficients from equation (1) into equation (2), the marginal revenue functions,including equal-proportion gate revenue sharing for both teams (equations 3 and 4), are obtained:

$$MR_L^{RS} = \propto a + c(\propto -1) + d(1-\propto) + [\propto (d-b) - d]W_L \quad (3)$$
$$MR_S^{RS} = \propto c + a(\propto -1) + b(1-\propto) + [\propto (b-d) - b]W_S \quad (4)$$

The condition for a parallel shift in a linear function is that the slope coefficient remains unchanged. For the shift in the MR function to be parallel after introducing equal-proportion gate revenue sharing for team L, the slope coefficients from equations(1) and (3) must be equal. This implies that α(d–b)–d = – b. Because α<1, this requirement is valid only when b = d. From equations1 and 4, the condition for a parallel shift for team S is that α(b–d)–b = – d. This occurs only when b = d. Therefore, parallel shifts in this model exist only when the slope coefficients for the two teams are equal.

Assumingthat the slope coefficients are equal forteams L and S, this is denoted as e (i.e., b = d = e). Hence, the slope coefficients are equal for the two marginal revenue curves, including gate revenue sharing (equations 5 and 6):

$$MR_L^{RS} = \propto a + c(\propto -1) + e(1-\propto) + (\propto e - \propto e - e)W_L$$
$$= \propto a + c(\propto -1) + e(1-\propto) - eW_L \quad (5)$$
$$MR_S^{RS} = \propto c + a(\propto -1) + e(1-\propto) + (\propto e - \propto e - e)W_S$$
$$= \propto c + a(\propto -1) + e(1-\propto) - eW_S \quad (6)$$

Therefore, equations (5) and (6) confirm parallel shifts in the marginal revenue functions in the special case when both teams have equal slope coefficients. Otherwise, the statement about parallel shifts in the marginal revenue functions of the teams when equal-proportion gate revenue sharing is not valid.

If the curves for the two teams shift parallel and identically, the absolute differences in both intercept terms should be identical and equal to the change of the equilibrium price.

Therefore, this part analysesthe differences in valuesof the intercept terms and the equilibrium pricewith and without the equal-proportion gaterevenue sharing system for the teams.[4]First, the values of the shifts in the intercept termsfor the two teams are as followswhen introducing the equal-proportion gaterevenue system (equations 7 and 8):

$$MR_L(W_L = 0) - MR_L^{RS}(W_L = 0) = a - [\propto a + c(\propto -1) + d(1-\propto)]$$
$$= (1-\propto)(a + c - d) \quad (7)$$

$$MR_S(W_S = 0) - MR_S^{RS}(W_S = 0) = c - [\propto c + a(\propto -1) + b(1-\propto)]$$
$$- (1-\propto)(a + c - b) \quad (8)$$

The absolute shifts in the marginal revenue curves in equations (7) and (8) will be identical for the two teams only when b = d.Thus, if b = d = e, both teams will have a shift that is equal to equation 9:

$$MR_L(W_L = 0) - MR_L^{RS}(W_L = 0) = MR_S(W_S = 0) - MR_S^{RS}(W_S = 0)$$
$$= (1-\propto)(a + c - e) \quad (9)$$

Furthermore, the value of the changed equilibrium price can be writtenas equation 10:

[4]For example, Vrooman (2007) shows a figure where marginal revenue curves, including revenue sharing, do not have parallel shifts. However, this revenue sharing is based on “a simple pool sharing formula” (p. 320).Therefore, further analysis is beyond the scope of this paper.

$$P^* - P^{RS*} = \frac{ad + bc - bd}{b + d} - \left[(2 \propto -1)\left(\frac{ad + bc - bd}{b + d}\right)\right]$$
$$= \frac{2(1-\propto)}{b+d}(ad + bc - bd) \quad (10)$$

If the slope coefficients for the marginal revenue curves are equal(= e), equation (10) can be written as follows (equation 11):

$$P^* - P^{RS*} = \frac{2(1-\propto)}{b + d}(ad + bc - bd) = \frac{2(1-\propto)}{2e}(ae + ec - ee)$$
$$= (1-\propto)(a + c - e) \quad (11)$$

Thus, when b = d = e, the absolute shift in the value of the equilibrium price (i.e., equation 11) will be equal to the absolute shift in the intercept term for both teams (i.e., equation 9).Hence, this situation produces both parallel and identical shifts in the marginal revenue functions of the two teams after the introduction of equal-proportion gate revenue sharing. However, as long as b < d, the statement aboutparallel and identical shifts is not valid. This can be expressed by comparing equations (7) and (10) for team Land equations (8) and (10) for team S.These differences will be zero if the shifts are parallel. Forteam L,the deviation of the two differences is written as:

$$(P^* - P^{RS*}) - [(MR_L(W_L = 0) - MR_L^{RS}(W_L = 0)]$$
$$= \left[\frac{2(1-\propto)}{b + d}(ad + bc - bd)\right] - [(1-\propto)(a + c - d)]$$
$$= \frac{(1-\propto)[a(d - b) + c(b - d) + d(d - b)]}{b + d} \quad (12)$$

The sign of equation (12) can be found by comparing different expressions in the square bracket of the equation. Typically, these types of models possess a>c and d>b. Then,equation (12)willbe positive, which indicates that the difference between the two curves (i.e., without and with the revenue sharing system) at the intersection with the y-axis is smaller than the difference at the intersection point. This is observed because the sum of the two first expressions in the square bracket is greater than zero because a > c, and the third expression is positive (i.e., d–b >0). Hence, in this situation, the marginal revenue function, including revenue sharing, has a slope coefficient that is higher than that of the original function. This is confirmed by the following relationship [– b –$(\propto d - \propto b - d)$] > 0, when comparing equations (1) and (3).

Similarly, the deviation in the differences of the intercept term and the equilibrium price for team Sisrepresented by equation (13):

$$(P^* - P^{RS*}) - [(MR_S(W_S = 0) - MR_S^{RS}(W_S = 0)]$$
$$= \left[\frac{2(1-\propto)}{b + d}(ad + bc - bd)\right] - [(1-\propto)(a + c - b)]$$
$$= \frac{(1-\propto)[a(d - b) + c(b - d) + b(b - d)]}{b + d}$$
$$= \frac{(1-\propto)[a(d - b) + (c + b)(b - d)]}{b + d} \quad (13)$$

When d>b, the expression is negative, as long as (c+d)>a. This indicates that the difference between the two curves (i.e., without and with the revenue sharing system) at the intersection with the y-axis is typically bigger than the difference at the intersection point. Therefore, the marginal revenue function, including revenue sharing, has a smaller slope coefficient than the function without revenue sharing. This is confirmed by comparing equations (1) and (4), which provide the following relationship for team S:[– d –$(\propto b - \propto d - b)$] < 0.

The effect on competitive balancedue to changes in government regulationsis an important topic in the sports economics literature. Here, it can be shown that the intersection point when $MR_L^{RS} = MR_S^{RS}$ is still at $W_L^{RS*} = W_L^* = \frac{a-c+d}{b+d}$, while the equilibrium price is $P^{RS*} = (2 \propto -1)\left(\frac{ad+bc-bd}{b+d}\right) = (2 \propto -1)P^*$, which is consistent with Vrooman (1995) and Késenne (2007). Hence, different slope coefficients do not alter the invariance proposition[5] and are,therefore, only relevant in a pedagogical context.

Numerical examples

In the following discussion, numerical examples are employed to emphasize the abovementioned theoretical relationships. Following Kringstad (2015) and by converting the numerical exercise 6.1 on page 124 in Késenne (2007) into winning percentages (W_L forteam Land W_S for team S), the revenue functions in the closed two-team league (i.e., $W_L = 1 - W_S$) can be written as follows:

$MR_L = 160 - 200W_L$ and $MR_S = 120 - 200W_S$

For the two profit-maximizing teams, the equilibrium point is at $W_L^* = \frac{a-c+d}{b+d} = \frac{3}{5}$, and the equilibrium price is equal to $P^* = \frac{ad+bc-bd}{b+d} = 40$.

Now, let us assume an80-20 gaterevenue sharing system. Thenthe MRcurves for the two teams, including this equal-proportion gate revenue sharing system, are written as:
$MR_L^{RS} = \propto a + c(\propto -1) + e(1-\propto) - eW_L$= 144 – 200$W_L$
$MR_S^{RS} = \propto c + a(\propto -1) + e(1-\propto) - eW_S$= 104 – 200$W_S$

Thewinning percentages at the equilibrium point remain the same, while the equilibrium price is reduced from 40 to$P^{RS*} = (2 \propto -1)\left(\frac{ad+bc-bd}{b+d}\right) = (2 \propto -1)P^* = 24$.

Hence, the differences in the intercept terms and in the equilibrium price are all 16, and therefore, the shifts in the marginal revenue curves for both teams are parallel and identical.This is illustrated in Figure 1:

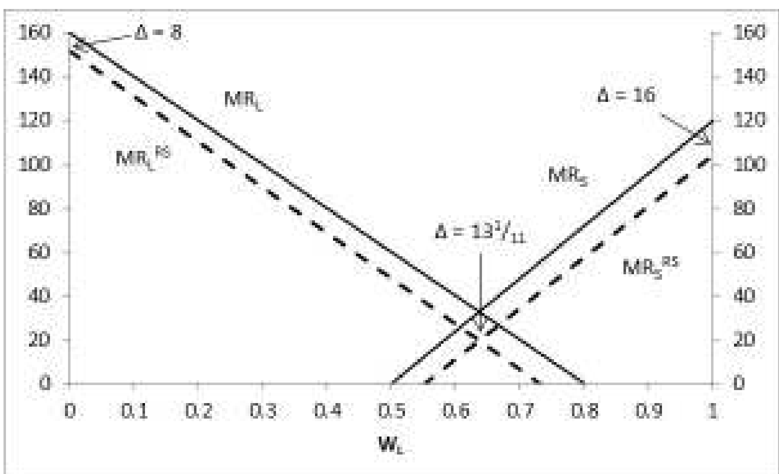

Figure 1: Marginal revenue functions in a closed two-team league with an equal slope coefficient with and without equal-proportion gate revenue sharing

Now, let us assume a different marginal revenue curve forteam S, for example,$MR_S = 120 - 240W_S$ (i.e.,a similar intercept term as in the first case but witha larger slope coefficient). The equilibrium without revenue sharing is changed to$W_L^* = \frac{a-c+d}{b+d} = \frac{7}{11}$and the equilibrium price is$P^* = \frac{ad+bc-bd}{b+d} = 32\frac{8}{11}$.

Furthermore, using the abovementioned revenue sharing system, the MRcurvesare written as:
$MR_L^{RS} = \propto a + c(\propto -1) + d(1-\propto) + (\propto d - \propto b - d)W_L = 152 - 208W_L$
$MR_S^{RS} = \propto c + a(\propto -1) + b(1-\propto) + (\propto b - \propto d - b)W_S = 104 - 232W_S$

Next, the equilibrium price after introducing equal-proportion gate revenue sharing is $P^{RS*} = (2 \propto -1)P^* = 19\frac{7}{11}$. In absolute values, the difference in the equilibrium price with and without the revenue sharing system is$P^* - P^{RS*} = 13\frac{1}{11}$. This difference deviates from the changedintercept term for the

[5] Further analysis of the invariance proposition is beyond the scope of this paper.

two teams. Forteam L, the intercept term is reduced by 160 – 152 = 8. Thus, the difference in the changes between the intercept term and the equilibrium price is $13\frac{1}{11}\quad 8 = 5\frac{1}{11}$. This result indicates that the MR function, including equal-gate revenue sharing,has a higher slope coefficient than that of the MR function without this revenue sharing.According to equation (12), the difference in the changes between the equilibrium price and the intercept term can be obtained directly as follows: $\frac{(1-\propto)[a(d-b)+c(b-d)+d(d-b)]}{b-d} = 5\frac{1}{11}$.

For team S, the difference in the intercept term is 120 – 104 = 16, while the difference between the changes in the equilibrium price and the intercept term is $13\frac{1}{11} - 16 = -2\frac{10}{11}$. As shown above, this difference can be obtained by applying equation (13):

$\frac{(1-\propto)[a(d-b)+(c+b)(b-d)]}{b+d} = -2\frac{10}{11}$.

Figure 2 illustrates that the shifts in the linear MR curves for the two teams are not parallel and identical.

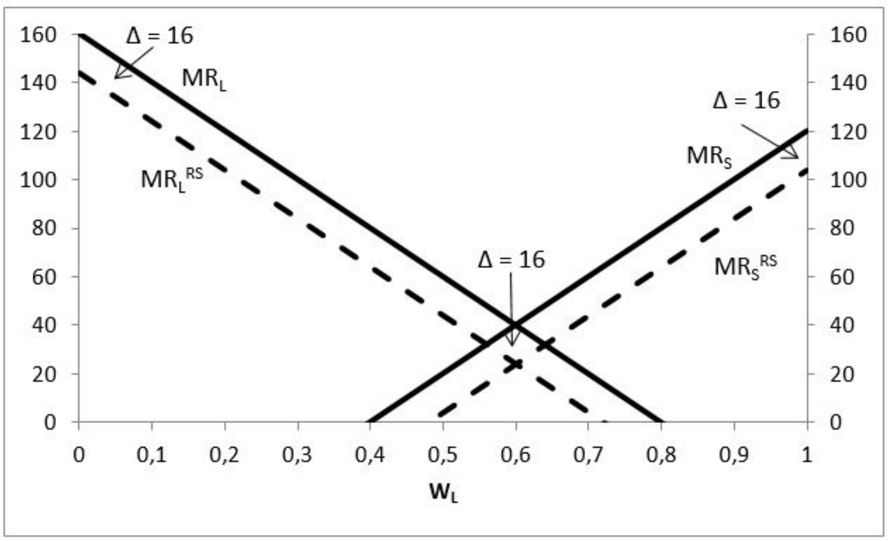

Figure 2: Marginal revenue functions in a closed two-team league with an unequal slope coefficient with and without equal-proportion gate revenue sharing

To summarize, in the case of equal slope coefficients for the marginal revenue functions of the two teams (i.e.,b = d), our example had the following marginal revenue curves: $MR_L = 160 - 200W_L$ and $MR_S = 120 - 200W_S$. After introducing an 80-20 gaterevenue sharing system, these curves can be described by the following formulas: $MR_L^{RS} = 144 - 200W_L$ and $MR_S^{RS} = 104 - 200W_S$. It is clear that the slope coefficients remain at the same level, 200.Because the shifts in the intercept terms are similar for both teams, the shifts are parallel and identical.

When b < d, ourexample has the following marginal revenue curves: $MR_L = 160 - 200W_L$ and $MR_S = 120 - 240W_S$. After introducing the same equal-proportion gaterevenue sharing system as in the first case, the new functions for the marginal revenue curves are: $MR_L^{RS} = 152 - 208W_L$ and $MR_S^{RS} = 104 - 232W_S$. The changes in the intercept terms for the teams are not identical, and thenew slope coefficients deviate from the origin.Therefore, these results show that the shiftswhen introducing equal-proportion gate revenue sharing are not parallel and identical when the two marginal revenue functions have different slope coefficients.

Conclusions

In this study, the traditional profit-maximizingtwo-team theoretical league model with linear marginal revenue functionswhen introducing an equal-proportion gate revenue sharing system in a closed labour market was analysed. The notion of parallel and identical reductions in the MR curves was scrutinised because the mathematical analysis clearly indicates that parallel and identical shifts are an exception rather than the general case. The mathematical analysis confirms that parallel and identical shifts appear only when the slope coefficients for the two marginal revenue functions are equal. Therefore, inconsistency between textbooks and numerical student tasks appear when the slope coefficients are different. In these cases, changes in the marginal revenue curvesdue to the introduction of equal-proportion gate revenue sharing are not parallel and identical.

The results of this paper show that caution should be taken in claiming parallel and identical shifts in the linear marginal revenue curves in a profit-maximizingtwo-team league with equal-proportion gate revenue sharing. It is suggestedto highlight that parallel and identical shiftsin the linear marginal revenue curves are a special case. Otherwise, one should avoid this case by applying different slope coefficients.

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