A fault tolerant control scheme using the feasible constrained control allocation strategy

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Abstract: This paper investigates the necessity of feasibility consideration in a fault tolerant control system using the constrained control allocation methodology where both static and dynamic actuator constraints are considered. In the proposed feasible control allocation scheme, the constrained model predictive control is employed as the main controller. This considers the admissible region of the control allocation problem as its constraints. Using the feasibility notion in the control allocation problem, provides the main controller with information regarding the actuators status which leads to the closed loop system performance improvement. Several simulation examples under normal and faulty conditions are employed to illustrate the effectiveness of the proposed methodology. The main results clearly indicate that closed loop performance and stability characteristics can be significantly degraded by neglecting the actuator constraints in the main controller. Also, it is shown that the proposed strategy substantially enlarges the domain of attraction of the MPC combined with the control allocation as compared to the conventional MPC.

Keywords: Control allocation, feasibility, fault tolerant control, model predictive control, domain of attraction

1 Introduction

IN recent years, the growing demand of safety, reliability, maintainability and survivability in critical safety systems such as aircrafts [1], automotive, ship and underwater systems [2] has motivated research in the field of fault tolerant control systems. Fault tolerant control systems can accommodate various faults such that closed loop system stability and acceptable performance are ensured [3, 4, 5].

Actuator redundancy is a widely adopted technical solution to achieve fault tolerant control systems. The systems designed with redundant actuators are called overactuated systems, where the number of actuators is greater than the number of degrees of freedom. Due to input redundancy, several configurations can lead to the same generalized force which is called the virtual control signal. It is desired to select the best solution based on the actuators' conditions. Control allocation is an approach to manage the redundant actuators such that the desired effort is produced and actuators' constraints are satisfied [6, 7]. Constrained control is an important challenge in the controller design procedure and control allocation can be specifically employed to deal with input constraints [8]. In the past two decades, a wide range of the control allocation approaches are presented such as the pseudo inverse approach[9], the redistributed pseudo inverse approach[10], the daisy chain approach[11], the direct allocation approach [12], the linear programming approach [13], the quadratic programming approach [1], and the multi parametric programming approach[14]. A comprehensive survey on the control allocation methods is presented in [9].

On the other hand, a noticeable number of the fault tolerant control strategies are introduced to handle actuator faults. Such techniques use the two step method which includes: 1) Estimation of the actuators' limits, and 2) Implementation of the control allocation algorithm with the estimated limits [6]. Different control allocation methods can be used for the second step. Most of the proposed schemes follow a modular control configuration program which divides the control system structure into the following two parts: The main controller that provides the specified desired virtual control and the control allocator unit that maps the total control demand onto individual actuator settings [9].

Most works in the control allocation field have assumed that actuators are fast enough to neglect their dynamics. In [7], unknown input observers are employed to detect and isolate actuator and effector faults. Then, the failed actuator and its corresponding column of the control effectiveness matrix are ignored and commands to the healthy actuators are determined by solving a reduced pseudo inverse problem. Smart actuators update the constraints in the control allocation unit. Mixed integer linear programming is used to solve the control allocation problem in [15] that proposes an adaptive control scheme as the main controller. In [16], a fault tolerant control system is designed for the underwater vehicles where the sliding mode control and linear programming techniques are employed as the main controller and the control allocator unit, respectively. A fault tolerant control strategy is presented in [17] which converts a control allocation problem into an optimization problem solved with quadratic programming. Also, [18] compares a robust control scheme with the control allocation method which is posed as a quadratic programming problem solved by a fixed point iteration algorithm. The pseudo inverse along the null space method is employed in [19] to achieve a fault tolerant control system. In this method, a vector of the null space of the control effectiveness matrix is added to the pseudo inverse solution if it does not satisfy the constraints. The corrective term is calculated such that elements of the

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control signal which violate the constraints are forced back to the admissible region. The main drawback of the methods following the modular configuration is that generation of the desired virtual control produced by the main controller is not guaranteed.

Some integrated methods are proposed which concurrently analyse the main controller and the control allocator. Fault tolerant control schemes using on-line control allocation based on sliding mode control concepts is presented in [20]. This develops a rigorous design procedure from a theoretical view and prove closed loop stability in the presence of some bounded uncertainties. A fault tolerant scheme which employs the adaptive control allocation is presented in [21]. A Lyapunov function based on the difference between outputs of the system and a predefined reference model is considered to update the control allocation matrix. Also, a sliding mode controller is employed as the main controller to compensate for the control allocation error. However, the integrated methods do not explicitly consider actuator constraints in their design, hence it is possible that the demand control signal is not in the admissible constraint limits.

In order to consider the actuators dynamics, several dynamic control allocation methods are presented. [22] proposes frequency apportioned control allocation method which distributes high frequency commands to the actuators with higher rate limits and low frequency commands to the highly effective controls. The desired virtual control is distributed among the actuators through either position-weighted or rate-weighted pseudo inverses depending on the command speed. The dynamic control allocation problem is posed into a sequential quadratic programming optimization problem in several works e.g. [23] which considers position and rate limits for the actuators. The first priority of the optimization problem is to produce the desired virtual control by the available actuators. The next step is to minimize the mismatch between the actual and the desired virtual control and the control signal deflection. There are some weighting matrices to tune the relative importance. The actuator dynamics are modeled by state space equations in [24, 25]. The actuators command and the generated virtual control are considered as the inputs and outputs of the model, respectively. A model predictive control (MPC) is employed to track the desired virtual control calculated by the main controller. The MPC problem is posed to a quadratic programming optimization. Also, the dynamic control allocation optimization problem is solved via an LMI formulation in [26].

It is obvious that actuators constraints consideration are critical in any practical control system implementations. As MPC can directly handle actuator limits, it is therefore a popular choice for any real control design application.

However, the control system containing a control allocation unit which uses MPC as the main controller is a nonlinear closed loop system with the control constraints. The concept of the domain of attraction is a critical issue in the nonlinear systems which determines the stability regions of the system. Finding and estimating the domain of attraction is an interesting and active research area [27, 28]. The MPC domain of attraction can be estimated by numerical methods. Enlarging the domain of attraction leads to the closed loop stability improvement. In the literature, limited papers are presented to enlarge the domain of attraction of MPC and these are designed for specific MPC methodologies. [29] employs a saturated local control law to expand the terminal set that results in the domain of attraction enlargement. [30] considers a prediction horizon larger than the control horizon in order to enlarge the domain of attraction. Also, [31] proposes a modification in the terminal cost and terminal constraints. The enlargement of the domain of attraction is achieved by implementing a dual MPC in [32].

In this paper, feasible control allocation is proposed as a solution for the control allocation problem. In the proposed method, the feasible region of the control allocation unit is considered as the control constraints of an MPC controller that plays the role of the main controller. Although the feasible region is invariant when the actuators dynamics are neglected, it is time varying when actuators dynamics are taken into account. Therefore, the control constraints of the MPC will be time varying. The feasibility consideration guarantees that the desired virtual control produced by the main controller will be distributed among the actuators such that their limits are satisfied. The simulation results confirm that the feasibility consideration in the main controller improves the control performance and maintains the stability under severe conditions. Also, the proposed scheme leads to enlarging the domain of attraction of the main controller compared to the conventional MPC. It is assumed that actuators fault information is received from smart actuators or a fault detection unit. Thus, fault detection and diagnosis is not considered in this paper.

In summary, the paper proposes a fault tolerant control scheme for the linear time invariant systems that guarantees the determination of the actuators' commands such that the input constraints are satisfied and the desired control effort is produced.

The paper is structured as follows. The problem statement is given in section 2. In section 3, methodology study is presented for static and dynamic cases. In section 4, several simulation results are illustrated to show the effectiveness of the presented method. Concluding remarks are given in section 5.

2 Problem statement

Consider a linear system described by the following discrete time state-space equations:

$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_u\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{1}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$, and $\mathbf{y} \in \Re^k$ are the state variables, inputs and outputs of the system, respectively. Also, $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{B}_u \in \Re^{n\times m}$, and $\mathbf{C} \in \Re^{k\times n}$ are the state, input and output matrices. It is assumed that the system has input redundancy and therefore the input matrix is rank deficient:

$$\operatorname{rank}(\mathbf{B}_u) = d < m \tag{2}$$

The virtual control signal is the total effect of the inputs and it is introduced as follows:

$$\mathbf{v}(t) = \mathbf{B}\mathbf{u}(t) \tag{3}$$

where $\mathbf{B} \in \Re^{d \times m}$ is called the control effectiveness matrix which is derived from the system structure and is related to the effectors type, size and location. Combining (1) and (3) yields the following state space representation:

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_v \mathbf{v}(t) \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{4}$$

and $\mathbf{B}_v$ satisties

$$\mathbf{B}_u = \mathbf{B}_v \mathbf{B} \tag{5}$$

The admissible space can be defined as follows:

$$\mathbf{u}(t) \in \Omega(t) \equiv \{\mathbf{u}_i(t) \mid \underline{\mathbf{u}}_i(t) \le \mathbf{u}_i(t) \le \bar{\mathbf{u}}_i(t); 1 \le i \le m\} \tag{6}$$

where the bounds $\underline{\mathbf{u}}(t)$ and $\bar{\mathbf{u}}(t)$ are the lower and upper limits and depend on the actuator health and status.

In the case of slow actuators, the rate limits are considered to model their dynamics as follow:

$$|\dot{\mathbf{u}}_i| \le \bar{r}_i;\ \forall 1 \le i \le m \tag{7}$$

where $\bar{r}_i$ is the rate limit of the ith actuator. It is possible to merge the constraints (6) and (7) into overall time varying input constraints as follows [23]:

$$\underline{\mathbf{u}}'_i(t) \le \mathbf{u}_i(t) \le \bar{\mathbf{u}}'_i(t); \forall 1 \le i \le m \tag{8}$$

where

$$\underline{\mathbf{u}}'_i(t) = \max\left[\underline{\mathbf{u}}_i(t), \mathbf{u}_i(t-1) - \bar{r}_i T\right] \\ \bar{\mathbf{u}}'_i(t) = \min\left[\bar{\mathbf{u}}_i(t), \mathbf{u}_i(t-1) + \bar{r}_i T\right] \tag{9}$$

where T is the sampling time, $\underline{\mathbf{u}}'_i(t)$ and $\bar{\mathbf{u}}'_i(t)$ are the actuators instantaneous lower and upper limits.

3 The feasible control allocation methodology (FCA)

The main idea of the proposed method is to consider the feasibility of the control allocation in the main controller. Feasibility notion will be subsequently defined. For this purpose, it is necessary to employ a controller which can handle the input constraints. Model predictive control (MPC) is an appropriate candidate because of its capability to deal with state and input constraints which is used in this paper. Hence, the proposed method divides the control system into two parts: 1) A model predictive controller in the d-dimensional space as the main controller which produces the desired virtual control and, 2) A constant control allocation unit which distributes the desired virtual control among the actuators. In order to guarantee the feasibility of the control allocation problem, the feasible region is considered as the input constraints of the MPC. The first step would be to determine the feasible region of the control allocation problem.

Definition: Feasible region is a subset of the virtual control space. If the desired virtual control signal is located there, it is guaranteed that the control allocation unit will map it into the control signal space such that actuator constraints are not violated.

The generalized pseudo inverse is employed as the control allocator in the proposed control structure which solves the following optimization problem:

$$\text{Min } J_{\text{CA}}(\mathbf{u}(t)) = (\mathbf{u}(t) - \mathbf{u}_d(t))^T \mathbf{W}_p (\mathbf{u}(t) - \mathbf{u}_d(t)) \\ \text{subject to :} \mathbf{v}_d(t) = \mathbf{B}\mathbf{u}(t) \tag{10}$$

where $\mathbf{u}_d$ and $\mathbf{W}_p$ are the desired values for the control signal and the weighting matrix, respectively. Also, $\mathbf{v}_d$ is the desired virtual control commanded by the main controller. The solution of the above mentioned optimization problem is:

$$\mathbf{u}(t) = (\mathbf{I} - \mathbf{F}\mathbf{B})\mathbf{u}_d(t) + \mathbf{F}\mathbf{v}_d(t) \tag{11}$$

where

$$\mathbf{F} = \mathbf{W}_p^{-1}\mathbf{B}^T\left(\mathbf{B}\mathbf{W}_p^{-1}\mathbf{B}^T\right)^{-1}. \tag{12}$$

3.1 Control allocation in the presence of static actuator constraints

When the actuators are much faster than the system dynamic, they can be modeled as a static gain. For the control allocation in the presence of static actuator constraints, $\mathbf{u}_d(t)$ is chosen as the zero vector. Let $\mathbf{B}^\dagger$ denote the pseudo inverse of the matrix $\mathbf{B}$:

$$\mathbf{F} = \mathbf{B}^\dagger = \mathbf{W}_p^{-1}\mathbf{B}^T\left(\mathbf{B}\mathbf{W}_p^{-1}\mathbf{B}^T\right)^{-1} \tag{13}$$

The solution of the control allocation problem using the pseudo inverse is:

$$\mathbf{u}(t) = \mathbf{F}\mathbf{v}_d(t) = \mathbf{B}^\dagger\mathbf{v}_d(t) \tag{14}$$

where $\mathbf{v}_d$ is the desired virtual control. To determine the feasible region, the following inequalities, which define a convex polyhedron, should be satisfied:

$$\underline{\mathbf{u}}_i(t) \le \mathbf{f}_i\mathbf{v}_d(t) \le \bar{\mathbf{u}}_i(t); \forall 1 \le i \le m \tag{15}$$

where $\mathbf{f}_i$ is the ith row of $\mathbf{F}$. In the two dimensional space, the feasible region is a convex polygon.

In the proposed method, an MPC plays the role of the main controller which minimizes the following cost function [33]:

$$J_v(\mathbf{V}_d) = \sum_{j=1}^{N}([\hat{\mathbf{y}}(t+j|t) - \mathbf{w}(t+j)]^T \mathbf{Q}[\hat{\mathbf{y}}(t+j|t) - \mathbf{w}(t+j)] \\ + [\mathbf{v}_d(t+j-1|t) - \mathbf{v}^*(t+j-1)]^T \mathbf{R} \\ [\mathbf{v}_d(t+j-1|t) - \mathbf{v}^*(t+j-1)]) \tag{16}$$

where $\hat{\mathbf{y}}(t+j|t)$ is an optimum j-step ahead prediction of the system output based on the data up to time t, N is the control horizon, $\mathbf{Q}$ and $\mathbf{R}$ are the weighting matrices, $\mathbf{w}(t+j)$ is the future reference trajectory, and $\mathbf{v}^*(t+j)$ is the ideal input value. To put the MPC problem in a suitable form for optimization, stacked vectors with future states and control inputs are defined as follows [33]:

$$\mathbf{Y}=\begin{bmatrix}\hat{\mathbf{y}}(t+1|t)\\ \hat{\mathbf{y}}(t+2|t)\\ \vdots\\ \hat{\mathbf{y}}(t+N|t)\end{bmatrix},\quad \mathbf{V}_d=\begin{bmatrix}\mathbf{v}_d(t|t)\\ \mathbf{v}_d(t+1|t)\\ \vdots\\ \mathbf{v}_d(t+N-1|t)\end{bmatrix},$$
$$\mathbf{V}^*=\begin{bmatrix}\mathbf{v}^*(t)\\ \mathbf{v}^*(t+1)\\ \vdots\\ \mathbf{v}^*(t+N-1)\end{bmatrix},\quad \mathbf{W}=\begin{bmatrix}\mathbf{w}(t+1)\\ \mathbf{w}(t+2)\\ \vdots\\ \mathbf{w}(t+N)\end{bmatrix} \tag{17}$$

The predicted outputs can be written as $\mathbf{Y}=\mathbf{Hx}(t|t)+\mathbf{SV}_d$ where:

$$\mathbf{H}=\begin{bmatrix}\mathbf{CA}\\ \mathbf{CA}^2\\ \vdots\\ \mathbf{CA}^N\end{bmatrix},$$
$$\mathbf{S}=\begin{bmatrix}\mathbf{CB} & 0 & \cdots & 0\\ \mathbf{CAB} & \mathbf{CB} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{CA}^{N-1}\mathbf{B} & \mathbf{CA}^{N-2}\mathbf{B} & \cdots & \mathbf{CB}\end{bmatrix} \tag{18}$$

Hence, the cost function can be rewritten as follows:

$$\begin{aligned}J(\mathbf{V}_d)=&(\mathbf{Hx}(t|t)-\mathbf{W})^T\bar{\mathbf{Q}}(\mathbf{Hx}(t|t)-\mathbf{W})\\ &+\mathbf{V}^{*T}\bar{\mathbf{R}}\mathbf{V}^*+2(\mathbf{Hx}(t|t)-\mathbf{W})\mathbf{Q}\bar{\mathbf{S}}\mathbf{V}_d\\ &+2\mathbf{V}^{*T}\bar{\mathbf{R}}\mathbf{V}_d+\left(\mathbf{S}^T\bar{\mathbf{Q}}\mathbf{S}+\bar{\mathbf{R}}\right)\mathbf{V}_d\end{aligned} \tag{19}$$

where $\bar{\mathbf{Q}}$ and $\bar{\mathbf{R}}$ are the extended weighting matrices. The desired virtual control signals are determined by solving the following constrained convex quadratic optimization problem:

$$\begin{aligned}&\min_{\mathbf{V}_d} J_v(\mathbf{V}_d)\\ &\text{subject to}:\underline{\mathbf{u}}_i(t)\le\mathbf{f}_i\mathbf{v}_d(t+j-1)\le\bar{\mathbf{u}}_i(t);\\ &1\le i\le m, 1\le j\le N-1\end{aligned} \tag{20}$$

The optimization problem is solved in the d-dimensional space with less computational complexity as compared to the conventional MPC which solves the control problem in the m-dimensional space and determines the actuators command, directly. The structure of the proposed method is shown in Fig.1.

It should be mentioned that existance of solution in the the MPC problem is not generally guaranteed. The key achievement of the feasible control allocation is to eliminate control allocation discrepancy which may occur in the other control allocation methods. The proposed approach guarantees that the virtual control signal generated by the actuators are equal with its desired value produced by the main controller.

Failure of an actuator may substantially reduce the feasible region space. Therefore, if an actuator is stuck at a fixed location, it cannot be controlled and the desired control effort would be provided by other actuators. For this purpose the value of the virtual control produced by the failed actuators (e.g. stuck in a fixed position) denoted by $\mathbf{v}_{st}$ should be calculated as follows:

$$\mathbf{v}_{st}(t)=\mathbf{B}_{st}\mathbf{u}_{st}(t) \tag{21}$$

where $\mathbf{B}_{st}$ and $\mathbf{u}_{st}$ contain the columns of $\mathbf{B}$ and the actuator positions corresponding to the failed actuators, respectively. The rest of the desired virtual control should be generated by the other actuators as follows:

$$\mathbf{u}_{sa}(t)=\mathbf{B}_{sa}^{\dagger}(\mathbf{v}_d(t)-\mathbf{v}_{st}(t)) \tag{22}$$

where $\mathbf{B}_{sa}$ and $\mathbf{u}_{sa}$ contain the columns of $\mathbf{B}$ and the actuator positions corresponding to the fault-free actuators. In this case, the constraints of the main controller should be changed based on the following inequality.

$$\underline{\mathbf{u}}_{sa}(t)+\mathbf{B}_{sa}^{\dagger}\mathbf{v}_{st}(t)\le\mathbf{B}_{sa}^{\dagger}\mathbf{v}_d(t)\le\bar{\mathbf{u}}_{sa}(t)+\mathbf{B}_{sa}^{\dagger}\mathbf{v}_{st}(t) \tag{23}$$

where $\underline{\mathbf{u}}_{sa}(t)$ and $\bar{\mathbf{u}}_{sa}(t)$ are lower and upper limits of the fault-free actuators. This procedure can be repeated until the rank of $\mathbf{B}_{sa}$ is equal or greater than d.

3.2 Control allocation in the presence of actuator rate constraints

In the feasible control allocation in the presence of actuator rate constraints, due to rate limits of the actuators, it is ideal to have the minimum variation in the control signal. Therefore, the last position of the actuators could be chosen as the desired control signal (i.e. $\mathbf{u}_d(t)=\mathbf{u}(t-1)$). Also, in order to have the maximum effectiveness the weighting matrix could be a diagonal matrix whose elements are the inverses of corresponding actuator rate limits:

$$\mathbf{W}_p=\begin{bmatrix}\bar{r}_1 & 0 & \cdots & 0\\ 0 & \bar{r}_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \bar{r}_m\end{bmatrix}^{-1} \tag{24}$$

It leads to more variations for fast actuators as opposed to the slower ones. The feasible region could be determined by the following inequalities which are yielded by combining (11) and (9):

$$\begin{gathered}\underline{\mathbf{u}}_i'(t)-\mathbf{u}_{p_i}(t)\le\mathbf{f}_i\mathbf{v}_d(t)\le\bar{\mathbf{u}}_i'(t)-\mathbf{u}_{p_i}(t)\\ 1\le i\le m\end{gathered} \tag{25}$$

where $\mathbf{f}_i$ is the ith row of $\mathbf{F}$ and $\mathbf{u}_p(t)=(\mathbf{I}-\mathbf{FB})\mathbf{u}(t-1)$.

The main controller is an MPC which considers dynamic feasible region as its control constraints:

$$\begin{aligned}&\min_{\mathbf{V}_d} J_v(\mathbf{V}_d)\\ &\text{subject to}:\underline{\mathbf{u}}_i'(t)\le\mathbf{f}_i\mathbf{v}_d(t+j-1|t)\le\bar{\mathbf{u}}_i'(t);\\ &1\le i\le m, 1\le j\le N-1\end{aligned} \tag{26}$$

where an approximation is employed such that the upper and lower limits for the future samples are replaced by the current values.

3.3 The domain of attraction

An important issue in investigating the model predictive control is the domain of attraction. The domain of attraction can show the stabilizing ability. In a regulation problem, the domain of attraction is defined as follows:

Definition: Domain of attraction is the set of all $\mathbf{x}_0$ in the state space such that the solution of (1) is defined for all $t > 0$ and converges to the origin as t tends to infinity [34].

This paper employs the MPC domain of attraction concept as a criterion to confirm the effectiveness of the feasible control allocation strategy. Using the proposed approach can enlarge the domain of attraction of the MPC compared to the standard MPC. This is due to the fact that because the optimization problem is solved in a lower dimensional space and the static relation between the control signals and the virtual control signals is considered. To the best of our knowledge, there is no analytical method to determine the MPC domain of attraction. Therefore, the domain of attraction can be estimated by repetitive simulation with different initial values. The binary search method [35] is employed to find the estimated domain of attraction. The comparison is done for several systems and it is obsereved that the feasible control allocation enlarges the domain of attraction. Note that the estimation method for domian of attraction in both strategy is similar.

4 Simulation results

In this section, several simulation results are presented to investigate the effectiveness of the proposed method. YALMIP [36] and the quadratic control allocation toolbox (QCAT) [37] are employed to solve the MPC optimization and the quadratic programming problems, respectively. The simulations are done in MATLAB software on a desktop computer with core$^{\mathrm{TM}}$i7 CPU (3.2GHz) and 16 GB RAM.

4.1 Example 1

In the first example, it is assumed that actuators are static. Two methods are selected for comparison studies in the sense of control performance and computational complexity. Standard MPC can control an overactuated system and reach the desirable performance with satisfied actuators constraints. Therefore, the main controller considers the actuators limits without using the control allocation unit. Another alternative is to employ an unconstrained MPC and the quadratic programming technique as the main controller and the control allocator, respectively. This strategy does not consider the feasibility of the control allocation problem. Consider the Admire flight system with one redundancy degree which is described by the following state space matrices. Note that there are four actuators (canard wings, left and right elevons, and rudder) to control the three system output (angle of attack, sideslip angle, and roll rate)[38].

$$
\mathbf{A} = \begin{bmatrix} -0.543 & 0.013 & 0 & 0.978 & 0 \\ 0 & -0.12 & 0.221 & 0 & -0.9661 \\ 0 & -10.52 & -0.997 & 0 & 0.6176 \\ 2.62 & -0.003 & 0 & -0.506 & 0 \\ 0 & 0.708 & -0.0939 & 0 & -0.213 \end{bmatrix}
$$
$$
\mathbf{B} = \begin{bmatrix} 0 & -4.24 & 4.24 & 1.487 \\ 1.653 & -1.27 & -1.27 & 0.0024 \\ 0 & -0.28 & 0.28 & -0.88 \end{bmatrix}
$$
$$
\mathbf{B}_v = \begin{bmatrix} 0_{2\times 3} \\ I_3 \end{bmatrix}
$$
$$
\mathbf{C} = \begin{bmatrix} 0_{3\times 2} & I_3 \end{bmatrix} \tag{27}
$$

The main goal of the control system is convergence of the outputs to zero under different actuator conditions. In the first 25 seconds, the actuators are fault-free and work in the nominal range. Then, the allowable range of some actuators is reduced for the next 25 seconds. Finally, the second actuator sticks at a position at $t = 50$. In order to compare different conditions, at $t = 30$ and $t = 60$, a disturbance, which resets the state variables to the initial values, is considered. The controllers parameters that affect the transient response quality and even stability are chosen as $\mathbf{Q} = 100I, \mathbf{R} = I, \mathbf{R}_v = I$, and $N = 5$ by trial and error.

Fig. 2 shows the results of employing an MPC without the control allocation unit. It is obvious that the controller manages the actuators in different modes well such that stability and an acceptable performance are maintained. Tighter ranges and sticking in a position for the actuators increases settling time and over or undershoots. Results of using a constrained MPC as the main controller with pseudo inverse as the control allocator are presented in Fig.3. The proposed method can handle actuators in the fault-free and faulty modes appropriately but with some larger transients than the conventional MPC. Employing an unconstrained MPC and quadratic programming (QP) as the main controller and the control allocator leads to the responses shown in Fig.4-a which shows instability after $t = 50$ when an actuator sticks in a position. It is because of the lack of feasibility consideration in the main controller that can be observed as the difference between the desired and generated virtual control which is obvious in Fig.4-b.

In order to compare the results quantitatively, Tables 1-3 present the computational time (Com. time) and cost function value for the three different conditions. The cost function is defined as follows:

$$
J_s = \tag{28}
$$
$$
\sum_{t=0}^{T_{end}} \Big((\mathbf{y}(t) - \mathbf{w}(t))^T \, \mathbf{Q} \, (\mathbf{y}(t) - \mathbf{w}(t)) + \mathbf{u}(t)^T \mathbf{R} \mathbf{u}(t) \Big) / T_{end}
$$

Although in fault-free operations, all strategies have the same performance, tightening of the actuators allowable range leads to performance degradation with an increase in the cost function value. In the case of the failure occurrence, unconstrained MPC with quadratic programming control allocation cannot maintain the closed loop system stability.

Table 1 Fault-free actuators

Method	Cost function	Com. time (s)
MPC without control allocation	5.61	0.098
Constrained MPC with FCA	5.71	0.08
Unconstrained MPC with QP	5.71	0.081

Table 2 Tighter range of the actuators

Method	Cost function	Com. time (s)
MPC without control allocation	18.89	0.091
Constrained MPC with FCA	27.33	0.071
Unconstrained MPC with QP	unstable	0.066

Table 3 Stuck actuator

Method	Cost function	Com. time (s)
MPC without control allocation	605.2	0.095
Constrained MPC with FCA	626.45	0.079
Unconstrained MPC with QP	unstable	0.079

In this example, MPC without control allocation and constrained MPC with feasible control allocation which consider the feasibility problem can maintain the closed loop stability. The proposed method needs less computational time but with a slightly worse closed loop performance.

In summary, feasibility consideration of the control allocation unit in the main controller can improve the closed loop stability and performance under severe conditions. It should be mentioned that the computational times are presented to compare the approaches and it can be reduced by choosing appropriate hardware and software.

4.2 Example 2

The second example is a planar robot shown in Fig.5 which could be employed in robot therapy. The system is made up of a planar surface and an end effector which can move on it. The end effector has two degrees of freedom. The force on the end effector is produced by four actuators [39]. Therefore, the redundancy degree of this system equals to 2. The system dynamics are driven from [40]:

$$\mathbf{M}\ddot{\mathbf{X}} = -\mathbf{v} \tag{29}$$

where $\mathbf{X} = [x \quad y]^{\mathrm{T}}$, $\mathbf{M}$ and $\mathbf{v} = [f_x \quad f_y]^{\mathrm{T}}$ are end effector's position, mass matrix and total force, respectively. The effectiveness matrix which shows the relation between motors forces and the total force can be yielded by projecting motors forces u_i onto the x and y axes as follows:

$$\mathbf{v} = \begin{bmatrix} \cos\theta_1 & \cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \sin\theta_1 & \sin\theta_2 & \sin\theta_3 & \sin\theta_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}. \tag{30}$$

The control effectiveness matrix depends on the position of the end effector and can be rewritten as follows:

$$b_{1i} = \cos\theta_i = \frac{x_i - x}{\sqrt{(x_i - x)^2 + (y_i - y)^2}} \tag{31}$$
$$b_{2i} = \sin\theta_i = \frac{y_i - y}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}; i = 1, 2, ..., 4$$

where $[x_i \quad y_i]^{\mathrm{T}}$ is the location of the ith motor in the plane.

Force and rate limits are considered as follows:

$$\mathbf{u}_{\min} = \begin{bmatrix} -1 & -1 & -0.5 & -0.75 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{u}_{\max} = \begin{bmatrix} 0.7 & 0.8 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$$
$$\bar{\mathbf{r}} = \begin{bmatrix} 0.2 & 0.01 & 0.04 & 0.14 \end{bmatrix}^{\mathrm{T}} \tag{32}$$

Also, the controller parameters are chosen as $\mathbf{Q} = 10\mathbf{I}, \mathbf{R} = \mathbf{I}$, and $N = 10$.

Dynamic quadratic programming control allocation [23] is employed to compare the proposed method results. In this scheme, an unconstrained MPC is used as the main controller. The results of applying this method is shown in Fig.6 where output tracking is done. As is shown in Fig.6(b) there is discrepancies between the desired and actual value of the virtual control signal which degrades the control performance. Fig.6(c) shows the control signals and their upper and lower limits which are time varying. Also, Fig.7 depicts results of using the proposed feasible control allocation in the same condition. It could be observed in Fig.7(a) that transient response is improved by employing the knowledge of the feasible region in the main controller. Also, there is no mismatch between the actual and the desired virtual control as is shown in Fig.7(b).

In the second scenario, more limited constraints are considered for the actuators such that:

$$\mathbf{u}_{\min} = \begin{bmatrix} -0.1 & -0.1 & -0.05 & -0.07 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{u}_{\max} = \begin{bmatrix} 0.07 & 0.08 & 0.1 & 0.1 \end{bmatrix}^{\mathrm{T}}$$
$$\bar{\mathbf{r}} = \begin{bmatrix} 0.01 & 0.005 & 0.005 & 0.007 & 0.002 \end{bmatrix}^{\mathrm{T}} \tag{33}$$

In this case, the control system using dynamic quadratic programming control allocation leads to closed loop instability. Fig.8 shows that employing the dynamic feasible control allocation scheme maintains system stability and with only slight performance degradation compared to the previous condition.

In the dynamic feasible control allocation scheme, the feasible region might change in each sample. The feasible area of the robotic control problem in the second scenario is shown in Fig.9.

4.3 Example 3

In order to compare two methods which consider the feasibility of the control allocation unit in the main controller more carefully, a regulation problem of the following system is considered:

$$\mathbf{G}(z) = \begin{bmatrix} \frac{0.74z^{-1}}{z-0.94} & \frac{-0.88z^{-3}}{z-0.95} \\ \frac{0.58z^{-7}}{z-0.91} & \frac{-1.4z^{-1}}{z-1.07} \end{bmatrix} \tag{34}$$

The system has the redundancy degree of 3 and the control effectiveness matrix is:

$$\mathbf{B} = \begin{bmatrix} 2 & -0.5 & -3 & 1 & 2 \\ 0.5 & 1 & -2 & -0.5 & -1 \end{bmatrix} \tag{35}$$

The domain of attraction of the controller is estimated for both approaches in different conditions. Although, the binary search is done in the state space, the domain of attraction is projected into the 2-dimensional output spaces. The estimated domain of attraction of the MPC in different conditions are shown in Fig. 10-12 which confirm that the domain of attraction of the proposed method is significantly larger than the standard MPC. It means that the feasible control allocation scheme surpasses the standard MPC in a regulation problem. Also, it can be observed that the actuators faults can dwindle the domain of attraction.

5 Conclusion

This paper investigates actuator fault tolerant control systems employing a control allocation unit with emphasis on the feasibility issue. It proposes to consider the feasibility of the control allocation problem in the main controller in order to improve the control performance and to maintain the closed loop stability under severe conditions. For this purpose, an MPC controller and the pseudo inverse method are used as the main controller and the control allocator. The key point shown in this paper is that the feasible region of the control allocation should be considered as the input constraints of the main controller. The proposed method can consider both position and rate constraints for the actuators. The simulation results confirm that considering the feasible area in the main controller improves the control system effectiveness under faulty conditions. Also, it is shown that considering the feasible region as the main controller constraints can enlarge the domain of attraction of the main controller.

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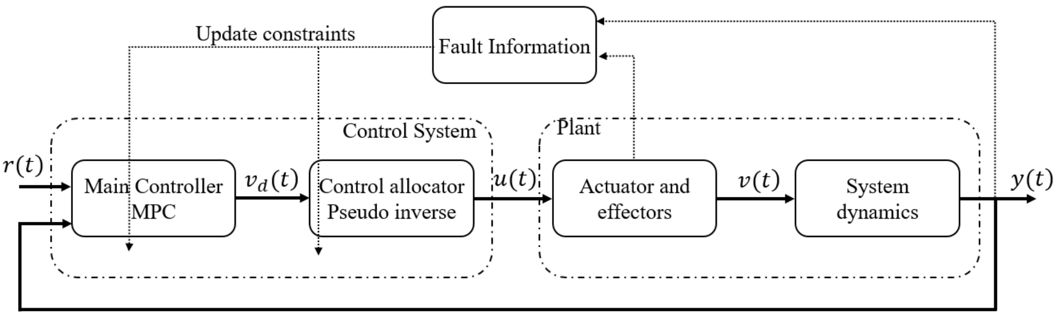
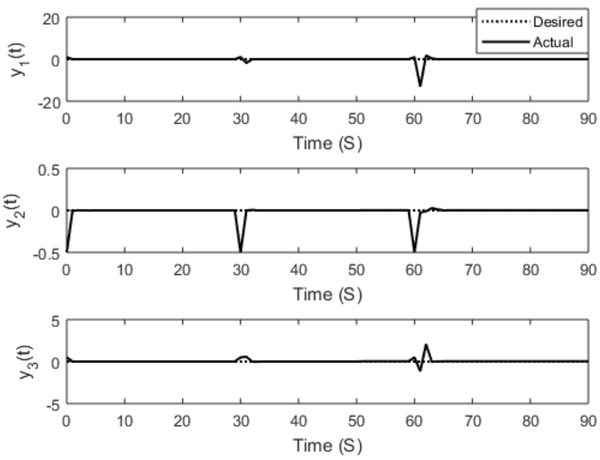


Figure 1 Structure of the fault tolerant control using MPC and the feasible control allocation



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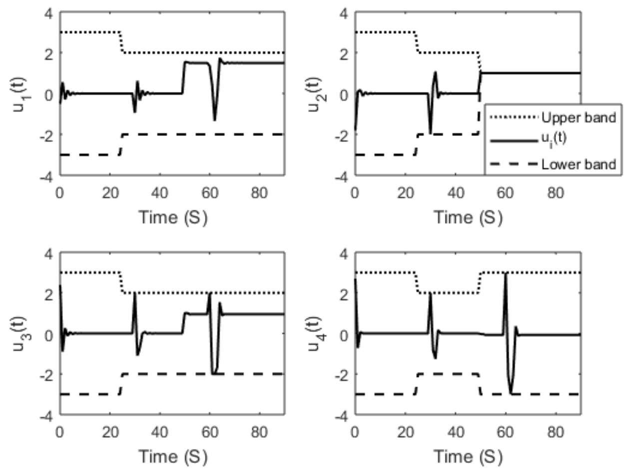
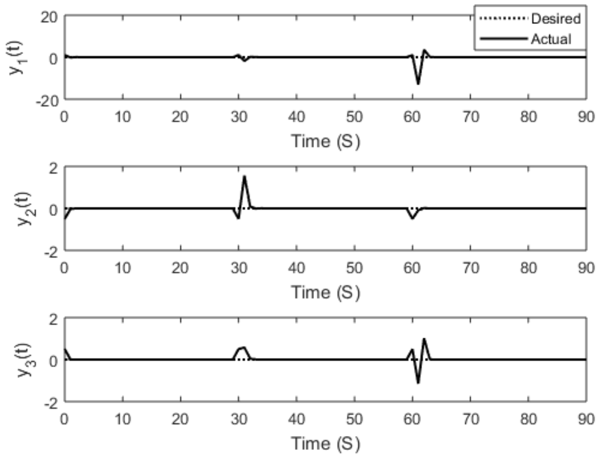


(b)

Figure 2 Example 1- MPC without control allocation, (a) System outputs, (b) Control signals



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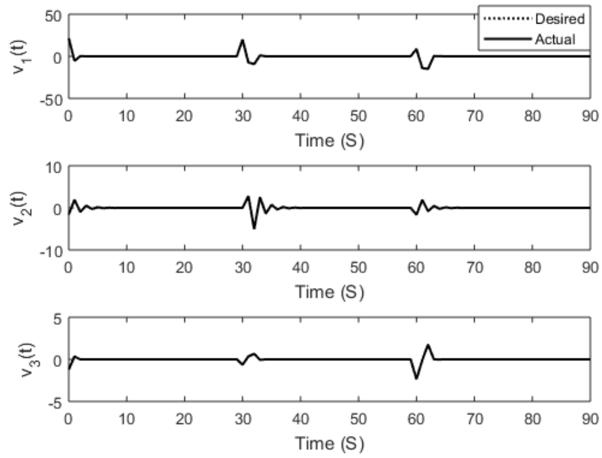

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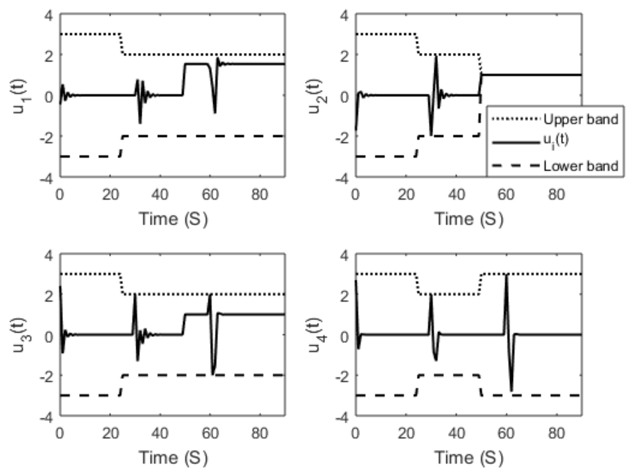

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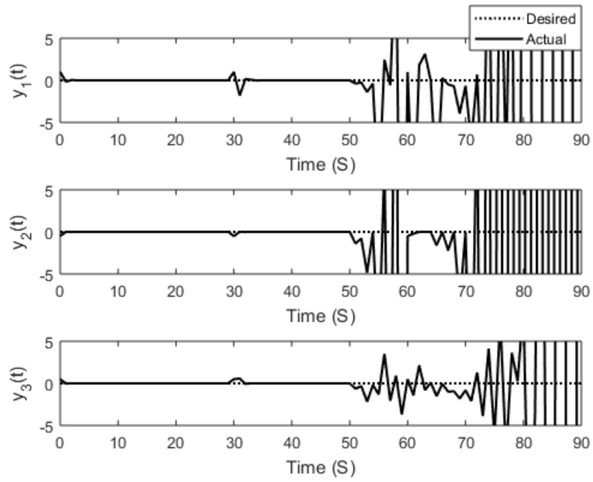
Figure 3 Example 1- MPC with feasible control allocation, (a) System outputs, (b) Virtual control signals, (c) Control signals



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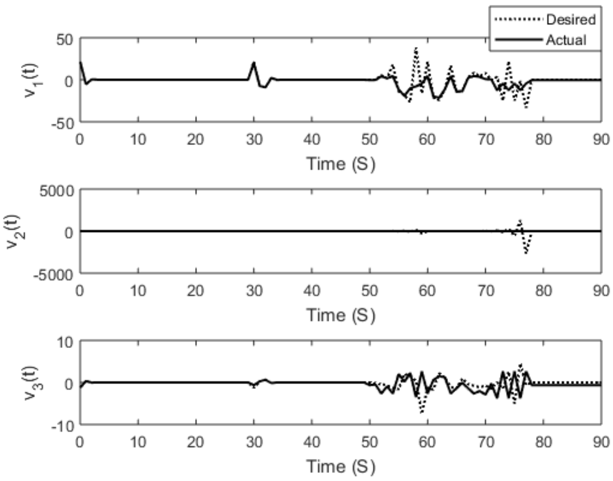

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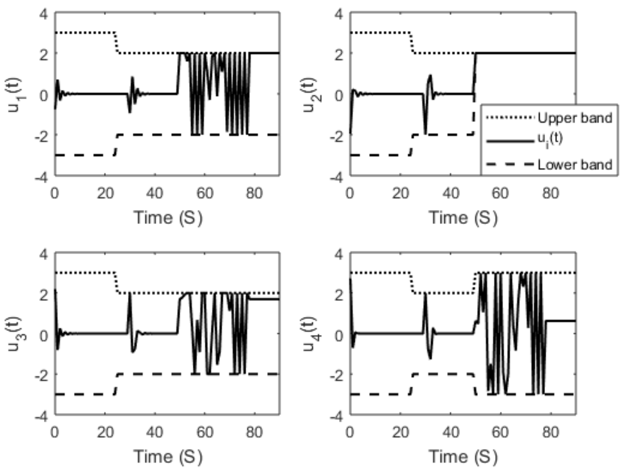

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Figure 4 Example 1- MPC with quadratic programming, (a) System outputs, (b) Virtual control signals, (c) Control signals

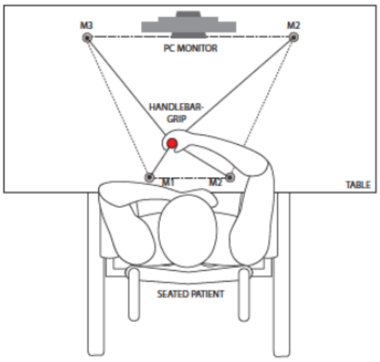
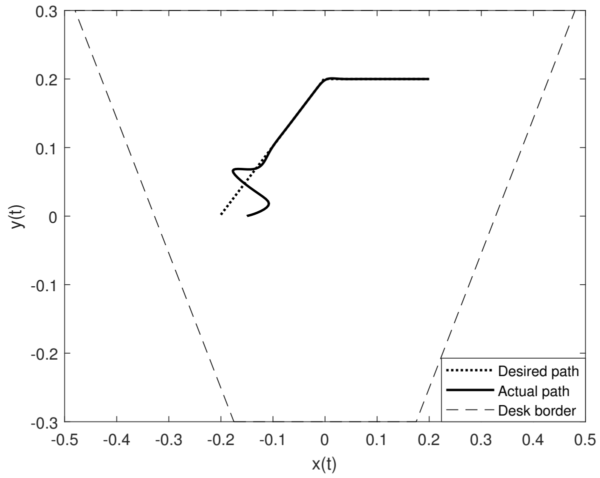


Figure 5 Structure of the planar robot [39]



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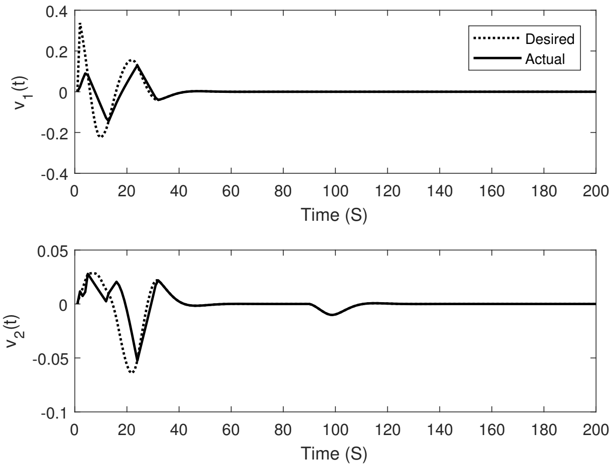

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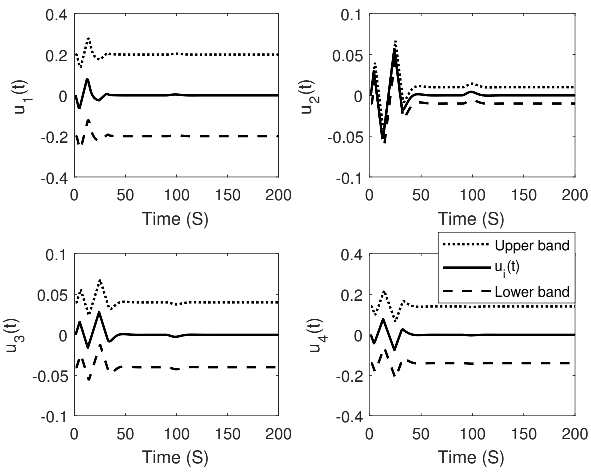
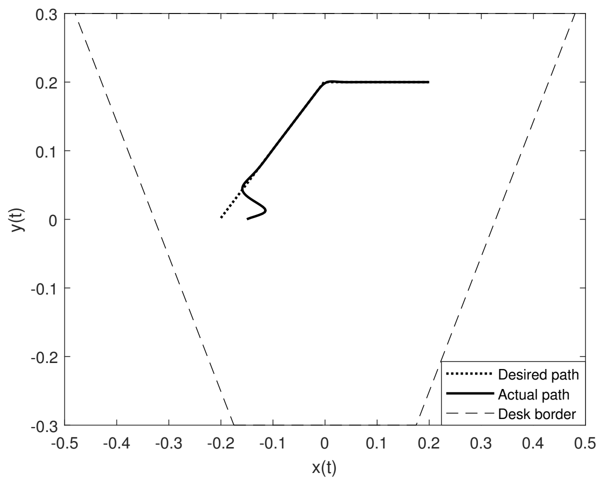


(c)

Figure 6 Dynamic quadratic programming control allocation, (a) System outputs, (b) Virtual control signals (c), Control signals



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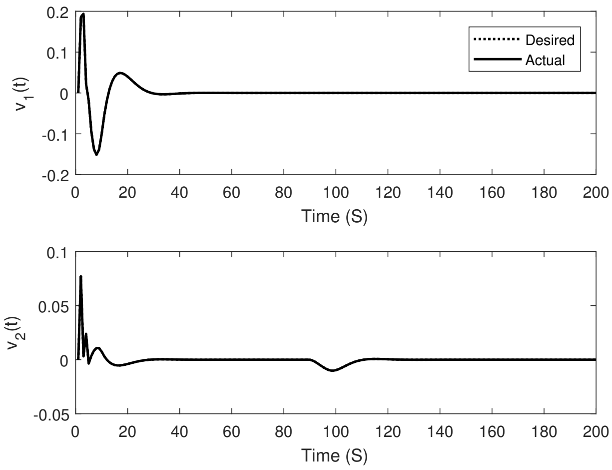

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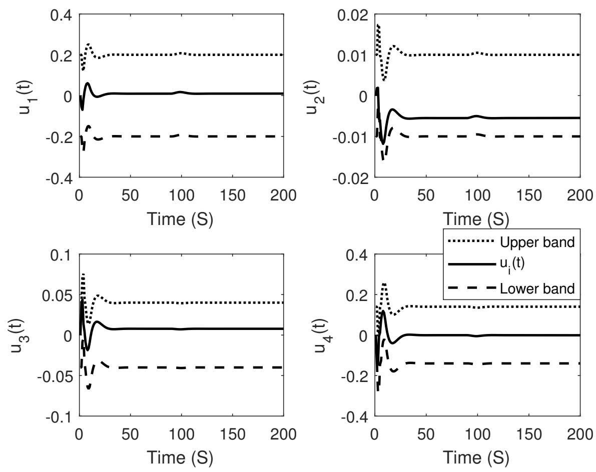
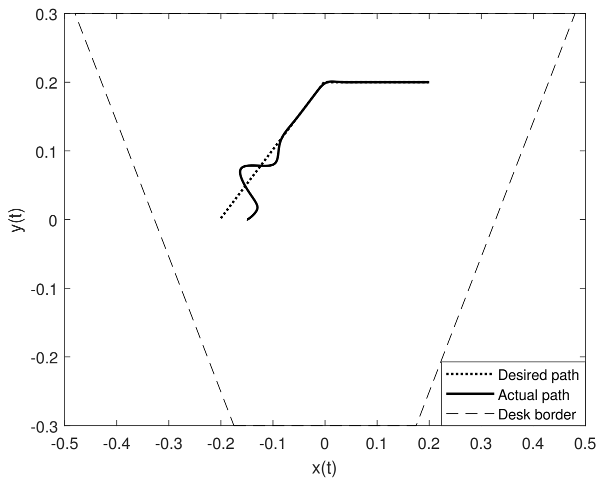


(c)

Figure 7 Dynamic feasible control allocation, (a) System outputs, (b) Virtual control signals (c), Control signals



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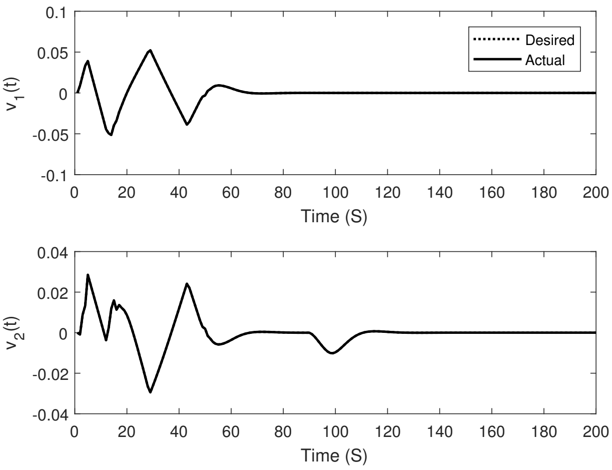

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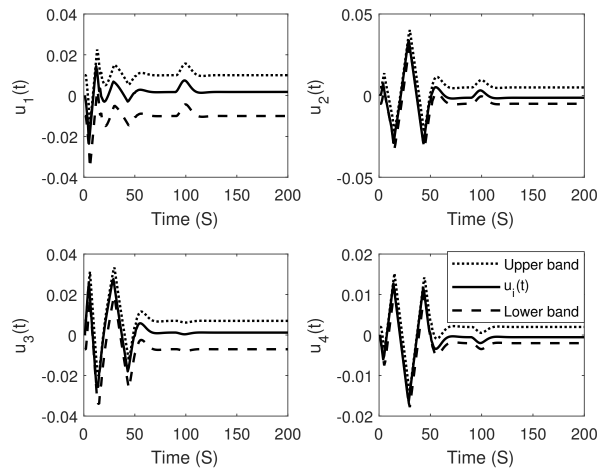

(c)

Figure 8 Dynamic feasible control allocation, (a) System outputs, (b) Virtual control signals (c), Control signals

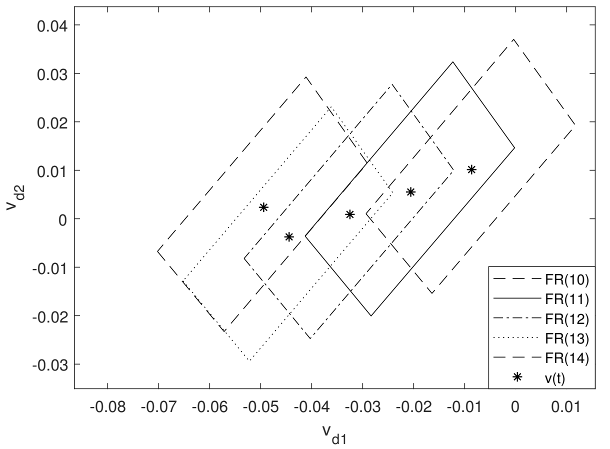

Figure 9 Dynamic feasible region $20 \leq t \leq 24$

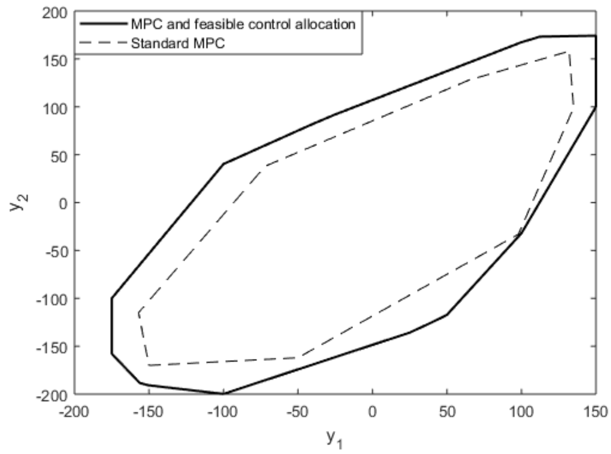

Figure 10 Domain of attraction of MPC controller- Fault free actuators

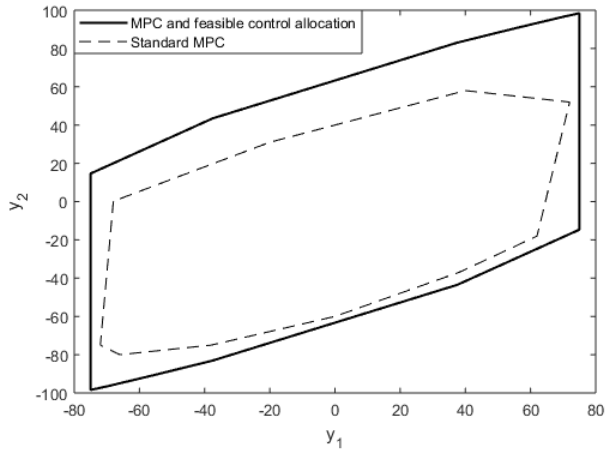

Figure 11 Domain of attraction of MPC controller-Tighter range of actuators

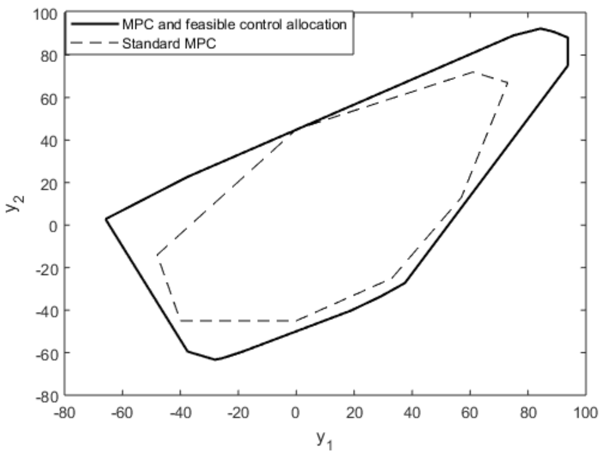

Figure 12 Domain of attraction of MPC controller- Stuck actuators