## Optimal pricing of production changes in cascaded river systems with limited storage

Hans Ole Riddervold<sup>1</sup>, Hans Ivar Skjelbred<sup>2</sup>, Jiehong Kong<sup>2</sup>, Ole Løseth Elvetun<sup>3</sup>, Prof. Magnus Korpås<sup>4</sup>

<sup>1</sup> Norsk Hydro & NTNU, Trondheim, Norway
<sup>2</sup> SINTEF Energy Research, Trondheim, Norway
<sup>3</sup> Nextbridge Analytics AS, Oslo, Norway
<sup>4</sup> NTNU, Trondheim, Norway
lncs@springer.com

**Abstract.** Optimization of hydroelectric power production is often executed for river systems consisting of several powerplants and reservoirs located in the same region. For hydropower stations located along the same river, the release from upstream reservoirs ends up as inflows to downstream stations. Calculating marginal cost for a string of powerplants with limited reservoir capacity between them, requires a new approach compared to heuristically calculating marginal cost for single plants in well-regulated hydrological systems. A new method, using marginal cost curves for individual powerplants to generate an overall marginal cost curve for interlinked power stations has been developed. Results based on a real-world case study demonstrate the advantage of the proposed method in terms of solution quality, in addition to significant insight into how optimal load distribution should be executed in daily operations.

Keywords: Heuristic algorithms, hydroelectric power generation, cascaded river systems

### **1** INTRODUCTION

In the planning process for production of hydroelectric power, the optimal solution associated with predicted prices and inflows can be used to create bids for the day-ahead spot market and generate production schedules [1]. Deviation from the original production scheduling, typically created 12-36 hours prior to actual production hour, is more frequent with increasing activity in the intraday-market and more volatility imposed by intermittent power production.

For a power producer, it can be tempting to optimize all power stations and reservoirs located in the same price area in one common model. A motivation for this could be distribution of obligations in the spot and reserve markets, and/or for financial hedging purposes [2]. For practical purposes, and to reduce calculation time, optimization is often carried out on an aggregation level where hydraulically coupled reservoirs and power stations are modeled together. For river systems with large reservoir capacity between power stations, the interdependency between production in the individual plants could be more important for long-term maneuvering than for short-term bidding. However, for cascaded river systems with limited storage capacity between plants, hereafter referred to as linked river systems, this is often the opposite case since production in a downstream plant is a direct result of upstream production.

The existing method used for comparison in the case study investigated in this paper is based on the "Single Plant Model" [3]. For application of the Single Plant Model, Hveding's conjecture [4] states that "in the case of many independent hydropower plants with one limited reservoir each, assuming perfect maneuverability of reservoirs, but plant-specific inflows, the plants can be regarded as a single aggregate plant and the reservoirs can be regarded as a single aggregate reservoir when finding the social optimal solution for operating the hydropower system". This model uses basic principles for energy calculations considering, head-loss, generator- and turbine efficiency to generate a combined production/waterflow relationship for the single plant, and thereby associate marginal cost to different levels of operation.

Other methods for scheduling and/or coordinated control in cascaded river systems have been described [5]-[7]. These approaches apply different optimization techniques, and are often tailored for individual river systems. They are not necessarily primarily designed for bidding in the spot-, balancing- and intraday market, but could represent an alternative approach to the method described in this paper. An important criterion in relation to finding an applicable method to be used in the bidding process, is the time used to generate bids.

A method for heuristically calculating the marginal cost for all the operating points of a power plant, covering the entire working area for the plant and including all the physical limitations and reserve obligations in other markets has been presented by SINTEF [8]. For a hydrological system with significant storage, the method has demonstrated to be computationally efficient.

In this paper, we present a short-term scheduling method for heuristically calculating the marginal cost in linked river systems where storage capacity between plants is limited.

The method has further been investigated on a large Norwegian river system consisting of five linked powerplants with varying degree of interim storage capacity. The results from the calculation have been used to generate dynamic bids for rapid response to opportunities in the intraday and balancing market as market prices, inflows, and other physical parameters in the river system change.

## **2 PROBLEM DESCRIPTION**

When heuristically calculating the marginal cost for plant production, the optimum production for one plant can be associated with a different waterflow than for another plant in the river system. Results from methods developed for hydrological systems with significant storage capacity between plants can therefore not be directly applied.

The main challenge associated with computing marginal cost for linked power plants to be used for bidding in the spot, balancing and/or intraday market, is that the water-flow for the power stations must be in balance at all time-steps.

# 2.1 Existing method used for calculating marginal cost for linked power plants

Water values as marginal cost for hydropower generation is a widespread means of assigning monetary values to the available water resources. The water value can be defined as the future expected value of the stored marginal kWh of water, i.e. its alternative cost [9], [10]. The water value for a power station is typically given by a seasonal model, and referred to the optimal point of operation ( $Q^*$ ) for the power plant.

$$Q^* = \underset{Q}{\operatorname{arg}\max} \ \frac{P(Q)}{Q} \tag{1}$$

From basic economic theory, the marginal cost for one operating point is the change in the opportunity cost of water (C) involved as a result of an infinitesimally small increase in the discharge of the units (P), which is expressed as:

$$mc = \frac{\partial C}{\partial P} \tag{2}$$

Combining the water value as reference for marginal cost at optimal point of operation with (1), the piecewise linear marginal cost ( $\epsilon$ /MWh) for changing production from production i to j (MW) is given by (3)

$$mc_{ij} = \frac{\Delta Q_{ij}}{\Delta P_{ij}} * \alpha * WV^*$$
<sup>(3)</sup>

Where WV\* is the water value ( $\notin$ /MWh) at optimal point of operation and  $\alpha$  is given by  $\partial P$  divided by  $\partial Q$  at optimum (Q\*). For discrete mc calculations,  $\alpha$  is fixed to a value such that mc is equal to WV\* at the point of operation where the highest production relative to the water consumption is defined. This method is used when the existing method calculates marginal cost for discrete change between predefined levels of production.

Assuming that a plant can operate independently, and a water value referred optimal production of  $30 \notin MWh$ , table 1 illustrates how marginal cost can be calculated using (3). Plant 1 would in this case produce 100 MW (35 m3/s) at a market price of 30  $\notin MWh$ 

Plant 1 P [MW]	Plant 1 Q [m3/s]	P/Q	$\frac{\Delta Q_{ij}}{\Delta P_{ij}}$	mc <sub>ij</sub>
70	25	2.80		
100	35 (Q*)	2.86	0.33	30.0
140	50	2.80	0.38	33.8
200	75	2.67	0.42	37.5

Table I Marginal cost for "independent" power station using existing method

In this example, we are calculating the marginal cost of increasing production from level i to j. The marginal cost for the initial P/Q level is therefore omitted.

If we introduce a second plant in the example with optimal production at another waterflow, and these plants are linked with limited or no intermediate storage capacity, a common waterflow must be chosen where one or both power stations must deviate from the plants optimal point of operation ( $Q^*$ ) to avoid flooding past one of the plants. The existing approach is to aggregate the production (MW) for the plants at the same waterflow (m3/s) to create a common production/waterflow relationship for the two plants.

Even though some of these river systems originally where designed to have common optimal waterflow, gradual plant upgrades and market developments affecting production patterns, might lead to the need of making tradeoffs between optimum production in the different plants.

#### 2.2 Proposed method

The existing method described in Section 2.1 defines a common production-waterflow curve for a linked river system. The main weakness of generating a curve based on this method concerns the dynamics that are associated with modeling of several power stations in a linked river system. One plant could consist of several generators where some are shut down for maintenance. There could also be temporary load restrictions, concessional requirements, or local inflow effecting operations. This would require a continuous update of the combined production-waterflow curve. It would also require maintenance of a model which is not representing the physical power system. Finally, when distributing load requirements, a separate model or optimization must be run to allocate production to the correct generators.

An improved method is described in the two following sections. The first section gives a general description of the best profit method, while the second describes how the method can be used for linked river systems.

#### 2.2.1 Heuristics, Best Profit

For completeness, we include a description of the way marginal cost curves are created by the best profit functionality in the Short-term Hydro Optimization Program (SHOP). SHOP is a software tool for optimal short-term hydropower scheduling developed by SINTEF Energy Research [1]. Interested readers can find more details about the best profit functionality in [8]. We assume that the water value and gross head for each plant is given. In real-world operation, the head loss in the main tunnel and the penstock that unit i connects to should not be neglected. It can be represented as a quadratic equation of the total flow going through the main tunnel/penstock. The net head, and therefore, is calculated as:  $NH_{ist} = GH_{st}$ 

$$-\alpha_{main} \cdot \left(\sum_{i \in I_{s,main}} q_{ist}\right)^2 - \alpha_{pen} \cdot \left(\sum_{i \in I_{s,pen}} q_{ist}\right)^2 \qquad (4)$$
$$i \in I_s, s \in S, t \in T$$

where:

 $I_s$  Set of units in plant s.

 $I_{s,main}$  Set of units that connect to main tunnel in plant s.

I<sub>s,pen</sub> Set of units that connect to penstock *pen* in plant *s*.

 $NH_{ist}$  Net head of unit *i* in plant *s* at period *t* (m).

 $\alpha_{main}$  Loss factor for main tunnel.

 $\alpha_{pen}$  Loss factor for penstock *pen*.

 $q_{ist}$  Flow going through unit *i* in plant *s* in period *t* (m<sup>3</sup>/s).

For a generating unit i in plant s, the power production, in (5), depends on the net head and the flow going through that unit. It also relies on the generator efficiency and head-dependent turbine efficiency.

 $mw_{ist} = 0.001$ 

$$\begin{aligned} &\cdot \eta_i^{GEN}(mw_{ist}) \cdot \eta_i^{TURB}(q_{ist}, NH_{ist}) \cdot G \cdot NH_{ist} \cdot q_{ist} \\ &\quad i \in I_s, s \in S, t \in T. \end{aligned}$$

where:

 $mw_{ist}$  Power produced by unit *i* in plant *s* in period *t* (MW).

- $\eta_i^{GEN}$  Generator efficiency of unit i, which is interpolated on the basis of production  $mw_{ist}$ .
- $\eta_i^{TURB}$  Turbine efficiency of unit *i*, which is interpolated on the basis of flow  $q_{ist}$  and net head  $NH_{ist}$ .

*G* Gravity value, default setting is  $9.81 \text{ (m/s}^2$ ).

Based on (4) and (5), if the discharge for each unit is given (i.e. one possible operating point for the plant), we can precisely calculate the corresponding production, taking the head loss into consideration. This transformation from the flow discharge to the power generation is implicitly done by the functionality in SHOP.

For a given operating point p in one specific unit combination c, the generation cost for this point is the opportunity cost of the water used. In the previous section, we have presented how the hourly water cost is defined, and how the production can be accurately obtained when the discharge of the units is decided, in (5). Therefore, we denote the average cost for this operating point by

$$ac_{cst}^{p} = \frac{3600 \cdot WC_{st} \cdot \sum_{i \in I_{c}} q_{ist}^{p}}{\sum_{i \in I_{c}} mw_{ist}^{p}}$$

$$p \in P_{c}, c \in C, s \in S, t \in T.$$
(6)

where:

- *C* Set of unit combinations.
- $I_c$  Set of units in unit combination c.
- $P_c$  Set of operating points in unit combination c.

 $ac_{cst}^p$  Average cost for the operating point p in unit combination c in plant s in period t ( $\in$ /MWh).

In economics, marginal cost is the change in the opportunity cost that arises when the quantity produced has an increment by one unit. In contrast to the transformation from discharge to production, it is much more complicated to find the discharge by a given production. In addition, the power produced is infinitely divisible. Therefore, we find the marginal cost by increasing the discharge by a small amount, expressed as

$$mc_{cst}^{p}$$

$$=\frac{3600 \cdot WC_{st} \cdot \sum_{i \in I_c} (q_{ist}^p + \Delta q_{ist}^p) - 3600 \cdot WC_{st} \cdot \sum_{i \in I_c} q_{ist}^p}{\sum_{i \in I_c} (\widetilde{m}\widetilde{w}_{ist}^p) - \sum_{i \in I_c} mw_{ist}^p}$$
(7)  
$$p \in P_c, c \in C, s \in S, t \in T.$$

where:

- $mc_{cst}^{p}$  Marginal cost for the operating point p in unit combination c in plant s in period t ( $\epsilon$ /MWh).
- $\Delta q_{ist}^{p} \quad \text{A small increment in the discharge of unit } i \text{ in unit combination } c \text{ in plant } s \text{ in } period \ t, \ \Delta q_{ist}^{p} = 0.001 \cdot \frac{q_{ist}^{p}}{\sum_{i \in I_{c}} q_{ist}^{p}}, \ (\text{m}^{3}/\text{s}).$
- $\widetilde{mw}_{ist}^{p}$  Power produced by unit *i* in plant *s* in period *t* when there is a small increment in the discharge of the units (MW).

After calculating the marginal cost for a large number of combinations of flows in the running units, we can find the optimal production distribution and the corresponding marginal cost curve. For each production level, the optimal distribution is the one resulting in the lowest discharge. This ensures that the most efficient units will always be used first.

Best profit curves generally contain information about the marginal cost of each production level, the optimal production distribution between the running units and at what price it is optimal to switch between unit combinations. Example of a Best profit curve can be found in figure 2. In this paper, it is assumed that the combination of running units at each plant is given. This means that the best profit curve only has to contain the information about marginal costs and optimal production distribution, as described above.

#### 2.2.2 Best Profit customized for linked river systems

To be able to apply the best profit method for linked river systems, output from the model must include a marginal cost [€/MWh] / waterflow [m3/s] relationship in addition to the marginal cost [€/MWh] / production [MW] relationship that is already produced by the existing models. Several requirements must be met for the proposed method to be applicable.

6

To keep focus on the main principles of the best profit method compared to the existing method, a requirement in this analysis is set that all generators in the river system must run.

A water-value is normally estimated for each power station in the river system, and the value is defined as the marginal cost of an incremental increase of production from the optimal point of operation for the specific power station. This leaves us with a challenge related to defining which water-value should be used for the aggregated power station. In the best profit calculations presented in this paper, one common watervalue is used for all plants, and is estimated as a weighted average of the water-values for the power stations at the combined plants optimal production.

If we assume there is no intermediate natural inflows between stations, calculating the aggregated marginal cost value for the linked river system requires that plant specific marginal costs are selected for identical waterflows for each power station. For power station 1 to 5,  $q1 = \cdots = q5$ . The method can be extended to handle inflows between stations.

To accommodate for the difference in output effect for the power stations, the marginal cost for each plant must be weighted according to the relative production of the plant at the selected waterflow to generate the overall marginal cost at a selected point of operation.

Each bidding point will have a unique waterflow and accumulated production associated with it. We can therefore select the price that should be used for bidding of production (MW) to the spot, intraday or balancing market. For the intraday and balancing market, the bidding price would typically be the price associated with deviating from the current point of operation.

## 3 Case study

The river system investigated in this paper consists of five plants with very limited intermediate storage. The main reservoir which is located upstream all power plants has a degree of regulation of approx. 1.5, meaning that yearly inflow is 1.5 times reservoir capacity. To produce the yearly inflow, the power stations must operate at full capacity approximately 70% of the available hours in a year. The time used from when water is released from the main reservoir until it reaches the lower reservoir is short and is disregarded in this analysis. The lower reservoir has sufficient storage capacity, and the linked river system can be assumed to operate independently of the water content in this reservoir. To reflect the alternative value of production, a water value of 31  $\notin$ /MWh from the main upstream reservoir is used as basis.

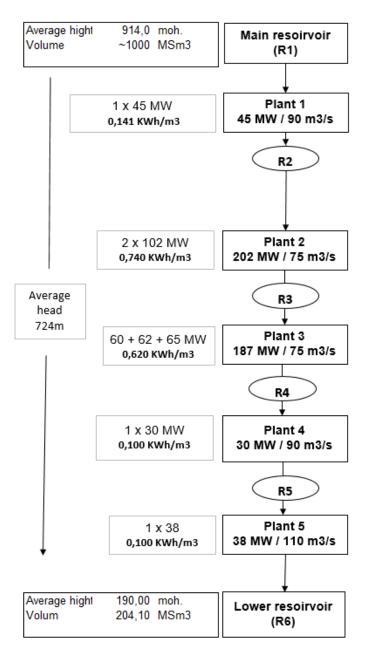


Fig. 1. Schematic overview of the investigated river system

The best profit values have been selected for production in an hour where market price is equal to the water-value (31  $\notin$  / MWh). At this price, all power stations are in operation. To ensure that all generators are running, q\_min = 60 m3/s and q\_max = 75 m3/s in further marginal cost calculations.

8

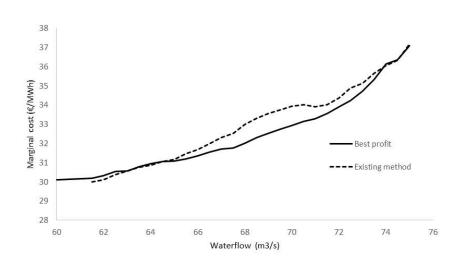


Fig. 2. Aggregated marginal cost curves for the linked river system

Fig. 2 illustrates how the best profit curve compares to the existing method described in Section 2.1. The graph shows that the result coincides well for production in the lower and upper range of the waterflow area. However, there is a deviation in production in the mid-range of water flows.

The observed deviation reveals a true benefit of using a dynamic best profit curves. Plant 3 is one of the larger power stations in the river system consist of 3 generators. This plant has been through several upgrades during the last years. These generators have different characteristics, and how these generators are uploaded will have significant impact on the plants total efficiency. In the existing method, these generators are uploaded in steps defined by a relatively simple algorithm, whilst the best profit utilize the complimentary characteristics of the generator to ensure optimal distribution of load for all represented waterflows. This results in a relatively low loss of efficiency for the plant for waterflows in the range from 60-67 m3/s compared to the other plants.

## 4 Conclusion

It has been demonstrated that the best profit method can be used to generate real-time marginal cost curves for linked river systems. These results can readily be used for bidding to the spot-, balancing and/or intraday market. Further, results from the best profit give significant insight into how optimal load distribution for linked river system should be executed in daily operations. Often, real-time regulation of complex linked river systems is carried out by SCADA-systems with limited user interaction from production planners handling the commercial process. Having a quick and robust method like best profit available, the traditional and often static approach to operation of linked

river systems can continuously be challenged. This will create additional values for the power producers, and ensure that pricing toward a gradually more complex market is as correct as possible.

In this paper, we calculate the marginal cost for a linked river system where all generators are in operation. This is not a limitation that applies for the method in general. A relatively trivial expansion is to investigate the best profit value when one or more generators are out for maintenance. This can be done by investigating the best profit values for an area of operation where the available generators are running. The existing existing-method however, has considerable challenges in handling these situations. For further analysis, it will also be of interest to investigate production behavior in ranges where different generators in the linked river system will be turned on and off. Defining more correctly the link between water values and the use in the best profit method, particularly to incorporate the coupling with cuts [11], will also be an issue for further improvement

#### References

- O. B. Fosso, M. M. Belsnes, "Short-term hydro scheduling in a liberalized power system,", in Proc. 2004 International Conference on Power System Technology - POWERCON, Vols 1 and 2, pp. 1321-1326.
- 2. Mo B, Gjelsvik A and Grundt A, Integrated risk management of hydropower scheduling and contract management. In IEEE Transactions on Power Systems 2001;16:216-221.
- 3. Hveding, V. (1968). Digital simulation techniques in power system planning. Economics of Planning, 8(1), 118-139.
- Forsund, F. (2015). Hydropower Economics (2nd ed., Vol. 112, International Series in Operations Research & Management Science). Boston, MA: Springer US., pp. 71-76
- Hamann, A., Hug, G., & Rosinski, S. (2017). Real-Time Optimization of the Mid-Columbia Hydropower System. Power Systems, IEEE Transactions on, 32(1), 157-165.
- O. Nilsson and D. Sjelvgren, "Mixed-integer programming applied to Short term planning of a hydro-thermal system," IEEE Trans. Power Syst., vol. 11, no. 1, pp. 281-286, Feb. 1996.
- E. K. Aasgard, G. S. Andersen, S. E. Fleten et al., "Evaluating a stochastic-programmingbased bidding model for a multireservoir system," IEEE Transactions on Power Systems, vol. 29, no. 4, pp. 1748-1757, Jul, 2014.
- Hans Ivar Skjelbred, Jiehong Kong, Tellef Juell Larsen, Fredd Kristiansen, "Operational use of marginal cost curves for hydropower plants as decision support in real-time balancing markets", European Energy Market (EEM) 2017 14th International Conference on the, pp. 1-6, 2017, ISSN 2165-4093.
- Wolfgang O, Haugstad A, Mo B, Gjelsvik A, Wangensteen I, Doorman G. Hydro reservoir handling in Norway before and after deregulation. Energy 34 (10); 2009. pp. 1642-1651
- 10. W Brovold, Skar, & Fosso. (2014). Implementing Hydropower Scheduling in a European Expansion Planning Model. Energy Procedia, 58, 117-122.
- Gjelsvik A., Mo B., Haugstad A.,"Long- and Medium-term Operations Planning and Stochastic Modelling in Hydro-dominated Power Systems Based on Stochastic Dual Dynamic Programming", Handbook of Power Systems I, pp. 33-56, Springer Berlin Heidelberg, 2010.