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SHORT COMMUNICATION

Addendum to “Stokes transport in layers in the water column based on long-term wind statistics: assessment using long-term wave statistics”

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Summary This article addresses the Stokes drift in layers in the water column for deep water random waves based on wave statistics in terms of the sea state wave parameters significant wave height and mean zero-crossing wave period. This is exemplified by using long-term wave statistics from the North Atlantic, and is supplementary to Myrhaug et al. (2018) presenting similar results based on long-term wind statistics from the same ocean area. Overall, it appears that the results based on long-term wave statistics and long-term wind statistics are consistent. The simple analytical tool provided here is useful for estimating the wave-induced drift in layers in the water column relevant for the assessment of the transport of, for example, marine litter in the ocean based on, for example, global wave statistics.

Myrhaug et al. (2018) (hereafter referred to as MWH18) provided estimates of the Stokes transport in layers in the water column based on long-term wind statistics. The results were exemplified by using the Phillips and the Pierson-Moskowitz model wave spectra together with open ocean long-term mean wind speed statistics from one location in the northern North Sea and from four locations in the North Atlantic. This article is supplementary to MWH18 with the purpose to demonstrate how similar results of the Stokes transport in layers in the water column can be obtained by using long-term wave statistics in terms of the sea state parameters significant wave height and mean zero-crossing wave period. Results are exemplified by

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using open ocean deep water wave data from five locations in the North Atlantic, including comparison with the MWH18 results based on long-term wind statistics from four locations in the same ocean area. Overall, it appears that the results are consistent.

The recent attention on environmental issues associated with plastic and microplastic litter in the oceans is the main motivation for this supplement. It is well documented that plastic litter occurs and is transported in different layers in the water column beneath the ocean surface; see, for example, the references in the Introduction of MWH18 as well as in Van Canwenberghe et al. (2015), Acampora et al. (2016), and Ruiz-Orejon et al. (2016). One important component, which is responsible for the transport of plastic and microplastic located in different layers in the water column, is the Stokes drift. The Stokes drift is the wave-average of the water particle trajectory in the wave propagation direction, i.e. corresponding to the Lagrangian velocity. Furthermore, the volume Stokes transport corresponds to the integral over the water depth of the Stokes drift (Rascle et al., 2008). The background and more details of the Stokes drift are found in, for example, Dean and Dalrymple (1984). Myrhaug et al. (2016) gives a brief review of the literature up to that date (see the references therein) and of the more recent works referred to in MWH18. Furthermore, Paprota et al. (2016) presented results from an experimental study of wave-induced mass transport, while Paprota and Sulisz (2018) developed a theoretical model of the kinematics of water particles and mass transport under nonlinear waves generated in a closed flume and verified their results against the data from Paprota et al. (2016). Grue and Kolaas (2017) presented experimental results from wave tank measurements on particle paths and drift velocity in steep waves at finite water depth. Song et al. (2018) derived a theoretical statistical distribution of wave-induced drift for long-crested random waves in finite water depth. A recent comprehensive review of wave-induced drift was given by van den Bremer and Breivik (2018).

The article is organized as follows. This introduction is followed by the theoretical background of the Stokes transport velocity in layers in terms of the sea state wave parameters significant wave height and mean zero-crossing wave period. Then examples of results for long-term wave statistics from the North Atlantic are presented, including comparison with the results in MWH18 based on long-term wind statistics from the same area. Finally, the main conclusions are given.

The theoretical background follows that given in Section 2 of MWH18, where more details are provided. Since the main issue here is the transport of material in the water column, the drift velocity associated with the Stokes transport in different layers of the water column is a quantity of interest, which within a sea state of random waves is given by Eq. (9) in MWH18 as (where ω_1 replaces ω_p = spectral peak frequency).

$$V=\frac{1}{\Delta h}\left(e^{-2(\omega_1^2/g)h_1}-e^{-2(\omega_1^2/g)h_2}\right)\frac{\pi H_s^2}{8T_1}. \tag{1}$$

Here g is the acceleration of gravity, $H_s=4\sqrt{m_0}$ is the spectral significant wave height, $T_1 = 2\pi/\omega_1 = 2\pi m_0/m_1$ is the spectral mean period, ω_1 is the spectral mean frequency, $\Delta h = h_2 - h_1$, h_1 and h_2 are two elevations in the water column, and the nth spectral moments are defined in terms of the wave spectrum $S(\omega)$ as

$$m_n=\int_0^\infty \omega^n S(\omega)\,d\omega;\quad n=0,1,2,\ldots \tag{2}$$

where ω is the cyclic wave frequency. Thus, V is defined in terms of the sea state parameters H_s and T_1 in deep water.

Long-term wave statistics are commonly given in terms of joint frequency tables of H_s and T_p (= spectral peak period) or H_s and T_z (= mean zero-crossing wave period). In the following, an example of results will be presented for long-term wave statistics, where the wave statistics is given in terms of joint distributions of H_s and $T_z=2\pi\sqrt{m_0/m_2}$. Thus, in this article T_1 is replaced with T_z, justified by that both these wave periods obtained from the wave spectrum are estimates of the mean zero-crossing wave period calculated from zero-crossing analysis of the time series of the wave elevation. For the Pierson-Moskowitz (PM) spectrum used in MWH18, $T_z = 0.92T_1$ (Tucker and Pitt, 2001).

The results are exemplified by the two locations $h_1 = 0$ and $h_2 = \lambda_z/s$ where $s \geq 2$, i.e. the Stokes transport velocity corresponds to the mean drift velocity over the thickness of the subsurface layer equal to a fraction of the wave length. The wave length is $\lambda_z = 2\pi/k_z$, where k_z is the wave number obtained from the deep water dispersion relationship $\omega_z^2=gk_z$; here $\omega_z = 2\pi/T_z$ and

$$\lambda_z=\frac{g}{2\pi}T_z^2. \tag{3}$$

For $s = 2$ the result represents the mean drift velocity over the whole water column since the wave motion in deep water penetrates down to about half the wavelength. Thus, by taking $h_1 = 0$, $h_2 = \lambda_z/s$ and substituting Eq. (3) in Eq. (1), Eq. (1) is given as

$$V=\frac{\pi^2 s}{4g}\left(1-e^{-4\pi/s}\right)\frac{H_s^2}{T_z^3};\quad s\geq 2. \tag{4}$$

Then, by defining $\hat{V}(H_s,T_z)$, Eq. (4) is rearranged to

$$\hat{V}(H_s,T_z)\equiv\frac{V}{\frac{\pi^2 s}{4g}\left(1-e^{-4\pi/s}\right)}=\frac{H_s^2}{T_z^3}. \tag{5}$$

Now $\hat{V}$ (and V) is defined in terms of the sea state parameters H_s and T_z in deep water (i.e. representing a sea state where each pair of H_s, T_z represents one storm condition with a duration of e.g. 3 h). Different parametric models for the joint probability density function (*pdf*) of H_s and T_z are given in the literature; a recent review is given by Bitner-Gregersen (2015). In this study, the statistical properties of $\hat{V}$ are exemplified by using the joint *pdf* of H_s and T_z fitted by Bitner-Gregersen and Guedes Soares (2007) (hereafter referred to as BGGS07) to five data sets from the North Atlantic, given in the Appendix.

Statistical quantities of interest are, for example, the expected (mean) value of $\hat{V}(H_s,T_z)$, $E[\hat{V}(H_s,T_z)]$, and the variance of $\hat{V}(H_s,T_z)$, $Var[\hat{V}(H_s,T_z)]$. This requires calculation of $E[\hat{V}^n(H_s,T_z)]$, obtained as (Bury, 1975)

$$E[\hat{V}^n(H_s,T_z)]=\int_0^\infty\int_0^\infty \hat{V}^n(H_s,T_z)p(H_s,T_z)\,dH_s\,dT_z. \tag{6}$$

Furthermore (Bury, 1975),

$$Var[\hat{V}(H_s, T_z)] = E[\hat{V}^2(H_s, T_z)] - (E[\hat{V}(H_s, T_z)])^2. \tag{7}$$

The BGGS07 distribution results are obtained from Eqs. (5), (6), (7), (A1), (A2), (A3) and (A4) and are given in Table 1, where Datasets 1 to 5 hereafter are referred to as Wa1 to Wa5. It appears that $E[V]$ is in the range of 0.00803 to 0.0216 m s^{-1}, and that the standard deviation to mean value ratios of $E[V]$ are large, that is, in the range 0.65 to 1.

Similarly, a characteristic value of λ_z, that is, $E[\lambda_z]$, can be obtained from Eq. (3) as

$$E[\lambda_z] = \frac{g}{2\pi} E[T_z^2] = \frac{g}{2\pi} \int_0^\infty \int_0^\infty T_z^2 p(H_s, T_z)\, dH_s\, dT_z. \tag{8}$$

From Table 1 it appears that $E[\lambda_z]$ is in the range of 82 m to 150 m.

A quantity of interest, for example, for the assessment of marine litter, is the volume Stokes transport per crest length. Table 1 gives the values of $M = E[V]E[\lambda_z]/2$, that is, the volume Stokes transport per crest length over the whole water column where there is wave activity, since the wave motion goes down to about half the wavelength. It appears that M is in the range of 0.6 m^2 s^{-1} (Wa5) to 0.9 m^2 s^{-1} (Wa1). For Wa1 this means, for example, that the mean volume Stokes transport $\pm$ one standard deviation is 0.13 m^2 s^{-1} and 1.7 m^2 s^{-1}, respectively, in the water column from the surface down to about 41 m. The corresponding intervals (i.e. the mean value $\pm$ one standard deviation) of the volume Stokes transport for the water columns from the surface ($z = 0$ m) to about $z = -21$ m, -10 m, -5 m, -2.5 m are (0.065, 0.85) m^2 s^{-1}, (0.033, 0.43) m^2 s^{-1}, (0.016, 0.21) m^2 s^{-1}, (0.008, 0.11) m^2 s^{-1}, respectively. This is of direct relevance, for example, to the volume transport of near neutrally buoyant litter as discussed in Fig. 3 of MWH18.

Now these results will be compared with those based on the PM spectrum given in Table 3 of MWH18. However, in order to be consistent with the present results, the MWH18 results should be based on T_z (instead of T_p). For a PM spectrum $T_p/T_z = 1.41$ (Tucker and Pitt, 2001), and thus $E[\lambda_z]/E[\lambda_p] = (T_z/T_p)^2 = 0.503$. Then, the $E[V]$ results for the PM spectrum given in Table 1 (referred to as Wi1 to Wi4) are obtained by multiplying the PM results in Table 2 of MWH18 by the factor $E[V_z]/E[V_p] = (T_p/T_z)^3 = 2.80$.

It appears that $E[V]$ is in the range 0.0143 m s^{-1} (Wi2) to 0.0223 m s^{-1} (Wi3), which overall agrees with the wave statistics results, except for Wa5, which is significantly smaller. It is noted that the standard deviation to the mean value ratios of $E[V]$ are in the range 0.43 to 0.46, i.e. smaller than those based on wave statistics. The three values in the columns for $E[V]$ represent $E[V]$ − 1SD (=st.dev.), $E[V]$, $E[V]$ + 1SD, respectively. It appears that the wave statistics and the wind statistics values for $E[V]$ are partly in the same range. This is also the case for the values of M, except for datasets Wa2 and Wi2 (see Table 1; the three values given in the column for M are based on using $E[V]$ − 1SD, $E[V]$, $E[V]$ + 1SD, respectively). Overall, the values of M are larger based on wave statistics than based on wind statistics. This is mainly due to that the standard deviation of V and $E[\lambda_z]$ are larger based on wave statistics, which might be attributed to the different inherent features of the data.

The main conclusions are as follows:

1. Overall, it appears that the present assessment of Stokes transport in layers in the water column based on long-term wave statistics from five deep water locations in the North Atlantic are consistent with similar results in Myrhaug et al. (2018) based on long-term wind statistics from four locations in the same ocean area.
2. More specifically, the mean values of the surface Stokes drift based on wave statistics agree with those based on wind statistics except for one of the wave statistics datasets, which is significantly smaller. However, by comparing the results including the mean values plus and minus one standard deviation there is overlap between the wave and wind statistics results. This is also the case for the Stokes transport in the water

Table 1 Example of results using wind and wave statistics from the North Atlantic. The three values in the columns for $E[V]$ represent $E[V]$ − 1SD (=st.dev.), $E[V]$, $E[V]$ + 1SD, respectively, and the three values of M are based on the three values in the column for $E[V]$, respectively.

Distribution	$E[V]$ [m s^{-1}]	st.dev./m.v.	$E[\lambda_z]$ [m]	$M = \frac{E[V]E[\lambda_z]}{2}$ [m^2 s^{-1}]
WAVES				
BGGS07				
Dataset 1, Wa1	0.0030, 0.0216, 0.0401	0.86	82	0.124, 0.886, 1.65
Dataset 2, Wa2	0.0054, 0.0154, 0.0254	0.65	96	0.259, 0.739, 1.22
Dataset 3, Wa3	0.0033, 0.0204, 0.0375	0.84	85	0.139, 0.867, 1.60
Dataset 4, Wa4	0, 0.0112, 0.0224	1	118	0, 0.661, 1.32
Dataset 5, Wa5	0.00016, 0.00803, 0.0159	0.98	150	0.012, 0.602, 1.19
WIND				
Location				
20°W 60°N, Wi1	0.0127, 0.0222, 0.0317	0.43	55	0.348, 0.610, 0.873
10°W 40°N, Wi2	0.0077, 0.0143, 0.0209	0.46	23	0.089, 0.165, 0.241
40°W 50°N, Wi3	0.0127, 0.0223, 0.0319	0.43	55	0.349, 0.613, 0.877
20°W 45°N, Wi4	0.0107, 0.0188, 0.0269	0.43	39	0.209, 0.366, 0.523

column when the mean values plus and minus one standard deviation are taken into account. However, overall the values of the Stokes transport in the water column are larger than those based on wind statistics, which is attributed to that the standard deviation and the characteristic wave length are larger based on wave statistics.

The strength of this work together with that of Myrhaug et al. (2018) is that it demonstrates how the presented methods can be used to assess the Stokes transport velocity and the volume Stokes transport for deep water random waves within sea states using available wind and wave statistics, which is important for assessing further the drift of, for example, marine litter in the ocean.

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Appendix. Joint distributions of H_s, T_z

The joint *pdf* by BGGS07 is given as

$$p(H_s, T_z) = p(T_z|H_s)p(H_s), \tag{A1}$$

where $p(H_s)$ is the marginal *pdf* of H_s, given by the following three-parameter Weibull distribution

$$p(H_s) = \frac{r}{s}\left(\frac{H_s - t}{s}\right)^{r-1} \exp\left[-\left(\frac{H_s - t}{s}\right)^{r}\right], \quad H_s \geq t, \tag{A2}$$

where r, s and t are the Weibull parameters given in BGGS07; see Table A1.

$p(T_z|H_s)$ is the conditional *pdf* of T_z given H_s, given by the lognormal distribution

$$p(T_z|H_s) = \frac{1}{\sqrt{2\pi}\sigma T_z} \exp\left[-\frac{(\ln T_z - \mu)^2}{2\sigma^2}\right], \tag{A3}$$

where μ and σ^2 are the mean value and the variance, respectively, of $\ln T_z$ given as

$$\mu = a_1 + a_2 H_s^{a_3}, \qquad \sigma = b_1 + b_2 H_s^{b_3}, \tag{A4}$$

where the parameters in μ,σ are given in BGGS07; see Tables A2 and A3. All the BGGS07 data represent wave conditions in the North Atlantic. Datasets 1, 2 and 3 are numerically generated wave data taken from global databases representing 44 years (1958—2004) at 59°00′N, 19°00′W. Dataset 4 refers to Global Wave Statistics (GWS) zone 9 (the zone located south of Iceland and west of UK) representing visual observations collected from ship in normal service all over the world in the period 1949—1986. Dataset 5 refers to Juliet Shipborne Wave Recorder (SBWR) representing data registered at the Ocean Weather Station Juliet during 13 years since 1952 at 52°00′N, 20°00′W. More details are given in BGGS07.

Table A1 Weibull parameters for H_s, see Eq. (A2).

BGGS07, North Atlantic	s	r	t
Dataset 1	3.104	1.357	0.906
Dataset 2	2.848	1.419	1.021
Dataset 3	2.939	1.240	0.896
Dataset 4	2.857	1.449	0.838
Dataset 5	2.420	1.169	1.258

Table A2 BGGS07: The parameters in the mean value of $\ln T_z$, see Eq. (4).

BGGS07, North Atlantic	a_1	a_2	a_3
Dataset 1	1.350	0.366	0.392
Dataset 2	1.365	0.375	0.453
Dataset 3	0.790	0.805	0.292
Dataset 4	0.835	1.139	0.119
Dataset 5	1.952	0.168	0.499

Table A3 BGGS07: The parameters in the standard deviation of $\ln T_z$, see Eq. (A4).

BGGS07, North Atlantic	b_1	b_2	b_3
Dataset 1	0.020	0.165	−0.166
Dataset 2	0.033	0.285	−0.752
Dataset 3	0.055	0.195	−0.269
Dataset 4	0.140	0.030	−0.958
Dataset 5	0.070	0.066	−0.081

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