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Evaluation of the Strain Energy Density Value without the Construction of the Control Volume in the Preprocessing Phase of the Finite Element Analysis

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Abstract

This work investigates the error in the evaluation of the Strain Energy Density value when it is evaluated without the construction of the control volume in the pre-processing phase of the finite element analysis. Several numerical simulations were carried out to evaluate this value both according to the conventional procedure of the Strain Energy Density method, that requires the construction of the control volume in the pre-processing phase of the FEM analysis, and according to the procedure shown in this work that can be applied directly in the post-processing phase without requiring particular devices in the pre-processing phase of the numerical analysis. The main advantage of this new procedure is that, accepting an error that depends on the refinement of the mesh, it is possible to apply this method also to those numerical simulations already done for other purposes.
This allows also to decrease considerably the effort of the researcher and the calculation time.

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Keywords: Strain Energy Density method; Finite element Analysis; Control Volume

Nomenclature

E	Young's modulus
I_1, I_2	mode 1 and 2 functions in the SED expression for sharp V-notches. Available in literature
K_{IC}	material toughness

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R_0	radius of the control volume
SED	strain energy density
W_C	critical strain energy
Greek	
2α	opening angle of V-notch
γ	supplementary angle of $\alpha : \gamma = \pi - \alpha$
λ_1, λ_2	mode 1 and 2 Williams' eigenvalues for stress distribution at V-notches
ν	Poisson's ratio

1. Introduction

The SED method is an energetic local approach that has been validated as a method to investigate both fracture in static condition and fatigue failure by Lazzarin et al. (2001; 2002 and 2008).

According to this method, the brittle fracture occurs when the local SED W, evaluated in a given control volume, reaches a critical value $\overline{W} = W_C$ independent of the notch opening angle and of the loading type as demonstrated by Lazzarin et Al. (2001). The mean SED critical value is evaluable through the conventional ultimate tensile strength σ_t in the case of an ideally brittle material through the following expression:

$$W_C = \frac{\sigma_t^2}{2E} \quad (1)$$

The concepts stated above represent the basic idea of the SED method. For more considerations about the analytic frame of this method we remand to Berto et al. (2014).
As regards the control volume, in plane problems, both in mode I and mixed mode (I+II) loading, it becomes a circle or a circular sector with radius R_0 respectively in the case of cracks and pointed V-notches, as shown in Fig. 1.

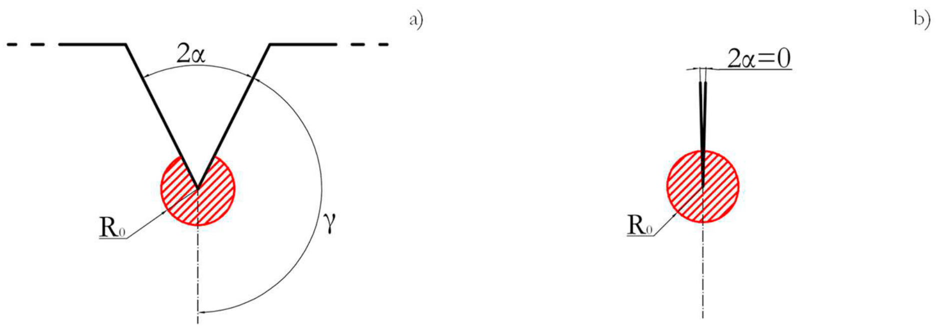

Figure 1: Control volume (area) for: a) sharp V-notch; b) crack.

In the case of the crack, the radius R_0 can be estimated both under plane strain and plane stress as reported by Lazzarin et al. (2005[1] and 2005[2]) and by Yosibach et al. (2004).

$$R_0 = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \qquad \textit{plane strain} \quad (2)$$

$$R_0 = \frac{(5-3\nu)}{4\pi}\left(\frac{K_C}{\sigma_t}\right)^2 \qquad \textit{plane stress} \tag{3}$$

While in the case of a pointed V-notch the critical radius can be assessed as shown by Lazzarin et al. (2001):

$$R_0 = \left[\frac{I_1 \cdot K_{1C}^2}{4\lambda_1(\pi-\alpha)EW_C}\right]^{\frac{1}{2(1-\lambda_1)}} = \left[\frac{I_1}{2\lambda_1(\pi-\alpha)}\left(\frac{K_{1C}}{\sigma_t}\right)^2\right]^{\frac{1}{2(1-\lambda_1)}} \tag{4}$$

One of the major limitations of this method, that restricts also its practical application, is the need to build, in the pre-processing phase of the FE analysis, the control volume.

The aim of this work is to demonstrate that an acceptable estimation of the SED value is possible also without the construction of the control volume in the model and that its low sensibility to mesh remains almost the same.

2. FE analysis

The aim of the present work is to prove that a good estimation of the SED value is possible without involving the construction of the control volume in the pre-processing phase of the FE analysis. In order to demonstrate what stated above, we carried out a series of FE analysis. The detail taken into account is a V-notched specimen, shown in Fig. 2 with its geometrical parameters. As regard the notch opening angle 2α , three different cases were considered: 90°; 120°; 135°. Four different models, shown in Fig. 3, are taken into account. As regards the mesh parameters, the control volume radius varies between 0,14 *mm* and 0,98 *mm* with step of 0,14 *mm* for a total of 7 different cases while the mesh refinement varies as ratio of the control volume radius between 1/6 and 1/20 for a total of 8 different cases.

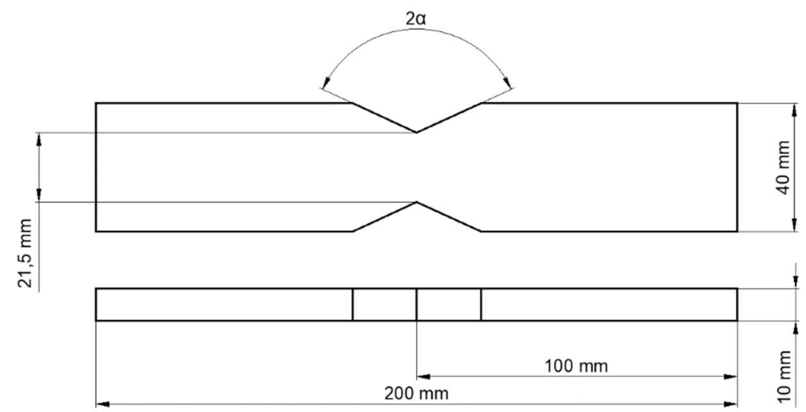

Figure 2: Geometry of the detail analysed.

The first model, Fig 3 a), is represented by a conventional FE model built to evaluate the SED value with the construction of the control volume. In the second model, Fig 3 b), the control volume is built in the pre-processing phase of the FE analysis, but the model has a free mesh. The third model, Fig 3 c), is built in order to have a mapped mesh without the construction of the control volume. For this model, a mapped mesh was considered to take into account those FE models built for any other purpose by designers or researchers in order to estimate the error in the evaluation of the SED value as a post-processing tool. The fourth model, Fig 3 d), has instead a completely free mesh with only a refinement in the notch tip in order to consider the error in the evaluation of the SED value considering the easiest way possible to build the model.

The SED value for the first two models, Fig 3 a) and b), is acquired following the conventional procedure exploited until now to apply the SED method as it is possible to see from Fig 4 a) and b). As regards the models shown in Fig 3 c) and d) the SED value is acquired through a selection of the elements close to the notch tips using a polar coordinate system centered in the notch tip with a radius equals to the control volume radius considered. The result of such a selection is shown both in Fig 4 c) for a mapped mesh and in Fig. 4 d) for a completely free mesh.

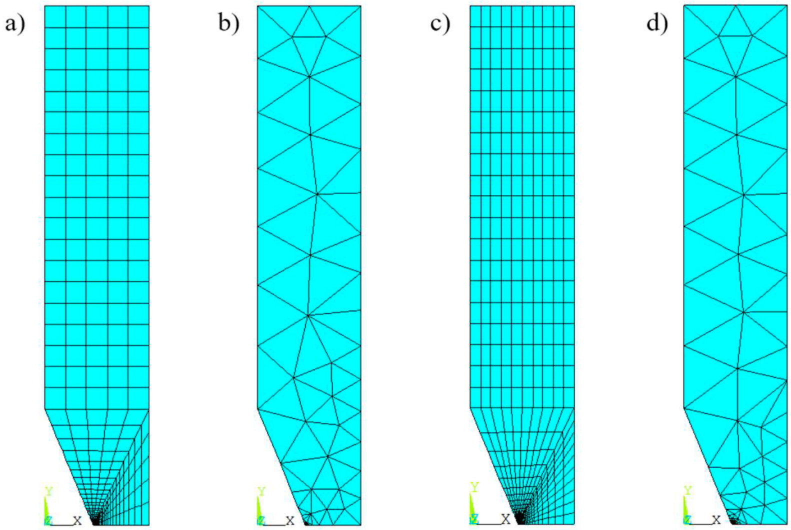

Figure 3:FE models for: a) mapped mesh with control volume; b) free mesh with control volume; c) mapped mesh without control volume; d) free mesh without control volume.

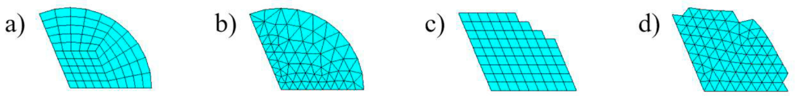

Figure 4: Control volume for: a) mapped mesh with control volume; b) free mesh with control volume; c) mapped mesh without control volume; d) free mesh without control volume.

3. Results

As stated above, the main aim of the present work is to estimate the error in evaluating the SED value without the construction of the control volume in the pre-processing phase of the FE analysis. The results reported are given in terms of the error in percentage with respect to a reference SED value.

For each notch opening angle and for each control volume radius considered, the reference case corresponds to the value acquired with the numerical analysis carried out with the most refined mesh with the model shown in Fig. 3 a) that corresponds to the conventional procedure utilised to estimate the SED value.

Considering the amount of data acquired with the numerical simulations, to avoid reporting the error in percentage for each simulation, we report in table 1 for each notch opening angle, for each control volume radius and for each model considered the minimum and the maximum error got, the mesh size in the control volume, expressed as ratio of the control volume radius, that leads to an error less than 1% and the error for this particular case.

Table 1: SED error in percentage

Model	Control Volume Radius [mm]	Angles 90° Err % min	Err % max	Mesh size	Err %	120° Err % min	Err % max	Mesh size	Err %	135° Err % min	Err % max	Mesh size	Err %
Fig. 6 a)	0.14	0,003	0,054	1/6	0,004	0,002	0,037	1/6	0,037	0,003	0,051	1/6	0,051
	0.28	0,002	0,048	1/6	0,011	0,003	0,039	1/6	0,039	0,004	0,038	1/6	0,038
	0.42	0,003	0,041	1/6	0,016	0,004	0,037	1/6	0,037	0,005	0,033	1/6	0,033
	0.56	0,004	0,033	1/6	0,012	0,005	0,041	1/6	0,041	0,007	0,026	1/6	0,027
	0.70	0,005	0,030	1/6	0,015	0,006	0,043	1/6	0,043	0,008	0,023	1/6	0,023
	0.84	0,006	0,024	1/6	0,006	0,007	0,042	1/6	0,042	0,009	0,026	1/6	0,026
	0.98	0,007	0,020	1/6	0,020	0,008	0,040	1/6	0,040	0,009	0,029	1/6	0,029
Fig. 6 b)	0.14	0,962	2,809	-	-	2,178	4,111	-	-	0,177	4,832	-	-
	0.28	0,338	0,809	1/6	0,338	1,692	2,838	-	-	0,844	2,605	-	-
	0.42	0,034	0,768	1/6	0,164	0,033	0,485	1/6	0,120	0,027	1,160	1/6	0,027
	0.56	0,008	0,119	1/6	0,037	0,010	0,807	1/6	0,393	0,066	1,186	1/6	0,252
	0.70	0,020	0,312	1/6	0,202	0,012	0,675	1/6	0,012	0,008	0,735	1/6	0,260
	0.84	0,006	0,238	1/6	0,006	0,056	0,945	1/6	0,141	0,009	0,853	1/6	0,052
	0.98	0,041	0,198	1/6	0,096	0,024	0,429	1/6	0,024	0,010	0,809	1/6	0,095
Fig. 6 c)	0.14	0,078	0,792	1/8	0,371	0,209	1,206	1/10	1,03	0,062	3,313	1/8	0,734
	0.28	0,072	1,030	1/8	0,430	0,201	1,215	1/10	1,04	0,063	3,306	1/8	0,730
	0.42	0,076	1,151	1/8	0,326	0,207	1,215	1/10	1,03	0,055	3,287	1/8	0,720
	0.56	0,210	1,048	1/8	0,481	0,207	1,226	1/10	1,03	0,060	3,293	1/8	0,729
	0.70	0,045	1,169	1/8	0,045	0,196	1,233	1/10	1,05	0,061	3,278	1/8	0,728
	0.84	0,202	0,803	1/6	0,315	0,205	1,234	1/10	1,03	0,060	3,274	1/8	0,715
	0.98	0,068	1,418	1/8	0,082	0,198	1,230	1/10	1,04	0,057	3,265	1/8	0,714
Fig. 6 d)	0.14	0,282	1,294	-	-	0,516	4,542	-	-	0,629	2,332	-	-
	0.28	0,044	0,533	1/6	0,533	0,061	0,602	1/6	0,198	0,013	0,789	1/6	0,646
	0.42	0,009	0,436	1/6	0,322	0,357	1,037	1/6	0,572	0,176	0,489	1/6	0,187
	0.56	0,086	0,670	1/6	0,670	0,160	0,807	1/6	0,750	0,033	0,384	1/6	0,113
	0.70	0,010	0,509	1/6	0,459	0,350	0,589	1/6	0,436	0,015	0,337	1/6	0,023
	0.84	0,006	0,500	1/6	0,500	0,049	0,818	1/6	0,360	0,069	0,293	1/6	0,103
	0.98	0,014	0,616	1/6	0,616	0,127	0,405	1/6	0,206	0,029	0,295	1/6	0,029

The data acquired show that a good estimation of the SED value is possible also without the construction of the control volume in the pre-processing phase of the FE analysis. An evaluation with an error less than 1% is possible with a mesh size of $1/8$ of the control volume radius. It is possible to state that the method has a low sensibility to the mesh refinement also with the procedure shown in this work.

These results show that the SED value can be evaluated also through a post-processing tool for every FE model that has in the critical zone a mesh size of at least 1/8 of the control volume radius.

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