SMACS MODEL, a stochastic multihorizon approach for charging sites management, operations, design, and expansion under limited capacity conditions

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\section*{ARTICLE INFO}

\textbf{Keywords:}
- Electric vehicles
- Stochastic optimisation
- Multihorizon
- Charging sites
- Battery degradation
- Design
- Expansion

\section*{ABSTRACT}

The increasing demand of electric vehicles creates challenges for the electric grid both on the transmission level and distribution level. Charging sites in particular will have to face strong challenges especially in those countries where a massive penetration of electric vehicles happened in the last years and even more is expected in the forthcoming future. Such an increased forecast demand will lead to a capacity lack within the existing charging sites, therefore new investments in design and expansion have to be planned. We propose the so called SMACS MODEL that stands for Stochastic Multihorizon Approach for Charging Sites Management, Operations, Design and Expansion under Limited capacity conditions. The model is built to analyse critical decisions in terms of transformer expansion, grid reinforcements, renewable installation and storage integration, over a time horizon of 10 years, with a particular focus on the long term uncertainty in the price variations of the available resources. Long term investment decisions and short term operational decisions are addressed simultaneously in a holistic approach that includes also battery degradation issues and is able to tackle the optimal trade off between battery replacements, grid reinforcements and renewable installations throughout the chosen time horizon. Compared to traditional decision approaches the model is able to take more precise decisions due to its higher insight on the long term costs projections, the inclusion of battery degradation issues and the inclusion of grid rules and regulations limits that affect the final decisions.

\section*{1. Introduction}

The increasing demand of electric vehicles creates challenges for the electric grid both on the transmission level and distribution level. Charging sites for several vehicles will face strong challenges, especially in those countries where a massive penetration of electric vehicles happened in the last years and even more is expected in the forthcoming future. In this case, it is not the energy consumption that is challenging, but the fact that the maximum capacity in the distribution network and station itself is limited. This motivates to study the problem of smart design and operation of charging sites. Here typical design elements can be the charging capacity, the local power generation capacity, the capacity of batteries installed to smooth out the residual load, the transformer capacity as well as potentially the need to increase the dimension on the distribution grid. The main objective of this paper is to show under which conditions storage integration and renewable integration becomes convenient compared to grid reinforcement investments.

In this paper we propose to address the problem of integrated planning of operations, design and capacity expansion by using a multistage stochastic programming approach. For this purpose we developed an integrated model for design and operation, a Stochastic Multihorizon Approach for Charging Sites Management, Operations, Design and Expansion under Limited capacity conditions (SMACS MODEL). This model is used to analyse critical decisions in terms of transformer expansion, grid reinforcements, renewable installation and storage integration, over a time horizon of 10 years. There is a novel focus on the long-term uncertainty in the price variations of the available resources and cost development: long-term investment decisions and short-term operational decisions are addressed simultaneously in a holistic approach. The expansion decisions are taken at the distribution level, which is the focus of the study.

The main contribution is the methodology to handle the trade-off between battery replacements, grid reinforcements and renewable...
installations throughout the chosen time horizon based on stochastic modelling. The multistage stochastic model includes both short-term and long-term uncertainty. The model framework is tested on realistic data, and results have been analysed to discuss the value of a multi-horizon perspective versus traditional approaches that are usually neglecting learning effects and uncertainty in investment costs.

Moreover, a second contribution is a detailed methodology to include battery technical properties and degradation in order to optimise battery stock replacements throughout the years according to residual lifetime throughput and capacity degradation.

A third contribution is the extensive computational experiments and sensitivity analyses with real world dataset that has been performed. We provide real world dataset related to the cost projection of different energy technologies, as well as electricity price and demand forecast development. Analysis are performed to investigate the trade-off between performance and costs over time. A particular focus has been put on investigating which combinations of storage replacement costs and grid reinforcement costs make the system economical under different demand assumptions. Further tests have been done to investigate the trade-off between cheaper batteries with lower performance and more expensive batteries with better performance.

The study shows that the ability to take decisions by considering the uncertainty in the future development of investment costs of energy units is crucial. Technology improvement drastically affect the timing, cost and performance of the charging site.

The structure of the paper is as follows: Section 2 will present a literature review in the field of charging sites operations, location-allocation and design; Section 3 will introduce the main technical aspects linked to battery properties, transformer properties and grid reinforcement tasks that are needed to understand the mathematical model presented in the following Section 4; the real world data set used for computational experiments will be discussed in Section 5 while testing and results will be presented in Section 6; finally, Section 7 will draw the conclusions.

2. Literature review

A few optimization approaches related to charging sites exist in the literature. We divide them into the following categories: charging site operation, charging site location-allocation and charging site design.

Charging site operation regards the problem of managing the local resources like batteries and available charging capacity (power) in shorter time horizons (often decided by the charging cycle of the batteries). A comprehensive review on scheduling methods for charging sites is proposed in [1] where conventional optimisation methods, game theory and heuristics algorithms are surveyed. A review and classification of method for smart charging of electric vehicles for fleet operators is presented also in [2]. A framework for optimizing charging and discharging of the electric drive vehicles, given the driving patterns of the fleet and the variations in market prices of electricity is presented in [3]. Moreover, a novel optimal charging scheduling strategy for different types of electric vehicles is proposed in [4] where analyses are based not only on transport system information, such as road length, vehicle velocity and waiting time, but also grid system information, such as load deviation and node voltage. An optimization framework for the operating model of battery swapping stations is proposed in [5] while a model which yields the rate of degradation of the battery as a function of both temperature and depth-of-discharge is proposed in [6]. The latter is then used in an electric vehicle energy management optimization problem, where the degradation suffered by the battery due to a controlled charge is minimized. In [7] authors present a model dealing with the simultaneous scheduling of electric vehicles and responsive loads to reduce operation costs and emissions in presence of wind and photovoltaic sources in a microgrid. Renewable sources are included also in [8] where a two-stage framework for the economic operation of a microgrid-like electric vehicle parking deck with on-site photovoltaic generation is presented. The uncertainty due to renewable resources is addressed through a two-stage stochastic optimisation approach in [9]. Moreover, the integration of renewable energy sources within charging sites is addressed in [10] where authors investigate the possibilities to integrate additional loads of uncertain renewable energy sources, by smart charging strategies of three different electric vehicle fleets (namely, commercial customers, commuters, and opportunity parkers). A stochastic approach for controlling electric vehicles charging is proposed in [11] where the focus is put on frequency regulation issues. The same authors propose a two stage stochastic optimisation approach in [12] for operational control of electric vehicles charging. The model captures the use of distributed energy resources and uncertainties around electric vehicles arrival times and charging demands upon arrival, non-electric vehicles loads on the distribution system, energy prices, and availability of energy from the distributed energy resources.

Charging site location-allocation problems are oriented towards the problem of locating in a geographical area a set of charging stations, and simultaneously deciding their capacity based on allocation of customers demand to each site. An optimisation approach for charging stations location is proposed in [13]. This paper proposes a new location model based on the set cover model taking the existing traditional gas station network as the candidate sites to determine the distribution of the charging and battery swap stations. An Integer Linear Programming approach for sitting and sizing of electric taxi charging stations is proposed in [14]. A queuing model is adopted to estimate the probability of taxis being charged at their dwell places. Input guiding the location and capacity decisions comes from large-scale GPS trajectory data collected from the taxi fleet. The optimal planning of electric vehicles charging/swap stations with MILP approach is proposed in [15]. Here models for location and capacity decisions are developed for rapid-charging stations and battery swap stations considering the distribution network and potential reinforcement. In [16] the charging station location problem includes where to locate the charging stations and how many chargers should be established in each charging station. In [17] authors present an analytical approach to estimate the optimal density of charging stations for certain urban areas, which are subsequently aggregated to city level planning. In [18] a multi-period model for strategic charging station location planning is presented, recognizing that CS will be introduced gradually over time. A study based on the real traffic flow data of the Korean Expressway network is presented. A stochastic model is proposed in [19] where location of fast-charging stations with uncertain electric vehicle flows is addressed.

These studies focus on location of multiple charging stations in big regions rather than design and expansion of particular charging sites. The charging site design problem addresses the more detailed design and capacity planning of a single site and investigates the optimal capacity based on deterministic or stochastic demand for charging services. If the model includes long-term dynamics so that investment decisions can be optimally timed we call it a design and expansion problem. In both cases the models include dimensioning of technical equipment like maximum charging power, local generation, transformers, batteries and local grid may be included. The charging site design problem is addressed in [20] through a simulation approach that makes use of the HOMER simulation software, and in [21] where authors proposed an algorithm for the optimal sizing of different units, but the uncertainty in prices and production both on the short term operational level and on the long term strategic level is not considered. In [22] authors present an approach that considers time and distance from electric vehicles to a charging station as well as construction costs of transformers, cables, chargers and operational grid (harmonic power loss and other) to determine the optimal placement and sizing of charging stations.

A key component within charging sites is represented by battery energy storage. Therefore the optimal design and operation of such devices is an important aspect to be considered, especially when it
comes to degradation issues involved in the charge/discharge cycles. A review on methodologies for the optimal placement, sizing and control of battery energy storage can be found in [23]. A mixed integer linear programming approach for optimal placement, sizing, and dispatch of battery energy storage in distribution networks is addressed in [24]. The authors compare the value of battery energy storage with grid reinforcement, but they focus on the technology state of the art here and now, without taking into account the effects of long term forecast future variations in the technology investment costs and in the technology performance. Battery degradation issues are also gaining more and more attention and included within mathematical optimisation models to analyse how they affect the value of such technologies. An example can be found in [25] where the degradation cost of batteries is included within microgrids optimal energy management.

The contribution of our work is on the charging site design and expansion problem, including the integration with the grid. We consider both local renewable energy sources production, batteries and transformers to balance the capacity of the charging site with the distribution system costs. In order to provide the needed detail for the capacity planning, representative operational time periods are added. A novel approach combining short-term uncertainty in the operational horizon (i.e. load, generation) and long-term uncertainty (i.e. investment cost development) is used. To make this computationally tractable, multihorizon stochastic programming is proposed. As far as the authors know, this is the first approach where the long-term design decisions are linked to this detail level on the operational level for charging site design including stochasticity in both the long run and short run.

While there is no literature on this for charging sites, the general literature on capacity expansion suggests that detailed modelling of short-term dynamics and stochasticity is a necessary approach. Below we give some examples on stochastic capacity expansion and multihorizon stochastic programming respectively. The stochastic capacity expansion problem is a well known and extensively studied. A three level MILP approach is proposed in [26] where optimal expansion of an electric network with demand uncertainty is proposed using an equilibrium model. A real application in the power system in Chile is presented. A stochastic methodology for capacity expansion planning for remote microgrids is presented in [27] where uncertainty is addressed with Monte Carlo approach and authors aim at investigating the economic benefits of including uncertainty in such kind of problems. Although operational uncertainty is considered, long term investment uncertainty is not taken into account, especially regarding different technologies investment prices. A stochastic multistage methodology is used also in [28] for transmission expansion planning and energy storage integration. Although the methodology is multistage with a longer time horizon, the uncertainty in the investment costs of energy units such as storage and renewables is not addressed, only operational uncertainty. A main issue with stochastic capacity expansion problems is the joint modelling of dynamics and stochasticity. Long-term dynamics is the process of repeated investment decisions in several time periods, hence the possibility to time investments. Short-term dynamics is the development of decisions in operational time periods where renewable energy sources generation, load and batteries are managed over a sequence of hours and/or days. When combining these dynamics with long-term uncertainty (costs development, demand trends) and short-term uncertainty (load, generation) in a stochastic programming model, the size of the scenario trees grows exponentially. Multihorizon stochastic programming is a modelling framework developed in particular to address this issue [29]. The main idea is that as long as the future long-term outcomes of stochastic variables are not linked to the history of the short-term uncertain variable outcomes, the long-term development can be separated from the short-term stochastic process using local end-of-horizons operationally. The main real world applications of multihorizon models found so far in literature are related to hydro plants management and load management in buildings. The first use was in modellling natural gas infrastructure development under uncertainty as shown in [30,31]. Another application is hydro power planning where [32,33] compare multihorizon models with alternative state-of-the-art modeling approaches. The method has been used in a TIMES energy system model to study short-term uncertainty in renewable energy sources generation and its effects on the energy system design [34,35]. Skar et al. uses a multihorizon approach in EMPIRE, a stochastic model for the European power system [36]. Moreover, the multihorizon methodology is used in [37] to analyse retrofitting opportunities for energy-efficient buildings on a long-term horizon by taking into account uncertainty in energy prices and technology costs stemming from deregulation.

To our knowledge this is the first attempt to handle both short-term and long-term dynamics and uncertainty in optimal charging site design and expansion.

3. Background on technical aspects

This section will give a broad overview of the main technical aspects linked to battery properties, transformer properties and grid reinforcement tasks. The objective is to outline the main features that needs to be taken into account when building mathematical optimisation models for the design and expansion of sites that are supposed to contain such technologies. This will facilitate the understanding of the technical constraints that are part of the mathematical model proposed in the following section.

Batteries are rated in terms of their nominal voltage and ampere-hour capacity (Ah). Assuming that the voltage is constant and equal to the nominal voltage, the battery capacity $B_{\text{cap}}$ is given in kWh and is calculated as the battery voltage multiplied by the Ah. The roundtrip efficiency $B_{\text{eff}}$ indicates the percentage of the energy going into the battery that can be extracted later. We assume that the efficiency in both directions is the same (see [38,39]). The minimum state of charge $B_{\text{ch}}$ defines a limit below which a battery must not be discharged to avoid permanent damage. The $B_{\text{rt}}$ defines the rate at which a battery is being discharged. It is defined as the ratio between the discharge current and the theoretical current under which the battery would deliver its nominal rated capacity in one hour. The state-of-charge of a battery is the percentage of its capacity available relative to the capacity when it is fully charged. For further readings about battery properties we refer to [40,41].

The life of a battery can be measured by the so called lifetime throughput $B_{\text{th}}$ that defines the total amount of energy in kWh that can be discharged before it is no longer able to deliver sufficient energy to satisfy the load requirements of the system. The lifetime curve provided by manufactures relate different depth of discharge with the number of residual cycles to failure. The deeper the discharge, the lower the number of related cycles to failure [42].

The state-of-health of a battery is the percentage of its capacity available when fully charged relative to its rated capacity. It takes into account the loss of capacity as the battery ages. Through the lifetime throughput calculation, manufactures guarantee that the capacity of the battery will not drop more than a certain percentage $B_{\text{fade}}$ as long as the total energy drawn is kept within the lifetime throughput. Academic battery literature has typically considered a battery degraded to the point of needing replacement when it is only able to provide 80% of its original capacity [43].

In a battery bank mixing batteries of different ages is not recommended as they interact with each other. The overall performance is only as good as the weakest link in the chain [44]. Therefore when one battery in a bank of two or more batteries needs to be replaced, they should all be replaced at the same time [45].

The transformer capacity $T_{\text{cap}}$ is given in kVA. The actual output power in kW is determined through the load power factor $F$ defined as the ratio of the real power flowing to the load to the apparent power in the circuit [46–48].
Installing a transformer that is too small may cause outages and voltage drops, while installing a transformer that is too large may bring unnecessary load losses. The core losses (also called no-load losses) in particular increase with the capacity of the transformer, hence proper sizing is crucial and a careful selection of the transformer capacity will ensure lowest core loss. As stated in [49-51] the no-load loss is considered as a critical factor as it has undesirable impacts on the aging of transformers.

It is anyway important to have a transformer bigger than the peak demand to avoid overload that affects negatively the lifetime and the quality of service [52,53].

Given the above technical details, the transformer capacity upgrade has to be limited to a certain value in order to avoid oversizing issues. Hence as a general guideline a transformer should not exceed the peak demand more than a certain percentage $T^\%$. This will put an upper bound on the size to avoid oversizing issues, but at the same time will leave some exceeding capacity to avoid overloading issues. For further reading about transformer proper sizing and selection it is possible to refer to [54].

Lines in the distribution grid are oversized to take into account both extreme peak loads and possible future additional needs. When the transformer capacity in the charging station increases to a certain level, the distribution grid supplying the charging station also needs to be strengthened. Therefore reinforcement costs for grids will typically occur when the needed capacity is greater than a certain percentage $G^\%$ of the existing one.

4. Mathematical model

The problem addressed in the mathematical model takes as a starting point a location for a charging site and either suggests an optimal expansion of its existing capacity, or performs the optimal design for a greenfield installation.

The need for expansion will be driven by higher charging demand forecasts. In order to handle the forecast loads that will exceed the current capacity of the site, it is necessary to select one or a combination of the following investments: a new transformer with a greater capacity; grid reinforcements with new upgraded cables; additional battery storage; additional renewable resources.

As this is a stochastic model, the future demand may be uncertain along with the cost developments. Investment decisions are hence made under uncertainty. In addition to the long-term uncertainty of investment costs, we also model short-term uncertainty that affects both the operations directly and the investments indirectly, through operational costs.

Another key characteristic is the need to balance the capacities in the demand (load), the local generation of renewable energy, the storage capacity (battery), the transformer capacity and the local grid. Both batteries and renewable energy generation reduce the need for grid, but they come at a cost. The effect of these investments depends on the variations in renewable energy generation and load as well as the correlations between these.

The problem is a combination of a design problem and an operational management problem that have to be addressed together. The design choices are affected by the short-term uncertainty in the renewable generation and demand. Clearly also the short-term management of the system is limited by the installed capacities. This is the purpose of using a stochastic multihorizon optimisation model where the main decisions are the long-term investments, but a number of operational representative periods are included to consider the expected effects on the operations. The main objective is to minimise the total expected investment and operational costs.

4.1. The multihorizon scenario tree

Fig. 1, depicts a multihorizon scenario tree [29] describing how the uncertainty (both in the short term and long term) is managed. Investment decisions in terms of units to be installed are made in strategic nodes (black bullets). A long-term scenario consists of all the strategic nodes on a path from the root node to a leaf. The depicted tree has a two-stage structure where long-term uncertainty is resolved once (at time $t = 1$) and the outcome of all long-term is known at that stage. Of course both a different time for resolving uncertainty, and a multistage structure is supported by the methodology. Each of the long term scenarios show a different development for the investment costs for the various units such as batteries, transformer, renewable plants (black arcs). Six long term scenarios are shown in Fig. 1 departing from the root node.

Fig. 2 shows the details of short term operational scenarios. From every strategic node (black bullet), two operational scenarios are shown (red arcs) to take into account the short-term uncertainty in the demand and renewable production. Operational decisions on energy flows are made in operational nodes (blue bullets) that are embedded in every operational scenario. The operational uncertainty is described in a two-stage tree, where uncertainty is partly resolved in the first branching and completely resolved in the operational leaf nodes. Other structures may of course be chosen for the operational tree, based on specific modelling needs.

Fig. 2 shows that for every scenario, the operational nodes are split into three different profiles that aim at representing typical periods of the year. Each period represents a typical week in a year, hence it has to be made of 168 operational nodes, that represents the hours in a week. The number of operational periods, should be decided by the need to capture time correlations and to manage the storage cycles. As we are including battery storage, we need to bear in mind that the state of charge of the storage unit will be denied as input at the beginning of every operational period. After some preliminary tests, it has been found that a weekly length is suitable as the model has the freedom to charge and discharge within seven days without using any strong assumption on the daily initial state of charge.

Every profile will be multiplied by a weight that defines how many times such typical profile occurs in a year. Not all the operational nodes are depicted in the tree in Fig. 2 which is meant as an example. Similarly, only two operational scenarios for each strategic node are represented.

The time horizon should be chosen in such a way to both obtain a problem that is computationally manageable and allow a proper observation of investment replacements throughout the strategic years. It is important to highlight that there is not limit on the number of years that can be included in the model for computational experiments, given that the computational time will increase. The time horizon has been chosen in order to have a computationally tractable problem and allow a proper observation of battery replacements. The average life for a battery pack that is used in combination with renewable resources, is around four to five years, or even less if the battery is subject to high and deep cycling. More information about the average life of a battery pack can be found in [55–59]. In addition, technological improvement together with demand increment might motivate earlier replacements to cover the additional demand and take advantage of the investment costs reduction. Therefore, after some preliminary tests and considerations, it has been decided that 10 years was a suitable time to keep the model tractable and observe how battery installation and replacements were going to behave according to degradation issues and long term variations in the technological investment costs and performance.

Given the parameters and variables listed in Table 1–3, the stochastic multihorizon mathematical model for a charging site design and management follows. For a general introduction about Multihorizon Stochastic Optimisation and Programming theory see [29].
4.2. Methodological notes on the solution approach

We apply a two-stage stochastic programming approach, where the uncertain parameters are represented by a discrete set of possible realisations, called scenarios [60]. This implies that the model decisions are split into two groups; first stage and second stage variables. The approach is designed such that the first stage variables are made before knowing the outcome of the uncertain parameters whereas the second-stage variables are made for a best adaption of each scenario. In our case, the first-stage decisions are investments in new capacity and the second-stage decisions are operational decisions [34].

The model is developed in AIMMS and Python using the pyomo algebraic language [61,62] and the CPLEX solver [63] under academic license.

One of the most common methods to solve a two-stage stochastic linear program is to build and solve the deterministic equivalent [64]. For stochastic linear programs the deterministic equivalent is represented by another potentially very large linear program. A stochastic
<table>
<thead>
<tr>
<th><strong>Renewable</strong></th>
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<tbody>
<tr>
<td>$R$</td>
<td>Unitary current investment cost of the renewable plant ($)</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Long term multiplier that defines variations in the renewable investment costs in every strategic node $n$</td>
</tr>
<tr>
<td>$R_n^w$</td>
<td>efficiency of the renewable plant of type $w$ (%)</td>
</tr>
<tr>
<td>$R_{p,s}$</td>
<td>forecast production of one unit of renewable source in time $t$, profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$R_{\text{lim}}$</td>
<td>upper limit in the possible renewable installation linked to available area (kWh)</td>
</tr>
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<thead>
<tr>
<th><strong>Transformer</strong></th>
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<tbody>
<tr>
<td>$T_i$</td>
<td>current investment cost of the transformer of type $i$ ($)</td>
</tr>
<tr>
<td>$T_n$</td>
<td>long term multiplier that defines variations in the transformer investment costs in every strategic node $n$</td>
</tr>
<tr>
<td>$T_i^{\text{eff}}$</td>
<td>capacity of the transformer of type $i$ (kVA)</td>
</tr>
<tr>
<td>$T_i^{\text{cap}}$</td>
<td>capacity of the existing transformer of type $i$ (kVA)</td>
</tr>
<tr>
<td>$T_i^{\text{peak}}$</td>
<td>percentage of the peak demand that limits the transformer capacity increment in every strategic node (%)</td>
</tr>
<tr>
<td>$T_{\text{lim}}$</td>
<td>upper bound on the maximum size allowed for a distribution transformer (kVA)</td>
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<tr>
<th><strong>Battery</strong></th>
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<tbody>
<tr>
<td>$B_j$</td>
<td>current investment cost of the battery of type $j$ ($)</td>
</tr>
<tr>
<td>$B_n$</td>
<td>long term multiplier that defines variations in the battery investment costs in every strategic node $n$</td>
</tr>
<tr>
<td>$B_j^{\text{eff}}$</td>
<td>efficiency of the battery of type $j$ (%)</td>
</tr>
<tr>
<td>$B_j^{\text{cap}}$</td>
<td>capacity of the battery of type $j$ (kWh)</td>
</tr>
<tr>
<td>$B_j^{\text{min}}$</td>
<td>minimum state of charge of the battery of type $j$ (%)</td>
</tr>
<tr>
<td>$B_j^{\text{ons}}$</td>
<td>desired state of charge of the battery at the beginning of every typical profile (%)</td>
</tr>
<tr>
<td>$B_j^{\text{offs}}$</td>
<td>desired state of charge of the battery at the end of every typical profile (%)</td>
</tr>
<tr>
<td>$B_j^{\text{th}}$</td>
<td>lifetime throughput of battery of type $j$ (kWh)</td>
</tr>
<tr>
<td>$B_j^{\text{fade}}$</td>
<td>capacity fade of battery of type $j$ when 100% of the total throughput is used (%)</td>
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<tr>
<th><strong>Demand</strong></th>
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<tbody>
<tr>
<td>$D_{t,n,p,s}$</td>
<td>forecast demand in time step $t$, profile $p$, operational scenario $s$ within strategic node $n$ (kWh)</td>
</tr>
<tr>
<td>$D_n$</td>
<td>long term multiplier that defines demand variations in every strategic node $n$</td>
</tr>
<tr>
<td>$D_{p,s}$</td>
<td>short term multipliers modelling the daily demand trend in time $t$, profile $p$, operational scenario $s$</td>
</tr>
<tr>
<td>$D_{\text{peak},n}$</td>
<td>peak demand in every strategic node $n$ (kWh)</td>
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<tr>
<th><strong>Electricity Price</strong></th>
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<tr>
<td>$P_{t,n,p,s}$</td>
<td>forecast electricity price in time step $t$, profile $p$, operational scenario $s$, within strategic node $n$ ($/kWh$)</td>
</tr>
<tr>
<td>$P_n$</td>
<td>long term multiplier that defines price variations in every strategic node $n$</td>
</tr>
<tr>
<td>$P_{p,s}$</td>
<td>short term multipliers modelling the daily price in time $t$, profile $p$, operational scenario $s$</td>
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<tr>
<th><strong>Grid reinforcement</strong></th>
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<tbody>
<tr>
<td>$G_{\text{cost}}$</td>
<td>unitary cost of digging for cable installation ($/km$)</td>
</tr>
<tr>
<td>$G_{\text{cap}}$</td>
<td>unitary cost of cables ($/km$)</td>
</tr>
<tr>
<td>$G_{\text{cap}}^{\text{amp}}$</td>
<td>cable capacity (amps)</td>
</tr>
<tr>
<td>$k$</td>
<td>distance of the charging site from the transformer substation (km)</td>
</tr>
<tr>
<td>$H$</td>
<td>factor for converting amps to kW in cable sizing</td>
</tr>
<tr>
<td>$G_{\text{lim}}$</td>
<td>percentage of additional transformer capacity at which grid reinforcement will occur (%)</td>
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<th><strong>Other</strong></th>
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<tr>
<td>$P_s$</td>
<td>probability of operational scenario $s$ (%)</td>
</tr>
<tr>
<td>$P_n$</td>
<td>probability of strategic node $n$ (%)</td>
</tr>
<tr>
<td>$Y_n$</td>
<td>year number associated to strategic node $n$</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>weight of profile $p$</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate ($$)</td>
</tr>
<tr>
<td>$\text{BigM}$</td>
<td>a very big number</td>
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A linear program is built from a collection of multi-period linear programs, each having the same structure but somewhat different data. In order to incorporate uncertainties in the second stage, it is necessary to assign probabilities to the different scenarios and then solve the corresponding deterministic equivalent. With a finite number of scenarios, two-stage stochastic linear programs can be modelled as large linear programming problems. This formulation is often called the deterministic equivalent linear program. Strictly speaking a deterministic equivalent is any mathematical program that can be used to compute the optimal first-stage decision, so these will exist for continuous linear programs, each having the same structure but somewhat different data. In practice, it might be possible to construct scenarios by eliciting experts’ opinions on the future. The number of constructed scenarios should be relatively modest so that the obtained deterministic equivalent can be solved with reasonable computational effort. In order to face the computational burden due to the necessity to handle both short term and long term uncertainty, the computational experiments have been carried out at the HPC lab; Solstorm.iot.ntnu.no (HPC standing for High Performance Computing at the NTNU lab for Computational Economics and Optimization).

### 4.3. Objective function

$$\text{MIN} \sum_n P_n^* \frac{1}{(1 + r)^n} \left( TC_n + TO_n \right)$$

(1)

$$TC_n = P_{n,Cost}^R + \sum_j B_{n, Cost}^j e_{n,j} + \sum_i T_{n, Cost}^i \theta_{n,i} + G_{n, Cost} \eta_n + G_{n, Cost}^b \eta_n$$

(2)

$$R_{n,Cost}^p = R_n^p + R \quad \forall n, p, s, t$$

(3)

$$B_{n,j} = B_{n,j}^p B_{n,j}^i \quad \forall n, p, s, t$$

(4)

$$T_{n,i} = T_{n,i}^t + T_{n,i}^t \quad \forall n, p, s, t$$

(5)

$$TO_n = \sum_j P_j^s \left( \sum_p \theta_p^s \left( \sum_{i,j} R_{i,n,p,j} \theta_{i,n,p,j} + \sum_{j} R_{i,n,p,j} \theta_{i,n,p,j} \right) + \sum_{i,j} R_{i,n,p,j} \theta_{i,n,p,j} \right)$$

(6)

$$P_{i,n,p,j} = P_{i,n,p,j}^s \quad \forall n, p, s, t$$

(7)

The objective function minimises the net present value of operational costs $TO_n$ and investment costs $TC_n$ that will occur in every year represented by strategic nodes $n$. As different long term scenarios are considered (see the different branches of the tree in Fig. 1), such actualised costs are multiplied by the related probability $P_n$.

As shown in Eq. (2) the total investment costs in every strategic node are related to renewable installation costs, battery installation costs, transformer upgrade costs and grid reinforcement costs. Eqs. (3), (4) and (5) define the investment costs of renewable plants, batteries and transformer in every strategic node $n$ according to long term multipliers that forecast their future investment cost trend.

Operational costs are shown in Eq. (6) as the summation of $(P_{i,n,p,j} \theta_{i,n,p,j})$ that is the cost of the energy that flows from the transformer $i$ to the load in every time step $t$ embedded in every strategic node $n$, plus $(\theta_{i,n,p,j}^s P_{i,n,p,j}^s)$ that is the cost of the energy that flows from the transformer $i$ to the battery $j$ in every time step $t$ embedded in

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<td>Nomenclature – sets and indexes.</td>
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| Sets and index | Description |
|---|
| $t \in T$ | set of operational time periods |
| $p \in P$ | set of operational profiles |
| $n \in N$ | set of strategic nodes |
| $s \in S$ | set of operational scenarios |
| $j \in J$ | set of battery types |
| $i \in I$ | set of transformer types |

---

<table>
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<th>Table 3</th>
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<tr>
<td>Nomenclature - variables.</td>
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<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
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<tbody>
<tr>
<td>$f_{i,n,p,s}$</td>
<td>energy flow from the renewable plant to the demand in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$f_{i,n,p,j}$</td>
<td>energy flow from the renewable plant to the battery $j$ in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$f_{i,j}$</td>
<td>energy flow from the transformer $i$ to the demand in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$f_{R,n,p,s}$</td>
<td>energy flow from the transformer $i$ to the battery $j$ in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$f_{n,p,s}$</td>
<td>energy flow from the battery $j$ to the demand in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$\theta_{n,j}$</td>
<td>integer variable defining the units of transformer of type $i$ installed on strategic node $n$</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>variable that keeps track of the actual installed transformer capacity in every strategic node $n$</td>
</tr>
<tr>
<td>$z$</td>
<td>cable size that can accommodate the transformer capacity (amp)</td>
</tr>
<tr>
<td>$\theta_{n,j}$</td>
<td>binary variable equal to 1 if the battery $j$ is installed on strategic node $n$</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>integer variable defining the number of batteries of type $j$ to be installed on strategic node $n$</td>
</tr>
<tr>
<td>$\theta_{n,j}$</td>
<td>variable that keeps track of the actual installed battery capacity in every strategic node $n$</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>state of charge of the battery $j$ in every time step $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>binary variable equal to 1 if the battery of type $j$ is charging in time $t$ of strategic node $n$ for profile $p$, operational scenario $s$ (kWh)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>continuous variable defining the additional renewable capacity to install in the strategic node $n$ (kW)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>variable that keeps track of the actual installed renewable capacity in every strategic node $n$ (kW)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>binary variable equal to 1 if grid reinforcement is needed on strategic node $n$</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>transformer reference capacity in very node $n$ used to decide if grid reinforcement is needed (kVA)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>residual lifetime throughput of battery of type $j$ at the beginning of the strategic period $n$ (kWh)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>residual lifetime throughput of battery of type $j$ at the end of the strategic period $n$ (kWh)</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>binary variable equal to 1 if a battery replacement of type $j$ occurs on strategic node $n$</td>
</tr>
<tr>
<td>$\eta_{n,j}$</td>
<td>battery residual throughput that is still available in the old battery once the replacement occurs (kWh)</td>
</tr>
</tbody>
</table>
every strategic node \( n \). As explained previously, we are sectioning the year into typical profiles \( p \), therefore the operational costs have to be multiplied by the weight \( w_p \) that represents how many times a typical profile of type \( p \) occurs in a year. As we are including also operational uncertainty, such energy flows have to be multiplied by \( P_t \) that represents the probability of every operational scenario \( s \).

The electricity price \( P_{n,p,s} \) is given by the forecast price \( P_{n,p,s}^F \), multiplied by \( P_{t} \) that is a long term multiplier aiming at representing the price variation in the forthcoming years \( n \).

### 4.4. Demand

\[ f^{RD}_{n,p,i} + \sum_j f^{RD}_{n,p,i,j} + \sum_i f^{TD}_{n,p,i} + T^{cap}_{n,p,i} = D_{n,p,i} \quad \forall \; n, \; p, \; s, \; t \]  
\[ (8) \]
\[ D_{n,p,i} = D_{n} + D^b_{p,i} \quad \forall \; n, \; p, \; s, \; t \]  
\[ (9) \]

Eq. (8) assures that the demand is met by the energy flows from the transformer, from the renewable and from the battery.

The demand \( D_{n,p,i} \) is given by the forecast demand \( D^b_{p,i} \) multiplied by \( D_{n} \) that is a long term multiplier aiming at representing the demand variation in the forthcoming years \( n \).

### 4.5. Renewable

\[ q^R_{n} = q^R_{n-1} + \Delta q^R_{n} \quad \forall \; n > 0 \]  
\[ (11) \]
\[ q^R_{0} = q^R_{REF} \]  
\[ (10) \]

Eq. (10) assures that the demand is met by the energy flows from the transformer, from the renewable and from the battery.

The variable \( q^R_{n} \) keeps track of the renewable capacity available in every node \( n \). In the first node, the available renewable capacity is equal to the number of units installed (Eq. (10)) while in the following nodes it is equal to the capacity available in the previous node plus the additional units installed in the current node (Eq. (11)). Space limitations might impose an upper bound in the maximum installable capacity (Eq. (12)). The renewable operations in terms of flows to the load and to the battery are limited by the units installed multiplied by the forecast unitary production \( R^F_{n,p,i} \) (Eq. (13)).

### 4.6. Transformer

\[ q^T_{n,i} = \theta_{n,i} + T^{cap}_{n,i} + T^{cap}_{n,i} \quad \forall \; i, \; n > 0 \]  
\[ (14) \]
\[ q^T_{n,i} = q^T_{n-1,i} + \theta_{n,i} \quad \forall \; i, \; n > 0 \]  
\[ (15) \]
\[ \sum_j f^{TB}_{n,p,i,j} + f^{TD}_{n,p,i} \leq q^T_{n,i} \quad \forall \; n, \; p, \; s, \; t, \; i \]  
\[ (16) \]
\[ \sum_j q^T_{n,j} \leq D^{peak}_{n} \quad \forall \; n \]  
\[ (17) \]
\[ \sum_i q^T_{n,i} \leq T^{lim} \quad \forall \; n \]  
\[ (18) \]

The variable \( q^T_{n,i} \) keeps track of the transformer additional capacity available in every node \( n \). In the first node the transformer upgrade is equal to the additional installed capacity, while in the following nodes it is equal to the capacity available in the previous nodes plus the additional installation in the current node (Eqs. (14) and (15) respectively). The flows out the transformer are limited by the available existing and new capacity in every node (Eq. (16)). As explained in details in Section 3, the total transformer capacity in every node has to be limited to a certain percentage \( T^{lim} \) of the peak demand. Constraint (17) guarantees a technically feasible choice to prevent the negative implications of oversizing and overloading described in Section 3. Moreover, distribution transformer sizes are supposed to lie within certain dimensions that impose upper bounds \( T^{lim} \) in the maximum allowed typical size (constraint (18)). Typical values for small, medium, large distribution transformers will be discussed in Section 5.6.

### 4.7. Grid reinforcement

\[ g_0 = 0 \rightarrow c_{0}^{REF} = \tilde{T}^{cap}F \quad \forall \; n, \; i = 1 \]  
\[ (19) \]
\[ g_0 = 0 \rightarrow c_{0}^{REF} = c_{0}^{REF} \quad \forall \; n, \; i > 1 \]  
\[ (20) \]
\[ g_0 = 1 \rightarrow c_{n}^{REF} = \sum_i q^T_{n,i} \quad \forall \; n \]  
\[ (21) \]
\[ \sum_i q^T_{n,i} > G^{cap}F \rightarrow g_0 = 1 \quad \forall \; n, \; i > 1 \]  
\[ (22) \]
\[ \sum_i q^T_{n,i} > G^{cap}F \rightarrow g_0 = 1 \quad \forall \; n, \; i > 1 \]  
\[ (23) \]
\[ \sum_i f^{TB}_{n,p,i,j} + f^{TD}_{n,p,i} \leq z^*G^{cap}H \quad \forall \; n, \; p, \; s, \; i \]  
\[ (24) \]

Grid reinforcement costs will occur when the transformer has to be upgraded above a certain limit. It is assumed that grid reinforcements occur when the transformer capacity has to be upgraded of an amount that is greater than a certain percentage \( G^{cap} \) of the current available capacity. The variable \( c_{n}^{REF} \) keeps track of the transformer reference capacity in every strategic node. As long as the transformer is not upgraded below a certain percentage of the current capacity, the reference variable \( c_{n}^{REF} \) will remain the same. When the transformer capacity will be upgraded above the limit, then the reference variable \( c_{n}^{REF} \) will be updated. This is achieved by the following set of constraints.

Constrains (19), (20) and (21) define what happens to the reference variable \( c_{n}^{REF} \) if grid reinforcements occur or not. If no grid reinforcements happen in the first strategic node, the reference variable \( c_{n}^{REF} \) is simply equal to the existing transformer capacity (constraint (19)). If no grid reinforcements happen in any other strategic node, then the reference variable \( c_{n}^{REF} \) remains unchanged and equal to the value it had in the previous strategic node (constraint (20)). If a grid reinforcement happens in any of the strategic nodes, then the reference variable \( c_{n}^{REF} \) is updated by adding the new capacity \( q^T_{n,i} \) to the existing capacity (constraint (21)).

Constraints (22) and (23) define in which circumstances grid reinforcements happen. In the first strategic node, if the upgraded transformer capacity is greater than a certain percentage \( G^{cap} \) of the initial existing capacity, then a grid reinforcement occurs and the binary variable \( g_0 \) has to be 1 (constraint (22)). In every other strategic node, if the upgraded transformer capacity is greater than a certain percentage \( G^{cap} \) of the reference capacity in the previous node \( c_{n-1}^{REF} \), then a grid reinforcement occurs (constraint (23)).

All the above statements can be written in a form suitable for optimisation models by using proper BigM formulations or by including indicator constraints. For further reading about handling indicator constraints in MIP problems see [65].

Finally constraint (24) ensures that the chosen cable capacity is suitable for the forecast energy flows that will be needed in every strategic node.

### 4.8. Battery choice and degradation

\[ q^B_{n} = q^B_{n} \quad \forall \; j, \; n = 1 \]  
\[ (25) \]
\[ h_{\text{INT}}^{\text{INIT}} = B_j^{\text{INT}} \cdot e_{\text{INT}} \quad \forall \ j, \ n/n = 1 \] (26)

\[ h_{\text{FIN}}^{\text{INIT}} = B_j^{\text{FIN}} \cdot e_{\text{INT}} - \sum_i P_i = \left( \sum_p \omega_p \sum_i f_{i,n,p,s,j}^{\text{BD}} B_j^{\text{INT}} \right) \quad \forall \ j, \ n/n = 1 \] (27)

\[ h_{\text{INT}}^{\text{REP}} = h_{\text{FIN}}^{\text{REP}} + B_j^{\text{INT}} \cdot e_{\text{INT}} \quad \forall \ j, \ n/n > 1 \] (28)

\[ \sum_j h_{\text{REP}}^{\text{FIN}} = 1 \rightarrow \sum_j h_{\text{FIN}}^{\text{REP}} = 0 \quad \forall \ n/n > 1 \] (29)

\[ h_{\text{INT}}^{\text{REP}} = h_{\text{INT}}^{\text{FIN}} + B_j^{\text{INT}} \cdot e_{\text{INT}} \quad \forall \ j, \ n/n > 1 \] (30)

\[ h_{\text{FIN}}^{\text{REP}} = h_{\text{FIN}}^{\text{FIN}} - \sum_i P_i = \left( \sum_p \omega_p \sum_i f_{i,n,p,s,j}^{\text{BD}} B_j^{\text{INT}} \right) + B_j^{\text{INT}} \cdot e_{\text{INT}} - h_{\text{REP}}^{\text{RE}} \quad \forall \ j, \ n \] (31)

\[ \sum_j e_{\text{INT}} \leq 1 \quad \forall \ n \] (32)

\[ e_{\text{INT}} \neq 0 \rightarrow e_{\text{INT}} = 1 \quad \forall \ n \] (33)

For the battery choice we refer to the real world situation in which different standard types of batteries are available in the market with certain properties (price, capacity, efficiency, rating, etc.). Hence the model chooses one type of battery \( j \) with a certain capacity \( B_j^{\text{INT}} \) and then the optimal number of units of that type to be installed \( e_{\text{INT}} \) in order to create the battery bank.

The variable \( q_{B,j}^p \) keeps track of the total storage capacity available in every node \( n \). The storage capacity available in the first year is equal to the capacity of the battery of type \( j \) multiplied by the number of units to be installed (constraint (25)).

Two main degradation issues are taken into account in the model: lifetime throughput and capacity fade.

The model keeps track of the residual lifetime throughput in every strategic node through the variables \( h_{\text{INT}}^{\text{REP}} \) and \( h_{\text{FIN}}^{\text{REP}} \) that define the lifetime throughput at the beginning of every year and at the end of every year respectively. On the first node the initial lifetime throughput is given by the throughput of the chosen battery multiplied by the number of units installed (constraint (26)); while the final throughput is given by the initial value minus the total energy that has been drawn in the different typical periods and for the different operational scenarios (constraint (27)).

In all the other nodes the lifetime throughput at the beginning of the year, is equal to the residual throughput at the end of the previous year, plus the additional throughput derived from new additional battery installation (constraint (28)).

However, as discussed in Section 3, when adding new batteries, the whole battery bank has to be replaced as it is not recommended to keep batteries of different age and type working together. Hence when a battery replacement occurs in a strategic node (\( h_{\text{REP}}^{\text{FIN}} = 1 \)), the lifetime throughput has to be zero at the end of the previous year (constraint (29)) and the battery installation is simply equal to the new installation that occurs in the node (constraint (30)).

In a multihorizon framework, a decreasing trend of battery costs together with an increasing trend of demand, may bring to decisions in which it is worthy to replace a small battery bank with a larger one in a certain year, instead of carrying on using the residual throughput of the existing bank from the previous year (that may be insufficient to fulfill the increased demand requirements of the forthcoming years). In order to give the model freedom in deciding if it is worthy to replace a battery bank that still has a residual throughput, a variable \( h_{\text{REP}}^{\text{REP}} \) is inserted in constraint (31). If the term \( e_{\text{INT}} \) is greater than zero, a replacement occurs. Therefore the residual throughput of the previous year has to be zero (constraints (29) and (30)) but the variable \( h_{\text{REP}}^{\text{REP}} \) can get a value greater than zero: this is the residual throughput that may still be there when doing such battery replacement. This way, whatever the residual throughput is, a battery replacement can occur if certain combinations of forecast demand increment and cost reductions arise.

Constraint (32) aims at including the capacity fade that occurs in the battery as a further degradation term in addition to the throughput discussed above. If no replacement occurs, the battery capacity will not remain the same, but it will decrease as a function of the used throughput. As discussed in Section 3, as long as the battery usage is kept within the throughput, the manufacturers guarantee that the capacity will not drop below a certain limit \( B_j^{\text{MIN}} \). Therefore it is possible to approximately quantify the capacity fade \( \% \text{fade} \) at the end of every year through a simple proportion \( B_j^{\text{MIN}} = B_j^{\text{MAX}} - h_{\text{INT}}^{\text{REP}} \cdot h_{\text{INT}}^{\text{REP}} \). This gives us informations about the percentage of capacity fade associated with a certain usage of throughput. This is inserted in constraint (32) to properly keep track of the available storage capacity and penalise it according to capacity fade issues.

Constraint (33) limits the choice of battery units to a one single type, so that batteries of the same type will be installed in the battery bank.

Finally constraint (34) links the two decisional variables \( e_{\text{INT}} \) and \( \theta_{j,n} \).

Constraints (29), (30) and (32) can be expressed using BigM formulations or indicator constraints. For further reading about handling indicator constraints in MIP problems see [65].

4.9. Battery operations

\[ h_{\text{SOC},n,p,s,i,j} = B_j^{\text{INT}} \cdot q_{\text{SOC},n,p,s,i,j} \quad \forall \ n, p, s, t, j \neq 0 \] (35)

\[ h_{\text{SOC},n,p,s,i,j} = B_j^{\text{FIN}} \cdot q_{\text{SOC},n,p,s,i,j} \quad \forall \ n, p, s, t, j = T \] (36)

\[ h_{\text{SOC},n,p,s,i,j} = h_{\text{SOC},n,p,s,i,j} - f_{i,n,p,s,j}^{\text{BD}} \cdot \frac{1}{B_j^{\text{INT}}} + \sum_i f_{i,n,p,s,j}^{\text{BD}} \cdot \frac{1}{B_j^{\text{INT}}} + f_{i,n,p,s,j}^{\text{RB}} \cdot \frac{1}{R_j^{\text{MIN}}} \] (37)

\[ f_{i,n,p,s,i,j}^{\text{BD}} \leq \frac{1}{B_j^{\text{INT}}} \frac{1}{R_j^{\text{MIN}}} \] (38)

\[ f_{i,n,p,s,i,j}^{\text{BD}} \leq \frac{1}{B_j^{\text{INT}}} \frac{1}{R_j^{\text{MIN}}} \] (39)

\[ h_{\text{SOC},n,p,s,i,j} \leq B_j^{\text{INT}} \cdot q_{\text{SOC},n,p,s,i,j} \quad \forall \ n, p, s, t, j \] (40)

\[ h_{\text{SOC},n,p,s,i,j} \leq B_j^{\text{FIN}} \cdot q_{\text{SOC},n,p,s,i,j} \quad \forall \ n, p, s, t, j \] (41)

The variable \( q_{\text{SOC},n,p,s,i,j} \) keeps track of the storage capacity available in every node \( n \).

Constraints (35) and (36) define the battery state of charge at the beginning and at the end of every typical profile respectively. Equation (37) defines the battery state of charge in every time step. Flows in and out the battery are mutually exclusive as imposed by constraints (38) and (39).

The battery maximum capacity and minimum state of charge to avoid permanent damage are defined in constraints (40) and (41).

The battery rating is defined in constraints (42) and (43).

5. Real world data collection

5.1. Notes on scenario generation

This section will outline the main approaches that have been used
for scenario generation purposes, both on a short term and long term basis.

Short term scenarios are generated to capture both daily and weekly uncertainty. Random sampling has been used. It is important to highlight that this is a strategic model; hence the main motivation for our scenario generation approach is to represent realistic operational situations in the model, not to forecast the future. Therefore we base scenarios on historical data and use no prediction model. The method presented is in general valid for long-term investment models with intermittent energy sources, as discussed in [35]. We use historical data in order to find realistic outcomes for the demand, renewable availability and electricity prices. It is not likely that the same realisations occur in the future, but they can be used as representative days. A sample of the historical data set is used to generate the stochastic scenarios. For each model period, a scenario is generated by a random sample of a historical year. When creating these operational scenarios, it is desirable to have a good statistical property match between the sample used for scenarios in the model and the historical data. Authors in [35] illustrate how to make scenarios by an iterative random sampling method. As illustrated in [66], the number of scenarios is primarily chosen to ensure manageable computational time, even though a higher number of scenarios can increase the quality of the model results. With too few scenarios, the description of uncertainty may not be good enough and the model may not be stable. With too many scenarios, the resulting model may become computationally intractable. Considering computational intractability, it is desirable to use as few scenarios as possible as long as we satisfy the stability requirements. As described in [66] in order to check if the number of scenarios is high enough to give a stable solution, it is possible to test in-sample and out-of-sample stability.

For long term scenario generation, there is more subjectivity than the short run. For short term scenarios generation, researchers would generally use historical data or statistical techniques. While for long term scenario generations, qualitative analyses are generally necessary, by combining contributions from expert opinions, information contained in reports provided by established energy agencies, as well as information about societal development and policy actions. For the proposed work, the material for the long term scenarios generation has been represented by the expert opinions received from the eSmart system company, which also provided access to the smart meter data within the charging site. In addition, the information provided within open available reports on energy, policy, and societal development have been used. Therefore the long term scenarios represent different long term realisations of policy actions, energy prices, technological development, and demand trends. The considered assumptions aim at capturing a range of possible realisations of the future energy prices, technological developments and those societal change that will affect the demand trends of electric vehicles charging. Even though the long term scenario generation is done more at a qualitative level, still it has to be highlighted the importance of capturing the covariations between parameters, in order to generate scenarios with a proper combination of long term projections that can show proper long term variations. Delving deeper into long term scenario generation methodologies is beyond the scope of the current paper that rather aims at proposing a methodology to make use of short term and long term input data for decision making under uncertainty. However, to gain more insight on long term scenario generation at a qualitative level, as well as future prices development and future effects of policy actions, it is possible to refer to the works proposed in [67], [68] and [34].

The following sections will give a comprehensive overview of the sources where real world data and reports have been collected for scenario generation purposes. It is important to highlight that different samples have been used in each strategic node to allow a better representation of uncertainty. The short term trends visible in the following Figs. 5, 6 and 9, are showing only one example of short term scenarios for illustrative purposes. Within the model and analyses, different samples have been used in each strategic node, by using the available real world dataset gathered from the smart meters installed at the charging site location, as well as the historical data for renewable generation and price variations. In addition, in each strategic node the short term scenario is multiplied by multiplicative factors in order to incorporate the long term variation that can happen in each of the future years.

5.2. Battery data

Current battery prices, throughput values and lifetime properties can be found in [69] where different data for various batteries from different manufacturers are proposed. According to this source, the battery cost per kWh is currently set in a range around 800 - 1000 $/kWh for a kWh throughput per kWh capacity in a range around 2000 - 3000 kWh. This is in line also with what is discussed in [70] and [71].

According to [69], higher throughput properties in the range of 7000 - 10000 kWh throughput per kWh capacity have a cost that is set around 2000 - 2500 $/kWh.

Future battery cost projections are discussed in [72] where different forecast trend proposed by BNEF, NAVIGANT and EIA are shown as well as an averaged curve. Long term multipliers that trace such forecasts are shown in Fig. 3 and used in the mathematical model to define the battery prices in different strategic nodes. Scenario01 is tracing BNEF optimistic prediction, Scenario03 is tracing EIA pessimistic prediction while Scenario02 is tracing the averaged curve. NAVIGANT prediction is not shown as it was overlapping with BNEF optimistic prediction.

![Fig. 3. Long term multipliers to calculate battery projected costs in the forthcoming years.](image-url)
For further readings about battery cost projections see [73] and [74].

5.3. Renewable data

Current and projected future costs for photovoltaic plants are discussed in [75], [76] and [77]. The former in particular analyse different scenarios from which suitable long term multipliers can be derived as shown in Fig. 4.

According to the mentioned sources, the current average cost of a photovoltaic plant (solar panel and installation) is set around 700 $/kW.

The unitary production of a photovoltaic plant can be found in the PVWatts calculator available online through the NREL website [78]. Fig. 5 shows the photovoltaic production for typical weeks in Norway.

5.4. Electricity price data

Both current and historical electricity price values are available from the Nord Pool website [79]. Fig. 6 shows the price trend for typical weeks in Norway.

Long term projection of electricity price in Norway has been discussed in [80], [81,82]. The former in particular shows three different scenarios for price forecasts. According to such study, long term price multipliers are shown in Fig. 7.

For an overview of electricity price projection in Europe it is possible to refer to [83].

5.5. Demand data

This paper addresses the effect of the increasing demand of electric vehicles charging, on the expansion decisions taken within existing charging sites. The considered load is therefore described by the aggregated power demand that is detected at the charging power station through smart meters. Short term demand data were collected through smart meters installed at a real charging site station, called Ladetorget, located in Moss (Norway). The data were collected for a whole year, with a resolution of one minute. Such data have been used to develop the short term demand with hourly resolution to be used in the model for the computational analyses. Random sampling from the existing dataset have been used to pick the short term scenarios. Fig. 9 shows the demand trend for typical weeks in different seasons.

The same data have been used also to develop the long term scenarios, by introducing multiplicative factors to describe the demand increment throughout the forthcoming years. As outlined in previous sections, qualitative analyses has to be performed for long term scenario generation. For the proposed work, the material for generating long term demand scenarios, has been represented by the expert opinions received from eSmartSystems, an energy company located in Halden (Norway) which also provided access to the smart meter data within the charging site. In addition, the information provided within open available reports on energy and societal development have been used to generate long term scenarios at qualitative level. In particular, references about electric vehicles demand development in Norway can be found in [84]. While for an overview related to electric vehicles demand development in Europe see [85,86].

According to the mentioned qualitative analyses, for long term demand development, the basic assumption is that the power needed will approximately double every second year, mainly due to charging at higher power levels, but also due to further increasing penetration of electric vehicles in the Norwegian transportation market. Fig. 8 shows the long term demand multipliers that describe two scenarios of future demand development. In particular, Scenario01 is optimistic and assumes that demand will double every second year as mentioned above. While Scenario02 is a pessimistic one, that mitigates the effect of Scenario01, assuming that the demand will keep on increasing for a limited period and then settle. This is made to take into account possible future policy changes. Indeed, in Norway electric vehicles have seen a very high diffusion mainly due to strong government incentives (i.e. electric vehicles are exempt road tax, public parking fees, toll payment as well as being able to use bus lanes). But for instance, these incentives were supposed to in effect until 2018 as further discussed in [87]. Hence how a change of incentives and policies will affect the market is unknown, but needs to be taken into account in future scenarios generation.

5.6. Transformer data

Different typical transformer sizes and related investment costs updated in 2016 can be found in [88] and are summarised in Table 4. According to the proposed numbers, it is reasonable to assume an approximated average unitary cost equal to 15 $/kVA that can be suitable for linear programming models.

The lifetime of a transformer can be realistically assumed around 20 years more or less [53].

We do not assume long term uncertainty in the transformer price.

![Fig. 4. Long term multipliers to calculate photovoltaic projected costs in the forthcoming years.](image1)

![Fig. 5. Example of renewable production for one kW photovoltaic panel, throughout typical weeks of summer, winter and the rest of the year. Different samples have been used in every strategic node to allow a better representation of uncertainty.](image2)
because this is a very established technology that have not changed very much for the last years.

5.7. Grid reinforcement data

Grid reinforcement costs can be found in the Sinterf Planbok [89] and are higher for installations inside the city compared to installations outside the city. In particular, the digging part for cable installation which is around 75000 $/km inside the city and 33000 $/km outside the city. As for the cost of cables, this varies with both the capacity and the length of the cable. A unitary cost can be defined according to data provided in [90] and summarised in Table 5.

As the lifetime of cables is very long, it is reasonable to assume that within a time horizon of 10 years, if such an investment is needed, it will be done once and cables will be oversized according to the load forecast. It is straightforward that the trade off between proper battery installation and proper cable oversizing according to the related expenses and load forecast is worthy to be investigated through the proposed optimisation model.

Grid reinforcement costs can include a wider variety of other costs related to electrical components, substation upgrade procedures, additional connections etc. However, the cable cost per km provided above, can be considered as a representative cost aimed at defining a reasonable starting point for further sensitivity analyses. In particular, sensitivity analyses assuming different grid reinforcement costs according to different distances from transformer and load can be performed to evaluate the different response and choice of the proposed optimisation model.

5.8. Scenarios selection

The six selected scenarios for testing are shown in Fig. 10 and aim at capturing different combinations of battery and renewable costs long term variations, as well as demand and electricity price long term development discussed in previous sections.

6. Computational experiments

The main research question investigated in the computational experiments, is related to an important trade-off that has to be addressed in the proposed decision problem: given that the investment costs of
battery storage and renewable resources will drop in the forthcoming
years, how should grid reinforcement be timed in terms of transformers
and cables? Cables represent one of the highest cost components when
upgrading such systems, and costs increase with the cable size and
length. Therefore, given a forecast for demand increment, we are in-
terested in analysing whether battery and renewable investment spread
out over the years, can mitigate - or even make us able to avoid - the
costs of grid reinforcements needed today. Moreover, even if batteries
and renewable might still be too expensive today, their costs may drop
through the years making them more convenient in the future. This may
allow savings today in terms of grid reinforcements because smaller
installations can be done recognizing that future demand increment
will then be covered by additional storage and renewable technologies
whose costs are supposed to become cheaper.

For testing purposes we assume an existing site that is undersized
compared to the forecast increment in demand, and where an upgrade
decision has to be taken already in the first year. That might include an
upgrade of the transformer, the installation of new cables, the in-
stallation of batteries or renewables. The decision can be any combi-
nation of these.

6.1. Effect of charging site location and distance from transformer
substation

The tests includes sensitivity analyses to understand how the cost of
cables linked to the distance between charging site and transformer
substation can affect the investment decisions.

As seen in the data section, the cable costs represent a high
component of the site upgrade costs. This is influenced not only by the size of cables (that is linked to the upstream transformer size), but also by the length (that is linked to the distance between the transformer substation to the charging site itself). Hence, closer substations will be cheaper than those which are far away. The following tables show how does this affect investment decisions in different energy units. The column Bat shows the investments in battery capacity, column Ren shows the investments in renewable photovoltaic plants, while the column Reinf shows the investment decisions in terms of grid reinforcement (this indicates the maximum power in watts which the new upgraded cables are able to carry. Transformer is upgraded accordingly).

When the distance from the charging site to the transformer increases, the investments in grid reinforcement on the first year become lower, resulting in higher and earlier investments in batteries along the following years.

Note that, even with very high grid reinforcement costs (Table 9), with the given scenarios the batteries are still too expensive to be installed in the first year. Large battery installations happen anyway around the 7th and 8th year when prices are likely to be more or less 80% of the current price (Table 6).

In Tables 8 and 9 results indicate that very small battery installations are done the year before a larger installation. In such cases the battery is small and quickly degraded in terms of throughput in order to allow a larger installation the year after, at a better price.

In scenario 6 of Tables 7–9, a large battery installation is proposed in the 9th year. This is due to the particular scenario structure that is visible in Fig. 10, where the stronger battery price decrement is combined with a strong demand increment. Hence to take advantage of the price reduction in batteries in combination with a demand increment that can allow a high investment, the model is postponing the large battery installation to the 9th year, but still installing a battery on the 7th year. This does not happen in scenario 3 because even if the battery learning curve is the same, the demand increment is smoother and does not justify larger investments in batteries.

In these tests it is clear that the demand curve is affecting the battery installation in terms of both timing and sizing. Battery installation happens later and in smaller capacity for scenarios 1, 2 and 3 where the demand curve is growing less, compared to scenarios 4, 5 and 6 where the demand curve is increasing rapidly and strongly throughout the whole time horizon. The grid reinforcement made on the root node (year 1) will have to cover all the future scenarios. In order to properly cover the highest demand curve of scenarios 4, 5 and 6, cables will result in being slightly oversized for the lower demand curve of scenarios 1, 2 and 3.

Finally, the renewable photovoltaic resource is installed very late, probably due to the fact that in this particular application with data from Norway, a good portion of the year is characterized by a very low solar production due to the long winter season.

### 6.2. Influence of demand forecast on the worthiness of batteries and renewable

In this section we propose sensitivity analyses to investigate how different demand curves affect the model decisions in terms of battery and renewable installation. Indeed, the tests discussed in the previous section, showed the long term investment decisions when the scenarios available in Fig. 10 are utilised as input data to describe the uncertainty in the long term trend of demand, energy prices, battery investment costs and renewable investment costs. The authors recognised during the testing that the high load increase visible in scenarios 4, 5 and 6 of Fig. 10, was strongly impacting the decisions in terms of battery storage installation. Therefore, this section will propose additional computational experiments by considering the effect of different long term load curves on the final decisions.

Fig. 11 summarises tests made by considering a charging site that is 1 km distant from the transformer substation. Different demand curves are shown. For every curve, a dot point indicates on which year a battery installation happens with the related installed capacity. Moreover, the related cables upgrade on the first year is indicated in the legend for every curve. For each series, the legend indicates the maximum power in kW that the new upgraded cables are able to carry (the transformer is upgraded consequently).

Results show that the higher the demand increment is, the later the battery installation occurs and the greater the battery capacity installed is. Higher demand increment means higher battery capacity needed and therefore higher investment. The higher the battery capacity, the later the investment in order to take advantage of the battery price forecast dropping. Hence the ability to look ahead in battery prices combined with the ability to properly forecast the demand trend, is crucial to make optimal choices in terms of cable size today and future battery installations. For this particular set of tests, no renewable has never been chosen within the optimal solution.

Figs. 12 and 13 show the results for a set of tests made by considering a charging site that is 5 km away from the transformer substation. Hence, a case study in which grid reinforcement costs are much higher compared to the previous one due to the longer distances involved. Compared to the previous set of tests, Fig. 12 shows not only higher and earlier investments in batteries but also battery bank replacements for the lowest demand curves (violet and green). Still the main trend of postponing battery installation for higher demand increment is kept.

Moreover in this case study renewable installations are shown in Fig. 13. Due to the high costs of grid reinforcement, the cable size is
lower compared to the previous case study, and renewable generation is chosen in addition to batteries to meet the higher demand in the later years.

The computational experiments proposed above, show that the proposed methodology is suitable not only for investment decision making, but also to analyse the sensitivity of the results to different input dataset and provide precious insight about the effect of long term uncertainty on the decision making process. It is also worthy it to note that the current model structure allows changes in the chosen resolution for the investment decision making. The proposed experiments have been run with a yearly resolution for the long term investment decisions, and a hourly resolution for the short term decisions However, the time structure of the model is flexible and can be adjusted. It is possible for instance to set investment decisions every 2 or 3 years or it is possible to have a finer resolution at the beginning where the uncertainty is lower, and a less fine resolution for the farther years where the knowledge about the future is less precise.

6.3. Influence of battery performance and related costs

In this set of tests we analyse the effect of battery performance and related costs on the optimal decisions. In particular, we run the model by considering a charging site that is 5km distant from the transformer substation and we include the lower demand curve (violet one) from the previous tests. We observed previously that two battery installations occurred throughout the time horizon for this particular case study (see Fig. 12 where battery is installed on the first year and then replaced on the 5th year for the lower demand curve in violet). Therefore we are now interested in looking deeper into this case study by providing the model with the choice between two batteries with different performances in terms of rating and efficiency. In particular, the model will choose between a battery of type 01 with better performance (0.8 efficiency and 0.5 rating), and a battery of type 02 with worse performance (0.7 efficiency and 0.25 rating). The cost of the battery 02 is assumed 800 $/kWh that is around the cheaper price that can be currently found in the market. Compared to battery 02, the cost of battery 01 is varied from slightly less than double, slightly more than double and three times larger in order to compare results. The objective is to give the reader an overview of how the costs and battery performance affect the model decisions.

Fig. 14 shows a case study where the cost of battery of type 01 with a better performance is almost double the cost of the battery of type 02 with lower performance. In this case the optimal choice is to always install the better battery of type 01 both in the first year and in the fifth year.

Fig. 15 shows a case study where the cost of battery of type 01 with a better performance is now slightly higher than double compared to the battery of type 02. In this case the optimal choice is the best battery of type 01 in the first year, that will be replaced by the lower performing battery of type 02 in the fifth year. This is happening because the battery price will increase with the capacity. Given the low installation required in the first year, the cost of battery 01 is still more than compensated by the better efficiency and rating. But in the fifth year, higher capacity is required and the better performance is no longer worthy the higher costs. Therefore investment in a cheaper unit is preferred.

### Table 7
Results: 2 km distance from site to transformer substation.

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<th>Scenario</th>
<th>Bat (kWh)</th>
<th>Ren (kW)</th>
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<td>01</td>
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<td>650</td>
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### Table 8
Results: 3 km distance from site to transformer substation.

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<th>Ren (kW)</th>
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Table 9
Results: 4-10 km distance from site to transformer substation.

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<th>Year</th>
<th>Scenarios</th>
<th>01 Bat (kWh)</th>
<th>02 Bat (kWh)</th>
<th>03 Reinf (kW)</th>
<th>04 Bat (kWh)</th>
<th>05 Reinf (kW)</th>
<th>06 Bat (kWh)</th>
<th>06 Reinf (kW)</th>
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Fig. 11. Battery installation and grid reinforcement decisions for different demand forecast trends considering a 1 km distance from the charging site to the transformer substation.

Fig. 12. Battery installation and grid reinforcement decisions for different demand forecast trends considering a 5 km distance from the charging site to the transformer substation.

Fig. 16 shows a case study where the cost of battery of type 01 with a better performance is assumed three times larger compared to the lower performing battery of type 02, hence a better battery performance is now very expensive. In this case the optimal choice is to go for the cheaper battery type both in the first year and in the fifth year when the bank is replaced. This is because the performance improvement in terms of efficiency and rating cannot defend the higher cost.

Hence, the previous examples showed how sensible such decisions can be and how important it is to have a proper tool to support investment decisions in this field. Analyse the trade-off between cheaper batteries and more expensive batteries and their cost difference in the market can be crucial for industries involved in expansion decisions like the proposed one.
7. Conclusions

A mathematical model for the optimal design, extension and management of electric vehicles charging sites has been presented. It uses a multihorizon approach which compared to traditional approaches, allows to include both long-term uncertainty and short-term uncertainty in the model without an explosion of the scenario tree size as a consequence. The long-term uncertainty is important to be able to model the uncertain long-term trends, allowing the model to delay decisions until more information is known. The short-term uncertainty is important to estimate the consequence of investments in terms of capacity utilization of equipment under different operational settings.
The paper provides a complete real world dataset which can be of interest for similar studies. Extensive computational experiments and sensitivity analyses have been presented to gain insight in the factors driving the decisions under different assumptions. The analysis shows that both the long-term uncertainty and the short-term uncertainty play a major role in both timing, technology choice, and capacity decisions. Compared to traditional decision support approaches the model is able to take more precise decisions due to its multihorizon approach, the inclusion of battery degradation and the inclusion of grid rules and regulations limits that affect the final results.

Acknowledgments

The authors would like to acknowledge Sintef Energy Research for the precious help in providing costs of grid reinforcements for charging sites. Authors acknowledge also eSmart Systems for providing the demand data from a real charging site in Moss, Norway.

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