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Strain energy density evaluation with free coarse mesh model

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Abstract
This work investigates the error in estimating the strain energy density value through a free coarse mesh model in a finite element analysis. Several numerical simulations were carried out to acquire this value both according to the conventional methodology of the strain energy density method and according to the procedure shown in the present work that can be applied directly in the post-processing phase of the simulation without requirements for the model. The main advantage of this new procedure is the possibility to apply the method also to those numerical simulations already done for other purposes decreasing considerably the effort of the researcher and the calculation time. The numerical simulations carried out show that the strain energy density value can be estimated with an error of 1% also with a free coarse mesh model having a mesh size near the notch tip of 1/8 of the control volume radius.

KEYWORDS
finite element simulation, strain energy density, control volume

1 | INTRODUCTION

The strain energy density (SED) method is an energy-based local approach that has been validated as a method to predict failure under both static and fatigue loading conditions.[1-4]

According to this method, the brittle fracture occurs when the local SED, W, evaluated in a given control volume, reaches a critical value $\overline{W} = W_C$ independent of the notch opening angle and of the loading type.[1] The SED critical value is evaluable in the case of an ideally brittle material, $W_C = \sigma_t^2/(2 \cdot E)$, through the conventional ultimate tensile strength σ_t and the Young modulus E of the material analysed.

As regards the control volume, in plane problems, both in mode I and mixed mode (I + II) loading, it becomes a circle or a circular sector with radius R_0 respectively in the case of cracks and pointed V-notches, as shown in Figure 1.

The value of the control volume radius, R_0, can be estimated for both cracks and sharp V-notches[1] under plane strain[5,6] and plane stress[7] using the material properties.

The method presents not only many advantages[4] but also some drawbacks such as the need of a finite element (FE) model build in order to have the control volume that limits also the practical application of this approach.

The aim of this work is to demonstrate that a good estimation of the SED value is achievable also without a specific FE model and that the low sensitivity to mesh of this method remains almost unchanged even if the control volume shape is roughly approximated in the FE analysis.

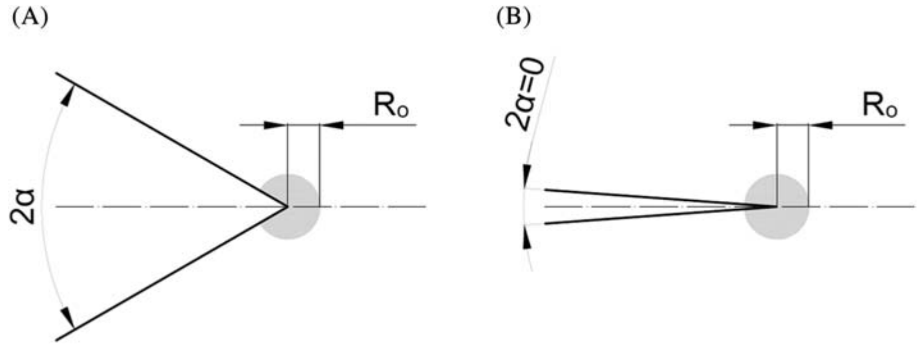

FIGURE 1 Control volume for (A) sharp V-notch and (B) crack

2 | FE ANALYSIS

In order to demonstrate that a good estimation of the SED value is possible without a specific FE model, we carried out a series of FE analysis on a simply V-notched geometry, reported in Figure 2, under mode I loading condition for the notch. The notch opening angles 2α considered are 90 ° ,120 ° ,135 ° . The control volume radius varies between 0.14 and 0.98 mm with step of 0.14 mm for a total of seven different cases, while the mesh refinement varies as ratio of the control volume radius between 1/6 and 1/20 for a total of eight different cases.

Four different models, shown in Figure 3, are taken into account. The first model, A, allows the application of the conventional methodology to evaluate the SED value having the model the control volume built in the pre-processing phase of the FE analysis. The second model, B, as the previous one allows a conventional application of the method, but a free mesh is used to discretize the component analysed. The third and the fourth models, C and D, are built without the construction of the control volume in the pre-processing phase of the FE analysis. They differ only for the type of mesh used to discretize the component; the third model has a mapped mesh, while the fourth one has a completely free mesh with only a refinement in the notch tip. These last two models allow the authors making some consideration about the possibility of applying the method directly as a post-processing tool.

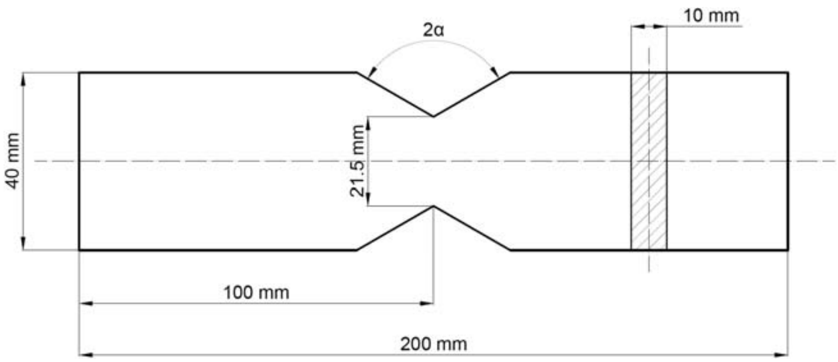

FIGURE 2 Geometry of the detail analysed

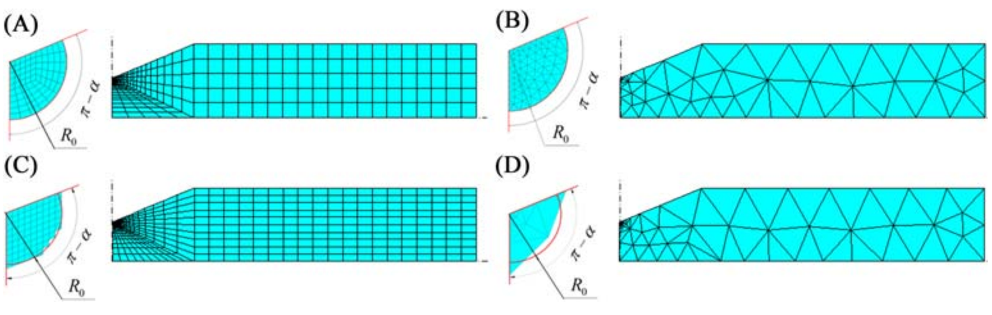

FIGURE 3 Control volume and finite element (FE) model for (A) mapped mesh with control volume, (B) free mesh with control volume, (C) mapped mesh without control volume, and (D) free mesh without control volume

TABLE 1 SED error percentage

Model	Control Volume Radius, mm	Angles								
		90°			120°			135°		
		Err % min	Err % max	Mesh[a] size	Err % min	Err % max	Mesh[a] size	Err % min	Err % max	Mesh[a] size
A	0.14	0.003	0.054	1/6	0.002	0.037	1/6	0.003	0.051	1/6
	0.28	0.002	0.048	1/6	0.003	0.039	1/6	0.004	0.038	1/6
	0.42	0.003	0.041	1/6	0.004	0.037	1/6	0.005	0.033	1/6
	0.56	0.004	0.033	1/6	0.005	0.041	1/6	0.007	0.026	1/6
	0.70	0.005	0.030	1/6	0.006	0.043	1/6	0.008	0.023	1/6
	0.84	0.006	0.024	1/6	0.007	0.042	1/6	0.009	0.026	1/6
	0.98	0.007	0.020	1/6	0.008	0.040	1/6	0.009	0.029	1/6
B	0.14	0.962	2.809	1/10	2.178	4.111	-	0.177	4.832	1/10
	0.28	0.338	0.809	1/6	1.692	2.838	-	0.844	2.605	1/10
	0.42	0.034	0.768	1/6	0.033	0.485	1/6	0.027	1.160	1/6
	0.56	0.008	0.119	1/6	0.010	0.807	1/6	0.066	1.186	1/6
	0.70	0.020	0.312	1/6	0.012	0.675	1/6	0.008	0.735	1/6
	0.84	0.006	0.238	1/6	0.056	0.945	1/6	0.009	0.853	1/6
	0.98	0.041	0.198	1/6	0.024	0.429	1/6	0.010	0.809	1/6
C	0.14	0.078	0.792	1/8	0.209	1.206	1/10	0.062	3.313	1/8
	0.28	0.072	1.030	1/8	0.201	1.215	1/10	0.063	3.306	1/8
	0.42	0.076	1.151	1/8	0.207	1.215	1/10	0.055	3.287	1/8
	0.56	0.210	1.048	1/8	0.207	1.226	1/10	0.060	3.293	1/8
	0.70	0.045	1.169	1/8	0.196	1.233	1/10	0.061	3.278	1/8
	0.84	0.202	0.803	1/6	0.205	1.234	1/10	0.060	3.274	1/8
	0.98	0.068	1.418	1/8	0.198	1.230	1/10	0.057	3.265	1/8
D	0.14	0.282	1.294		0.516	4.542	1/10	0.629	2.332	1/10
	0.28	0.044	0.533	1/6	0.061	0.602	1/6	0.013	0.789	1/6
	0.42	0.009	0.436	1/6	0.357	1.037	1/6	0.176	0.489	1/6
	0.56	0.086	0.670	1/6	0.160	0.807	1/6	0.033	0.384	1/6
	0.70	0.010	0.509	1/6	0.350	0.589	1/6	0.015	0.337	1/6
	0.84	0.006	0.500	1/6	0.049	0.818	1/6	0.069	0.293	1/6
	0.98	0.014	0.616	1/6	0.127	0.405	1/6	0.029	0.295	1/6

Abbreviation: SED, strain energy density.

[a]Mesh size in the control volume, expressed as ratio of the control volume radius, that leads to an error less than 1%.

For the models A and B, the SED value is acquired following the conventional methodology exploited until now to apply the SED method. As regards the models C and D, the SED value is evaluated using a polar coordinate system, centred in the notch tip, to select the elements close to the notch tip within a distance from the notch tip equal to the control volume radius. The result of such a selection is shown both in Figure 3C for a mapped mesh and in Figure 3D for a completely free mesh.

3 | RESULTS AND CONCLUSIONS

The results reported give the error percentage of the acquired values from all the numerical simulations carried out with respect to a reference SED value that, for each notch opening angle and for each control volume radius considered, corresponds to the value acquired with the numerical analysis carried out with the most refined mesh with model A that corresponds to the conventional methodology utilised to acquire the SED value. Considering the amount of acquired data, to avoid reporting the error percentage for each simulation, we report in Table 1 for each notch opening angle, for each control volume radius, and for each model considered the minimum and the maximum error got and the mesh size in the control volume, expressed as ratio of the control volume radius, which leads to an error less than 1% . Considering the acquired data, it is possible to state that a good estimation of the SED value is possible also without a specific FE model built for the application of this method. Besides an evaluation with an error less than 1% is possible with a mesh size of 1/8 of the control volume radius leaving the method low sensitivity to the mesh refinement almost unchanged also with the procedure shown in this work.

The methodology proposed, considering the results shown in the present work, allows a more practical application of the SED method that requires in this way less expertise from the user; besides, it makes the application of this method more suitable to be incorporated as a post processing tool in the commercial software for the assessment of structural integrity and fatigue life.

CONFLICT OF INTEREST

I, here, confirm that the submission of the present article is novel and done in absence of conflict of interest for all the authors.

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