Models for Inventory Management, Scheduling and Routing in Fish Feed Distribution

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Objective and scope

The thesis addresses routing of ships, used in order to serve an industrial network of nodes covering aquaculture location along the Norwegian coastline. Nodes are served with feed from a central factory. The aim of the thesis is making a feasible routing model that minimizes operating costs, while ensuring feed deliveries that meet the feed requirements at the fish farms at any given time.

Only the distribution system itself and feed consumption is considered in the optimization process. The distribution system includes loading and discharge from ships to ports and the sailing between locations. Feed production and earlier stages of the supply chain are not considered.

Introduction

Fish feed is one of the main cost drivers in the salmon farming industry.
The Norwegian fish feed industry totaled NOK 24.5 billions in revenues in 2016, with a EBITDA margin of about 11% [5]. Transportation of feed is one of the key cost drivers. Failure to provide sufficient amount of feed at the right time can also impact the growth of the salmon and therefore also reduce revenues. Reliable deliveries are therefore crucial in order to keep unit costs low in the salmon farming industry. As the number of fish farming locations have increased, intuition is no longer sufficient when planing routing of the ships used when distributing feed. Thus optimization techniques implemented in the form of computer algorithms is necessary in order to evaluate and analyze routing problems in an efficient manner.

Maritime routing problems have been present for as long as ships have existed, and in recent years mathematical formulations have been used in order to solve these problem. Important work includes Fagerholt (1999) [6] that solves a multi-trip vehicle routing problem VRPMT using a route generation algorithm and Christiansen (1999) [3] which is formulating a maritime inventory routing problem. For transportation of fish feed, Haugland and Thygesen (2017) [7] provides insight to the feed production and distribution in the aquaculture industry. Haugland and Thygesen formulates a VRP that is solved using the route generation algorithm laid out by Fagerholt.

Multiple model formulations can be suitable to ensure efficient deliveries of fish feed to the industrial network of nodes. The two main alternatives are Vehicle Routing Problem (VRP) and Inventory Routing Problem (IRP) formulations. The VRP describes how a set of vehicles with given capacities may be utilized in order to serve costumers. The objective is to reduce the total cost of serving the costumers. The problem was first described by Dantzig and Ramser (1959)[4] and was focused on petrol deliveries. The main advantage of the VRP formulation is that it is less computational complex than the IRP. Thus, problems can be solved to optimality in a shorter time than the IRP formulations. The main advantage of the IRP is that routing, scheduling and inventory management are all considered in the same model. The IRP ensures that inventory levels at network nodes never run out.

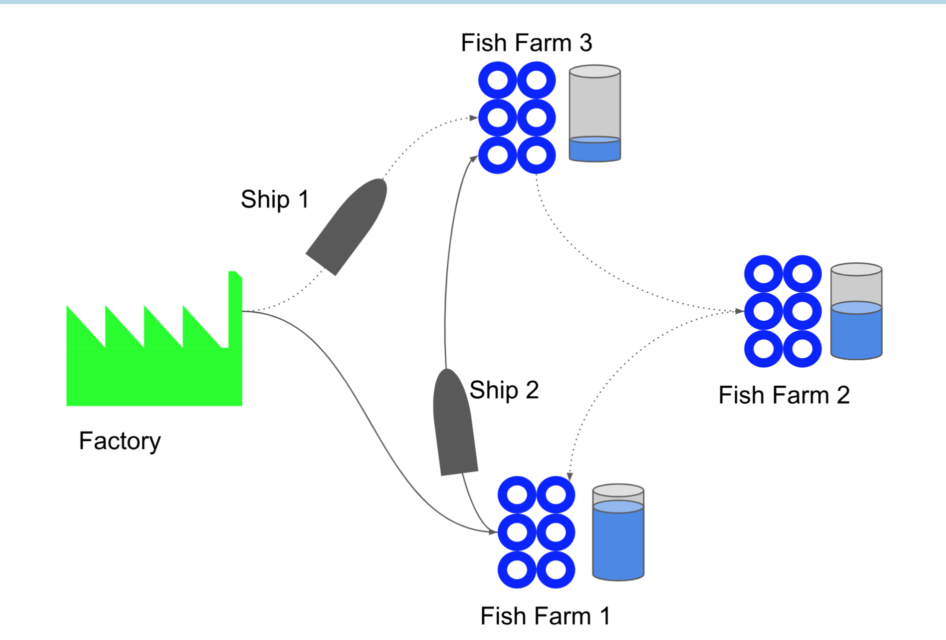

Illustration of an IRP model. The model prioritizes deliveries to nodes before they run out of inventory. This is done while optimizing the total costs for the deliveries throughout the planning period.

The IRP formulations complexity and solution time increases quickly with additional harbours in the IRP. As the IRP problem can potentially include multiple nodes for each harbour in order to resemble separate arrivals, the number of possible arcs grows much faster for the IRP formulation, than the VRP formulation. Thus, model simplifications are more essential in the modelling of a IRP. Model simplifications helps improve the computational time, but it also limits the model, as real world implications are neglected. Such real world implications can include changing consumption rates, variable sailing times due to weather changes or operational changes due to viruses and bacteria in the fish.

The computational complexity of the the IRP makes formulation improvements more important. Such formulation improvements may include adding additional constraints as sub-tour elimination constraints[1] and dynamic cut inequalities[2]. Alternative model formulations can potentially also be utilized to improve computational result. Different model formulations include arc-load formulations, arc-flow formulations and Dantzig-Wolfe decomposition.

References

[1] Yossiri Adulyasak, Jean-François Cordeau, and Raf Jans. Benders decomposition for production routing under demand uncertainty. *Operations Research*, 63(4):851–867, 2015.

[2] Agostinho Agra, Marielle Christiansen, Alexandrino Delgado, and Luidi Simonetti. Hybrid heuristics for a short sea inventory routing problem. *European Journal of Operational Research*, 236(3):924–935, 2014.

[3] Marielle Christiansen. Decomposition of a combined inventory and time constrained ship routing problem. *Transportation science*, 33(1):3–16, 1999.

[4] George B Dantzig and John H Ramser. The truck dispatching problem. *Management science*, 6(1):80–91, 1959.

[5] E. Moe EY. The norwegian aquaculture analysis 2017. *The Norwegian aquaculture analysis*, 2017.

[6] K. Fagerholt. Optimal fleet design in a ship routing problem. 1999.

[7] M. G. Haugland and S. Thygesen. Use of clusters in a route generation heuristic for distribution of fish feed. 2017.

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Modelling

The problem is modelled as an Inventory Routing Problem, with three different model formulations. The models includes an arc-load formulation, an arc-flow formulation and a Dantzig-Wolfe decomposition.

1 Arc-Load Model

The arc-load formulation can be solve independently, but it also forms the foundation for the other formulations. Complete descriptions for parameters and variables can be found in the thesis.

1.1 Objective

The first term of the objective function adds the sailing costs between destinations. The second term adds the marginal cost if feed is from external sources. In other words, the difference between the cost of feed from the self-owned factory and the cost for feed at other sources. This includes all feed consumed at the given node during the planning period. The third term includes transportation costs for nodes served by external vessels.

$$minz = \sum_{v \in V} \sum_{A_v \in (i,m,j,n)} C_{ijv} x_{imjnv} + T \cdot E \sum_{i \in H_T} R_i (1 - u_i) + \sum_{i \in H_T} E_i^T R_i (1 - u_i) \tag{1}$$

1.2 Network constraints

The network constraints are defined in order to ensure that ships are generated, routed correctly and the eliminated. The constraint given by equation 2 ensures flow conservation, such that the numbers of vessels arriving and being generated at node, equals the number of vessels being eliminated and leaving the same node. Equation 3 ensures that a given harbour is only visited one time for each possible arrival slot.

$$W_{imv} + \sum_{j \in H_v} \sum_{n \in M_{jv}} x_{jnimv} - \sum_{j \in H_v} \sum_{n \in M_{jv}} x_{imjnv} - z_{imv} = 0, \forall v \in V, i \in H_v, m \in M_{iv} \tag{2}$$

$$\sum_{v \in V} \sum_{j \in H_v} \sum_{n \in M_{jv}} x_{jnimv} + y_{im} + \sum_{v \in V} W_{imv} = 1, \forall i \in H_T, m \in M_{Ti} \tag{3}$$

1.3 Loading and unloading constraints

Equation 4 ensures that vessel cargo is in accordance with the loading and unloading at ports visited. Equation 5 ensures that the maximum capacity of the ship is not surpassed. Equation 6 and 7 sets upper and lower bounds on how much feed that have to be unloaded during one port visit.

$$x_{imjnv}(l_{imv} + J_j q_{jnv} - l_{jnv}) = 0, \forall v \in V, (i,m,j,n) \in A_v \tag{4}$$

$$l_{imv} - \sum_{j \in H_v} \sum_{n \in M_{jv}} C_v^{AP} x_{jnimv} \leq 0, \forall v \in V, i \in H_v, m \in M_{iv} \tag{5}$$

$$q_{imv} - \sum_{j \in H_v} \sum_{n \in M_{jv}} Q_{imv}^{MAX} x_{jnimv} \leq W_{imv} Q_{imv}^{MAX}, \forall v \in V, i \in H_v, m \in M_{iv} \tag{6}$$

$$\sum_{v \in V} q_{imv} + Q_{im}^{MIN} y_{im} \geq Q_{im}^{MIN}, \forall i \in H_T, m \in M_{Ti} \tag{7}$$

1.4 Time constraints

Equation 8 ensures that the end of operations at a port for a given arrival happens at a time equal to the loading or discharge process time after the start of the process. Equation 9 ensures sailing routes and schedules comply. Equation 10 ensures that the start of a loading or unloading process at a port can not happen before the given process is finished for the previous arrival. Equation 11 ensures that operations happen within the given time windows.

$$t_{im}^S + \sum_{v \in V} T_i^Q q_{imv} - t_{im}^E = 0, \forall i \in H_T, m \in M_{Ti} \tag{8}$$

$$x_{imjnv}(t_{im}^E + T_{ijv}^S - t_{jn}) \leq 0, \forall v \in V m(i,m,j,n)\ inA_v \tag{9}$$

$$t_{im}^S - t_{Ei(m-1)} + T_i^B y_{im} \geq T_i^B, \forall i \in H_T, m \in M_{Ti} \tag{10}$$

$$T_{im}^{WS} \leq t_{im}^S \leq T_{im}^{WE}, \forall i \in H_T, m \in M_{Ti} \tag{11}$$

1.5 Inventory constraints

Equation 12 ensures that inventory levels at the end of a loading process is correct with respect to the inventory level at the start of the process. Equation 13 ensures correct changes in inventory levels at ports between arrivals. Equation 14 and 15 ensures that inventory levels are within predefined limits at arrivals, and equation 16 ensures that inventory levels are still within predefined levels at the end of the period.

$$s_{im}^S - \sum_{v \in V} J_i q_{imv} + R_i t_{im}^E - R_i t_{im}^S - s_{im}^E = 0, \forall i \in H_T, m \in M_{Ti} \tag{12}$$

$$s_{Ei(m-1)} + R_i t_{im}^S - R_i t_{Ei(m-1)} - s_{im}^S = 0, \forall i \in H_T, m \in M_{Ti} \tag{13}$$

$$S_i^{MIN} u_i \leq s_{im}^S \leq S_i^{MAX}, \forall i \in H_T, m \in M_{Ti} \tag{14}$$

$$S_i^{MIN} u_i \leq s_{im}^E \leq S_i^{MAX}, \forall i \in H_T, m \in M_{Ti} \tag{15}$$

$$S_i^{EMIN} u_i \leq s_{im}^E + R_i(T u_i - t_{im}^E) \leq S_i^{MAX}, \forall i \in H_T, m \in |M_{Ti}| \tag{16}$$

$$S_i^S + R_i(t_{i1}^E) = s_{i1}^S, \forall i \in H_T \tag{17}$$

2 Arc-Flow Model

An alternative way of linearizing constraint 4 can be achieved by reformulating the model from an arc-load model into an arc-flow model. Instead of assigning variables l_{imv} for the load onboard a ship when leaving a node, the arc-flow model uses variables l_{imjnv} for the load carried on each arc. By replacing constraint (4) - (5) with the following constraints, the model is reformulated into an arc-flow model.

$$\sum_{j \in H_v} \sum_{n \in M_{jv}} l_{jnimv} + J_i q_{imv} - \sum_{j \in H_v} \sum_{n \in M_{jv}} l_{imjnv} = 0, \forall v \in V, i \in H_v, m \in M_{iv} \tag{18}$$

$$l_{imjnv} \leq C_{APv} x_{imjnv}, \forall (i,m,j,n) \in A_v, v \in V \tag{19}$$

3 Dantzig-Wolfe Decomposition

A Dantzig-Wolfe decomposition is achieved by separating the ship specific and harbour specific constraints into separate subproblems. In addition to constraint (3), two new constraints are imposed.

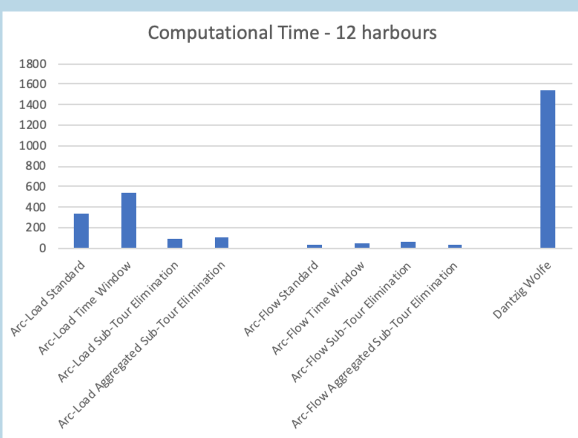
$$t_{im}^S - \sum_{v \in V} t_{imv} = 0 \tag{20}$$

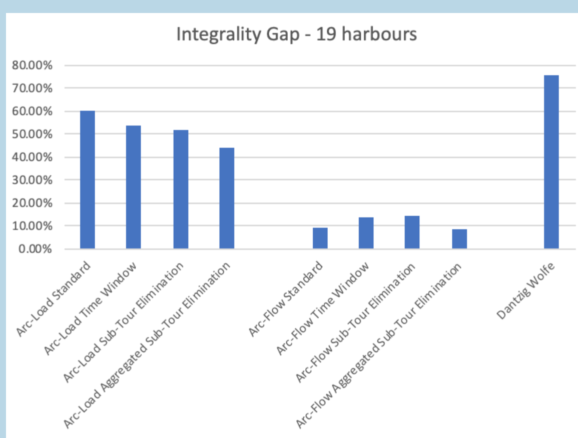
$$q_{im} - \sum_{v \in V} q_{imv} = 0 \tag{21}$$

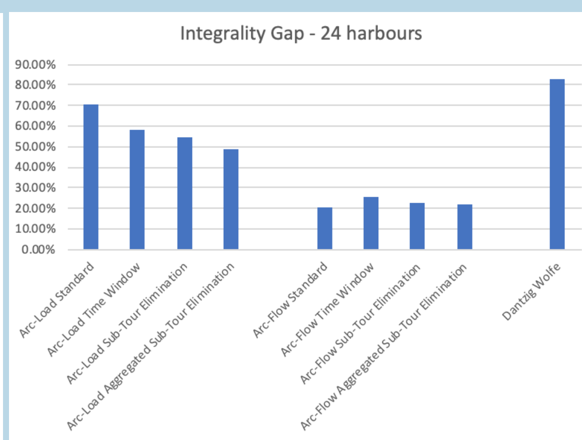
These are the common constraints that constitutes the master problem. A complete transformation for an IRP with similar properties can be found in [3]. As all ship and harbour constraints for the subproblems are independent of each other, each ship and harbour can be treated as a separate subproblem.

Optimization Results

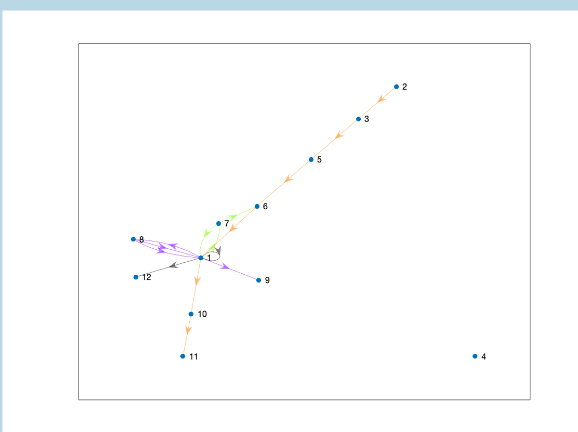
The arc-load and arc-flow formulations with different formulation improvements, as well as a standard Dantzig-Wolfe decomposition, were tested. All formulation combinations were tested on three different test cases, with respectively 12, 19 and 24 harbours. Computational time is plotted for the 12 harbour test case, since all model formulations solved the problem to optimality. The 19 harbour and 24 harbour test cases were not solved to optimality, and integrality gaps are therefore plotted. Scripts were manually stopped after 30 seconds.

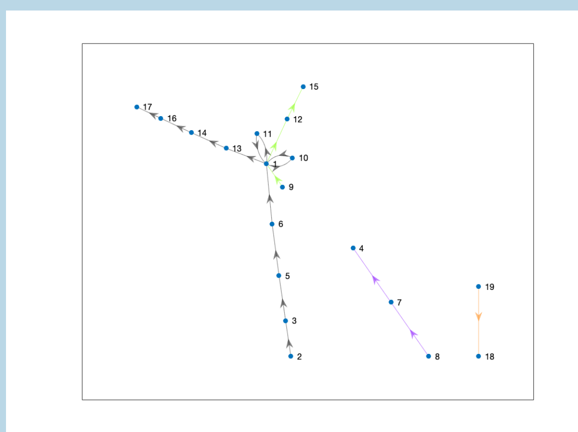


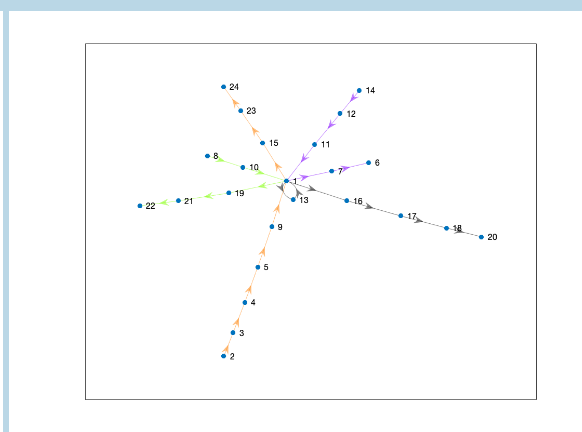




The routing paths for the best found solution for the respective test cases are plotted below. For the 12 harbours test case, all algorithms found the optimal solution, and the path is thus equal for all models. The standard arc-flow formulation provided the best solution for the 19 harbours test case, while the arc-flow model with aggregated sub-tour elimination constraints found the best solution for the 24 harbours test case.







Conclusion

The arc-flow formulations performs consistently better than the arc-load formulations, providing better solutions and bounds for all model improvement formulations tested. It also outperformes the Dantzig-Wolfe decomposition algorithm. Adding split or aggregated sub-tour elimination constraints improves computational time significantly for the arc-load formulation, but have limited impact on the arc-flow formulation. Problem instances with 24 nodes was solved with an integrality gap of about 20% within 30 minutes. Lower integrality gaps were achieved by limiting ship to harbour compatibility. However, the solution found were inferior as arcs of lower cost were neglected. Consumption scenarios were also tested. As expected, increased consumption lead to in-creased costs, as external deliveries were needed in order to serve all fish farms.