Electrical Power and Energy Systems 109 (2019) 38–48

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Review

Fundamentals and recent developments in stochastic unit commitment

Martin Håberg*

Department of Electric Power Engineering, Norwegian University of Science and Technology, Trondheim, Norway

ARTICLE INFO

Keywords:
Optimization
Power system modeling
Stochastic programming
Decomposition methods
Unit commitment

ABSTRACT

Optimal scheduling of power generating units must take into account possibilities of forecast errors and equipment failure. The stochastic unit commitment problem addresses scheduling of generating units under uncertainty. Focusing primarily on scenario-based approaches, this article summarizes the fundamental concepts of stochastic unit commitment, including the representation of uncertainty, different problem formulations and the most common decomposition techniques applied to solve the problem. It also provides a survey of the latest research on stochastic unit commitment, including an in-depth review of selected important recent advances in decomposition based methods.

1. Introduction

The well-established unit commitment problem concerns the optimal scheduling of power generating units, and is used for cost minimization in vertically integrated environments, as well as for market clearing, reliability assessments and intra-day operations in market environments. The possibility of forecast error or equipment failure means some of the parameters in the problem are uncertain. In particular, the rapidly increasing amounts of power generation from intermittent sources have motivated research on efficiently managing uncertainty in power system operations and unit commitment decisions. Numerous configurations of the unit commitment problem have been proposed, differing in how uncertainty is represented, as well as how to formulate and solve the problem [1]. An important family of approaches is *stochastic unit commitment*, where uncertainty is explicitly taken into account inside the model using a set of discrete scenarios with associated probabilities.

For systems of realistic size, unit commitment problems involve large numbers of binary decision variables, and even deterministic problems can be challenging to solve efficiently. The potential for cost savings have, however, led to intensive investigation of the unit commitment problem over several decades [2], and thousands of research articles have been published. Although the vast majority of unit commitment literature concerns deterministic problem formulations, there are also hundreds of articles discussing formulations and methodology for solving stochastic programs for unit commitment, including sporadic review papers. Zheng et al. [3] classify literature written until 2014 on stochastic optimization methods for unit commitment into three main categories: *stochastic programming*, *robust optimization*, and (approximate) *stochastic dynamic programming*. They further distinguish between two-stage, multi-stage and variations of risk-averse stochastic programming model, while also providing a discussion for each of these methods and a useful comparison of their advantages and disadvantages. Dai et al. [4], on the other hand, distinguish stochastic programming-based formulations for unit commitment between *vertically integrated* and *deregulated* market environments. Most of the literature is written in from a vertically integrated point of view, and they further classify these approaches into *basic two-stage* stochastic programming, *basic multi-stage* stochastic programming, *security-constrained* and *chance-constrained two-stage* formulations. Here, the *basic* formulations do not take security or risk considerations into account. Tanahan et al. [5] survey the broader area of what they call *uncertain unit commitment*, list three approaches for dealing with uncertainty inside the unit commitment model, *stochastic optimization*, *robust optimization*, and *chance-constrained optimization*. A recent update of this survey also focuses on heuristic methods [6]. Also focusing heavily on heuristics and other non-traditional methods, Aburajad et al. [1] provide a practical summary of a range of heuristic methods, and compare these with more traditional approaches.

While recognizing the increasingly important roles of robust [7], interval [8], and other risk-averse optimization approaches [3], the focus of this article is formulating and solving scenario-based stochastic programming approaches to the unit commitment problem. Scenario generation [9,10] and reduction [11,12] impact the performance of stochastic programming models, but are considered outside the scope of this paper.

This article does not attempt to duplicate the efforts of earlier reviewers by exhaustively surveying all available literature on unit

* Address: O. S. Bragstads plass 2E, 7034 Trondheim, Norway.
E-mail address: martin.haberg@ntnu.no.

https://doi.org/10.1016/j.ijepes.2019.01.037
Received 14 August 2018; Received in revised form 14 December 2018; Accepted 28 January 2019
Available online 04 February 2019
0142-0615/

commitment under uncertainty, but includes a synopsis of the fundamental literature, with explanations of the two-stage and multi-stage stochastic programming formulations, as well as methods for decomposing and solving such problems. Furthermore, the article surveys and classifies relevant literature from recent years. The article also includes a detailed review of some the most important developments in decomposition-based methods for stochastic unit commitment, aiming to highlight key algorithmic innovations and efficient solution strategies for the future.

Section 2 gives an introduction to the deterministic unit commitment problem, while different formulations for the stochastic unit commitment problem are presented in Section 3. Decomposition methods are key to solving many stochastic unit commitment problems, and Section 4 provides an overview of commonly used methods. Recent developments in decomposition-based methods are presented in Section 5, based on in-depth reviews of selected books and articles. A survey and classification of new literature on stochastic unit commitment is included in Section 6, before Section 7 summarizes the most important conclusions.

2. The unit commitment problem

In power generation scheduling, the *unit commitment decision* indicates, for each point in time over the scheduling horizon, what generating units are to be used [13]. Then, the most economic dispatch, i.e. the distribution of load across generating units for each point in time, is then determined to meet system load and reserve requirements [14]. Start-up and production costs are taken into account, and together with a host of operating constraints, this forms the unit commitment problem. The operating constraints could include local level status restrictions, minimum up time and down time requirements, and a variety of start-up, fuel usage and ramping constraints [2]. After making the unit commitment decisions, these can be passed on as 0/1 parameters to the economic dispatch problem, which can then be formulated and solved efficiently using continuous variables. The unit commitment problem, on the other hand, contains a potentially large number of binary decision variables.

2.1. General formulation

Assume we are to find a schedule for $|I|$ generating units over a horizon of $|T|$ time periods. Then the generic unit commitment problem can be formulated as

$$\min_{x,u} \sum_{i\in I} \sum_{t\in T} f_i\left(x_{it}, u_{it-1}, u_{it}\right) \tag{1}$$

$$\text{s. t.} \quad \sum_{i\in I} x_{it} \geqslant D_t \quad t \in T \tag{2}$$

$$\sum_{i\in I} \overline{X}_i u_{it} - x_{it} \geqslant R_t \quad t \in T \tag{3}$$

Here, f_i is the cost function of each generating unit i. This function typically includes operation and maintenance cost as a convex and quadratic function of the generation level x_{it}, as well as start-up and shut-down costs when the state u_{it} of a unit changes between zero and one or vice versa. A solution to (1) must, in addition to satisfying the demand D_t in (2) and reserve requirement R_t in (3) for each time period t, also satisfy operating constraints for each individual unit. These usually include minimum and maximum operating levels $\underline{X}_i$ and $\overline{X}_i$ in (4), as well as meeting constraints for minimum up time and down time. This means a unit has to be on for at least L_i^{up} periods, or similarly, off for L_i^{dn} periods before the state of the unit can be changed. Other operating constraints, such as ramp-rate limits, deration and status of the units, are sometimes included [15]. For all $i \in I, \ \ t \in T$,

$$\underline{X}_i u_i \leqslant x_{it} \leqslant \overline{X}_i u_{it} \tag{4}$$

$$u_{it} - u_{it-1} \leqslant u_{i\tau}, \quad \tau = t+1, \ ..., t + L_i^{\mathrm{up}} \tag{5}$$

$$u_{it-1} - u_{it} \leqslant 1 - u_{i\tau}, \quad \tau = t+1, \ ..., t + L_i^{\mathrm{dn}} \tag{6}$$

2.2. Solution methods

Decades ago, Sheble [15] provided an literature synopsis showing the diversity in methods and approaches to formulating and solving the unit commitment problem, starting with Garver [16] in the early 1960s. Based on their solution methods, Sheble classified the unit commitment literature available at that time into *exhaustive enumeration*, *priority list*, *dynamic programming*, *integer and mixed-integer programming*, *branch-and-bound*, *linear programming*, *dynamic and linear programming*, *separable programming*, *network flow programming*, *Lagrangian relaxation*, *expert systems/artificial neural networks*, *risk analysis*, *simulated annealing*, and *decision analysis*.

In particular, Lagrangian relaxation has been widely applied to unit commitment problems. This well-established method was first applied to unit commitment in [13], exploits the separable structure of the problem, and decomposes the problem into smaller, more manageable subproblems. Leaving out the reserve constraint (3) for simplicity, the Lagrangian relaxation procedure from [17] provides a demonstration. First, Lagrangian multipliers λ_t are associated with each of the demand constraint (2). Dualizing these constraints leads to the Lagrangian dual function $\mathcal{L}(\lambda)$, given as

$$\mathcal{L}(\lambda) = \min_{x,u} \sum_{i\in I} \sum_{t\in T} f_i\left(x_{it}, u_{it-1}, u_{it}\right) - \sum_{t\in T} \lambda_t \left(\sum_{i\in I} x_{it} - D_t\right), \tag{7}$$

subject to (4)–(6). The Lagrangian dual function, evaluated for any selected $\lambda \geqslant 0$ provides a lower bound to the solution of the unit commitment problem. The Lagrangian dual problem is to find the set of multipliers λ providing the *best* lower bound, i.e.

$$\max_{\lambda\geqslant 0} \mathcal{L}(\lambda) \tag{8}$$

To show its separable structure, (7) can be rewritten as

$$\mathcal{L}(\lambda) = \min_{x,u} \sum_{i\in I} \sum_{t\in T} \left(f_i\left(x_{it}, u_{it-1}, u_{it}\right) - \frac{\lambda_t}{|I|}\left(x_{it} - D_t\right)\right), \tag{9}$$

which is equivalent to $\mathcal{L}(\lambda) = \sum_{i\in I} F_i(\lambda)$, i.e. separate programs for each generating unit i:

$$F_i(\lambda) = \min_{x,u} \sum_{t\in T} \left(f_i\left(x_{it}, u_{it-1}, u_{it}\right) - \frac{\lambda_t}{|I|}\left(x_{it} - D_t\right)\right) \tag{10}$$

subject to the operating constraints of unit i.

The Lagrangian dual function $\mathcal{L}(\lambda)$ is concave and bounded, so a global optimum can be found. The non-differentiability of the function inhibits solution by conventional gradient descent methods, and several other methods are used. A subgradient procedure based on iteratively updating the Lagrangian multipliers based on the subgradients, given as $D_t - \sum_{i\in I} x_{it}$ (the slack of the demand constraints) has been widely used [2].

3. Stochastic unit commitment formulations

The demand for electricity is not known with certainty in the planning phase. Also, there is the possibility of generating units unexpectedly becoming unavailable due to equipment failure. Furthermore, the accuracy of forecasts for power generation from intermittent renewable energy sources is limited. These are the three most important sources of uncertainty that must be managed in the unit commitment problem. In the deterministic unit commitment formulation, this uncertainty is managed by including a reserve requirement

(3), to ensure sufficient flexibility in dispatch decisions taken at a later stage. Sometimes, price uncertainty is also taken into account in the unit commitment problem, e.g. to identify optimal bidding strategies [18], but such formulations are not included here.

3.1. Scenario representations of uncertainty

Assuming knowledge of the underlying distribution for the sources of uncertainty, the set of all possible realizations of the uncertain parameters can be approximated by a set of discrete scenarios S and their associated probabilities π_s. Then, D_{ts} would represent the realized electricity demand in time period t for each scenario $s \in S$. Availability of generation capacity is used to represent both outages and uncertainty in generation from intermittent sources, and can be included as $\overline{X}_{its}$, such that capacity for each generating unit depends on time period and scenario.

The problem can be formulated as a two-stage program [19,20], in which commitment decisions u_{it} must be made in the first stage, and dispatch decisions x_{its} are made in the second stage, after the realization of the uncertain parameters. The problem can also be formulated as a multi-stage problem using a scenario tree [17,21], where commitment decisions taken at a given node o are required to be equal across all scenario paths including o. A third possibility is to include demand uncertainty in chance constraints [22]. Other methods, such as robust optimization [7] and interval optimization [8] are sometimes also considered useful in representing uncertainty inside the unit commitment model, cf. [23].

3.2. Two-stage formulation

The two-stage formulation of the stochastic unit commitment problem takes uncertainty into account through scenarios. For the case with uncertain demand, the generation level x_{its} can be decided after information on demand D_{ts} is obtained, i.e. in the second stage. The commitment decision u_{it} however, must be made prior to obtaining this information, and is hence a first stage decision. Splitting the cost function f_i into first-stage ($f_i^{(1)}$) and second-stage ($f_i^{(2)}$) parameters emphasizes the two stages in the model:

$$\min \sum_{i\in I}\sum_{t\in T}\sum_{s\in S}\pi_s\left(f_i^{(1)}\left(u_{i(t-1)},\,u_{it}\right)+\sum_{s\in S}\pi_s f_i^{(2)}(x_{its})\right) \tag{11}$$

$$\text{s. t.}\ \sum_{i\in I} x_{its} \geqslant D_{ts},\quad t\in T,\ s\in S \tag{12}$$

The reserve constraint is often discarded, as uncertainty is considered to be managed by the scenario representation of the uncertain parameters. The objective function is often quadratic or piecewise linear.

As before, operating constraints must be satisfied. For the upper and lower capacities, this becomes

$$\underline{X}_i u_{it} \leqslant x_{its} \leqslant \overline{X}_{its} u_{it},\quad i\in I,\ t\in T \tag{13}$$

Often, only the upper bound is considered time variant and scenario dependent. Minimum uptime and downtime constraints are unchanged from (5) and (6). A similar formulation, using scenario dependent commitment variables u_{its} and non-anticipativity constraints is also possible.

The unit commitment problem was first formulated as a two-stage stochastic programming problem in 1977 in [24]. However, the considerably larger computational effort required to solve stochastic programs with integer variables impeded research of stochastic unit commitment until 1996, starting with the multi-stage formulations in [21,17]. Carøe and Schultz [19,25] developed a two-stage formulation and decomposed it by scenario, proposing a dual decomposition method based on Lagrangian relaxation of the non-anticipativity constraints. They then used a branch-and-bound procedure is used to identify primal feasible solutions. Dentcheva and Römisch [20] instead chose a Lagrangian relaxation of the demand constraints to obtain single-generator subproblems and search for feasible solutions using so-called Lagrangian heuristics, see also [26].

3.3. Multi-stage formulation

Just as in the two-stage formulation, uncertain parameters are represented by scenarios in the multi-stage formulation. Here, however, information on uncertain parameters is not given all at once, but is obtained at intervals throughout the planning horizon. This allows a more accurate representation of the decision process, especially for longer time horizons. Scenarios representing different outcomes are branched in a scenario tree at the node after which they are no longer equivalent. Up to this point, the scenarios belong to the same *bundle*, and the multi-stage formulation requires that decisions taken in a node must be equal for all scenario paths in which the node is included. I.e., if scenario s belongs to bundle $B(s,\,t)$ in time period t, then we have the non-anticipativity constraints

$$u_{its} = u_{it\sigma},\quad \sigma\in B(s,\,t),\ i\in I,\ t\in T \tag{14}$$

The multi-stage stochastic unit commitment problem can then be formulated as

$$\min \sum_{i\in I}\sum_{t\in T}\sum_{s\in S}\pi_s f_i\left(u_{i(t-1)s},\,u_{its},\,x_{its}\right) \tag{15}$$

$$\text{s. t.}\ \sum_{i\in I} x_{its} \geqslant D_{ts},\quad t\in T,\ s\in S \tag{16}$$

All of the operating constraints include commitment variables, which are now scenario dependent. For all $i\in I$, $t\in T$, $s\in S$,

$$\underline{X}_i u_{its} \leqslant x_{its} \leqslant \overline{X}_{its} u_{its} \tag{17}$$

$$u_{its} - u_{i(t-1)s} \leqslant u_{i\tau s},\quad \tau = t+1,\ \ldots, t+L_i^{\mathrm{up}}, \tag{18}$$

$$u_{i(t-1)s} - u_{its} \leqslant 1 - u_{i\tau s},\quad \tau = t+1,\ \ldots, t+L_i^{\mathrm{dn}}, \tag{19}$$

Carpentier et al. [21] provides an early example of a multi-stage formulation, letting the possibility of disturbances due to random events be represented in a scenario tree using a node formulation. The problem is decomposed using an Augmented Lagrangian relaxation on the demand constraints. Then, a stochastic dynamic subproblem is solved for each generating unit and iteration. Takriti et al. (1996) [17] also represent uncertain load and generator availability in a scenario tree, but decouple the problem by scenario and solve the problem using a progressive hedging algorithm [27] on the deterministic subproblems. Another multi-stage formulation is used by [28] for a hydro-thermal problem under load uncertainty, using a Lagrangian decomposition algorithm. In addition to uncertainty in demand, Takriti et al. (2000) [29] takes uncertain spot market and fuel prices into account using Benders decomposition, passing cuts from a fuel allocation problem to the unit commitment problem, which is solved using stochastic dynamic programming. Also, [20,30] include both two-stage and multi-stage formulations, both of which are decomposed into single-generator subproblems. A similar, but notably different approach was proposed in [31], where a multi-stage stochastic program is decomposed into single-generator subproblems using a column generation algorithm. Dynamic programming is used for the subproblems.

3.4. Chance-constrained formulation

For comparison, an alternative way to recognize the stochasticity of demand in the unit commitment problem is by requiring the demand constraints to be met with a high probability. Here, uncertain parameters are represented by their distributions, rather than scenarios, which can be seen as samples of these distributions. Therefore, random variables representing demand are not limited to discrete values, and the correlation between demand in consecutive hours is explicitly accounted for in the chance-constrained formulation.

A chance constrained formulation based on [22] is provided for comparison with the two-stage and multi-stage formulations in Sections 3.2 and 3.3. Starting from the deterministic unit commitment formulation in (1)–(6), we make $\mathbf{D}_t$ a random variable, and the demand constraints in (2) is rewritten as a single probabilistic constraint,

$$P\left[\sum_{i\in I} x_{it} \geqslant \mathbf{D}_t, \quad t\in T\right] \geqslant 1-\alpha \tag{20}$$

Here, (20) can be reformulated to obtain the deterministic equivalents used when solving the problem. First, let A_t denote the event that demand is met in time period t, i.e. $\sum_{i\in I} x_{it} \geqslant \mathbf{D}_t$, while A_t^c is the complement event. First, note that

$$P[\bigcap_{t\in T} A_t] = 1 - P[\bigcup_{t\in T} A_t^c] \tag{21}$$

Then, since we always have

$$P[\bigcup_{t\in T} A_t^c] \leqslant \sum_{t\in T} P[A_t^c] \tag{22}$$

we also have

$$P[\bigcap_{t\in T} A_t] \geqslant 1 - \sum_{t\in T} P[A_t^c] \tag{23}$$

and if we require $P[A_t^c] \leqslant \frac{\alpha}{T}$, we have

$$P[\bigcap_{t\in T} A_t] \geqslant 1-\alpha \tag{24}$$

Furthermore, assuming $\mathbf{D}_t$ to be normally distributed with mean μ_t and standard deviation σ_t, the requirement that $P[A_t^c] \leqslant \frac{\alpha}{T}$ is equivalent to $P[\mathbf{D}_t > \sum_{i\in I} x_{it}] \leqslant \frac{\alpha}{T}$. Denoting by $z_{\frac{\alpha}{T}}$ the $100(1-\frac{\alpha}{T})$th percentile of the standard normal distribution, we replace $\mathbf{D}_t$ and obtain the linear deterministic equivalents of (20)

$$\sum_{i\in I} x_{it} \geqslant \mu_t + z_{\frac{\alpha}{T}}\sigma_t, \quad t\in T \tag{25}$$

These constraints can be used in the otherwise deterministic unit commitment formulation to account for uncertainty in demand.

3.5. Extended formulations

Compared to the basic formulations presented in previous sections, various modeling extensions have been proposed. Typically incorporating additional constraints, notable example models include reserve and ramping requirements, transmission network constraints and generator fuel allocation.

3.5.1. Reserve requirements

Stochastic unit commitment models can take uncertainty in forecasts and availability of generation and transmission into account through representative scenarios. This approach represents modelling of the sources of uncertainty explicitly. Deterministic unit commitment also manages uncertainty in power system operations, but relies on multistage decision making and operating reserve requirements [32].

Ruiz et al. [32] point out that most earlier stochastic unit commitment studies [17,21,24,29,33–38] leave out the reserve requirement constraints in (3), considering uncertainty to be explicitly considered in the stochastic programming formulation. This can be problematic, since most of the same research, including [26], assess the performance of the commitment found by stochastic models using sets of scenarios already included in the optimization. In [32], on the other hand, it is acknowledged that the realization of the uncertain parameters can (and will most likely be) different from all of the scenarios used in the decision process. Therefore, performance is assessed in a Monte Carlo simulation for a number of realizations much larger than the amount of scenarios considered in the optimization. The authors argue that “a few scenarios in the unit commitment formulation cannot capture the whole spectrum of uncertainty” [32], and based on this recognition they choose to include reserve requirements in their stochastic program. Their combination of the stochastic and reserve methods outperforms both the deterministic and regular stochastic formulations in terms of both cost and reliability. It is not obvious, however, how to determine the optimal reserve requirement. Later, [39] has followed a similar approach, using both scenarios and a reserve requirement to obtain a risk-averse management of uncertainty.

3.5.2. Ramping constraints

Ramping constraints are sometimes included in unit commitment formulations. In [40], they are formulated as

$$-R_i^- \leqslant x_{its} - x_{i(t-1)s} \leqslant R_i^+, \quad i\in I,\ t\in T\backslash\{1\},\ s\in S \tag{26}$$

Here, $R_i^{+/-}$ denote the maximum allowed change in generator output between time periods.

Ramp-rate constraints are problematic when decomposing the stochastic unit commitment by time period, as they link consecutive time periods. Nasri et al. [41] propose a heuristic decomposition by time period, relaxing the ramping constraints, processing time periods successively, and enforcing ramp rate constraints only locally with respect to the previous time period.

3.5.3. Transmission network constraints

Different formulations have been proposed for transmission constrained stochastic unit commitment. Papavasiliou et al. (2013) [40] include transmission constraints using linearized (DC) power flow equations. Here, decision variables e_{lts} denote the power flow on line l in time period t and scenario s, and for each node n in the network, LI_n is the set of lines ending at node n, while lines in LO_n start at node n. Then, we have separate demand constrains at each node $n\in N$, as well as, $t\in T$, $s\in S$:

$$\sum_{l\in LI_n} e_{lts} + \sum_{i\in I} x_{its} = D_{nts} + \sum_{l\in LO_n} e_{lts} \tag{27}$$

Power flows e_{lts} are calculated based on the susceptance parameter B_{ls} (which is scenario dependent to model line failures), and the voltage phase angles θ_{nts}, which are decision variables.

$$e_{lts} = B_{ls}(\theta_{nts} - \theta_{mts}), \quad l=(m,n)\in L,\ t\in T,\ s\in S \tag{28}$$

Finally, each transmission line l have capacity limits TC_l due to thermal considerations,

$$-TC_l \leqslant e_{lts} \leqslant TC_l, \quad l\in L,\ s\in S,\ t\in T \tag{29}$$

When using AC power flow equations, the problem becomes mixed-integer nonlinear, and is hard to solve, although this has been done using Benders decomposition in [41].

Stochastic Security-Constrained Unit Commitment is an often used approach in taking into the possibility of network contingencies. In a large family of models [38,42–48], the usual approach is to solve the unit commitment without network constraints for each scenario, before assessing whether network constraints have been violated, and iteratively adding feasibility cuts in the master problem, e.g. using a Benders decomposition [42].

3.5.4. Fuel constraints

Takriti et al. (2000) [29] take uncertain fuel and spot market into account in a *fuel allocation problem* inside the multi-stage stochastic unit commitment formulation. Here, at each stage in the scenario tree, a two-stage problem is solved. The first-stage decision in this two-stage problem is the commitment decisions, while the second stage is a fuel allocation problem, which uses scenario specific price information. This is solved efficiently using Benders decomposition, in which the optimal fuel allocation subproblem for a given set of commitment decisions is added as a cut in the master problem, and through an iterative procedure, the optimal commitment decisions are found.

4. Decomposition methods for stochastic unit commitment

The computational effort required to solve stochastic mixed integer programming problems leads to intractability for large-scale problems. This is one of the key challenges in applying a stochastic programming formulation to the unit commitment problem [49]. Still, some authors have solved their stochastic unit commitment problems using mixed integer linear programming techniques [50–52]. The structure of the optimization problem, however, allows for decomposition into manageable subproblems, and various techniques have been proposed.

4.1. Scenario decomposition

A widely used family of approaches is *Scenario decomposition*, in which the stochastic problem decomposes into separate deterministic unit commitment problems for each scenario. Methods for scenario decomposition are well described in [53], and briefly summarized below.

4.1.1. Dual decomposition

Solving the stochastic unit commitment problem with this technique was first proposed in [25]. It uses Lagrangian relaxation of the non-anticipativity constraints to obtain a Lagrangian dual function which is separable for each scenario. Solving the Lagrangian dual problem (as in Section 2.2) provides a lower bound to the solution of the primal problem. The Lagrangian dual problem can be solved using e.g. the subgradient method [54] or variations of the cutting-plane method (including the bundle method). Notable applications of the dual decomposition are also found in [40,49,53,55,56].

4.1.2. Progressive hedging

This method was pioneered by Rockafellar and Wets [27], and was among the techniques used to decompose the first stochastic unit commitment problems [17]. The subproblems in progressive hedging have structural connections with the augmented Lagrangian [53]. Progressive Hedging algorithms have also been applied to the stochastic unit commitment problems in [57–59].

4.2. Unit decomposition

As an alternative to scenario decomposition, this approach decomposes the problem into single-generator stochastic programs, which can be solved separately, e.g. using dynamic programming. The decomposition itself is often achieved through Lagrangian relaxation, although this is not the only possibility.

4.2.1. Lagrangian relaxation

Also including *augmented* Lagrangian relaxation [21], this technique can be applied to the demand constraint to decompose the problem into single-generator problems, as is done in [20,28,30], to mention but a few. A similar method was also used in [29], coupled with a Benders decomposition for the uncertainty in fuel and electricity prices.

4.2.2. Danzig-Wolfe decomposition

This technique is less common, but it is used by [31] in a column generation approach. Here, new schedules are created in a Lagrangian subproblem by solving it using DP on a scenario tree.

4.3. Benders-like decomposition

The basic version of Benders decomposition adds cuts in the master (first-stage) problem, based on the evaluation of the subproblem (second-stage) for given values of the first-stage decision variables. Both multi-cut and single-cut methods appear in literature. Benders-like decomposition has been applied to the stochastic unit commitment problem in several models, including [29,41,42,52,60–62]. The L-shaped method, based on Benders decomposition, is, in contrast to scenario and unit decomposition, a *time stage-based* decomposition technique, and has a less uniform distribution of the difficulty of the subproblems compared to scenario decomposition, which reduces the benefit of solving subproblems in parallel [57,63].

5. Advances in decomposition-based methods for stochastic unit commitment

Effectively applying stochastic programming in unit commitment decisions requires the ability to solve such problems efficiently. Several methods aim to improve the computational efficiency through advanced concepts for decomposing and solving stochastic unit commitment problems, possibly establishing new best practices for the future. This section summarizes and explains some of the most important concepts developed in the past few years.

5.1. Parallel distributed algorithms

When performing a Lagrangian relaxation on the non-anticipativity constraints of the stochastic unit commitment problem, the Lagrangian dual function is decomposable across scenarios. This separable structure encourages a parallel implementation of the solution of deterministic subproblems [17]. Computations can then be parallelized at each iteration of the dual problem, and parallel algorithms have been used in combination with scenario decomposition methods in [40,55–57,64,65], exploiting the increase in distributed computing capacity.

5.1.1. Dual decomposition algorithm for parallel implementation

A dual decomposition algorithm was developed in [40] to be used for the two-stage stochastic unit commitment problem on a high-performance computing cluster. The formulation is in many ways similar to (11)–(13), but they use both first stage and second-stage (scenario specific) commitment and startup variables, w_{it}, z_{it} and u_{its}, v_{its}, respectively. Then they enforce non-anticipativity using

$$\pi_s u_{its} = \pi_s w_{it} \quad i \in I,\ t \in T,\ s \in S \tag{30}$$

$$\pi_s v_{its} = \pi_s z_{it} \quad i \in I,\ t \in T,\ s \in S \tag{31}$$

Also they use a linear objective function, and include ramp-rate constraints and power flow on lines in a transmission network in the model.

By dualizing the non-anticipativity constraints (30) and (31) with associated dual variables μ_{its} and ν_{its}, they obtain the Lagrangian dual function

$$\begin{aligned}\mathcal{L} = &\sum_{i\in I}\sum_{t\in T}\sum_{s\in S}\pi_s(K_i u_{its} + S_i v_{its} + C_i x_{its})\\ &+\sum_{i\in I}\sum_{t\in T}\sum_{s\in S}\pi_s(\mu_{its}(u_{its} - w_{it}) + \nu_{its}(v_{its} - z_{it}))\end{aligned} \tag{32}$$

Decomposing (32) into one subproblem for each scenario, we can find the optimal second-stage decisions for scenario s using

$$\begin{aligned}\left(\mathrm{P2}_s\right):\ \min &\sum_{i\in I}\sum_{t\in T}\pi_s(K_i u_{its} + S_i v_{its} + C_i x_{its})\\ &+\sum_{i\in I}\sum_{t\in T}\pi_s(\mu_{its} u_{its} + \nu_{its} v_{its})\end{aligned} \tag{33}$$

subject to energy balance, line flow, unit capacity, ramp-rate, startup logic constraints, as well as non-negativity for x_{its} and v_{its}, and $u_{its} \in \{0, 1\}$.

Optimal first-stage decisions can be found using a single subproblem:

$$\left(\text{P1}\right): \ \min \ -\sum_{i\in I}\sum_{t\in T}\sum_{s\in S}\pi_s(\mu_{its}w_{it}+\nu_{its}z_{it}) \tag{34}$$

subject to minimum up and down-time constraints and binary requirement for w_{it} and logic constraints, including bounds, for z_{it}.

Finally, they update the dual variables after each iteration k using

$$\mu_{its}^{k+1} = \mu_{its}^{k} + \alpha_k\pi_s(w_{it}^k - u_{its}^k), \quad i\in I,\ t\in T,\ s\in S \tag{35}$$

$$\nu_{its}^{k+1} = \nu_{its}^{k} + \alpha_k\pi_s(z_{it}^k - v_{its}^k), \quad i\in I,\ t\in T,\ s\in S \tag{36}$$

While it would have been possible to relax only (30), the authors choose also to relax (31). This means non-anticipativity of the startup variables is handled in (P2$_s$) $s\in S$, which is a smaller problem compared to (P1), which contains constraints on the unit commitment. Also, this division allows the commitment schedule w^k found in (P1) in iteration k to be used as input for an economic dispatch calculation (ED$_s$) for each scenario s, thereby generating a feasible solution to the original problem. Hence, an upper bound is obtained in every iteration. The second-stage subproblems (P2$_s$) and the economic dispatch calculations (ED$_s$) can be solved in parallel.

5.1.2. Asynchronous algorithm

Parallel algorithms using traditional iteration methods, such as subgradient, gradient or proximal-point methods, run subprocesses in parallel at each iteration, and move to the next iteration when all subprocess are finished. Aravena and Papavasiliou [55] highlights this need for *synchronization points* in previous approaches as an important drawback, as parts of the computing capacity is idle while waiting for other subprocess to finish.

To better utilize the parallel computing infrastructure, they propose an *asynchronous* algorithm, in which the traditional iteration method is replaced by an *incremental* method [66]. An incremental method uses information from parts of the objective function when calculating the update direction. They choose an incremental subgradient method and apply it asynchronously in combination with the dual decomposition technique [40] summarized above into a two-stage formulation [65] of the stochastic unit commitment problem. They also handle primal recovery solutions in an asynchronous manner, feeding feasible commitment schedules obtained in each incremental iteration to a queue for primal recovery.

In the incremental subgradient methods, we consider concave, non-differentiable *component functions* $h_j(\cdot)$ and convex optimization problems of of the form

$$\max_{y\in Y}\sum_{j\in J}h_j(\mathbf{y}), \tag{37}$$

with Y being a nonempty, closed and convex set. Only a subset of the component functions are used for updating the multipliers. For iteration k, $\mathbf{y}$ is updated as

$$y^{k+1} := \mathcal{P}_Y(\mathbf{y}^k + \alpha^k\mathbf{q}_j^\kappa), \quad j\in J,\ \mathbf{q}_j^\kappa\in\partial_y h_j(\mathbf{y}^\kappa),\ \kappa\leqslant k \tag{38}$$

Here, $\mathbf{q}_j^\kappa$ is a subgradient of the component function $h_j(\cdot)$ at the κth iteration. $\mathcal{P}_Y$ is the projection onto Y.

However, due to delays, different components will be evaluated using different sets of multipliers, complicating the calculation of a lower bound. Denoting by f_s the component function given by (P2$_s$) in (33), we let θ be the index of the most recently evaluated component function $f_\theta(\cdot)$, and at the k-th iteration, all other component functions $f_t(\cdot)$ have been evaluated using multipliers $\mu_t^{j(t)}$, $\nu_t^{j(t)}$ with iteration $j(t) < k$. Concatenating the most recent multipliers used for all component functions into multiplier vectors $\boldsymbol{\mu}^j$, $\boldsymbol{\nu}^j$,

$$\boldsymbol{\mu}^j := (\mu_{t_1}^{j(t_1)},\ldots,\mu_\theta^k,\ldots,\mu_{t_n}^{j(t_n)}), \quad t_l\neq\theta \tag{39}$$

$$\boldsymbol{\nu}^j := (\nu_{t_1}^{j(t_1)},\ldots,\nu_\theta^k,\ldots,\nu_{t_n}^{j(t_n)}), \quad t_l\neq\theta \tag{40}$$

and the lower bound LB^k can be calculated by evaluating the second stage component functions using their latest multiplier values, and adding the evaluation $g(\cdot)$, corresponding to the (P1) subproblem in (34) using the concatenated multiplier vectors:

$$LB^k = f_\theta\left(\mu_\theta^k,\ \nu_\theta^k\right) + \sum_{t\in S,t\neq\theta}f_t\left(\mu_t^{j(t)},\ \nu_t^{j(t)}\right) + g\left(\mu^j,\ \nu^j\right) \tag{41}$$

The subgradients of $f_\theta(\cdot)$ and $g(\cdot)$ are available after evaluating (41), and multipliers μ and ν are updated using

$$\mu_\theta^{k+1} := \mu_\theta^k + \alpha_k\pi_\theta(\mathbf{w}^k - \mathbf{u}_\theta^k) \tag{42}$$

$$\mu_t^{k+1} := \mu_t^k, \quad \forall\ t\in S,\ t\neq\theta \tag{43}$$

$$\nu_\theta^{k+1} := \nu_\theta^k + \alpha_k\pi_\theta(\mathbf{z}^k - \mathbf{v}_\theta^k) \tag{44}$$

$$\nu_t^{k+1} := \nu_t^k, \quad \forall\ t\in S,\ t\neq\theta \tag{45}$$

This incremental step is used for updating multipliers, in contrast to updating using the regular subgradient method in (35) and (36). Compared to an equivalent synchronous algorithm, the authors achieved convergence three times faster using the asynchronous algorithm.

5.2. Other dual decomposition algorithms

As noted in Section 4.1, stochastic programs for unit commitment have often been decomposed into scenarios using dual decomposition or progressive hedging algorithms. Kim and Zavala [53] explain and elaborate on both of these categories.

The differences between dual decomposition algorithms are related to the solution of the Lagrangian dual problem, i.e. after we obtain a separable expression of the Lagrangian dual function. This was shown in (10) for a decomposition by generator. Similarly, a separate program in terms of the dual variables λ for each scenario in the Lagrangian dual function can be denoted F_s, and then we have the dual problem

$$\max_{\lambda\geqslant 0}\mathcal{L}(\lambda) = \sum_{s\in S}F_s(\lambda) \tag{46}$$

The *subgradient method*, as described before, updates the dual variables λ using information on the slack of the dualized constraints, as in (35). The step sizes α_k at each iteration must be appropriately selected, and different rules have been studied. Finite termination cannot be proved in the subgradient method [54].

The *cutting-plane method* also solves the Lagrangian dual problem iteratively. By adding linear inequalities, an outer representation is formed, given for each iteration k by the Lagrangian master problem

$$m_k := \max_{\theta_s,\lambda}\sum_{s\in S}\theta_s \tag{47}$$

$$\text{s. t.}\ \ \theta_s\leqslant F_s(\lambda^j) + (\mathbf{u}_s^j - \mathbf{w}_s^j)^T(\lambda - \lambda^j), \quad s\in S,\ j=0,\,1,\,\ldots,k \tag{48}$$

Here, the difference between $\mathbf{u}_s^j$ and $\mathbf{w}_s^j$ is the slack in the dualized non-anticipativity constraints in scenario s. Hence, at this point, each variable θ_s in the Lagrangian master problem is constrained by $k+1$ hyperplanes. New multipliers λ^{k+1} are found by solving (47) and (48). The authors [53] point out that this problem exhibits a block-angular structure, and can thus be solved in parallel using an interior-point solver [67]. By terminating the algorithm when $m_k - 1\leqslant\mathcal{L}(\lambda^k)$, the cutting-plane method will find an optimal solution to the Lagrangian dual function in a finite number of steps.

The instability of the cutting-plane method causes solutions of the master problem to oscillate in the first iterations, when the linear inequalities added do not adequately approximate the dual problem. The solution also suffers from degeneracy. Kim and Zavala [53] present two variants of the cutting-plane addressing these issues.

The *interior-point cutting-plane method*, proposed in [56] uses an early termination criterion, allowing a duality gap in the solution of (47) and (48). This is suboptimal, but interior feasible solutions are used to find stronger cuts and avoid degeneracy. Their algorithm is proved to terminate after a finite number of iterations with an optimal solution of the Lagrangian dual problem [56].

The *bundle method* stabilizes the master problem in (47) by adding a quadratic term. The master problem is then reformulated as

$$m_k := \max_{\theta_s,\lambda} \sum_{s\in S} \theta_s + \frac{1}{2\tau}\left|\left|\lambda - \lambda^+\right|\right|^2 \tag{49}$$

$$\text{s. t.}\quad \theta_s \leqslant F_s(\lambda^j) + (\mathbf{u}_s^j - \mathbf{w}_s^j)^T(\lambda - \lambda^j),\qquad s\in S,\ j=0,1,\ldots,k \tag{50}$$

Here, two parameters are added. $\tau > 0$ defines the *stabilization effect*, and λ^+ is a *stability center*. The parameter value τ typically needs to be adjusted at every iteration [67], and to avoid this, [56] propose using a trust-region constraint as an alternative way of stabilizing the solution of the Lagrangian master problem.

5.3. Advanced progressive hedging algorithms for stochastic unit commitment

Progressive hedging algorithms have been used for scenario decomposition in the stochastic unit commitment problem since the beginning. In [17], a progressive hedging algorithm proposed by Rockafellar and Wets [27] is applied by first performing a Lagrangian relaxation of the bundle (non-anticipativity) constraints, associating with it multipliers μ_{its} and adding a penalty term $\mu_{its}(u_{its} - c_i^k)$ to the objective function. Here, for all scenarios *s* that share the same bundle Ω_k, the target value will be the same and given as the weighted average of the decisions

$$c_i^k = \frac{\sum_{s\in B(s,t)=\Omega_k} \pi_s u_{its}}{\sum_{s\in B(s,t)=\Omega_k} \pi_s} \tag{51}$$

The $|S|$ deterministic unit commitment problems are solved in an iterative procedure where the penalties μ_{its} are updated until the solutions of the subproblems satisfy the bundle constraints. As [17] remarks, there is no guarantee that the progressive hedging algorithm will converge, and if it does, it may terminate at a local minimum.

As Takriti et al. [17] use a ordinary Lagrangian relaxation of the non-anticipativity constraints, the added penalty term is linear. Rockafellar and Wets [27] note that the *augmented Lagrangian* combines features of multipliers and penalties, is not limited to convex problems and is more powerful, “if one can work with it”. The augmented Lagrangian adds, in addition to the linear dualized constraints, a quadratic proximal term $\frac{\rho}{2}||\sum_{s\in S}\pi_s\mathbf{u}_s - \mathbf{c}||$ is added to the objective function. This term is not decomposable into separate terms for each scenario, and to achieve decomposition, the progressive hedging algorithm solves, for each scenario *s*, separate subproblems of the form

$$P_s\left(c,\mu,\rho\right) := \min F_s + \frac{\rho}{2}\left|\left|\mathbf{u}_s - \mathbf{c}\right|\right|^2 \tag{52}$$

This appears to be the common approach when applying progressive hedging to stochastic unit commitment programs [53,57,63].

Ryan et al. [57] include *accelerators*, techniques to improve convergence of the progressive hedging algorithm. This includes *variable fixing* and *slamming*, as described in [68], together with techniques for detecting and breaking cyclic behaviour caused by integer decision variables.

Furthermore, [57] highlights the importance of the penalty parameter ρ in the quadratic proximal term, as poor choices can lead to slow convergence, or no convergence at all. The authors employ variable-specific ρ values, using a *cost-proportional* parametrization based on locational marginal prices of the economic dispatches of the scenario subproblems, evaluated after the 0th iteration.

The authors also report on successfully reducing solution times for the subproblems is to group scenarios into bundles forming small-scale stochastic programs, and solve them directly using commercial MIP solvers. However, the merit of this aggregation depends on the number of scenarios in the bundles, which is an empirical question [57].

Table 1
Classification of recent literature on scenario-based stochastic programming for the unit commitment problem.

Decomposition	Method	Two-stage	Multi-stage
Scenario	Dual decomposition	[53,55,69,70]	
	Progressive hedging	[10,53,59,71,72]	[59]
	Primal-dual decomposition	[73]	
	Column generation	[74–76]	[74–76]
Unit	Spatial decomposition	[70]	
Benders-like	Benders decomposition	[41,77–80]	
None	Solved directly	[39,81–87]	[83,88–91]
	Cutting-plane		[92]
	Dynamic formulation		[93]
	Heuristic	[94–96]	

6. Survey and classification of new stochastic unit commitment literature

This paper reports on new literature on scenario-based stochastic programming for unit commitment and classifies it into *multi-stage* and *two-stage formulations*, and further distinguish between the main decomposition approaches, namely *scenario decomposition*, *unit decomposition*, *Benders-like decomposition* and *No decomposition* approaches. Table 1 provides a compact summary of this classification.

6.1. Two-stage formulations

As described in Section 5.1, Aravena and Papavasiliou [55] present a distributed asynchronous algorithm, using an incremental method to solve the Lagrangian dual problem following a dual decomposition. Papavasiliou et al. (2015) [69] describe, for a related model, the parallel implementation on a high-performance computing cluster. Kim and Zavala (2016) [53] also decompose the problem by scenarios, but do so using different dual decomposition methods and progressive hedging for decomposing a two-stage formulation by scenarios into subproblems solvable in parallel. They also provide the generic algorithms for the methods, as well as some recent innovations.

Also using progressive hedging, Feng et al. [10] and Cheung et al. [71] report on novel customizations to the progressive hedging algorithm for stochastic unit commitment, proving able to solve “realistic, moderate-scale problems with reasonable numbers of scenarios in no more than 15 min of wall clock time on commodity compute platforms”. Unlike the Lagrangian relaxation method, progressive hedging seeks to find a feasible solution, thus obtaining an upper bound [53]. Gade et al. [59] present a method for obtaining lower bounds in two-stage and multi-stage formulations of the stochastic unit commitment using a progressive hedging algorithm. Here, lower bounds can be computed using dual prices calculated during execution of the progressive hedging algorithm. A final contribution is Rachunok et al. [72]. They apply a progressive hedging algorithm using a relatively large grid model, while investigating the impact of the number of scenarios used on computational complexity, and identify an unexpected transition at a given threshold. This was found to be caused by wind power scenarios with zero or very low power trajectories being “very difficult to reconcile with more typical scenarios”.

Apparently, the only recent application of unit decomposition is made by Scuzziato et al. [70]. For a system complicated by cascaded hydropower plants, they evaluate different configurations of the decomposition, differing in the spatial representation of hydro plants and reservoirs. Through a formalized benchmarking procedure, they find one of these configurations computationally outperforming a scenario decomposition procedure based on Lagrangian relaxation, while performing almost as well in terms of duality gaps.

Benders decomposition is still popular in recent literature on stochastic unit commitment. It is used by Nasri et al. [41] to decompose an AC network-constrained unit commitment with uncertain wind power production by both scenario and time period. The original problem is a MINLP, but though a convexification, Benders decomposition can be applied to the complicating (first-stage) decisions, giving a MILP master problem for the first stage, and separate, nonlinear problems for each scenario, which are decomposed by time period using a Benders decomposition and heuristic relaxation technique on the intertemporal constraints. Solution of the subproblems for each scenario and time period provides cuts to the master problem, which updates the fixed values of the first-stage decisions in an iterative procedure. Lopez-Salgado et al. [80] also decompose the problem into single time periods subproblems using Benders decomposition. However, complex hydropower topologies in their model make the dispatch problem non-linear. To optimize the binary commitment decisions, outer approximation is applied, providing candidate commitment schedules to the dispatch problem and making use of its results to improve the MILP approximation of the non-linear problem.

Another application of Benders decomposition is incorporating network constraints in security-constrained unit commitment models. In two similar models, Vatanpour et al. [78,79] extend the use of feasibility cuts to also ensure first-stage decisions to be feasible in all scenarios. Yet another subproblem is used to generate optimality cuts based on the optimal schedules across all scenarios. This is different from the approach taken by Mehrtash et al. [77]. They also decompose a stochastic security-constrained unit commitment problem into a master problem and several subproblems, but use a point estimation method is used rather than a conventional scenario representation. In a point estimation method, the probability density functions of random variables are concentrated into a few points, provided by their first central moments, and all probable combinations of these points constitute the set of scenarios considered in the optimization. Imani et al. [97] incorporate vehicle-to-grid (V2G) participation and uncertain wind production in what they claim to be a security-constrained unit commitment model. However, it is highly unclear how the small set of wind power scenarios is used to find optimal commitment decisions, if it is done at all.

In [73], van Ackooij and Malick provide a holistic introduction to stochastic unit commitment, including an interesting discussion on the limitations of existing decomposition approaches. In their words, “the existing primal (Benders) or dual (Lagrangian) approaches make unclear how to recover feasible first stage solutions, or make significant changes to the set of technical constraints”. They propose a new decomposition algorithm, a *primal-dual approach*, using several *hot-started bundle methods* together with cutting plane methods and primal recovery heuristics. Another example is the hybrid formulation by Dvorkin et al. [81]. Using both stochastic and interval unit commitment, the first hours of the horizon are scheduled using the full scenario representation, while interval optimization is applied to the remaining hours. This reduces the computational burden significantly, and for their test cases, the formulation also outperform a stochastic programming formulation in terms of expected actual operating cost. No decomposition is applied. Shi and Oren [89,90] incorporate topology control, which are binary decisions and further increase the computational challenge of solving large stochastic unit commitment problems. Facing this, they apply decomposition, but not in the traditional sense. Rather than decomposing by unit or scenario, they simply partition a large network into separate zones and solve these separately, regarding cross-zonal flows as fixed parameters. Acknowledging that this is suboptimal compared to treating such flows with decision variables, they use the term *decomposition heuristic* to describe the procedure and, in essence, solve their smaller problems directly on a solver. In a different approach to improve computational tractability, Du et al. [87] propose using *scenario maps* an efficient alternative to a the traditional set of scenarios typically found using scenario reduction. As for scenarios, higher solution quality can be traded against computational performance by increasing the number of states in the scenario map, and numerical tests with no decomposition applied shows the scenario-map method to be faster than a traditional approach using the necessary number of scenarios to obtain the same solution quality.

A considerable share of recent papers report on solving two-stage stochastic unit commitment problems without applying decomposition techniques. Bakirtzis et al. [84] use a two-stage model over a rolling horizon to manage storage energy levels. The scale of the model is limited by two important factors. The scheduling horizon is limited, varying from 12 h to 36 h in each optimization process, using a variable time resolution of 15 m, 30 m and hourly periods. Moreover, only 15 scenarios are used to represent uncertainty in wind and consumption. When applying the procedure to a model of the Greek power system, an average optimization run takes about 10 s without applying any decomposition or parallel architecture. Valinejad et al. [85] apply a rather traditional two-stage configuration to account for uncertainty in wind power, forecasted demand and random failures. Including a representation of aggregated demand response participation, the authors apply the method on a relatively small test network without reporting on solution methods or computational experience. Rather than aggregating demand response, Gomes et al. [86] use a two-stage model to optimize their bidding strategy for a generation portfolio consisting of wind power, PV and thermal units. Interestingly, not only wind and PV power, but also uncertainty in market and imbalance prices are represented through different scenarios. The model is tested for a short horizon without a network model, and also provide no details on implementation or computational experience. In an interesting real-life application, Abbaspourtorbati and Zima [82] describe the two-stage stochastic unit commitment model used for clearing the reserve market in Switzerland. However, the computational challenge is limited, as they use only a small number of scenarios. Asensio and Contreras [39] also appear to be able to solve their program without decomposition. Their two-stage model manages uncertainty in demand, wind and photovoltaic power production in an isolated island system through both scenarios and a reserve requirement. This is used to obtain a risk-averse solution to the problem, and they also include a constraint on the conditional value-at-risk in the model.

Solving stochastic unit commitment problems solely using various kinds of heuristics has become more commonplace. Although outside the main focus of this survey, a few examples will be mentioned. Shahbazitabar and Abdi [94] reduce computation time by circumventing the traditional MILP procedure of optimizing binary commitment decisions. Instead, they generate candidate commitment schedules using priority lists, which is a simple classical heuristic. While the heuristic method is found to reduce costs compared to a deterministic model, the performance is not benchmarked against a formal stochastic optimization procedure. Jo and Kim [96] also solve their stochastic program with a priority list, boldly claiming it to be an “optimal solution procedure”. To their defence, efforts are mainly focused on uncertainty representation through so-called multi-scenario trees. A comparable, but more sophisticated approach is made by Wang et al. [95]. They apply and compare a range of different metaheuristics to a stochastic unit commitment problem, and conclude that the best performance is achieved using a *binary artificial sheep algorithm*. However, the value of this insight is questionable, as they do not evaluate these heuristics against traditional methods, which they claim to have such deficiencies as “poor quality results” [95].

6.2. Multi-stage formulations

Compared to two-stage models, the list of recent literature on multi-stage stochastic unit commitment models is sparse. However, several of the efforts that have been made are well documented and appear to meet high quality standards. By studying the substructures of the constraints, Jiang et al. [92] introduce strong valid inequalities to strengthen the linear programming relaxation of the multi-stage stochastic unit commitment program. The valid inequalities are incorporated as cutting planes into a branch-and-cut framework, and the authors also claim their proposed cutting planes can be integrated with other decomposition methods. Attacking the problem from a different angle, the key innovation by Analui and Scaglione [93] is not in the solution method, but in their multi-stage formulation. Unit commitment programs typically require fairly long horizons to ensure minimum up- and down-time constraints are satisfied. Their formulation avoids this by representing the on-time of units in state variables, allowing the nodes in the scenario tree to be solved in a dynamic manner, only using information from the next time-step, rather than an entire scenario. This reduces the problem to a set of much smaller problems, allowing parallelization, and they apply no further decomposition in their case studies.

Schulze and McKinnon [75] compare two-stage and multi-stage stochastic unit commitment models to their deterministic counterparts to evaluate their added value in day-ahead and intra-day operations, using an innovative scenario decomposition based on Dantzig-Wolfe decomposition first described in [74]. The decomposition is combined with a new techniques to for initialization and primal recovery. The model is also further extended in [76], and applied to a model based on the British national grid, achieving significantly faster convergence compared to solving the extensive form directly. Although not compared directly to alternative decomposition methods, the reported computational performance is very promising.

While most unit commitment studies consider scheduling of slow-starting units, Wang and Hobbs [88] focus on a short-start unit commitment to simulate real-time markets, solving it directly using the CONOPT solver, without applying decomposition techniques. However, the problem instances appear to be relatively small in size.

Uckun and Botterud [83] introduce a dynamic decision process for unit commitment with uncertain wind power production. This is slightly different from a traditional multi-stage formulation, not relying on scenario trees but rather scenario *buckets*, representing similar wind power levels. This allows using non-anticipativity constraints, giving a dynamic solution, but at a considerably lower computational cost compared to a multi-stage formulation. Comparative simulations on the same input data confirm advantageous computational performance compared to the multi-stage formulation, while it outperforms the two-stage formulation in terms of solution quality. Another model disobeying established norms of structure is the three-stage program formulated by Du et al. [91]. The main difference from a traditional two-stage model is in the third stage or look-ahead stage, extending the horizon of some of the operational decisions using an economic dispatch model.

7. Conclusion

This article has explained the fundamental concepts for stochastic programming methods for the unit commitment problem. The increasing penetration of intermittent power generation in many power systems increases uncertainty in day-ahead generation decisions. In the stochastic unit commitment problem, uncertainty is often handled explicitly by representing uncertain parameters by scenarios. These problems can be formulated as two-stage or multi-stage programs, which are often intractable due to large number of integer decision variables. To be able to solve stochastic unit commitment programs efficiently, the structure of the problem is usually exploited to decompose the problem into manageable subproblems.

The rapidly growing list of literature on stochastic unit commitment can be categorized in different ways. This article proposes a classification of the newest stochastic programming literature based on the formulation and decomposition methods used, dividing literature by *two-stage* and *multi-stage* models and further distinguishing into *scenario decomposition*, *unit decomposition*, *Benders-like decomposition* and approaches using *No decomposition*.

During the last few years, there are several articles proposing algorithmic innovations for decomposing and solving stochastic unit commitment problems more efficiently. In particular, new techniques for primal recovery and calculation of lower bounds are available, and there is significant focus on distributed algorithms utilize parallel computing resources. There also seems to be a preference of scenario decomposition and two-stage formulations in the newest literature. Unit decomposition, which was widespread in pioneering stochastic unit commitment research, appears not to prevail. At the same time, there is also a clear tendency of stochastic unit commitment problems being solved directly, without applying decomposition. Some of these models are hybrid formulations or limited in size, whereas others can be of considerable scale.

Interestingly, several recent stochastic unit commitment models embrace factors such as demand response and various kinds of storage. Compared to the traditional scheduling of slow-starting generating units under uncertain load and renewable generation, there is a trend of applying the techniques in a broader set of contexts, including real-time market operations and scheduling of energy storage.

Declaration of interest

None.

Acknowledgements

The author wishes to thank Gerard Doorman at Statnett for valuable feedback and support.

Funding: This work was supported by Statnett, Oslo, Norway and the Norwegian Research Council, Oslo, Norway, grant number 248291.

References

[1] Abujarad SY, Mustafa MW, Jamian JJ. Recent approaches of unit commitment in the presence of intermittent renewable energy resources: a review. Renew Sustain Energy Rev 2017;70(November 2016):215–23. https://doi.org/10.1016/j.rser.2016.11.246.

[2] Bard JF. Short-term scheduling of thermal-electric generators using Lagrangian relaxation. Oper Res 1988;36(5):756–66. https://doi.org/10.1287/opre.36.5.756.

[3] Zheng QP, Wang J, Liu AL. Stochastic optimization for unit commitment—a review. IEEE Trans Power Syst 2015;30(4):1913–24. https://doi.org/10.1109/TPWRS.2014.2355204.

[4] Dai H, Zhang N, Su W. A literature review of stochastic programming and unit commitment. J Power Energy Eng 2015;03(04):206–14. https://doi.org/10.4236/jpee.2015.34029.

[5] Tahanan M, van Ackooij W, Frangioni A, Lacalandra F. Large-scale unit commitment under uncertainty. 4OR 2015;13(2):115–71. https://doi.org/10.1007/s10288-014-0279-y.

[6] van Ackooij W, Danti Lopez I, Frangioni A, Lacalandra F, Tahanan M. Large-scale unit commitment under uncertainty: an updated literature survey. Ann Oper Res 2018. https://doi.org/10.1007/s10479-018-3003-z.

[7] Bertsimas D, Litvinov E, Sun XA, Zhao J, Zheng T. Adaptive robust optimization for the security constrained unit commitment problem. IEEE Trans Power Syst 2013;28(1):52–63. https://doi.org/10.1109/TPWRS.2012.2205021.

[8] Pandzic H, Dvorkin Y, Qiu T, Wang Y, Kirschen DS. Toward cost-efficient and reliable unit commitment under uncertainty. IEEE Trans Power Syst 2016;31(2):970–82. https://doi.org/10.1109/TPWRS.2015.2434848.

[9] Osório GJ, Lujano-Rojas JM, Matias JCO, Catalão JPS. A new scenario generation-based method to solve the unit commitment problem with high penetration of renewable energies. Int J Electr Power Energy Syst 2015;64:1063–72. https://doi.org/10.1016/j.ijepes.2014.09.010.

[10] Feng Y, Rios I, Ryan SM, Spürkel K, Watson J-P, Wets RJ-B, et al. Toward scalable stochastic unit commitment. Part 1: load scenario generation. Energy Syst 2015;6(3):309–29. https://doi.org/10.1007/s12667-015-0146-8.

[11] Morales J, Conejo A, Perez-Ruiz J. Economic valuation of reserves in power systems with high penetration of wind power. IEEE Trans Power Syst 2009;24(2):900–10.

https://doi.org/10.1109/TPWRS.2009.2016598.
[12] Growe-Kuska N, Heitsch H, Romisch W. Scenario reduction and scenario tree construction for power management problems. 2003 IEEE Bologna power tech conference proceedings, vol. 3. IEEE; 2003. p. 152–8. https://doi.org/10.1109/PTC.2003.1304379.
[13] Muckstadt JA, Koenig SA. An application of Lagrangian relaxation to scheduling in power-generation systems. Oper Res 1977;25(3):387–403. https://doi.org/10.1287/opre.25.3.387.
[14] Wood W. Spinning reserve constrained static and dynamic economic dispatch. IEEE Trans Power Apparat Syst 1982;PAS-101(2):381–8. https://doi.org/10.1109/TPAS.1982.317118.
[15] Sheble GB, Fahd GN. Unit commitment literature synopsis. IEEE Trans Power Syst 1994;9(1):128–35. https://doi.org/10.1109/59.317549.
[16] Garver LL. Power generation scheduling by integer programming-development of theory. Trans Am Inst Electr Eng Part III: Power Apparat Syst 1963;81(3). https://doi.org/10.1109/AIEEPAS.1962.4501405.
[17] Takriti S, Birge J, Long E. A stochastic model for the unit commitment problem. IEEE Trans Power Syst 1996;11(3):1497–508.
[18] Wang Q, Wang J, Guan Y. Price-based unit commitment with wind power utilization constraints. IEEE Trans Power Syst 2013;28(3):2718–26. https://doi.org/10.1109/TPWRS.2012.2231968.
[19] Carøe CC, Schultz R. A two-stage stochastic program for unit commitment under uncertainty in a hydro-thermal power system. Konrad-Zuse-Zentrum fur Informationstechnik 1998;11(February).
[20] Dentcheva D, Römisch W. Optimal power generation under uncertainty via stochastic programming. Stochastic programming methods and technical applications 1998. p. 22–56. https://doi.org/10.1007/978-3-642-45767-8_2.
[21] Carpentier P, Gohen G, Culioli J-C, Renaud A. Stochastic optimization of unit commitment: a new decomposition framework. IEEE Trans Power Syst 1996;11(2):1067–73. https://doi.org/10.1109/59.496196.
[22] Ozturk U, Mazumdar M, Norman B. A solution to the stochastic unit commitment problem using chance constrained programming. IEEE Trans Power Syst 2004;19(3):1589–98. https://doi.org/10.1109/TPWRS.2004.831651.
[23] Melamed M, Ben-Tal A, Golany B. A multi-period unit commitment problem under a new hybrid uncertainty set for a renewable energy source. Renew Energy 2018;118:909–17. https://doi.org/10.1016/j.renene.2016.05.095.
[24] Wiebking R. Stochastische Modelle zur optimalen Lastverteilung in einem Kraftwerksverbund. Z Oper Res 1977;21(6):B197–217. https://doi.org/10.1007/BF01918361.
[25] Carøe CC, Schultz R. Dual decomposition in stochastic integer programming. Oper Res Lett 1999;24(1-2):37–45. https://doi.org/10.1016/S0167-6377(98)00050-9.
[26] Gollmer R, Nowak MP, Römisch W, Schultz R. Unit commitment in power generation – a basic model and some extensions. Ann Oper Res 2000;96:167–89. https://doi.org/10.1023/A:1018947401538.
[27] Rockafellar RT, Wets RJB. Scenarios and policy aggregation in optimization under uncertainty. Math Oper Res 1991;16(1):119–47.
[28] Nowak MP, Römisch W. Stochastic Lagrangian relaxation applied to power scheduling in a hydro-thermal system under uncertainty. Ann Oper Res 2000;100(1/4):251–72. https://doi.org/10.1023/A:1019248506301.
[29] Takriti S, Krasenbrink B, Wu LS-Y. Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem. Oper Res 2000;48(2):268–80. https://doi.org/10.1287/opre.48.2.268.12379.
[30] Gröwe-Kuska N, Kiwiel KC, Nowak MP, Römisch W, Wegner I. Power management in a hydro-thermal system under uncertainty by Lagrangian relaxation. Decision making under uncertainty, vol. 128. 2002. p. 39–70. https://doi.org/10.1007/978-1-4684-9256-9_3.
[31] Shiina T, Birge JR. Stochastic unit commitment problem. Int Trans Oper Res 2004;11(1):19–32. https://doi.org/10.1111/j.1475-3995.2004.00437.x.
[32] Ruiz P, Philbrick C, Zak E, Cheung K, Sauer P. Uncertainty management in the unit commitment problem. IEEE Trans Power Syst 2009;24(2):642–51. https://doi.org/10.1109/TPWRS.2008.2012180.
[33] Barth R, Brand H, Meibom P, Weber C. A stochastic unit-commitment model for the evaluation of the impacts of integration of large amounts of intermittent wind power. 2006 International conference on probabilistic methods applied to power systems IEEE; 2006. p. 1–8. https://doi.org/10.1109/PMAPS.2006.360195.
[34] Nowak MP, Schultz R, Westphalen M. A stochastic integer programming model for incorporating day-ahead trading of electricity into hydro-thermal unit commitment. Optim Eng 2005;6(2):163–76. https://doi.org/10.1007/s11081-005-6794-0.
[35] Galiana F, Bouffard F, Arroyo J, Restrepo J. Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves. Proc IEEE 2005;93(11):1970–83. https://doi.org/10.1109/JPROC.2005.857492.
[36] Bouffard F, Galiana F, Conejo A. Market-clearing with stochastic security— Part I: formulation. IEEE Trans Power Syst 2005;20(4):1818–26. https://doi.org/10.1109/TPWRS.2005.857016.
[37] Bouffard F, Galiana F, Conejo A. Market-clearing with stochastic security— Part II: case studies. IEEE Trans Power Syst 2005;20(4):1827–35. https://doi.org/10.1109/TPWRS.2005.857015.
[38] Wu L, Shahidehpour M, Li T. Stochastic security-constrained unit commitment. IEEE Trans Power Syst 2007;22(2):800–11. https://doi.org/10.1109/TPWRS.2007.894843.
[39] Asensio M, Contreras J. Stochastic unit commitment in isolated systems with renewable penetration under CVaR assessment. IEEE Trans Smart Grid 2016;7(3):1356–67. https://doi.org/10.1109/TSG.2015.2469134.
[40] Papavasiliou A, Oren SS. Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network. Oper Res 2013;61(3):578–92. https://doi.org/10.1287/opre.2013.1174.
[41] Nasri A, Kazempour SJ, Conejo AJ, Ghandhari M. Network-constrained AC unit commitment under uncertainty: a Benders's decomposition approach. IEEE Trans Power Syst 2016;31(1):412–22. https://doi.org/10.1109/TPWRS.2015.2409198.
[42] Wang J, Shahidehpour M, Li Zuyi. Security-constrained unit commitment with volatile wind power generation. IEEE Trans Power Syst 2008;23(3):1319–27. https://doi.org/10.1109/TPWRS.2008.926719.
[43] Wu Lei, Shahidehpour M, Fu Yong. Security-constrained generation and transmission outage scheduling with uncertainties. IEEE Trans Power Syst 2010;25(3):1674–85. https://doi.org/10.1109/TPWRS.2010.2040124.
[44] Wu L, Shahidehpour M, Li Z. Comparison of scenario-based and interval optimization approaches to stochastic SCUC. IEEE Trans Power Syst 2012;27(2):913–21. https://doi.org/10.1109/TPWRS.2011.2164947.
[45] Wu H, Shahidehpour M. Stochastic SCUC solution with variable wind energy using constrained ordinal optimization. IEEE Trans Sustain Energy 2014;5(2):379–88. https://doi.org/10.1109/TSTE.2013.2289853.
[46] Sahin C, Shahidehpour M, Erkmen I. Allocation of hourly reserve versus demand response for security-constrained scheduling of stochastic wind energy. IEEE Trans Sustain Energy 2013;4(1):219–28. https://doi.org/10.1109/TSTE.2012.2213849.
[47] Wu H, Ela E, Krad I, Florita A, Zhang J, Hodge B-M, et al. An assessment of the impact of stochastic day-ahead SCUC on economic and reliability metrics at multiple timescales. 2015 IEEE power & energy society general meeting, vol. 2015-Septe. IEEE; 2015. p. 1–5. https://doi.org/10.1109/PESGM.2015.7286542.
[48] Wu H, Shahidehpour M, Alabdulwahab A, Abusorrah A. Demand response exchange in the stochastic day-ahead scheduling with variable renewable generation. IEEE Trans Sustain Energy 2015;6(2):516–25. https://doi.org/10.1109/TSTE.2015.2390639.
[49] Papavasiliou A, Oren SS, Neill RPO. Reserve requirements for wind power integration: a stochastic programming framework. IEEE Trans Power Syst 2011;26(4):2197–206. https://doi.org/10.1109/TPWRS.2011.2121095.
[50] Valenzuela J, Mazumdar M. Commitment of electric power generators under stochastic market prices. Oper Res 2003;51(6):880–93. https://doi.org/10.1287/opre.51.6.880.24924.
[51] Philpott A, Schultz R. Unit commitment in electricity pool markets. Math Program 2006;108(2–3):313–37. https://doi.org/10.1007/s10107-006-0713-9.
[52] Cerisola S, Baillo a, Fernandez-Lopez JM, Ramos a, Gollmer R. Stochastic power generation unit commitment in electricity markets: a novel formulation and a comparison of solution methods. Oper Res 2009;57(January 2015):32–46. https://doi.org/10.1287/opre.1080.0593.
[53] Kim K, Zavala VM. Large-scale stochastic mixed-integer programming algorithms for power generation scheduling. In: Martín M, editor. Alternative energy sources and technologies Cham: Springer International Publishing; 2016. p. 493–512. https://doi.org/10.1007/978-3-319-28752-2_18.
[54] Fisher ML. An applications oriented guide to Lagrangian relaxation. Interfaces 1985;15(2):10–21. https://doi.org/10.1287/inte.15.2.10.
[55] Aravena I, Papavasiliou A. A distributed asynchronous algorithm for the two-stage stochastic unit commitment problem. 2015 IEEE power & energy society general meeting IEEE; 2015. p. 1–5. https://doi.org/10.1109/PESGM.2015.7285913.
[56] Kim K, Zavala VM. Algorithmic innovations and software for the dual decomposition method applied to stochastic mixed-integer programs. Optimization Online. https://doi.org/10.13140/RG.2.1.2062.3202.
[57] Ryan SM, Wets RJ-B, Woodruff DL, Silva-Monroy C, Watson J-P. Toward scalable, parallel progressive hedging for stochastic unit commitment. 2013 IEEE power & energy society general meeting IEEE; 2013. p. 1–5. https://doi.org/10.1109/PESMG.2013.6673013.
[58] Ordoudis C, Pinson P, Zugno M, Morales JM. Stochastic unit commitment via Progressive Hedging - extensive analysis of solution methods. 2015 IEEE Eindhoven PowerTech PowerTech; 2015. https://doi.org/10.1109/PTC.2015.7232629.
[59] Gade D, Hackebeil G, Ryan SM, Watson J-P, Wets RJ-B, Woodruff DL. Obtaining lower bounds from the progressive hedging algorithm for stochastic mixed-integer programs. Math Program 2016;157(1):47–67. https://doi.org/10.1007/s10107-016-1000-z.
[60] Xiong P, Jirutitijaroen P. Stochastic unit commitment using multi-cut decomposition algorithm with partial aggregation. 2011 IEEE power and energy society general meeting IEEE; 2011. p. 1–8. https://doi.org/10.1109/PES.2011.6039516.
[61] Wang J, Wang J, Liu C, Ruiz JP. Stochastic unit commitment with sub-hourly dispatch constraints. Appl Energy 2013;105:418–22. https://doi.org/10.1016/j.apenergy.2013.01.008.
[62] Zheng QP, Wang J, Pardalos PM, Guan Y. A decomposition approach to the two-stage stochastic unit commitment problem. Ann Oper Res 2013;210(1):387–410. https://doi.org/10.1007/s10479-012-1092-7.
[63] Guo G, Hackebeil G, Ryan SM, Watson J-P, Woodruff DL. Integration of progressive hedging and dual decomposition in stochastic integer programs. Oper Res Lett 2015;43(3):311–6. https://doi.org/10.1016/j.orl.2015.03.008.
[64] Lubin M, Petra CG, Anitescu M, Zavala V. Scalable stochastic optimization of complex energy systems. Proceedings of 2011 international conference for high performance computing, networking, storage and analysis on - SC '11 New York, USA: ACM Press, New York; 2011. p. 1. https://doi.org/10.1145/2063384.2063470.
[65] Papavasiliou A, Oren SS. Large-scale integration of deferrable demand and renewable energy sources. IEEE Trans Power Syst 2014;29(1):489–99. https://doi.org/10.1109/TPWRS.2013.2238644.
[66] Nedic A, Bertsekas DP. Incremental subgradient methods for nondifferentiable optimization. SIAM J Optim 2001;12(1):109–38. https://doi.org/10.1137/S1052623499362111.
[67] Lubin M, Martin K, Petra CG, Sandıkçı B. On parallelizing dual decomposition in stochastic integer programming. Oper Res Lett 2013;41(3):252–8. https://doi.org/

10.1016/j.orl.2013.02.003.

[68] Watson J-P, Woodruff DL. Progressive hedging innovations for a class of stochastic mixed-integer resource allocation problems. CMS 2011;8(4):355–70. https://doi.org/10.1007/s10287-010-0125-4.

[69] Papavasiliou A, Oren SS, Rountree B. Applying high performance computing to transmission-constrained stochastic unit commitment for renewable energy integration. IEEE Trans Power Syst 2015;30(3):1109–20. https://doi.org/10.1109/TPWRS.2014.2341354.

[70] Scuzziato MR, Finardi EC, Frangioni A. Comparing spatial and scenario decomposition for stochastic hydrothermal unit commitment problems. IEEE Trans Sustain Energy 2018;9(3):1307–17. https://doi.org/10.1109/TSTE.2017.2781908.

[71] Cheung K, Gade D, Silva-Monroy C, Ryan SM, Watson J-P, Wets RJ-B, et al. Toward scalable stochastic unit commitment. Energy Syst 2015;6(3):417–38. https://doi.org/10.1007/s12667-015-0148-6.

[72] Rachunok B, Woodruff DL. Stochastic unit commitment performance considering monte carlo wind power scenarios. 2018 IEEE international conference on probabilistic methods applied to power systems (PMAPS). 2018. p. 1–6.

[73] van Ackooij W, Malick J. Decomposition algorithm for large-scale two-stage unit-commitment. Ann Oper Res 2016;238(1–2):587–613. https://doi.org/10.1007/s10479-015-2029-8.

[74] Schulze T, Grothey A, Mckinnon KIM. A stabilised scenario decomposition algorithm applied to stochastic unit commitment problems; 2015. p. 1–26.

[75] Schulze T, McKinnon K. The value of stochastic programming in day-ahead and intra-day generation unit commitment. Energy 2016;101:592–605. https://doi.org/10.1016/j.energy.2016.01.090.

[76] Schulze T, Grothey A, McKinnon K. A stabilised scenario decomposition algorithm applied to stochastic unit commitment problems. Eur J Oper Res 2017;261(1):247–59. https://doi.org/10.1016/j.ejor.2017.02.005.

[77] Mehrtash M, Raoofat M, Mohammadi M, Zareipour H. Fast stochastic security-constrained unit commitment using point estimation method. Int Trans Electr Energy Syst 2016;26(3):671–88. https://doi.org/10.1002/etep.2107.

[78] Vatanpour M, Sadeghi Yazdankhah A. The impact of energy storage modeling in coordination with wind farm and thermal units on security and reliability in a stochastic unit commitment. Energy 2018;162:476–90. https://doi.org/10.1016/j.energy.2018.07.181.

[79] Vatanpour M, Sadeghi Yazdankhah A. Application of Benders decomposition in stochastic scheduling of thermal units with coordination of wind farm and energy storage system considering security constraint. J Energy Manage Technol 2018;2(1):11–9. https://doi.org/10.22109/JEMT-2018-108690.

[80] Lopez-Salgado CJ, Ano O, Ojeda-Esteybar DM. Stochastic unit commitment and optimal allocation of reserves: a hybrid decomposition approach. IEEE Trans Power Syst 2018;33(5):5542–52. https://doi.org/10.1109/TPWRS.2018.2817639.

[81] Dvorkin Y, Pandžić H, Ortega-vazquez MA, Kirschen DS. A hybrid stochastic/interval approach to transmission-constrained unit commitment. IEEE Trans Power Syst 2015;30(2):621–31. https://doi.org/10.1109/TPWRS.2014.2331279.

[82] Abbaspourtorbati F, Zima M. The swiss reserve market: stochastic programming in practice. IEEE Trans Power Syst 2016;31(2):1188–94. https://doi.org/10.1109/TPWRS.2015.2415467.

[83] Uckun C, Botterud A, Birge JR. An improved stochastic unit commitment formulation to accommodate wind uncertainty. IEEE Trans Power Syst 2016;31(4):2507–17. https://doi.org/10.1109/TPWRS.2015.2461014.

[84] Bakirtzis EA, Simoglou CK, Biskas PN, Bakirtzis AG. Storage management by rolling stochastic unit commitment for high renewable energy penetration. Electr Power Syst Res 2018;158:240–9. https://doi.org/10.1016/j.epsr.2017.12.025.

[85] Valinejad J, Oladi Z, Barforoushi T, Parvania M. Stochastic unit commitment in the presence of demand response program under uncertainties. Int J Eng Trans B 2017;30(8):1134–43. https://doi.org/10.5829/ije.2017.30.08b.04.

[86] Gomes IL, Laia R, Pousinho HM, Melicio R, Mendes VM. Aggregation platform for wind-PV-thermal technology in electricity market. SPEEDAM 2018 - proceedings: international symposium on power electronics, electrical drives, automation and motion 2018. p. 799–804. https://doi.org/10.1109/SPEEDAM.2018.8445348.

[87] Du E, Zhang N, Kang C, Xia Q. Scenario map based stochastic unit commitment. IEEE Trans Power Syst 2018;33(5):4694–705. https://doi.org/10.1109/TPWRS.2018.2799954.

[88] Wang B, Hobbs BF. Real-time markets for flexiramp: a stochastic unit commitment-based analysis. IEEE Trans Power Syst 2016;31(2):846–60. https://doi.org/10.1109/TPWRS.2015.2411268.

[89] Shi J, Oren SS. Flexible line ratings in stochastic unit commitment for power systems with large-scale renewable generation. Energy Syst 2018. https://doi.org/10.1007/s12667-018-0306-8.

[90] Shi J, Oren SS. Stochastic unit commitment with topology control recourse for power systems with large-scale renewable integration. IEEE Trans Power Syst 2018;33(3):3315–24. https://doi.org/10.1109/TPWRS.2017.2772168.

[91] Du E, Zhang N, Hodge BM, Wang Q, Lu Z, Kang C, et al. Operation of a high renewable penetrated power system with CSP plants: a look-ahead stochastic unit commitment model. IEEE Trans Power Syst 2018;PP(c):1. https://doi.org/10.1109/TPWRS.2018.2866486.

[92] Jiang R, Guan Y, Watson J-P. Cutting planes for the multistage stochastic unit commitment problem. Math Program 2016;157(1):121–51. https://doi.org/10.1007/s10107-015-0971-5.

[93] Analui B, Scaglione A. A dynamic multistage stochastic unit commitment formulation for intraday markets. IEEE Trans Power Syst 2018;33(4):3653–63. https://doi.org/10.1109/TPWRS.2017.2768384.

[94] Shahbazitabar M, Abdi H. A novel priority-based stochastic unit commitment considering renewable energy sources and parking lot cooperation. Energy 2018;161:308–24. https://doi.org/10.1016/j.energy.2018.07.025.

[95] Wang W, Li C, Liao X, Qin H. Study on unit commitment problem considering pumped storage and renewable energy via a novel binary artificial sheep algorithm. Appl Energy 2017;187:612–26. https://doi.org/10.1016/j.apenergy.2016.11.085.

[96] Jo KH, Kim MK. Stochastic unit commitment based on multi-scenario tree method considering uncertainty. Energies 2018;11(4). https://doi.org/10.3390/en11040740.

[97] Imani MH, Yousefpour K, Ghadi MJ, Andani MT. Simultaneous presence of wind farm and V2G in security constrained unit commitment problem considering uncertainty of wind generation. 2018 IEEE Texas power and energy conference, TPEC 2018 2018-Febru 2018. p. 1–6. https://doi.org/10.1109/TPEC.2018.8312082.