Andreas Sanne

Buckling of Non-spherical Moss-LNG tank

Master's thesis in Marine Technology Supervisor: Jørgen Amdahl June 2019

Norwegian University of Science and Technology Faculty of Engineering Department of Marine Technology



Andreas Sanne

Buckling of Non-spherical Moss-LNG tank

Master's thesis in Marine Technology Supervisor: Jørgen Amdahl June 2019

Norwegian University of Science and Technology Faculty of Engineering Department of Marine Technology



MASTER THESIS 2019

for

Stud. techn. Andreas Sanne

Buckling of Non-spherical MOSS-LNG Carriers G

In recent years the demand for large LNG ships has increased. Ship owners are now

requesting larger LNG ships with a cargo capacity up to $180\ 000\ m^3$, with ship dimensions being compliant with the limitations of the new locks of the Panama Canal. A challenge with the Moss LNG tanks is that increase in cargo capacity is most efficiently dealt with by

increasing the tank diameter. Spherical tank ships of cargo capacity 165 000 m^3 are in compliance with the Panama canal restrictions, but for larger capacities modification of tank shape is necessary, as discussed below.

The Moss-type LNG tank is an independent aluminium tank, supported by a cylindrical skirt, which provides the structural connection to the ship double bottom structure. The cylindrical skirt connects to the LNG tank through a central horizontal ring (the equator profile). The Moss LNG tank is very robust and is preferred by many ship owners, with more than 20 vessels in construction in early 2017.

As discussed above, increasing the cargo capacity without increasing number of tanks is challenging because the maximum ship width must remain within the Neo-panamax limitations. An option for increasing the cargo capacity is to modify the shape of the tank. An example of an altered tank shape is the apple-shaped tanks designed by Mitsubishi shown in Figure 1. The apple shape gives the tank a larger capacity than a spherical tank, while maintaining the ship width within the limitations of the Panama Canal. The center of gravity of the tank is also lower than for a vertically stretched tank, which makes it easier to meet the stability requirements. Although the tank shape is altered, the tank support system is not. Many of the important characteristics of the spherical Moss LNG tank are therefore maintained.



Figure 1 Illustration of apple shaped tanks on a LNG Ship.

Changing the tank shape to a non-spherical shape introduces some challenges with regard to calculating the tanks structural capacity for the Ultimate Limit State (ULS) especially. Simplified equations for structural capacity available in Classification codes are only valid for purely spherical tanks. When the tank shape deviates from a pure sphere more complex

analyses must be performed to verify the tanks ULS capacity. For the Mitsubishi design, nonlinear finite element analyses (NLFEA) were performed to verify the tank's structure integrity. One should however note that non-linear finite element analyses are very time consuming both with regard to modelling and computation time, and may not be an efficient tool in the design phase.

An accurate estimate of the buckling capacity of the tank shell when the shell has a different curvature in the different directions is needed. This should also include the secondary effects of thermal contraction, as the tank shape deviate from the initial shape once it is filled. Estimates for second order geometrical loads are easily included when performing NLFEA, so the focus should first be on establishing a method for determining the buckling capacity of a non-spherical tank.

NLFEA is not a very efficient method for designing structures even with the recent increases in computation capacities. Simple estimates are preferred in the design stage. The availability and validity of these simple methods is however not known.

The work shall address the following topics:

- 1. A brief description of various type of various types of gas carriers. Outline the design of Moss Rosenberg spherical tank and extension of this into alternative shapses.
- 2. Based on the work carried out in the Project thesis, provide a summary of relevant code and/or literature formulas for estimation of the buckling capacity of spherical shells. Focus should be placed on estimates of the critical pressure for different shapes.
- 3. Conduct modal analysis on the sphere to determine the tank's deformation patterns.
- 4. Perform non-linear analysis in LS-DYNA to determine the critical pressure on a sphere with *sep* realistic imperfections. Perform the analysis with both elastic and elasto-plastic materials. Compare the results from the finite element analyses to simplified theory.
- 5. Perform non-linear analysis of tank shapes deviating from a spherical shell, and compare results with available buckling theories.
- 6. Condense the results and observations from the nonlinear calculations into extensions of analytical formula or propose new empirical calculation methods for future use.
- 7. Conclusions and recommendations for further work

Moss Maritime will support the work with relevant background data and discussions throughout the work execution, such as examples of tank size/scantlings, measured as-built imperfections, proposed altered tank shape geometry etc.

Literature studies of specific topics relevant to the thesis work may be included.

The work scope may prove to be larger than initially anticipated. Subject to approval from the supervisor, topics may be deleted from the list above or reduced in extent.

In the thesis the candidate shall present his personal contribution to the resolution of problems within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction.

The candidate should utilize the existing possibilities for obtaining relevant literature.

The thesis should be organized in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Telegraphic language should be avoided.

The thesis shall contain the following elements: A text defining the scope, preface, list of contents, summary, main body of thesis, conclusions with recommendations for further work, list of symbols and acronyms, references and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisor may require that the candidate, in an early stage of the work, presents a written plan for the completion of the work. The plan should include a budget for the use of computer and laboratory resources which will be charged to the department. Overruns shall be reported to the supervisor.

The original contribution of the candidate and material taken from other sources shall be clearly defined. Work from other sources shall be properly referenced using an acknowledged referencing system.

The report shall be submitted in two copies:

- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints which cannot be bound should be organised in a separate folder.

Supervisor: Prof. Jørgen Amdahl

Contact person at Moss Maritime Martin Slagstad

Deadline: June 11???, 2019

Trondheim, January 11, 2019

engen Andalel

Jørgen Amdahl

Preface

This report is a master's thesis written as the last work of a master's degree in marine technology, with marine structures as specialization. The thesis is a continuation of a project thesis written during the fall semester of 2018. The work with the project thesis made it possible to get familiar with the subject on an early basis. This was important in order to understand the concepts of the subject. This master's thesis counts as thirty points of study, and the entire spring semester of 2019 was spent on this report.

I will like to thank my supervisor Jørgen Amdahl for his help and guidance during the work with this master's thesis. His knowledge was most important for this work. This thesis was also written by the help of Moss Maritime. I will therefore like to thank Martin Slagstad as the contact person at the company. He was always willing to help with problems along the way, and he provided scantlings of design and important knowledge for this thesis. I will thank the postdoctoral fellows Yanyan Sha and Zhaolong Yu, in addition to the support team at Dynamore Nordic for their help in understanding the FEM software LS-Dyna. The analyses conducted in this report would not have been possible without their help. Finally I will thank Frank Klæbo at Sintef Ocean for his help in creating the geometrical models in Patran.

The subject of this master's thesis is the buckling strength of non-spherical LNG storage tanks in LNG carriers. Consequently the buckling strength of spherical tanks are considered, then the knowledge is extended to non-spherical shapes.

9/6-19 / Troudheim

Andreas Same

Date/Place

Author: Andreas Sanne

Summary

The demand of LNG carriers with larger capacities has increased in the recent years according to Moss Maritime. In order to carry more LNG in each ship, the spherical independent tank type B from Moss Maritime need to be modified. The tank can not be extended in the transverse ship direction because of limitations in the requirements from the Panama Canal. The idea is to extend the tank in the longitudinal direction of the ship, with spherical end caps and a cylindrical middle section. The buckling capacity of this new shape need to be analysed and compared to existing rules, because existing rules are only valid for spherical LNG tanks. If existing rules are insufficient, a proposition should be made on modification of the rules.

Finite Element Analyses are used to analyse the spherical and non-spherical LNG tank. Patran was used to generate the geometric models. These were extracted to LS-Prepost, where specifications for each analysis were added. Finally the model was analysed in the finite element software LS-Dyna. The first model was a half-sphere. This was analysed and extended further to a full sized spherical tank. Then a non-spherical tank was analysed. The models were verified against literature and existing formulas when possible, before it was extended further until the desired complexity was obtained.

The scope of work was divided in different steps. The first steps include a description of different gas carriers, and the design of the spherical tank from Moss Maritime. Then a summary of relevant codes and literature is given. The finite element analyses include a modal analysis of a sphere and a non-linear analysis of the spherical tank with realistic imperfections with elastic and elasto-plastic material. A non-linear analysis of the non-spherical tank is performed and these results are compared against relevant theory. Finally, a suggestion for modification of the rules is given.

The main results of this thesis are given with respect to two loading conditions. First loading condition contains only external pressure. The second loading condition contains external pressure, weight of isolation on the tank, additional acceleration on the material due to ship movements, and a sloshing load. The sloshing load arises due to ship movements, and this load is modelled as a static, sloped liquid surface. This surface causes a hydrostatic pressure with a constant height and density for each point in the tank, but with an increasing acceleration. The magnitude of this acceleration before the tank buckles is analysed in the second loading condition.

The results from the first loading condition can be seen in table 0.0.1. The results from the non-linear analysis are presented because these are more realistic than the linear results. A linear analysis was first conducted, the ten lowest buckling modes from the linear analysis was used as imperfections in the non-linear analysis for the spherical and non-spherical tank in both loading conditions. It can be seen that the non-spherical tank has a 0.02 MPa lower capacity than the spherical tank in the first loading condition.

Table 0.0.1: Results from non-linear analyses of spherical and non-spherical tank in aluminium exposed to external pressure

Model	Largest init. imp.	Material model	Buckling pressure
Spherical	0.0393 m	Elasto-plastic	0.16 MPa
Non-spherical	0.040 m	Elasto-plastic	$0.14 \mathrm{MPa}$

The results from the second loading condition are shown in table 0.0.2. The non-spherical tank can be seen to endure a lower acceleration on the fluid than the spherical tank. This causes the rules in DNVGL (1997) to estimate a buckling point at 75% of the capacity compared to LS-Dyna. While this point was estimated to 50% of the buckling capacity for the sphere according to LS-Dyna. By replacing the equation for the elastic buckling pressure for a sphere with the elastic buckling pressure for an ellipse in DNVGL (1997), the utilization was decreased to 62.5%. This gives a larger safety factor for the non-spherical tank. It is emphasized that additional analyses should be conducted in order to confirm the results. The analyses conducted in this thesis are too few to determine the exact safety level.

Table 0.0.2: Results from non-linear analyses of spherical and non-spherical tank in aluminium exposed to all loads

Model	Largest init. imp.	Ext. Pres.	Accel. on mat.	Critical acceleration on fluid
Spherical	0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$36 m/s^2$
Non-spherical	0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$24 \ m/s^2$

Sammendrag

Etterspørselen etter skip som kan frakte mer LNG har økt de siste årene ifølge Moss Maritime. Den uavhengige sfæriske tanken av type B fra Moss Maritime må modifiseres hvis et skip skal frakte mer LNG. Tanken kan ikke utvides tverrskips på grunn av regler i henhold til Panamakanalen. Derfor er ideen å strekke tanken langskips med en sylindrisk del på midten, og en halvkule på hver ende. Kapasiteten med tanke på knekking må analyseres for denne tanken. Denne kapasiteten må sammenliknes med den ordinære sfæriske tanken, og med det gjeldende regelverket. Regelverket er i dag kun gjeldende for sfæriske LNG tanker. Hvis dette regelverk er utilstrekkelig for den avlange tanken skal derfor et forslag til modifikasjon gis.

Elementmetoden ble brukt for å analysere den sfæriske og den avlange tanken. Patran ble brukt for å lage de geometriske modellene. Disse modellene ble eksportert til LS-Prepost, her ble spesifikasjoner for hver analyse lagt til. Deretter ble modellene analysert i LS-Dyna. Den første modellen som ble brukt var en halvkule. Denne ble analysert og utvidet til en full sfærisk LNG tank. Den sfæriske tanken ble så erstattet av en avlang tank. Underveis ble resultatene sammenliknet med litteratur og regelverk hvis mulig. Hvis resultatene ble bekreftet ble modellene utvidet til de inneholdt tilstrekkelig mengde detaljer.

Omfanget av arbeidet ble delt inn i forskjellige steg. Det første steget innebærer en beskrivelse av forskjellige typer LNG skip, og designet til den sfæriske tanken fra Moss Maritime. Etter dette blir det presentert et sammendrag av relevante regler og litteratur. Analysene etter elementmetoden inkluderer en lineær knekkingsanalyse av en kule, og en ulineær analyse av en sfærisk LNG tank med realistiske imperfeksjoner samt elastisk og elasto-plastisk material. Neste steg innebærer en ulineær analyse av den avlange tanken, hvor resultatene blir sammenliknet med relevant teori. Et forslag til endring av reglene blir gitt til slutt for å tilpasse dem til den avlange tanken.

Hovedresultatene i denne oppgaven blir presentert i forhold til to lasttilfeller. Første lasttilfelle innebærer kun ytre trykk. Andre lasttilfelle innebærer ytre trykk, i tillegg til vekten av isolasjonen i tanken, akselerasjon av materialet på grunn av skipets bevegelser i sjøen, og en last som følge av bevegelser i væsken ved lavere fyllingsgrader enn 100%. Denne siste lasten blir mod-

ellert i analysene som en statisk, skrå væskeflate. Denne væskeflaten forårsaker et hydrostatisk trykk med konstant høyde og tetthet, men en økende akselerasjon for hvert punkt i tanken. Størrelsen på denne akselerasjonen før tanken knekker blir undersøkt i det andre lasttilfellet. De mest realistiske resultatene kommer fra de ulineære analysene, derfor er disse presentert her.

Resultatene fra første lasttilfelle vises i tabell 0.0.3. En lineær analyse ble gjort først med første lasttilfellet. De ti laveste knekkmodene fra den lineære analysen ble brukt som imperfeksjoner i både den sfæriske og den avlange tanken for begge lasttilfeller. Den avlange tanken har 0.02 MPa mindre kapasitet enn den sfæriske tanken for det første lasttilfellet.

Table 0.0.3: Resultater fra ulineær analyse av sfærisk og avlang tank i aluminium fra første lasttilfellet

Modell	Største imperf.	Materialmodell	Knekktrykk
Sfærisk	$0.0393 { m m}$	Elasto-plastisk	0.16 MPa
Avlang	0.040 m	Elasto-plastisk	$0.14 \mathrm{MPa}$

Resultatene fra det andre lasttilfellet vises i tabell 0.0.4. De viser at den avlange tanken tåler en mindre akselerasjon på væsken før den knekker enn den sfæriske tanken. Dette gjør at reglene i DNVGL (1997) estimerer punktet for knekking på 75% av kapasiteten i forhold til LS-Dyna. Dette punket for knekking ligger på 50% av kapasiteten for den sfæriske tanken i forhold til LS-Dyna. Ved å erstatte likningen for elastisk knekklast for kule i DNVGL (1997), kan punktet for knekking senkes fra 75% til 62.5%. Dette gir en større sikkerhetsfaktor for den avlange tanken. Men analysene som ble utført i denne oppgaven er for få til å bekrefte nøyaktig sikkerhetsfaktor. Flere analyser burde derfor gjennomføres for å bekrefte resultatene.

Table 0.0.4: Resultater fra ulineær analyse av sfærisk og avlang tank i aluminium fra andre lasttilfellet

Modell	Største imperf.	Ytre trykk	Aksel. av mat.	Kritisk akselerasjon av fluid
Sfærisk	0.04 m	0.0075 MPa	18.06 m/s^2	$36 \ m/s^2$
Avlang	0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$24 \ m/s^2$

Nomenclature

Λ neuliced sienderness ratio	λ	Reduced	slenderness	ratio
------------------------------	-----------	---------	-------------	-------

- ν Poisson's ratio
- σ Stress
- σ_{cr} Elasto-plastic buckling stress
- σ_E Elastic buckling stress
- σ_y Yield stress
- E Young's modulus
- *P* Pressure
- R Radius
- t Thickness

Contents

\mathbf{P}_{1}	refac	e	i
Sı	ımm	ary	ii
Sa	ammo	endrag	iv
N	omer	nclature	vi
1	Intr	roduction	1
	1.1	Objective	1
	1.2	Scope of Work	2
	1.3	Structure of Thesis	2
	1.4	Background	3
		1.4.1 Natural Gas	4
		1.4.2 Transport of LNG	4
		1.4.3 Types of Gas Carriers	5
	1.5	Design of Spherical LNG Tank	8
	1.6	Design of Non-spherical tank	11
2	Me	thod	13
	2.1	Software	13
	2.2	General Apporach	14
3	Lite	erature Study on Buckling of Spherical and Non-spherical Shells	15
	3.1	Buckling of Shells	15
	3.2	Spherical Shells	16
		3.2.1 "Elastoplastic buckling and collapse of spherical shells under combined	
		loadings" (Tall et al., 2018) \ldots	16

		3.2.2	"Design buckling pressure for thin spherical shells: Development and validation" (Eulin and Lythachova, 2018)	17
	2 2	Non s	valuation (Even and Eyenachova, 2018)	18
	0.0	331	"Buckling of egg-shaped shells subjected to external pressure" (Jasion	10
		0.0.1	et al 2015)	18
		3.3.2	"Elastic buckling of clothoidal–spherical shells under external pressure –	10
		0.0.2	theoretical study" (Zhang et al., 2017)	19
		3.3.3	"Estimation of elastic buckling strength of a non-spherical tank in the	
			partially filled condition" (Sano et al., 2017)	20
	3.4	Rules	for design	21
		3.4.1	Classification Notes 30.1	22
		3.4.2	Classification Notes 30.3	23
		3.4.3	Class Guideline, DNVGL-CG-0134	24
		3.4.4	Comments and Considerations on Formulas in Classifications Notes 30.1	
			and 30.3	28
4	Brie	ef on F	Finite Element Analysis	31
	4.1	Linear	Buckling	31
	4.2	Non-li	near Buckling	31
	4.3	Implic	tit and Explicit Solution Procedure	32
5	Line	ear Bu	ckling Analysis of Half-sphere	33
	5.1	Introd	luctory Analysis and Comparison with Abaqus	34
	5.2	Eigen	value Buckling Analysis of Half-sphere with Realistic Dimensions	38
6	Nor	n-linea	r Buckling Analysis of Half-sphere	41
	6.1	Imper	fection Type nr. 1	42
	6.2	Imper	fection Type nr. 2, Elastic and Elasto-plastic Material	46
	6.3	Evalua	ation of Elasto-plastic Results	50
	6.4	Imper	fection Type nr. 3, Elastic and Elasto-plastic Material	53
7	Buc	kling	Analysis of Spherical LNG Tank	56
	7.1	Linear	Buckling Analysis	60
	7.2	Non-li	near Buckling Analysis	61
		7.2.1	Steel S235	62
		7.2.2	Aluminium Alloy 5083	63

	7.3	Additi	onal Loads	64
		7.3.1	Procedure for Calculating Sloshing Load	64
		7.3.2	Linear Buckling Analysis	67
		7.3.3	Non-linear Buckling Analysis	70
		7.3.4	Extraction of Stresses in Spherical Coordinates	74
		7.3.5	Evaluation of Results	76
8	Buc	kling A	Analysis of Non-spherical LNG Tank	83
	8.1	Extern	nal Pressure	85
		8.1.1	Linear Buckling Analysis	85
		8.1.2	Non-linear Buckling Analysis	88
	8.2	All Lo	ads Applied	90
		8.2.1	Linear Buckling Analysis	90
		8.2.2	Non-linear Buckling Analysis	94
		8.2.3	Evaluation of Results	97
9	Buc	kling A	Analysis of Spherical LNG Tank with Different Thicknesses	99
	9.1	Extern	nal Pressure	100
		9.1.1	Linear Buckling Analysis	101
		9.1.2	Non-linear Buckling Analysis	102
	9.2	All Lo	ads Applied	105
		9.2.1	Linear Buckling Analysis	106
		9.2.2	Non-linear Buckling Analysis	107
10	Buc	kling A	Analysis of Non-spherical LNG Tank with Different Thicknesses	110
	10.1	Extern	nal Pressure	112
		10.1.1	Linear Buckling Analysis	112
		10.1.2	Non-linear Buckling Analysis	113
11	Disc	cussion	ı	116
	11.1	Procee	lures	116
		11.1.1	Simplifications in the Analyses	116
		11.1.2	Extraction of Stresses in Spherical Coordinates	117
		11.1.3	Determination of Buckling Point	118
		11.1.4	Application of Imperfections	119
	11.2	Result	s	120

	11.3	11.2.1 External Pressure 1 11.2.2 All Loads Applied 1 11.2.3 Non-spherical Tank Compared with Class Guideline DNVGL-CG-0134 1 Suggestion Regarding Rules for Non-spherical Tank 1	20 22 26 27
12	Con	clusion 1	29
13	Furt	ther Work 1	32
Re	efere	nces 1	35
A	ppe	ndix	i
A	Pyt	hon Scripts	ii
	A.1	Computation of Sloshing Load	ii
	A.2	Buckling Check According to DNVGL (1997)	v
	A.3	Buckling Check According to DNVGL (2004)	vi
	A.4	Buckling Check of Cylinder According to DNVGL (2017)	vii
	A.5	Plotting of Force-displacement Curve	viii
	A.6	Reading of Largest Resultant Imperfection From File	ix
	A.7	Generation of Stress-strain Curve for Elasto-plastic Aluminium	х
в	Rou	tines from LS-PrePost	xi
	B.1	Function for Application of Static Sloped Liquid Surface	xi
\mathbf{C}	Pers	sonal Communication	xii
	C.1	Communication with Dynamore Nordic on Extraction of Stresses	xii
	C.2	Communication with Dynamore Nordic on Direction of Spherical Stresses	civ
	C.3	Communication with Dynamore Nordic on Material Model	vii

List of Tables

0.0.1	Results from non-linear analyses of spherical and non-spherical tank in alu-	
	minium exposed to external pressure	111
0.0.2	Results from non-linear analyses of spherical and non-spherical tank in alu-	
	minium exposed to all loads	iii
0.0.3	Resultater fra ulineær analyse av sfærisk og avlang tank i aluminium fra første	
	lasttilfellet	v
0.0.4	Resultater fra ulineær analyse av sfærisk og avlang tank i aluminium fra andre	
	lasttilfellet	v
1.5.1	Dimensions of different zones of the tank provided by Moss Maritime	9
3.4.1	Design pressures for buckling analysis of spherical tanks (DNVGL, 2016b, p. 39)	25
5.1.1	Parameters used in simulation of a half-sphere in Dyna	35
5.1.2	Results from linear buckling analysis in Dyna, analytic result and Abaque $\ .$.	38
5.2.1	Parameters used in simulation of one half of a sphere in Dyna	39
5.2.2	Results from linear buckling analysis of half-sphere with realistic dimensions in	
	Dyna	40
6.0.1	Parameters in non-linear analysis	42
6.1.1	Largest displacement in each buckling mode before scaling	42
6.1.2	Results from non-linear analysis of half-sphere with imperfection type nr. 1 and	
	elastic material	44
6.2.1	Parameters for elasto-plastic steel S235	46
6.2.2	Summary of results on non-linear analysis of half-sphere	48
6.3.1	Calculated critical pressure by Classification Notes 30.1	52
6.3.2	Critical pressure from non-linear analysis in Dyna	52
6.4.1	Results with buckling mode nr. 1 as imperfection	53
7.0.1	Properties of Aluminium A5083, annealed condition	59

7.1.1	Results from linear buckling analysis of full spherical tank \ldots \ldots \ldots \ldots	61
7.2.1	Displacement and contribution for each mode with steel S235 \ldots	62
7.2.2	Results from non-linear buckling analysis with steel S235 \ldots	62
7.2.3	Displacement and contribution for each mode with a luminium	63
7.2.4	Results from non-linear buckling analysis with a luminium alloy 5083 \ldots	63
7.3.1	Results from linear buckling analysis with several loads $\ldots \ldots \ldots \ldots \ldots$	69
7.3.2	Displacement and contribution from each buckling mode with all the loads	
	combined \ldots	70
7.3.3	Results from non-linear analysis with all loads combined \ldots \ldots \ldots \ldots	73
7.3.4	Evaluation of results from non-linear analysis with several loads	80
8.1.1	Results from linear analysis of non-spherical tank exposed to external pressure .	87
8.1.2	Results from linear analysis of non-spherical tank under external pressure with	
	different thickness on cylindrical section $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	88
8.1.3	Results from non-linear analysis of non-spherical tank $\ldots \ldots \ldots \ldots \ldots$	90
8.2.1	Results from linear anlysis of non-spherical tank with all loads applied $\ . \ . \ .$	94
8.2.2	Displacement and contribution from each buckling mode with all the loads	
	combined for non-spherical tank	94
8.2.3	Results from non-linear analysis of non-spherical tank with all loads $\ldots \ldots \ldots$	96
8.2.4	Evaluation of results from non-linear analysis of non-spherical tank with several	
	loads	98
9.0.1	Approximated dimensions of the zones of the spherical tank	00
9.1.1	Results from linear analysis of spherical tank with different sections exposed to	
	external pressure	102
9.1.2	Displacement and contribution from each buckling mode with all the loads	
	combined for spherical tank with different sections	02
9.1.3	Results from non-linear analysis of spherical tank with different thicknesses	
	exposed to external pressure	105
9.2.1	Results from non-linear analysis of spherical tank with different thicknesses and	
	all loads applied	109
10.0.1	Dimensions of different zones of the non-spherical tank	11
10.1.1	Results from linear analysis of non-spherical tank with different thicknesses,	
	exposed to external pressure	13

10.1.2	Displacement and contribution from each buckling mode with all the loads
	applied for non-spherical tank with different sections $\ldots \ldots \ldots$
10.1.3	Results from non-linear analysis of non-spherical tank with different thicknesses,
	exposed to external pressure
11.2.1	Results from linear buckling analysis of spherical tank with uniform thickness
	with several loads $\ldots \ldots \ldots$
11.2.2	Results from non-linear buckling analysis of spherical tank with several loads $~$. 124
11.3.1	Evaluation of results from non-linear analysis of non-spherical tank with several
	loads
12.0.1	Results from non-linear analyses of spherical and non-spherical tank in alu-
	minium exposed to external pressure
12.0.2	Results from non-linear analyses of spherical and non-spherical tank in alu-
	minium exposed to all loads

List of Figures

1.4.1	Movement of the natural gas	5
1.4.2	Cross-section of one spherical tank type B, provided by Moss Maritime	7
1.4.3	Cross-section of ship with spherical tank type B (Zhan et al., 2015)	7
1.4.4	Cross-section of ship with membrane tanks (Zhan et al., 2015)	7
1.5.1	Outline of spherical tank type B (Odland, 1991)	8
1.5.2	Scantling of spherical LNG tank provided by Moss Maritime	9
1.5.3	Nomenclature of Spherical LNG Tank (DNVGL, 2016b)	10
1.5.4	Nomenclature of cargo hold (DNVGL, 2016b)	11
1.6.1	Illustration of non-spherical tank	12
3.1.1	Stress-strain relation for perfect and imperfect shell (Amdahl, 2010)	16
3.3.1	Buckling mode of an egg exposed to external pressure (Zhang et al., 2017)	18
3.3.2	Clothoidal shell shape	19
3.3.3	Illustration of non-spherical tank \ldots	20

3.4.1	Stiffened cylindrical shell (DNVGL, 1997)	27
5.1.1	Regular mesh generated by Abaqus	34
5.1.2	Axisymmetric mesh generated by Patran	35
5.1.3	Model of half-sphere with boundary conditions	36
5.1.4	Eigenvalue nr. 1 (left) and nr. 5 (right) from Abaqus with axisymmetric mesh	
	from Tall et al. (2018) \ldots	36
5.1.5	Buckling mode 1 in Abaqus and Dyna	37
5.1.6	Buckling mode 5 in Abaqus and Dyna	37
5.1.7	Buckling mode nr. 1 with element formulation 1 and 2 in Dyna \ldots .	38
5.2.1	Buckling mode nr. 1 and 2 for half-sphere with realistic dimensions	39
5.2.2	Buckling mode nr. 3 and 4 for half-sphere with realistic dimensions	40
5.2.3	Buckling mode nr. 5 for spherical tank with realistic dimensions	40
6.1.1	Pressure-displacement curve for node nr. 5945	43
6.1.2	Pressure-displacement curve for node nr. 4274	44
6.1.3	Resultant displacement, applied load of $0.7178~\mathrm{MPa},$ displacement scaling factor	
	$= 100 \ldots \ldots$	45
6.1.4	Resultant displacement, applied load of 0.583 MPa (post-buckling), displace-	
	ment scaling factor = $100 \dots $	45
6.2.1	Stress-strain curve for steel based on Ramberg-Osgood equation from Misovic,	
	Tadic, and Lucic (2016)	47
6.2.2	Pressure-displacement curve for elastic and elasto-plastic material, largest im-	
	perfection of 0.0088 m	48
6.2.3	Resultant displacement with elastic material, applied load of 0.879 MPa \ldots	49
6.2.4	Resultant displacement with elasto-plastic material, applied load of 0.640 MPa $$	49
6.2.5	Resultant displacement with elasto-plastic material, applied load of 0.063 MPa	
	(post-buckling)	50
6.3.1	Pressure-displacement curve for elasto-plastic material with different imperfec-	
	tions	53
6.4.1	Pressure-displacement curve for elastic and elasto-plastic material, buckling	
	mode nr. 1 as imperfection	54
7.0.1	Model of the spherical tank with skirt	56
7.0.2	Model of spherical tank with boundary conditions $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$	57
7.0.3	Close-up picture of the skirt	58

7.0.4	Stress-strain curve for elasto-plastic aluminium based on Ramberg-Osgood from				
	Misovic, Tadic, and Lucic (2016)	59			
7.1.1	Resultant displacement on buckling mode nr. 1 (top-view) and nr. 5 (bottom				
	view)	60			
7.1.2	Resultant displacement on buckling mode nr. 9, top-view	60			
7.3.1	Hydrostatic pressure from sloped liquid surface				
7.3.2	Calculation of resulting acceleration				
7.3.3	Resultant displacement on bucking mode nr. 1 from linear analysis with all				
	loads applied, external pressure of 0.05 MPa, bottom-view	69			
7.3.4	Resultant displacement on buckling modes for all loads combined, bottom view.	70			
7.3.5	Explanation of application of loads in non-linear analysis	72			
7.3.6	Resultant displacement at buckling with ext. pres. of 0.0075 MPa, initial				
	imperfection of 0.04 m.	74			
7.3.7	Post-buckling shape with external pressure of 0.0075 MPa, initial imperfection				
	of 0.04 m	74			
7.3.8	Stress in θ - and ϕ -direction when exposed to external pressure only	75			
7.3.9	Stress in θ - and ϕ -direction when exposed to all loads $\ldots \ldots \ldots \ldots \ldots \ldots$	76			
7.3.10	Force displacement curve for first analysis in table 7.3.3	77			
7.3.11	Normal stress components in the shell when exposed to all loads \ldots	81			
7.3.12	Von Mises Stress on skirt with deformation scale factor of 70, all loads applied				
	on spherical tank	82			
0.0.1		0.9			
8.0.1	Model of the non-spherical tank	83			
8.0.2	Model of non-spherical tank with boundary conditions	84			
8.1.1	Buckling modes with deformation scaling factor of 200	85			
8.1.2	Buckling mode nr. 3 with deformation scaling factor of 200	86			
8.1.3	Buckling modes for non-spherical tank with thickness of 0.15 m on cylindrical	07			
014		87			
8.1.4	Buckling modes for non-spherical tank with thickness of 0.25 m on cylindrical	00			
015		88			
8.1.5	Resultant displacement after buckling for non-spherical tank	89			
8.1.6	Force-displacement relation for non-linear buckling of non-spherical tank	89			
8.2.1	Rotated model and not	91			
8.2.2	Buckling modes from linear analysis of non-spherical tank with several loads,	0.2			
	bottom view	92			

8.2.3	Buckling modes from linear analysis of non-spherical tank with several loads,			
	bottom view	92		
8.2.4	Buckling mode nr. 10 from linear analysis of non-spherical tank with several			
	loads, bottom view	93		
8.2.5	Deformed shape at buckling for the non-spherical tank, first analysis in table			
	8.2.3	95		
8.2.6	Deformation of non-spherical tank before buckling with scaling factor of 200,			
	first analysis in table 8.2.3	95		
8.2.7	7 Force-displacement relation for non-spherical tank with all loads applied			
8.2.8	B Deformed shape at buckling for the non-spherical tank			
8.2.9	Force-displacement curve for non-spherical tank with all loads applied \ldots .	97		
9.0.1	Model of spherical tank with different sections	99		
9.1.1	Resultant displacement on buckling mode nr. 1 from linear analysis with exter-			
	nal pressure	101		
9.1.2	Resultant displacement at buckling for spherical tank with different thicknesses,			
	largest initial imperfection of $0.4 \text{ m} \dots $	103		
9.1.3	Force-displacement relation for spherical tank with different thicknesses, largest			
	initial imperfection of 0.04 m \hdots	104		
9.1.4	Force-displacement relation for spherical tank with different thicknesses $\ . \ . \ .$	104		
9.2.1	Buckling mode nr. 1 for non-linear analysis of spherial tank with different			
	thicknesses with all loads applied	107		
9.2.2	Resultant displacement after buckling for sphere with different thicknesses with			
	all loads applied	108		
9.2.3	Force-displacement curve for non-linear analysis of spherical tank with different			
	thicknesses and no imperfections	109		
10.0.1	Model of non-spherical tank with different sections	110		
10.1.1	Buckling modes from linear analysis of non-spherical tank with different thick-			
	nesses exposed to external pressure	112		
10.1.2	Resultant displacement at buckling for non-spherical tank with external pressure	e114		
10.1.3	Force-displacement relation for non-spherical tank with external pressure $% \left({{{\bf{x}}_{{\rm{s}}}} \right)$	114		

Chapter 1

Introduction

A large amount of the transported gas is done at sea by LNG (liquefied natrual gas) carriers. LNG ships carry natural gas in different types of tanks. One example of a tank is the Moss LNG tank. This is a spherical container for liquid natural gas. According to Moss Maritime, the requests has increased for larger LNG tanks to carry larger amounts of gas per ship. In order to do this, the spherical tank must be modified in order to comply with the restrictions of the Panama Canal. A wider ship would violate the restrictions. The challenge by modifying the shape of the tank is to predict the buckling capacity. Accessible rules today are mostly for spherical tanks. A modified shape of the tank with different curvatures would need a modified set of rules to predict the buckling load of the tank.

1.1 Objective

The objective of this thesis is to conduct finite element analyses on the spherical tank from Moss Maritime, and to extend these analyses to account for a non-spherical tank. Existing design formulas should be utilized with respect to the tanks. If the existing rules are not applicable on the non-spherical tank, a proposition should be made on modification of the rules. This modification should adjust the rules to account for the non-spherical shape.

1.2 Scope of Work

What is mainly considered in this report is the buckling capacity of the tank. The rules and regulations in DNVGL (2016b) defines a set of loading conditions that should be considered when evaluating the strength of the spherical LNG tank. Mainly two loading conditions are used in the analyses. The first loading condition contains only external pressure, while the second loading condition contains several loads. Both linear and non-linear analyses are performed for both loading conditions. The work is split up in the following steps according to the task description given after the title page in this thesis:

- 1. A brief description of various types of gas carriers is given, in addition to an outline of the design of Moss Rosenberg spherical tank and the extension of this into alternative shapes.
- Based on the work carried out in the project thesis, a summary of relevant code and literature formulas for estimation of the buckling capacity of spherical shells are provided. Focus is placed in estimates of the critical pressure for different shapes.
- 3. A modal analysis is conducted on the sphere to determine the tank's deformation patterns.
- 4. A non-linear analysis is performed in LS-Dyna to determine the critical pressure on a sphere with realistic imperfections. The analysis is performed with both elastic and elasto-plastic materials. The results are then be compared to simplified theory.
- 5. A non-linear analysis of the tank shapes deviating from a spherical shell is performed and the results are compared with available buckling theories.
- 6. The results and observations from the non-linear calculations are condensed into modifications of analytic formulas.
- 7. Conclusions and recommendations for further work are then given.

1.3 Structure of Thesis

This thesis starts by an introduction of natural gas and how it is transported today. A review is given on the different gas carriers, in addition to the design of the Moss LNG tank. The shape of the non-spherical tank is then explained.

The thesis continues by stating the method used followed by a literature study conducted on the topic. This literature study includes buckling on different geometrical shapes, both spherical shells and non-spherical shells. A compressed chapter on finite element theory is given after this before the analyses are presented.

The main part of this thesis consists of several finite element analyses. These analyses build upon each other. The first analysis in chapter 5 uses a simple half-sphere to verify the approach in the finite element software. A linear analysis is performed to start with, and this is followed by a non-linear analysis in chapter 6. When the analysis of the half-sphere is finished, the model is extended to a full spherical LNG tank. The same types of analyses are performed on this model, both linear and non-linear analysis. An additional loading condition containing several loads is introduced on the spherical LNG tank. The linear and non-linear analyses are performed for this loading condition also. The analyses on the spherical LNG tank are presented in chapter 7. A model of the non-spherical LNG tank is made next and analysed in chapter 8. This model is analysed when exposed to the two different loading conditions that were applied on the spherical tank. Both linear and non-linear analyses are performed for each loading condition. The last models that were analysed are the spherical and non-spherical tank split into sections. Each section in the models has an individual thickness. These are the most realistic models used in this thesis. Linear and non-linear analyses are attempted for both loading conditions. Discussion, conclusion and recommendations for further work are given at the end.

1.4 Background

This section will give an overview of the production and transport of natural gas today. After a short overview of the movement of the natural gas, the focus will be on the transport of natural gas. Types of gas-carrying ships will then be outlined.

1.4.1 Natural Gas

Natural gas is a hydrocarbon product used for fuel in different types of engines. Natural gas mainly consists of methane, but other hydrocarbon gases may also be a part of the mixture. The gas is found deep down in the soil under numerous layers of rocks. It is formed by organic material that is buried by layers of rock and sand. During millions of years, the layers on top of the organic material will grow, and the pressure on the organic material increases. Finally, hydrocarbons are made from the process. Among these hydrocarbons may natural gas be a component. In order to find the gas, the most common way is to search for pockets of gas in the soil. These pockets are formed because the gas travels up in the soil until it meets an impermeable rock layer. The gas will stop under this layer, and the amount of gas will grow as more gas finds the way up to this point. In order to collect the natural gas, a well can be drilled through to the pocket, and the gas can be transported further (IGU, 2018).

1.4.2 Transport of LNG

In order to make use of the collected gas from the soil, it must be processed, thereafter transported to the desired place and customer. The gas may be drilled up from a well on land, or at sea. Either way, the gas is transported to a processing facility. After the gas is processed, it should be transported. The transport may happen in several ways, two of the ways are either by pipelines, or by shipping of the gas. Shipping of gas can be done with liquefied gas, or by pressurized gas. Figure 1.4.1 gives a short illustration of the movement of the natural gas (IGU, 2018). The orange circle in figure 1.4.1 indicates the position of the LNG tankers in the movement of the natural gas from collecting to usage.

The international gas union made a "world report" for 2017 (IGU, 2017). This report stated that there were 439 LNG carriers world wide at the end of 2017. During 2017, 31 new builds entered the market. This report illustrates the growing need for LNG vessels as transport of natural gas increases. The following quot from page 38 in the IGU-report substantiates the statement from Moss Maritime about the need for larger transport capacity: "Tanker storage capacity continues to grow as charterers prefer larger tankers that reduce the unit cost of transported LNG."



Figure 1.4.1: Movement of the natural gas

1.4.3 Types of Gas Carriers

The gas carriers are often classified by the type of cargo they carry. Different types of carriers are designed to carry different types of cargo. The types of carriers are given below according to Central Commission for the Navigation of the Rhine and Oil Comapnies International Marine Forum (2010):

- fully pressurized tankers.
- semi-pressurized tankers.
- fully refrigerated tankers.

The fully pressurized tankers use containment systems that carry pressurized gas, and not liquid. Because the gas is pressurized, and not liquefied, the amount of gas transported is smaller than for liquefied gas transport. This type of ships are suitable for transport of gas between small terminals. Semi-pressurized tankers are ships carrying liquefied gas at low temperatures. Fully refrigerated tankers use a containment system that can carry liquefied gas at atmospheric pressure and low temperatures. The different ships carrying gas has different types of containment systems. The distinctions between the different types of tanks can be made on several properties of the construction. Some tanks are an integrated part of the ship hull, and will contribute to stiffness of the total ship hull. Some tanks are independent of the hull and self-supporting. These tanks do not contribute to the strength of the hull. The main distinctions are made between an independent tank and integrated tank. As well as the total arrangement of the tank. Some of the tanks are explained according to Central Commission for the Navigation of the Rhine and Oil Comapnies International Marine Forum (2010):

- type A.
- type B.
- type C.
- membrane tank.

Independent tank type A is prismatic, fully refrigerated tank. This tank consists of flat parts, and carry cargo near atmospheric pressure in fully refrigerated condition. This tank needs a secondary containment system in order to ensure safety because the tank is not safe enough itself against leakage. Between the tank and the secondary barrier is a space filled with inert gas in order to prevent fires and explosions.

Type B can be a tank consisting of flat surfaces, or it can be of spherical shape. This type of tank does not need the secondary containment in the same extend as type A tanks. If the tank is spherical, the part over deck is covered by a steel dome, while the part below deck has a partly secondary barrier. The space between the tank and the barrier is filled with inert gas or dry air. A spherical tank of type B is the most common shape of type B, and this is only used for transport of LNG. The independent tank type B is considered in this report.

Type C tanks are most often spherical or cylindrical in shape, and this type has a higher capacity of maximum pressure. The tanks are either fully or semi-pressurized. It is not a requirement for this tank to have a secondary barrier.

Membrane tanks are constructed with a primary barrier and secondary barrier. The primary

barrier is for membrane tanks very thin, and is mostly supported by the isolation of the tank. This tank is not self-supporting, therefore the ship hull is strengthened by the tank. Figure 1.4.3 and 1.4.4 show a ship with independent tanks type B and membrane tanks respectively. Figure 1.4.2 shows a cross-section of one independent tank type B.



Figure 1.4.2: Cross-section of one spherical tank type B, provided by Moss Maritime



Figure 1.4.3: Cross-section of ship with spherical tank type B (Zhan et al., 2015)



Figure 1.4.4: Cross-section of ship with membrane tanks (Zhan et al., 2015)

1.5 Design of Spherical LNG Tank



Figure 1.5.1: Outline of spherical tank type B (Odland, 1991)

The independent tank type B is the tank Moss Maritime use in their LNG carriers. One example of this tank can be seen in figure 1.5.1. The tank itself is a sphere with a tower in the middle. This tower is used to load, and unload the tank with LNG. The tank is supported along the equator line. From the equator line and down to the ship bottom is a cylindrical skirt supporting the tank. Between the tank and the bottom of the ship is a spacing that is filled with inert gas or dry air to prevent explosions of leakage from the tank. Below the bottom of the tank is a "drip tray". This is used to collect any LNG that leaks from the tank. The top of the tank is covered by a hemispherical dome that is a part of the ship structure, and not the tank itself. These covers support the strength of the ship (DNVGL, 2016b). Figure 1.5.2 shows a scantling of the tank and the skirt. The tank is divided into different zones. Each zone has a separate height, thickness and weight. One reason for this is that the loads on the tank is different in the different areas of the tank. Along the equator line and just below this line is a thicker section than in the rest of the structure. Table 1.5.1 shows the thickness, height and weight of the different zones.

Zone	Height [mm]	Thickness [mm]	Weight [tonnes]
1	1871	53	40
2	7173	53	152
3	10806	66	285
4L	3006	71	85
Equator-line	1350	195	76
4U	3006	57	68
5	10806	51	220
6	7173	32	92
7	1871	40	30

Table 1.5.1: Dimensions of different zones of the tank provided by Moss Maritime



Figure 1.5.2: Scantling of spherical LNG tank provided by Moss Maritime

Typical nomenclature for the spherical tank can be seen in figure 1.5.3. The figure shows a cross section of the tank including the ship double bottom and the cover for the top of the tank.

The space filled with dry air or inert gas can be seen between the tank and the ship structure. Figure 1.5.4 illustrates the ship structure below the tank were the tank has been removed. The nomenclature is also included in this figure.



Figure 1.5.3: Nomenclature of Spherical LNG Tank (DNVGL, 2016b)



Figure 1.5.4: Nomenclature of cargo hold (DNVGL, 2016b)

1.6 Design of Non-spherical tank

The main objective of this thesis treats a non-spherical LNG tank. The purpose of the nonspherical tank is to carry more LNG per ship, and the idea of this shape is to extend the tank in the longitudinal direction of the ship. This can be done by introducing a cylindrical section in the middle, with spherical or elliptic end caps. An illustration of the non-spherical tank can be seen in figure 1.6.1. This figure has a cylindrical middle section with one half-sphere at each end.

The tank is composed by the shape of a sphere/ ellipsoid and a circular cylinder. This will cause the supporting skirt to have an oval shape, instead of a circular cylinder as for the spherical tank. The end caps of the tank may be elliptic, but only spherical end caps are used in this thesis.



Figure 1.6.1: Illustration of non-spherical tank

One important remark regarding the non-spherical shape is made on the load carrying behaviour of the different sections. The spherical sections are curved in two directions normal to each other, but the cylindrical section is only curved in one direction. This makes the stresses higher in the cylinder than in the sphere. The circumferential stress in the cylinder is specifically two times the circumferential and meridional stress in the spherical sections. The circumferential and meridional stress in the sphere is given by equation 1.6.1, while the circumferential stress in the cylinder is given by equation 1.6.2. Because of this, the cylindrical section needs to have twice the thickness of the spherical sections when modelling the tank. The circumferential direction of the cylinder goes around the circumference of the cylinder, while the circumferential direction of the sphere goes in the horizontal direction in figure 1.6.1. The meridional direction in the sphere is normal to the circumferential direction.

$$\sigma_{Circumferential}^{Sphere} = \frac{PR}{2t} \tag{1.6.1}$$

$$\sigma_{Circumferential}^{Cylinder} = \frac{PR}{t} \tag{1.6.2}$$

Chapter 2

Method

This chapter presents the method used in this thesis. A description of the software that is applied will be given first, then a general explanation on how the software was used to produce reliable results is given.

2.1 Software

The programs used were Patran, Abaqus CAE and LS-Dyna in connection with LS-PrePost. Python was used for interpretation of results and verification. Geogebra was used in a small extent to visualize geometric challenges.

The finite element software used in this thesis were mostly LS-Dyna. The work was conducted in collaboration with Moss Maritime, and they requested that LS-Dyna was used as finite element solver. To solve a finite element model in LS-Dyna, three steps were used in this thesis. First Patran was used to create the geometric model. This program has an user interface that makes it easy to design different kinds of spherical shapes, along with minor details in the model. Patran also has an option to convert the model into a key-file that can be interpreted by LS-PrePost. The geometric model was therefore transported from Patran to LS-Prepost. The different specifications for the analysis were specified in LS-Prepost, then the finite element problem was solved using LS-Dyna finite element solver.
The other finite element software used in this report was Abaqus CAE. Abaqus was used in some of the introductory analyses. The reason for this was to compare the results from the introductory analysis between LS-Dyna, Abaqus and analytic results. This would hopefully verify that the results from LS-Dyna were reliable. The problems analysed in this thesis were considered to be of static behaviour. A main distinction between Abaqus and LS-Dyna is that Abaqus started out as a static software, and were expanded to dynamic application later (Eworks Global, 2005). While LS-Dyna started out as a dynamic software, and were expanded to static application (Livermore Software Technology Corporation, 2019). To perform a static analysis in LS-Dyna may therefore by slightly more challenging than in Abaqus. But the two finite element softwares seem to give approximately the same result. This is discussed in section 5.1.

Python was used for interpretation of results and calculations. This was useful for producing different plots, and for doing the same calculations for different problems. Several python scripts can be found in appendix.

2.2 General Apporach

The finite element software is used to produce the results in this thesis. A simple case is used in the first analysis in order to verify the results. When these results are verified, the model can be extended to include more details. The detailed model is then compared against analytic results, before it is extended further. The general approach in this thesis is therefore to utilize the finite element software, but to verify these results before the model is extended until the desired complexity of the problem is reached.

With respect to the literature study in this thesis, it will primarily consist of primary and secondary literature according to Schembri, 2007. Primary literature is most often published scientific work, and it has been peer-reviewed. The rules and regulations provided by Moss Maritime in connection with this thesis are examples of secondary literature. While the articles presented are an example of primary literature. Tertiary literature is used in a small extent in the form of textbooks.

Chapter 3

Literature Study on Buckling of Spherical and Non-spherical Shells

A compressed section on buckling of spherical shells is presented first. Then a summary of some relevant articles concerning buckling strength of spherical and non-spherical shells is presented. A summary of the relevant rules and guidelines from DNV GL will be given last.

3.1 Buckling of Shells

A shell can be made from a plate by making the surface curved in one or two directions. Shells are usually thin structures, and can experience buckling if exposed to compressing forces. The load carrying of a shell differs from a plate. A plate carry lateral load by bending stresses, while a shell can carry lateral load by a larger part of membrane stresses than bending stresses (Moan, 2003). This makes the shells suitable for pressure vessels for instance. Because of the small thickness in shells, they may experience buckling when exposed to large forces and pressures. Figure 3.1.1 illustrates the relation between the normalized stress and the normalized strain for a structure that buckles. The path for both a perfect and imperfect shell is shown in the figure. The "B" in the figure shows where the bifurcation point is located. This is the point for buckling of a shell that has no imperfections. The "L" shows the point of buckling load for a shell that has initial geometric, and possibly material imperfections.



Figure 3.1.1: Stress-strain relation for perfect and imperfect shell (Amdahl, 2010)

3.2 Spherical Shells

Two relevant articles were chosen that were treating buckling strength of spherical shells. The articles were "Elastoplastic buckling and collapse of spherical shells under combined loadings" by Tall et al. (2018), and "Design buckling pressure for thin spherical shells: Development and validation" by Evkin and Lykhachova (2018).

3.2.1 "Elastoplastic buckling and collapse of spherical shells under combined loadings" (Tall et al., 2018)

The objective of the article by Tall et al. (2018) was to develop a set of buckling capacity curves for spherical shells exposed to a combination of external pressure and torsional loading. The curves were constructed by plotting buckling capacity against slenderness. Slenderness is defined in this article as the square root of buckling stress in material non-linear analysis, divided by buckling stress from linear bifurcation analysis. These curves can be of assistance when designing spherical shells against the specified loading condition. The article used a set of parametric equations to describe these curves. The relevant part of the article for this report were concerning the elastic buckling strength. The article uses an analytic expression for the elastic buckling strength derived by Zoelly (1915).

when considering elastic buckling is given in equation 3.2.1.

$$P_{cr} = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{R}\right)^2$$
(3.2.1)

The article uses five different analyses to analyse the sphere exposed to external pressure. The analysis procedures were:

- linear bifurcation analysis.
- material non-linear analysis.
- geometrically non-linear analysis.
- geometrically non-linear imperfect analysis.
- geometrically, and materially non-linear imperfect analysis.

The linear analysis gave an error of less than 1 % compared to the Zoelly critical pressure (Tall et al., 2018). The other approaches were used to establish the buckling curves based on numerical simulations in Abaqus.

3.2.2 "Design buckling pressure for thin spherical shells: Development and validation" (Evkin and Lykhachova, 2018)

This article focuses on clamped spherical caps exposed to external pressure. The article starts by defining the Zoelly critical pressure given in equation 3.2.1. The article concludes by defining an equation for the buckling capacity of a spherical cap. This formula includes effect of imperfections, plasticity and geometric non-linearities. It is therefore able to describe a more accurate design buckling pressure than the elastic buckling pressure predicted by Zoelly. This is wanted because thin shell structures experience bifurcation buckling which implies a significantly higher linear buckling load than the non-linear buckling load. The resulting expression for the buckling capacity was extensive and can be found in Evkin and Lykhachova (2018).

3.3 Non-spherical Shells

Three articles were found to treat buckling of non-spherical shells. The article by Sano et al. (2017) treats buckling of a non-spherical LNG tank. While the article by Zhang et al. (2017) treats elastic buckling of egg-shaped shells. Finally the article by Jasion et al. (2015) treats elastic buckling of a compounded shell. This shell is composed by a middle section of a clothoid, and the end sections of a sphere. Figure 3.3.2 illustrates this shape.

3.3.1 "Buckling of egg-shaped shells subjected to external pressure" (Jasion et al., 2015)

The different shapes of eggs and their capacities were studied in this article. The authors used Abaque to determine the buckling capacity of these eggs. The buckling mode for an egg analysed in the article can be seen in figure 3.3.1.



Figure 3.3.1: Buckling mode of an egg exposed to external pressure (Zhang et al., 2017)

A previously derived formula by Babich (1993) was used to determine the linear buckling load for the eggs. The formula can be seen in equation 3.3.1 to 3.3.3. The critical pressure is given by equation 3.3.1, while the input parameters for this equation is given in equation 3.3.2 and 3.3.3. The parameters "a" and " β " defines the geometry of the egg. A value of $\beta = 0$ would create a circle, while increasing β gives an egg-shape. Parameter "a" is defined as half of the length of the axis of revolution for the shell. While \bar{x} and \bar{r} are $\bar{x} = \frac{x}{a}$ and $\bar{r} = \frac{r}{a}$ respectively, which are the normalized two-dimensional coordinates.

It can be seen from equation 3.3.1 when $2R_1 - R_2$ are equal to 1, this formula reduces to the Zoelly critical pressure in equation 3.2.1, with R_2 as the governing radius. This can for instance

be for $R_1 = R_2 = 1$. This formula for the elastic buckling pressure of egg-shaped shells can therefore be interpreted as an extension of the Zoelly critical pressure. Were the extension accounts for the geometry of the egg.

$$P_{cr} = \frac{2Et^2}{(2R_1 - R_2)R_2} \frac{1}{\sqrt{3(1 - \nu^2)}}$$
(3.3.1)

$$\frac{1}{R_1} = -\frac{\bar{r}''}{a[1+(\bar{r}')^2]^{3/2}}, \frac{1}{R_2} = \frac{1}{a\bar{r}[1+(\bar{r}')^2]^{1/2}}$$
(3.3.2)

$$\bar{r} = \frac{\sqrt{3}}{2} \sqrt{\bar{x}(2-\bar{x}) \left[1 - \frac{\beta^2}{(1+\bar{x})^2} \right]}$$
$$\bar{r}' = \frac{3}{4\bar{r}} \left[1 - \bar{x} + \frac{(2\bar{x}-1)\beta^2}{(1+\bar{x})^3} \right]$$
$$\bar{r}'' = \frac{1}{\bar{r}} \left(\frac{3}{4} \left[-1 + \frac{(5-4\bar{x})\beta^2}{(1+\bar{x})^4} \right] - (\bar{r}')^2 \right)$$
(3.3.3)

3.3.2 "Elastic buckling of clothoidal–spherical shells under external pressure – theoretical study" (Zhang et al., 2017)

The behaviour of the compounded shell is investigated in this article. Figure 3.3.2 illustrates the shape of the shell. A large part of the article is devoted to analyse the geometry and the stresses in the shell.



Figure 3.3.2: Clothoidal shell shape

The results of the investigation of these shells shows that an analytic formula can be used to determine the elastic buckling capacity of the shell. The formula can be seen in equation 3.3.4. This formula can again be seen as an extension of the critical pressure from Zoelly in equation 3.2.1, where the thickness-to-radius relation has been replaced by a ratio including mass (m_s) of the shell, volume (V_s) of the shell, and density of the metal (ρ_s) used in clothoidal shell. The article showed an important relation between the buckling strength of spheres compared to the clothoidal shape. Spheres was seen to have higher buckling capacities than the clothoid. The capacity gradually decreases as the sphere becomes more like a clothoid, and finally to the resemblance of a cylinder.

$$P_{cr} = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t_s}{R}\right)^2$$
(3.3.4)

$$\frac{t_s}{R} = \frac{m_s}{3V_s\rho_s} \tag{3.3.5}$$

3.3.3 "Estimation of elastic buckling strength of a non-spherical tank in the partially filled condition" (Sano et al., 2017)



Figure 3.3.3: Illustration of non-spherical tank

A non-spherical LNG tank is considered in this article. An illustration of the tank can be seen in figure 3.3.3. The loading conditions used in this article were a tank with partially filled fluid levels. The surface of the fluid will be located in the toroidal shell segment indicated in figure 3.3.3. An analytic expression for the elastic buckling load was derived in this article for the partially filled loading condition. Finite element analyses were performed in order to compare the analytic results with numerical calculations in Abaqus. The load was applied as a pressure located in the region of the toroidal segment in the simulations.

The analytic expression for the elastic buckling stress can be seen in equation 3.3.6 and 3.3.7. Equation 3.3.6 gives the elastic buckling stress (σ_E) for the non-spherical tank. The parameter K' is a factor containing a number of parameters concerning the forces in the shell, geometry of the shell and material constants. The parameter r_2 is a radial parameter used to describe the geometry of the elliptic bottom of the tank. Equation 3.3.7 can again be seen to be very similar to equation 3.2.1 which is the Zoelly critical pressure. Hutchinson (2016) presents the exact same formula as in equation 3.3.7 in connection with the Zoelly critical pressure. These formulas are presented as the elastic critical value for a perfect sphere. Hutchinson (2016) shows that both equations fulfill the differential equation based on Donnel-Mushtari-Vlasov theory for the elastic buckling problem. Equation 3.3.7 gives the elastic critical stress while equation 3.2.1 gives the elastic critical pressure. The connection is shown in section 3.4.4. In the case of the non-spherical tank, the critical level is modified by a factor (K') to account for the shape deviation from a sphere. The total expression for K' can be found in Sano et al. (2017).

$$\sigma_E = K' \sigma_{cl} \tag{3.3.6}$$

$$\sigma_{cl} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{r_2}$$
(3.3.7)

3.4 Rules for design

The relevant rules for designing the spherical LNG tank are Classification Notes 30.1 (DNVGL, 2004), 30.2 (DNVGL, 2009) and 30.3 (DNVGL, 1997), in addition to Class Guideline DNVGL-CG-0134 with the title "Liquified gas carriers with spherical cargo tank of type B" (DNVGL, 2016b). Most of 30.2 is included in the Class Guideline, consequently only the Class Guideline is presented in this thesis. These rules contain formulas that predict the buckling capacity of the spherical tank. A summary of these formulas will be given in this section.

3.4.1 Classification Notes 30.1

DNVGL (2004) presents a buckling criteria for spherical shells in general. The formula for the critical stress of the spherical tank is defined based on the φ -method. This approach modifies the critical stress because of plasticity. The expression for the critical stress can be found by Merchant-Rankine formula shown in equation 3.4.1 (Odland, 1991). The first fraction accounts for elastic buckling, while the second accounts for elasto-plastic buckling.

$$\left(\frac{\sigma_{cr}}{\sigma_E}\right)^2 + \left(\frac{\sigma_{cr}}{\sigma_y}\right)^2 = 1 \tag{3.4.1}$$

By introducing the reduced slenderness (λ) in equation 3.4.2, the formula for the critical stress for a sphere used in DNVGL (2004) can be found. This formula is shown in equation 3.4.3. The buckling capacity is calculated with respect to both elastic, and elasto-plastic buckling when using this approach. Elastic buckling refers to when a structure buckles before yield stress is reached in the material. While elasto-plastic buckling refers to when a structure buckles when the material reaches yielding (DNVGL, 1997).

$$\lambda = \sqrt{\frac{\sigma_y}{\sigma_E}} \tag{3.4.2}$$

$$\sigma_{cr} = \frac{\sigma_y}{\sqrt{1+\lambda^4}} \tag{3.4.3}$$

The equation for the elastic buckling stress (σ_E) can be seen in equation 3.4.4. This includes a knock-down factor (ρ), and this is defined in equation 3.4.5. The knock-down factor is needed because the realistic buckling load is significantly smaller than the elastic buckling load. The reason for this is because shells in general, and especially spherical shells are very imperfection sensitive (Tall et al., 2018).

$$\sigma_E = 0.606\rho E \frac{t}{R} \tag{3.4.4}$$

$$\rho = \frac{0.5}{\sqrt{1 + \frac{R}{t}}}$$
(3.4.5)

3.4.2 Classification Notes 30.3

DNVGL (1997) includes formulas for computation of buckling stress for both the cylindrical skirt supporting the tank, and the spherical tank itself. The formula for the elasto-plastic critical buckling level for the sphere (Λ_{CR}) is given in equation 3.4.6. This can be seen to be the same as equation 3.4.3. The differences are that Λ_{CR} is normalized with respect to equivalent stress including load factors (σ_{e0}), and the equation for the slenderness (λ_E) is extended. This can be seen in equation 3.4.7 where λ_E is the equivalent reduced slenderness.

$$\Lambda_{CR} = \frac{1}{\sqrt{1 + \lambda_E^4}} \frac{\sigma_y}{\sigma_{e0}} \tag{3.4.6}$$

$$\lambda_E = \sqrt{F_E \frac{\sigma_y}{\sigma_{e0}}} \tag{3.4.7}$$

The equation for F_E can be seen in equation 3.4.8. Here ρ is a knockdown factor to account for geometrical imperfections, and Λ_{CR} is the normalized elastic critical buckling value. Equation 3.4.10 gives the elastic buckling stress, and can be seen to be the same as in section 3.3.3 about the non-spherical LNG tank. This formula is shown to be equivalent to the Zoelly critical pressure in section 3.4.4.

$$F_E = \frac{1}{\rho \Lambda_{CL}} \tag{3.4.8}$$

$$\Lambda_{CL} = \frac{\sigma_{CL}}{\sigma_{10}} \tag{3.4.9}$$

$$\sigma_{CL} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R}$$
(3.4.10)

The skirt buckling has an equivalent equation for the buckling capacity (equation 3.4.6) with a different calculation of F_E and different stress components included in σ_{e0} . F_E is calculated according to equation 3.4.11. Here a, b and c forms a quite large set of formulas covering all the different forms of buckling of a stiffened cylindrical shell. The expression for Λ_E seems to be the solution of a second degree equation, but which equation is not specified.

$$F_E = \frac{1}{\Lambda_E}$$

$$\Lambda_E = \frac{1}{2a} \left[b \pm \sqrt{b^2 - 4ac} \right]$$
(3.4.11)

3.4.3 Class Guideline, DNVGL-CG-0134

The Class Guideline gives an extensive explanation of the approach for analysing the spherical cargo tank. The guideline starts by defining how to do a global analysis of the ship and the cargo hold. This is to establish the overall strength of the ship and the arrangement of the cargo. When the cargo hold is analysed, the individual tank can be considered. Stresses and loads from the global analysis are used to determine the loads on the tank. A brief outline of the global analysis will first be done, then a more detailed review will be done on the part of the guideline that focuses on the strength assessment of the spherical tank.

The global analysis of the ship and cargo hold is mainly done to determine the interaction forces between the tank and the hull. One global analysis should be done for the whole ship. This is needed to determine the strength of the total structure. In addition, one analysis is needed in order to consider the cargo hold. To analyse the cargo hold, at least three of the tanks need to be considered simultaneously in the analysis, or the two tanks in the front can be considered in connection with the front of the ship. The loading conditions needed to consider the strength of the cargo hold are given in table 2 on page 22 in the guideline. This table includes 12 different loading conditions with different combinations of empty and full tanks. Detailed instructions are given in order to preform a finite element analysis of the structure. The tanks are covered by a steel dome that is connected to the hull. The strength of this cover also needs to be considered individually. This is the next section in the guideline after the cargo hold and the full ship is considered. The guideline uses four different loading conditions that need to be investigated. Formulas for stress calculations and estimation of buckling loads on the dome are given at page 33 to 36 in the guideline.

Section 5 in the guideline deals with strength analysis of the spherical cargo tank. The different analysis procedures that need to be conducted for the tank are given here. There are ten of

these on page 37 in the guideline (DNVGL, 2016b), and they are:

- Wave load analysis of the ship.
- Assessment of interaction forces.
- Analysis of sloshing loads in cargo tank.
- Analysis of skirt and tank structure including stationary thermal loads.
- Buckling analysis.
- Fatigue analysis.
- Crack propagation analysis.
- Leak rate analysis.
- Steady state temperature and stress analysis to determine the temperature distribution in the tank system. The temperature gradient in the upper part of the skirt is of particular significance.
- Transient thermal stresses (cool down analysis). This is not a design analysis as such but has to be carried out on the final tank design in order to ensure that the tank is not overstressed due to too rapid cool down and filling up of the tank.

Where the most relevant point for this report is the buckling analysis in combination with sloshing loads and thermal loads. Table 2 on page 39 (DNVGL, 2016b) gives relevant design pressures for the buckling capacity check. This is the static loads that need to be considered. At the point when the tank is empty and partly filled, the loading conditions are given in table 3.4.1.

Table 3.4.1: Design pressures for buckling analysis of spherical tanks (DNVGL, 2016b, p. 39)

Loading case	Pressure
Empty tank	External overpressure $P_0 > 0.005$ MPa
Partly filled tank	External overpressure $P_0 > 0.005$ MPa

Procedures for calculating dynamic loads, interaction forces and design loads for partial filled tanks are given next. The design loads for partial filled condition includes one important case. This is the sloshing load from the free fluid surface. Different sloshing behaviors can happen inside the tank, but the result of the sloshing is anyway a transverse force acting at the side of the tank. The guideline presents an approach for calculating the circumferential force as a result of the sloshing. Figure 13 on page 51 in the guideline (DNVGL, 2016b) illustrates the approach. The approach is described later in this thesis in section 7.3.1.

The next part of the guideline deals with the finite element procedure of modelling the spherical tank. Detailed instructions are given on type of elements and location of stress concentrations that need to be considered more carefully. Boundary conditions are also suggested. Load cases for stress assessment is then given, and subsequently are load cases for buckling assessment given. This is given in table 8 on page 63 (DNVGL, 2016b). The load cases are:

- 1. Tank test condition.
- 2. Sea going with empty tank.
- 3. Sea going with partly filled tank.

The first load case includes few loads to be evaluated. The loads to be evaluated are:

- 1. Self weight of system.
- 2. Partial filled with fresh water.
- 3. Static interaction force due to still water bending moment and external pressure.

The second load case includes load 1 and 3 of the first, but in addition the following needs to be evaluated:

- 1. External pressure of 0.005 MPa.
- 2. Dynamic interaction force due to wave bending moment and external wave pressure.

The third load case includes all of the loads from the second case, but the partial filling need to be considered by:

1. Static and dynamic loads combined based on resulting skewed acceleration a_R .

All of these cases should be evaluated with respect to buckling of the spherical tank. After the finite element analysis is complete, the results should be verified by the DNV rules. The rules relevant for this case is Class Notes 30.3 (DNVGL, 1997), the same rules are given in appendix D in the Class Guideline (DNVGL, 2016b).

The last part of section 5 deals with non-spherical tanks. The tank has the same shape as was studied in Tall et al. (2018), which can be seen in figure 3.3.3. The shape considered in this section is a sphere with a cylindrical part around the equator with a certain height. The suggestion by the guideline when checking the capacity of this structure is to consider each part by itself. Buckling of the cylindrical part can be checked by formulas for buckling of a cylinder. While the buckling of the sphere can be checked by the formulas for buckling of a sphere. This was the part of the guideline that treats the cargo tank. What also need to be considered is the strength of the skirt supporting the tank. This is done in section 7 of the guideline.

The first part states which loads that need to be considered for the skirt. These includes both static and dynamic loads, and they can be found in the table on page 90 (DNVGL, 2016b). A series of formulas for the stress components are then introduced in order to assess the stresses and confirm that they are below the limit stated by DNV GL. With respect to the buckling capacity of the skirt, this should also be verified by Class Note 30.3 (DNVGL, 1997). Table 2 on page 97 (DNVGL, 2016b) illustrates the different buckling modes that need to be considered. Because the skirt is a stiffened cylindrical shell, it can buckle in several different ways. For instance the buckling can happen between stiffeners in the unstiffened shell segment, it can be global shell buckling, or buckling of stiffeners. Figure 3.4.1 shows a stiffened cylindrical shell. The full sized skirt supporting the spherical tank will look similar to this.



Figure 3.4.1: Stiffened cylindrical shell (DNVGL, 1997)

The remaining parts of the guideline treats the pump tower in the middle of the cargo tank, and fatigue assessment of the structures. This is not within the subject of this report and will therefore not be included in this review.

3.4.4 Comments and Considerations on Formulas in Classifications Notes 30.1 and 30.3

A comparison can be made for equation 3.4.4 against the elastic buckling stress for an unstiffened cylindrical shell under axial compression. The expression for the elastic buckling stress without the correction factor (ρ) can be seen to be the same for a sphere under external pressure, and an unstiffened cylindrical shell exposed to axial compression according to Amdahl (2010) and DNV-RP-C202 (DNVGL, 2017). This can be seen from eguation 5.62 in (Amdahl, 2010) and in section 3.3.2 in (DNVGL, 2017). It is emphasized that the similarities hold for elastic buckling capacity without knockdown factors. The equations for the elastic buckling strength of an unstiffened cylindrical shell can be seen in equation 3.4.12. Here L is the length of the cylinder. It can be seen that the elastic buckling stress is the same as for the sphere in equation 3.4.4 without knock-down factor (ρ). This is true even though a cylindrical shape is curved in one direction, and a spherical shape is curved in both directions. This shows that buckling of shells in general are related problems.

$$\sigma_E = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{L}\right)^2,$$

$$C = 0.702 \frac{L^2}{Rt} \sqrt{1-\nu^2} \Rightarrow$$

$$\sigma_E = 0.605 \frac{Et}{R}$$
(3.4.12)

By simple considerations it can be seen that the elastic buckling stress is equivalent in equation 3.4.4 (DNVGL, 2004) and 3.4.10 (DNVGL, 1997). Which is again the same as the elastic buckling stress for the cylinder in equation 3.4.12. This is shown by inserting $\nu = 0.3$ and calculating the fraction as shown in equation 3.4.13. Hence equation 3.4.10 and equation 3.4.4 are equivalent when disregarding geometrical knockdown factor ρ .

$$\sigma_{CL} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R} = 0.6052 \frac{Et}{R} = \sigma_E \tag{3.4.13}$$

It can also be shown be simple considerations that equation 3.4.10 is equivalent to the Zoelly critical pressure in equation 3.2.1. By inserting the principal membrane stresses for a sphere

exposed to lateral pressure (DNVGL, 1997) as shown in equation 3.4.14, the Zoelly critical pressure is found. Here σ_1 and σ_2 are the membrane stresses in the two different directions in the shell. The connection between the Zoelly critical pressure and equation 3.4.10 is known from Hutchinson (2016).

$$\sigma_{1} = \sigma_{2} = \frac{PR}{2t} \Rightarrow$$

$$\frac{P_{cr}R}{2t} = \frac{E}{\sqrt{3(1-\nu^{2})}} \frac{t}{R} \Rightarrow$$

$$P_{cr} = \frac{2E}{\sqrt{3(1-\nu^{2})}} \left(\frac{t}{R}\right)^{2}$$
(3.4.14)

In addition to this, it can be seen that the equation for the critical stress in Classification Notes 30.1 (DNVGL, 2004), 30.3 (DNVGL, 1997) and DNVGL-RP-C202 (DNVGL, 2017) are similar. All of them are based on the φ -approach as described in the beginning of section 3.4.1. And the critical level is given as shown in equation 3.4.15 below.

$$\sigma_{cr} = \frac{\sigma_y}{\sqrt{1+\lambda^4}} \tag{3.4.15}$$

The reduced slenderness for a cylinder is defined in DNVGL-RP-C202 as in equation 3.4.16. All the parameters denoted with σ and f are stresses. The fractions in the bracket-parenthesis are stress divided by elastic buckling stress for respectively axial stress, bending stress, circumferential stress and shear stress.

$$\lambda^2 = \frac{\sigma_y}{\sigma_{j,Sd}} \left[\frac{\sigma_{a0,Sd}}{f_{Ea}} + \frac{\sigma_{m0,Sd}}{f_{Em}} + \frac{\sigma_{h0,Sd}}{f_{Eh}} + \frac{\tau_{Sd}}{f_{E\tau}} \right]$$
(3.4.16)

For the sphere, the elastic buckling stress is the same in all directions, and we disregard shear loading. This makes it possible to consider the elastic buckling stress as one parameter denoted σ_E . The numerator will then be a sum of stresses in different directions, we denote this σ'_{10} . We can then rearrange the expression as shown in equation 3.4.17.

$$\lambda = \sqrt{\frac{\sigma_{10}' \sigma_y}{\sigma_E \sigma_j}} \tag{3.4.17}$$

If the knock-down factor (ρ) is disregarded in the formula from Classification Notes 30.3, the reduced slenderness can be written as shown in equation 3.4.18. When inserted for F_E in this formula, σ_{e0} is the equivalent stress for the sphere, while σ_j in equation 3.4.17 is equivalent stress for the cylinder. The relevant stress components for the spherical and cylindrical shell is summed up in respectively σ_{10} and σ'_{10} .

$$\lambda_E = \sqrt{\frac{\sigma_{10}}{\sigma_E} \frac{\sigma_y}{\sigma_{e0}}} \tag{3.4.18}$$

From this it can be seen that the approach to determine the critical buckling load in cylindrical shells and spherical shells are similar. The approach in DNGGL-RP-C202, Classification Notes 30.3 and 30.1 can be seen to be similar. One important difference between the three is that 30.1 does not include the equivalent stress σ_{e0} . This is included in 30.3 and RP-C202 for the cylinder. Classification Notes 30.1 assumes that the stresses in circumferential and meridional direction is the same in the spherical shell. This is not assumed in 30.3. Therefore 30.3 seems to contain a more accurate approach to determine the critical buckling level.

This section has shown that there are a lot of similarities between the different approaches to determine the critical buckling load on shells in general. It seems that all the different approaches use the same foundation to determine the different elastic buckling levels. Hutchinson (2016) shows that the Zoelly critical pressure can be derived based on Donnel-Mushtari-Vlasov theory. This theory uses a differential equation with initial conditions to derive the elastic buckling load. The Donnel equation is also used in Sano et al. (2017) and Amdahl (2010). It therefore seems that this equation is the basis for the approaches to establish the elastic buckling level. And that the φ -method is used to extend the approach to account for elasto-plastic behaviour.

Chapter 4

Brief on Finite Element Analysis

A brief introduction on finite element analysis is presented in this chapter. This chapter is especially focused on describing the theory with respect to LS-Dyna as finite element software.

4.1 Linear Buckling

The linear buckling load can be illustrated by point B in figure 3.1.1. No imperfections are included in this analysis. This can be computed by solving for the eigenvalues in the system. The eigenvalues can be found from the stiffness matrix K of the system. The equation for the eigenvalue problem can be seen in equation 4.1.1. The matrix " K_M " is the material tangent stiffness matrix, and " K_G " is the initial stress or geometric stiffness matrix. The eigenvalue, or the buckling load is denoted by " Λ ", and the corresponding buckling mode, or eigenvector is "u" (Livermore Software Technology Corporation, 2017).

$$(K_M + \Lambda K_G)u = 0 \tag{4.1.1}$$

4.2 Non-linear Buckling

To be able to find the buckling load L on the non-linear curve in figure 3.1.1, another solution procedure must be used. The solution procedure in LS-Dyna to solve non-linear buckling problems is "the arc-length method". Dyna uses a combination of Riks and Wempner with BFGS method. The arc-length method introduces one new equation that includes both displacement and load increment. The arc-length can be calculated by this equation, which makes it possible to trace a non-linear buckling curve (Livermore Software Technology Corporation, 2017).

The non-linearities that can be introduced in a non-linear problem are non-linear material and geometric non-linearity. The material non-linearity can be introduced by elasto-plastic material. This material model introduces yield stress of the material. The stress-strain relationship will become non-linear when stresses become larger than yield stress in this material model. Geometric non-linearity takes the deformation of the structure into account (Mathisen, 2012).

4.3 Implicit and Explicit Solution Procedure

A step-wise method is needed to solve the system of equations in non-linear problems. A distinction can be made between two types of solution methods. One method is the explicit method, and the other is implicit method. The explicit method considers the equation at the old step, and uses this to compute the new step in the analysis. While the implicit method considers the equations in the new step to compute the new step in the analysis. The implicit solution method requires an iterative solution procedure. It disregards inertial effects, and is therefore suitable for static analysis. Explicit solution procedure includes inertial effects in the solution, which makes it suitable for dynamic analysis (Erhart, 2016).

Chapter 5

Linear Buckling Analysis of Half-sphere

This section includes two different analyses. An introductory eigenvalue buckling analysis was conducted first in order to understand the analysis procedure and verify results. Then the same type of analysis was conducted on a structure with realistic dimensions.

The introductory analysis conducted in LS-Dyna was compared with results from Abaqus. In order to perform the buckling analysis in Dyna, a set of keywords had to be used. Keywords are commands that are added for the specific analysis. The essential keywords for this analysis are:

- Control_Implicit_General
- Control_Implicit_Solution
- Control_Implicit_Buckle
- Control_Implicit_Eigenvalue

"Control_Implicit_General" activates the implicit solver that should be used in this analysis. "Control_Implicit_Solution" lets the user define which type of non-linear solver that should be used. A linear solver was chosen in this case. "Control_Implicit_Buckle" specifies the number of desired buckling modes, and the command writes a file ("eigout") that contains the eigenvalues of the buckling problem. "Control_Implicit_Eigenvalue" computes the eigenfrequencies, but the computation of von-Mises stress can be activated by this command.

5.1 Introductory Analysis and Comparison with Abaqus

Two different meshes were considered in this introductory analysis. Figure 5.1.1 and 5.1.2 show these meshes. Figure 5.1.1 shows a regular mesh generated in Abaqus, while figure 5.1.2 shows an axisymmetric mesh generated by Patran. The results from Dyna in the eigenvalue buckling analysis was compared with the article from Tall et al. (2018), and results from Abaqus. It was difficult to reproduce realistic buckling modes for the regular mesh in Dyna. The axisymmetric mesh in figure 5.1.2 was therefore chosen.



Figure 5.1.1: Regular mesh generated by Abaqus

Tall et al. (2018) says on page 117 in the article that the theoretical fundamental buckling mode will not be axisymmetric, and that this is shown by Koiter (1969). The axisymmetric buckling modes are most frequently occurring with the mesh in figure 5.1.2. A regular mesh (figure 5.1.1) gives different buckling modes than an axisymmetric mesh. Tall et al. (2018) show by this that the buckling modes are very dependent of the mesh used on the sphere. It is established both in DNVGL (1997) and Tall et al. (2018) that spheres are very imperfection sensitive. This causes a large number of buckling modes to be very close to each other with respect to critical buckling loads. Several modes may therefore interact in real life. It is therefore concluded that the modes identified by numerical analysis in Dyna with axisymmetric mesh may not be the fundamental buckling modes in real life structures. But the corresponding load should be the

exact linear buckling load.



Figure 5.1.2: Axisymmetric mesh generated by Patran

The sphere with the axisymmetric mesh was exposed to a uniform external pressure. The boundary conditions were determined to be zero translation and rotation around the equatorline. Only one half of the sphere was modelled because of the symmetry the buckling mode will have around the equator-line. The boundary conditions were chosen because the spherical tank is supported along the equator by the skirt. It may be more accurate to just fix against translation because the equator can have some movements. But all six degrees of freedom were fixed in this case. Specific parameters of the simulation in Dyna of one half of the sphere can be found in table 5.1.1. The same model was analysed in Abaqus with element type "S4".

Entity	Magnitude
Radius	1000 mm
Thickness	$1 \mathrm{mm}$
Uniform external pressure	$0.2 \mathrm{MPa}$
Young's modules	210e3 MPa
Poisson's ratio	0.3
Element formulation	21
Nr. of elements	24964

Table 5.1.1: Parameters used in simulation of a half-sphere in Dyna



Figure 5.1.3: Model of half-sphere with boundary conditions

The model of the half-sphere is shown in figure 5.1.3 with the boundary conditions as white crosses along the equator. Figure 5.1.5 and 5.1.6 show the first and fifth buckling mode respectively from Abaqus and Dyna. The von Mises stress is plotted on the figures for illustrative purposes. From these figures it can be seen that the buckling modes are the same in the two different finite element programs. Figure 5.1.4 shows mode shape nr. 1 and 5 in the article from Tall et al. (2018). Here it can be seen that the buckling modes are similar to the ones from Abaqus and Dyna. Only difference is the number of waves occurring in the shell. Different radius and thickness between the spheres may cause this difference. This substantiates the fact that the mesh generated by Patran is fine to use. The reliability of the results is therefore strengthened.



Figure 5.1.4: Eigenvalue nr. 1 (left) and nr. 5 (right) from Abaqus with axisymmetric mesh from Tall et al. (2018)



Figure 5.1.5: Buckling mode 1 in Abaqus and Dyna



Figure 5.1.6: Buckling mode 5 in Abaqus and Dyna $\,$

The results from the simulations are summarised in table 5.1.2. It can be seen that both Abaqus and Dyna overestimates the linear buckling load slightly compared to the Zoelly critical pressure.

Method	Critical pressure
Dyna	$0.254 \mathrm{MPa}$
Abaqus	$0.255 \mathrm{MPa}$
Zoelly critical pressure (Equation 3.2.1)	$0.250 \mathrm{MPa}$

Table 5.1.2: Results from linear buckling analysis in Dyna, analytic result and Abaqus

To compare the different element formulations in Dyna, two additional elements were tested. Figure 5.1.7 shows buckling mode nr. 1 when element formulation 1 and 2 were used. Element formulation 1 is called "Hughes-Liu", while element formulation 2 is called "Belytschko-Tsay". All the element formulations tested (1, 2 and 21) are shell elements in Dyna. Element formulation nr. 2 is the default element in Dyna. It can be seen that the buckling mode generated by element formulation nr. 1 and 2 are different from the first buckling mode in figure 5.1.5. One important difference between the elements is that element formulation 21 is specifically designed for linear analysis, but formulation 1 and 2 can be used in non-linear analysis. Formulation nr. 2 was implemented in Dyna to make a computational faster alternative to formulation nr. 1 (Hallquist, 2006). Based on the buckling shape, element formulation nr. 1 and 2 were not used further in the analyses.





(a) Element formulation nr. 1
(b) Element formulation nr. 2
Figure 5.1.7: Buckling mode nr. 1 with element formulation 1 and 2 in Dyna

5.2 Eigenvalue Buckling Analysis of Half-sphere with Realistic Dimensions

Next the eigenvalue buckling analysis was performed on a half-sphere with realistic dimensions. An axisymmetric mesh, as shown in figure 5.1.2 was chosen with element formulation 21 ("Fully integrated linear C0 shell (5DOF)") in Dyna. The diameter was taken to be 43 meters, which is a typical diameter of a spherical LNG tank according to Moss Maritime. The thickness of the tank was chosen based on table 1.5.1, which contains the thickness over the different sections of the tank. The thickness was chosen as an approximate average of 55 mm. Detailed parameters of the simulation can be seen in table 5.2.1. Figure 5.2.1, 5.2.2 and 5.2.3 show buckling mode nr. 1 to 5 for the half-sphere with realistic dimensions. It can be seen that mode nr. 1 is similar to mode nr. 1 in the introductory analysis, while nr. 5 is different. The tank with realistic dimensions has a different t/R-relationship then in the introductory analysis. This may be a reason for the different deformed shape in buckling mode nr. 5. This assumption is based on the elastic buckling pressure from Zoelly in equation 3.2.1. The t/R-relation affects the elastic buckling pressure. It is therefore assumed to affect the deformation pattern.

Entity	Magnitude
Radius	$21~500~\mathrm{mm}$
Uniform thickness	$55 \mathrm{~mm}$
Uniform external pressure	$0.5 \mathrm{MPa}$
Young's modules	210e3 MPa
Poisson's ratio	0.3
Element formulation	21
Nr. of elements	22500

Table 5.2.1: Parameters used in simulation of one half of a sphere in Dyna



(a) Buckling mode nr. 1



(b) Buckling mode nr. 2

Figure 5.2.1: Buckling mode nr. 1 and 2 for half-sphere with realistic dimensions





(a) Buckling mode nr. 3

(b) Buckling mode nr. 4

Figure 5.2.2: Buckling mode nr. 3 and 4 for half-sphere with realistic dimensions



Figure 5.2.3: Buckling mode nr. 5 for spherical tank with realistic dimensions

The results of the eigenvalue buckling analysis can be seen in table 5.2.2. It can be seen that the Zoelly critical pressure gives the same result in this case. When a smaller tank was used in section 5.1, the finite element software overestimated the critical buckling value with approximately 2-3%. This indicates that the finite element analysis and the analytic formula gives more similar results for increasing t/R-relationship.

Table 5.2.2: Results from linear buckling analysis of half-sphere with realistic dimensions in Dyna

Method	Critical pressure
Dyna	1.664 MPa
Zoelly critical pressure (Equation 3.2.1)	$1.664 \mathrm{MPa}$

Chapter 6

Non-linear Buckling Analysis of Halfsphere

A non-linear buckling analysis is conducted in this section. This analysis is conducted as a continuation of the linear eigenvalue buckling analysis. The main characteristics of the model in the eigenvalue analysis is therefore used in this section, but some adaptions were needed in order to perform the new analysis. The method used in this non-linear analysis is the arc-length method. A series of keywords is used in order to conduct this analysis in LS-Dyna. Some of them will be mentioned here. The non-linear solver is chosen to be nr. 12, which is a nonlinear solver with BFGS updates + optional arclength. The keyword "control_implicit_solution" is used to choose this. The implicit solution method is activated by the keyword "control_implicit_general", the time step is chosen and the geometric stiffness is included. Static analysis is used by "control_implicit_dynamics". Automatic time step is activated by "control_implicit_auto". Elastic material is used in the first analysis in section 6.1. While elasto-plastic material is used in section 6.2 and 6.4. Material nr. 18 in Dyna is used as model for elasto-plastic material. Only parameters for the material are included by this material model, and not an entire stress-strain curve. And the element formulation is chosen to be nr. 16, which is a fully integrated shell element for non-linear analysis. Number of integration points through the thickness of the shell is chosen to be 5. Three different types of imperfections were tested in this chapter. Each imperfection is described in the current section. Table 6.0.1 summarise the parameters of the model.

Entity	Magnitude
Radius	$21~500~\mathrm{mm}$
Uniform thickness	$55 \mathrm{~mm}$
Uniform external pressure	$2 \mathrm{MPa}$
Young's modules	210e3 MPa
Poisson's ratio	0.3
Element formulation	16
Nr. of elements	22500
Size of element	$0.5 \mathrm{m}$

Table 6.0.1: Parameters in non-linear analysis

6.1 Imperfection Type nr. 1

The five buckling modes with the lowest buckling load found in section 5.2 are used as initial imperfections in this non-linear analysis. The first buckling mode is not scaled when used as an imperfection. The second mode is scaled by using 10% of the displacement, while 5% of the third mode is used, 2.5% of the fourth and 1.25% of the fifth. It is assumed that the first buckling mode will be most important in the deformation of the shell. And that each of the modes will affect less as the corresponding buckling load increase. However many of the modes have very similar buckling load. An alternative could therefore have been to not scale the contribution from the individual mode. But the scaling is chosen in this analysis. The five different buckling mode and the largest displacement in each mode before they are scaled. The resultant displacement is the displacement when x-, y- and z-coordinates are combined to form the displacement. The largest value in resultant displacement for the initial imperfection was 0.0175 m.

Mode nr.	Largest resultant displacement
1	0.001734 m
2	$0.001757 {\rm \ m}$
3	0.001927 m
4	$0.001949 {\rm m}$
5	$0.002967 {\rm m}$

Table 6.1.1: Largest displacement in each buckling mode before scaling

To identify the buckling load from the non-linear analysis, the displacement and the load were extracted from the simulation. It is a known relation that the applied pressure in MPa on the half-sphere is proportional to the time multiplied by two. This is because the force is applied linearly so that 2 MPa is applied after 1 second. This was used to plot the force displacement curve to identify the buckling load. The force displacement curve shown in figure 6.1.1 shows the relation for node nr. 5945. From this figure it can be seen that the buckling load was 0.7178 MPa, and the maximum displacement was 0.0091 m.

When another node was chosen to plot the force-displacement relation, the curve looked different. The curve for node nr. 4274 can be seen in figure 6.1.2. From this curve it can look like the solution has some instabilities. A more smooth curve is expected as in figure 6.1.1 according to Amdahl (2010). The reason for the unstable curve for node 4274 may be found by looking at the displacement shape for the half-sphere. This can be seen in figure 6.1.3 and 6.1.4.



Figure 6.1.1: Pressure-displacement curve for node nr. 5945



Figure 6.1.2: Pressure-displacement curve for node nr. 4274

It can be seen from the two figures of the displacement shape that a dent is initiated in the red area in figure 6.1.3. This dent is moving from figure 6.1.3 to figure 6.1.4. It can be seen that the dent is moving towards the top of the half-sphere as the simulation proceeds. This would mean that the first dent is straightened out, and a new one is made. Node nr. 5945 is far away from these dents, while node nr. 4274 is in the neighbourhood of the dents. This may cause force-displacement curves with complicated shapes as shown in figure 6.1.2. From this analysis it can be seen from figure 6.1.4 that a dent at the top of the half-sphere will be governing for the post-buckling shape. The results are summarized in table 6.1.2.

Table 6.1.2: Results from non-linear analysis of half-sphere with imperfection type nr. 1 and elastic material

Largest initial imperfection	Material model	Buckling load
0.0175 m	Elastic	0.7178 MPa

LS-DYNA user input Time = 0.35888 Contours of Resultant Displacement min=0, at node# 1 max=0.0355663, at node# 6300 max displacement factor=50



ž_x

Figure 6.1.3: Resultant displacement, applied load of 0.7178 MPa, displacement scaling factor = 100



Figure 6.1.4: Resultant displacement, applied load of 0.583 MPa (post-buckling), displacement scaling factor = 100

6.2 Imperfection Type nr. 2, Elastic and Elasto-plastic Material

In order to obtain a force-displacement curve that corresponded to the theoretical curves as in Amdahl (2010) and Tall et al. (2018) for instance, it was attempted to reduce the initial imperfections with a factor of 0.5. This means that the relative contribution from each buckling mode is the same, but the resultant displacement is scaled by a factor. The smaller imperfections gave a maximum displacement in the resultant initial imperfections of 0.0088 m. The analysis was then repeated with elastic material. Another analysis was conducted with elasto-plastic material. The parameters for the elasto-plastic material were taken from DNVGL-RP-C208 (DNVGL, 2016a), with exception of the plastic failure strain. This is set to a high value in order to prevent rupture in the model, because rupture is not studied in this thesis. A table is included in (DNVGL, 2016a) for structural steel type S235 on page 22 that can be used in this case. The parameters used for the elasto-plastic material in this analysis are shown in table 6.2.1.

Table 6.2.1: Parameters for elasto-plastic steel S235

Entity	Magnitude
Density	$7850 \ kg/m^3$
Young's modulus	$210 { m ~GPa}$
Poisson's ratio	0.3
Strenght coefficient, K	$520 \mathrm{MPa}$
Hardening exponent, n	0.166
Yield stress	$235 \mathrm{MPa}$
Plastic failure strain	0.15

The stress-strain curve can be generated by the Ramberg-Osgood equation from Misovic, Tadic, and Lucic (2016) by equation 6.2.1. The stress-strain curve can be seen in figure 6.2.1.

$$\varepsilon = \frac{\sigma_y}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \tag{6.2.1}$$



Figure 6.2.1: Stress-strain curve for steel based on Ramberg-Osgood equation from Misovic, Tadic, and Lucic (2016)

The pressure-displacement curve can be seen in figure 6.2.2 for the elastic material and the elasto-plastic material with the same imperfection. These curves can be seen to have a clear top which represents the maximum load. They can be seen to follow each other in the elastic (linear) region, while the elasto-plastic material causes the buckling load to be smaller. This is because the elasto-plastic material takes the yield stress of the material into account. This is not accounted for in the elastic material model. The structure is then able to buckle because the stresses reaches yield with elasto-plastic material. If the elastic critical stress is higher than yield stress, the elastic material model causes a higher buckling load as shown by the curves.



Figure 6.2.2: Pressure-displacement curve for elastic and elastoplastic material, largest imperfection of 0.0088 m

The results of the non-linear analyses conducted in this section are summarized in table 6.2.2. Here the buckling load of the three different analyses are shown. It can be seen that the buckling load becomes larger when the imperfections are smaller. The elasto-plastic material causes a lower buckling load than for elastic material with the same imperfections as shown in figure 6.2.2.

Largest imperfection displacement	Material model	Buckling load
$0.0175 { m m}$	Elastic	0.7178 MPa
0.0088 m	Elastic	$0.9514 \mathrm{MPa}$
0.0088 m	Elasto-plastic	0.7606 MPa

Table 6.2.2: Summary of results on non-linear analysis of half-sphere

The location of the node where the curves are plotted is in the dent that originates as shown in figure 6.2.3. The red area shows were the dent appears for the elastic material. The deformation of the half-sphere for the elasto-plastic material contain several dents more at the top. The deformation of the half-sphere for elasto-plastic material can be seen in figure 6.2.4.

LS-DYNA user input Time = 0.43961 Contours of Resultant Displacement min=0, at node# 1 max=0.0592747, at node# 17635



×

Figure 6.2.3: Resultant displacement with elastic material, applied load of 0.879 MPa $\,$





The deformation of the half-sphere can be simulated for a longer period after buckling because elasto-plastic material is taken into account. Figure 6.2.5 shows how the deformation looks like for an applied load of 0.063 MPa. This is from the last step in the analysis, which corresponds
to the end of the orange curve in figure 6.2.2. It can be seen that the structure is far beyond buckling and a total collapse would most likely have happened before this state. The maximum displacement is here 15 m.



Figure 6.2.5: Resultant displacement with elasto-plastic material, applied load of 0.063 MPa (post-buckling)

6.3 Evaluation of Elasto-plastic Results

Because shells are very imperfection sensitive (Tall et al., 2018), an evaluation of the imperfections was considered to be necessary. The aim was to determine the size of the imperfections used in the analyses relative to fabrication tolerance according to DNV. This can be done by following the instructions in DNVGL (2016b). The imperfection tolerance is given as in equation 6.3.1. The parameter g was taken to be the length of one wave in the first buckling mode of the shell. This buckling mode can be seen in figure 5.2.1a. Inserting values gives the imperfection tolerance as shown. It is emphasized that g is approximated, and this cause an approximated imperfection tolerance.

$$\delta = \frac{0.01g}{1 + g/R} = \frac{0.01 \cdot 4222}{1 + 4222/21500} = 35.3 \ mm \tag{6.3.1}$$

The rest of the procedure to calculate the critical load is done according to Classification Notes 30.1 (DNVGL, 2004). The procedure was written in a python script which can be found in appendix A.3. The elastic buckling stress can be calculated by equation 6.3.2, and the knock-down factor by equation 6.3.3.

$$\sigma_E = \frac{E}{\sqrt{3(1-\nu^3)}} \frac{t}{R} = 325 \ MPa \tag{6.3.2}$$

$$\rho = \frac{0.5}{\sqrt{1 + 0.01\frac{R}{t}}} = 0.22 \tag{6.3.3}$$

Then we can find the elastic buckling stress with knock-down factor by multiplying σ_E and ρ . Or we can keep the elastic buckling stress without knockdown factor. The knock-down factor takes initial imperfections into account in the calculation. By excluding this, the calculation disregards initial imperfections. The reduced slenderness and the critical stress can be found by equation 6.3.4 and 6.3.5. And the critical pressure can be found by equation 6.3.6.

$$\lambda = \sqrt{\frac{\sigma_y}{\sigma_E \rho}} \tag{6.3.4}$$

$$\sigma_{cr} = \frac{\sigma_y}{\sqrt{1+\lambda^4}} \tag{6.3.5}$$

$$P_{cr} = \frac{\sigma_{cr} \cdot 2t}{R} \tag{6.3.6}$$

The calculated values are summarized in table 6.3.1. These values were compared with two new non-linear analyses in Dyna. The results from the two analyses are shown in table 6.3.2 and figure 6.3.1. A scaling factor (SCF) was used to scale the initial imperfections. It follows that the relative contribution between the five different buckling modes are the same as shown in table 6.1.1, but the resultant initial imperfection is scaled. The value for this scaling factor is given in table 6.3.2 by "SCF".

Knock-down factor	Material model	Calculated critical pressure
$\rho = 0$	Elasto-plastic	$0.97 \mathrm{MPa}$
$\rho = 0.22$	Elasto-plastic	$0.35 \mathrm{MPa}$

Table 6.3.1: Calculated critical pressure by Classification Notes 30.1

Table 6.3.2: Critical pressure from non-linear analysis in Dyna

Imperfection SCF	Material model	Largest initial imperfection	Critical pressure from Dyna
1e-9	Elasto-plastic	1.8e-11 m	0.96 MPa
2	Ealsto-plastic	$0.035 \mathrm{~m}$	0.42 MPa

When the initial imperfections are disregarded, it can be seen that the calculated value (0.97 MPa) is approximately the same as the one from Dyna (0.96 MPa). When initial imperfections are taken into account, the two procedures differ. The non-linear analysis in Dyna gives a critical pressure of 0.42 MPa, while the calculated value is 0.35 MPa. On reason for this deviation may be the approximated "g" when calculating the imperfection tolerance in equation 6.3.1. This is calculated according to DNVGL-CG-0134 (DNVGL, 2016b). Another reason may be that the calculated knock-down factor (ρ) from Calssification Notes 30.1 (DNVGL, 2004) does not account for the same initial imperfection as in DNVGL (2016b). The knock-down factor in DNVGL (2004) may use a different upper tolerance for initial imperfections than the given δ from DNVGL (2016b). This means that the calculated critical pressure according to the procedure in DNVGL (2004) may not use the same imperfection as in DNVGL (2016b). Which causes the procedures to differ, because the calculated imperfection of 0.035 m is used in the simulation in Dyna. It is not said in DNVGL (2004) how large imperfections the knock-down factor (ρ).

The deformed shape in the post-buckling range is governed by a concentrated dent near the equator of the half-sphere. The equator is were the boundary conditions are applied and the assumed symmetry plane of the buckling mode. The node location in figure 6.3.1 is specified as "dent", this refers to the dent that governs the post buckling shape of the structure.



Figure 6.3.1: Pressure-displacement curve for elasto-plastic material with different imperfections

6.4 Imperfection Type nr. 3, Elastic and Elasto-plastic Material

The non-linear analysis was also conducted with only buckling mode nr. 1 as imperfection. The amplitude of this buckling mode was scaled so that it had the same largest imperfection as calculated in equation 6.3.1 (0.035 m). The analysis was performed with elastic and elastoplastic material. The force-displacement curve can be seen in figure 6.4.1, and the results can be seen in table 6.4.1.

Table 6.4.1: Results with buckling mode nr. 1 as imperfection

Material model	Largest initial imperfection	Critical pressure from Dyna
Elastic	0.035 m	0.46 MPa
Elasto-Plastic	$0.035 \mathrm{~m}$	$0.41 \mathrm{MPa}$

It can be seen that the critical pressure of 0.41 MPa with elasto-plastic material is very similar to the critical pressure of 0.42 in table 6.3.2. Only difference between the two models is that

the results from table 6.3.2 use the five buckling modes as imperfection.



Figure 6.4.1: Pressure-displacement curve for elastic and elastoplastic material, buckling mode nr. 1 as imperfection

The values from this analysis can also be compared with the elastic and the elasto-plastic critical pressure. Which can be found by equation 6.4.1 and 6.4.2 respectively. Non-linear analysis with elastic material was not compared with analytic results in previous section. But the non-linear buckling pressure of 0.35 MPa with elasto-plastic material was calculated in the previous section also. Here 235 MPa is the yield stress of the material. The magnitude of σ_E and ρ were calculated in the previous section.

$$\sigma_E \cdot \rho = 71.5 \ MPa \Rightarrow P_{cr,Elastic} = \frac{71.5 \cdot 2t}{R} = 0.37 \ MPa \tag{6.4.1}$$

$$\sigma_{cr} = \frac{235}{\sqrt{1 + \left(\sqrt{\frac{235}{71.5}}\right)^4}} = 68.4 \ MPa \Rightarrow P_{cr,Ealsto-Plastic} = \frac{68.4 \cdot 2t}{R} = 0.35 MPa \qquad (6.4.2)$$

It can be seen that the elastic critical pressure from Dyna, and the calculated elastic pressure both are larger than the calculated elasto-plastic critical pressure. When the elastic material is used in Dyna, this analysis overestimates the calculated elastic buckling pressure of 0.37 MPa by 24%. When the elasto-plastic material is used in Dyna, this analysis overestimates the calculated elasto-plastic buckling pressure of 0.35 MPa by 17%. It is emphasized once more that it is not specified which imperfections that are accounted for in the knock-down factor ρ in DNVGL (2004). This may cause the deviation between Dyna and the calculated critical pressures.

Chapter 7

Buckling Analysis of Spherical LNG Tank



Figure 7.0.1: Model of the spherical tank with skirt

A linear and non-linear buckling analysis of a full sized spherical LNG tank exposed to uniform external pressure will be conducted in this chapter. The previous chapter performed a linear and non-linear analysis of a half-sphere made of steel S235. In order to be able to compare with the half-sphere, steel is first used in this chapter. But since the tank is originally made in aluminium, both the linear and non-linear analysis is performed with both materials. The results for aluminium will therefore be representing more realistic result.

The last part of this chapter introduces additional loads for another loading condition. A linear and non-linear analysis is performed with the additional loads in order to investigate the buckling load for this loading condition.

The model of the spherical tank can be seen in figure 7.0.1. The thickness and diameter is the same as for the half-sphere. A uniform thickness of 55 mm is applied for both skirt and sphere, and a diameter of 43 m. The boundary conditions are applied to the bottom of the skirt by fixing all the nodes in all six degrees of freedom. Figure 7.0.2 shows the model with the boundary conditions as white crosses.



Figure 7.0.2: Model of spherical tank with boundary conditions

The spherical tank is modelled with the supporting skirt. The skirt is modelled only with the top two meters, which is the distance before the first stiffener in the skirt is introduced. Including stiffeners in the model was disregarded because the sphere is the main aspect of this thesis. The skirt is modelled in order to have some mobility of the equator of the sphere. If the skirt

was disregarded, the boundary conditions would have to be applied to the equator. This would decrease the mobility of the sphere. A transition piece is inserted after the first two meters of the skirt, then the rest of the skirt is made of steel instead of aluminium. Consequently it is convenient to model the first two meters.

The skirt is modelled with an offset from the equator line of the tank. Figure 7.0.3 shows a close-up picture of the skirt. A triangle is used to model the offset between the skirt and the sphere. The distance from the sphere to the skirt is 150 mm in horizontal direction, and the distance from equator to the top of the triangle is 1000 mm. The mesh was generated with 38 896 elements on the entire model which implies an element size of 0.5 m.



Figure 7.0.3: Close-up picture of the skirt

The parameters for elasto-plastic steel are given previously in this report. These parameters can be found in table 6.2.1. The parameters for elasto-plastic aluminium can be found in table 7.0.1. Two sources were used to determine these material properties; a web page with material properties of aluminium alloy 5083 (AZO Materials, 2005), and the article from Misovic, Tadic, and Lucic (2016). The plastic failure strain is set to a high value in order to prevent rupture in the model. The analyses will not be conducted so far that rupture will occur.

Entity	Magnitude
Young's modulus	7.1e10 Pa
Density	$2650 \ kg/m^{3}$
Yield stress	$134 \mathrm{MPa}$
Poisson's ratio	0.3
Strength Coefficient, K	$426 \mathrm{MPa}$
Plastic Failure strain	0.15
Hardening exponent, n	0.2004

Table 7.0.1: Properties of Aluminium A5083, annealed condition

The Ramberg-Osgood equation can be used according to Misovic, Tadic, and Lucic (2016) to define the stress-strain curve for the aluminium alloy. For the elasto-plastic aluminium used in this thesis, a stress-strain curve was generated based on Ramberg-Osgood model. This can be seen in figure 7.0.4. The python script in appendix A.7 was used to make this curve.



Figure 7.0.4: Stress-strain curve for elasto-plastic aluminium based on Ramberg-Osgood from Misovic, Tadic, and Lucic (2016)

$$\varepsilon = \frac{\sigma_y}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \tag{7.0.1}$$

The parameters used for elasto-plastic aluminium was investigated with respect to the Ramberg-Osgood model in Misovic, Tadic, and Lucic (2016). The strength coefficient K and hardening exponent n is defined on page 185 in this article. K is defined as the stress where the strain is equal to one, and n is the strain hardening rate. Equation 7.0.1 was used to generate this curve. This is the Ramberg-Osgood equation which was also used in section 6.2 to generate the stress-strain curve for steel S235.

(a) Buckling mode nr. 1(b) Buckling mode nr. 5

7.1 Linear Buckling Analysis





Figure 7.1.2: Resultant displacement on buckling mode nr. 9, top-view

A linear buckling analysis was performed to begin with. This was done in order to compare with the analysis of the half-sphere, because the total sphere should have the same linear buckling pressure. In addition, the buckling modes from the linear analysis should be used as initial imperfections in the non-linear analysis. Buckling mode 1, 5 and 9 can be seen in figure 7.1.1 and 7.1.2. Buckling mode nr. 2 to 10 resembled a version of nr. 5 and nr. 9.

The linear buckling pressure was found by Dyna to be 1.646 MPa. This is approximately the same result as for the half-sphere in section 5.2 with the same diameter and thickness. The buckling pressure for the half-sphere was 1.664 MPa. The results from the linear buckling analysis of the spherical tank is summarised in table 7.1.1 for steel and aluminium, along with the analytic buckling pressure according to Zoelly (1915). When aluminium is used as material, the capacity decreases significantly. This is because the Young's modulus of aluminium is approximately one third of the Young's modulus for steel.

Table 7.1.1: Results from linear buckling analysis of full spherical tank

Approach	Material	Buckling Pressure [MPa]
Dyna	Steel S235	1.646
Dyna	Aluminium Alloy 5083	0.557
Zoelly crit. pres.	Steel S235	1.664
Zoelly crit. pres.	Aluminium Alloy 5083	0.544

7.2 Non-linear Buckling Analysis

This non-linear analysis was first performed with steel, then with aluminium. The buckling modes from the linear analysis are used as imperfections in this non-linear analysis. From the deformation pattern, it can be seen that the top and bottom of the sphere have the largest resultant displacement in each mode. When all of the modes are combined to form the initial imperfection, it is expected that the top and bottom of the sphere will have the largest imperfection. Buckling of the sphere will therefore most likely happen here.

7.2.1 Steel S235

Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	2.02e-2	1
2	2.0e-2	0.1
3	1.14e-2	0.05
4	1.12e-2	0.025
5	1.11e-2	0.0125
6	1.13e-2	0.00625
7	6.09e-3	0.00313
8	5.76e-3	0.00156
9	5.83e-3	0.00078
10	6.13e-3	0.00039

Table 7.2.1: Displacement and contribution for each mode withsteel S235

The initial imperfections in the following analyses were buckling mode nr. 1 to 10. The resulting initial imperfection is a linear combination of these modes. By multiplying the largest resultant displacement with the scale factor in table 7.2.1, the displacement contribution from each mode is found. The initial imperfection is found by adding the contribution from each mode.

The analysis was conducted with different size on largest initial imperfection. Table 7.2.2 show the results from these analyses. It can be seen that the buckling pressure of 1.05 MPa is approximately the same as was calculated in section 6.3 to 0.97 MPa with no imperfections. And the non-linear analysis of the half-sphere gave 0.96 MPa in buckling pressure. It can also be seen that the buckling pressure of 0.43 MPa with largest initial imperfection of 0.0344 m corresponds to the calculated pressure in section 6.3 of 0.35 MPa. And to the analysis of the half-sphere, which gave a buckling pressure of 0.42 MPa. The results of the full-size spherical tank therefore seems to correspond with the analysis of the half-sphere, and analytic values.

Table 7.2.2: Results from non-linear buckling analysis with steel S235

SCF	Largest Imp. [m]	Material Model	Load Case	Buckling Pressure [MPa]
1e-10	1.75e-11	Elasto-Plastic	Ext.Pressure	1.05
0.17	0.0344	Elasto-Plastic	Ext. Pressure	0.43

7.2.2 Aluminium Alloy 5083

Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	3.48e-2	1
2	3.45e-2	0.1
3	1.96e-2	0.05
4	1.93e-2	0.025
5	1.91e-2	0.0125
6	1.94e-2	0.00625
7	1.05e-2	0.00313
8	9.91e-3	0.00156
9	1.00e-2	0.00078
10	1.05e-2	0.00039

Table 7.2.3: Displacement and contribution for each mode with aluminium

The buckling modes look the same for aluminium, but the buckling load is different. The largest resultant displacement in each of the 10 buckling modes are therefore different. This is shown in table 7.2.3. The scale factor for each of the modes are the same for both steel and aluminium.

The results of the analysis conducted with aluminium instead of steel can be seen in table 7.2.4. A pressure of 0.7 MPa was applied linearly from zero to one second. The buckling pressure is found from the force-displacement curve. It can be seen that the non-linear buckling pressure is significantly lower than the linear buckling pressure of 0.557 MPa. The size of the imperfection is 5 mm larger compared to the last result for steel in table 7.2.2. Still the buckling pressure for aluminium is only 37% of the buckling pressure for steel in table 7.2.2. This shows that the tank has much larger capacity for steel than for aluminium.

Table 7.2.4: Results from non-linear buckling analysis with aluminium alloy 5083

SCF	Largest Imp. [m]	Material Model	Load Case	Buckling Pressure [MPa]
0.31	0.0393	Elasto-Plastic	External Pressure	0.16

7.3 Additional Loads

Up to this point, only external pressure has been considered as load on the structure. Some additional loads need to be considered in order to asses the capacity of the structure. The following loads will be considered in this thesis:

- Gravity, including weight of isolation in the tank.
- Sloshing load.
- External pressure.
- Additional acceleration on the tank due to ship motions.

A gravity load of 9.81 m/s^2 should be applied on the structure. In addition to the weight of the metal, the isolation on the tank should also be considered. This isolation is assumed to have a weight of 15 kg/m^2 . The weight of the isolation is included by increasing the density of the metal. The density of the metal per square meter is found by dividing by the thickness of the model. The thickness is uniform over the model with a size of 55 mm. A factor can be found to increase the density of the metal by dividing the density of the metal with isolation, by the density of the metal per square meter. External pressure should also be applied in addition to this. Next is a sloshing load that arises when the tank is partly filled with liquid gas. The ship motions causes the liquid to move around in the tank. This causes a load on the tank due to sloshing. The sloshing load can be computed based on DNVGL (2016b). The same motions of the ship that causes sloshing also causes an acceleration of the material in the tank. This also needs to be applied.

7.3.1 Procedure for Calculating Sloshing Load

The procedure for calculating the sloshing load is found on page 50 in DNVGL (2016b). A Python script was generated in order to compute the force due to sloshing. This script can be found in appendix A.1.

Sloshing may occur in the tank when it is partly filled with liquid gas. Because sloshing is dependent on the movements of the tank, the sloshing load is a dynamic load. The procedure in DNVGL (2016b) is developed in order to be able analyze this load statically. The result of

the procedure is an acceleration that is computed, and this acceleration affects the hydrostatic pressure from a sloped liquid surface. Figure 7.3.1 illustrates the partly filled tank with a sloped liquid surface.



Figure 7.3.1: Hydrostatic pressure from sloped liquid surface

The hydrostatic pressure due to the sloped liquid surface is calculated by equation 7.3.1. Where ρ is the density of the liquid, h is the height of the liquid normal to the surface, and a_R is the acceleration of the liquid. This is different from gravity because the movements of the ship causes movements of the tank, which will accelerate the liquid.

$$P = \rho \cdot h \cdot a_R \tag{7.3.1}$$

The aim for the procedure in DNVGL (2016b) is to calculate the resulting acceleration a_R . This resulting acceleration should account for the sloshing forces in the tank. The procedure starts by calculating the accelerations on a reference ship. Then the specifications on the desired ship is introduced, such as speed, width and GM, in addition to information on the tank. Four types of filling conditions are evaluated. These are full, h/D of 0.65, 50% filled and h/D of 0.29. Here D is diameter and h is the distance from the bottom to the liquid surface. The volume of the liquid is calculated for each case. A rolling angle and amplitude is calculated, these are used to calculate a tangential force. The formulas for this is found in DNVGL (2009). The transverse acceleration a_y is calculated based on ship specifications and filling condition. Then the force on the tank in the reference ship is calculated, and this is used to calculate the forces on the desired tank. The forces on the tank is used to define a transverse acceleration a_y^* that includes sloshing effects. A heel angle of the surface is chosen, and this is used to calculate the final acceleration a_R .

Figure 7.3.2 illustrates how a_R is computed based on the transverse acceleration a_y^* and the vertical acceleration a_z . Where a_z is the vertical acceleration of the ship. The acceleration components a_y^* and a_z defines an ellipse. The length of the resulting acceleration a_R is found by the second point where the vector crosses the ellipse. The length of a_R needs to be computed by inserting the equation for a straight line into the equation of the ellipse defined by the accelerations. The length of the vertical component of a_R can then be found, and the total resulting acceleration can be found by using the heel angle of the liquid surface named β .



Figure 7.3.2: Calculation of resulting acceleration

The angle β is taken to be the angle from the horizontal surface to the sloped surface measured from the horizontal axis. The acceleration a_R was calculated for a filling level of h/D = 0.29 and an angle of 20 degrees. It was found to be 13.4 m/s^2 .

According to DNVGL (2016b), an iteration procedure should be conducted in order to find the slope and acceleration of the liquid that causes the worst stresses in the shell. The iteration is

not used in this report, a slope is chosen and used in the analyses. The object of calculating this load was to give an indication of the loads on the tank to compare with results from Dyna.

7.3.2 Linear Buckling Analysis

The additional loads need to be applied by keywords in LS-PrePost. One individual load curve is made for each load, except for the sloshing load. To make the modelling of the loads easier, the geometric model is rotated the same angle as the sloped liquid surface should be rotated. In this way, the geometry is rotated, but the coordinate system stays the same. The acceleration on the material can then be applied by "Load_Body_X" in connection with the correct load curve. The external pressure is applied by "Load_Segment_Set" and the corresponding load curve. The sloped liquid surface is applied by first defining a function. This function is programmed to apply a hydrostatic pressure below a specified value in the vertical direction. The function is then applied to the whole model by "Load_Segment_Set". The function can be found in appendix B.1.

The linear buckling analysis in this section includes all the loads explained in section 7.3. The analysis calculates an eigenvalue. All of the loads applied is scaled by this eigenvalue in order to find the critical value for each load. Linear elastic aluminium is used with an increased density because of the weight of isolation on the tank. The density of the aluminium is now taken to be 2923 kg/m^3 . The rest of the material properties can be found in table 7.0.1.

An external pressure is applied, so is the acceleration of the tank due to ship movements. The resulting acceleration on the tank includes gravity in the vertical component. The acceleration is therefore applied in one command. This acceleration is applied to the material of the tank and is only caused by ship movements. It does not include sloshing effects. The acceleration components included in this is a_z , a_y and g. All of the components are constants because they are only dependent on the specifications of the ship. Pythagoras is then used to find the resulting acceleration on the material. Typical values for a_y and a_z are given from Moss Maritime to be approximately 0.5g and 0.45g respectively. This gives a resulting acceleration on the liquid surface, in the same direction as the hydrostatic pressure.

The hydrostatic pressure because of the sloped liquid surface is applied separately. This is the approximation for the sloshing force on the tank. The sloped liquid surface is applied with a constant angle of 20 degrees. The tank was considered to be filled so that the h/D-ratio (height of liquid/diameter) was equal to 0.29. The liquid is taken to be water with a density of 1000 kg/m^3 . The surface of the liquid is assumed to have constant heel angle, but the surface would have an increasing heel angle for an increasing acceleration in real life. To increase the acceleration on the liquid without increasing the heel angle of the liquid surface is an approximation in this analysis. To summarize, the following loads have been applied:

- Weight of isolation and tank by increasing density of the material to 2923 kg/m^3 .
- Sloshing load by applying a hydrostatic pressure due to a sloped liquid surface. The liquid is taken to be water with a density of 1000 kg/m^3 and an acceleration of 30 m/s^2 .
- External pressure of 0.1 MPa.
- Additional acceleration on the material of 15.05 m/s^2 due to ship movements.

Performing an eigenvalue analysis of the hydrostatic pressure force alone would be the best way to perform this analysis, but this approach produced negative eigenvalues. Which indicates that the hydrostatic pressure moves in the opposite direction. This does not happen in real life. When applying the uniform external pressure in addition to the hydrostatic pressure, a more reasonable buckling mode was generated by the linear analysis. That is why all of the loads are applied in this linear analysis. It is emphasized that all of the loads are considered to contribute to failure of the tank. If an eigenvalue analysis was possible for the hydrostatic pressure force alone, the results would most likely have been higher for this load. Because it would be the only load contributing to failure of the structure. Figure 7.3.3 shows the buckling mode for the linear analysis with all of the loads applied, but an external pressure of 0.05 MPa instead of 0.1 MPa. This was considered to not be a realistic buckling mode for this load case. The same buckling mode was generated if only hydrostatic pressure was considered. It was therefore concluded that the linear analysis should be performed with all of the loads applied, and an external pressure of 0.1 MPa or higher.



Figure 7.3.3: Resultant displacement on bucking mode nr. 1 from linear analysis with all loads applied, external pressure of 0.05 MPa, bottom-view

Table 7.3.1 lists the linear critical values for all the loads from the analysis. The combination of these loads is the loading condition that makes the structure fail. This does not mean that it does not exists other combinations. But to investigate this, each combination would have to be analyzed. The combination used in this analysis contained the lowest uniform external pressure that produced reasonable buckling modes. If the external uniform pressure was set lower, then the eigenmode in figure 7.3.3 was produced. The eigenvalue from this analysis was found to be approximately 2.

Table 7.3.1: Results from linear buckling analysis with several loads

Critical Pressure	Critical Acceleration on fluid	Critical Acceleration on material
0.20 MPa	$60.8 \ m/s^2$	$30.51 \ m/s^2$

Buckling mode 1 and 10 can be seen in figure 7.3.4. All the modes looked like a version of these. It can be seen that the structure buckles around the area of the liquid surface.



Figure 7.3.4: Resultant displacement on buckling modes for all loads combined, bottom view.

7.3.3 Non-linear Buckling Analysis

This analysis was performed with elasto-plastic aluminium as material. The density included the weight of the isolation as in the linear analysis. The density was taken to be 2923 kg/m^3 .

Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	1.05e-2	1
2	1.05e-2	0.1
3	1.051e-2	0.05
4	1.055e-2	0.025
5	7.73e-3	0.0125
6	7.70e-3	0.00625
7	8.76e-3	0.00313
8	8.77e-3	0.00156
9	9.78e-3	0.00078
10	1.02e-2	0.00039

Table 7.3.2: Displacement and contribution from each bucklingmode with all the loads combined

A combination of the ten buckling modes from the linear analysis was used as imperfections in this non-linear analysis. Table 7.3.2 lists the modes, the largest displacement in each mode, and the scaling factor for contribution on the initial imperfection.

To determine the size of the largest initial imperfection in the model, DNVGL (2016b) was used to calculate the tolerance for the imperfection. Equation 7.3.2 shows how this is calculated, where g is given in equation 7.3.3. These equations give a conservative estimate for the imperfection which becomes 0.036m. The highest imperfections used in this analysis was 0.04, which can be seen to be a conservative value, 11% higher than 0.036 m. A new calculation of the imperfection was made because the loading condition was changed from section 6.3 where the calculation was last performed.

$$\delta = \frac{0.01g}{1+g/R} \tag{7.3.2}$$

$$g = 4\sqrt{Rt} \tag{7.3.3}$$

The non-linear analysis was also performed with all the loads applied on the model. But in this procedure, only one load was increased until failure. Therefore all the other loads were increased up to a certain value, then the non-linear analysis was started when the final load was initiated. This makes the linear and the non-linear analysis not fully comparable. Because the linear analysis scale all of the loads until failure, but the non-linear analysis only increases the final load to failure, while the others have a constant value. Figure 7.3.5 shows how the load-time relation is for the loads applied in the non-linear analysis.

The force-displacement curves are plotted in the non-linear analyses in this thesis to determine the buckling level. Appendix A.5 shows how the force-displacement relation can be plotted. The size of the largest initial imperfection was also important to know. A file named "pert_node_res" is generated by Dyna each time an analysis is conducted. Appendix A.6 shows how the largest initial imperfection can be extracted from this file. Both are scripts written in Python.



Figure 7.3.5: Explanation of application of loads in non-linear analysis

Table 7.3.3 shows the results for the non-linear analyses that were conducted. According to DNVGL (2016b), the tank should be checked against buckling for an external pressure of 0.005 MPa. A pressure of 0.05 MPa is therefore 10 times the size that the rules requires. Different pressures were used to investigate the influence on the critical acceleration, and to compare with the linear analysis. The linear analysis had a critical pressure of 1.94 MPa, and critical acceleration on fluid of 60.8 m/s^2 . It can be seen when the largest initial imperfection is 1.13e-12 m, which is approximately zero, the critical acceleration becomes 54 m/s^2 for an external pressure of 0.05 MPa. This approaches the linear critical value of 60.8 m/s^2 . When the pressure is increased to 0.1 MPa, the critical acceleration can be seen to have the same value of 54 m/s^2 . And when the pressure is increased to 0.2 MPa, the critical acceleration decreases to 53 m/s^2 . All of these are still not very far from the linear value of 60.8 m/s^2 . The external pressure can be seen to not have a very high influence on the critical acceleration if it is increased from 0.1to 0.2 MPa. It is important to keep the external pressure below the buckling value for external pressure. If it approaches the buckling value, the structure might buckle because of external pressure instead. From the analysis of only external pressure, the non-linear buckling load was found to be 0.16 MPa for a largest imperfection of 0.039 m, while the linear buckling load was

0.56. If the external pressure is increased to more than 0.2 MPa, the structure may begin to buckle because of the external pressure. The aim of including all the loads was to determine the critical acceleration for the sloshing load. It was therefore decided not to increase the pressure further.

External Pressure	Accel. on material	Largest Initial Imp.	Critical Acceleration
$0.0075 \mathrm{MPa}$	$18.06 \ m/s^2$	0.04 m	$36 m/s^2$
$0.05 \mathrm{MPa}$	$15.05 \ m/s^2$	0.04 m	$26 m/s^2$
$0.05 \mathrm{MPa}$	$15.05 \ m/s^2$	$0.005 \mathrm{~m}$	$44 m/s^2$
$0.05 \mathrm{MPa}$	$15.05 \ m/s^2$	1.13e-12~m	$54 \ m/s^2$
$0.1 \mathrm{MPa}$	$15.05 \ m/s^2$	1.13e-12 m	$54 \ m/s^2$
$0.2 \mathrm{MPa}$	$15.05 \ m/s^2$	1.13e-12~m	$53 m/s^2$

Table 7.3.3: Results from non-linear analysis with all loads combined

It can be seen that when the imperfections are increased, for a pressure of 0.05 MPa, the critical acceleration is lowered to 44 m/s^2 , then to 26 and m/s^2 . This seems plausible because larger imperfections will lower the capacity of the structure.

One analysis was performed with an external pressure of 0.0075 MPa, an acceleration on the material of 18.06 m/s^2 , and with largest initial imperfection of 0.04 m. These values were used in order to compare the result with DNVGL (1997). A load factor of 1.5 is applied to the external pressure of 0.005 MPa, and a factor of 1.2 is applied to the acceleration on the material of 15.05 m/s^2 . The critical acceleration then became 36 m/s^2 . The buckling shape can be seen in figure 7.3.6. The orange line in figure 7.3.6a indicates the approximate position of the liquid surface. All of the buckling modes from the analyses in table 7.3.3 buckles around the surface of the liquid like in figure 7.3.6a. The post-buckling shape is mostly characterized by a dent either on one, or both sides of the tank. Figure 7.3.7 shows the post-buckling shape for the first analysis in table 7.3.3. The view is from the side as in figure 7.3.6. The same dent appears symmetrically on the other side of the tank. It can be seen that the deformation of the tank is clearly affected by the buckling modes from the linear analysis.



Figure 7.3.6: Resultant displacement at buckling with ext. pres. of 0.0075 MPa, initial imperfection of 0.04 m.



Figure 7.3.7: Post-buckling shape with external pressure of 0.0075 MPa, initial imperfection of 0.04 m $\,$

7.3.4 Extraction of Stresses in Spherical Coordinates

LS-Prepost was used in the post-processing phase to extract the stresses from the analysis. By "general setting", a new coordinate system could be defined in the center of the sphere. The z-axis would then be in vertical direction. By clicking "sph" and "apply", and choosing "user" instead of "d3plot" in "fringe component", the stresses were transformed from x-, yand z-direction to r-, θ - and ϕ -direction. Figure 7.3.8 shows the stresses in θ - and ϕ -direction for the spherical tank exposed to only external pressure.



The largest compression in both directions (blue area) were compared to analytic stress in equation 7.3.4. The analytic stress could be seen to be 11.97 MPa for applied pressure of 0.0613 MPa. The largest compressive stress in ϕ -direction was 11.97 MPa, while the largest compressive stress in θ -direction was 12.27 MPa. This confirmed that the stresses were successfully transformed to spherical coordinates.

$$\sigma = \frac{PR}{2t} = \frac{0.0613 \cdot 21.5}{2 \cdot 0.055} = 11.97 \ MPa \tag{7.3.4}$$

It should also be determined which direction is circumferential, and which is meridional. This was determined by looking at the plots of the stresses in ϕ - and θ -direction of the sphere when it was exposed to all the loads. The documentation on LS-Prepost was not found on this topic, physical considerations were therefore used. The stress components can be seen in figure 7.3.9. Pure compression was expected in the circumferential direction in the area of the waves in the structure. This was because the bottom of the tank was stretched in vertical direction because of the hydrostatic pressure. Because of transverse contraction, the tank will contract in the area of buckling, which causes compressive stresses in the area of buckling in circumferential direction. This can be seen for the θ -direction, and not for ϕ -direction. It was therefore concluded that

circumferential direction corresponds to θ -direction in LS-Prepost, and ϕ -direction corresponds to meridional direction. This implies that ϕ is measured from the z-axis, and θ is measured in the x-y-plane in the local cartesian coordinate system. This was also confirmed by Dynamore Nordic. The emails regarding the communication on this topic can be found in appendix C.2.



Figure 7.3.9: Stress in θ - and ϕ -direction when exposed to all loads

It can be seen in figure 7.3.8 that the stresses occurs in different zones in the tank. The resemblance is staring compared to the zones in figure 1.5.2. The blue area in the figure can be seen to contain the largest compressive stresses. And the stresses decrease towards the top and bottom of the sphere. The blue area in the figure has approximately the same position as zone 4U/4L and zone 5/3. While the green area has the approximate position of zone 6 and 2, and the red and yellow area has the approximate position of zone 7 and 1. This image of the stresses therefore gives an explanation for why the tank is divided into zones as in figure 1.5.2.

7.3.5 Evaluation of Results

All of the results from Dyna for the critical acceleration on the fluid is significantly higher than the calculated value for a_R of 13.4 m/s^2 . This is the size of the sloshing load according to DNVGL (2016b) that should be used in calculations. It may not be the highest value. Because the procedure in DNVGL (2016b) is an iterative process. But it gives an indication of the expected tolerance level that should be checked. The analyses therefore show that the structure survives higher sloshing loads than the recommended value from DNVGL (2016b).

The result of 36 m/s^2 as critical acceleration is found from the force-displacement curve from the analysis. This can be seen in figure 7.3.10. The stresses were extracted at buckling when the acceleration was 36 m/s^2 . This would be point 1 in the figure. These stresses were used to do a buckling check according to DNVGL (1997). The stresses extracted were in the meridional (σ_{20}) and in the circumferential (σ_{10}) direction. The stress in Dyna corresponding to σ_{10} was the stress in θ -direction, and the stress corresponding to σ_{20} in Dyna was the stress in ϕ -direction. The stresses were also extracted at point 2 and 3 in the figure. The check was performed at all three points. The procedure was programmed in a python script that can be found in appendix A.2. A summary of the approach is also provided in section 3.4.2 of this report. A more detailed explanation will be given here.



Figure 7.3.10: Force displacement curve for first analysis in table 7.3.3

The buckling check according to DNVGL (1997) starts by calculating the elastic buckling stress according to equation 7.3.5. Then this is normalized with equation 7.3.6.

$$\sigma_{CL} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R}$$
(7.3.5)

$$\Lambda_{CL} = \frac{\sigma_{CL}}{\sigma_{10}} \tag{7.3.6}$$

Then the knock-down factor ρ is found by iteration in equation 7.3.7. Where b, δ and γ_p is found by equation 7.3.8, 7.3.9 and 7.3.10 respectively. The parameters δ_1 and δ_2 are different sizes of imperfection tolerances.

$$\rho = 1 - \left(\frac{3\sqrt{3}}{2}\gamma_p\sqrt{-b}\frac{\delta}{t}\rho\right)^{2/3}$$
(7.3.7)

$$b = -0.5e^{1.15(\sigma_{20}/\sigma_{10})} \tag{7.3.8}$$

$$\delta = \delta_1 + (\delta_2 - \delta_1) e^{2.5[(\sigma_{20}/\sigma_{10}) - 1]}$$
(7.3.9)

$$\gamma_p = 1 / \left(1 - 0.375 \sqrt{\frac{\delta}{t} \sqrt{-b}} \right) \tag{7.3.10}$$

Depending on the ratio σ_{20}/σ_{10} , δ_1 has two different definitions. These are shown in equation 7.3.12. If the stress ratio σ_{20}/σ_{10} was found to be less than -1, it was assumed that the last definition of g in equation 7.3.12 could be used. Although it says that it is valid for the stress ratio between -1 and 0. The parameter δ_2 is defined in quation 7.3.11 for aluminium tanks.

$$\delta_2 = \frac{R}{750} \tag{7.3.11}$$

$$\delta_{1} = \frac{0.01g}{1 + \frac{g}{R}},$$

$$g = 4\sqrt{Rt}, \ \frac{\sigma_{20}}{\sigma_{10}} > 0$$

$$g = (4 + 2\frac{\sigma_{20}}{\sigma_{10}})\sqrt{Rt}, \ -1 < \frac{\sigma_{20}}{\sigma_{10}} < 0$$
(7.3.12)

The reduced slenderness is found by equation 7.3.13. Where σ_y is yield stress, F_E and σ_{e0} is defined in equation 7.3.14 and 7.3.15 respectively.

$$\lambda_E = \sqrt{F_E \frac{\sigma_y}{\sigma_{e0}}} \tag{7.3.13}$$

$$F_E = \frac{1}{\rho \Lambda_{CL}} \tag{7.3.14}$$

$$\sigma_{e0} = \sqrt{\sigma_{10}^2 + \sigma_{20}^2 - \sigma_{20}\sigma_{10}} \tag{7.3.15}$$

The criterion to be satisfied is that g defined by equation 7.3.17 should be larger than zero. Where the size of κ depends on the reduced slenderness, while γ_m is 1.15. The definition of κ can be seen in equation 7.3.16.

$$\kappa = 1.0, \ \sqrt{\frac{\sigma_y}{\sigma_{e0}\Lambda_{CL}}} < 0.2,$$

$$\kappa = 0.925 + 0.375 \sqrt{\frac{\sigma_y}{\sigma_{e0}\Lambda_{CL}}}, \ 0.2 < \sqrt{\frac{\sigma_y}{\sigma_{e0}\Lambda_{CL}}} < 1.0$$

$$\kappa = 1.3, \ \sqrt{\frac{\sigma_y}{\sigma_{e0}\Lambda_{CL}}} > 1.0$$
(7.3.16)

$$g = \Lambda_{CR} - \gamma_{sum} = \frac{1}{\sqrt{1 + \lambda_E^4}} \frac{\sigma_F}{\sigma_{e0}} - \kappa \gamma_m > 0 \tag{7.3.17}$$

Table 7.3.4 summarises the results of the buckling check for the three different points in figure 7.3.10. It can be seen that point 1 does not satisfy the requirements from DNVGL (1997) that g should be higher than 0. Point 2 does not satisfy the requirement either, but point 3 satisfies the requirements. From this it can be seen that the structure has some post-critical capacity compared to DNVGL (1997). It therefore seems that the rules from DNV are very conservative. The critical acceleration at buckling according to DNVGL (1997) would be between point 2 and 3. An interpolation can be done between the acceleration of $27 \ m/s^2$ (point 2) and $17 \ m/s^2$ (point 3) with the corresponding value for g. This interpolation gives a critical acceleration of $18.3 \ m/s^2$. According to figure 7.3.10, the structure can be seen clearly to buckle at $36 \ m/s^2$. If point 1 is taken as reference for when the structure buckles, the structure has only used 50.8% of the capacity at the tolerance limit from DNVGL (1997).

several loads

Table 7.3.4: Evaluation of results from non-linear analysis with

Point in figure 7.3.10	Acceleration	σ_{10}	σ_{20}	g
1	$36 \ m/s^2$	5.0e7 Pa	3.0e7 Pa	-0.91
2	$27 \ m/s^2$	3.0e7 Pa	$1.34\mathrm{e7}$ Pa	-0.405
3	$17 \ m/s^2$	2.0e7 Pa	8.3e6 Pa	0.06

Plot of the circumferential and meridional stress components can be seen in figure 7.3.11. Blue is negative compression, while red is positive tension. The orange circle indicates where the stresses were extracted. This is the point in the structure that has largest resultant displacement. The stresses are therefore extracted at this point. Both stresses are in compression as can be seen from the figure.



Figure 7.3.11: Normal stress components in the shell when exposed to all loads

The supporting skirt of the tank has the same thickness as the rest of the structure of 55 mm. This is assumed to be a structure with a quite high buckling capacity. A simple check can be performed on the skirt in order to see if this assumption holds. The elastic buckling strength of a cylinder according to DNVGL (2017) is given in equation 7.3.18, where L is the length of the cylinder. The cylinder is in this case 2 meters long. The procedure was written in Python and can be found in appendix A.4.

$$\sigma_E = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{L}\right)^2 \tag{7.3.18}$$

$$C = 2\sqrt{1 + \left(\frac{0.6 \cdot 1.04\sqrt{Z}}{2}\right)^2} \tag{7.3.19}$$

$$Z = \frac{L^2}{Rt}\sqrt{1-\nu^2}$$
(7.3.20)

The capacity of a cylinder exposed to hydrostatic pressure was considered. The elastic buckling stress of the cylinder then becomes 112 MPa. The Von Mises stress in the skirt was maximum

59 MPa. It can therefore be seen that the skirt is far away from buckling. Figure 7.3.12 shows the deformation of the skirt with a scale factor of 70. This is also from the analysis with an external pressure of 0.0075 MPa and the largest imperfection of 0.04 m.



Figure 7.3.12: Von Mises Stress on skirt with deformation scale factor of 70, all loads applied on spherical tank

Chapter 8

Buckling Analysis of Non-spherical LNG Tank



Figure 8.0.1: Model of the non-spherical tank

The spherical tank has been the focus of this thesis to this point. The objective of this thesis was to analyse a non-spherical LNG tank, and compare this with the spherical version and analytic expressions. The non-spherical tank can be seen in figure 8.0.1. The end caps are half-spheres with radius of 21.5 m, and the middle section (red area) is a cylinder with length of 7 m. This tank is also supported by a skirt, which is marked with blue, yellow and green. The thickness of the different parts of the model is 55 mm for the spherical end caps in addition to the skirt. But the cylindrical middle section has a thickness of 110 mm. The thickness is applied outside the tank, so the inside should be a smooth surface. The skirt supporting the spherical tank had an offset from the sphere, but because of difficulties with meshing, this was not possible for the non-spherical model. Therefore the offset is neglected in the model of the non-spherical tank. The number of elements generated on the model was 41 700. Each element therefore has a length of 0.5 m. This is the same mesh size as was used for the sphere. Aluminium was proceeded with as material, and the boundary conditions were applied on the bottom of the supporting skirt. All nodes here were fixed against all six degrees of freedom. Figure 8.0.2 shows the model with boundary conditions as white crosses.



Figure 8.0.2: Model of non-spherical tank with boundary conditions

The thickness of the cylindrical middle section is set to be twice as large as the spherical thickness. The reason for this is the difference in stresses in the cylinder and the sphere. This is explained in section 1.6. Two different analyses will be conducted in this section. One type where the sphere is only exposed to external pressure, and no other loads. The other analysis will be performed with the loading conditions described in section 7.3. These loads were:

- Gravity, including weight of isolation in the tank.
- Sloshing load.
- External pressure.
- Additional acceleration on the material due to ship motions.

8.1 External Pressure

The non-spherical tank was in this analysis exposed to uniform external pressure. The pressure was applied linearly from zero up to a specified value during one second. The material properties for aluminium in this analysis are the same as in table 7.0.1, which is found in the beginning of chapter 7. Elastic aluminium was used in linear analysis, and elasto-plastic aluminium was used in non-linear analysis. The analysis procedure in Dyna is the same as for the spherical LNG tank exposed to uniform external pressure in section 7.1 for the linear analysis, and section 7.2 for the non-linear analysis.



8.1.1 Linear Buckling Analysis

Figure 8.1.1: Buckling modes with deformation scaling factor of $200\,$


Figure 8.1.2: Buckling mode nr. 3 with deformation scaling factor of 200

A uniform external pressure of 0.2 MPa was applied linearly from 0 to 1 second, and the linear buckling analysis was performed. Buckling mode nr. 1, 2 and 3 can be seen in figure 8.1.1 and 8.1.2. The deformation pattern is scaled by a factor of 200 to illustrate the buckling shape more clearly. It can be seen that the cylindrical part buckles in this analysis. All of the ten lowest buckling modes looked like a version of these three modes, or a combination of them.

The article from Jasion et al. (2015) confirms that this is a likely buckling shape for a sphere that is stretched, and approaching a cylinder under external pressure. A simple calculation can also be done with respect to elastic buckling pressure. By using equation 7.3.18, 7.3.19 and 7.3.20 with a length of 7 m, thickness of 110 mm and radius of 21.5 m, the elastic buckling stress can be found. The elastic buckling pressure can be found by equation 1.6.2. The linear critical pressure then becomes 0.277 MPa for the cylindrical section. The lowest buckling pressure from Dyna was 0.287 MPa. The result is summarised in table 8.1.1. From section 7.1, it was found that the elastic buckling pressure for a spherical LNG tank in aluminium was 0.56 MPa according to Dyna. It can be seen that the cylinder has a lower buckling pressure than the sphere, this substantiates the buckling pressure for the non-spherical tank is very similar to the elastic buckling pressure for a cylinder under hydrostatic pressure. The value from Dyna can be seen to be 0.01 MPa higher. A higher value than the analytic one can possibly be explained by the spherical end caps. They seem to stiffen the cylindrical middle section slightly and give a higher capacity.

Approach	Thickness of cyl. sect.	Linear buckling pressure
Dyna	0.110 m	0.287 MPa
Analytic (cylinder)	0.110 m	$0.277 \mathrm{MPa}$

Table 8.1.1: Results from linear analysis of non-spherical tank exposed to external pressure

The thickness of the structure can be seen to be crucial for the elastic buckling pressure. If the thickness is increased to 150 mm from 110 mm, the elastic buckling pressure becomes 0.64 MPa. Then it is larger than the buckling pressure for the spherical end caps. The length also affects the linear critical value. If the length was reduced from 7 to 4 m for instance, the buckling pressure would become 0.63 MPa, and again be higher than the value for the sphere. Some additional analyses were performed to investigate the effect of changing thickness.

The linear analysis was conducted with a thickness of 0.15 m on the cylindrical section. Figure 8.1.3 shows buckling mode nr. 1 and 10 from this analysis. It can be seen that mode nr. 1 buckles in the cylindrical section. But there are some indications that the load is approaching the limit for the spherical part at the top of the tank. The analytic buckling pressure for the cylindrical section is 0.64 MPa with a thickness of 0.15 m. This exceeds the buckling pressure of the sphere of 0.56 MPa. Nevertheless the cylindrical section buckles with a buckling load of 0.53 MPa. This is between the analytic buckling load for the sphere and cylinder. Mode nr. 10 can be seen to buckle in the spherical section. The buckling load is then increased to 0.56 MPa, which is the value for the elastic buckling pressure of the spherical section.



Figure 8.1.3: Buckling modes for non-spherical tank with thickness of 0.15 m on cylindrical section

The analysis was also conducted with a thickness of 0.250 m on the cylindrical section. The analytic elastic buckling pressure is then increased to 2.59 MPa, which is significantly larger than 0.56 MPa for the spherical part. From figure 8.1.4 that shows buckling mode nr. 1 and 10, it can clearly be seen that the spherical section buckles. The results are summarised in table 8.1.2. From these examples it seems that the elastic buckling pressure can be used to identify the weakest part of the non-spherical tank.

_			
	Approach	Thickness of cyl. sect.	Linear buckling pressure
	Dyna	0.150 m	0.53 MPa
	Dyna	$0.250 \mathrm{~m}$	$0.56 \mathrm{MPa}$
	Analytic (cylinder)	$0.150 { m m}$	$0.64 \mathrm{MPa}$
	Analytic (cylinder)	$0.250 { m m}$	2.59 MPa

Table 8.1.2: Results from linear analysis of non-spherical tank under external pressure with different thickness on cylindrical section



Figure 8.1.4: Buckling modes for non-spherical tank with thickness of 0.25 m on cylindrical section

8.1.2 Non-linear Buckling Analysis

As a continuation of the linear buckling analysis, a non-linear analysis was performed. An external pressure of 0.3 MPa was applied linearly from 0 to 1 second. The buckling shape can be seen in figure 8.1.5. The figure shows the bottom of the tank, where it can be seen that a dent is beginning to characterize the post-buckling shape in the middle. The shape is plotted for the last step in the analysis.



Figure 8.1.5: Resultant displacement after buckling for non-spherical tank

The force-displacement relationship was plotted in the area of this dent. Figure 8.1.6 shows this relation. The buckling pressure was from this figure chosen to be 0.14 MPa. Around this value the curve starts to flatten out. It could also be possible to define the buckling pressure as the top of the curve with 0.15 MPa, or at for instance 0.12 MPa. But 0.14 is chosen in this case.



Figure 8.1.6: Force-displacement relation for non-linear buckling of non-spherical tank

Table 8.1.3 summarises the results from this analysis. The linear analysis gave a buckling pressure of 0.287 MPa. The results from the non-linear analysis can be seen to be approximately 50% of the linear buckling pressure with an imperfection of 40 mm at maximum. Previous analyses have shown that the critical value can drop more than 50% from linear to non-linear analysis. This was shown for the half-sphere in chapter 5 and 6. And it was verified to be likely in section 6.3 by calculations. A decrease in capacity by 50% does therefore not seem unlikely. The decrease in capacity is mainly caused by the imperfections as shown previously. The capacity will increase if the imperfections were decreased, and opposite.

Table 8.1.3: Results from non-linear analysis of non-spherical tank

Largest initial imperfection [m]	Material Model	Load Case	Buckling Pressure [MPa]
0.04	Elasto-Plastic	External Pressure	0.14

8.2 All Loads Applied

All of the loads are applied in this analysis. This includes the sloshing load, gravity, external pressure and acceleration on material. The same density of 2923 kg/m^2 is assumed for the non-spherical tank. This is the density of aluminium including isolation. A linear analysis is performed first, then a non-linear analysis.

8.2.1 Linear Buckling Analysis

All of the loads could be applied in the same way for the non-spherical tank, as for the spherical tank. But the sloshing load needed some modifications. The static sloped liquid surface is applied by rotating the model, and keeping the coordinate system fixed. By specifying a value for the axis in vertical direction, the sloped liquid surface can be defined. The model is rotated the same angle as the desired angle for the liquid surface. The distance from the origin of the coordinate system in the model to the sloped liquid surface therefore needs to be calculated. This is different from the distance in the sphere. The tank is filled so that h/d = 0.29 for the horizontal liquid surface. Where h is the largest height of the liquid, and d is the diameter of 43 m. When the model is rotated, the volume under the liquid surface should be the same as before it was rotated. Therefore the volume needs to be calculated for the sloped surface, and for a horizontal liquid surface. The horizontal liquid surface is illustrated by the yellow line in figure 8.2.1a, while the sloped surface is illustrated by the yellow line in figure 8.2.1b. To calculate the volume below both liquid surfaces, the middle cross-section of the model was plotted in Geogebra by analytic expressions. Geogebra is able to integrate between functions, which was used to determine the area between the functions in two dimensions. This was used since the model is symmetric around the center cross-section. When the liquid surface denoted by the yellow line was rotated 20 degrees around the center of the model, the area below the line was 0.94% larger than for the horizontal surface. This was accepted to determine the distance from the coordinate system origin to the sloped liquid surface. This distance was determined to be 7.83 m. The coordinate system of the model is shown in figure 8.2.1b by the yellow arrows ("CS in model"). The distance calculated by Geogebra is shown by the blue line ("Desired distance").



Figure 8.2.1: Rotated model and not

Buckling mode nr. 1, 2, 7, 8 and 10 can be seen in figure 8.2.2, 8.2.3 and 8.2.4. Similarities can be seen to when the sphere was exposed to all the loads in section 7.3.2. The sphere could be seen to buckle in the area around the liquid surface. This can also be seen to happen for the non-spherical shape. The lowest buckling modes have a smaller area of generated waves, while the waves propagate in the higher modes. This was seen also for the sphere.



Figure 8.2.2: Buckling modes from linear analysis of non-spherical tank with several loads, bottom view



Figure 8.2.3: Buckling modes from linear analysis of non-spherical tank with several loads, bottom view



Figure 8.2.4: Buckling mode nr. 10 from linear analysis of non-spherical tank with several loads, bottom view

The results are summarised in table 8.2.1. The spherical tank had a linear critical external pressure of 0.2 MPa, a critical acceleration on material of 30.51 m/s^2 , and a critical acceleration on fluid of 60.8 m/s^2 . It can be seen that both accelerations are lower for the non-spherical tank, than for the spherical tank. The critical pressure is 0.02 MPa higher for the non-sphere than for the sphere. But it is still below the linear buckling pressure of 0.287 MPa for the non-sphere when it is exposed to external pressure. From previous analyses performed in this thesis, it has been seen that the external pressure does not seem to be decisive for the buckling of the structure when the sloshing load is applied. The pressure could be increased, without affecting the critical value for the sloshing load in a considerable amount. This may explain why the critical value for the external pressure is 0.02 MPa higher for the non-sphere than for the sphere. Simply that it does not matter very much for the final buckling. The hydrostatic pressure seems to govern the buckling of the structure.

An external pressure of 0.15 MPa was applied, an acceleration on the material of 15 m/s^2 , and an acceleration on the fluid of 30 m/s^2 . It was tried with an external pressure of 0.1 MPa as was used for the sphere. But this produced negative eigenvalues. A pressure of 0.15 MPa was the lowest external pressure that did not produce negative eigenvalues. See section 7.3.2 for explanation on why negative eigenvalues are not wanted.

Table 8.2.1:	Results i	from linear	r anlysis	of non	-spherical	tank	with
all loads app	olied						

Critical External Pressure	Critical acceleration on material	Critical acceleration on fluid
0.22 MPa	$22.38 \ m/s^2$	$44.61 \ m/s^2$

8.2.2 Non-linear Buckling Analysis

The non-linear analysis was performed with the ten buckling modes as initial imperfections. Table 8.2.2 lists the modes along with the contribution factor. From the linear buckling analysis, it could be seen that the buckling modes formed couples with approximately equal buckling load. That is why couples of buckling modes have the same scaling factor. When the buckling load is approximately the same, they are assumed to have equal influence on the non-linear buckling.

Table 8.2.2: Displacement and contribution from each bucklingmode with all the loads combined for non-spherical tank

Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	2.32e-2	1
2	2.31e-2	1
3	2.31e-2	0.1
4	2.33e-2	0.1
5	1.99e-2	0.05
6	1.99e-2	0.05
7	2.25e-2	0.025
8	2.25e-2	0.025
9	1.75e-2	0.0125
10	1.75e-2	0.0125

The deformed shape at buckling for the non-sphere can be seen in figure 8.2.5. The orange line in figure 8.2.5a indicates the approximate position of the sloped liquid surface. When the deformation scaling factor is increased as in figure 8.2.5b, it becomes more clear that the liquid surface stretches the tank because of the hydrostatic pressure. The non-spherical tank can be seen to buckle in the same manner as the spherical tank. The wave pattern originates around the liquid surface were the height of the liquid seems to be approaching maximum. The buckling shape of the spherical tank can be seen in figure 7.3.6. The non-spherical tank can be seen to buckle in the spherical end caps. The cylindrical part seems to withstand wave pattern from developing. But the resultant displacement is still significant in the cylindrical part. The large thickness of the cylindrical section may be the reason for the wave pattern not continuing. Although the wave pattern does not continue in the cylindrical section, figure 8.2.6 shows that this section seems to have only slightly lower deformations than in the spherical section.



(b) Deformation scaling factor of 100





Figure 8.2.6: Deformation of non-spherical tank before buckling with scaling factor of 200, first analysis in table 8.2.3

The acceleration was plotted versus the displacement in the model to identify at which point the structure buckles. From figure 8.2.7a it can be seen that the structure buckles at an acceleration of 20 m/s^2 for the first analysis in table 8.2.3. The acceleration is increased further to approximately 28 m/s^2 before it decreases rapidly to zero. This indicates that the structure has some post-critical capacity as mentioned for the spherical tank. Figure 8.2.7b shows the force-displacement relationship for the second analysis in table 8.2.3.



Figure 8.2.7: Force-displacement relation for non-spherical tank with all loads applied

The first analysis in table 8.2.3 can be compared with the critical acceleration on the spherical tank of 26 m/s^2 . This critical acceleration for the sphere had the same initial imperfection, external pressure and acceleration on material. It can be seen that the sphere has 6 m/s^2 higher capacity than the non-spherical tank.

The second analysis in table 8.2.3 can be compared with the critical acceleration on the spherical tank of 36 m/s^2 . The critical acceleration can be seen to be 12 m/s^2 lower for non-spherical tank in this analysis. The buckling shape looks the same as in figure 8.2.5.

Table 8.2.3: Results from non-linear analysis of non-spherical tank with all loads

Largest initial imperfection	Ext. Pres.	Acc. on material	Critical acceleration on fluid
0.04 m	0.05 MPa	$15.05 \ m/s^2$	$20 m/s^2$
0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$24 \ m/s^2$

8.2.3 Evaluation of Results





(a) Circumferential stress in non-spherical tank
 (b) Meridional stress in non-spherical tank
 Figure 8.2.8: Deformed shape at buckling for the non-spherical tank



Figure 8.2.9: Force-displacement curve for non-spherical tank with all loads applied

97

In order to evaluate the results from the non-linear analysis, the stresses were extracted in the circumferential and meridional direction of the spherical cap. The stresses were extracted in the orange circle in figure 8.2.8. This is the point in the structure that has largest resultant displacement. The stress state at this point is compression for both components.

The stresses were extracted at several accelerations in this analysis. The points are marked in the force-displacement curve in figure 8.2.9. The approach used to assess the capacity of the structure is found in DNVGL (1997). The approach is also explained in more detail in section 7.3.5.

It can be seen from table 8.2.4 that point 3 satisfies the rules from DNVGL (1997). Interpolation between point 2 and 3 gives a critical acceleration of 18 m/s^2 . If point 1 is taken as buckling for this structure, it buckles at an acceleration of 24 m/s^2 . The capacity of the structure is then utilized by 75% at the limit from DNVGL (1997). The value of 18 m/s^2 is the same value as was calculated for the spherical tank. The rules should predict the same capacity for similar structures. The non-spherical tank is treated as a sphere by the rules, that is why the same capacity is found. The same stresses are needed in the spherical cap and the spherical tank in order for it to buckle. But since the spherical tank has a higher capacity than the non-spherical tank, the utilization at buckling according to DNVGL (1997) is different. This is 75% for the non-spherical tank, and 50% for the spherical tank.

Point in figure 8.2.9	Acceleration	σ_{10}	σ_{20}	g
1	$24 \ m/s^2$	3.52e7	1.18e7	-0.59
2	$20 m/s^2$	2.5e7	6.0e6	-0.19
3	$17 m/s^2$	2.0e7	4.0e6	0.15

Table 8.2.4: Evaluation of results from non-linear analysis of non-spherical tank with several loads

The same stresses appear in the the spherical cap at a lower acceleration than for the spherical tank. The explanation for this may be that the volume below the surface is larger for the non-spherical tank. If the pressure along the wet surface in the non-spherical shape is integrated, this will be larger than for the spherical shape. Which may cause the capacity to decrease.

Chapter 9

Buckling Analysis of Spherical LNG Tank with Different Thicknesses



Figure 9.0.1: Model of spherical tank with different sections

A new model was created in order to approach the real life spherical LNG tank. This model is divided into sections according to figure 1.5.2. Table 1.5.1 shows the height and thickness of each section. The sections in the model in figure 9.0.1 has some approximations. The first is that the equator-line is defined to be the part of the sphere that is located behind the skirt. This has a height of 1 meter and is located from the original equator and 1 m upwards. This gives zone 4U a height of 2 m, while zone 4L has a height of 3 m. Zone 3 and 5 have a height of 10 m, and zone 2 and 6 have a height of 7 m. Finally zone 1 and 7 have a height of 1.5 m. The thicknesses are listed in table 9.0.1. The rest of the model is the same as was studied in chapter 7.

Zone	Height [mm]	Thickness [mm]
1	1500	53
2	7000	53
3	10000	66
4L	3000	71
Equator-line	1000	195
4U	2000	57
5	10000	51
6	7000	32
7	1500	40

Table 9.0.1: Approximated dimensions of the zones of the spherical tank

The same procedure is followed in this chapter with respect to analyses. The procedure starts by load case number one, which is only external pressure. Linear and non-linear buckling analysis is performed for this loading condition. Then all of the loads are applied in load case number two. Linear and non-linear buckling analyses are performed on this loading condition as well. The buckling modes from the linear analysis are used as imperfections in the non-linear analysis. Each thickness was applied in order to make the inner surface of the tank smooth. This can be controlled by the parameter "NLOC" in "Section_Shell" for each of the sections.

9.1 External Pressure

Linear elastic aluminium was used in the linear analysis and elasto-plastic aluminium in nonlinear analysis. These properties can be found in table 7.0.1. The mesh size is the same as before with a length of 0.5 m for each element.

9.1.1 Linear Buckling Analysis

An external pressure of 0.7 MPa was applied linearly during one second. The linear buckling analysis gave a deformation pattern as seen in figure 9.1.1. Only buckling mode nr. 1 is shown here, but all ten buckling modes had the same shape. Only difference between them was a small deviation in the number of waves occurring in the shell. From the figure, it can be seen that zone 6 is the part of the structure that buckles. This is the second zone from the top, and it has the smallest thickness. The thickness of this zone is 32 mm.



Figure 9.1.1: Resultant displacement on buckling mode nr. 1 from linear analysis with external pressure

The results from this analysis can be compared with the equation for the elastic buckling pressure according to Zoelly (1915). This formula is shown in equation 3.2.1. The thickness of zone 6 is used in the equation. Table 9.1.1 summarizes the results from the linear analysis with analytic results. It can be seen that Dyna gave a critical pressure of 0.191 MPa, while Zoelly critical pressure gave 0.190 MPa. This confirms the result from Dyna. It can also be seen that the zone that buckles can be treated as a full sphere in the formula with radius equal to the spherical tank.

Table 9.1.1: Results from linear analysis of spherical tank with different sections exposed to external pressure

Approach	Critical Pressure
Dyna	0.191 MPa
Zoelly critical pressure (equation 3.2.1)	0.190 MPa

9.1.2 Non-linear Buckling Analysis

An external pressure of 0.15 MPa was applied linearly from zero to one second in this non-linear analysis. The ten buckling modes from the linear analysis were used as imperfections. Largest resultant displacement and contribution scale factor can be seen in table 9.1.2.

Table 9.1.2: Displacement and contribution from each buckling mode with all the loads combined for spherical tank with different sections

Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	1.18e-2	1
2	1.18e-2	0.1
3	1.15e-2	0.05
4	1.14e-2	0.025
5	1.17e-2	0.0125
6	1.16e-2	0.00625
7	1.07e-2	0.00313
8	1.06e-2	0.00156
9	1.14e-2	0.00078
10	1.14e-2	0.00039

The buckling modes from the linear analysis are quite similar. This causes all the buckling modes to contribute to an imperfection pattern that is very concentrated in zone 6 of the spherical tank. From this it can be concluded that the largest initial imperfection definitely occurs in zone 6. The deformation pattern at buckling for an imperfection of 0.04 m can be seen in figure 9.1.2. This shows that one of the waves dominates the deformation pattern. It can be seen throughout this thesis that the post-buckling shape of the structure is governed by one dent that develops to a larger deformation.



Figure 9.1.2: Resultant displacement at buckling for spherical tank with different thicknesses, largest initial imperfection of 0.4 m

Three different sizes of the largest resultant displacement were tested in this analysis. The largest of these was 0.04 m. This imperfection was calculated based on a thickness of 0.055 m of the sphere. But the model used in this section has different thicknesses in the shell. The thinnest part of this model is zone 6 with a thickness of 0.032 m. The imperfection tolerance limit for a sphere with a thickness of 0.032 m will be lower than for a thickness of 0.055 m. So two additional sizes were used for largest initial imperfection. These were 0.029 m and 0.010 m. The imperfection of 0.029 m was found based on calculations according to the procedure in equation 7.3.12. Where the definition of g for $\sigma_{20}/\sigma_{10} > 0$ was used. This definition of g would give the largest tolerance for the imperfection, and therefore lowest capacity. This calculation gave a maximum tolerance limit for the largest initial imperfection of 0.029 m. The last size of the imperfection was chosen to be significantly lower then 0.029 m, therefore 0.01 m.



Figure 9.1.3: Force-displacement relation for spherical tank with different thicknesses, largest initial imperfection of 0.04 m



Figure 9.1.4: Force-displacement relation for spherical tank with different thicknesses

The force-displacement relationship can be seen in figure 9.1.3 for a largest imperfection of 0.04 m. The relation can be seen to become non-linear before the top of the curve is reached. The

exact point of buckling is therefore not easily identified. But based on this curve, the buckling point is chosen to be 0.05 MPa. This is not the top of the curve, but a compromise of the top and the increasing non-linearity of the curve. This buckling pressure is only 26% of the linear buckling pressure. The force-displacement curve for the two other imperfections can be seen in figure 9.1.4a and 9.1.4b. The top of the curve was chosen as the point of buckling for figure 9.1.4a because this curve is steeper. The top was also chosen for the curve in figure 9.1.4b as point of buckling.

Table 9.1.3 summarises the results of the analyses performed in this section. The change in largest imperfection from 0.04 m to 0.029 m did not affect the capacity in a large extent. The buckling pressure increased with 0.005 MPa for a decreasing imperfection to 0.029 m. It could be argued that the increase actually is larger. A larger increase can be obtained if a lower buckling pressure is chosen from the curve in figure 9.1.3. The imperfection of 0.010 m caused a buckling pressure of 0.08 MPa. This is approaching half of the capacity from the linear analysis which was 0.191 MPa. From the analysis of the spherical tank with uniform thickness, the reduction in capacity was approximately 30% from linear to non-linear analysis. The capacity is also seen to increase for decreasing imperfection for the spherical tank with different thicknesses therefore seem reliable.

Table 9.1.3: Results from non-linear analysis of spherical tank with different thicknesses exposed to external pressure

Largest initial imperfections	Material Model	Load Case	Buckling Pressure
0.04 m	Elasto-Plastic	External Pressure	$0.05 \mathrm{MPa}$
0.029 m	Elasto-Plastic	External Pressure	$0.055 \mathrm{MPa}$
0.010 m	Elasto-Plastic	External Pressure	$0.08 \mathrm{MPa}$

9.2 All Loads Applied

It was attempted to extend the analysis to the second loading condition as done previously in this thesis. First a linear analysis was attempted, then a non-linear analysis. The aim of analysing this loading condition is to determine how big the sloshing load can be before the tank buckles. The magnitude of the sloshing load is investigated by the acceleration that is used to determine the hydrostatic pressure.

9.2.1 Linear Buckling Analysis

The linear buckling analysis applies all of the four loads. All loads are ramped up to a specified value during one second for each load. Then the linear buckling analysis is performed. It was attempted with an acceleration on the material of 18.06 m/s^2 . The density was increased to 2923 kg/m^3 for the aluminium, and the acceleration on the fluid was 30 m/s^2 . The external pressure was tested for 0.099 MPa and 0.095 MPa.

A limitation was found in this linear analysis. The external pressure needed to be larger than a specific value in order for the analysis to produce reasonable results. For the spherical tank with uniform thickness, the external pressure needed to be above 0.1 MPa. And for the non-spherical tank it needed to be above 0.15 MPa. If the external pressure was lower than this, negative eigenvalues were produced for both models. Negative eigenvalues indicate that the loads are applied in the opposite direction than the direction defined by the user of the software. This implies that the hydrostatic pressure also acts in the opposite direction, which is not wanted. The same problem arose for the spherical tank with different thicknesses.

If the external pressure is too low, the deformation pattern in figure 9.2.1a is found. If the external pressure is too high, the deformation pattern in figure 9.2.1b is found. The deformation pattern for too low external pressure is the same as in figure 7.3.3 for the spherical tank with uniform thickness and too low external pressure. This buckling shape produce negative eigenvalues. From the previous analyses of the spherical, and the non-spherical tank, it is known that the tank should buckle in the area around the liquid surface with a positive eigenvalue. This does not happen for either of the figures.

For too high external pressure, the thinnest zone of the tank buckles because of the external pressure (figure 9.2.1b). This can be seen to be zone 6. If we compare the buckling load with the buckling pressure from the analysis with only external pressure in section 9.1.1, they can be seen to be quite similar. The buckling load from the analysis in this section becomes 0.187 MPa, while the buckling pressure from the analysis with only external pressure in section 9.1.1 was 0.191 MPa. The eigenvalue is positive in this case, indicating external pressure. It was

therefore concluded that a pressure did not seem to exist with respect to producing reliable buckling modes. Because too high external pressure cause zone 6 to buckle due to the external pressure, and too low external pressure cause negative eigenvalues. The expected results would be a buckling pattern around the liquid surface due to the hydrostatic pressure, with positive eigenvalues. This could be achieved for the previous model with uniform thickness by increasing the external pressure. But the problem of buckling in zone 6 arises in the model with different thicknesses.



(a) External pressure of 0.095 MPa applied, bottom view (b) External pressure of 0.099 MPa applied, side view

Figure 9.2.1: Buckling mode nr. 1 for non-linear analysis of spherial tank with different thicknesses with all loads applied

9.2.2 Non-linear Buckling Analysis

The linear analysis is usually conducted before the non-linear analysis. This is to get an upper estimate of the critical level, and to get an impression on the buckling modes that may govern the post-buckling shape. The buckling modes are also used in this report to apply initial imperfections in the non-linear analysis. But the non-linear analysis is still possible to solve without inserting imperfections. It is emphasized that the critical level will be severely overestimated without introducing imperfections in the non-linear analysis. A non-linear analysis was performed without imperfections. This was because the linear analysis appeared to be difficult to accomplish. The deformed shape just after buckling can be seen in figure 9.2.2. The orange lines indicate the liquid surface. The surface stretches from the red area and into the paper. The dotted line illustrates that the surfaces goes on the other side of the tank. The dent occurs in the transition between zone 2 and 3, with thickness of 53 and 66 mm respectively. The spherical tank with uniform thickness had a thickness of 55 mm. Table 9.2.1 summarise the results from the non-linear analysis in this section. The value of 59 m/s^2 can be compared with the linear critical acceleration for the sphere with uniform thickness in section 7.3.2. This was found to be 61 m/s^2 . It therefore seems reasonable that the critical acceleration on the fluid becomes approximately the same when it buckles in the transition between zone 2 and 3. To get a more realistic estimate, the non-linear analysis need to be performed with initial imperfections. A decrease in capacity of approximately 50% would then be expected. A similar decrease was seen in section 7.3.3.



Figure 9.2.2: Resultant displacement after buckling for sphere with different thicknesses with all loads applied

The buckling shape can be seen to be similar to the shape in figure 7.3.7 from the non-linear analysis of the spherical tank with uniform thickness. The deformation is governed by a dent that appears in the area of the liquid surface. The location of the dent is somewhat different. A 90 degree angle differs the two locations in the circumferential direction of the tank. The force displacement curve can be seen in figure 9.2.3. This shows a clear peak where the structure is said to buckle.

Table 9.2.1: Results from non-linear analysis of spherical tank with different thicknesses and all loads applied

Largest initial imperfection	Ext. Pres.	Acc. on material	Critical acceleration on fuid
0 m	0.0075 MPa	18.06	$59 \ m/s^2$



Figure 9.2.3: Force-displacement curve for non-linear analysis of spherical tank with different thicknesses and no imperfections

Chapter 10

Buckling Analysis of Non-spherical LNG Tank with Different Thicknesses



Figure 10.0.1: Model of non-spherical tank with different sections

The last model used in this thesis is shown in figure 10.0.1. This LNG tank has the same geometry as the one studied in chapter 8, but it is divided into different sections with different thicknesses. The skirt is modelled without offset as it was for the non-spherical tank with two thicknesses. The sections in figure 10.0.1 has a thickness on the spherical parts according to table 1.5.1. Each cylindrical part with same position and height as the spherical part has twice the thickness of the spherical part. Some simplifications were done with respect to the height of the zones. Table 10.0.1 shows each zone with corresponding height. The zones follow figure 1.5.2 by splitting the spherical tank in the middle along the vertical axis, and inserting a cylindrical part. The thickness was attempted to be put outside the tank to make a smooth inner surface. This was obtained only to some extent.

Zone	Height [mm]	Thickness [mm]
1	1500	53
2	7000	53
3	10000	66
4L	2350	71
Equator-line	1300	195
4U	2350	57
5	10000	51
6	7000	32
7	1500	40

Table 10.0.1: Dimensions of different zones of the non-spherical tank

The mesh on this model can be seen to be different than the other models in previous chapters. An axisymmetric mesh has been used previously in this report. The choice of mesh was discussed when the half-sphere was analysed in chapter 5. The axisymmetric mesh was then found to be most reliable. The mesh in figure 10.0.1 is generated by Patran with a method called "Paver". This mesh becomes quite irregular, and may affect buckling shapes and stresses that occurs. The geometry was too complicated for Patran to generate a regular axisymmetric mesh. The different sections that should have different thicknesses cause Patran to give an error that says the surface is not "biparametric". Consequently the axisymmetric mesh can not be generated. The mesh size is still 0.5 m.

The same approach is used in this section for the analyses. The first load case is only external pressure. Linear and non-linear buckling analysis is performed for this load case. The purpose

was to conduct the analyses on the second load case as well. This was not accomplished in this chapter.

10.1 External Pressure

An external pressure of 0.1 MPa was applied linearly from zero to one second in both the linear and non-linear analysis. Linear elastic aluminium is used in the linear analysis, while elasto-plastic aluminium is used in the non-linear analysis. Material data are found in table 7.0.1.

10.1.1 Linear Buckling Analysis

Buckling mode nr. 1 and 5 can be seen in figure 10.1.1. It can be seen that the cylindrical part buckles in zone 6. The same zone buckled for the spherical tank in chapter 9. The non-spherical tank also seem to buckle in some extent in zone 7 (the top). This seems likely because the cylinder has lower capacity than the sphere, and zone 6 and 7 have the smallest thicknesses.



Figure 10.1.1: Buckling modes from linear analysis of nonspherical tank with different thicknesses exposed to external pressure

The results from the analysis can be seen in table 10.1.1. The results from Dyna were compared with the analytic value for the elastic buckle pressure of a cylinder under hydrostatic pressure from DNVGL (2017). This can be seen in the table to be 0.066 MPa, which is about 60% of the value from Dyna. This is a much larger error than for the non-spherical tank with two thicknesses. In that case the value from Dyna were 3.6% from the analytic value according to DNVGL (2017). One cause of this difference may be the change in mesh. The paver-mesh is more irregular than the axisymmetric mesh.

Table 10.1.1: Results from linear analysis of non-spherical tank with different thicknesses, exposed to external pressure

Approach	Critical Pressure
Dyna	0.105 MPa
Analytic	$0.066 \mathrm{MPa}$

10.1.2 Non-linear Buckling Analysis

A non-linear buckling analysis was performed in the same manner as previously. The ten buckling modes from the linear analysis were used as imperfections in the non-linear analysis. Table 10.1.2 lists the largest displacement in each mode, and the contribution factor for each mode.

1		
Buckling mode nr.	Largest resultant displacement [m]	Contribution Scale Factor
1	4.71e-2	1
2	4.72e-2	0.1
3	5.43e-2	0.05
4	5.31e-2	0.025
5	6.86e-2	0.0125
6	6.81e-2	0.00625
7	7.13e-2	0.00313
8	7.04e-2	0.00156
9	3.62e-2	0.00078
10	3.43e-2	0.00039

Table 10.1.2: Displacement and contribution from each buckling mode with all the loads applied for non-spherical tank with different sections

Figure 10.1.2 shows the deformation of the tank just after buckling. It can be seen that the dent occurs in the red area. This will develop further if the analysis is conducted for a longer period. This does not look like the deformation pattern that was seen in the non-linear analysis

of the non-spherical tank in chapter 8. The non-spherical tank in chapter 8 developed a dent in the middle of the cylindrical section. The non-spherical tank with different sections and thicknesses develop a small dent in the transition between the sphere and the cylinder in zone 6. The force-displacement relation can be seen in figure 10.1.3.



Figure 10.1.2: Resultant displacement at buckling for nonspherical tank with external pressure



Figure 10.1.3: Force-displacement relation for non-spherical tank with external pressure

The results are summarised in table 10.1.3. The linear buckling pressure and the deformation in the non-linear analysis may indicate that this model is not reliable. The deviation between analytic and numerical value in the linear analysis was quite large. The analytic value was 63% of the numerical value. Buckling mode nr. 5 in figure 10.1.1b seems to indicate a problem with the mesh. Buckling in the transition between spherical and cylindrical part did not occur when it was only two different thicknesses. There is no obvious reason for this to happen for this model either. Based on these results, the model of the non-spherical tank with different thicknesses does not seem reliable. It was therefore found counter-intuitive to proceed with the analyses of this model. The analyses with external pressure should be confirmed before it is proceeded with the second loading condition.

Table 10.1.3: Results from non-linear analysis of non-spherical tank with different thicknesses, exposed to external pressure

Largest init. imperfection	Material Model	Load Case	Buckling Pressure
0.029 m	Elasto-plastic	Ext. Pressure	$0.056 \mathrm{MPa}$

Chapter 11

Discussion

This chapter discusses the produced results and some of the utilized methods. The first part discusses different methods that were used, and the second part discusses the actual results from the analyses. The non-spherical tank is compared to DNVGL (2016b) in the end, where the analyses of this tank is discussed further. Followed by a suggestion for modification of the rules to account for a non-spherical shape.

11.1 Procedures

Different procedures were used on different parts of the analyses throughout the work. Some of them are discussed below.

11.1.1 Simplifications in the Analyses

Two considerable simplifications were made in the analyses. One of them was made with respect to the geometric model, and the other was in the setup of the analyses.

The geometric simplification was the tower in the middle of the spherical tank. In the real spherical tank, a tower is placed in the middle of the tank to load and unload with LNG. Because of this, the tower has to penetrate the sphere in the top. It therefore seems reasonable that this tower may affect the stresses and capacity of the tank. The tank analysed in this

thesis has been a continuous shell. It may had been more accurate to model the sphere with a reinforcement in the top, or a hole to account for the discontinuity in the shell. In addition to affect the top, it would most likely also affect the bottom of the tank. With respect to the sloshing load, the tower may affect the capacity because the tank is stretched in vertical direction. The tower may be an additional load that pushes the bottom of the tank in the same direction as the hydrostatic pressure. Or it may stiffen the tank and prevent large deformations in the bottom.

The temperature is the other simplification. The LNG is carried in liquid form, which implies that the temperature inside the tank is quite low. The low temperature will cause the metal in the tank to contract. This will affect the original shape in the tank and cause stresses in the shell. This should be possible to include in Dyna, but it was not investigated in this thesis.

11.1.2 Extraction of Stresses in Spherical Coordinates

This section discusses the evaluation of stresses that was made in section 7.3.4, 7.3.5 and 8.2.3. The extraction of the stresses are done according to colour coding in the structure. The colour coding indicates the stresses at specific points in the structure. But one colour can contain stresses with a range of approximately 10 MPa. Depending on if the highest or lowest stress are used in the evaluation, the capacity of the structure according to DNVGL (1997) can be quite different.

If the largest stresses are chosen for instance, the limit according to DNVGL (1997) can be as low as 11 m/s^2 for the non-spherical tank. This implies a utilization of the capacity of 46% at the limit from DNVGL (1997) compared to the buckling acceleration of 24 m/s^2 according to Dyna. If the lowest stresses are chosen in the evaluation, the critical acceleration can become as large as 25 m/s^2 . Which can be seen to be larger than the critical value predicted by Dyna. But the spherical tank had a more narrow gap between smallest and largest value.

If the largest stresses were used in the evaluation of the capacity for the spherical tank, the critical acceleration according to DNVGL (1997) became 16 m/s^2 . If the lowest stresses were used, a critical acceleration of 22 m/s^2 was found. The middle value of 18 m/s^2 was chosen as the critical limit according to DNVGL (1997). Because the rules treats spherical shapes,

the critical acceleration should be the same for the spherical and non-spherical shape. The uncertainty explained regarding extraction of stresses for the non-spherical shape in the previous paragraph is therefore reduced.

It was attempted to extract the most accurate stresses at the specific point, but it can be seen that this may vary according to the user of the software. However, it seemed clearer which stresses to extract on the spherical tank, than on the non-spherical tank. Consequently, the approach to extract the stresses can be seen to have some uncertainties. The reason why this approach was chosen is that the transformation from cartesian to spherical coordinates needed to be done in the post-processing phase. The best way to extract the stresses would be if Dyna could calculate the stresses in the sphere in spherical coordinates during the analysis, and write these stresses to a file for a specific point in the structure. The procedure to model this was note found. And the approach that was used was suggested by Dynamore Nordic. The email from Dynamore Nordic can be found in appendix C.1.

11.1.3 Determination of Buckling Point

The determination of the buckling point is a central part of both the linear and non-linear analyses. The linear buckling analysis gives an output file that contains each buckling mode with the corresponding eigenvalue. The buckling load is found by multiplying the eigenvalue with applied load. The source of error is therefore very small. The non-linear buckling analysis requires more interpretation of results. The force-displacement curve is plotted from the nonlinear analysis, and the buckling point is chosen to where the curve becomes significantly non-linear. For classical bifurcation buckling, this will be the top of the curve because the curve is linear until the top is reached. But some curves becomes non-linear before the top is reached. In this case, the buckling point is said to be where the curve becomes significantly non-linear. This point may vary between the curves. A more systematic and consistent method for determination of the buckling point could have been preferable. When comparing the nonlinear buckling load between analyses, it could be said more certain which criteria that are used in order to compare.

11.1.4 Application of Imperfections

The non-linear analyses in this thesis include initial imperfections. The imperfections are made of the buckling modes from the linear analyses. Each linear analysis search for ten buckling eigenmodes, which are used as imperfections. Equation 11.1.1 shows how the eigenmodes are added in order to produce the imperfection in most of the analyses. The letters a to j indicate the number of eigenmode where a is mode nr. 1, b mode nr. 2 and so on. The way of combining the contributions are specified for each non-linear analysis. In order to implement the imperfections in the analysis, three files were written. Each file containing the displacement in x-, y- and z-direction. In order to produce the resultant initial imperfection, these three files are combined in the non-linear analysis.

$$a + (b \cdot 0.1) + (c \cdot 0.05) + (d \cdot 0.025) + (e \cdot 0.0125) + (f \cdot 0.00625) + (g \cdot 0.00313) + (h \cdot 0.00156) + (i \cdot 0.00078) + (i \cdot 0.00039)$$
(11.1.1)

The contribution from each mode is scaled by a factor. This is because the buckling load corresponding to each mode will increase for increasing number of the buckling mode. The assumption is that the buckling mode with lowest load (mode 1), will have more influence on the deformation than higher modes. That is why this scaling is used. But there may exist a better way to scale the different contributions. For instance it can be seen that the contribution from mode nr. 1 (a) is not scaled at all. But the contribution from mode nr. 2 (b) is scaled by 10%. The percentage is then bisected for the following modes. It may be argued that mode nr. 1 should not have that much influence compared to the other modes. In some analyses, the buckling loads are quite similar. Then it can be argued that the x-, y- and z-files should be combined of equal contribution for all the modes.

It is also possible to generate user specified imperfections in LS-Prepost. These can be written to three files in the same manner and used as imperfections in the analysis in Dyna. However, the linear analysis points out the lowest buckling eigenmode of the structure. It therefore seems reasonable that this deformation pattern is the most sensitive for the structure. And consequently this would trigger the lowest buckling load in the non-linear analysis. But how much each mode should contribute is not determined by a linear analysis. It is emphasized that the largest value of the initial imperfection is controlled by a single parameter when inserting the imperfections in LS-Prepost. The largest value of the initial imperfection can be scaled by this parameter. The different contributions are therefore not decisive for the largest value of the imperfection. But they will affect were the imperfections are inserted on the structure depending on the buckling mode.

11.2 Results

This section discusses the results from the analyses. Two loading conditions are used through this thesis. Number one is only external pressure, while number two includes external pressure, sloshing load, weight of isolation in the tank and acceleration on material due to ship movements. The results from the first loading condition is discussed first, followed by the second loading condition.

11.2.1 External Pressure

Several analyses have been conducted on different models exposed to external pressure. A halfsphere was used in the beginning, then a full sized spherical tank followed by a non-spherical tank. Finally the spherical and non-spherical tank were analysed with different sections of different thickness.

The half-sphere was started with in order to confirm the analysis procedure in Dyna. The buckling modes from the linear analysis was the same in Dyna and the article from Tall et al. (2018). The Zoelly critical pressure shown in the article also corresponded well with the critical pressure from Dyna. Dyna overestimated the critical pressure by 1.6%. The axisymmetric mesh was chosen because of the correlation between the results from Dyna, Tall et al. (2018) and Abaqus. A non-linear analysis was proceeded with in order to estimate a more realistic result for the critical pressure. It was found that elasto-plastic material gave a cleaner force-displacement curve than elastic material. And the curves corresponded well with theoretical curves as in Amdahl (2010). The non-linear results were finally evaluated based on DNVGL (2004). When the knock-down factor was neglected in the calculations, this gave the same critical pressure as Dyna without imperfections. But when the knock-down factor was included, Dyna overestimated the critical pressure by 20%. The largest resultant imperfection were chosen based on equations from DNVGL (2016b). But it was not clear what imperfections that were accounted for in the knock-down factor from the calculations. This may cause the deviation of 20%. The analysis of the half-sphere therefore seemed to be reliable.

The next model was a full sized spherical LNG tank. The linear analysis from Dyna is within 95% of the Zoelly critical pressure both for aluminium and steel. The analysis of the half-sphere gave exactly the same result as Zoelly critical pressure. Since the full sized tank is within 95% of the Zoelly critical pressure and the results from the half-sphere, the results from the linear analysis seemed reliable for the full sized tank. From the non-linear analysis, the steel tank can be seen to have 2.7 times the capacity of the aluminium tank for approximately the same imperfection. It can therefore be concluded that the aluminium tank has a severely lower capacity than the steel tank. This is mainly caused by the decrease in Young's modulus and yield stress. These parameters for aluminium is approximately one third of the values for steel. The sphere buckles at the top with a small dent in the beginning, this develops further as the simulation runs. The shape of the deformation seems to correspond with the article from Tall et al. (2018) when they study the geometrically non-linear analysis.

The buckling modes from the linear analysis of the full sized spherical tank were slightly different from the analysis of the half-sphere. In the full sized tank, the waves originate from the top of the sphere, and decreases as they approach the equator. For the half-sphere, the waves originates from one side along the equator. The waves seem to have rotated an angle of 90 degrees. This may be explained by the mesh on the sphere. The waves seem to be dependent on the starting point of the mesh. For the full sized sphere, the starting point is located at the top of the sphere where the colour becomes more black. This is because the size of each element decrease as it approach the top. This point is located at the equator for the half-sphere. The location of this point therefore seems to be vital for the deformation pattern in the buckling modes.

The linear buckling pressure for the non-spherical tank according to Dyna overestimates the analytic value by 3.6%. The analytic value to compare with here is the elastic buckling pressure of a cylinder exposed to hydrostatic pressure according to DNVGL (2017). The reason for the overestimation may be because the spherical end caps strengthen the cylindrical section.

Based on elastic calculations, it can be evaluated if the spherical or cylindrical section will
buckle. The section with lowest elastic buckling pressure will most likely buckle. By changing thickness of the sections or the length of the cylindrical section, it can be adjusted which part that buckles. As the sphere stretches it becomes more like a cylinder. And according to Jasion et al. (2015) the middle section will buckle if the sphere is stretched to become more like a cylinder. This can be explained by the stresses that occur in the shell. The circumferential stress in the cylindrical section will be twice as large as the stress in the spherical section if they have the same thickness. The cylindrical section will therefore have lower capacity. The non-linear analysis gave a decrease in the critical pressure of 50%. The decrease can be seen to be coherent with other linear to non-linear transitions in the thesis.

When the spherical and non-spherical tank is split into sections, the thinnest zone buckles. The spherical tank with different thicknesses buckles in the second zone from the top (zone 6) which is the thinnest. The linear elastic value from Dyna is approximately the same as the Zoelly critical pressure for the thickness in zone 6. The non-spherical tank with different thicknesses was analysed with a more irregular mesh. This seemed to affect the results considerably. The linear buckling pressure for zone 6 in the cylindrical part was compared with elastic buckling pressure according to DNVGL (2017) as before. But the analytic value became approximately 60% of the value from Dyna. This is a significant deviation compared to previous results in this report. The mesh is most likely what causes this problem. However, it may be possible that the results are roughly correct. But further considerations must be made in order to confirm this. The model is therefore deemed to not be reliable until further confirmations can be made. A more regular mesh as the axisymmetric mesh may give more reliable results.

The thicknesses of the spherical and non-spherical tanks with several sections should be applied outside the tank. Then the inner surface of the tank will be smooth. This was achieved for the spherical tank, but only to some extent for the non-spherical tank. This may also affect the results from the analyses of the non-spherical tank with different thicknesses.

11.2.2 All Loads Applied

The loads applied in this loading condition are:

• External pressure.

- Acceleration on material due to ship movements.
- Weight of isolation in the tank.
- Sloshing load.

The aim of the analysis is to investigate the capacity of the tank with respect to the sloshing load. The sloshing load is modelled as a static, sloped liquid surface according to DNVGL (2016b). This is an approximation of the sloshing load in order to be able to model it statically. An approximation is made in this thesis compared to DNVGL (2016b). The angle of the surface is held constant while the acceleration is increased. For increased acceleration, the angle of the surface should also be increased, but this was neglected in this thesis.

When several loads are applied on the structure, each load has to be applied separately. The linear analysis scale up the loads until failure, while the non-linear analysis scale the last load until failure. The last load is in this case the sloshing load. The best approach would be for the linear analysis to use only the sloshing load. But the result from this approach was negative eigenvalues, which would indicate negative hydrostatic pressure. A negative hydrostatic pressure will not be physically correct. Why all of the loads are needed in order to produce purposeful results in the linear analysis is not clear. If this could be solved, the linear analysis could be conducted with the sloshing load alone. This would give more accurate results for the linear critical value for the acceleration on the fluid. It it also important to notice that the external pressure needed to be above a certain value in order to produce reliable results in the linear analysis. The amount of external pressure needed was different on each model.

Before the analyses were started, the resulting acceleration on the fluid was calculated by the procedure on page 48 in DNVGL (2016b). The python script used to calculate this can be found in appendix A.1. The calculated resulting acceleration was $13.4 \ m/s^2$. The results from the linear analysis of the spherical tank with uniform thickness can be seen in table 11.2.1. It can be seen that the critical acceleration on the fluid according to Dyna is significantly larger than the value calculated according to DNVGL (2016b). This may be explained by the procedure in DNVGL (2016b). An iteration should be conducted in order to find the worst stresses in the shell from the liquid, so it may increase in further iterations. A linear analysis also overestimates realistic values. The deformation of the structure occurred in the area of the liquid surface. The article by Sano et al. (2017) analyses a standing non-spherical tank in

partially filled condition. In that case the tank buckles in the area around the liquid surface. The deformation pattern corresponds to the deformation of the spherical tank in figure 7.3.4.

Table 11.2.1: Results from linear buckling analysis of spherical tank with uniform thickness with several loads

Critical Pressure	Critical Acceleration on fluid	Critical Acceleration on material
0.20 MPa	$60.8 \ m/s^2$	$30.51 \ m/s^2$

The linear analysis was followed by a non-linear analysis. Several non-linear analyses were performed. The difference between them was either the amount of external pressure, or the size of the largest initial imperfection. These parameters were found to affect the capacity in some extent. The main result from these analyses are shown in table 11.2.2. The critical acceleration can now be seen to be 36 m/s^2 , while the other loads are at a constant level throughout the analysis. This is a decrease of 40% from the linear analysis. This result was then compared with the rules in DNVGL (1997). The critical acceleration was according to these rules found to be 18 m/s^2 . The rules are therefore very conservative compared to the critical level according to Dyna. According to Dyna, the spherical tank with uniform thickness has only used 50% of the capacity at the critical level from DNV.

Table 11.2.2: Results from non-linear buckling analysis of spherical tank with several loads

External Pressure	Accel. on material	Largest initial imp.	Critical Acceleration on fluid
0.0075 MPa	$18.06 \ m/s^2$	0.04 m	$36 \ m/s^2$

Some of the potential sources of error are mentioned already. The method used to extract the stresses in spherical coordinates is one source. These stresses are used in the capacity check according to DNVGL (1997). The size of the largest initial imperfection is another source of error. It could be argued that a smaller imperfection should be used, which would give a larger capacity in Dyna. Or a larger imperfection which would give a smaller capacity, and a decreasing deviation between the rules in DNVGL (1997) and Dyna. But the imperfections were chosen based on calculations according to DNVGL (2016b), so they should be of reasonable size.

The non-spherical tank was also analysed with all of the loads applied. The linear analysis was

conducted in the same manner as for the spherical tank. A lower critical acceleration on the fluid was found at $45 m/s^2$ compared to $61 m/s^2$. A possible explanation for this was believed to be the volume of liquid in the two tanks. The integrated pressure along the wet surface in the non-spherical tank will be larger than for the spherical tank. This may explain the decrease in capacity. The non-linear analysis found a critical acceleration of $24 m/s^2$ compared to $36 m/s^2$ for the spherical tank. The difference between the two can be seen to be significant. The results from this analysis was also compared with the rules from DNVGL (1997). It was then found that the critical level from DNVGL (1997) was at 75% of the level from Dyna for the non-spherical tank. This is 25% higher utilization than for the spherical tank. If the same level of safety factors are wanted, they should be decreased for the non-spherical tank compared to DNVGL (1997) according to these findings. The utilization should be decreased to 50% from 75% in order to apply the same level of safety.

The spherical tank was also analysed with different thicknesses. The linear analysis with all of the loads applied was not possible to conduct in this case with reliable results. The external pressure applied was either too low, or to high to produce reliable results. For a too low external pressure, negative eigenvalues were produced. The same problem occurred for the spherical tank with uniform thickness. But if a high pressure was applied to the spherical tank with different sections, the thinnest part of the tank buckled because of external pressure. This did not happen for the spherical tank with uniform thicknesses was not possible to conduct in this case. The expected buckling shape would be buckling around the liquid surface, which did not occur. The non-linear analysis was therefore performed with no initial imperfections. The critical acceleration from this analysis was 1 m/s^2 below the linear critical value for the spherical tank with uniform thickness. From this it can be seen that the capacities with respect to the sloshing load are quite similar with same filling of h/d = 0.29. This is expected because the thicknesses in the spherical tank with uniform thickness of 55 mm.

The non-spherical tank with different thicknesses was not analysed with all of the loads applied. This was because the analysis conducted with external pressure was deemed not reliable. To proceed with analysis with additional loads would therefore not produce significant results. The analysis with external pressure should be verified in order to proceed.

11.2.3 Non-spherical Tank Compared with Class Guideline DNVGL-CG-0134

Section 5 in DNVGL (2016b) gives guidelines for design evaluation of a non-spherical LNG tank. The tank used as example is a standing non-spherical tank as was analysed in Sano et al. (2017). According to these guidelines, the capacity of the spherical and cylindrical section can be assessed individually according to existing rules for spheres and cylinders.

Some interesting aspects were found with respect to the linear analysis of the non-spherical tank exposed to uniform external pressure. This is analysed in section 8.1.1. It can be seen from these analyses that the elastic buckling pressure can be used to identify the weakest section in the tank. When the elastic buckling pressure for the cylindrical section is lower than the elastic buckling pressure for the spherical section, the cylindrical part buckles in Dyna. This was tested with a thickness of 0.11 m on the cylindrical section. The elastic buckling pressure of the cylindrical section is then 0.28 MPa compared to 0.56 MPa for the spherical section according to the formulas. The buckling pressure from Dyna on the non-spherical tank was 3.6% higher than the analytic buckling pressure for a cylinder.

When the elastic buckling pressure for the cylindrical section is higher than the elastic buckling pressure for the spherical section, the spherical part buckles. This was tested with a thickness of 0.25 m on the cylindrical section. According to formulas, the elastic buckling pressure for the cylindrical section becomes 2.56 MPa compared to 0.56 MPa for the spherical section. Dyna estimated the linear buckling value for the non-spherical tank to the elastic buckling pressure of the spherical section with insignificant error.

It seems to be a transition area when the elastic buckling pressures are close to each other for the spherical and cylindrical section. If a thickness of 0.15 m is applied on the cylindrical section, the analytic buckling pressure is 0.64 MPa. This is close to the analytic buckling value for the spherical section of 0.56 MPa. The buckling load then became 0.53 MPa for the nonspherical tank according to Dyna. It may therefore be some interaction between the sections if the elastic buckling values for the sections are close to each other. It can be seen that the guidelines from DNVGL (2016b) seems to be correct in the assumption that the cylindrical and spherical section can be investigated as individual parts. At least when the elastic buckling pressures are different from each other. The rules for a cylinder (DNVGL, 2017) can be applied for the cylindrical section, and the rules for a sphere (DNVGL, 1997) can be applied for the spherical section. This holds when the elastic buckling stresses are different from each other. But when the pressures are close, this approach does not seem to give the exact result.

With respect to design of this structure, it should evaluated which part of the tank to strengthen in order to increase the capacity. If the elastic buckling pressure is close to each other for the two sections, the whole structure may be affected. But if there is a difference in the pressures, the weakest section can easily be identified. Knowing which part is the weakest may be an advantage in a design procedure, because then this part can be strengthened. Strengthening of the cylindrical part can be achieved by increasing thickness, or decreasing length. Strengthening spherical part can also be achieved by increasing thickness or decreasing radius. The ways of strengthening the structure must be compared to the cost of strengthening in terms of added material. By considering the buckling modes in figure 8.1.3 and 8.1.4, it can be seen that the whole cylinder buckles in figure 8.1.3. An area of equal length seems to buckle on the spherical section. It therefore seems cheaper to strengthen the spherical section because of decreasing radius for increasing length. But this occurs if the cylindrical section already is quite strong. This involves adding a lot of material to the cylindrical section. Unless the cylindrical section is designed to be quite short, it seems that this has to be strengthened in any case.

11.3 Suggestion Regarding Rules for Non-spherical Tank

With respect to the second loading condition, the utilization according to DNVGL (1997) compared to Dyna were 50% for the spherical tank, and 75% for the non-spherical tank. Both tanks had uniform thickness in spherical and cylindrical section. An attempt should be made on adapting the rules to have the same safety factor for the non-spherical tank.

The non-spherical tank resembles the shape of an ellipse more than a sphere and a cylinder. The elastic buckling stress for the sphere in DNVGL (1997) can be replaced by the elastic buckling stress of an ellipse. Then the same capacity check can be conducted as in section 8.2.3. Table 11.3.1 summarises the stresses used in the capacity check when the elastic buckling stress according to an ellipse was used in DNVGL (1997).

Point in figure 8.2.9	Acceleration	σ_{10}	σ_{20}	g
1	$24 \ m/s^2$	3.52e7	1.18e7	-0.77
2	$20 \ m/s^2$	2.5e7	6.0e6	-0.44
3	$17 \ m/s^2$	2.0e7	4.0e6	-0.16
4	$13 \ m/s^2$	$1.39\mathrm{e}7$	5.6e6	0.19

Table 11.3.1: Evaluation of results from non-linear analysis of non-spherical tank with several loads

The equation for the elastic buckling stress of an ellipse was found in Ruiz-Teran and Gardner (2008), and can be seen in equation 11.3.1. By interpolating between point 3 and 4, the critical level according to DNVGL (1997) is estimated to an acceleration of 15 m/s^2 . This implies an utilization compared to Dyna of 62.5%. This is a significant decrease from 75%. But the safety factor is not as large as for the sphere, which had a utilization of 50%. The parameter "a" was taken to be the radius added with half of the length of the cylinder, while "b" was taken to be the radius.

$$\sigma_E = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{a^2/b}$$
(11.3.1)

Chapter 12

Conclusion

The spherical LNG tank from Moss Maritime was analysed in this thesis. The tank was exposed to two loading conditions. First loading condition consisted of external pressure, while the second contained external pressure, weight of isolation on the tank, additional acceleration on material due to ship motions and a sloshing load. The non-spherical tank was analysed when exposed to the same loading conditions. The purpose was to compare the capacity of the non-spherical tank with the spherical tank and existing formulas. If existing formulas were not applicable on the non-spherical tank, a proposition should be made on how to modify the rules.

This conclusion is based on the analyses of the tanks with uniform thickness. Both of the tanks were exposed to external pressure in the first loading condition. The elastic buckling pressure from Dyna seemed to correspond well with the analytic elastic buckling pressure. The linear elastic buckling pressure from Dyna corresponded to analytic formula from Zoelly (equation 3.2.1) with over 95% accuracy for the spherical tank. The non-spherical tank was able to buckle due to external pressure in the cylindrical section, or in the spherical section. The equations for the elastic buckling pressure for a cylinder and a sphere was found to correspond with over 95% accuracy with the elastic buckling pressure for the non-spherical tank. The lowest analytic buckling pressure governed the buckling of the tank. This holds when the difference between the analytic values are large. If the analytic values are close to each other, the buckling load on the non-spherical tank is affected by buckling of both cylindrical and spherical section. Consequently, the difference between analytic and numerical values from Dyna increase.

The main results from the first loading condition were obtained by a thickness of 55 mm on the spherical tank, and on the spherical section of the non-spherical tank. The cylindrical section in the non-spherical tank had a thickness of 110 mm. The results are shown in table 12.0.1. It can be seen that the non-spherical tank had a capacity of 0.02 MPa less than the spherical tank for approximately same imperfection size. The buckling pressure from the non-linear analysis will vary depending on the size of the initial imperfection. The size of the imperfection was chosen based on calculations according to DNVGL (2016b). The non-linear analysis of the half-sphere was compared with DNVGL (2004) to verify the non-linear analysis procedure in Dyna.

Table 12.0.1: Results from non-linear analyses of spherical and non-spherical tank in aluminium exposed to external pressure

Model	Largest init. imp.	Material model	Buckling pressure
Spherical	0.0393 m	Elasto-plastic	0.16 MPa
Non-spherical	0.040 m	Elasto-plastic	$0.14 \mathrm{MPa}$

For the second loading condition, the same tanks were analysed. It can be seen from the table 12.0.2 that the non-spherical tank has $12 m/s^2$ smaller capacity than the spherical tank with respect to the sloshing load. These critical accelerations were compared to critical levels according to DNVGL (1997). The spherical tank was found to have a utilization of 50% at critical level according to DNVGL (1997) compared to Dyna. While the non-spherical tank had a utilization of 75%. The existing rules do therefore not apply the same level of safety on the non-spherical tank compared to the spherical tank.

Table 12.0.2: Results from non-linear analyses of spherical and non-spherical tank in aluminium exposed to all loads

Model	Largest init. imp.	Ext. Pres.	Accel. on mat.	Critical acceleration on fluid
Spherical	0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$36 \ m/s^2$
Non-spherical	0.04 m	0.0075 MPa	$18.06 \ m/s^2$	$24 \ m/s^2$

A modification of the existing rules should therefore be implemented. A suggestion for this modification was to replace the elastic buckling pressure for a sphere in DNVGL (1997) by the elastic buckling pressure of an ellipse. This caused a utilization of 62.5% at critical level according to DNVGL (1997) for the non-spherical tank. This gives a larger safety factor than

75% utilization. But not as large as 50% utilization which was found for the spherical tank.

It is emphasized that these results should be verified by further analyses in order to confirm the utilization at critical level from DNVGL (1997) compared to Dyna. The analyses conducted in this thesis are too few to confirm that these utilizations are exact. The same analyses should also be conducted on the spherical and non-spherical tank with different thicknesses. These tanks are more realistic than a tank with uniform thickness. The analyses of these tanks were not fully completed to produce reliable results in this thesis.

Chapter 13

Further Work

According to both Amdahl (2010) and Odland (1991), the starting point to describe elastic buckling of shells are most often an equation named "Donnel's equation". When the standing non-spherical tank is investigated in Sano et al. (2017), Donnel's equation is also used to derive the elastic buckling stress. To adapt the existing rules to the non-spherical tank investigated in this thesis, Donnel's equation could be used to derive an elastic buckling stress for this tank.

Another aspect that should be proceeded with is analyses of the spherical and non-spherical tank with different thicknesses. The analysis of the spherical tank with different thicknesses was not fully finished for the second loading condition. The linear analysis was not successfully conducted, and the non-linear analysis was conducted without imperfections. The non-spherical tank with different thicknesses seemed to have a problem with the mesh. It should be investigated how to mesh this with axisymmetric mesh, and it should be analysed as the other models.

The linear analyses produce a set of buckling modes. The largest displacement in this mode is not 1. How this displacement is calculated could also be investigated. Although this does not have a major impact on the analyses. The material model in the non-linear analyses could also be investigated further. Material nr. 18 is used in this report. It was noticed that material nr. 24 maybe is better to use, because here you can specify the stress-strain curve for the material. The tip came from Dynamore Nordic. Email can be found in appendix C.3. The Ramberg-Osgood equation can be used according to Misovic, Tadic, and Lucic (2016) to define the stress-strain curve for an aluminium alloy. This curve can be used in the material model.

References

- Amdahl, Jørgen (2010). TMR4205 Buckling and Ultimate Strength of Marine Structures. Trondheim: NTNU.
- AZO Materials (2005). Aluminium Alloys Aluminium 5083 Properties, Fabrication and Applications. URL: https://www.azom.com/article.aspx?ArticleID=2804 (visited on 03/14/2019).
- Babich, D V (1993). "Stability of shells of revolution with multifocal surfaces". In: International Applied Mechanics 29.11, pp. 935–938. ISSN: 1573-8582. DOI: 10.1007/BF00848278. URL: https://doi.org/10.1007/BF00848278.
- Central Commission for the Navigation of the Rhine and Oil Comapnies International Marine Forum (2010). International Safety Guide for Inland Navigation Tank-bargers and Terminals. First edit. Central Commission for the Navigation of the Rhine. Chap. 33, pp. 505–518. ISBN: 978-2-7466-2185-5.
- DNVGL (1997). Buckling Criteria of LNG Spherical Cargo Tank Containment System Skirt and Sphere, Classification Notes No. 30.3. DNVGL.
- (2004). Buckling Strength Analysis of Bars and Frames, and spherical shells, Classification Notes No. 30.1. DNVGL.
- (2016a). DNVGL-RP-C208: Determination of structural capacity by non-linear finite element analysis methods. September. DNVGL. URL: www.dnvgl.com..
- (2017). DNV-RP-C202: Buckling Strength of Shells. DNVGL.
- (2016b). Liquefied gas carriers with spherical tanks of type B, DNVGL-CG-0134. DNVGL.
- (2009). Strength Analysis of LNG Carriers with spherical tanks, Classification Notes No. 30.2. DNVGL.
- Erhart, Tobias (2016). Tips and tricks for successful implicit analyses with LS-DYNA. Dynamore GmbH.
- Evkin, Alexander Yu. and Olga V Lykhachova (2018). "Design buckling pressure for thin spherical shells: Development and validation". In: *International Journal of Solids and Structures*. ISSN: 0020-7683.
- Eworks Global (2005). The New Age for ABAQUS. URL: http://www.eworksglobal.com/ reports_127794587201093750.htm (visited on 05/06/2019).
- Hallquist, John O. (2006). LS-Dyna Theory Manual. Livermore Software Technology Corporation. ISBN: 0-9778540-0-0.

- Hutchinson, J W (2016). "Buckling of spherical shells revisited". In: Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 472.2195. ISSN: 13645021.
- IGU (2018). Natural Gas Facts & Figures, Natural Gas 101. URL: https://www.igu.org/ resources-data (visited on 06/07/2019).
- (2017). World LNG Report. International Gas Union.
- Jasion et al. (2015). "Elastic buckling of clothoidal–spherical shells under external pressure theoretical study". In: *Thin-Walled Structures* 86, pp. 18–23. ISSN: 0263-8231.
- Koiter, W. (1969). "The nonlinear buckling problem of a complete spherical shell under uniform external pressure". In: *Proc.Kon.Ned.Akad.Wetenschappen* 72, pp. 40–123.
- Livermore Software Technology Corporation (2019). *LS-Dyna*, *About*. URL: http://www.lstc. com/products/ls-dyna (visited on 05/06/2019).
- (2017). LS-DYNA Keyword User's Manual. Vol. I. Livermore Software Technology Corporation. ISBN: 9254492507. DOI: 10.1016/j.euromechsol.2010.02.010.
- Mathisen, Kjell Magne (2012). Solution Methods for Nonlinear Finite Element Analysis (NFEA). Lecture 11. Norwegian University of Science and Technology, pp. 1–39.
- Misovic, M, N Tadic, and D Lucic (2016). "Deformation characteristics of aluminium alloys". In: *Gradevinar* 68.3, pp. 179–189. ISSN: 0350-2465.
- Moan, Torgeir (2003). TMR4190-Finite Element Modelling and Analysis of Marine Structures. Vol. UK-03-98. NTNU.
- Odland, Jonas (1991). Dimensionering av skallkonstruksjoner. NTNU.
- Ruiz-Teran, A M and L Gardner (2008). "Elastic buckling of elliptical tubes". In: *Thin-Walled Structures* 46.11, pp. 1304–1318. ISSN: 0263-8231.
- Sano, Atsushi et al. (2017). "Estimation of elastic buckling strength of a non-spherical tank in the partially filled condition". In: Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering - OMAE 3.
- Schembri, Patrick J. (2007). THE DIFFERENT TYPES OF SCIENTIFIC LITERATURE. URL: https://www.um.edu.mt/__data/assets/file/0006/42981/The_different_types_ of_scientific_literature.pdf (visited on 05/30/2019).
- Tall, M et al. (2018). "Elastoplastic buckling and collapse of spherical shells under combined loadings". In: *Thin-Walled Structures* 123, pp. 114–125. ISSN: 0263-8231.
- Zhan, R. H. M. Huijsmans et al. (2015). "Natural Gas Storage and Transportation". In: 19th International Ship and Offshore Structures Congress 2, pp. 591–618.
- Zhang, Jian et al. (2017). "Buckling of egg-shaped shells subjected to external pressure". In: *Thin-Walled Structures* 113, pp. 122–128. ISSN: 0263-8231.

Zoelly, Robert (1915). Ueber ein Knickungsproblem an der Kugelschale. ETH Zurich.

List of Appendices

A Python Scripts	ii
A.1 Computation of Sloshing Load	ii
A.2 Buckling Check According to DNVGL (1997)	v
A.3 Buckling Check According to DNVGL (2004)	vi
A.4 Buckling Check of Cylinder According to DNVGL (2017)	vii
A.5 Plotting of Force-displacement Curve	viii
A.6 Reading of Largest Resultant Imperfection From File	ix
A.7 Generation of Stress-strain Curve for Elasto-plastic Aluminium	x
B Routines from LS-Prepost	xi
B.1 Function for Application of Static Sloped Liquid Surface	xi
C Personal Communication	xii
C.1 Communication with Dynamore Nordic on Extraction of Stresses	xii
C.2 Communication with Dynamore Nordic on Direction of Spherical S	Stresses xiv
C.3 Communication with Dynamore Nordic on Direction on Material N	Model xvii

Appendix A

Python Scripts

A.1 Computation of Sloshing Load

```
1
  import numpy as np
2
3
   #Prelocating
4
5 | Vol = np.array([0.0, 0.0, 0.0])
6 | FxRef = np.array([0.0, 0.0, 0.0])
7 aY = np.array([0.0,0.0,0.0])
8 Fx = np.array([0.0, 0.0, 0.0])
9 FTheta = np.array([0.0, 0.0, 0.0])
10 FyStar = np.array([0.0,0.0,0.0])
  aYStar = np.array([0.0,0.0,0.0])
11
12
13 | a = np.array([0.0, 0.0, 0.0])
14 a_2 = np.array([0.0, 0.0, 0.0])
15 | c = np.array([0.0, 0.0, 0.0])
16 | b = np.array([0.0, 0.0, 0.0])
   NegativexPositionBeta1 = np.array([0.0,0.0,0.0])
17
   PositivexPositionBeta1 = np.array([0.0,0.0,0.0])
18
   NegativexPositionBeta2 = np.array([0.0,0.0,0.0])
19
20 PositivexPositionBeta2 = np.array([0.0,0.0,0.0])
   aResultBeta1 = np.array([0.0,0.0,0.0])
21
22
   aResultBeta2 = np.array([0.0,0.0,0.0])
23
24 | z = np.array([10, 13, 15])
25 FxR = np.array([0.054,0.117,0.143]) #FxR from fig 6 on page 45 in CG-0134
26
27 #Reference ship
```

```
28
29
   L0 = float(276)
30 | V0 = 19.5 * np.sqrt(L0/237)
31 \mid B = float(40)
   GMR = float(7)
32
33
34
   aORef = (0.2*(VO/np.sqrt(L0)))+((34.0-(600.0)/(L0))/(L0))
35
36
  aYRef = 0.912 * a0Ref
37
38 #Actual Ship
39
40 du = float(46.3) #Outer diameter of tank
41 di = float(42.5) #Inner diameter of tank
42 df = du-di
43 |V = float(19) #Speed in knots
44 | B = float(49)
45 g = 9.81
46 | az = 0.45
47 | ay = 0.5
48
   rho = 500.0 #Cargo density
49 kr = 0.39 * B
50 | xL = 0.3
51 GM = 5.0
52
53 #Volumes of cargo and tank
54
   #Total volume of tank
55
56 \mid Vol100 = 45585
   #Volume of cargo when h/d = 29\%
57
   Vol[0] = ((np.pi*(0.29*du)**(2.0))/(3.0))*((3.0*di/2)-(0.29*du))
58
   #Volume of tank when 50% filled
59
60 \ Vol[1] = Vol100/2
   #Volume when 65% filled
61
62 Vol[2] = Vol100-((np.pi*(0.35*du)**(2.0))/(3.0))*((3.0*di/2.0)-(0.35*du))
63
64 #Calculations ----
65
66 | a0 = (0.2*(V/np.sqrt(L0))) + ((34.0 - (600.0)/(L0))/(L0))
67
68 \mid \text{kappa} = np.\text{maximum}(13*(GM/B), 1)
69
70 |Tr = (2*kr)/(np.sqrt(GM))
71
72 Theta = (50.0*(1.25-0.025*Tr))/(B+75.0)
73
74 for i in range(0,3):
```

```
75
        aY[i] = a0*np.sqrt(0.6+2.5*((xL+0.05)**2.0)+kappa*((0.6*kappa*(z[i]/B))\
76
          **2.0))
77
78
   #Forces on reference ship. Step 5 in procedure! Read FxR from
79
   #graph, rearrange equation and calculate Fx based on the
80
   #same equation as in step 5. Gamma is
81
   #equal to the density*g
82
83
        FxRef[i] = FxR[i]*rho*g*(du**3.0)
84
85
86
   #Forces on actual ship
87
88
        Fx[i] = FxRef[i]*(aY[i])/(aYRef)
89
        FTheta[i] = rho*g*Theta*Vol[i]
90
91
92
        FyStar[i] = np.sqrt(Fx[i]**2+FTheta[i]**2)
93
        aYStar[i] = FyStar[i]/(rho*g*Vol[i])
94
95
96
   #Resulting acceleration with ellipse
97
   beta1 = 20*(np.pi/180)
98
   beta2 = 21*(np.pi/180)
99
100
101
   #Beta1
   for i in range(0,3):
102
103
        a[i] = aYStar[i] **(2)+(np.tan(beta1)*az)**(2)
104
        b[i] = -2*aYStar[i]**(2)
        c[i] = aYStar[i] **(2)*(1-az**2)
105
106
107
        NegativexPositionBeta1[i] = (-b[i]-np.sqrt(b[i]**(2)-4*a[i]*c[i]))/\
        (2*a[i])
108
        PositivexPositionBeta1[i] = (-b[i]+np.sqrt(b[i]**(2)-4*a[i]*c[i]))/\
109
110
        (2*a[i])
111
112
        if NegativexPositionBeta1[i]<PositivexPositionBeta1[i]:</pre>
            aResultBeta1[i] = PositivexPositionBeta1[i]/np.cos(beta1)
113
114
        else:
115
            aResultBeta1[i] = NegativexPositionBeta1[i]/np.cos(beta1)
116
   #Beta2
117
118
        a2[i] = aYStar[i]**(2)+np.tan(beta2)*az**(2)
119
        NegativexPositionBeta2[i] = (-b[i]-np.sqrt(b[i]**(2)-4*a2[i]*c[i]))/
120
        (2*a2[i])
121
```

```
122 PositivexPositionBeta2[i] = (-b[i]+np.sqrt(b[i]**(2)-4*a2[i]*c[i]))/\
123 (2*a2[i])
124
125 if NegativexPositionBeta2[i]<PositivexPositionBeta2[i]:
126 aResultBeta2[i] = PositivexPositionBeta2[i]/np.cos(beta2)
127 else:
128 aResultBeta2[i] = NegativexPositionBeta2[i]/np.cos(beta2)</pre>
```

A.2 Buckling Check According to DNVGL (1997)

```
1
  import numpy as np
2 import matplotlib.pyplot as plt
3
4 #Input-----
                              -----
5
6 | \text{Sigma10} = 2.77*10.0**(7.0)
7
  Sigma20 = -4.4*10.0**(7.0)
8
9 E = 7.1 \times 10.0 \times (10.0)
10 nu = 0.3
  SigmaF = 134.0*10.0**(6.0)
11
12
13 | t = 0.055
14 R = 21.5
15
16 |LambdaF = Sigma20/Sigma10
17
18 |SigmaCL = ((E)/(np.sqrt(3.0*(1.0-nu**(2.0)))))*(t/R)
19
20 LambdaCL = (SigmaCL)/(Sigma10)
21
22
  #Knock-down Factor-----
                                   ______
23
24 | b = -0.5*np.exp(1)**(1.15*LambdaF)
25
26
  if LambdaF < 0.0:</pre>
27
       g = (4.0+2.0*LambdaF)*np.sqrt(R*t)
28
       delta1 = (0.01*g)/(1+(g/R))
29
30 if LambdaF >0.0:
31
       g = 4.0*np.sqrt(R*t)
32
       delta1 = (0.01*g)/(1+(g/R))
33
34 \mid #delta1 = 0.042
  delta2 = R/750.0
35
36
```

```
37
  delta = delta1+((delta2-delta1)*(np.exp(1)**(2.5*(LambdaF-1.0))))
38
39
  #Iterate to find rho!-----
40
  my = (delta/t)*np.sqrt(-b)
41
42
  Psi = 1.0 - (0.375 * np.sqrt(my))
43
44
45
  GammaP = 1.0/Psi
46
  47
48
49
  for i in range(0,9):
50
      rhoiterations[i+1] = 1-(((3.0*np.sqrt(3.0))/2.0)*GammaP*np.sqrt(-b)*\
      (delta/t)*rhoiterations[i])**(2.0/3.0)
51
      rho = rhoiterations[i+1]
52
53
54 | #rho = 0.74
  #Final calculations------
55
56
57 | Fe = 1.0/(rho*LambdaCL)
58
  Sigmae0 = np.sqrt(Sigma10**2.0+Sigma20**2.0-Sigma10*Sigma20)
59
60
  LambdaE = np.sqrt(Fe*(SigmaF/Sigmae0))
61
62
63 |LambdaCR = (1.0/(np.sqrt(1.0+LambdaE**4.0)))*(SigmaF/Sigmae0)
64
65
  # Safety factors
66
  LambdaECheck = np.sqrt((SigmaF/Sigmae0)*(Sigma10/SigmaCL))
67
68
  if LambdaECheck <1.0:</pre>
69
70
      kappa = 0.925+0.375*LambdaECheck
71
      GammaSum = kappa*1.15
72
73 if LambdaECheck>1.0:
74
      kappa = 1.3
75
      GammaSum = kappa*1.15
```

A.3 Buckling Check According to DNVGL (2004)

```
1
2 import numpy as np
3 import matplotlib.pyplot as plt
4
```

```
5
  R = 21.5
   t = 0.032
6
7 | E = 7.1 * 10.0 * * (10.0)
   SigmaY = 1.34*10.0**(8.0)
8
9
10 | rt = R/t
11
12 |rho = (0.5)/np.sqrt(1.0+((rt)/(100.0)))
13
   SigmaE = rho*0.606*E*(t/R)
                                    #Pa
14
15
   Lambda = np.sqrt(SigmaY/SigmaE)
16
17
18
   SigmaCR = SigmaY/(np.sqrt(1.0+Lambda**(4.0)))
19
20 Pcr = ((SigmaCR * 2.0 * t)/R) * 10.0 * * (-6.0)
                                                    #MPa
21
22 SigmaENoIMP = 0.606*E*(t/R)
23
24 | PNoIMP = (SigmaENoIMP*2.0*t)/R*10.0**(-6.0)
```

A.4 Buckling Check of Cylinder According to DNVGL (2017)

```
1 import numpy as np
2
3 #Hydrostatic Pressure - DNVG1-RP-C202------
4
5 | L = 4.0
  t = 0.11
6
7
  R = 21.5
  nu = 0.3
8
  E = 7.1 * 10 * * (10)
9
10
  psi = 2.0
11
12 | rho = 0.6
13
14
15 Z = ((L**(2.0))/(R*t))*np.sqrt(1.0-nu**(2.0))
16
17 |xi = 1.04*np.sqrt(Z)
18
  C = psi*np.sqrt(1+((rho*xi)/(psi))**(2.0))
19
20
21 #Elastic Buckling stress-----
```

```
22
23 SigmaE0 = (C*((np.pi**(2.0)*E)/(12.0*(1.0-(nu**(2.0)))))*((t/L)**(2.0)))*10**
24
25 #Elstic Buckling Pressure------
26
27 Pe = (SigmaE0*t)/R
```

A.5 Plotting of Force-displacement Curve

1

```
2
  import matplotlib.pyplot as plt
  import numpy as np
3
4
  import pandas as pd
5
  #Rean in file nr. 1-----
6
7
8
  df = pd.read_csv('DisplacementSectionsAllLoads.csv', skiprows=[0,1,2,3,4,5,6,7
  df = df.drop(df.index[-1])
9
10
11
  #Convert first column from string to float
12
13 df.Load = df.Load.astype(float)
14
                            -----
  #Read in file nr. 2-----
15
16
17
  #df2 = pd.read_csv('displ-EalstoP-IMPmode1.csv', skiprows=[0,1,2,3,4,5,6,7], set
18
  #df2 = df2.drop(df2.index[-1])
19
20
  #Convert first column from string to float
21
22
  #df2.Load2 = df2.Load2.astype(float)
23
24 #Compute load forboth files
25
26 Load = (df.Load*100)-200
27
  #Load2 = df2.Load2*2
28
29 #Set displacement vector
30
31
  Displacement = df.Displacement
32
  #Displacement2 = df2.Displacement2
33
34 #Plot the results
35
36 |line1 = plt.plot(Displacement, Load, label='Elasto-Plastic Material')
37 | #line2 = plt.plot(Displacement2[1:30], Load2[1:30], label='Elasto-Plastic Mat
```

```
38 plt.show()
39 #plt.title('Applied Acceleration = 70 m/s<sup>2</sup>',fontsize=20)
40 plt.xlabel('Resultant Displacement [m]',fontsize=15)
   plt.ylabel('Acceleration [m/s<sup>2</sup>]',fontsize=15)
41
42
   plt.grid()
43 plt.xlim(0,1)
   plt.ylim(0,60)
44
45 | plt.legend(framealpha=1, frameon=True);
46
   plt.show()
47
   #Extract max and min values
48
49
50 LoadMax = max(Load)
51 DisplMax = max(Displacement)
52
53 #Load2Max = max(Load2)
54 #Displ2Max = max(Displacement2)
55
56 DisplMin = min(Displacement)
```

A.6 Reading of Largest Resultant Imperfection From File

```
import matplotlib.pyplot as plt
1
2
   import numpy as np
3
  import pandas as pd
4
   #Read in resultant imperfection file
5
6
7
   df = pd.read_csv('pert_node_resNonLinBucklIMP.csv', skiprows=[0,1,2,3,4,5], sep
8
   df = df.drop(df.index[-1])
9
   #df.Index = df.Index.astype(float)
10
11
   #Find minimum displacement
12
13
  ImpMin = 1
14
15
16
   for i in range(0,len(df.Imperfection)):
17
18
       if ImpMin > df.Imperfection[i] and df.Imperfection[i] > 0:
19
           ImpMin = df.Imperfection[i]
20
21
22 | #Find maximum displacement
```

```
23
24 ImpMax = max(df.Imperfection)
```

A.7 Generation of Stress-strain Curve for Elasto-plastic Aluminium

```
1
2
   import matplotlib.pyplot as plt
3
   import numpy as np
4
5 | K = 426.0
6 n = 0.2004
   E = 71000
7
8
9
   Sigmay = 134.0
10
   epsilon = np.linspace(0.0,0.2)
11
12
   sigma = K*(epsilon-((Sigmay)/(E)))**n
13
14
  line1 = plt.plot(epsilon,sigma)
15
16
17 plt.show()
18 plt.title('Ramberg-Osgood curve, Aluminium',fontsize=20)
19 plt.xlabel('Strain ',fontsize=15)
20 | plt.ylabel('Stress [MPa]',fontsize=15)
21 plt.grid()
22 plt.xlim(0,0.25)
23 plt.ylim(0,350)
24 plt.legend(framealpha=1, frameon=True);
25 | plt.show()
```

Appendix B

Routines from LS-PrePost

B.1 Function for Application of Static Sloped Liquid Surface

```
1
2 #refx: surface of liquid
3 #The hydrostatic pressure should be applied after 2 seconds
4
5 float hpres(float t, float x, float y, float z, float x0, float y0,\
6 float z0)
7 {float fac, trise, refx, rho, acc, h, time;
8 trise = 0.1; refx = -9.03; rho = 1000.0; acc = 30.0; time = 0.0;
9 if(t>=2.0) time = t-2.0;
10 fac = 1.0;
11 h = refx-x;
12 if(h<0.0) h = 0.0;
13 return time*rho*acc*h;}</pre>
```

Appendix C

Personal Communication

C.1 Communication with Dynamore Nordic on Extraction of Stresses

Re: [DmN Support Ticket#526004] Stresses in spherical shell

Support <support@dynamore.se>

to. 11.04.2019 16.07

Til: Andreas Sanne <andrsann@stud.ntnu.no>

HI Andreas!

There is no single correct way of doing it. I'll give you some alternatives. Depending on your situation, you can choose your method.

1. Load d3plot and the keyword file. Create a spherical coordinate system in General Settings - Local Coord System. Use the "Sph" (spherical) option. Select the system in the list and make it active by clicking Apply. Then fringe plot the stress and switch the option at the bottom in "Fringe Component" from d3plot to User. The stress will then be transformed to local (r, phi, z) spherical system.

2. if the elements in the sphere have their local coordinate system aligned with the lateral and longitudinal directions, you can plot the stresses in "Elem" system directly without having to create any additional coord system.

3. If you use an material model with material directions, you can plot the stress in "Mtrl" system. All these (User, Elem and Mtrl) are found in the option at the bottom of the "Fringe Component" interface.

regards, Anders Jernberg

DYNAmore Nordic AB Mail: support@dynamore.se Phone: +46 (0)13 236680

11/04/2019 14:40 - Andreas Sanne wrote: Hello!

Do you know if it is possible to extract the stresses in the two different directions in the spherical shell? There exists two stresses that act normal to each other: one in meridional direction, and one in circumferential direction. I want to be able to extract these stresses for each time step in the non-linear analysis, if it is possible.

Can this be done by "fringe component" and "1st (2nd) principal stress"? I assume the x, and y- stresses refer to global coordinate-system? Or can it be done "database_ASCII", and "elout"? What would be the correct way to do this?

Best regards Andreas Sanne

C.2 Communication with Dynamore Nordic on Direction of Spherical Stresses

Re: [DmN Support Ticket#526059] Sv: Sv: Directions in Spherical Coordinate System

Support <support@dynamore.se> ma. 13.05.2019 14.08 Til: Andreas Sanne <andrsann@stud.ntnu.no> Hi Andreas,

Theta is in the x-y plane in LSPP.

regards, Anders

DYNAmore Nordic AB Mail: support@dynamore.se Phone: +46 (0)13 236680

13/05/2019 09:30 - Andreas Sanne wrote:

Okey, so you do not know exactly how they are defined? Do you have any idea how I can find this out? I try to google it, but it seems that physics and mathematics normally use opposite definitions. And I cannot find out how Prepost defines these coordinates on google...

Fra: Support <support@dynamore.se>
Sendt: mandag 13. mai 2019 06:02
Til: Andreas Sanne
Emne: Re: [DmN Support Ticket#526059] Sv: Directions in Spherical Coordinate System

Hej!

OK. Now I understand. LS-DYNA only has Cartesian Coordinate system in 3D. So, the method you are using is to calculate everything in a Cartesian system in LS-DYNA and then transform the stresses to a spherical system. But this is done in the post-processor (LS-PREPOST) and not in LS-DYNA. The spherical coordinate system depends on how you define it in LS-PREPOST, so it might very well be according the system that you are describing.

I hope this helps.

Best regards,

/Mikael

DYNAmore Nordic AB Mail: support@dynamore.se Phone: +46 (0)13 236680

11/05/2019 17:00 - Andreas Sanne wrote:

I use the approach to transform the stresses in cartesian coordinates tp spherical coordinates by "general setting", creating a local coordinate system in the center of my sphere, I choose "sph" and "apply". By choosing "user" in "fringe componenet", the stresses appear in r, theta and phi direction. Then my question is how these directions of phi and theta are defined with respect to my local coordinate system in cartesian coordinates?

The manual(Vol.1) on page 2424 has some desription on spherical coordinate system, also on page 2836. But the manual seems to say the opposite of what I would assume...

It seems that the manual sais the theta-direction is measured from the z-axis, and phi-direction is measured in x-y-plane. From my results, I would think the opposite was more likely...

Was that more clear?

Fra: Support <support@dynamore.se>
Sendt: fredag 10. mai 2019 12.48
Til: Andreas Sanne
Emne: Re: [DmN Support Ticket#526059] Directions in Spherical Coordinate System

Hej Andreas,

Now I am a bit confused. To my knowledge here is no spherical coordinate system in LS-DYNA. Could you refer to the manual page where you found this?

DYNAmore Nordic AB Mail: support@dynamore.se Phone: +46 (0)13 236680

10/05/2019 14:20 - Andreas Sanne wrote: Hello!

I'm discussing stresses in my spherical shell, and I just wondered if it is correct that the phidirection in dyna is in the y-z plane, while the theta-direction is in the x-y plane for a spherical coordinate system? This seems correct from my analysis, but not from the manual...

Best regards Andreas Sanne

C.3 Communication with Dynamore Nordic on Material Model

Re: [DmN Support Ticket#525915] Sv: Sv: Sv: Dynamic Relaxation and Gravity

Support <support@dynamore.se>

ma. 11.03.2019 11.05

Til: Andreas Sanne <andrsann@stud.ntnu.no>

1 vedlegg (435 byte)

mat18.m;

Hej Andreas,
regarding MAT_18: I don't know if this is a material model that is widely used.
But what I know is that really everyone used MAT_24 for isotropic plasticity in LS-DYNA.
MAT_24 is a very reliable material model, both for implicit and explicit.
Perhaps MAT_18 works just as fine, but instead of spending time on further investigations of that issue, I simply switched to MAT_24.
I generated the curve in octave (matlab) I have attached the code, mat18.m.

In coming LS-DYNA versions, there is a keyword *DEFINE_CURVE_STRESS that can create this type of common hardening curves automatically. I will try to close this ticket now. If you have further questions to this ticket, you can answer this mail. In case you have new questions regarding LS-DYNA please don't hesitate to contact us via e-mail support@dynamore.se or phone.

Best regards, Anders

DYNAmore Nordic AB Mail: support@dynamore.se Phone: +46 (0)13 236680

11/03/2019 10:40 - Andreas Sanne wrote: Thank you for your suggestion, this seems to work.

Do you have reason for why you would not use Mat18 for elastic-plastic material in implicit? Did you use a separate program to generate the curve for Mat24?

Fra: Support <support@dynamore.se> Sendt: torsdag 7. mars 2019 06.29 Til: Andreas Sanne Emne: Re: [DmN Support Ticket#525915] Sv: Sv: Dynamic Relaxation and Gravity</support@dynamore.se>
Hej Andreas, I think I found a solution to this. I believe you can run the whole sequence in "physical time". First apply gravity from 0 to 1. Then apply external pressure from 1 to 2. Set arctim = 1.1 on *CONTROL_IMPLICIT_SOLUTION. Then the gravity loading stays constant (at least in this case). In this case dnorm = 1 seems to give better results than dnorm = 2.
You can download a version from <u>https://project.dynamore.se/public/d853bd</u>
I could not found the y-perturbation, so I ran only with X and Y.
A few other notes: - I would not use MAT18 for an elastic-plastic material in implicit. So I switched it to MAT24 using a power-law load curve based on the same parameters. - For non-linear materials, NIP >= 3 is required in the shells. I think that in many cases NIP = 5 is a good option (*SECTION_SHELL).
Best regards, Anders



