Adrian Opheim

# Enhancing the Analysis Possibilities of Fedem by Performing Model Reduction in ANSYS 

Master's thesis in Produktutvikling og produksjon
Supervisor: Terje Rølvåg
June 2019

## - NTNU

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## Abstract

When performing dynamic analyses, model reduction is a method used that will reduce the computational cost and storage requirements in finite element analysis programs. The dynamic analysis software Fedem uses the same model reduction techniques as the finite element analysis software ANSYS. As ANSYS has a wider range of element types available for analyses, Fedem would benefit from utilizing this when performing dynamic analyses.

To enable this integration between Fedem and ANSYS, the ANSYS ACT extension "ModRed" has been developed. It offers the possibility of calculating and exporting the matrices needed by Fedem for performing dynamic analyses. In order to test performance of the extension, CMS model reduction of the same models has been performed in both ANSYS and Fedem for investigating the equality of the resulting matrices.

The results show that the full mass matrices of both systems are nearly identical, but the reduced mass matrices show poor similarity, with $10 \%$ to $15 \%$ difference from the Fedem reference matrix. The calculated gravity vectors give very varied equality to the used Fedem reference; from being close to identical to having a $36.5 \%$ difference in the worse test case.

As an issue in partitioning ANSYS matrices has been discovered, and since this issue is believed to be a cause for the varied results, it is too early to make a conclusion on the accuracy acquired from the extension. More testing using a wider range of test models and element types, as well as resolving the issue of matrix partitioning is needed. Despite varying results in testing, the ModRed extension has proven to be a valuable and easy-touse method for enhancing the analysis capabilities of Fedem.

## Sammendrag

Ved utførelse av dynamisk analyse er modellreduksjon en metode ofte tatt i bruk. Denne metoden vil redusere utregningstiden og lagringsbehovene for elementmetode-program. Dynamisk analyse-programvaren Fedem benytter de samme teknikkene for modellreduksjon som det som blir brukt i elementmetode-programmet ANSYS. Ettersom ANSYS har et større utvalg av elementtyper tilgjengelig, vil det kunne være nyttig for Fedem å utnytte dette i dynamiske analyser.

ANSYS ACT-programtillegget "ModRed" er blitt utviklet for å gjøre en slik integrering mellom ANSYS og Fedem mulig. Programtillegget gjør det mulig å regne ut og eksportere de nødvendige matrisene som skal til for at Fedem skal kunne utføre dynamiske analyser. For å teste ytelsen til programtillegget, har en CMS-modellreduksjon av identiske modeller blitt utført i både ANSYS og Fedem, og de resulterende matrisene er blitt sammenliknet.

Resultatene viser at de fulle massematrisene fra begge systemer er så godt som identiske. Det viser seg imidlertid at de reduserte massematrisene kun i liten grad er like, med $10 \%$ til $15 \%$ forskjell fra Fedems referansematrise. De utregnede gravitasjonsvektorene viser svært varierende likhet til referansen fra Fedem; fra å være tilnærmet identiske, til å være $36.5 \%$ forskjellige i tilfellet med størst forskjell.

Ettersom et problem med partisjonering av ANSYS-matriser har blitt oppdaget, og siden dette antas å være en årsak til de varierende resultatene, er det for tidlig å fastslå nøyaktigheten man kan oppnå ved å bruke programtillegget. Ytterligere testing med bruk av et større utvalg test-modeller og element-typer, samt å rette opp i matrise-partisjonerings-problemet er nødvendig. Til tross for varierende resultater ved testing, har ModRed-programtillegget vist seg å være en verdifull metode som det er lett å ta i bruk for å utvide Fedems muligheter for analyse.

## Preface

This master's thesis is the product of work done during the spring term in 2019 as the final part of a five-year master's degree in mechanical engineering. The thesis is written at the Norwegian University of Science and Technology (NTNU) in Trondheim, at the Department of Mechanical and Industrial Engineering.

The project was chosen based on my own interest in both mechanical engineering and software development. As this project would allow me to explore both fields, the potential learning outcome was a key driver. I would like to thank my supervisor, Professor Terje Rølvåg from the Department of Mechanical and Industrial Engineering for introducing me to the project and providing constructive feedback. In addition, I would like to thank Knut Morten Okstad from Fedem Technology for valuable support on the Fedem software.

Trondheim, June 2019

Adrian Opheim

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## Abbreviations

ACT $=$ ANSYS Customization Toolkit<br>APDL = ANSYS Parametric Design Language<br>CAD $=$ Computer Aided Design<br>CAE $=$ Coumpter Aided Engineering<br>CMS $=$ Component Mode Synthesis<br>DOF $=$ Degree of Freedom<br>FE $=$ Finite Element<br>FEM $=$ Finite Element Method<br>GUI $=$ Graphical User Interface<br>WBEX = WorkBench Extension<br>XML = Extensible Markup Language

## Chapter 1

## Introduction

When performing dynamic analyses of structures, similar repeated matrix calculations must be performed for every time step in the analysis. Dynamic analyses are therefore generally much more expensive in terms of computational time and storage space than performing static analyses. In order to reduce the cost of performing dynamic analyses, model reduction methods that reduce the number of degrees of freedom in the system may be applied. Guyan reduction and the component mode synthesis method are examples of such methods. The finite element software ANSYS have both these methods implemented in its APDL solver. In order to access these methods through the ANSYS Mechanical software, an ANSYS ACT extension must be developed, as model reduction techniques are currently not available through ANSYS Mechanical.

The dynamic analysis software Fedem uses the same model reduction techniques as ANSYS. Due to the extensive finite element library available, it is of interest to perform model reduction in ANSYS. The reduced models may thereafter be imported to Fedem for further dynamic analyses. This thesis will present such an extension that enables this integration.

## Chapter 2

## Theory

### 2.1 The Finite Element Method in Linear Dynamics

When analyzing continuous problems, very few of them have an exact, closed-form solution, implicating that discretization techniques should be applied in order to make an approximated system. The behavior of the discretized system could then be computed and used as an approximation for the continuous system. The finite element method is an discretization method that has become the preferred computational method for analyzing structures. Dividing the a structure into a finite number of elements, then assembling them while keeping continuity in the displacements of connected elements for representing the global displacements is the core principle of the finite element method.

For illustrating the principle of the finite element method, we consider a bar of length $L$ in axial deformation subjected to axial loads $P_{1}$ and $P_{2}$. The bar is divided into $N$ finite elements of length $l$, as shown in Figure 2.1.


Figure 2.1: Bar in axial deformation

For each element, the displacement field $u(x, t)$ is linearly interpolated by

$$
\begin{equation*}
u(x, t)=u_{1}(t) \phi_{1}(x)+u_{2}(t) \phi_{2}(x) \tag{2.1}
\end{equation*}
$$

where $\phi_{1}(x), \phi_{2}(x)$ are so-called shape functions of the element. The shape functions are chosen in such a way that they fullfill the boundary conditions

$$
u(0, t)=u_{1}(t) \quad u(l, t)=u_{2}(t)
$$

$u_{1}$ and $u_{2}$ are axial displacements at the element ends, referred to as nodes.
For the bar in axial deformation, the shape functions are linear, given as

$$
\begin{equation*}
\phi_{1}(x)=1-\frac{1}{l} \quad \phi_{2}(x)=\frac{x}{l} \tag{2.2}
\end{equation*}
$$

(2.1) can be written in matrix form

$$
\begin{equation*}
u(x, t)=\mathbf{F}_{e}(x) \mathbf{q}_{e}(t) \tag{2.3}
\end{equation*}
$$

where $x$ must be within the element $e$, and

$$
\begin{align*}
\mathbf{F}_{e}(x) & =\left[\begin{array}{ll}
\phi_{1}(x) & \phi_{2}(x)
\end{array}\right] \\
\mathbf{q}_{e}(t) & =\left[\begin{array}{ll}
u_{1}(t) & u_{2}(t)
\end{array}\right]^{T} \tag{2.4}
\end{align*}
$$

Using this, we can compute the elementary mass and stiffness matrices $\mathbf{M}_{e}$ and $\mathbf{K}_{e}$ given by

$$
\begin{align*}
\mathbf{M}_{e} & =\int_{0}^{l} m \mathbf{F}_{e}^{T} \mathbf{F}_{e} d x \\
\mathbf{K}_{e} & =\int_{0}^{l} E A \frac{d \mathbf{F}_{e}^{T}}{d x} \frac{d \mathbf{F}_{e}}{d x} d x \tag{2.5}
\end{align*}
$$

For the axial bar, we obtain the matrices

$$
\mathbf{K}_{e}=\frac{E A}{l}\left[\begin{array}{cc}
1 & -1  \tag{2.6}\\
-1 & 1
\end{array}\right] \quad \mathbf{M}_{e}=\frac{m l}{6}\left[\begin{array}{ll}
2 & 1 \\
1 & 2
\end{array}\right]
$$

The force element vector $\mathbf{g}_{e}$ is equal to to the end loads of the element:

$$
\mathbf{g}_{e}^{(1)}=\left[\begin{array}{l}
P_{1}  \tag{2.7}\\
P_{2}
\end{array}\right]
$$

### 2.1.1 Assembling elements to the global structure

For expressing dynamic equilibrium for whole bar, we need to know the topology of every element $e$ : Where is it located with respect to the global model and with respect to the other elements of the structure? This is solved by a topology matrix $\mathbf{L}_{e}$. It is constructed so that

$$
\begin{equation*}
\mathbf{q}^{T}=\mathbf{L}_{e} \mathbf{q} \tag{2.8}
\end{equation*}
$$

where $\mathbf{q}$ is a matrix containing all $(N+1)$ nodal displacements:

$$
\mathbf{q}=\left[\begin{array}{lll}
u_{0} & u_{1} & u_{2} \cdots u_{N}
\end{array}\right]^{T}
$$

For instance, element 1 and 2 of Figure 2.1 have the topology matrix

$$
\begin{align*}
\mathbf{L}_{1} & =\left[\begin{array}{llllll}
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & 0 & \cdots & 0
\end{array}\right] \\
\mathbf{L}_{2} & =\left[\begin{array}{llllll}
0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & \cdots & 0
\end{array}\right] \tag{2.9}
\end{align*}
$$

Using the topology matrix, we can assemble all $N$ elements into a global system, and we can then define

- The mass matrix of the assembled system:

$$
\begin{equation*}
\mathbf{M}=\sum_{e=1}^{N} \mathbf{L}_{e}^{T} \mathbf{M}_{e} \mathbf{L}_{e} \tag{2.10}
\end{equation*}
$$

- The stiffness matrix of the assembled system:

$$
\begin{equation*}
\mathbf{K}=\sum_{e=1}^{N} \mathbf{L}_{e}^{T} \mathbf{K}_{e} \mathbf{L}_{e} \tag{2.11}
\end{equation*}
$$

- The load vector of the assembled system:

$$
\begin{equation*}
\mathbf{g}=\sum_{e=1}^{N} \mathbf{L}_{e}^{T} \mathbf{g}_{e} \tag{2.12}
\end{equation*}
$$

This corresponds to an assembly as seen in Figure 2.2. From the figure, one can observe that

- The diagonal terms of the stiffness and mass matrix add up two and two along the diagonal of the matrix.
- The shaded zone correspond to the clamped end of the bar, and could therefore be set to 0 .


Figure 2.2: Assembly of the global matrices

For the axially loaded bar, the global structural matrices become

$$
\begin{align*}
\mathbf{K}=\frac{E A}{l}\left[\begin{array}{cccccc}
2 & -1 & & & & \\
-1 & 2 & -1 & & \mathbf{0} & \\
& -1 & 2 & \ddots & & \\
& & \ddots & \ddots & -1 & \\
& \mathbf{0} & & -1 & 2 & -1 \\
& & & & -1 & 1
\end{array}\right]  \tag{2.13}\\
\mathbf{M}=\frac{m l}{6}\left[\begin{array}{llllll}
4 & 1 & & & & \\
1 & 4 & 1 & & \mathbf{0} & \\
& 1 & 4 & \ddots & \\
& & \ddots & \ddots & 1 & \\
& \mathbf{0} & & 1 & 4 & 1 \\
& & & & 1 & 2
\end{array}\right] \tag{2.14}
\end{align*}
$$

### 2.2 Reduction Methods for Dynamic Problems

When a finite element analysis of a static system is performed, static deformations and stress levels in small details of the system is usually of interest. The static FEM models will therefore usually have a highly refined mesh and a corresponding very high number of degrees of freedom. One are able to solve these models fairly efficiently with static solvers, but if one wishes to find the dynamic behavior of the system, the computing time required is often unacceptable. However, the highly refined meshes used in static analyses are often not needed to capture the dynamic behavior of the system. If one were to find the free vibration modes of the system, we know that the first free vibration modes have a rather smooth deformation, meaning that a coarser mesh would be sufficient to capture these
modes. The reduction methods for dynamic problems addresses this problem. Because creating a fine mesh that captures the required static solutions is an important and timeconsuming part of FE analysis, the reduction methods do not modify the fine mesh created for static analyses, but instead reduces the size of the dynamic problem to solve.

Model reduction can in general be expressed as

$$
\begin{equation*}
\mathbf{v}=\mathbf{H q} \tag{2.15}
\end{equation*}
$$

where $\mathbf{v}$ is the full set of degrees of freedom for the fine mesh and is of size $(n \times 1) . \mathbf{H}$ is the reduction matrix and $\mathbf{q}$ is the reduced set of displacements used for capturing the dynamic behavior and is of size $(m \times 1)$. The aim for the model reduction is to achieve $m \ll n$ without a significant loss in accuracy for the stress results.

The matrix equation that governs system dynamics is expressed as

$$
\begin{equation*}
\mathbf{M} \ddot{\mathbf{v}}+\mathbf{K v}=\mathbf{Q} \tag{2.16}
\end{equation*}
$$

if one neglects damping. When reduction methods are applied to the system equations, the equation is partitioned as follows

$$
\left[\begin{array}{ll}
\mathbf{M}_{e e} & \mathbf{M}_{e i}  \tag{2.17}\\
\mathbf{M}_{i e} & \mathbf{M}_{i i}
\end{array}\right]\left[\begin{array}{c}
\ddot{\mathbf{v}}_{e} \\
\ddot{\mathbf{v}}_{i}
\end{array}\right]+\left[\begin{array}{ll}
\mathbf{K}_{e e} & \mathbf{K}_{e i} \\
\mathbf{K}_{i e} & \mathbf{K}_{i i}
\end{array}\right]\left[\begin{array}{c}
\mathbf{v}_{e} \\
\mathbf{v}_{i}
\end{array}\right]=\left[\begin{array}{c}
\mathbf{Q}_{e} \\
\mathbf{Q}_{i}
\end{array}\right]
$$

where subscript $e$ represents external nodes and subscript $i$ represents internal nodes. External nodes in the model will typically be chosen to be joints, springs, dampers, external loads, control input, points of interest, etc. There should be as few as possible external nodes to reduce the simulation model as much as possible. As will be shown, the reduction methods will eliminate the internal nodes from the FE model.

From (2.17), the stiffness relation for a sub-structure can be written as

$$
\left[\begin{array}{ll}
\mathbf{K}_{e e} & \mathbf{K}_{e i}  \tag{2.18}\\
\mathbf{K}_{i e} & \mathbf{K}_{i i}
\end{array}\right]\left[\begin{array}{c}
\mathbf{v}_{e} \\
\mathbf{v}_{i}
\end{array}\right]=\left[\begin{array}{l}
\mathbf{Q}_{e} \\
\mathbf{Q}_{i}
\end{array}\right]
$$

Writing out the equations gives

$$
\begin{align*}
\mathbf{K}_{e e} \mathbf{v}_{e}+\mathbf{K}_{e i} \mathbf{v}_{i} & =\mathbf{Q}_{e} \\
\mathbf{K}_{i e} \mathbf{v}_{e}+\mathbf{K}_{i i} \mathbf{v}_{i} & =\mathbf{Q}_{i} \tag{2.19}
\end{align*}
$$

Solving the second equation for $\mathbf{v}_{i}$ gives

$$
\begin{aligned}
\mathbf{v}_{i} & =\mathbf{K}_{i i}^{-1} \mathbf{Q}_{i}-\mathbf{K}_{i i}^{-1} \mathbf{K}_{i e} \mathbf{v}_{e} \\
& =\mathbf{K}_{i i}^{-1} \mathbf{Q}_{i}+\mathbf{B} \mathbf{v}_{e}
\end{aligned}
$$

where $\mathbf{B}$ is denoted as the influence matrix, given by $\mathbf{B}=-\mathbf{K}_{i i}^{-1} \mathbf{K}_{i e}$.
The internal displacements $\mathbf{v}_{i}$ can then be expressed as

$$
\begin{equation*}
\mathbf{v}_{i}=\mathbf{v}_{i}^{\text {dyn }}+\mathbf{v}_{i}^{\text {stat }} \tag{2.20}
\end{equation*}
$$

where $\mathbf{v}_{i}^{\text {dyn }}$ represents internal displacements with external DOFs fixed, the dynamic part of the internal node's displacements

$$
\begin{equation*}
\mathbf{v}_{i}^{\text {dyn }}=\mathbf{K}_{i i}^{-1} \mathbf{Q}_{i} \tag{2.21}
\end{equation*}
$$

and $\mathbf{v}_{i}^{\text {stat }}$ represents internal displacements as a function of external displacements, called the "static" part, because the internal nodes respond quasi-statically to the external nodes' displacements.

$$
\begin{equation*}
\mathbf{v}_{i}^{\text {stat }}=-\mathbf{K}_{i i}^{-1} \mathbf{K}_{i e} \mathbf{v}_{e}=\mathbf{B} \mathbf{v}_{e} \tag{2.22}
\end{equation*}
$$

When applying model reduction techniques, two methods are commonly used, namely Guyan reduction and Component Mode Synthesis (CMS). The two methods differ in how they are expressing the internal displacements $\mathbf{v}_{i}$. When applying Guyan reduction, the "dynamic" part $\mathbf{v}_{i}^{i}$ is neglected, and the reduction is based purely on the static response of the internal nodes from displacements at the external nodes, while the Component Mode Synthesis method uses both the dynamic and static part of the internal nodes' displacements

### 2.2.1 Guyan Reduction

Guyan reduction utilizes the fact that the quasi-static response of the internal nodes is often sufficient for describing the substructure's displacements. The reduction method neglects the dynamic part of the internal node's displacements, thus setting $\mathbf{v}_{i}^{\text {dyn }}=\mathbf{0}$. It is also assumed that no forces are applied on the internal nodes, $\mathbf{Q}_{i}=\mathbf{0}$. The reduction is then built by

$$
\mathbf{v}=\left[\begin{array}{l}
\mathbf{v}_{e}  \tag{2.23}\\
\mathbf{v}_{i}
\end{array}\right]=\mathbf{H}_{\text {Guyan } \mathbf{v}_{e}}=\left[\begin{array}{l}
\mathbf{I} \\
\mathbf{B}
\end{array}\right] \mathbf{v}_{e}
$$

where $\mathbf{I}$ is the identity matrix, and $\mathbf{B}=-\mathbf{K}_{i i}^{-1} \mathbf{K}_{i e}$ is the influence matrix.
The reduced stiffness and mass matrices from the Guyan reduction, respectively $\mathbf{k}_{\text {Guyan }}$ and $\mathbf{m}_{\text {Guyan }}$, are then found by premultiplying (2.16) with $\mathbf{H}_{\text {Guyan }}^{T}$ and inserting (2.23) for v . The reduced stiffness and mass matrices are then

$$
\begin{align*}
\mathbf{k}_{\text {Guyan }} & =\mathbf{H}_{\text {Guyan }}^{T} \mathbf{K} \mathbf{H}_{\text {Guyan }} \\
& =\mathbf{K}_{e e}-\mathbf{K}_{e i} \mathbf{K}_{i i}^{-1} \mathbf{K}_{i e} \\
\mathbf{m}_{\text {Guyan }} & =\mathbf{H}_{\text {Guyan }}^{T} \mathbf{M} \mathbf{H}_{\text {Guyan }}  \tag{2.24}\\
& =\mathbf{M}_{e e}-\mathbf{M}_{e i} \mathbf{K}_{i i}^{-1} \mathbf{K}_{i e}-\mathbf{K}_{e i} \mathbf{K}_{i i}^{-1} \mathbf{M}_{i e}+\mathbf{K}_{e i} \mathbf{K}_{i i}^{-1} \mathbf{M}_{i i} \mathbf{K}_{i i}^{-1} \mathbf{K}_{i e}
\end{align*}
$$

Using this, the dynamic equation for the whole system (2.16) can be reduced to the following for an undamped system

$$
\begin{equation*}
\mathbf{m}_{\text {Guyan }} \ddot{\mathbf{v}}_{e}+\mathbf{k}_{\text {Guyan }} \mathbf{v}_{e}=\mathbf{Q}_{e} \tag{2.25}
\end{equation*}
$$

This results tells us that the whole system now is being represented only by the external node's accelerations, displacements and loads. Applying the Guyan reduction technique will therefore reduce the number of degrees of freedom in the system drastically.

If the Guyan reduction is applied to static problems, the exact solution is found. It is also computationally efficient, thus being a widely used reduction technique. However, if the structure has a dynamic behaviour, an approximation is introduced by neglecting the dynamic response of the internal nodes.

### 2.2.2 CMS Model Reduction

The Component Mode Synthesis (CMS) model reduction was introduced by Craig and Bampton [1] in 1968. The method is therefore also called the "Craig-Bampton Method". When applying CMS reduction, the term $\mathbf{v}_{i}^{\text {dyn }}$ is no longer neglected. Instead it is approximated by a linear combination of eigenvalues. In the following, we assume that we originally have $n$ degrees of freedom. By partitioning through (2.17), we get $p$ external and $n-p$ internal degrees of freedom.
In order to approximate $\mathbf{v}_{i}^{\text {dyn }}$, we remember that $\mathbf{v}_{i}^{\text {dyn }}$ correspond to the displacements in the substructure when the external degrees of freedom are fixed. This corresponds the the case where

$$
\begin{equation*}
\mathbf{Q}_{i}=\mathbf{v}_{e}=\mathbf{0} \tag{2.26}
\end{equation*}
$$

Inserting this into (2.17) gives the equation

$$
\begin{equation*}
\mathbf{M}_{i i} \ddot{\mathbf{v}}_{i}^{\mathrm{dyn}}+\mathbf{K}_{i i} \mathbf{v}_{i}^{\mathrm{dyn}}=\mathbf{0} \tag{2.27}
\end{equation*}
$$

When considering simple harmonic motion, the displacement $\mathbf{v}_{i}^{\text {dyn }}$ may be expressed as

$$
\begin{equation*}
\mathbf{v}_{i}^{\mathrm{dyn}}=\phi \sin \omega t \tag{2.28}
\end{equation*}
$$

where $\phi$ is the eigenvector defined by the eigenvalue problem

$$
\begin{equation*}
\left(\mathbf{K}_{i i}-\omega^{2} \mathbf{M}_{i i}\right) \boldsymbol{\phi}=\mathbf{0} \tag{2.29}
\end{equation*}
$$

It is possible to describe an arbitrary displacement as a linear combination of the $(n-p)$ eigenmodes. Using a selection $s$ of the eigenmodes, $\mathbf{v}_{i}^{\mathrm{dyn}}$ can be approximated as

$$
\begin{equation*}
\mathbf{v}_{i}^{\mathrm{dyn}}=\sum_{k=1}^{s} \boldsymbol{\phi}_{k} y_{k}=\mathbf{\Phi} \mathbf{y} \quad s<n-p \tag{2.30}
\end{equation*}
$$

where

$$
\boldsymbol{\Phi}=\left[\begin{array}{llll}
\phi_{1} & \phi_{2} & \cdots & \phi_{s} \tag{2.31}
\end{array}\right]
$$

is the eigenvector matrix with dimensions $(n-p) \times s$.
Using this result, the displacement vector $\mathbf{v}$ can be expressed as

$$
\mathbf{v}=\left[\begin{array}{c}
\mathbf{v}_{e}  \tag{2.32}\\
\mathbf{v}_{i}
\end{array}\right]=\left[\begin{array}{cc}
\mathbf{I} & \mathbf{0} \\
\mathbf{B} & \mathbf{\Phi}
\end{array}\right]\left[\begin{array}{c}
\mathbf{v}_{e} \\
\mathbf{y}
\end{array}\right]=\mathbf{H}_{\mathrm{CMS}}\left[\begin{array}{c}
\mathbf{v}_{e} \\
\mathbf{y}
\end{array}\right]
$$

When combining the substructure dynamic equation (2.17) with the time derivatives of (2.32) and pre-multiplying with $\mathbf{H}_{\mathrm{CMS}}^{T}$ we get

$$
\left[\begin{array}{cc}
\mathbf{m}_{11} & \mathbf{m}_{12}  \tag{2.33}\\
\mathbf{m}_{21} & \mathbf{I}
\end{array}\right]\left[\begin{array}{c}
\ddot{\mathbf{v}}_{e} \\
\ddot{\mathbf{y}}
\end{array}\right]+\left[\begin{array}{cc}
\mathbf{k}_{11} & \mathbf{0} \\
\mathbf{0} & \mathbf{k}_{22}
\end{array}\right]\left[\begin{array}{c}
\mathbf{v}_{e} \\
\mathbf{y}
\end{array}\right]=\left[\begin{array}{l}
\mathbf{q}_{1} \\
\mathbf{q}_{2}
\end{array}\right]
$$

where

$$
\begin{align*}
\mathbf{m}_{11} & =\mathbf{M}_{e e}+\mathbf{B}^{T} \mathbf{M}_{i e}+\mathbf{M}_{e i} \mathbf{B}+\mathbf{B}^{T} \mathbf{M}_{i i} \mathbf{B} \\
\mathbf{m}_{12} & =\mathbf{m}_{21}^{T}=\mathbf{M}_{e i} \boldsymbol{\Phi}+\mathbf{B}^{T} \mathbf{M}_{i i} \boldsymbol{\Phi} \\
\mathbf{k}_{11} & =\mathbf{K}_{e e}+\mathbf{K}_{i e}^{T} \mathbf{B} \\
& {\left[\begin{array}{llll}
\omega_{1}^{2} & & & \\
& \omega_{2}^{2} & & \\
& & \ddots & \\
& & & \omega_{n-p}^{2}
\end{array}\right] }  \tag{2.34}\\
\mathbf{k}_{22} & =\left[\begin{array}{l} 
\\
\mathbf{q}_{1}
\end{array}=\mathbf{Q}_{e}+\mathbf{B}^{T} \mathbf{Q}_{i}\right. \\
\mathbf{q}_{2} & =\boldsymbol{\Phi}^{T} \mathbf{Q}_{i}
\end{align*}
$$

For the $\mathbf{k}_{22}$ matrix, $\omega_{1}^{2}, \omega_{2}^{2}, \ldots, \omega_{n-p}^{2}$ are the eigenvalues corresponding to the eigenmodes of eigenvector matrix $\boldsymbol{\Phi}$.

The substructure will be fully represented - meaning that no approximations are introduced - if all eigenmodes of the substructure is taken into account, thus setting $s=n-p$. Setting $s=0$ will give the Guyan reduction presented in Section 2.2.1. When applying CMS reduction, one tries to keep $s$ as small as possible for reducing the problem size. A criteria often used in practice is to include all eigenmodes having frequencies up to 1.8 or 2 times the highest frequency one wants to compute in the global structure. [2]

### 2.3 Using ANSYS for Solving Finite Element Problems

ANSYS is an American company founded in 1970 which develops and sells finite element simulation software. Their key product, the Mechanical APDL program, has the analysis capability to solve static and dynamic structural analyses, steady-state and transient heat transfer problems, mode-frequency and buckling eigenvalue problems, static or time-variying magnetic analyses, and various types of field and coupled-field applications.

### 2.3.1 ANSYS Parametric Design Language (APDL)

The Mechanical APDL program uses a command-line syntax called ANSYS Parametric Design Language (APDL) for writing commands to the FE solver. The language offers the possibility to use features like repeating commands, macros, if-then-else-branching, for-loops and matrix operations with APDL Math. In the last years, Mechanical APDL
has been replaced by, among others, the new products ANSYS Workbench and ANSYS Mechanical. However, all new ANSYS products still use the the FE solver implemented in Mechanical APDL. Learning to use the APDL language for writing FE solver commands is therefore of great importance. An example of APDL syntax can be seen in Section 2.3.2 and in the onExportData () method of ModRed.py, Section A.1. However, specifics of the APDL syntax will not be presented here. The Mechanical APDL Element Reference [3] serves as a great guide for referencing specific commands.


Figure 2.3: Example of a modal analysis set up in ANSYS Workbench that will be performed in the ANSYS Mechanical program.

Figure 2.3 shows the ANSYS Workbench environment that is used to analyse a simple beam constructed by shell elements. The ANSYS Workbench environment makes it easy to perform multi-domain analyses by dragging-and-dropping the analysis type seen to the left. When an analysis is opened, the user is taken to the ANSYS Mechanical program, where the analyses will be performed. When an analysis is correctly defined and the user hits "Solve", the following happens:

1. All items in the ANSYS Mechanical tree are converted to APDL commands, and written to the file ds. dat located in the ANSYS solver directory, illustrated in Figure 2.4.
2. Mechanical APDL is invoked in the background, performs the commands defined in the ds.dat file and returns results written in the file file.rst
3. The result file is read by ANSYS Mechanical, and displayed graphically to the user.

This means that it would be possible to write custom APDL commands directly to the ds.dat file for solving your specific problem. This is what is utilised in the ModRed application, Section 3.1


Figure 2.4: When the user hits "Solve" in ANSYS Mechanical, all analysis settings are translated to APDL code and sent to APDL in the background.

### 2.3.2 Performing Model Reduction in ANSYS

Model reduction is currently (Version 19.2) not available in the ANSYS Mechanical program. However, multiple model reduction techniques are available through APDL commands, among them Guyan (static) reduction and CMS reduction. Guyan reduction could be applied to both structural and non-structural analysis types. If Guyan reduction is applied to non-structural analyses, the $\mathbf{K}$ and $\mathbf{M}$ matrices are no longer expressing stiffness and mass quantities. Instead, they are matrices representing zero order terms (K) and second order terms (M). The CMS reduction implemented in APDL is not as versatile, and can only be applied to structural analyses. When performing model reduction in ANSYS, the process is divided into three separate steps, namely the generation pass, use pass and expansion pass.

## The Generation Pass

The generation pass could further be divided into two parts: model generation and superelement generation.

The model generation part involves defining the element types, material properties, model geometry and specifying the jobname for reuse in the later use and expansion passes. If the model is already defined, as will be the case in the ModRed application, this part would involve importing the model and its properties correctly into the ANSYS environment.

The superelement creation part involves condensing the full model into one or multiple superelements. The superelement is defined by selecting external (master) nodes that will serve as the interface between other elements and the superelement. The master nodes
need to be defined even though the superelement is not to be connected to other nonsuperelements. Master nodes need to be selected with care, as forces or constraints will later be applied only to the master nodes. The master nodes are therefore typically selected to be at the joints or extremities of the full model. Figure 2.5 shows how one can model a bridge using super elements. It is modeled with bar elements taking axial deformation, as described in Section 2.1. By selecting green nodes as external nodes, 9 super elements can be created, one for every truss in the bridge. Their ID is marked with a square in Figure 2.5. The external nodes are selected at the joints, as this is where forces or constraints will be applied later. The external nodes' ID is marked with a circle.


Figure 2.5: Structure simplified into finite elements

When the solve command is issued at the generation pass, multiple files are generated. Some are written in ASCII format, making it possible to read and edit them. However, the majority of files are written in the non-editable binary form for preserving the calculations and making them transportable between different computer systems. The most important generated binary files during a model reduction are
. sub: The superelement matrix file, which contains the reduced matrices $\mathbf{m}=\mathbf{H}^{T} \mathbf{M H}$, $\mathbf{k}=\mathbf{H}^{T} \mathbf{K H}$ and load vectors if any load is applied during the generation pass.
. emat : Element matrices file containing matrices for all elements in the model.
.esav : Element saved data files
. seld: Superelement load vector data from generation pass.
. In22 : Factorized stiffness matrix from the sparse solver
.full : Assembled global stiffness and mass matrices for the full model.
. db : Database file.
Among these, the . sub file is of particular interest, because it contains the mass and stiffness matrix for the reduced system. At a later stage, this file will be read and the
matrices converted and written out in ASCII format.

The following APDL code snippet shows a small example of a generation pass implemented in APDL code. It shows how one can define variables, as well as modelling a line, meshing it and selecting the element type to be used for the analysis.

Example of a generation pass in APDL

```
!
! Example of a generation pass in APDL
!
/ title, Cantilever Beam
/filname, gen ! Changing filename for clarity
save ! Saving
! Defining parameters
*set,youngs,20.58e10! Young's modulus [N/m^2]
* set, density,7800 ! Density [kg/m^3]
*set, length, 0.6 ! Length [m]
* set, nElements,10 ! Number of elements used
/prep7
k,1,0,0 ! Enter Keypoints
k,2, length,0
1,1,2 ! Create Line between keypoints
et ,2, beam188 ! Element Type for non-super elements
mp,ex,1,youngs ! Defining Young's Modulus
mp,prxy,1,0.33 ! Poisson's Ratio
mp,dens,1, density ! Density
sectype,1, beam, rect, , 0 ! Setting the beam's cross section
secoffset, cent ! Setting section offset
secdata, 0.08, 0.005,0,0,0,0,0,0,0,0,0,0 ! Defining cross-section
lesize , all ,,, nElements ! Number of elements
lmesh, all, , nElements ! Mesh Line
/ solu
antype, substr ! Defining a substructuring / CMS analyis type.
seopt,gen ,2,0,0,0 ! Defining options for substructuring analysis . Generating K
        and M matrices
lumpm,0 ! Not using lumped mass matrix.
m,1, all ! Selecting node 1 and 2 as master nodes. "Giving them access" to all
    degrees of freedom.
m, 2, all
save
solve ! Issuing the solve command
finish
/ quit
```


## The Use Pass

In the use pass, the superelement is used in analyses. Either by being a part of the model, connected to other non-superelements, or the entire model may be a superelement. Any analysis type except explicit dynamics analysis could be performed in the use pass, just like any other analysis. The difference being that yourm model now contains the superelement generated in the generation pass. The superelement portion of the model needs to be defined with the ANSYS element type MATRIX50 for the solver to know it is a super element. When the solve command is issued, the results will consist of a complete solution for the non-superelements, and a reduced solution containing the solution just at the selected master nodes, for the super elements. The reduced solution will be written to the file use.dsub.

## The Expansion Pass

In the expansion pass, the reduced solution use.dsub is expanded, and results at all degrees of freedom in the superelement is calculated. Multiple expansion passes could be issued if multiple superelements were created in the use pass. The solver uses the files gen.asav, gen.full, gen.sub, gen.ln22, gen.db and gen.seld from the generation pass and use. dsub from the use pass. The type of expansion pass method is automatically detected by the solver.

### 2.3.3 Accuracy of Model Reduction Performed in ANSYS

The Mechanical APDL program has an extensive database of verification test cases used for quality assurance of its element types and solution algorithms. The test cases are based on validated results from published work, and are used for validating new versions of the program with extended functionality. A full overview of available test cases can be seen in [4].

Multiple test cases are available for testing the model reduction methods of Mechanical APDL. This section will present one of them, namely an automotive suspension system seen in Figure 2.6. The model is used for demonstrating the benefits of CMS reduction. Three super elements are created; left and right wheel and the main frame. The main frame is then constrained in the top bolts. Next, a modal analysis is performed, and the first 100 eigenfrequencies are extracted.


Figure 2.6: Automotive suspension system used for testing CMS reduction accuracy

| Mode \# | Full Model [Hz] | CMS Model [Hz] | \% Diff |
| :---: | :---: | :---: | :---: |
| 1 | 31.12 | 31.31 | 0.59 |
| 2 | 33.13 | 33.12 | 0.03 |
| 3 | 44.66 | 44.69 | 0.08 |
| 4 | 53.17 | 53.10 | 0.12 |
| 5 | 87.88 | 88.18 | 0.34 |

Table 2.1: Comparison of found eigenfrequencies for the automotive suspension assembly in the full model vs the CMS reduced model

An excerpt of the results is presented in Table 2.1, where the resulting 5 first eigenfrequencies for the corresponding modes are shown. It shows a very little loss of accuracy a maximum of $0.59 \%$ - between analysing the full model and the model created by super elements. The full results could be seen in [5].

| Full Model [s] | CMS Model |  |
| :---: | :---: | :---: |
|  | Generation Pass [s] | Use + Expansion Pass [s] |
| 1291.0 | 1336.0 | 3.0 |

Table 2.2: Elapsed time for solving the full model and applying CMS reduction. Note that the generation pass is only necessary to do once when a geometry is set.

The improvement in solution time is also significant when using CMS reduction, as can be seen in Table 2.2, where the elapsed time for the modal analysis to calculate the first 100 frequencies of the model is shown. The results show that creating the super elements in the generation pass is a relatively expensive process, with an elapsed time close to solving the
full model. However, once the super elements are created, solving the modal analysis in the use and expansion passes is over 400 times faster than solving the full model. Applying CMS reduction to finite element models can therefore be a major advantage, as it enables a more agile development process, where multiple analyses could be performed rapidly, with a high accuracy for one design.

### 2.3.4 ANSYS ACT

As model reduction is currently not available through the ANSYS Mechanical application, one needs to create a custom way of accessing the model reduction techniques of APDL through the graphical user interface of ANSYS Mechanical. ANSYS Customization Toolkit (ACT) provides this possibility by enabling users to create apps for the ANSYS environment that could be tailored to fit their engineering problem. ACT can currently (Version 19.2) be used to customize the following ANSYS products:

- AIM
- DesignModeler
- DesignExplorer
- Electronics Desktop
- Fluent
- Mechanical
- SpaceClaim
- Workbench


### 2.3.5 Structure of an ACT Application

Two basic parts make up an ACT extension:

- An XML file defining context, custom GUIs and callbacks to functions.
- An IronPython script that contains functions responding to user interactions and GUI events, as well as the app's behavior. The IronPython language is an opensource implementation of the Python programming language. Its strength is its tight integration with the .NET Framework. Using IronPython, the user can use both the .NET Framework and Python libraries. In addition, other .NET languages can use the IronPython code. Further documentation can be seen in [6]

The extension may also contain components like external Python libraries, input files, and images to be displayed in the app. Once the extension is developed, it can be shared in two different formats, seen in Figure 2.7.


Figure 2.7: The two formats used for sharing ACT extensions: scripted or binary. ${ }^{1}$

The scripted extension contains the XML and IronPython script, as well as subdirectories containing additional code, images and image files. The contents of the scripted extension is all editable, allowing easy modification of functions in the extension.

The binary extension is a binary WBEX (WorkBench Extension) file that is generated when the scripted extension is built and compiled. The file could easily be shared and installed on another computer with an ANSYS license. The contents of the extension can not be edited, meaning that a new version of the WBEX file needs to be built, compiled and shared for the users to see changes made in the extension.

For the ANSYS Mechanical program, ANSYS ACT offers a wide range of possibilities for customisation:

- APDL macros can be run in the background, opening for automation of repetetive solver steps
- Having access to APDL scripting through ANSYS Mechanical enables the user to use solver capabilities of APDL that are not exposed in ANSYS Mechanical.
- Pre-processing features can be added, like custom loads and boundary conditions.
- Post-processing features could be developed in order to show custom results specifically tailored for the problem to solve.
- Third-party solvers are possible to integrate in the ANSYS environment for solver customisation.
- Graphics in ANSYS Mechanical is possible to tailor, opening possibilities to display custom information like drawing lines, surfaces and text descriptions.


### 2.3.6 ANSYS Element Library

The ANSYS element library contains all possible element types ANSYS offers for finite element analysis. Currently (Version 19.2) it contains 146 different elements, each specialized for its intended use. Elements have the following characteristics:

[^0](a)

(c)

(b)

(d)

Figure 2.8: The four main element types: (a) Mass element, (b) Line element, (c) Area element, (d) Volume element. Element illustrations taken from [8]

- Element name: All element types have a unique name on a maximum of eight characters such as BEAM188. It consists of a group label (BEAM) as well as a unique number (188).
- 2D/3D: The model is set in 2D or 3D depending on what elements to use. 2D models are defined in the $\mathrm{X}-\mathrm{Y}$ plane and run faster than equivalent 3D models. The model becomes 3D if any 3D elements are used. A 2D element may be used in a 3D model.
- Element Shape: All elements are in general divided into four shapes, seen in Figure 2.8:
- Point elements are defined by one node. Point elements are typically mass elements.
- Line elements are represented by a line or arc that is connecting two or three nodes. Typical elements are beams, pipes and axisymmetric shells.
- Area elements have triangular or quadrilateral shape. It might be a 2D solid element or a shell element.
- Volume elements have tetrahedral or brick shape. Typical elements are 3D solid elements.
- Discipline: Specialized elements for the following disciplines are available:

| - Structural | - Electric Circuit | - Load |
| :--- | :--- | :--- |
| - Thermal | - Coupled-Field | - Meshing |
| - Acoustic | - Contact | - Reinforcing |
| - Diffusion | - Combination | - User-defined |
| - Fluid | - Matrix |  |
| - Magnetic Electric | - Infinite |  |

## SHELL181

The SHELL181 element (Figure 2.9 is a 4-node ( $I, J, K, L$ ) strucural shell element for 3D space. It has six degrees of freedom at each node: Translation in $x, y, z$ and rotation about the $x, y, z$ axis. It is suitable for analyzing thin to moderately-thick shell structures. Its shell thickness needs to be defined with the APDL commands

Commands for Setting Thickness at SHELL181 Element

```
setcype,, shell
```

secdata, thickness


Figure 2.9: The SHELL181 element from the ANSYS element library [9]

For defining the shape functions of SHELL181, the following definitions and notations are used, referencing Figure 2.9: [8]

- As this is a shell element, the element coordinate system is not the same as the global Cartesian system. For SHELL181, $u$ and $v$ are in-plane motions and $w$ is out-of-plane motion.
- Coordinates $s, t$ and $r$ are normalized, going from -1.0 on one side of the element, to +1.0 on the other side. They are not necessarily orthogonal to each other.
- Subscripted variables such as $u_{J}$ refer to the $u$ motion at node $J$.

For the stiffness and mass matrix, the following shape functions are used:

$$
\begin{align*}
u=\frac{1}{4} & \left(u_{I}(1-s)(1-t)+u_{J}(1+s)(1-t)\right.  \tag{2.35}\\
& \left.+u_{K}(1+s)(1+t)+u_{L}(1-s)(1+t)\right)
\end{align*}
$$

$$
\begin{align*}
v= & \frac{1}{4}\left(v_{I}(1-s)(1-t)+\ldots(\text { analogous to } u)\right.  \tag{2.36}\\
w= & \frac{1}{4}\left(w_{I}(1-s)(1-t)+\ldots(\text { analogous to } u)\right.  \tag{2.37}\\
\theta_{x}= & \frac{1}{4}\left(\theta_{x, I}(1-s)(1-t)+\theta_{x, J}(1+s)(1-t)\right.  \tag{2.38}\\
& \left.\quad+\theta_{x, K}(1+s)(1+t)+\theta_{x, L}(1-s)(1+t)\right) \\
\theta_{y}= & \frac{1}{4}\left(\theta_{y, I}(1-s)(1-t)+\ldots\left(\text { analogous to } \theta_{x}\right)\right.  \tag{2.39}\\
\theta_{z}= & \frac{1}{4}\left(\theta_{z, I}(1-s)(1-t)+\ldots\left(\text { analogous to } \theta_{x}\right)\right. \tag{2.40}
\end{align*}
$$

If a lumped mass approximation is selected, only (2.35) - (2.37) is used.

### 2.3.7 Degree of Freedom Ordering

ANSYS use an internal ordering method for the set of degrees of freedom (DOFs). When assembling the full mass and stiffness matrix $\mathbf{M}, \mathbf{K}$, ANSYS saves matrix data in a binary file located in the solver directory with the extension .FULL. Items saved in the .FULL file uses an internal solver ordering. The ordering is obtained by an internal ANSYS algorithm that reorders the equations in order to minimize the solver time and disk requirements. The DOF ordering is specified in the file filename.mapping, written out by setting the Mapping parameter to Yes in the APDL command HBMAT. An example of the .mapping file for the simple twoQUAD4 model seen in Figure 4.1 is shown here.

Mapping file for the twoQUAD4 model

| Matrix Eqn | Node | DOF |
| :---: | :---: | :--- |
| 1 | 5 | UX |
| 2 | 5 | UY |
| 3 | 5 | UZ |
| 4 | 5 | ROTX |
| 5 | 5 | ROTY |
| 6 | 5 | ROTZ |
| 7 | 4 | UX |
| 8 | 4 | UY |
| 9 | 4 | UZ |
| 10 | 4 | ROTX |
| 11 | 4 | ROTY |
| 12 | 4 | ROTZ |
| 13 | 1 | UX |
| 14 | 1 | UY |
| 15 | 1 | UZ |
| 16 | 1 | ROTX |
| 17 | 1 | ROTY |
|  |  |  |


| 18 | 1 | ROTZ |
| :--- | :--- | :--- |
| 19 | 2 | UX |
| 20 | 2 | UY |
| 21 | 2 | UZ |
| 22 | 2 | ROTX |
| 23 | 2 | ROTY |
| 24 | 2 | ROTZ |
| 25 | 6 | UX |
| 26 | 6 | UY |
| 27 | 6 | UZ |
| 28 | 6 | ROTX |
| 29 | 6 | ROTY |
| 30 | 6 | ROTZ |
| 31 | 3 | UX |
| 32 | 3 | UY |
| 33 | 3 | UZ |
| 34 | 3 | ROTX |
| 35 | 3 | ROTY |
| 36 | 3 | ROTZ |

### 2.4 Fedem

Fedem, an acronym for Finite Element Dynamics in Elastic Mechanisms, is a computer program that provides features for creating, solving and post-processing a model in a 3D graphical environment. The program has multiple solver modules for performing different types of calculations. Detailed descriptions of each module can be seen in [10] (Fedem Version 7.2), and are briefly discussed here:

- Reducer: Performs a CMS reduction of the mass and stiffness matrices of a FE model for faster simulation of nonlinear dynamics.
- Dynamics Solver: Performs a non-linear dynamics simulation of the superelements' reaction over time to displacements and control system output.
- Stress Recovery: Recovers stresses and strain in the internal nodes from the deformations of the external nodes imposed at the model.
- Mode Shape Recovery: Recovers mode shapes from the eigenvalue results of the Dynamics solver.
- Strain Rosette Analysis: Applies virtual strain gauges on the FE model and outputs strain and stresses from the model over time similar to output from real strain gauges.
- Strain Coat Analysis: Recovers stresses and strains from the coat elements in a FE model, outputting the recovered stresses and strains for the model over the entire time history.
- Curve Export Utility: Allows the user to automatically export result curves to a single ASCII file.


### 2.4.1 Model Reduction in Fedem

The Fedem Reducer has implemented the Guyan reduction and CMS reduction techniques presented in Section 2.2 for effectively solving dynamic problems. The reduction technique is well suited for flexible mechanism analyses due to it preserving the effective masses and inertias of the model. The external nodes needed for the reduction are defined during modelling of the mechanism as "triads". The triads are defined on the connection points for joints, springs, dampers, external loads external load, control inputs or other points of interest. The reduction begins automatically when a dynamics simulation is started, and Fedem determines which parts that need to be reduced based on how the triads are chosen and their connection to the rest of the model. The number of component modes to use for the reduction is specified by the user before reduction. It is recommended to include the lowest modes of vibration in order to achieve good results.

### 2.4.2 Importing reduced models to Fedem

As Fedem does not support modelling or meshing of finite element models, they need to be created in external CAE systems, stored in separate files and imported into Fedem. Fedem supports importing files using the Nastran Bulk Data Format (.nas or .bdf), SESAM Input Interface File Format (.fem) as well as the older Fedem Link Model format (.flm). After reading these files, Fedem stores the info retrieved in the Fedem Technology Link Format (.ftl).

The .ftl file format contains all data needed for defining FE parts. It is defined in ASCII format, and can thus easily be edited using a text editor. The file contains a set of identifiers and parameters expressed with the same syntax:

```
identifier{id value1 value2 ... valueN {reference id text}}
```

where the parameters are listed in Table 2.3 taken from [10]
An example of an identifier with attributes is

$$
\text { QUAD } \left.4\left\{\begin{array}{lllllll}
4 & 22 & 34 & 12 & 32\{\text { PMAT } & 1
\end{array}\right\}\right\}
$$

This defines a 4-noded tetrahedron element with ID=4 that is coupled to the nodes 22, 34, 12 and 32. The element uses an attribute of type PMAT with $\mathrm{ID}=1$.

$$
\operatorname{PMAT}\{1 \quad 2.10 \mathrm{e}+11 \quad 8.00 \mathrm{e}+10 \quad 2.90 \mathrm{e}-01 \quad 7.82 \mathrm{e}+03\}
$$

This defines the material property that is referred to in the QUAD4 element. The decimal numbers describe material parameters like Young's modulus, shear modulus, Poisson's ratio and density. A comprehensive guide of the available identifiers can be found in [10, p. 298]

| Name | Description |
| :--- | :--- |
| identifier* | Specifies field type (e.g., element <br> type, attribute type). |
| id* $^{*}$ | Unique ID for the field (relative to the <br> other fields with the same identifier). |
| value1 ... valueN | Primary values for the object (can be <br> text, integers, or decimal digits). |
| references | Additional data or other fields can be <br> referred to using this field. |
| reference and id | Field reference (reference specified in <br> combination with a valid ID). |
| text | Can be used as additional informa- <br> tion for a field reference or as an <br> optional tag (e.g., a group name) |

Table 2.3: Syntax of Technology Link Format

In order to perform calculations on imported models, Fedem does also need matrix files from the reduced models. Those are saved internally as binary .fmx files after model reduction is performed. Binary .fmx files are created for the reduced stiffness and mass matrices, as well as for the gravity vector described in Section 2.4.3

### 2.4.3 Gravity Vectors in Fedem

Gravitational forces in Fedem are calculated from unit gravitational acceleration vectors, as described in [11, p. 118]. Unit acceleration in the $x, y$ and $z$ direction is denoted $\mathbf{U}_{x}, \mathbf{U}_{y}$ and $\mathbf{U}_{z}$ respectively. The unit vectors are constructed so that for all degrees of freedom in $\mathbf{U}_{x}$ that correspond to $x$ translation, the acceleration component of $\mathbf{U}_{x}$ is set to 1 , otherwise $0 . \mathbf{U}_{x}$ will be of size $\left.\left(\left(n_{e}+n_{i}\right) \cdot D O F s\right) \times 1\right)$ where $n_{e}$ is the number of external nodes, and $n_{i}$ is the number of internal nodes. The total number of nodes in the model is $n=n_{e}+n_{i}$. For an element with 6 DOFs per node, $\mathbf{U}_{x}, \mathbf{U}_{y}$ and $\mathbf{U}_{z}$ will then be

$$
\begin{aligned}
& \mathbf{U}_{x}=\left[\begin{array}{lllllllllll}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0
\end{array}\right]^{T} \\
& \mathbf{U}_{y}=\left[\begin{array}{lllllllllllll}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0
\end{array}\right]^{T} \\
& \mathbf{U}_{z}=\left[\begin{array}{lllllllllll}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0
\end{array}\right]^{T}
\end{aligned}
$$

The gravitational forces $\mathbf{G}_{x}$ corresponding to $\mathbf{U}_{x}$ are calculated from

$$
\left[\begin{array}{l}
\mathbf{G}_{x e}  \tag{2.41}\\
\mathbf{G}_{x i}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{M}_{e e} & \mathbf{M}_{e i} \\
\mathbf{M}_{i e} & \mathbf{M}_{i i}
\end{array}\right]\left[\begin{array}{l}
\mathbf{U}_{x e} \\
\mathbf{U}_{x i}
\end{array}\right]=\left[\begin{array}{l}
\mathbf{M}_{e e} \mathbf{U}_{x e}+\mathbf{M}_{e i} \mathbf{U}_{x i} \\
\mathbf{M}_{i e} \mathbf{U}_{x e}+\mathbf{M}_{i i} \mathbf{U}_{x i}
\end{array}\right]
$$

The forces are then reduced by the CMS tranformation matrix $\mathbf{H}$ to $\mathbf{g}_{x}$ :

$$
\left[\begin{array}{l}
\mathbf{g}_{x e}  \tag{2.42}\\
\mathbf{g}_{x g}
\end{array}\right]=\left[\begin{array}{cc}
\mathbf{I} & \mathbf{B}^{T} \\
\mathbf{0} & \boldsymbol{\Phi}
\end{array}\right]\left[\begin{array}{c}
\mathbf{G}_{x e} \\
\mathbf{G}_{x i}
\end{array}\right]=\left[\begin{array}{c}
\mathbf{G}_{x e}+\mathbf{B}^{T} \mathbf{G}_{x i} \\
\boldsymbol{\Phi}^{T} \mathbf{G}_{x i}
\end{array}\right]
$$

When inserting (2.41), and taking into account the symmetry property

$$
\begin{equation*}
\mathbf{M}_{e i}=\mathbf{M}_{i e}^{T} \tag{2.43}
\end{equation*}
$$

the full expression for $\mathbf{g}_{x e}$ and $\mathbf{g}_{x g}$ becomes

$$
\begin{align*}
& \mathbf{g}_{x e}=\mathbf{M}_{i e}^{T} \mathbf{U}_{x i}+\mathbf{M}_{e e} \mathbf{U}_{x e}+\mathbf{B}^{T} \mathbf{M}_{i i} \mathbf{U}_{x i}+\mathbf{B}^{T} \mathbf{M}_{i e} \mathbf{U}_{x e} \\
& \mathbf{g}_{x g}=\boldsymbol{\Phi}^{T} \mathbf{M}_{i i} \mathbf{U}_{x i}+\boldsymbol{\Phi}^{T} \mathbf{M}_{i e} \mathbf{U}_{x e} \tag{2.44}
\end{align*}
$$

Finding gravitational forces in the $y$ and $z$ direction is an equivalent process by change of indexes:

$$
\begin{align*}
& \mathbf{g}_{y e}=\mathbf{M}_{i e}^{T} \mathbf{U}_{y i}+\mathbf{M}_{e e} \mathbf{U}_{y e}+\mathbf{B}^{T} \mathbf{M}_{i i} \mathbf{U}_{y i}+\mathbf{B}^{T} \mathbf{M}_{i e} \mathbf{U}_{y e}  \tag{2.45}\\
& \mathbf{g}_{y g}=\boldsymbol{\Phi}^{T} \mathbf{M}_{i i} \mathbf{U}_{y i}+\boldsymbol{\Phi}^{T} \mathbf{M}_{i e} \mathbf{U}_{y e} \\
& \mathbf{g}_{z e}=\mathbf{M}_{i e}^{T} \mathbf{U}_{z i}+\mathbf{M}_{e e} \mathbf{U}_{z e}+\mathbf{B}^{T} \mathbf{M}_{i i} \mathbf{U}_{z i}+\mathbf{B}^{T} \mathbf{M}_{i e} \mathbf{U}_{z e}  \tag{2.46}\\
& \mathbf{g}_{z g}=\boldsymbol{\Phi}^{T} \mathbf{M}_{i i} \mathbf{U}_{z i}+\boldsymbol{\Phi}^{T} \mathbf{M}_{i e} \mathbf{U}_{z e}
\end{align*}
$$

The matrix used by Fedem for calculating gravitational forces is then the assembled matrix G:

$$
\mathbf{G}=\left[\begin{array}{lll}
\mathbf{g}_{x e} & \mathbf{g}_{y e} & \mathbf{g}_{z e}  \tag{2.47}\\
\mathbf{g}_{x g} & \mathbf{g}_{y g} & \mathbf{g}_{z g}
\end{array}\right]
$$

$\mathbf{G}$ will be of size $((p \cdot D O F s+s) \times 3)$ where $p$ is the number of external nodes, DOFs is the number of degrees of freedom per node and $s$ is the number of selected component modes.

## Calculating the Gravity Vector from the Reduced Mass Matrix

The gravity vectors in Fedem could also be calculated from the reduced mass matrix $\mathbf{m}=$ $\mathbf{H}^{T} \mathbf{M H}$. The gravity vectors could then simply be calculated as

$$
\left[\begin{array}{l}
\mathbf{g}_{x e}  \tag{2.48}\\
\mathbf{g}_{x g}
\end{array}\right]=\left[\begin{array}{ll}
\mathbf{m}_{11} & \mathbf{m}_{12} \\
\mathbf{m}_{21} & \mathbf{m}_{22}
\end{array}\right]\left[\begin{array}{l}
\mathbf{U}_{x e} \\
\mathbf{U}_{x c}
\end{array}\right]=\left[\begin{array}{l}
\mathbf{m}_{11} \mathbf{u}_{x i}+\mathbf{m}_{12} \mathbf{u}_{x e} \\
\mathbf{m}_{21} \mathbf{u}_{x i}+\mathbf{m}_{22} \mathbf{u}_{x e}
\end{array}\right]
$$

The displacement vector $\mathbf{U}_{x e}$ is the same as described in Section 2.4.3. $\mathbf{U}_{x c}$ is of size ( $s \times 1$ ) and is set to 0 . Expanding (2.48) gives for all directions:

$$
\begin{align*}
\mathbf{g}_{x} & =\mathbf{H}^{T} \mathbf{M} \mathbf{H} \mathbf{U}_{x}^{\mathrm{red}} \\
\mathbf{g}_{y} & =\mathbf{H}^{T} \mathbf{M H U} \mathbf{U}_{y}^{\mathrm{red}}  \tag{2.49}\\
\mathbf{g}_{z} & =\mathbf{H}^{T} \mathbf{M} \mathbf{H} \mathbf{U}_{z}^{\text {red }}
\end{align*}
$$

Comparing this to what is given when using the full mass matrix, we see that gravity vectors from the full mass matrix are given by

$$
\begin{align*}
\mathbf{g}_{x}^{\text {full }} & =\mathbf{H}^{T} \mathbf{M} \mathbf{U}_{x}^{\text {full }} \\
\mathbf{g}_{y}^{\text {full }} & =\mathbf{H}^{T} \mathbf{M} \mathbf{U}_{y}^{\text {full }}  \tag{2.50}\\
\mathbf{g}_{z}^{\text {full }} & =\mathbf{H}^{T} \mathbf{M} \mathbf{U}_{z}^{\text {full }}
\end{align*}
$$

where $\mathbf{U}_{i}^{f u l l}$ are defined as in Section 2.4.3.

### 2.4.4 The FFQ4 Shell Element in Fedem

FFQ4, seen in Figure 2.10, is a 4-node quadrilateral shell element used for modelling shell structures in Fedem. It is composed of a Quadrilateral plate Bending Element with Shear deformation (QBESH) and a Quadrilateral Membrane element with Rotational degrees of Freedom (QMRF). The element nodes are numbered clockwise 1-2-3-4, referring to Figure 2.10. Each of the four nodes have six degrees of freedom: $u, v, w, r_{x}, r_{y}, r_{z}$ where $r_{i}$ refer to rotation about axis $i$.


Figure 2.10: FFQ4 element used in Fedem ${ }^{2}$

The FFQ4 shell element is unfortunately poorly documented, but is based on free formulation theory developed by Pål Bergan, Magne Nygård et.al. where an element is constructed

[^1]of two parts: one for describing rigid body modes, and one describing higher order terms [13, 14]. From Fedem version 7.3, the FFQ4 shell element is replaced with a new element: ANDES, proposed by Felippa, Militello in [15] and researched by Haugen, Skallerup in [16].

### 2.4.5 Degree of Freedom Ordering in Fedem

The mass and stiffness matrix in Fedem is, similarly to ANSYS, restructured by the Fedem solver, in order to reduce disk requirements and make the solving process more efficient. The mapping of equation number to corresponding degree of freedom is defined in the file MEQN. res, which is written to the Fedem solver directory by the additional solver option -debug 3. This mapping is used in the CMS reduction process, when the full mass matrix is partitioned into

$$
\mathbf{M}=\left[\begin{array}{ll}
\mathbf{M}_{e e} & \mathbf{M}_{e i} \\
\mathbf{M}_{i e} & \mathbf{M}_{i i}
\end{array}\right]
$$

The partitioning process is easier understood by a small example. We create a custom mass matrix where every element in the matrix is set to its corresponding index number (row-column):

$$
\mathbf{M}=\left[\begin{array}{lllll}
11 & 12 & 13 & 14 & 15 \\
21 & 22 & 23 & 24 & 25 \\
31 & 32 & 33 & 34 & 35 \\
41 & 42 & 43 & 44 & 45 \\
51 & 52 & 53 & 54 & 55
\end{array}\right]
$$

The mapping of degree of freedom to the corresponding equation is given by the following Python-style dictionary:

```
mapping ={
    1: 3,
    2: 4,
    3: 5,
    4: 1,
    5: 2
}
```

The dictionary maps DOF number to the corresponding equation: mapping [dof] = equation, e.g. mapping[1] = 3, mapping[4] $=1$.

When the matrix is partitioned, the equations corresponding to the external nodes are moved to $\mathbf{M}_{e e}$, while the equations corresponding to the internal nodes are moved to the $\mathbf{M}_{i i}$ part of the matrix. The $\mathbf{M}_{e i}, \mathbf{M}_{i e}$ parts of the matrix will be a combination of the equations from the external and internal nodes. In addition, Fedem sorts the partitioned parts by ascending equation number.

The partitioning process will first move the rows corresponding to the equation numbers of the external and internal degrees of freedom. If the external degrees of freedom are chosen to be $\left[\begin{array}{ll}2 & 5\end{array}\right]$ and internal degrees of freedom are $\left[\begin{array}{lll}1 & 3 & 4\end{array}\right]$, the $\mathbf{M}$ matrix will be restructured to the following after moving the rows:

$$
\left[\begin{array}{lllll}
21 & 22 & 23 & 24 & 25 \\
41 & 42 & 43 & 44 & 45 \\
11 & 12 & 13 & 14 & 15 \\
31 & 32 & 33 & 34 & 35 \\
51 & 52 & 53 & 54 & 55
\end{array}\right]
$$

Next, the columns are moved, resulting in the partitioned matrix

$$
\left[\begin{array}{lllll}
22 & 24 & 21 & 23 & 25 \\
42 & 44 & 41 & 43 & 45 \\
12 & 14 & 11 & 13 & 15 \\
32 & 34 & 31 & 33 & 35 \\
52 & 54 & 51 & 53 & 55
\end{array}\right]
$$

The partitions of the matrix will then be

$$
\begin{array}{ll}
\mathbf{M}_{e e}=\left[\begin{array}{ll}
22 & 24 \\
42 & 44
\end{array}\right] & \mathbf{M}_{e i}=\left[\begin{array}{lll}
21 & 23 & 25 \\
41 & 43 & 45
\end{array}\right] \\
\mathbf{M}_{i e}=\left[\begin{array}{ll}
12 & 14 \\
32 & 34 \\
52 & 54
\end{array}\right] & \mathbf{M}_{i i}=\left[\begin{array}{lll}
11 & 13 & 15 \\
31 & 33 & 35 \\
51 & 53 & 55
\end{array}\right]
\end{array}
$$

### 2.5 Software Development

When developing software, many related processes lead to the production of a software system; specifications for the software's functionality are set, the software is developed and it is validated to ensure it meets the set specifications. Hence, the process of how one develops software should be investigated in order to optimize the process. Multiple models for software processes exist. For simplification, it is possible to divide the type of processes in two: plan-driven processes and agile processes. In plan-driven processes all process activities are planned in advance of the development. Progress made in the project as well as the resulting product is then measured against the plans set before development has started. In agile processes, the planning is continuous, and is a subject of change through the whole development process. Incremental development is one type of agile process.


Figure 2.11: Incremental development process.

When software is developed incrementally, it is based on the idea that one develops an initial version of the software. This is then tested and feedback is generated. The software will then evolve in incremental versions until a system meeting the requirements set has been developed. The specification, development and validation phases of the project will always be subject for change through the whole development. This is illustrated in Figure 2.11. In this way it reflects the way humans solve problems: a complete solution to a problem is rarely known in advance, but we move closer to a solution in small steps, and backtrack if a mistake is discovered. Sommerville [17] states three major advantages to incremental development:

- There is little cost related to implementing changes in requirements, as the specification, development and validation processes are always continuous and open for change. This means the amount of analysis and documentation that has to be redone when requirements change is reduced.
- It is easier to get feedback on work in development, as one at an early stage of the development process tries to have a working, initial version of the product. For people outside the development process, it is easier to judge a product and perhaps edit its specifications if they can have it demonstrated.
- It is possible to gain value from a product at an early stage in the process, as some stand-alone features might be available for use.


### 2.5.1 Validating Software by Unit Testing

Unit testing is the process of testing the small units that make up a computer program. Individual functions are usually seen as the simplest type of component, and is therefore usually the target for unit testing. When performing unit testing, the aim is simply to test if your functions behave as expected. This is accomplished by calling these routines with a range of different input parameters, and montoring if the expected result is returned or produced. Ideally, functions should be tested in isolation. This means that the test should
be free of dependencies to other parts of the program. In this way, we ensure that we are actually testing the desired unit of the program.
IronPython offers an automated unit testing framework unittest [18] for writing and running tests. It gives the possibility to run all tests written for a program, and reports the outcome of each individual test. The automated tests of unittest consist of three parts:

1. Setup part: Here, one can set up the initial state of the program. The setup is run before each single test, ensuring equal initial conditions for each test.
2. Call part: This is where the object or function to test is being called from the testing framework.
3. Assertion part: The behavior of the function is compared to what is expected. An evalutaion of the assertion that evaluates to True means that the test has been successful, if it is False, it has failed.
The following example, taken from [18] with small modifications, shows a small, basic example of testing the behavior of Python's random module

## Basic Unit Test Example

```
import random
import unittest
class TestSequenceFunctions( unittest .TestCase):
    def setUp( self ):
        self .seq = range(10)
    def test_shuffle ( self ):
        # make sure the shuffled sequence does not lose any elements
        random. shuffle ( self .seq)
        self .seq. sort ()
        self . assertEqual ( self .seq, range(10))
    def test_choice ( self):
        element = random.choice( self .seq)
        self . assertTrue (element in self .seq)
if __name__ == '__main__':
    unittest .main()
```


## Chapter 3

## Method

ANSYS supports the same CMS model reduction as implemented in Fedem. This means that ANSYS is able to generate the same reduced mass, stiffness and transformation matrix $\mathbf{H}$ as needed by Fedem to perform dynamics analyses. And this again means that the reduced models from ANSYS could be used in the Fedem environment, allowing the following benefits:

- Creating models could be performed in ANSYS with the CAD programs ANSYS SpaceClaim or ANSYS DesignModeler. The CAD programs support parametrization of the models, which in return facilitates the analysis of design changes.
- The ANSYS Meshing program could be used to create finite element meshes. The program supports advanced mesh configuration tools like automated inflation, physicsaware meshing and controls for moving, merging and editing nodes and elements.
- When meshing is performed in ANSYS, the full finite element library [9] of ANSYS is available. Currently (Version 19.2) it contains 146 different elements. Each element type is thoroughly documented, and specific elements for different applications are available.
- The extensive material data and material designer capabilities of ANSYS could be integrated in the models. This makes it possible to model composite materials, anisotropic material and more in the analysis.
- Using a combination of ANSYS and Fedem for analyses would make it possible to verify results to a larger extent than what is currently possible. Identical analyses could be performed using both systems, thus extending the possibilities for verifying results.

This chapter will introduce the ANSYS ACT extension "ModRed" that will enable an easy-to-use integration of CMS model reduction performed in ANSYS into Fedem. Its implementation details will be explained, and different use cases will be presented.

### 3.1 The ModRed Extension for Performing CMS Model Reduction in ANSYS

In order to open and analyze models in Fedem, the following matrices and data is needed:

- The reduced mass matrix by CMS reduction: $\mathbf{m}=\mathbf{H}^{T} \mathbf{M H}$
- The reduced stiffness matrix by CMS reduction: $\mathbf{k}=\mathbf{H}^{T} \mathbf{K H}$
- A FE mesh in . nas, . fem or .ftl format
- The assembled gravity vectors $\mathbf{G}$

The ACT extension "ModRed" is able to create all of these matrices and data, enabling easier integration between ANSYS and Fedem.

### 3.1.1 Installation

The ModRed extension will be available both as a compiled WBEX file, as well as a scripted extension containing the full source code. Installing the extension is an easy procedure, but differs slightly depending on what type is to be installed:

## Compiled WBEX extension

In ANSYS Workbench, go to

$$
\text { Extensions } \rightarrow \text { Install Extension }
$$

and select the . wbex file in the file dialog. This will install the extension to the ANSYS environment. If the extension does not show, try checking the "Loaded" box for ModRed under Extensions $\rightarrow$ Manage Extensions

## Scripted extension

In ANSYS Workbench, open the ACT Start Page tab. From there, go to Manage Extensions, click the Settings icon and select Add Folder. Then select the folder containing the file ModRed.xml. This will install the extension into the environment, and also allows editing the source code while using the extension. When opening the ANSYS Mechanical program, the ModRed extension is added to the toolbar, as seen in Figure 3.1

## Fedem Reduction $\%$ 五

Figure 3.1: The ModRed extension loaded into the toolbar. The left button places ModRed in the ANSYS Mechanical solution tree. The right button exports Fedem data.

Figure 3.1 shows the ModRed app loaded into the ANSYS Mechanical model tree, as a new "Solution" type. This allows the extension to edit solver commands as well as performing post-processing manipulation of solver data.


Figure 3.2: The ModRed extension loaded into ANSYS Mechanical

### 3.1.2 Importing a model into ANSYS Mechanical

There are multiple ways to import models into the ANSYS Mechanical program. Common for all is that this process is handled by ANSYS Workbench.

## Importing an Existing Mesh

If the model and mesh is already defined, ANSYS may import the mesh file through the External Model module, Figure 3.3


Figure 3.3: Importing an existing mesh into ANSYS Mechanical

From the External Model module, the file formats possible to import are listed in Table 3.1

| APDL Common <br> Database | Abaqus <br> Input | NASTRAN <br> Bulk Data | Fluent <br> Input | ICEM CFD | LS-DYNA |
| :--- | :--- | :---: | :---: | :--- | :--- |
| . cdb | .inp | .bdf, .dat, .nas | .msh, .cas | .uns | .k |

Table 3.1: Supported mesh file formats for import to ANSYS Mechanical

When using the External Model module to import a meshed model, ANSYS will convert the element type used in the external system to the most similar ANSYS element type in order to be able to solve the system.

## Modelling From Scratch in ANSYS

If the modelling is to be performed in ANSYS, a Geometry component is used to import the model from the modelling program and into ANSYS Mechanical. Figure 3.4 shows a model made in DesignModeler being imported into ANSYS Mechanical. This method is the most versatile, as the geometry made in ANSYS is possible to parametrize, making it possible to perform analyses on multiple geometrical variations.


Figure 3.4: Import a Native Model

### 3.1.3 Defining Settings for the ModRed Extension

| Details of "Fedem Reduction" |  | $\square$ |
| :---: | :---: | :---: |
| - | Selection of Master Nodes |  |
|  | Pick Master Nodes By... | Geometry |
|  | Scoping Method | Named Selection |
|  | Named Selection | eNodes |
| $\exists$ | Model Reduction Method |  |
|  | Reduction Method | CMS |
|  | Num. of Component Modes to Extract | 2 |
|  | File Name | master |
|  | Use lumped matrices? | No |
| $\exists$ | Export Options |  |
|  | Export Matrices to Fedem? | Yes |
|  | Fedem File Directory | C |
| Users\adrian\Documents\Dok... |  |  |

Figure 3.5: The ModRed extension loaded into ANSYS Mechanical

## Selecting external nodes

As ANSYS is not going to perform any analyses, only calculate the reduced stiffness and mass matrices, one only needs to perform the generation pass of CMS reduction in ANSYS. Here, the user selects what nodes to be used as external or "triads" in Fedem. ANSYS will then mark these as master nodes when the APDL solver is invoked. The external nodes may be selected in two ways:

Geometry Selection: Using the integrated ANSYS tool for selecting geometry, the user may simply select the nodes that will be set as triads in Fedem. Using the node selection tool, Figure 3.6, changes the view to render the mesh, and allows only nodal points to be selected. Press Ctrl while selecting nodes to select multiple nodes.


Figure 3.6: Node selection tool

Named Selection: ANSYS also allows selecting the desired nodes and saving them to a Named Selection which may be accessed from the ModRed settings. In this way, the user can select and save multiple sets of external nodes in order to decide what will yield the best results.

## Setting Model Reduction Settings

The ModRed extension allows the user to select two types of model reduction: Guyan and CMS reduction. If the Guyan reduction option is selected, no additional component modes
are selected, and the transformation matrix is $\mathbf{H}=\mathbf{H}_{\text {Guyan }}$ from Section 2.2.1. If the $C M S$ reduction method is chosen, the user also gets the possibility of setting the number of component modes to be extracted from the model. The transformation matrix will then be set as $\mathbf{H}=\mathbf{H}_{\mathrm{CMS}}$, from Section 2.2.2.

The File name tab allows the user set a file name that will be given to the .ftl file and binary . fmx files to be generated. This is useful if multiple parts is to be reduced. A unique name for each part will then make the process of importing the data into Fedem easier.

The user can also select whether or not to use lumped matrices. Setting the option to Yes will force the APDL solver to use the LUMPM command that makes the solver use a lumped mass approximation for reducing the disk space needed to save the matrices.

## Setting Export Options

In the Export Options, the user can select whether the matrices are to be exported or not. This is beneficial if the user wants to force ANSYS to not export new matrices on every Solve command. In the Fedem File Directory tab, the user selects to which folder to output the generated. ftl and .fmx files.

### 3.1.4 The APDL Commands Written by ModRed

Once all necessary settings to perform the model reduction has been set, the model is ready to be solved. This is done by hitting the Solve button. This will then invoke the onSolve () method in ModRed.py, which adds APDL commands to the ds. dat file to be read by the solver, as explained in Section 2.3.1. The code below shows an excerpt from the ds . dat file that is generated when solving a model with nodes $1,2,3,4$ selected as the external nodes. See the comments marked with! for more information about each command. Extensive documentation for each command is found in [19].

Excerpt from ds.dat

```
! ****** Begin Command Snippet ******
Commands written by ModRed
!
finish
/filname, master ! Setting the file name
save
/ solu
antype, substr !Defining a substructure analysis (includes both Guyan and CMS)
outpr, nsol, all ! Print command that must be defined if iokey=="tcms"
cmsopt, fix ,2,,,, tcms !Fixed interface normal modes with 2 component modes
seopt, master, 2, 0, 0, , ! Super element name and generating mass+ stiffness matrix
m,1, all ! Setting node 1-4 as master nodes, "having access to" all degrees of
    freedom
```

```
m, 2, all
m, 3, all
m, 4, all
alls ! Selecting all nodes before solving
solve
save
finish
! Master node selection finished
! Exporting matrices ..
*dmat, cst, d, import, tcms, master.tcms, cst ! Importing the CST matrix
*dmat, nor, d, import, tcms, master.tcms, nor ! Importing the NOR matrix
save
*export, cst, mmf, CST.mmf ! Exporting CST and NOR matrices to MMF format
*export, nor, mmf, NOR.mmf
/aux2
fileaux2, master, sub ! Specifying to dump the reduced matrices
hbmat, M_red, hbmat, , ascii , mass, no !Dumping reduced mass and stiffness matrix to
    Harwell-Boeing format (16-decimal precision)
hbmat, K_red, hbmat, , ascii , stiff , no
fileaux2, master, full ! Specifying to dump full matrices
hbmat, M_full, hbmat, ascii, mass, no, yes ! Dumping full mass matrix in
    Harwell-Boeing format (16-decimal precision)
finish
*smat, M_red, d, import, hbmat, M_red.hbmat, ascii ! Importing Harwell-Boeing format
    matrices
*smat, M_full, d, import, hbmat, M_full.hbmat, ascii
*smat, K_red, d, import, hbmat, K_red.hbmat, ascii
*export, M_red, mmf, M_red.mmf ! Exporting matrices in MMF format (maintains
    16-decimals precision)
*export, M_full, mmf, M_full.mmf
*export, K_red, mmf, K_red.mmf
save
```


### 3.1.5 Matrix Export

When the model has been solved with the added APDL commands seen in Section 3.1.4, matrices are then exported to the ANSYS solver directory. This can be accessed by rightclicking the Solution in the ANSYS Mechanical tree and then selecting Open Solver Files Directory. The following matrices are written to this directory:

- M_full.mmf: This is the full mass matrix, saved in a sparse format, and formatted in the MatrixMarket file format. Note also the following:
- The matrix is saved column-first.
- In order to utilize symmetry and reducing storage space needed, only the lower triangular part of the matrix is saved in the. mmf file.
- M_full.mapping: Mapping file for the full mass matrix. This relates node to the corresponding equation in the matrix.
- M_red.mmf, K_red.mmf: The reduced mass and stiffness matrix, saved in sparse format.
- CST.mmf: The static part of the transformation matrix H, namely

$$
\mathrm{CST}=\left[\begin{array}{l}
\mathbf{I}  \tag{3.1}\\
\mathbf{B}
\end{array}\right]
$$

CST is saved in dense form, column-first

- NOR.mmf: The component modes part of the transformation matrix $\mathbf{H}$, namely

$$
\mathrm{NOR}=\left[\begin{array}{c}
\mathbf{0} \\
\mathbf{\Phi}
\end{array}\right]
$$

NOR is saved in dense form, column-first

### 3.1.6 Generation of files to be read by Fedem

When the model has been solved, the user can click the Export Fedem Data button in the ModRed toolbar, thus invoking the onExportData() method in modRed.py. The full mass matrix will then be partitioned into external and internal parts as described in Section 2.4.5.

For visualising the solved model in Fedem, a . ftl file of the model is created and saved in the specified Fedem directory. Binary . fmx files containg the reduced mass and stiffness matrix and gravity vectors are also generated.

### 3.2 Verifying Data From the ModRed Extension

In order to verify that all matrix handling is performed correctly, an IronPython Unit Test framework has been set up. Here, all methods that are involved in matrix calculations are tested for common errors for ensuring correct code and matrix calculations. An excerpt of what is tested is shown in the following section.

## Verification of Matrices

The exported matrices from ANSYS in MatrixMarket Format need to be read by the ModRed extension in order to generate the needed Fedem files. As the used math library Math. NET.Numerics.LinearAlgebra [20] does not have any built-in functions for reading MatrixMarket formatted files, this method has to be written customly for the ModRed app. Thus, it is important that the matrices are read correctly before further calculations.

Testing that the matrix is read properly could be done through unit testing. The following code snippet shows how it is verified that the matrix is correctly read into IronPython by verifying the number of rows and columns on a test file, as well as checking some entries custom-picked from the ASCII-formatted .mmf file.

```
def test_readMMFMatrix_full( self):
    path = self . resourcesFolder_ + "\\beam.nas" +"\\Ansys" +"\\M_full.mmf"
    mat = ModRed.readMMFMatrix(path, "full")
    # Checking if dimensions are correct
    self . assertEqual (mat.RowCount, 630)
    self . assertEqual (mat.ColumnCount, 630)
    # Checking some values at the boundaries:
    self.assertEqual (mat [0,0], 8.688888888888170E-02)
    self . assertEqual (mat[0, 629], 0)
    self . assertEqual (mat[629, 0], 0)
    self . assertEqual (mat[629, 629], 1.448148148148030E-16)
    # Testing random values:
    self . assertEqual (mat[23,23], 1.448148148148030E-16)
    self . assertEqual (mat[285,243], 3.620370370370130E-07)
    self . assertEqual (mat[291,291], 5.792592592592110E-06)
```


## Testing for Symmetry

The generated mass and stiffness matrix should be symmetrical, both for the full and for the reduced versions. This is verified by the following simple test, which is also performed on the reduced mass matrix:
def test_readMMFMatrix full_isSymmetric ( self ):
path $=$ self. resourcesFolder_ + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Ansys" $+" \backslash \backslash$ M_full.mmf " mat $=$ ModRed.readMMFMatrix(path, "full")
\# Checking if matrix is symmetric:
for row in range ( 0 , mat.RowCount):
for col in range ( 0 , mat.ColumnCount):
self. assertEqual (mat[row, col ], mat[col, row])

## Testing for Consistent Mass

In order to verify that the mass matrix has a consistent mass after reduction, one can compute the total mass of the system by pre- and post-multiplying the system with unit translation in $x, y$ and $z$ direction:

$$
\begin{equation*}
m_{\text {total }}=\mathbf{u}_{\text {trans }}^{T} \mathbf{m u} \mathbf{u}_{\text {trans }} \tag{3.2}
\end{equation*}
$$

This is then compared with the mass of the model computed by Fedem. The same test is done with the full mass matrices by the following unit test:

```
def test_readMMFMatrix_red_correctMass( self ):
    ","
    Applying unit translation in x, y, z direction
    and verifying that it is the same mass as computed in Fedem
    path = self . _resourcesFolder_ + "\\beam.nas" + "\\\Ansys" + "\\M_red.mmf"
    mat = ModRed.readMMFMatrix(path, "sub")
    correct_mass =3.12800E +01
    for i in [1, 2, 3]:
        u = ModRed.createUnitVector(i, 4, 2)
        m}=\textrm{u}.\operatorname{Transpose() .Multiply (mat).Multiply (u) [0,0]
        self .assertAlmostEqual(correct_mass, m)
```

Note that the assertAlmostEqual method is used, as there will be a small difference in results because of floating point precision. The assertAlmostEqual method tests for equality down to 7 decimal places by default.

## Testing if matrix is diagonal

As both mass and stiffness matrices will be diagonal, one could check if elements not on the diagonal are zero. This is best tested after the matrix has been partitioned, as the partitioned matrix should remain diagonal.

```
def test_massMatrix_diagonality (self):
        #
        # Verifying that a lumped matrix is diagonal
        #
        mat = ModRed.readMMFMatrix(self._resourcesFolder + "\\beam.nas" + "\\\Ansys" +
    \M_full_lumped.mmf", "full")
    for row in range(0, mat.RowCount):
        for col in range(0, mat.ColumnCount):
            if row == col:
```

```
    self . assertTrue (mat[row, col] != 0.0 )
else :
    self . assertTrue (mat[row, col] == 0.0)
```


## Testing if Matrices are Partitioned Correctly

Testing if the matrices are partitioned correctly may only be partially done. This is due to the fact that Fedem, which is used as a reference, does only export the full, unpartitioned mass matrix $\mathbf{M}$, the $\mathbf{M}_{e e}$ part of the partitioned matrix, and the reduced mass matrix $\mathbf{m}$. ANSYS does only export the full mass matrix $\mathbf{M}$ and the reduced mass matrix $m$. This implies that it could only be verified that the $\mathbf{M}_{e e}$ part of the matrix is partitioned correctly. It should be noted that this only tells us that $\mathbf{M}_{e e}$ is partioned in an identical way as done by the Fedem reducer, not that it is identical to what ANSYS does, because ANSYS does not export the partitioned matrix. An example of testing the partitionMatrix function is shown in the following:

## Test of partitionMatrix function

```
def test_partitionMatrix_beam_Mee ( self):
    # Testing with M_full from beam.nas (medium model)
    mapping = ModRed.readMappingFileFedem(self.resourcesFolder + "\\beam.nas" +
    "\\Fedem" + "\\MMEQN.res")
        M_full = ModRed.readFedemMatrix(self._resourcesFolder + "\\beam.nas" +
    "\\\Fedem" + "\\M_full.res")
        eNodes = [1, 2, 3, 4]
        iNodes = [i for i in range(5, 106)]
        M_full_partitioned = ModRed.partitionMatrix(M_full, eNodes, iNodes, mapping, 6)
        # Using M_ee from Fedem as reference
        M_ee = ModRed.readFedemVector(self. _resourcesFolder + "\\\beam.nas" +
    "\\\Fedem" + "\\M_ee.res")
        for row in range(0, M_ee.RowCount):
            for col in range(0, M_ee.ColumnCount):
                self .assertAlmostEqual( M_full_partitioned [row, col ], M_ee[row, col ])
```


## Chapter 4

## Results

Testing has been performed with the three FE models listed in Table 4.1. All models are NASTRAN . nas files created in an external CAD system. For acquiring matrices from ANSYS, the model is imported through the External Module component and opened in ANSYS Mechanical. The ModRed extension is then used for selecting external nodes and exporting the needed matrices. To acquire matrices from Fedem, the same external nodes as defined in ANSYS have been selected, and the fedem_reducer module is run. The matrices are then read from the result file fedem_reducer. res for analyses.

|  | Small | Medium | Large |
| :--- | :---: | :---: | :---: |
| Fedem element | QUAD4 | QUAD4 | QUAD4 |
| ANSYS element | SHELL181 | SHELL181 | SHELL181 |
| Number of nodes | 6 | 105 | 7869 |
| Number of elements | 2 | 80 | 7680 |
| Material | Steel | Steel | Aluminium |
| Thickness [m] | 0.02 | 0.02 | 0.01 |
| Number of selected external nodes | 2 | 4 | 4 |
| File name | twoQUAD4 | beam | plate |

Table 4.1: FE models used for testing the ModRed extension

The small model can be seen in Figure 4.1. It is very simple, containing two 4 -node shell elements, with node 1 and 3 chosen to be external. The model is used because it facilitates easier debugging of its corresponding matrices and degree of freedom ordering.


Figure 4.1: The small test model put togeter of two 4-node shell elements. The external nodes are marked green.

### 4.1 Similarity of Mass Matrices between ANSYS and Fedem

When analyses of dynamics is performed in Fedem, it is vital that the structural matrices for the models yield identical results, independent of what the origin of matrices are. When the ModRed app is to be used for enabling analyses in Fedem, FE models and their associated structural matrices are likely to be of two different origins:

1. The FE model will be made in an external CAD software and exported to a file format that could be imported by Fedem. The external nodes (triads) will then be selected in Fedem, and the Fedem reducer will be used to generate the reduced mass matrix $\mathbf{m}$ used in calculations.
2. The FE model is created or imported to ANSYS. The external nodes will then be selected in ANSYS Mechanical, and the internal ANSYS APDL solver will be used for calculating the reduced mass and stiffness matrix. These matrices will then be saved in a binary . fmx format and read by Fedem. The Fedem reducer will not be used.

In order to test the similarity of structural matrices in ANSYS and Fedem, structural matrices for the models seen in Table 4.1 have been generated in both ANSYS and Fedem and then compared. In order to quantify their internal difference, the Euclidean distance $d$ between the sum of the two matrices is calculated. Then the relative difference $\alpha$ of the distance compared to the reference matrix is calculated:

$$
\begin{aligned}
& d=\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n}\left(\mathbf{a}_{i j}-\mathbf{b}_{i j}\right)^{2}} \\
& \alpha=\frac{d}{\sum_{i=1}^{n} \sum_{j=1}^{n} \mathbf{b}_{i j}}
\end{aligned}
$$

Here, $\mathbf{a}_{i j}$ is the elements of the matrix that will be compared, and $\mathbf{b}_{i j}$ is the matrix used as reference. $n$ is the total number of rows in the matrices. When using this method for comparing matrices, $\alpha=0$ means the two matrices are identical. $\alpha=1$ would imply there is a $100 \%$ difference between the matrices. I.e. a matrix filled with 2's compared to a matrix filled with 1's used as a reference, would give $\alpha=1$. In the analysis, Fedem mass matrices are used as a reference in all models.


Figure 4.2: Percentage difference of full and reduced mass matrices from Fedem and ANSYS in relation to the number of nodes in the model

Figure 4.2 shows the percentage difference $\alpha$ of the full and reduced mass matrices of the test models. The full mass matrices yield good results for models with $>100$ nodes, with $\alpha<10^{-5}$. For the small model with only six nodes, about $1 \%$ difference is obtained. The reduced mass matrices however, show overall relatively poor results, with $\alpha$ values indicating $10-15 \%$ difference between the reduced ANSYS and Fedem matrices. This means that if one were to set an acceptance limit of $1 \%$ difference, indicated in the figure, none of the reduced matrices would pass. Despite their poor $\alpha$ results, the reduced matrices are used in the following results, where they are, surprisingly, proven to yield some very good results. The results are further discussed in Section 5.1.

### 4.2 Testing for Consistent Mass in Models

As the full mass matrix is reduced in order to be used by Fedem, a key test for verifying that the reduced mass matrices are reduced correctly is to calculate the total mass of the model from the reduced mass matrix, and compare it to the total mass calculated from the full model. The model's total mass could be found by pre- and post-multiplying the mass matrix with unit translation in the $x, y$ and $z$ direction, similar to what is done in Section 2.4.3:

$$
\begin{equation*}
m_{\text {total }}=\mathbf{U}_{x, \text { red }}^{T} \mathbf{m} \mathbf{U}_{x, \text { red }} \tag{4.1}
\end{equation*}
$$

The calculated total mass from reduced mass matrices in ANSYS is then compared to the calculated total mass from the full mass matrix from Fedem for all models. The full Fedem mass matrix is used as a reference in all calculations. The reference weight is found as an average of resulting weight of the full Fedem model after $x, y$ and $z$ translation. Mass calculated from the reduced mass matrices is also found as an average of $x, y$ and $z$ translation.

|  | Ref. [kg] | Diff. [\%] |
| :--- | :---: | :---: |
| Small | 312.8 | $-2.24 \times 10^{-12}$ |
| Medium | 31.28 | $-1.14 \times 10^{-10}$ |
| Large | 5.184 | $8.22 \times 10^{-10}$ |

Table 4.2: Mass of test models calculated from the reduced mass matrix compared to mass calculated from the full mass matrix

Table 4.2 shows resulting percentage difference of mass calculated from the reduced mass matrix compared to mass calculated from the full mass matrix. For all models, there is close to no difference between the calculated masses. This indicates that the reduction process has been successful.

### 4.3 Calculation of Gravity Vectors

Gravity vectors have been calculated for all three test models and compared to the gravity vector given by Fedem, which is calculated from the full mass matrix. For quantifying the vectors' similarity to the Fedem gravity vector, the relative difference $\alpha$ based on the Euclidean distance (4.1) is used. However, because we are now comparing two vectors, namely $\mathbf{g}_{x}$ from Fedem and ANSYS, the Euclidean distance $d$ is found by summing all entries in the 1 -dimensional vector. The same is done in $y$ and $z$ direction. The gravity vector from Fedem is used as reference for all models.

Both the full and reduced mass matrix from ANSYS have been used for calculating gravity vectors, following the equation set described in Section 2.4.3.


Figure 4.3: Relative difference of the gravity vector from Fedem and ANSYS, calculated with Fedem vectors used as reference

Figure 4.3 presents the resulting differences $\alpha$ between the gravity vectors. Resulting vectors when using the full mass matrix $\mathbf{M}$ are shown in filled black color, while resulting vectors when using the reduced mass matrix $\mathbf{m}$ are shown in un-filled grey. The results are very varied: some gravity vectors are nearly identical to gravity vectors from Fedem, namely the small model's $x$ vector, the medium model's $x$ and $y$ vector and the large model's $z$ and $x$ vector, all from the reduced mass matrix. They have all $\alpha<10^{-11}$. If one were to set $\alpha=0.01$, corresponding to a $1 \%$ difference between ANSYS and Fedem vectors, as an acceptance limit, 12 out of 18 vectors are considered to have acceptable results. The worse result are obtained in the $z$ vector from the medium model. Here, using both the full and reduced mass matrix gives $36.5 \%$ difference between ANSYS and Fedem vectors. Interestingly, using the reduced mass matrix generally gives better results than using the full mass matrix, with 5 vectors having $\alpha<10^{-11}$.

There seem to be no ovious patterns regarding what direction gives the best results: both $x, y$ and $z$ vectors have $\alpha$ values close to the $1 \%$ difference mark, but are also almost identical to the Fedem reference in other models. Surprisingly, the number of nodes in the model seems to have almost no effect on the $\alpha$ values, although the large model gives acceptable results, with $\alpha<3.9 \cdot 10^{-3}$ for all vectors.

### 4.4 Replicating Fedem Gravity Vectors

Due to the varied results when calculating gravity vectors from ANSYS matrices, gravity vectors using $\mathbf{M}$ and $\mathbf{H}$ matrix from Fedem have been calculated in order to replicate the results given by Fedem. The medium model is used for testing, and results are shown in Table 4.3. ${ }^{1}$

|  | $\mathbf{g}_{x}$ | $\mathbf{g}_{y}$ | $\mathbf{g}_{z}$ |
| :--- | :--- | :--- | :--- |
| $\alpha$ | $8.15 \times 10^{-9}$ | $8.94 \times 10^{-9}$ | $3.34 \times 10^{-8}$ |

Table 4.3: Verifying the calculations made in the ModRed extension by replicating gravity vectors from Fedem by using Fedem matrices in all calulations

The results are very close to what Fedem gives, with all vector directions having $\alpha<$ $3.34 \times 10^{-8}$. Even better results could probably be obtained if a higher precision on the exported Fedem matrices were chosen. Here, six decimals were used. These results verify that the calculations performed in the ModRed extension are able to replicate calculations performed by Fedem. Further comments to why the gravity vectors from using ANSYS matrices do not show better results are discussed in Section 5.1.

### 4.5 Generation of FTL Files in ANSYS

For models to be displayed in Fedem, a .ftl file of the model needs to be generated. It defines node coordinates as well as nodal and element connectivity in the model. When the Export button in the ModRed extension is pressed, a . ftl file of the model is generated along with binary .fmx files containing the reduced stiffness, mass and gravity vectors. For the simple twoQUAD4 model, the generated .ftl file is given as

FTL file for the twoQUAD4 model

```
FTLVERSION{4 ASCII}
#
# FTL file generated by ModRed at 2019-06-07 08:59:07
#
#
# Nodal coordinates
#
NODE{1 1 0.0 0.0 0.0}
NODE{2 0 1.0 0.0 0.0}
NODE{3 1 2.0 0.0 0.0}
NODE{4 0 0.0-1.0 0.0}
```

[^2]```
NODE{5 0 1.0-1.0 0.0}
NODE{6 0 2.0-1.0 0.0}
#
# Element definitions
#
QUAD4{15412{PTHICK 1}{PMAT 1}}
QUAD4{26523{PTHICK 1}{PMAT 1}}
#
# Local coordinate systems
#
PCOORDSYS{2 0000011 00}
#
# Material properties
#
PMAT{1 2.07e+11 8.02e+10 0.29 7820}
#
# Shell thicknesses
#
PTHICK{1 0.02}
#
# End of file
```

Currently, dummy values for the coordinate system, material properties and shell thickness is inserted, but should in the future be read from the ANSYS API. A future version of Fedem should also be able to detect that the model is reduced externally, and then use the . fmx files created by ModRed in analyses. This could for example be accomplished by reading the header of the .ftl file. If it says "FTL file generated by ModRed at ...", Fedem should recognize that it is an externally reduced model, and not start its own reducer.

Figure 4.4 shows the twoQUAD4 element opened in Fedem from the ModRed generated FTL file. The external nodes in the top left and right, with node ID 1 and 3, are correctly read and marked with a green triad.

### 4.6 Screencast Demonstration of the ModRed Extension

A video demonstrating the features of ModRed has been made and can be watched at https://youtu.be/-E5c-ZojjsE. It is demonstrated how to import models, the different settings for the extension, as well as how to import the generated .ftl file into Fedem.


Figure 4.4: Visualisation of the twoQUAD4 element in Fedem, read from the ModRed generated FTL file

## Chapter 5

## Discussion and Future Work

### 5.1 Instability of Results

When observing the results obtained from the current version of the ModRed extension, there are some interesting observations to note:

- The full mass matrix $\mathbf{M}$ from ANSYS is shown to have much similiarity with the mass matrix generated by Fedem when the number of nodes is sufficiently high (over $10^{2}$ ).
- The reduced mass matrices from ANSYS show less similarity, from 10 to $15 \%$ difference from Fedem matrices. However, when the same reduced mass matrices are used for calculating the gravity vector, some very good results are obtained - in fact, the gravity vectors from the reduced mass matrix are more similar to what Fedem gives than the gravity vectors calculated from the full mass matrix.
- Testing the mass matrices for mass consistency after partitioning shows us that no element data is lost in the process.
- If using $\mathbf{H}$ and $\mathbf{M}$ from Fedem when calculating gravity vectors, identical results are obtained from the ModRed extension as what is calculated by Fedem, indicating that the ModRed extension is replicating the calculations performed in Fedem.

From this, it seems likely that the reason for the inconsistent results comes from how ANSYS is structuring its mass matrices, and that this is not handled correctly by the ModRed extension. In the following, it is shown that partitioning of an ANSYS mass matrix, with the identical partitioning method as used by a Fedem mass matrix, does not generate an ANSYS mass matrix partitioned identically as the Fedem reference.

Test of Matrix Partitioning

[^3]```
    Testing if Fedem and ANSYS matrices are partitioned correctly.
    Using the partitioned Fedem matrix as reference, as this is proven to yield
    the identical gravity vector as Fedem calculates.
    ""
    dict_fedem = ModRed.readMappingFileFedem(self._resourcesFolder +
"\\twoQUAD4" + "\\\Fedem" + "\\MEQN.res")
    dict_ansys = ModRed.readMappingFile(self. _resourcesFolder + "\\twoQUAD4" +
"\\\Ansys" + "\\M_full_lumped.mapping")
    M_full_fedem = ModRed.readFedemMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Fedem" + "\\\M_full.res")
    M_full_ansys = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Ansys" + "\\\M_full_lumped.mmf", "full")
    eNodes = [1, 3]
    iNodes = [2, 4, 5, 6]
    fedem = ModRed.partitionMatrix(M_full_fedem, eNodes, iNodes, dict_fedem, 6,
    sorting ="equation")
    ansys = ModRed.partitionMatrix(M_full_ansys, eNodes, iNodes, dict_ansys , 6,
    sorting ="equation")
    for row in range(0, ansys.RowCount):
        for col in range(0, ansys.ColumnCount):
            if row==col: # both matrices are diagonal
                self .assertAlmostEqual(fedem[row, col ], ansys[row, col ], places=1)
```

Result of Matrix Partitioning Test

| $\mathrm{r}: 0, \mathrm{c}: 0$, fedem: 39.1, | ansys: 39.1 |
| :---: | :---: |
| $\mathrm{r}: 1, \mathrm{c}: 1$, fedem: 39.1, | ansys: 39.1 |
| $\mathrm{r}: 2$, c: 2, fedem: 39.1, | ansys: 39.1 |
| r: 3, c: 3, fedem: 0.278214, | ansys: 0.00130333333333 |
| r: 4, c: 4, fedem: 0.393552, | ansys: 0.00130333333333 |
| $\mathrm{r}: 5, \mathrm{c}: 5$, fedem: 0.651667, | ansys: $1.30333333333 \mathrm{e}-13$ |
| r: 6, c: 6, fedem: 39.1, | ansys: 39.1 |
| r: 7, c: 7, fedem: 39.1, | ansys: 39.1 |
| $\mathrm{r}: 8, \mathrm{c}: 8$, fedem: 39.1, | ansys: 39.1 |
| $\mathrm{r}: 9, \mathrm{c}: 9$, fedem: 0.278214 , | ansys: 0.00130333333333 |
| $\mathrm{r}: 10, \mathrm{c}: 10$, fedem: 0.393552, | ansys: 0.00130333333333 |
| r: 11, c: 11, fedem: 0.651667, | ansys: $1.30333333333 \mathrm{e}-13$ |
| $\mathrm{r}: 12$, $\mathrm{c}: 12$, fedem: 39.1, | ansys: 78.2 |
| $\mathrm{r}: 13, \mathrm{c}: 13$, fedem: 39.1, | ansys: 78.2 |
| $\mathrm{r}: 14, \mathrm{c}: 14$, fedem: 39.1, | ansys: 78.2 |
| $\mathrm{r}: 15, \mathrm{c}: 15$, fedem: 0.278214 , | ansys: 0.00260666666667 |
| r: 16, c: 16, fedem: 0.393552, | ansys: 0.00260666666667 |
| $\mathrm{r}: 17, \mathrm{c}: 17$, fedem: 0.651667 , | ansys: $2.60666666667 \mathrm{e}-13$ |
| r: 18, c: 18, fedem: 39.1, | ansys: 39.1 |
| $\mathrm{r}: 19, \mathrm{c}: 19$, fedem: 39.1, | ansys: 39.1 |
| r: 20, c: 20, fedem: 39.1, | ansys: 39.1 |
| r: 21, c: 21, fedem: 0.278214, | ansys: 0.00130333333333 |

```
r: 22, c: 22, fedem: 0.393552,
:22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
: 22, c: 22, fedem: 0.393552,
ansys: 0.00130333333333
ansys: 1.30333333333e-13
ansys: 78.2
ansys: 78.2
ansys: 78.2
ansys: 0.00260666666667
ansys: 0.00260666666667
ansys: 2.60666666667e-13
ansys: 39.1
ansys: 39.1
ansys: 39.1
ansys: 0.00130333333333
ansys: 0.00130333333333
ansys: 1.30333333333e-13
```

The resulting printout of every diagonal item in the mass matrix shows that the partitioning of the two matrices is not identical. Using the partitioned Fedem matrix as a reference, one can see on line 13 that the partitioning of the ANSYS matrix can not be correct. The matrix entry should equal what Fedem gives (39.2), but is set to 78.2. This obviously wrong, as the translational DOFs from Fedem and ANSYS are similar (39.1 and 78.2) and should be set to the same position in both matrices. Attempts to resort the partitioned ANSYS matrix from other parameters than what Fedem uses - namely ascending equation number - has not yet proven successful.

The reason this issue is not straight-forward to resolve, is that it is limited what matrices could be exported from Fedem and ANSYS. From Fedem, one can obtain the full mass matrix, the $\mathbf{M}_{e e}$ part of the partitioned matrix and the reduced mass matrix. From ANSYS, you can only obtain the full and reduced mass matrices. This means that the only reference available for how to partition the mass matrix correctly, is $\mathbf{M}_{e e}$ from Fedem. It has therefore been assumed through the whole development process that ANSYS partitions the mass matrix identical to how Fedem does it. However, when it is proven that partitioning is handled differently in ANSYS, there is no $\mathbf{M}_{e e}$ matrix from ANSYS to use as reference, verifying that the partitioning has been done correctly.

This unresolved issue in matrix partitioning is believed to be the cause for the unstable results when calculating gravity vectors. The reason some of the gravity vectors gave very precise results is probably because large portions of the mass matrix is partitioned correctly, as seen in the example. The gravity vectors that gave very good results are therefore believed to have most parts of the mass matrix used for calculating them partitioned correctly.

The issue of matrix partitioning could also explain why the gravity vectors calculated from the reduced mass matrix give better results than using the full mass matrix for this. That is because when using the reduced mass matrix, the issue of matrix partitioning is removed, as the matrix is already partitioned internally by ANSYS before the reduced mass matrix is created from pre- and post-multiplying with $\mathbf{H}$

Due to the unstable results, it is difficult to make a general conclusion for the accuracy
of results obtained from ModRed. However, it is shown that the extension has the capability of producing precise results, with $\alpha<10^{-11}$ for some of the gravity vectors. If the assumption that the root of unstable results lies in handling matrix partitioning, there should not be too much work in resolving the issue. In any way, more testing is needed, as well as some extra functionality discussed next, for the ModRed extension to be used effectively.

### 5.2 Future Work

In the following, suggestions for future development of the ModRed extension are listed. The list is based on experiences from developing the current version of the app, and will hopefully be a helpful reference in the future.

- Extending the elements supported by the ModRed extension should be a priority. Not only will it enable extra functionality for the extension, but will also extend the testing possibilities, thus making it easier to determine its accuracy compared to Fedem.
- When using ANSYS' automatic meshing capabilities, ANSYS is likely to create a mesh containing many different element types for the same model. Shell/membrane elements are automatically used for thin surfaces, different types of solid elements could be used for thicker surfaces, and contact elements are used where ANSYS finds it likely to be a contact point between two surfaces. If the model is to be exported to a . ftl file, the different elements created by ANSYS need to be mapped to a corresponding, similar Fedem element. This is currently not supported, as only SHELL181 elements are handled by the extension. "
- When importing the .ftl files into Fedem, it must be detected that the model has been externally reduced by ANSYS. This could for example be performed by reading the header of the FTL file, which will say FTL file created by ModRed at [Date and time]. If the text is detected, Fedem knows that the model is already reduced.

In the current version of ModRed, the reduced mass matrix is used for calculating the gravity vectors needed by Fedem. Due to this, the calculations can be performed faster, as the $\mathbf{H}$ matrix is no longer needed for calculations, and thus does not have to be read line-for-line into the program. However, a future version of the ModRed extension should probably use the full mass matrix for gravity vector calculations as this should give better results.

- For future development, it is highly recommended to implement the code base in $\mathrm{C} \#$ instead of the current IronPython language. The reasons for this are many:
- Visual Studio, the text editor used for developing the ModRed extension, has a very limited support for the IronPython framework. For example is no autocompletion available, and method headers will normally not appear when calling methods written in the extension. This means that every input parameter
for methods used must be manually checked, resulting in tedious manual work, as well as code that is much more likely to contain errors.
- The ANSYS ACT API can be read by a C\# environment, meaning that every method available through the API will be listed as auto-completion suggestions when handling ANSYS ACT objects. As the available ACT extension examples are relatively sparse, the development process often contains much trial-and-error for figuring out what methods from the API to use. Due to the lack of auto-completion in Visual Studio, all calls to the API must either be investigated through the ACT Console in ANSYS Mechanical, looked up online or in the Developer's Guide. This is very time-consuming, but the added help from the IDE if using C\# would make this much easier.
- The Visual Studio debugger for IronPython is very unstable and is very likely to crash when connected to ANSYS Mechanical for debugging. Debugging the code is therefore often easier to perform with print statements.
- Overriding ANSYS' choice of elements for a mesh is proven to be a more complicated process than imagined. One can add the APDL command

APDL command for overriding ANSYS' choice of mesh elements

```
et, matid, 181
```

However, as this is a command sent directly to the APDL solver, the override of element type is not detected by ANSYS Mechanical, meaning there is no graphical feedback telling the user that the material type has been changed. In addition, the override of element type is not reflected in ANSYS ACT, meaning that internal node and element numbering could be altered at solve time.

- When performing model reduction through the ModRed extension, ANSYS Mechanical tells the user that an unknown error occured. However, the solver output from the APDL solver contains no errors, and all internal APDL solver files are generated. The error message is therefore probably occuring because of two reasons:

1. When defining CMS as the solution type in APDL, we are accessing a solution type that is not supported by ANSYS Mechanical. Thus it is likely that the program defaults to show an error message, because it has no graphical interface for showing the CMS solution type.
2. Many additional files are exported from ANSYS through APDL commands. Additionally, ANSYS moves all solver files to a temporary folder while the APDL solver is running. The export operations are therefore likely to be conflicting with ANSYS' internal file movement, which could be the cause for a default error message.

- In order to obtain 16-decimal precision on the exported mass and stiffness matrix, the full mass and the reduced stiffness and mass matrices are currently exported twice. This is because it is found that 16 -decimal precision matrices are only exported in the Harwell-Boeing (.hbmat) format, accessed through the fileaux2
-> hbmat command. After export to the .hbmat format, the matrices are imported to an APDL Math matrix, and again exported to the Matrix Market Format. This format was found to be much easier to handle than the . hbmat format, because custom import methods for matrices had to be written, as the Math.NET package did not support matrix import in any of the formats. However, this double export is definitely a time- and space-consuming operation, and a method for reading . hbmat matrices should be made. Optionally, the APDL command $\star$ MWRITE should do exactly what we want: writing a matrix to a file in a user-formatted sequence. Unfortunately, it has not been possible to make this command work when writing commands directly to the ds . dat file.


## Chapter 6

## Conclusion

The ANSYS ACT extension ModRed has been developed in order to perform model reduction in ANSYS. By using it, one can access the reduced matrices as well as the needed transformation matrix after model reduction in ANSYS Mechanical. The matrices from ANSYS have been tested for equality against reference matrices from Fedem. The obtained results were varied:

- For large models with more than 100 nodes, the full mass matrices from Fedem and ANSYS are close to identical. However, the reduced mass matrices show overall poorer results, with the comparison method used indicating $10 \%$ to $15 \%$ difference between the reduced ANSYS and Fedem matrices.
- Despite being relatively unequal, the reduced matrices have identical mass to the full matrices, indicating that no information is lost in the reduction process. When calculating gravity vectors the reduced mass matrices surprisingly give overall better results than when using the full mass matrices in calculations.

A possible explanation to the varied results may be found in the way ANSYS structures their mass matrices differently than Fedem. It has been discovered that matrix partitioning in ANSYS is performed differently than what is the case in Fedem. This may explain why the results seem partially inconsistent. Extended testing using a wider range of test models, as well as resolving the issue on matrix partitioning should therefore be performed before one can conclude on the accuracy of results acquired from the extension. Despite the varied results, the extension has proven the potential to become an easy-to-use method for integrating ANSYS solver capabilities in the Fedem software.

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## Appendix

## A

## Source Code for the ModRed ACT Extension

## A. 1 ModRed.py

## ModRed.py

```
import clr
clr .AddReferenceToFileAndPath(r"C:\Program Files \ANSYS
    Inc\v192\Addins\ACT\bin\Win64\MathNet.Numerics.dll") # Using the Math.NET dll.
    Can not use NumPy, as this is not supported by IronPython.
import MathNet.Numerics.LinearAlgebra as la # math library
import os
import subprocess
import datetime # For writing current date in generated FTL file
import ctypes
import cPickle as pickle # For saving object to file
import System
def createUnitVector (pos, nnodes, cmodes=0):
    Creates a vector with unit length at the specified position pos.
    :param pos: Int of position to give unit length. 1=UX, 2=UY, 3=UZ, 4=ROTX,
    5=ROTY, 6=ROTZ
    :param nnodes: Int of number of nodes to create unit length for.
    :param cmodes: Number of component nodes to add unit length
    return matrix of size (( nnodes*6)+cmodes x 1)
    if (pos <= 0):
```

raise Exception("pos argument must be int $>0$ ")
$\mathrm{u}=$ la.Matrix[System.Double].Build.Dense(nnodes*6+cmodes, 1) \# initializing with zeroes
for i in range(pos -1 , nnodes $* 6,6$ ): \# Adding 1 at position pos
$u[i, 0]=1$
for i in range(nnodes $* 6$, nnodes $* 6+$ cmodes): $\quad$ ( Filling component mode
entries with 0s
$u[i, 0]=0$
return $u$

```
def onCreateModRed(analysis):
    Adding the ModRed model object in the model tree
    :param analysis: The currently active analysis
    ""
    if str ( analysis .AnalysisType) == "Modal":
        # Use the analysis to create the APDL Based Result Evaluation result .
        analysis .CreateLoadObject("modred", "ModRed")
    else :
        # Display an error message in the Mechanical message log.
        ExtAPI.Application.LogError("Can only select ModRed for Modal analysis.")
def saveObject(obj, fname):
    Saves the current object to fedemDir_
    testResourcesPath = ExtAPI.ExtensionManager.CurrentExtension. InstallDir +
        "\\test-resources"
    with open(testResourcesPath +"\\" + fname + ".p", "wb") as input:
            pickle.dump(obj, input, pickle.HIGHEST_PROTOCOL)
def onExportData(analysis ):
    :param analysis: Ansys.ACT.Automation.Mechanical.Analysis object.
    - Calculates gravity vectors from reduced mass matrix
    - Creates FTL file for Fedem
    - Creates FMX files from fmxWriter DLL
    - All files are saved in solverData._fedemDir
    ExtAPI.Log.WriteMessage(''Exporting data for Fedem...")
    load = analysis .GetLoadObjects("ModRed")[0]
```

    solverData \(=\) SolverData \((\) analysis , load \() \quad\) \# Creating a SolverData object
    for the current analysis.
    \# Collecting exported matrices in Math.NET format
    M_red \(=\) readMMFMatrix(analysis.WorkingDir + "M_red.mmf", "sub")
    K_red = readMMFMatrix(analysis.WorkingDir + "K_red.mmf", "sub")
    \(\mathrm{G}=\) calculateGravityVectorReduced (M_red, len(solverData . eNodes),
        int (solverData ._cModes)) \# Using reduced mass matrix
    \# Generating FTL file for Fedem
    ftlFile \(=\) open(os.path. join (solverData._fedemDir, solverData._filename \(+"\). ftl"),
    "w") \# Creating a new ftl file
    generateFTL(solverData.getNodes(), solverData.getElements (), solverData._eNodes,
    ftlFile ) \# Generating FTL file to be placed in the solver directory.
    ExtAPI.Log.WriteMessage("FTL file " \(+\mathbf{s t r}\) (solverData. filename) \(+"\). ftl written to
    " + solverData.fedemDir)
    \# Generating FMX file for Fedem
    generateFMX(mathNET2list(G), mathNET2list(M_red), mathNET2list(K_red), solverData)
    def calculateGravityVectorReduced (M_red, nENodes, cmodes):
'"",
Calculates gravity vectors for Fedem from the reduced mass matrix.
:param M_red: The reduced mass matrix .
:param nENodes: Number of selected external nodes
\# Creating reduced unit vectors
$\mathrm{u}_{-} \mathrm{x}=$ createUnitVector (1, nENodes, cmodes)
$u_{-} y=$ createUnitVector ( 2, nENodes, cmodes)
$\mathrm{u}_{\mathrm{z}} \mathrm{z}=$ createUnitVector (3, nENodes, cmodes)
$\mathrm{u}=\mathrm{u}_{-} \mathrm{x} \cdot \operatorname{Append}\left(\mathrm{u}_{-} \mathrm{y}\right) \cdot \operatorname{Append}\left(\mathrm{u}_{-} \mathrm{z}\right)$
G_red = M_red.Multiply (u)
return G_red
def calculateGravityVector (M_full, H, nENodes, nINodes, dofs) :
Calculates gravity vectors for Fedem from the full mass matrix and H matrix.
Follows equations from page 118 in "Virtual Testing of Mechanical Systems"
:param M_full: The full, partitioned mass matrix.
: param H: The H matrix
$\mathrm{H}=$
[[llll $\left.\begin{array}{ll}\text { I } & 0\end{array}\right]$,
[B, phi ] $]$
:param nENodes: Number of selected external nodes
: param nINodes: Number of selected internal nodes

```
:param dofs: Number of degrees of freedom for the element
"",
# Extracting the partitions of the M matrix
    partitions = extractPartitions (M_full, nENodes, dofs)
M_ee = partitions [0]
M_ei = partitions [1]
M_ie = partitions [2]
M_ii = partitions [3]
# Creating unit vectors:
u_x_e = createUnitVector (1, nENodes)
u_x_i = createUnitVector (1, nINodes)
u_y_e = createUnitVector (2, nENodes)
u_y_i = createUnitVector (2, nINodes)
u_z_e = createUnitVector (3, nENodes)
u_z_i = createUnitVector (3, nINodes)
# Gravitational forces:
    G_x_i = M_ii.Multiply(u_x_i).Add(M_ie.Multiply(u_x_e)) # M_i*u_x_i + M_ie *
    u_x_e
    G_x_e = M_ei.Multiply(u_x_i).Add(M_ee.Multiply(u_x_e))
    G_y_i = M_ii.Multiply(u_y_i).Add(M_ie.Multiply(u_y_e))
    G_y_e = M_ei.Multiply(u_y_i ).Add(M_ee.Multiply(u_y_e))
    G_z_i = M_ii.Multiply(u_z_i).Add(M_ie.Multiply(u_z_e))
    G_z_e = M_ei.Multiply(u_z_i ).Add(M_ee.Multiply(u_z_e))
    # Collecting in one matrix:
    G_x = G_x_e.Stack(G_x_i)
    G_y = G_y_e.Stack(G_y_i)
    G_z = G_z_e.Stack(G_z_i)
    # Reducing to unit gravitational acceleration on the full matrix:
    G_x = H.Transpose().Multiply (G_x)
    G_y = H.Transpose() . Multiply (G_y)
    G_z = H.Transpose() . Multiply (G_z)
    # Collecting gravity vectors in one matrix
    G = G_x.Append(G_y).Append(G_z)
    return G
def extractPartitions (mat, numENodes, dofs):
    Extracts the partitions
    M=
    [[M_ee, M_ei],
    [M_ie, M_ii]]
    from the matrix mat
    M_ee of size (numENodes*dofs, numENodes*dofs)
```

    :param mat: Symmetric Math.NET matrix already partitioned into M_ee, M_ei, ...
    :param eNodes: Number of selected external nodes
    :param dofs: Number of degrees of freedom per node
    nRows \(=\) mat.RowCount
    nCols \(=\) mat.ColumnCount
    M_ee \(=\) mat.SubMatrix ( 0 , numENodes*dofs, 0 , numENodes*dofs) \# Top left of M
    M_ei \(=\) mat.SubMatrix \((0\), numENodes \(*\) dofs, numENodes \(*\) dofs, nCols - numENodes \(*\) dofs \()\)
        \# Top right
    M_ie \(=\) mat.SubMatrix (numENodes \(*\) dofs, nRows - numENodes \(* \operatorname{dofs}, 0\),
    numENodes*dofs) \# Bottom left
    M_ii \(=\) mat.SubMatrix(numENodes \(*\) dofs, nRows - numENodes \(*\) dofs, numENodes \(*\) dofs,
    nCols - numENodes*dofs) \# Bottom right
    return [ M_ee, M_ei, M_ie, M_ii ]
    def mathNET2list(matrix):
Returns a Python list of the items in the matrix
:param matrix: Math.NET matrix
return One-dimensional list of items in matrix saved column-first.
list $=[0] *($ matrix.ColumnCount $*$ matrix.RowCount $)$
index $=0$
for col in range( 0 , matrix. ColumnCount):
for row in range( 0 , matrix. RowCount):
list $[$ index $]=$ matrix $[$ row, col $]$
index $+=1$
return list
def onSolve(load, stream):
Adds the required APDL commands to the solver input (ds.dat) file. Activates when
the user hits "Solve"
:param load: the load associated to the callback. Interface IUserLoad
:param stream: a System.IO. StringWriter object, to which solver commands should be
appended (represents the ds.dat file )
\# Collecting user input:
nodeSelectionMethod = load. Properties ["Generation"]. Properties ["Geometry"].Value
method = load. Properties ["Method"]. Properties ["ReductionMethod"].Value
filename $=$
load. Properties ["Method"]. Properties ["ReductionMethod"]. Properties ["Name"].Value
\# string
nmode $=$
load. Properties ["Method"]. Properties ["ReductionMethod"]. Properties ["Nmode"].Value
\# string
isUseLumped $=$
load. Properties ["Method"]. Properties ["ReductionMethod"]. Properties ["LumpedMatrix"].Value
\# string
eNodes $=[]$
if (nodeSelectionMethod=="'Geometry"):
eNodes $=$ getENodes(load) \# List [int]
if (nodeSelectionMethod=="RBE2 Import"):
eNodes $=$ getRBE2Nodes(load)
isExport $=$ load. Properties ["Export"]. Properties ["IsExport"]. Value
if (isExport == "Yes"):
isExport $=$ True
else :
isExport $=$ False
stiffnessName = "Kmat"
massName $=$ " Mmat"
matrixFormat $=$ "MMF" \# Set this to ". matrix" for lower precision, but more
readable matrices
\#
\# Writing solver commands to the ds. dat file
\#
stream. WriteLine(" finish ")
stream.WriteLine("/filname, " + filename) \# Name of the super element file to
be generated
stream. WriteLine("save")
stream. WriteLine("/solu")
stream.WriteLine("antype, substr") \# Defining a substructure analysis type
(includes both Guyan and CMS)
if (method $==$ "CMS"): \# If CMSOPT needs to be set
cmsmeth $=$ "fix"
nmode $=\mathbf{s t r}($ nmode $)$
freqb $=" "$
freqe $=" "$
fbddef $=" "$
fbdval $=$ ""
iokey = "tcms"
if (iokey=="tcms"):
stream. WriteLine("outpr, nsol, all") \# Print command that must be
defined if iokey==tcms
stream. WriteLine $("$ cmsopt, $"+$ cmsmeth $+", "+$ nmode $+", "+$ freqb $+", "+$ freqe
$+", "+$ fbddef $+", "+$ fbdval $+", "+$ iokey $)$
sename $=$ filename \# name of superelement matrix file
sematr $=\boldsymbol{\operatorname { s t r }}(2) \quad$ \# generate $\quad$ stiffness and mass matrices
sepr $=\boldsymbol{\operatorname { s t r }}(0) \quad$ \# Do not print superelement matrices or load vectors
sesst $=\boldsymbol{\operatorname { s t r }}(0) \quad$ \# Do not save space for stress stiffening
stream. WriteLine("seopt, " + sename $+", "+$ sematr $+", "+$ sepr $+", "+$ sesst +
", ,")
for nodeID in eNodes:
stream. WriteLine(" $m, "+\boldsymbol{s t r}$ (nodeID) $+"$, all")
if (isUseLumped == "Yes"):
stream.WriteLine("lumpm, on") \# Using a lumped mass matrix
stream. WriteLine(" alls")
stream. WriteLine("solve")
stream. WriteLine("save")
stream. WriteLine(" finish ")
stream. WriteLine("! Master node selection finished")
if (isExport): \# Exporting matrices if selected
stream.WriteLine("! Exporting matrices ... ")
\# Creating H matrix. $\mathrm{H}=$ [CST NOR]
stream.WriteLine(" $*$ dmat, cst, D, import, tcms, $"+$ filename $+"$. tcms, cst")
\# Constraint mode data
stream.WriteLine(" $*$ dmat, nor, D, import, tcms, " + filename + ".tcms, nor")
\# Fixed-interface normal mode data
stream. WriteLine("save")
if (matrixFormat =="MMF"):
stream.WriteLine("*export, cst, MMF, CST.mmf") \# Exporting
constraint modes
stream.WriteLine("*export, nor, MMF, NOR.mmf") \# Exporting
fixed - interface normal mode data
stream.WriteLine("*smat, nod2solv, D, import, full ," + filename + ". full,
nod2solv") \# Importing the mapping vector internal $\rightarrow$ solver ordering
stream.WriteLine("*vec, mapback, I, import, full, " + filename + ". full,
back") \# Importing the BACK nodal mapping vector for external $->$ internal
ordering.
stream.WriteLine("/aux2") \# Manipulating binary files
stream. WriteLine("FILEAUX2," + filename + ", sub") \# Specifying
to dump reduced matrices file
stream.WriteLine("HBMAT, M_red, hbmat, , ASCII, MASS, NO") \#
Dumping reduced mass matrix to Harwell-Boeing format for 16-decimal precision
stream. WriteLine("HBMAT, K_red, hbmat, , ASCII, STIFF, NO") \# Dumping
reduced stiffness matrix
stream. WriteLine("FILEAUX2," + filename + ", full") \# Specifying to
dump full mass matrix
stream.WriteLine("HBMAT, M_full, hbmat, , ASCII, MASS, NO, YES") \#
Dumping full mass matrix in Harwell-Boeing format
stream. WriteLine(" finish ")
stream.WriteLine("*smat, M_red, D, import, HBMAT, M_red.hbmat, ASCII") \#
Creating sparse matrix from Harwell-Boeing format. Preserving 16-decimal precision
stream.WriteLine("*smat, M_full, D, import, HBMAT, M_full.hbmat, ASCII")
stream.WriteLine("*smat, K_red, D, import, HBMAT, K_red.hbmat, ASCII")

```
            stream.WriteLine("*export, M_red, MMF, M_red.mmf") # Exporting
        to Matrix Market Format. Preserves 16-decimal precision
            stream.WriteLine("*export,M_full,MMF, M_full.mmf")
            stream.WriteLine("*export, K_red, MMF, K_red.mmf")
        if (matrixFormat == ". matrix"):
            stream.WriteLine("*PRINT, Kmat, " + stiffnessName + ". matrix") #
        Printing matrix directly in easy-to-read format
            stream.WriteLine("*PRINT, Mmat, " + massName + ".matrix")
        stream.WriteLine("save")
    stream.WriteLine("/eof") # Quits correctly
def generateFMX(G, M_red, K_red, solverData):
    This will save FMX files of G, M_red and K_red at the solver directory.
    :param G: One-dimensional list of gravitational vectors saved column-first.
    :param M_red: One-dimensional list of reduced mass matrix saved column-first.
    :param K_red: One-dimensional list of reduced stiffness matrix saved column-first.
    :param solverData:SolverData object.
    # Converting to double slashes and in bytes representation
    file = solverData._fedemDir + "\\" + solverData. _filename
    file = file . replace ("\\",""\\\\")
    file = bytes( file ,' utf -8')
    # Skriver stivhetsmatrise
    status = writeFMX(file, 1, K_red)
    # Skriver massematrise
    status = writeFMX(file, 2, M_red)
    # Skriver gravitasjonskrefter
    status = writeFMX(file, 3, G)
    ExtAPI.Log.WriteMessage("FMX files written to " + solverData.fedemDir)
def writeFMX(file, ityp, data):
    Writes a square matrix as a binary FMX - file for FEDEM, using the fmxWriter DLL
    arg file : Full path to the fmx-file to be written
```

    arg ityp : 1= stiffness matrix, 2=mass matrix, 3=gravity force vectors
    arg data : Matrix content, column-wise storage
    return : Zero on success, otherwise negative
    "",
    \# Loading DLL for writing FMX files:
    extensionDir = ExtAPI.ExtensionManager.CurrentExtension. InstallDir
    dll \(=\) ctypes.cdll .LoadLibrary(extensionDir + " \(\backslash \backslash\) fmxWriter \(\backslash \backslash \mathrm{fmxWriter} . \mathrm{dll} ")\)
    cfil \(=\) file
    ctyp \(=\) ctypes. c_int (ityp)
    cdat \(=(\) ctypes.c_double \(* \operatorname{len}(\) data \())()\)
    cdat [:] = data
    clen \(=\) ctypes. c_int (len(data))
    return dll.WRITEFMX(cfil, ctypes.byref(ctyp), cdat, ctypes.byref(clen), len( file ))
    def getENodes(load):
Returning list with node numbers of selected external nodes
:param load: The current load object
: return List[int] Sorted list with integers representing node numbers of current
selected nodes.
selectedIDs $=$
load. Properties ["Generation"]. Properties ["Geometry"]. Properties ["Geometry"].Value.Ids
return sorted ( selectedIDs ) \# List [int ]
def generateFTL(nodes, elements, eNodes, file ):
Generates the FTL file for the current model to be used by Fedem.
The FTL file is placed in the solverData.fedemDir_
:param nodes: list [Node] of all nodes in the model. Sorted ascending by id
:param elements: list [Element] of all elements in the model. Sorted ascending by id
:param eNodes: list [int] of ids of external nodes
:param file : FTL file object
\#
\# Writing file header
\#
date $=$ datetime. datetime. now(). strftime ("\%Y-\%m-\%d \%H:\%M:\%S")
file . write ("FTLVERSION $\{4$ ASCII $\} \backslash \mathrm{n} "$ )
string $=" \# \backslash n \#$ FTL file generated by ModRed at " $+\boldsymbol{s t r}$ (date) $+" \backslash n \# \backslash n \backslash n "$
file . write ( string )
string $=" \# \backslash \mathrm{n} \#$ Nodal coordinates $\backslash \mathrm{n} \# "$
file . write ( string $+" \backslash \mathrm{n} "$ )
\#

```
# Writing NODE part
#
for node in nodes:
        nodeString = getNodeString(node, eNodes)
        file . write (nodeString + "\n")
    #
    # Writing element definitions part
    #
    thicknessId = 1
    materialId = 1
    file . write("#\n# Element definitions \n#\n")
    for element in elements:
        elString = getElementString(element, thicknessId, materialId)
        file . write ( elString + "\n")
    #
    # Writing coordinate system part
    #
    file . write("#\n# Local coordinate systems\n#\n")
    file . write('"PCOORDSYS{20000011100}" + "\n")
    #
    # Writing material properties part
    #
    file . write ("#\n# Material properties \n#\n")
    file . write ("PMAT{1 2.07e+11 8.02e+10 0.29 7820}" + "\n")) # Dummy material
    #
    # Writing shell thicknesses part
    #
    file . write ("#\n# Shell thicknesses \n#\n")
    file . write (''PTHICK{1 0.02}" + "\n")
    #
    # End of file
    #
    file . write ("#\n# End of file ")
    file . close () # Closing the created FTL file
def getNodeString(node, eNodes):
    Returns node string to write in FTL file.
    :param id: id of node to write to string
    :param nodes: sorted list [Node] of all nodes in the model. Sorted by node id
    :param eNodes: list [int] of ids of selected external nodes
    : return string of NODE part to be written in FTL file
    ,,",
```

```
    string = "NODE{"
    string += str(node.id)
    string +=""
    if (node. id in eNodes):
        string +="1"
    else :
        string += "0"
    string +=""
    string += str(node._x) +"" + str(node..y) +"" + str (node._z)
    string += "}"
    return string
def getElementString(element, thicknessId, materialId):
    Returns string of ELEMENT part to be written in FTL file
    :param element: Element object of the element to write
    :param thicknessID: id of thickness PTHICK
    :param materialID: id of material PMAT
    string = "QUAD4{"
    string += str(element. _id) +""
    string += str(element._nodes [0]. _id) + ""
    string += str(element._nodes [1]. _id) + ""
    string += str(element._nodes [2]. _id ) + " "
    string += str(element._nodes [3]. _id) + " "
    string +="{PTHICK"
    string += str(thicknessId) + "}"
    string +=" "
    string +="{PMAT"
    string += str(materialId ) + "}"
    string +="}"
    return string
def isValidNoModes(entity, property):
    Returns true if valid input to component modes property. False if not.
    :param entity: IUserObject. The entity containing the property.
    :param property: ISimProperty. The property to check.
    :return True if property is positive int. False if not.
    try:
        #ExtAPI.Log.WriteMessage(str(int (property.Value) >= 0))
```

```
        return int(property.Value) >=0
    except ValueError:
        #ExtAPI.Log.WriteMessage(str(False))
        return False
def readMappingFileFedem(path):
    Reads a Fedem mapping file that maps degree of freedom to equation.
    The MEQN file is structured
    column 0: dof column 1: equation
    1 25
    2 26
    3 27
    This means dof 1 equals equation }25
    :param path: File path of MEQN.res file from Fedem
    : return Dictionary on the form
    dict = {
    "dof": equation
    },,,,
    file = open(path, "r")
    line = file .readline () # Reading header line
    dict ={}
    lines = file . readlines ()
    for line in lines:
        line = line. split ()
        dof = int (line [0])
        eq = int(line [1])
        dict[dof] = eq
    return dict
def readMappingFile(path):
    Reads an ANSYS .mapping file that maps equation to corresponding node.
    The .mapping file is structured :
    column 0: equation column 1: node column 2: dof
    Example:
    1 5 UX
    2 5 UY
    3 5 UZ
    This means equation 1 is related to node 5, dof UX
    Relation node }->\mathrm{ dof is
    node 1, UX => dof 1
    node 1, UY => dof 2
```

    node 1 , UZ \(\quad=>\operatorname{dof} 3\)
    node 2 , ROTX \(=>\operatorname{dof} 4 \ldots\)
    node 2, UX \(\quad=>\operatorname{dof} 7\)
    :param path: File path of ".mapping" file
    :param dofDict: Dictionary relating internal dof for a node to it corresponding
    numbering.
    Example: " \({ }^{\prime} X^{\prime}=1, " U Y "=2, " U Z "=3, " R O T X "=4\)
    : return Dictionary
    dict \(=\{\)
        "dof": "equation"
    \}
    where "dof" is in ascending order from node numbers, i.e .:
    dof \(=1=>\) Node 1, degree of freedom nr 1 (UX)
    dof \(=2=>\) Node 1, degree of freedom nr 2 (UY)
    dof \(=7=>\) Node 2, degree of freedom nr 1 (UX)
    dof \(=12=>\) Node 2, degree of freedom nr 6 (ROTZ)
    "",
    dofDict \(=\{\)
        "UX": 1 ,
        "UY": 2,
        "UZ": 3,
        "ROTX": 4,
        "ROTY": 5,
        "ROTZ": 6
    \}
    dict \(=\{ \}\)
    file \(=\) open(path, "r")
    line \(=\) file. readline ()\(\quad\) \# Skipping first line
    lines \(=\) file . readlines ()
    for line in lines:
        line \(=\) line. \(\operatorname{split}()\)
        \(\mathrm{eq}=\operatorname{int}(\) line [0])
        node \(=\operatorname{int}(\) line [1] \()\)
        dofText \(=\) line [2]
        dof \(=\) node \(2 \operatorname{dof}(\) node, dofDict, 6 , dofText \# Currently hard-typing in
    6 dofs
        \(\operatorname{dict}[\mathrm{dof}]=\mathrm{eq}\)
    return dict
    def node2dof(node, dofDict, dofs, dofText):
Relates node number to dof number.
:param node: int of node number
:param dofs: Number of dofs for the node
:param dofText: string of dof in ANSYS .mapping file. Ex.: "UX", "UY",
",",
$\operatorname{dof}=($ node -1$) * \operatorname{dofs}+\operatorname{dofDict}[\operatorname{dofText}]$
return dof
def nodes2Dofs(nodes, dofs) :
\# Relation dof $->$ node is
$\#$ dof $=($ node -1$) *$ dofs +a
\# where $\mathrm{a}=[1$, dofs $]$
\#
\# Example:
\# nodes $=[1,5,10]$ in elements with 6 dofs
$\#=>$ returnDofs $=[1,2,3,4,5,6,25,26,27,28,29,30,55,56,57,58,59,60]$
\#
\# Example 2:
\# nodes $=[1,3]$
\# dofs $=2$
$\#=>$ returnDofs $=[1,2,5,6]$
",",
returnDofs $=$ []
for node in nodes:
partDofs $=\operatorname{range}(($ node -1$) * \operatorname{dofs}+1,($ node -1$) * \operatorname{dofs}+(\operatorname{dofs}+1))$
returnDofs += partDofs
return returnDofs
def dofs2Eqs(dofs, dict):
Returns a list with the corresponding equation for a node.
\# Example:
\# dofs $=[1,2,3]$
\# dict $=\{$
1: 2,
2: 3,
3: 1
\}
returns $[2,3,1]$
:param dofs: List of dofs. 1-indexed. Length $n$
:param dict: Dictionary coupling dof to equation like
$\operatorname{dict}[\operatorname{dof}]=$ eq
:param return: List of equations for the specified dofs. Length n. 1-indexed.
"",
eqs $=[]$
for dof in dofs:
$\mathrm{eq}=\boldsymbol{\operatorname { d i c t }}[\mathrm{dof}]$
eqs.append(eq)
return eqs
def partitionMatrix (mat, eNodes, iNodes, dict, dofs, sorting ="equation"):
Creates a partitioned matrix on the form
M =
[[Mee, Mei],
[Mii, Mii]]
:param mat: The Math.NET matrix to partition
:param eNodes: list [int] of the selected external nodes. Indexing starting at 1.
Sorted.
:param iNodes: list [int] of internal nodes. 1-indexed. Sorted.
:param dict: Dictionary with the mapping of DOF to equation number in the matrix.
Example: dict [dof] = eq
NOTE: dof and equation in the dictionary is 1 -indexed!
: param DOFs: Number of degrees of freedom per node.
:param sorting: How to sort the partitioned matrix.
sorting ="equation" means the partitioned matrix is sorted after ascending equation
numbers. This is what Fedem uses.
This is the default selection.
sorting $=$ "dof" means the partitioned matrix is sorted after ascending degree of
freedom numbers.
: return partitioned matrix
,,,
\# Getting the dof numbering for the external (e) and internal (i) nodes
\# Relation dof $->$ node is
\# dof $=($ node -1$) *$ dofs +a
\# where $\mathrm{a}=[1$, dofs $]$
\# Example:
\# eNodes $=[1,5,10]$ in elements with 6 dofs
$\#=>$ eDofs $=[1,2,3,4,5,6,31,32,33,34,35,36,61,62,63,64,65,65]$
eDofs $=$ nodes 2 Dofs (eNodes, dofs $)$
iDofs $=$ nodes 2 Dofs(iNodes, dofs $)$
\# Mapping to solver ordering :
eEqs $=$ dofs2Eqs $($ eDofs, dict $) \quad$ \# equations corresponding to external
nodes. 1-indexed.
iEqs $=\operatorname{dofs} 2 E q s(i D o f s$, dict $) \quad \#$ equations corresponding to internal
nodes. 1-indexed.
if ( sorting $==$ "equation"): \#eEqs and iEqs are already sorted by dof number.
eEqs $=\boldsymbol{\operatorname { s o r t e d }}(\mathrm{eEqs})$
iEqs $=\boldsymbol{\operatorname { s o r t e d }}(\mathrm{iEqs})$
\# Initializing partitioned matrix

```
    nRows \(=(\operatorname{len}(\mathrm{eNodes})+\boldsymbol{l e n}(\mathrm{iNodes})) *\) dofs
    nCols \(=\) nRows
    matPart \(=\) la.Double.SparseMatrix(nRows, nCols) \# Initializing with zeroes
    \# Moving rows
    row \(=0\)
    for e in eEqs:
        matPart.SetRow(row, mat.Row(e-1))
        row+=1
    for i in iEq :
        matPart.SetRow(row, mat.Row(i-1))
        row+=1
    matCopy \(=\) la.Double.SparseMatrix(nRows, \(n\) Cols) \# Initializing with zeroes
    matPart.CopyTo(matCopy) \# Using a copy because values are accesed by reference.
    \# Moving columns
    \(\mathrm{col}=0\)
    for e in eEqs:
        matPart.SetColumn(col, matCopy.Column(e-1))
        \(\mathrm{col}+=1\)
    for i in iEqs :
        matPart.SetColumn(col, matCopy.Column(i-1))
        \(\mathrm{col}+=1\)
    return matPart
```

def readFedemMatrix(filename):
Reading a Fedem matrix to MathNETmatrix for easier comparison with ANSYS matrices.
Written to import the full mass and stiffness matrix from Fedem
:param filename: Path to matrix.
: return a sparse Math.NET matrix with contents of the matrix in path
file $=$ open(filename, " $r "$ ) \# Reading data
line $=$ file. readline ()
line $=$ line. split ()
nRows $=$ int $($ line [0]) \# Number of columns and rows are stored in the
first line after the comments
nCols $=$ int (line [1])
mat $=$ la. Double.SparseMatrix(nRows, nCols) \# Initializing with zeros
row $=0$
$\mathrm{col}=0$
content $=$ file. readlines ()
for line in range $(0$, len (content)):
splitted $=$ content [line ]. split ()
if (len( splitted) $!=0$ ):
if ( content [ line ]. split () [0]. startswith ("+++")):
row $=\operatorname{int}($ content $[$ line ]. split () [2]) -1
colLine $=$ content $[$ line +1$]$. split ()
valLine $=$ content $[$ line +2$]$. split ()
$\operatorname{col}=\boldsymbol{\operatorname { i n t }}(\operatorname{colLine}[0])-1$
val $=$ float $($ valLine [0])
$\operatorname{mat}[$ row, col $]=$ val
file . close ()
return mat
def readFedemVector(path):
Reads vector from fedem_reducer. res file for comparison with vectors from ANSYS
Reads gravity vectors, as well as BMAT.res and PHI.res
:param path: Path to Fedem vector to read.
: return Sparse math.NET matrix of the Fedem vector in path
"""
file $=$ open(path, " $r$ ")
lines $=$ file . readlines ()
firstLine $=$ lines [0]. split ()
nRows $=\operatorname{int}($ firstLine [0])
nCols $=\operatorname{int}($ firstLine [1])
mat $=$ la.Double.SparseMatrix(nRows, nCols) \# Initializing with zeros
colIndexes $=[]$
nextNumCols $=0$
currNumCols $=0$
row $=0$
$\operatorname{col}=0$
startCol $=0$
for in inge( 0 , len( lines )):
col $=$ startCol
line $=$ lines [i]. split ()
if $(i \quad!=(\operatorname{len}($ lines $)-1))$ : $\quad \#$ if not at end of file
nextNumCols $=\operatorname{len}($ lines $[i+1]$. split ()$)$
currNumCols $=$ len (line $)$
if (currNumCols > nextNumCols): \# if new columns
startCol $=\operatorname{int}($ lines $[i+1]$. split ()$[0])-1$
continue
row $=\operatorname{int}($ line [0]) -1
vals $=$ list $(\operatorname{map}($ float, line $))[1:]$
for val in vals:
$\operatorname{mat}[$ row, col $]=\mathrm{val}$
col $+=1$
file . close ()
return mat
def readMMFMatrix(filename, format):

Reading a dense or sparse Matrix Market file format into a Math.NET array. NOTE:

- The matrices are saved column first!
- Matrices created from the .SUB binary ANSYS file are saved with the full ( $\mathrm{n} \times \mathrm{n}$ ) matrix.

This is the case for M_red and K_red files

- Matrices created from the .FULL binary ANSYS file have only the lower triangular part of the matrix saved.

```
:param filename: String of file name to the .mmf file. Dense .mmf matrices are
    saved column-first!
    :param format: Format of the MMF matrix. "sub" for the reduced matrices . " full" for
    the full matrices.
    : return Dense Math.NET matrix read from filename. Saved row-first .
    isFirstLine = False
    nRows = 0
    nCols = 0
    file =open(filename, "r") # Reading data
    #
    # Reading the lines defining dimensions of the matrix
    #
    dimLine = None
    while ( isFirstLine == False):
        line = file . readline ()
        if (not line . startswith ('%')):
            isFirstLine = True
            line = line . split ()
            nRows = int(line [0]) # Number of columns and rows are stored in
```

    the first line after the comments
            nCols \(=\operatorname{int}(\) line [1])
            dimLine \(=\) line \(\quad\) \# Saving for use in the next for loop
    \# Initializing Math.NET matrix
    mat = la. Double.SparseMatrix (nRows, nCols)
    isDenseFull = False
    isSparseFull = False
    isSparseSub = False
    if \((\) len \((\) dimLine \()==2\) and format=="full"):
        isDenseFull = True
    if \((\) len \((\) dimLine \()==3\) and format=="full"):
        isSparseFull \(=\) True
    if \((\) len (dimLine) \(==3\) and format=="sub"):
        isSparseSub = True
    \# Filling Math.NET matrix with values from the file
    row \(=0\)
    ```
    col = 0
    for line in file :
        if (isDenseFull): # If we are reading a dense MMF matrix from ANSYS
    SUB or full file
                val = float (line)
                mat[row, col] = val
                if (row != nRows - 1 ): # Updating counting variables
                row += 1
        else :
            col += 1
                row = 0
        if (isSparseSub or isSparseFull ): # We are reading a sparse MMF matrix
    from ANSYS FULL file
        line = line. split ()
        i = int(line [0]) - 1 # Reading indexes. Is 1-indexed in the
    MMF file
        j = int (line [1]) - 1
        val = float (line [2])
        mat[i, j] = val
        mat[j, i] = val # sparse MMF matrix is saved in lower
        triangular form
    file . close ()
    return mat
def getRBE2Nodes(load):
    Imports RBE2 nodes from NASTRAN file and returns list node ids of RBE2 nodes.
    "CompID" is the string from the Workbench site, when pressing on external model,
        under "General" tab. Typically "Setup X"
    " Identifier" is the string under identifier tab when double-clicking the external
    model. Typically "File1".
    :param load: The current load object
    : return List[int] Sorted list with integer nodal ids of found RBE2 nodes in
        NASTRAN file.
    compID =
        load. Properties ["Generation"]. Properties ["Geometry"]. Properties ["CompID"].Value
        identifier =
        load. Properties ["Generation"]. Properties ["Geometry"]. Properties [" Identifier "]. Value
    commands = ExtAPI.DataModel.Project.Model.GetFECommandsRepository(compID,
        identifier)
    rbe2Comms = commands.GetCommandsByName('RBE2")
    rbe2Count = rbe2Comms.Count
    nodeList = []
    for i in range (0, rbe2Count):
```

```
        cmd = rbe2Comms[i+1]
        nodeList.append(int(cmd.GetArgument[2])) # Node ID is on argument index
        two. Convert to int.
        return sorted(nodeList)
class Element():
    Class used for storing info on elements in a mesh
    def __init_- ( self, id, nodes):
        self ._nodes = nodes # list [Node] connected to the element
        self . id = id # Element id
    def getNodes(self ):
        return self._nodes
    def getId( self ):
        return self.id
class Node:
    Data storage class for one node from a mesh
    def __init__ ( self, id, dofs, elements, x, y, z):
        :param id: Int of nodal id
        :param dofs: Int of number of dofs for the node
        :param elements: List [int] of element ids to the elements connected to the node
        :param x: x coordinate
        :param y: y coordinate
        :param z: z coordinate
        self . id = id
        self . _dofs = dofs
        self._elements = elements
        self._x = x
        self._y = y
        self._z = z
    def getId ():
        return self._id
```

```
def getDofs():
    return self ._dofs
    def getElements():
    return self ._elements
class SolverData:
    #
    # Class used to store data from the solver and ModRed application.
    #
    def __init_- ( self, analysis, load):
        #
        # Collecting user input
        #
        self . _dofs = 6 # Currently hard-typing in number of dofs
        self . filename =
    load. Properties ["Method"]. Properties ["ReductionMethod"]. Properties ["Name"].Value
    # string
        self ._workingDir = analysis .WorkingDir
        self ._fedemDir =
    load. Properties ["Export"]. Properties ["IsExport"]. Properties ["FileDir"]. Value
        self . _testResourcesDir = ExtAPI.ExtensionManager.CurrentExtension. InstallDir +
    "\\test-resources"
        self ._eNodes = getENodes(load) # List[int ] of selected external nodes.
    Sorted
        self ._cModes =
    int(load. Properties ["Method"]. Properties ['ReductionMethod"]. Properties ["Nmode"].Value)
        # Number of component modes
        self ._elements = self .getElementsFromMesh(analysis.MeshData)
    #List [Element]
        self ._nodes = self .getNodesFromElements(self. _elements) # list [Node]
        self ._nRows = len( self . nodes) * self . _dofs # Number of rows in the full
    mass matrix
    self ._nCols = self .nnRows # Quadratic matrix
    def saveObject( self, obj, fname):
    Saves the current object to _fedemDir
    with open(self . _testResourcesDir + "\\" + fname + ".p", "wb") as file :
        pickle.dump(obj, file , pickle.HIGHEST_PROTOCOL)
    def getNodes(self ):
```

```
        Return sort list [Node] of nodes in the model. Sorted ascending by id
        ","
        return sorted(self ._nodes, key=lambda x: x._id)
def getElements( self ):
    Return sorted list [Element] of elements in the model. Sorted ascending by id
    return sorted( self . _elements, key=lambda x: x._id)
def getNodesFromElements(self, elements):
    Creates list [Node] of nodes connected to the element.
    :param elements: List[Element]
    : return List[Node] with all nodes in the model
    ",
    nodes = []
    for e in elements:
        for n in e.getNodes():
            if not any(x. _id == n. _id for }\textrm{x}\mathrm{ in nodes):
            nodes.append(n)
    return nodes
def getNodeIds(self ):
    : return Sorted list [int] of all node ids in the model
    "",
    ids = []
    for node in self._nodes:
        ids.append(node._id)
    return sorted(ids)
def getElementsFromMesh(self, mesh):
    Returns list of custom Element objects containing all elements in the model.
    :param mesh: ANSYS MeshData object
    :return List[Element]
    ","
    returnElements = []
    ansysElements = mesh.Elements
    for e in ansysElements:
        id = e.Id
        nodeList = []
```

```
        for node in e.Nodes:
            nodeId = node.Id
            elements = node.ConnectedElementIds #List[int] of ids of
        the connected elements
            node = Node(nodeId, 6, elements, node.X, node.Y, node.Z) #
    Currently hard-typing in dofs per node. Should be changed to be read from ANSYS
    API
            nodeList.append(node)
            element = Element(id, nodeList)
            returnElements .append(element)
        return returnElements
class Results:
    Class for calculating results from the model reduction extension
    ",
    def __nit_- (self):
        self. resourcesFolder = os.path.dirname(os.path.realpath ( __file_- )) +
        "\\ test -resources\\"
    def quadraticSumDifference( self, mat, refMat):
        Quadratic sum of the difference of all elements in n x n matrix
        :param mat: Matrix that will be compared
        :param refMat: Matrix used as reference for comparison
        : return percentage difference on quadratic sum of mat, compared to refMat
        matSum = self.matrixSum(mat)
        refMatSum = self .matrixSum(refMat)
        squared = (matSum - refMatSum )}*(\mathrm{ matSum - refMatSum)
        diff = math.sqrt (squared)
        percentage = diff /refMatSum
        return percentage
    def quadraticSumDifferenceOfVector(self, vec, refVec):
    Quadratic sum of the difference of two vectors of size (n x 1)
        :param : Vector that will be compared
        :param refVec: Vector used as reference for comparison
        : return percentage difference on quadratic sum of vec, compared to refVec
        ","
        vecSum = vec.Sum()
```

        refVecSum \(=\) refVec.. \(\operatorname{Sum}()\)
        squared \(=(\) vecSum \(-\operatorname{refVecSum}) *(\) vecSum \(-\operatorname{refVecSum})\)
        diff \(=\) math. sqrt (squared)
        percentage \(=\) diff \(/\) refVecSum
        return percentage
    def matrixSum(self, mat):
        Calculating the matrix sum of a mat.NET matrix
        :param mat: The matrix to calculate the sum of
        : return float Sum of all elements in the matrix
        ", "
        rowSums = mat.RowSums()
        return sum(rowSums)
    def calculateMassFromMassMatrix(self, mat, reduced=True, cmodes=2):
        Calculates the mass of a mass matrix mat by applying unit translation :
    m_tot \(=\mathrm{u}^{\wedge} \mathrm{T} *\) mat \(* \mathrm{u}\)
    :param mat: The mass matrix
    :param reduced: Indicating if it is a reduced mass matrix. Defaults to True.
    : param cmodes: Number of component modes in the reduced mass matrix. Defaults
    to 2
    if (reduced==True):
        nRows \(=\) mat.RowCount
        nNodes \(=\mathbf{i n t}((\) nRows - cmodes \() / 6) \quad\) \# Currently assuming 6 DOFs per
    node
        \(\mathrm{u}_{-} \mathrm{x}=\) ModRed.createUnitVector(1, nNodes, cmodes)
        \(\mathrm{u}_{-} \mathrm{y}=\) ModRed.createUnitVector (2, nNodes, cmodes)
        \(\mathrm{u}_{\mathrm{z}}=\) ModRed.createUnitVector(3, nNodes, cmodes)
        mass_x \(=u_{-}\)x. Transpose () . Multiply (mat). Multiply (u_x) [0,0]
        mass_y \(=u_{-} y . \operatorname{Transpose}() \cdot\) Multiply (mat). Multiply (u_y) [0,0]
        mass_z \(=u_{\text {_ }}\). Transpose () . Multiply (mat). Multiply (u_z) [0,0]
        return [mass_x, mass_y, mass_z]
    else :
        nRows \(=\) mat. RowCount
        nNodes \(=\boldsymbol{i n t}(\) nRows/6) \(\quad \#\) currently assuming 6 dofs per node
        \(u_{\_} \mathrm{x}=\) ModRed.createUnitVector (1, nNodes)
        \(u_{-} \mathrm{y}=\) ModRed.createUnitVector(2, nNodes)
        \(u_{\_} \mathrm{z}=\) ModRed.createUnitVector(3, nNodes)
        mass_x \(=u_{-} x\). Transpose ()\(\cdot \operatorname{Multiply}(m a t) \cdot \operatorname{Multiply}\left(u_{-} x\right)[0,0]\)
        mass_y \(=u_{-} y . \operatorname{Transpose}() \cdot\) Multiply (mat).Multiply (u_y) [0,0]
        mass_z \(=u_{-}\)z. Transpose ()\(\cdot\) Multiply (mat). Multiply (u_z) [0,0]
        return [mass_x, mass_y, mass_z]
    
## A. 2 ModRed.xml

ModRed.xml

```
<extension version="1" name="ModRed">
    \(<!--\) This is the GUID for the ModRed extension. It must remain the same independently
        \(\hookrightarrow\) of
    the extension version or name. It is used to uniquely identify this extension. The
        \(\hookrightarrow\) shortId
    attribute is needed for compatibility with old projects, it is the name of the
        \(\hookrightarrow\) extension
    before adding this GUID. \(-->\)
    <guid shortid="modred">D0B4EDE5-4151-4EC1-96B4-BDA4410E0CDE</guid>
        <script src="ModRed.py" compiled="true"/>
        <interface context="Mechanical">
            \(<\) images \(>\) images \(<\) /images \(>\)
            <toolbar name="modred" caption="Fedem Reduction">
                                    <entry name="Fedem Reduction" icon="fedem-transp" >
        <callbacks>
                <onclick>onCreateModRed</onclick>
            </callbacks >
                                    </entry>
        <separator />
        <entry name="Export" caption="Generate Fedem Data" icon="fedem-export" >
            <callbacks>
                <onclick>onExportData</onclick>
            \(</\) callbacks \(>\)
        </entry >
                            </toolbar>
            \(</\) interface \(>\)
            <simdata context="Mechanical">
                    \(<\) !-- defining the object that is inserted under "Model" in the project
        \(\hookrightarrow\) tree \(-->\)
            <load name="modred" version="1" caption="Fedem Reduction"
        \(\hookrightarrow\) icon="fedem-transp"
                                    isload ="true" color="\#0000FF" contextual="true">
                    <callbacks>
            \(<\) !-- Allowing the user to right - click Fedem icon in the tree and select "Export
        \(\hookrightarrow\) Fedem Data" -->
            <action name="onExportData" caption="Export Fedem Data"
        \(\hookrightarrow\) icon="fedem-export" \(>\) onExportData</action \(>\)
                            \(<\) getsolvecommands \(>\) onSolve \(</\) getsolvecommands \(><\) ! --
        \(\hookrightarrow\) Running the specified function when the user hits "solve" \(-\gg\)
                    </callbacks >
                    <propertygroup name="Generation"
                        caption \(="\) Selection of Master Nodes"
                    display ="caption">
```

<propertygroup name="'Geometry" caption="Pick Master
\(\hookrightarrow\) Nodes By..." control =" select" display ="Property" default="Geometry"> < attributes options="Geometry,RBE2 Import" /> <property name="Geometry" control="scoping" caption="Geometry Selection"
$\hookrightarrow$ visibleon $=$ "'Geometry" $>$ < attributes selection_filter ="node"/>
</property>
<property name="CompID" control="String"
$\hookrightarrow$ caption="Component ID" visibleon="RBE2 Import" />
<property name=" Identifier" control ="String"
$\hookrightarrow$ caption =" Identifier " visibleon ="RBE2 Import" />
</propertygroup>
</propertygroup>
<propertygroup name="Method" caption="Model Reduction Method" display ="Caption" $>$
<propertygroup name="'ReductionMethod"
$\hookrightarrow$ caption="Reduction Method" control="select" display ="Property" default="CMS" $>$ $<$ attributes options="Guyan,CMS"/>
<property name="Nmode" caption="Num. of
$\hookrightarrow$ Component Modes to Extract"

```
                                    control ="int" default="'2"
```

$\hookrightarrow$ visibleon $=$ "'CMS" $>$
$\hookrightarrow<$ isvalid $>$ isValidNoModes $<$ /isvalid $>$
$\hookrightarrow$ control="string"
default="master"
$\hookrightarrow$ visibleon ="Guyan $\mid$ CMS" / $>$
<property name="LumpedMatrix" caption="Use lumped matrices?" control="select"
$\hookrightarrow$ default $=$ "No" $>$
$<$ attributes options="Yes,No"/>
</property>
</propertygroup>
</propertygroup>
<propertygroup name="Export"
caption $=$ "Export Options"
display ="Caption" $>$
<propertygroup name="IsExport" control=" select"
$\hookrightarrow$ caption="Export Matrices to Fedem?" display="Property"
default="Yes">
$<$ attributes options="No,Yes" />
$<$ property name="'FileDir" caption="Fedem File
$\hookrightarrow$ Directory"
control ="folderopen"

```
                                    default=""
                                    visibleon ="Yes">
                                    </property>
                                    </propertygroup>
                </propertygroup>
            </load>
        </simdata>
</extension>
```


## A. 3 test_ModRed.py

test_ModRed.py
\#
\# Unit tests for ModRed
\#
import clr
clr .AddReferenceToFileAndPath(r"C: $\backslash$ Program Files $\backslash$ ANSYS
Inc $\backslash v 192 \backslash$ Addins $\backslash$ ACT $\backslash$ bin $\backslash$ Win64 $\backslash$ MathNet.Numerics.dll") \# Using the Math.NET dll.
Can not use NumPy, as this is not supported by IronPython.
import MathNet.Numerics.LinearAlgebra as la
import System
import unittest
import os
import pickle
from System import Array as sys_array
import sys
sys. path. insert ( 0 ,
r'C: $\backslash$ Users $\backslash$ adrian $\backslash$ Documents $\backslash$ Dokumenter $\backslash N T N U \backslash$ Master $\backslash$ masters $\backslash$ ModRed $\backslash$ ModRed')
import ModRed
from ModRed import Node
from ModRed import Element
from ModRed import SolverData
class TestModRed(unittest.TestCase):
def setUp( self ):
self . resourcesFolder $=$ os.path.dirname(os.path. realpath ( __file_- $)$ ) +
$" \backslash \backslash$ test -resources $\backslash \backslash "$
\#
\# Defining the twoQUAD4 model
\#
self . .node1 $=\operatorname{Node}(1,6,[1], \quad 0.0, \quad 0.0,0.0)$
self . _node $2=\operatorname{Node}(2,6,[1,2], 1.0,0.0,0.0)$
self . _node3 $=\operatorname{Node}(3,6,[2], 2.0, \quad 0.0,0.0)$
self ._node4 $=\operatorname{Node}(4,6, \quad[1], \quad 0.0,-1.0,0.0)$
self ._node5 $=\operatorname{Node}(5,6,[1,2], 1.0,-1.0,0.0)$
self . _node6 $=\operatorname{Node}(6,6,[2], 2.0,-1.0,0.0)$
self . _element1 = Element(1, [ self ._node1, self ._node2, self . _node5, self ._node4]) \# clockwise
self . .element $2=\operatorname{Element}\left(2, \quad\left[\right.\right.$ self.$\_$node2, self.$\_$node3, self.$\_n o d e 6$, self . node5])
self . _elements $=$ [ self . .element1, self . _element2]
self . nodes $=$ [ self . node1, self . node2, self . node3, self . node4, self . node5, self . node6]
self ._unsortedNodes $=$ [ self . node2, self . _node1, self . node3, self . node6, self . node5, self .node4]
def array $(* x)$ : return sys_array [ float ] $(x)$ \# Helper function to create custom Math.NET matrices
self ._mat_ = la .Double.Matrix.Build.DenseOfRowArrays(
$\operatorname{array}(1,2,3,4,5,6,7,8,9)$,
$\operatorname{array}(10,11,12,13,14,15,16,17,18)$,
array (19, 20, 21, 22, 23, 24, 25, 26, 27),
array (28, 29, 30, 31, 32, 33, 34, 35, 36),
array (37, 38, 39, 40, 41, 42, 43, 44, 45),
array (46, 47, 48, 49, 50, 51, 52, 53, 54),
$\operatorname{array}(55,56,57,58,59,60,61,62,63)$,
array $(64,65,66,67,68,69,70,71,72)$,
array $(73,74,75,76,77,78,79,80,81)$
)
self.. indexMat $\quad=$ la.Double.Matrix.Build.DenseOfRowArrays(
$\operatorname{array}(11,12,13,14,15,16,17,18,19)$,
array $(21,22,23,24,25,26,27,28,29)$,
array $(31,32,33,34,35,36,37,38,39)$,
array (41, 42, 43, 44, 45, 46, 47, 48, 49),
array (51, 52, 53, 54, 55, 56, 57, 58, 59),
array $(61,62,63,64,65,66,67,68,69)$,
array $(71,72,73,74,75,76,77,78,79)$,
array $(81,82,83,84,85,86,87,88,89)$,
array (91, 92, 93, 94, 95, 96, 97, 98, 99)
)
self ._indexMatSmall_ = la .Double.Matrix.Build.DenseOfRowArrays(

```
        array (11, 12, 13),
        array (21, 22, 23),
        array (31, 32, 33)
        )
        self ._indexMat5x5_ = la.Double.Matrix.Build.DenseOfRowArrays(
        array (11, 12, 13, 14, 15),
        array (21, 22, 23, 24, 25),
        array (31, 32, 33, 34, 35),
        array (41, 42, 43, 44, 45),
        array (51, 52, 53, 54, 55)
    )
def tearDown(self):
    if os.path. exists ( self . resourcesFolder + "\\\testFTL. ftl "):
        os.remove(self. resourcesFolder + "\\testFTL. ftl ")
    def saveObject( self, obj, fname):
    ","
    Saves the current object to fedemDir
    with open(self . _resourcesFolder + "\\" + fname + ".p", "wb") as input:
        pickle.dump(obj, input, pickle.HIGHEST_PROTOCOL)
def test_generateFTL ( self ):
    file = open(self. resourcesFolder + "\\\testFTL. ftl", "w")
    eNodes = [1, 3]
    ModRed.generateFTL(self._nodes, self ._elements, eNodes, file )
    # Need to open the closed file again
    with open(self . resourcesFolder + "\\testFTL. ftl", "r") as file:
        fileLines = file.readlines ()
        file . close ()
    with open(self . resourcesFolder + "\\demoFTL.ftl", "r") as file:
        correctFileLines = file . readlines ()
        file . close ()
    print ( fileLines )
    print ( correctFileLines )
    self . assertEqual (len(fileLines ), len( correctFileLines )) # Should be the
    same length
    self . assertEqual ( fileLines [3:], correctFileLines [3:]) # Content is equal
    (except for the "generated at ..." part
```

```
def test_getNodeString ( self ):
    node = self ._node1
    eNodes = [1, 3]
    string = ModRed.getNodeString(node, eNodes)
    correctString = "NODE{111 0.0 0.0 0.0}"
    self .assertEqual (string, correctString )
def test_getNodeString_notENode( self ):
    If the node id is not in eNodes, it should be marked with 0
    ",
    node = self ._node2
    eNodes = [1, 3]
    string = ModRed.getNodeString(node, eNodes)
    correctString = "NODE{2 0 1.0 0.0 0.0}"
    self . assertEqual (string, correctString )
def test_getElementString ( self ):
    element = self . _element1
    thicknessId = 1
    matId = 1
    string = ModRed.getElementString(element, thicknessId, matId)
    correctString = "QUAD4{1 1254{PTHICK 1}{PMAT 1}}"
    self . assertEqual (string, correctString )
    def test_getElementString_element2 ( self ):
    element = self ._element2
    thicknessId = 1
    matId = 1
    string = ModRed.getElementString(element, thicknessId, matId)
    correctString = "QUAD4{2 2 365 {PTHICK 1}{PMAT 1}}"
    self . assertEqual (string , correctString )
    def test_createUnitVector ( self ):
    #"
    Should return matrix of size ((nnodes*6)+cmodes x 1)
    ","
    nnodes = 10
    cmodes =2
    u = ModRed.createUnitVector(1, nnodes, cmodes)
```

$$
\text { self . assertEqual (u }[0,0], 1)
$$

self . assertEqual (u [1,0], 0)
self . assertEqual (u [6,0], 1)
self . assertEqual (u.ColumnCount, 1)
self . assertEqual (u.RowCount, (nnodes $* 6$ ) + cmodes)
\# Checking if component mode positions are 0 :
self . assertEqual (u[nnodes $* 6,0], 0)$
self . assertEqual (u[nnodes $* 6+1,0], 0)$
$\mathrm{u}=$ ModRed.createUnitVector(2, nnodes, cmodes)
self . assertEqual (u [0,0], 0)
self . assertEqual (u [1,0], 1)
self . assertEqual (u [7,0], 1)
self . assertEqual (u.ColumnCount, 1)
self . assertEqual (u.RowCount, (nnodes $* 6$ ) + cmodes)
\# Checking if component mode positions are 0 :
self . assertEqual (u[nnodes $* 6,0], 0)$
self . assertEqual (u[nnodes $* 6+1,0], 0$ )
def test_extractPartitions ( self):
def $\operatorname{array}(* x)$ : return sys_array [ float ](x) \# Helper function to create custom
Math.NET matrices

```
mat = self ._indexMat_
```

nENodes = 1
dofs $=3$
parts $=$ ModRed. extractPartitions (mat, nENodes, dofs)
parts_ee $=$ parts [0]
parts_ei $=$ parts [1]
parts_ie $=$ parts [2]
parts_ii $=$ parts [3]
corr_ee $=$ la.Double.Matrix.Build.DenseOfRowArrays(
array (11, 12, 13),
array (21, 22, 23),
array $(31,32,33)$
)
self . assertEqual (parts_ee, corr_ee)
corr_ei $=$ la.Double.Matrix.Build.DenseOfRowArrays(
array $(14,15,16,17,18,19)$,
array $(24,25,26,27,28,29)$,
array $(34,35,36,37,38,39)$
)
self . assertEqual ( parts_ei , corr_ei )
corr_ie $=$ la.Double.Matrix.Build.DenseOfRowArrays(
array (41, 42, 43),
array (51, 52, 53),
array $(61,62,63)$,
array (71, 72, 73),
array (81, 82, 83),
array (91, 92, 93),
)
self . assertEqual ( parts_ie , corr_ie )
corr_ii = la.Double.Matrix.Build.DenseOfRowArrays(
array (44, 45, 46, 47, 48, 49),
array (54, 55, 56, 57, 58, 59),
array (64, 65, 66, 67, 68, 69),
array (74, 75, 76, 77, 78, 79),
array (84, 85, 86, 87, 88, 89),
array (94, 95, 96, 97, 98, 99)
)
self . assertEqual ( parts_ii , corr_ii )
def test_extractPartitions_FedemMatrix ( self ):
with open( self . resourcesFolder +"<br>beam.nas" + " $\backslash \backslash$ Fedem" +
$\backslash$ M_full_partitioned .p") as file:
M_full $=$ pickle. load (file $)$
mat $=$ ModRed. extractPartitions (M_full, 4, 6)
M_ee $=\operatorname{mat}[0]$
M_ei $=\operatorname{mat}[1]$
M_ie $=$ mat[2]
M_ii $=\operatorname{mat}[3]$
self . assertEqual (M_ee.RowCount, 24)
self . assertEqual (M_ee.ColumnCount, 24)
self . assertEqual (M_ee [0,0], 9.775000E-002)
self . assertEqual (M_ee[0,1], 0.0)
self . assertEqual (M_ee[1,0], 0.0)
self . assertEqual (M_ee[23,0], 0.0)
self . assertEqual (M_ee[0,23], 0.0)
self .assertAlmostEqual(M_ee[23,23], 4.072916E-006)
self . assertEqual (M_ei.RowCount, 24)
self . assertEqual (M_ei.ColumnCount, 606)
self . assertEqual (M_ei [0,0], M_full [0, 24])
self . assertEqual (M_ei[0, 605], M_full [0, 629])
self . assertEqual (M_ei[23, 0], M_full [23, 24])
self . assertEqual (M_ei[23, 605], M_full [23, 629])
self . assertEqual (M_ie.RowCount, 606)
self . assertEqual (M_ie.ColumnCount, 24)
self . assertEqual (M_ie [0,0], M_full [24, 0])
self . assertEqual (M_ie[0, 23], M_full [24, 23])
self . assertEqual (M_ie[605, 0], M_full[629, 0])
self . assertEqual (M_ie[605, 23], M_full[629, 23])
self . assertEqual (M_ii.RowCount, 606)
self . assertEqual (M_ii. ColumnCount, 606)
self . assertEqual (M_ii [0,0], M_full [24, 24])
self . assertEqual (M_ii [0, 605], M_full[24, 629])
self . assertEqual (M_ii[605, 0], M_full [629, 24])
self . assertEqual (M_ii[605, 605], M_full[629, 629])
def test_massMatrix_correct_mass_beam ( self ):
,","
Testing if the mass matrix is correct by applying
$\mathrm{u}_{-}$trans ${ }^{\wedge} \mathrm{T} * \mathrm{M} * \mathrm{u}_{-}$trans $=\mathrm{m}$ _tot
, ",
\# Validating M_full from Fedem
M_full $=$ ModRed.readFedemMatrix(self._resourcesFolder + " $\backslash \backslash$ beam.nas" +
" $\backslash \backslash$ Fedem" + " $\backslash \backslash$ M_full.res")
correct_mass $=3.12800 \mathrm{E}+01$ \# Taken from fedem_reducer. res. In kg
for i in $[1,2,3]$ :
$\mathrm{u}=\operatorname{ModRed} . c r e a t e \operatorname{Unit} \operatorname{Vector}(\mathrm{i}, 105)$
$\mathrm{m}=\mathrm{u}$. Transpose () . Multiply (M_full) . Multiply (u) [0,0]
self .assertAlmostEqual (correct_mass, m)
\# Testing mass matrix from ANSYS
M_full $=$ ModRed.readMMFMatrix(self._resourcesFolder + " $\backslash \backslash$ beam.nas" + " $\backslash \backslash$ Ansys"

+ "<br>M_full.mmf", "full")
for i in $[1,2,3]$ :
$\mathrm{u}=\operatorname{ModRed} . c r e a t e \operatorname{Unit} \operatorname{Vector}(\mathrm{i}, 105)$
$\mathrm{m}=\mathrm{u}$. Transpose () . Multiply (M_full) . Multiply (u) $[0,0]$
self .assertAlmostEqual (correct_mass, m)
\# Testing the lumped mass matrix from ANSYS
M_full = ModRed.readMMFMatrix(self._resourcesFolder + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Ansys" + " $\backslash \backslash$ M_full_lumped.mmf", "full")
for i in [1, 2, 3]: $\mathrm{u}=\operatorname{ModRed} . c r e a t e \operatorname{Unit} \operatorname{Vector}(\mathrm{i}, 105)$ $\mathrm{m}=\mathrm{u} . \operatorname{Transpose}()$. Multiply (M_full) . Multiply (u) [0,0] print (m) self .assertAlmostEqual ( correct_mass , m)

```
def test_partitionMatrix_correct_mass_beam ( self ):
```

    ","
    Testing if the mass is unchanged after partitionMatrix method
"川,
M_full = ModRed.readMMFMatrix(self._resourcesFolder + " $\backslash \backslash$ beam.nas" + " $\backslash \backslash$ Ansys"

+ " $\backslash \backslash$ M_full.mmf", "full")
eNodes $=[1,2,3,4]$
iNodes $=[\mathrm{i}$ for i in range $(5,106)]$

```
    dict = ModRed.readMappingFile(self. resourcesFolder + "\\\mathrm{ beam.nas" +"\\\Ansys"}
+"\\M_full.mapping")
    M_full = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict , 6)
    correct_mass = 3.12800E+01 # Taken from fedem_reducer.res. In kg
    for i in [1, 2, 3]:
        u = ModRed.createUnitVector(i, 105)
        m}=\textrm{u}.\mathrm{ Transpose() . Multiply (M_full) . Multiply (u) [0,0]
        print (m)
        self .assertAlmostEqual(correct_mass, m)
    def test_partitionMatrix_smallMatrices ( self ):
    Testing the partitionMatrix method with small matrices
    "",
    def array (*x): return sys_array [ float ](x) # Helper function to create custom
Math.NET matrices
    dict ={ # dof = equation - 1
        1:2,
        2: 3,
        3:1
    }
    # Remember: 3 DOFs per node
    eNodes = [1]
    iNodes = [2, 3]
    partitioned = ModRed.partitionMatrix( self ._indexMatSmall_, eNodes, iNodes,
    dict, 1, sorting ="dof")
    correct = la.Double.Matrix.Build.DenseOfRowArrays(
        array (22, 23, 21),
        array (32, 33, 31),
        array (12, 13, 11)
    )
    self . assertEqual ( correct, partitioned )
    for row in range (0, correct.RowCount):
        for col in range ( }0\mathrm{ , correct. ColumnCount):
            self . assertEqual ( partitioned [row, col ], correct [row, col ])
        def test_partitionMatrix_5x5 ( self):
    #
    # Testing of partitioning a custom 5x5 matrix
    #
    def array (*x): return sys_array [ float ](x) # Helper function to create custom
Math.NET matrices
    dict = { # dict [dof] = eq
        1:3,
```

```
        2: 4,
        3: 5,
        4: 1,
        5:2
    }
        eNodes = [2,5]
        iNodes = [1, 3, 4]
        partitioned = ModRed.partitionMatrix( self ._indexMat5x5_, eNodes, iNodes, dict,
        1, sorting ="equation")
        print ( partitioned )
    def test_partitionMatrix_9x9 ( self ):
    #
    # Testing with 9x9 matrix
    #
    def array (*x): return sys_array [ float ](x) # Helper function to create custom
Math.NET matrices
    dict = {
        1: 4,
        2: 5,
        3: 6,
        4: 7,
        5: 8,
        6: 9,
        7: 1,
        8: 2,
        9:3
    }
    eNodes = [1]
    iNodes = [2, 3]
    partitioned = ModRed.partitionMatrix( self ._indexMat_, eNodes, iNodes, dict, 3,
    sorting ="dof")
    tmp = la.Double.Matrix.Build.DenseOfRowArrays( # Moving rows
        array (41, 42, 43, 44, 45, 46, 47, 48, 49),
        array (51, 52, 53, 54, 55, 56, 57, 58, 59),
        array (61, 62, 63,64,65,66,67, 68,69),
        array (71, 72, 73, 74, 75, 76,77, 78, 79),
        array (81, 82, 83, 84, 85, 86, 87, 88, 89),
        array (91, 92, 93, 94, 95, 96, 97, 98, 99).
        array (11, 12, 13, 14, 15, 16, 17, 18, 19),
        array (21, 22, 23, 24, 25, 26, 27, 28, 29),
        array (31, 32, 33, 34, 35, 36, 37, 38, 39)
    )
    ,",",
    correct = la.Double.Matrix.Build.DenseOfRowArrays( # Moving cols
        array (44, 45, 46, 47, 48, 49, 41, 42, 43),
        array (54, 55, 56, 57, 58, 59, 51, 52, 53),
        array (64, 65, 66,67,68,69,61, 62, 63),
```

            array \((74,75,76,77,78,79,71,72,73)\),
            array \((84,85,86,87,88,89,81,82,83)\),
            array (94, 95, 96, 97, 98, 99, 91, 92, 93),
            \(\operatorname{array}(14,15,16,17,18,19,11,12,13)\),
            array \((24,25,26,27,28,29,21,22,23)\),
            array (34, 35, 36, 37, 38, 39, 31, 32, 33)
        )
        for row in range ( 0 , correct . RowCount):
        for col in range ( 0 , correct . ColumnCount):
            self . assertEqual ( partitioned [row, col ], correct [row, col ])
    def test_dofs2Eqs ( self ) :
dofs $=[1,2,3]$
dict $=\{$
1: 2,
2: 3,
3: 1
\}
correct $=[2,3,1]$
eqs $=$ ModRed.dofs2Eqs(dofs, dict)
self . assertEqual (eqs, correct)
self. assertEqual (len(dofs), len(eqs))
with open( self . _resourcesFolder + "<br>\beam.nas" + " $\backslash \backslash$ Ansys" +
$\backslash$ M_full.mapping.p") as file:
dict $=$ pickle. $\operatorname{load}($ file $)$
dofs $=\operatorname{range}(1,631)$
eqs $=$ ModRed.dofs2Eqs(dofs, dict)
self. assertEqual (len(dofs), len(eqs))
self . assertEqual (eqs [0], 625) \# dof 1 coupled to equation 624
self . assertEqual (eqs [1], 626)
self . assertEqual (eqs [282], 1)
def test_nodes2Dofs ( self ) :
\# Example:
\# eNodes $=[1,5,10]$ in elements with 6 dofs
$\#=>$ eDofs $=[1,2,3,4,5,6,25,26,27,28,29,30,55,56,57,58,59,60]$
,","
eNodes $=[1]$
dofs $=6$
correct $=[1,2,3,4,5,6]$
eDofs $=$ ModRed.nodes2Dofs(eNodes, dofs)
self . assertEqual (eDofs, correct)
eNodes $=[1,5,10]$
correct $=[1,2,3,4,5,6,25,26,27,28,29,30,55,56,57,58,59,60]$
eDofs $=$ ModRed.nodes 2 Dofs(eNodes, dofs)
self . assertEqual (eDofs, correct)
eNodes $=[1,3]$
dofs $=2$
correct $=[1,2,5,6]$
eDofs $=$ ModRed.nodes2Dofs(eNodes, dofs)
self . assertEqual (eDofs, correct)

```
def test_partitionMatrix_difference_twoQUAD4 ( self ):
    Testing if Fedem and ANSYS matrices are partitioned correctly.
    Using the partitioned Fedem matrix as reference, as this is proven to yield
    the identical gravity vector as Fedem calculates.
    dict_fedem = ModRed.readMappingFileFedem(self._resourcesFolder +
    "\\twoQUAD4" + "\\\Fedem" + "\\MEQN.res")
    dict_ansys = ModRed.readMappingFile(self. _resourcesFolder + "\\twoQUAD4" +
    "\\\Ansys"+"\\M_full_lumped.mapping")
    M_full_fedem = ModRed.readFedemMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Fedem" + "\\\M_full.res")
    M_full_ansys = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Ansys" + "\\M_full_lumped.mmf", "full")
    eNodes = [1, 3]
    iNodes =[2, 4, 5, 6]
    fedem = ModRed.partitionMatrix(M_full_fedem, eNodes, iNodes, dict_fedem, 6,
    sorting ="equation")
    ansys = ModRed.partitionMatrix(M_full_ansys, eNodes, iNodes, dict_ansys , 6,
    sorting ="equation")
        for row in range(0, ansys.RowCount):
            for col in range(0, ansys.ColumnCount):
                if row==col:
                print ("r:" + str (row) +",\ tc:" + str (col) +",\tfedem: " +
    str (fedem[row, col ]) +",\t\tansys:"+\operatorname{str}(\mathrm{ ansys [row, col ]) )}
                self .assertAlmostEqual(fedem[row, col ], ansys[row, col ], places=1)
        self . fail ("Not correct")
```

```
def test_massMatrix_diagonality ( self ):
```

def test_massMatrix_diagonality ( self ):
\#

```
    #
```

```
    # Verifying that a lumped matrix is diagonal
    #
    mat = ModRed.readMMFMatrix(self._resourcesFolder + "\\\beam.nas" + "\\\Ansys" +
    "\\M_full_lumped.mmf", "full")
    for row in range(0, mat.RowCount):
        for col in range(0, mat.ColumnCount):
            if row == col:
                self . assertTrue (mat[row, col] != 0.0 )
            else:
                self . assertTrue (mat[row, col] == 0.0)
    def test_partitionMatrix_twoQUAD4_Ansys( self ):
        #
        # Testing with mass matrix from twoQUAD4.
    #
        dict = ModRed.readMappingFileFedem(self._resourcesFolder + "\\\twoQUAD4" +
    "\\\Fedem" + "\\\MEQN.res")
    M_full = ModRed.readFedemMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Fedem" + "\\\M_full.res")
    eNodes = [1, 3]
    iNodes = [2, 4, 5, 6]
    mat = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict , 6)
    # Checking if diagonal:
    for row in range (0, mat.RowCount):
        for col in range (0, mat.ColumnCount):
            if row == col:
                self . assertTrue (mat[row, col] != 0.0 )
            else:
                self . assertTrue (mat[row, col] == 0.0)
        # It seems that some of the precision is lost in Fedem when comparing M_full
        and M_ee from Fedem.
    # Therefore the assertAlmostEqual
    self .assertAlmostEqual(mat [0,0], 3.910000E+001)
    self .assertAlmostEqual(mat [1,1], 3.910000E+001)
    self .assertAlmostEqual (mat [2,2], 3.910000E+001)
    self .assertAlmostEqual(mat [3,3], 2.782139E-001, places=6)
    self .assertAlmostEqual(mat [5,5], 6.516665E-001, places=6)
    self .assertAlmostEqual (mat [11,11], 6.516665E-001, places=6)
        def test_partitionMatrix_twoQUAD4_Fedem(self):
    #
    # Testing with mass matrix + dictionary from ANSYS. Compared to M_ee from
        Fedem
    #
```

```
    dict = ModRed.readMappingFile(self. resourcesFolder + "\\twoQUAD4" +
    "\\Ansys" + "\\M_full.mapping")
    M_full = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\\Ansys" + "\\\M_full_lumped.mmf", "full")
    eNodes = [1, 3]
    iNodes = [2, 4, 5, 6]
    mat = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict, 6)
    correct = ModRed.readFedemVector(self. _resourcesFolder + "\\twoQUAD4" +
"\\Fedem" + "\\M_ee.res")
    # Checking if diagonal:
    for row in range (0, mat.RowCount):
        for col in range(0, mat.ColumnCount):
            if row == col:
                self . assertTrue (mat[row, col] != 0.0 )
            else:
                self . assertTrue (mat[row, col] == 0.0)
    # It seems that some of the precision is lost in Fedem when comparing M_full
and M_ee from Fedem.
    # Therefore the assertAlmostEqual
    self .assertAlmostEqual(mat [0,0], 3.910000E+001, places=2)
    self .assertAlmostEqual(mat [1,1], 3.910000E+001, places=2)
    self .assertAlmostEqual(mat [2,2], 3.910000E+001, places=2)
    self .assertAlmostEqual (mat [3,3], 2.782139E-001, places=2)
    self .assertAlmostEqual (mat [5,5], 6.516665E-001, places=2)
    self .assertAlmostEqual(mat [9,9], 2.782139E-001, places=5)
    self .assertAlmostEqual(mat [11,11],6.516665E-001, places=2)
def test_partitionMatrix_Fedem_beam ( self ):
    #
    # Testing with mass matrix from beam.nas. Using the Fedem matrices,
    # because here we know the resulting matrix
    #
    dict = ModRed.readMappingFileFedem(self._resourcesFolder + "\\beam.nas" +
    \Fedem" + "\\MEQN.res")
    M_full = ModRed.readFedemMatrix(self._resourcesFolder + "\\beam.nas" +
    "\\\Fedem" + "\\\M_full.res")
    eNodes = [1, 2, 3, 4]
    iNodes = [i for i in range (5, 106)]
    mat = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict, 6)
    # Checking if diagonal:
    for row in range ( 0, mat.RowCount):
        for col in range(0, mat.ColumnCount):
        if row == col:
            self . assertTrue (mat[row, col] != 0.0 )
        else:
```

```
                self . assertTrue (mat[row, col] == 0.0)
    # It seems that some of the precision is lost in Fedem when comparing M_full
and M_ee from Fedem.
    # Therefore the assertAlmostEqual
    self .assertAlmostEqual(mat [0,0], 9.775000E-002)
    self . assertEqual (mat [1,0], 0.0)
    self . assertEqual (mat [0,1], 0.0)
    self . assertEqual (mat [1,1], 9.775000E-002)
    self .assertAlmostEqual(mat [3,3], 1.982601E-006)
    self .assertAlmostEqual(mat [4,4], 3.048196E-006)
    self .assertAlmostEqual(mat [5,5], 4.07292E-06)
    self .assertAlmostEqual(mat [11,11], 4.072916E-006)
    self .assertAlmostEqual(mat[15, 15], 1.982601E-006)
    self .assertAlmostEqual(mat[22, 22], 3.048196E-006)
    self .assertAlmostEqual(mat[23, 23], 4.072916E-006)
    self . assertEqual (mat[24, 24], 1.95500E-01)
    self . assertEqual (mat[629, 629], 1.62917E-05)
def test_partitionMatrix_Fedem_unsrtENodes ( self ):
    # beam.nas model from Fedem, with selected eNodes = [22, 28, 6, 44]
    dict = ModRed.readMappingFileFedem(self._resourcesFolder + "\\beam.nas" +
    \\Fedem" + "\\uunsrtENodes" + "\\MMEQN.res")
    M_full = ModRed.readFedemMatrix(self._resourcesFolder + "\\\beam.nas" +
    "\\Fedem" + "\\\unsrtENodes" + "\\M_full.res") # NOTE: The full mass matrix is
    not the same
    eNodes = [6, 22, 28, 44]
    iNodes = [i for i in range (1, 106)]
    for e in eNodes:
    iNodes.remove(e)
    mat = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict, 6)
    # Checking if diagonal:
    for row in range(0, mat.RowCount):
    for col in range(0, mat.ColumnCount):
        if row == col:
            self . assertTrue (mat[row, col] != 0.0 )
        else:
            self . assertTrue (mat[row, col] == 0.0)
    # Checking entries in M_ee:
    self .assertAlmostEqual(mat [0,0], 1.955000E-001)
    self .assertAlmostEqual(mat [3,3], 3.965202E-006)
    self .assertAlmostEqual(mat [6,6], 1.955000E-001)
```

self .assertAlmostEqual (mat [16,16], 6.096392E-006)

```
def test_partitionMatrix_beam_Mee ( self):
    # Testing with M_full from beam.nas (medium model)
    mapping = ModRed.readMappingFileFedem(self._resourcesFolder + "\\\beam.nas" +
    "\\\Fedem" + "\\\MEQN.res")
    M_full = ModRed.readFedemMatrix(self._resourcesFolder + "\\beam.nas" +
    "\\Fedem" + "\\\M_full.res")
    eNodes = [1, 2, 3, 4]
    iNodes = [i for i in range (5, 106)]
    M_full_partitioned = ModRed.partitionMatrix(M_full, eNodes, iNodes, mapping, 6)
    # Using M_ee from Fedem as reference
    M_ee = ModRed.readFedemVector(self. _resourcesFolder + "\\beam.nas" +
    "\\Fedem" + "\\\M_ee.res")
    for row in range(0, M_ee.RowCount):
        for col in range(0, M_ee.ColumnCount):
            self .assertAlmostEqual( M_full_partitioned [row, col ], M_ee[row, col ])
```

    def test_partitionMatrix_noDict ( self ) :
    Testing the partitionMatrix method with dictionary dof \(=\) equation
    def \(\operatorname{array}(* x)\) : return sys_array [ float ] \((x)\) \# Helper function to create custom
    Math.NET matrices
    \# Dictionary coupling dof to equation:
    \# dict[dof] = eq
    noDict \(=\{\quad \#\) dof \(=\) equation
        1: 1,
        2: 2,
        3: 3,
        4: 4,
        5: 5,
        6: 6,
        7: 7,
        8: 8,
        9: 9
    \}
    \# Remember: 3 DOFs per node
    eNodes \(=[1]\)
    iNodes \(=[2,3]\)
    partitioned \(=\) ModRed.partitionMatrix (self._mat_, eNodes, iNodes, noDict, 3,
        sorting \(=\) "dof")
    \# Dictionary with dof \(=\) equation and eNodes, iNodes in ascending order
    ```
    # should return the same matrix
    self . assertEqual( self ._mat_, partitioned )
    partitioned = ModRed.partitionMatrix( self ._mat_, eNodes, iNodes, noDict, 3,
sorting ="dof")
    # Testing first if partitionMatrix works with no dictionary mapping
    correct = la.Double.Matrix.Build.DenseOfRowArrays( # Moving rows
        array (31, 32, 33, 28, 29, 30, 34, 35, 36),
        array (40, 41, 42, 37, 38, 39, 43, 44, 45),
        array (49, 50, 51, 46, 47, 48, 52, 53, 54),
        array (4, 5, 6, 1, 2, 3, 7, 8, 9),
        array (13, 14, 15, 10, 11, 12, 16, 17, 18),
        array (22, 23, 24, 19, 20, 21, 25, 26, 27),
        array (58, 59, 60, 55, 56, 57, 61, 62, 63),
        array (67, 68, 69, 64, 65, 66, 70, 71, 72),
        array (76, 77, 78, 73, 74, 75, 79, 80, 81)
    )
    eNodes = [2]
    iNodes = [1, 3]
    partitioned = ModRed.partitionMatrix( self ._mat_, eNodes, iNodes, noDict, 3,
sorting ="dof")
    self .assertEqual ( partitioned, correct)
def test_mathNET2list ( self ):
    list = ModRed.mathNET2list(self._mat_)
        correct_list =[lll.0, 10.0, 19.0, 28.0}
    wrong_list =[1.0, 2.0, 3.0,4.0] # Should be returned column-first
    print ("my list: " + str ( list [0:4 ]))
    self . assertEqual ( list [0:4 ], correct_list )
    self .assertNotEqual ( list [0:4 ], wrong_list)
    self.assertEqual (len( list ), 81) # 9x9 matrix
def test_readMMFMatrix_red(self):
    M_red_path = self . resourcesFolder + "\\\beam.nas" + "\\\Ansys" + "\\\M_red.mmf"
    mat = ModRed.readMMFMatrix(M_red_path, "sub")
    self . assertEqual (mat.RowCount, 26)
    self . assertEqual (mat.ColumnCount, 26)
    self . assertEqual (mat [0,0], 9.037402289784589E+00)
    self . assertEqual (mat [0,2], 0)
    self . assertEqual (mat [2,0], 0)
```

self . assertEqual (mat [0,5], 4.528581487170890E-05)
self . assertEqual (mat [5,0], 4.528581487170890E-05)
self . assertEqual (mat [25,25], 1.0)
M_red_path $=$ self . _resourcesFolder + " $\backslash \backslash$ plate. nas" + " $\backslash \backslash$ Ansys" + " $\backslash \backslash$ M_red.mmf" mat $=$ ModRed.readMMFMatrix(M_red_path, "sub")
self . assertEqual (mat.RowCount, 26)
self . assertEqual (mat.ColumnCount, 26)
self . assertEqual (mat [0,0], 5.871600155722680E-01)
self . assertEqual (mat [0,2], 4.381258188504480E-01)
self . assertEqual (mat[24, 20], 2.027417791055110E-17)
def test_readMMFMatrix_full( self ):
path $=$ self. resourcesFolder + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Ansys" $+" \backslash \backslash$ M_full.mmf" mat $=$ ModRed.readMMFMatrix(path, "full")
\# Checking if dimensions are correct
self . assertEqual (mat.RowCount, 630)
self . assertEqual (mat.ColumnCount, 630)
\# Checking some values at the boundaries:
self . assertEqual (mat [0,0], 8.688888888888170E-02)
self . assertEqual (mat[0, 629], 0)
self . assertEqual (mat[629, 0], 0)
self . assertEqual (mat[629, 629], $1.448148148148030 \mathrm{E}-16$ )
\# Testing random values:
self . assertEqual (mat [23,23], 1.448148148148030E-16)
self . assertEqual (mat[285,243], 3.620370370370130E-07)
self . assertEqual (mat[291,291], 5.792592592592110E-06)
def test_readMMFMatrix_full_isSymmetric ( self ):
path $=$ self . _resourcesFolder + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Ansys" $+" \backslash \backslash$ M_full.mmf"
mat $=$ ModRed.readMMFMatrix(path, "full")
\# Checking if matrix is symmetric:
for row in range ( 0 , mat.RowCount): for col in range ( 0 , mat.ColumnCount): self . assertEqual (mat[row, col ], mat[col, row])

```
def test_readMMFMatrix_full_correctMass ( self ):
    path = self . _resourcesFolder + "\\beam.nas" +"\\\Ansys" +"\\M_full.mmf"
    mat = ModRed.readMMFMatrix(path, "full")
    correct_mass = 3.12800E+01 # Taken from Fedem_reducer
    for i in [1, 2, 3]:
        u = ModRed.createUnitVector(i, 105)
        m = u.Transpose() . Multiply (mat).Multiply (u) [0,0]
        print (m)
        self .assertAlmostEqual(correct_mass, m)
    def test_readMMFMatrix_red_isSymmetric(self):
    path = self . resourcesFolder + "\\\beam.nas" + "\\\Ansys" + "\\\M_red.mmf"
    mat = ModRed.readMMFMatrix(path, "sub")
    # Checking if matrix is symmetric:
    for row in range (0, mat.RowCount):
        for col in range (0, mat.ColumnCount):
            self . assertEqual (mat[row, col ], mat[col, row])
def test_readFedemVector_red_correctMass ( self ):
    path = self . _resourcesFolder + "\\\beam.nas" + "\\\Fedem" + "\\\M_red.res"
    mat = ModRed.readFedemVector(path)
    print (mat)
    correct_mass = 3.12800E+01
    for i in [1, 2, 3]:
        u = ModRed.createUnitVector(i, 4, 2)
        m}=u.Transpose () . Multiply (mat).Multiply (u) [0,0
        self .assertAlmostEqual(correct_mass, m, places=5)
    def test_readMMFMatrix_red_correctMass( self ):
    ","
    Applying unit translation in x, y, z direction and verifying it is the same
    mass
    as computed in Fedem
    ","
    path = self . _resourcesFolder + "\\\mathrm{ beam.nas" + "\\\Ansys" + "\\\M_red.mmf"}
    mat = ModRed.readMMFMatrix(path, "sub")
    correct_mass = 3.12800E+01
    for i in [1, 2, 3]:
        u = ModRed.createUnitVector(i, 4, 2)
        m}=u.Transpose() . Multiply (mat).Multiply (u) [0,0
        self .assertAlmostEqual(correct_mass, m, places=6)
```

```
#
# Commented out for being able to run all tests faster
def test_readMMFMatrix_largeFiles ( self ):
        M_full_path = self . _resourcesFolder + "\\lca.nas" + "\\\M_full.mmf" # Large
    file (230 000 lines)
        M_full = ModRed.readMMFMatrix(M_full_path, 'full")
        self . assertEqual (M_full .RowCount, 35553)
        self . assertEqual (M_full .ColumnCount, 35553)
        self . assertEqual (M_full [0, 0], 6.837606621530309E-05) # start
        self . assertEqual (M_full[165, 12], 2.229360838501640E-05) # random place
        self . assertEqual (M_full[35552, 35552], 2.580374027760560E-07) # end
    def test_readMMFMatrix_largeFiles2(self ):
        M_full_path = self . resourcesFolder + "\\ reactor" + "\\M_full.mmf" # Large
    file (450 000 lines )
        M_full = ModRed.readMMFMatrix(M_full_path, "full")
        self . assertEqual (M_full .RowCount, 69438)
        self . assertEqual (M_full .ColumnCount, 69438)
        self . assertEqual (M_full [0, 0], 9.966062445579789E-05) # start
        self . assertEqual (M_full[26236, 26236], 7.549508902862550E-04) # random place
        self .assertEqual (M_full[69437, 69437], 8.306723475513540E-04) # end
    #"
    def test_readFedemMatrix ( self ):
    path = self . resourcesFolder + "\\\beam.nas" + "\\\Fedem" + "\\\M_full.res"
    mat = ModRed.readFedemMatrix(path)
    # Checking some entries
    self . assertEqual (0.1955, mat [0,0])
    self . assertEqual (mat [0,1], 0.000)
    self . assertEqual (mat [1,1], 0.1955)
    self . assertEqual (mat[202, 202], 6.09639E-06)
    self . assertEqual (mat[605, 605], 1.62917E-05)
    self . assertEqual (mat[606, 606], 9.77500E-02)
    self . assertEqual (mat[629, 629], 4.07292E-06)
    def test_readFedemVector_twoQUAD4(self):
    path = self . _resourcesFolder + "\\twoQUAD4" + "\\\Fedem" + "\\gravVec.res"
    mat = ModRed.readFedemVector(path)
    self . assertEqual (mat [0,0], 1.564000E+002)
    self . assertEqual (mat [0,1], -3.556089E+001)
    self . assertEqual (mat [13,2], -9.710526E-015)
    self . saveObject(mat, "gravVec")
```

```
def test_readFedemVector(self ):
    path = self . _resourcesFolder + "\\\mathrm{ beam.nas" + "\\\Fedem" + "\\\Bmat.res"}
    mat = ModRed.readFedemVector(path)
    self . assertEqual (mat [0,0], 2.270180E-001)
    self . assertEqual (mat [0,1], 8.015493E-002)
    self . assertEqual (mat [0,9], 0.0)
    self . assertEqual (mat [0,17], -3.023801E-004)
    self . assertEqual (mat [0,23], -7.732552E-003)
    self . assertEqual (mat [601,0], 1.497216E-002)
    self . assertEqual (mat [604,0], 0.0)
    self . assertEqual (mat[605, 23], -2.384053E-002)
    self . assertEqual (mat[605,23], -2.384053E-002)
    self . assertEqual (mat[604,9], -2.839266E-004)
    self . assertEqual (mat[605,9], 0.0)
    self . assertEqual (mat[605,10], 4.054158E-003)
    self . assertEqual (mat[605,11], -3.068604E-002)
    self . assertEqual (mat [605,5], 4.054158E-003)
    self . assertEqual (mat [605,1], 1.572045E-001)
    self . assertEqual (mat [605,0], 5.364874E-001)
    mat = ModRed.readFedemVector(self. _resourcesFolder + "\\\beam.nas" + "\\\Fedem"
+ "\\phi_fedem.res")
    self . assertEqual (0.0, mat [0,0])
    self . assertEqual (6.392561E-016, mat[0,1])
    self . assertEqual (-8.829041E-002, mat[2,0])
    self . assertEqual (3.321254E-016, mat[605,0])
    self . assertEqual (-5.843641E-014, mat[605,1])
    vec = ModRed.readFedemVector(self. resourcesFolder + "\\\beam.nas" + "\\\Fedem"
    + "\\gravVec.res")
    self . assertEqual (vec [0,0], 7.820000E+000)
    self . assertEqual (vec [0,1], 1.297520E+001)
    self . assertEqual (vec [25,0], 1.826227E-014)
    self . assertEqual (vec [25,1], 4.104385E-014)
    self . assertEqual (vec [25,2], -1.456509E-013)
    def test_readMappingFileFedem( self ):
    dict = ModRed.readMappingFileFedem(self.resourcesFolder + "\\\beam.nas" +
        "\\\Fedem" + "\\MEQN.res")
    dof = 1
    eq = 607
    self . assertEqual (eq, dict [dof])
    dof =2
    eq = 608
```

self . assertEqual (eq, dict [dof])
dof $=630$
$\mathrm{eq}=258$
self . assertEqual (eq, dict [dof])
def test_calculateGravityVector_Fedem_beam ( self ):
with open( self . _resourcesFolder + "<br>beam.nas" + "<br>Fedem" + "<br>gravVec.p") as file: correct $=$ pickle. $\operatorname{load}($ file $)$
with open( self . resourcesFolder + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Fedem" $+" \backslash \backslash$ NOR.p") as file: NOR = pickle.load ( file )
with open( self . resourcesFolder + " $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Fedem" +
" $\backslash \backslash$ identityMatrix.p") as file:
ident $=$ pickle $\cdot \operatorname{load}($ file $)$
B $=$ ModRed.readFedemVector(self. resourcesFolder + " $\backslash \backslash$ beam.nas" + " $\backslash \backslash$ Fedem" + " $\backslash \backslash$ Bmat.res")

CST $=$ la. Double.SparseMatrix $(630,24)$
CST.SetSubMatrix(0, 24, 0, 24, ident)
CST.SetSubMatrix(24, 606, 0, 24, B)
$\mathrm{H}=\mathrm{CST}$.Append(NOR) $\quad \# \mathrm{H}=[\mathrm{CST}$ NOR]
\# Building M_full:
M_full $=$ ModRed.readFedemMatrix(self._resourcesFolder + " $\backslash \backslash$ beam.nas" +
" $\backslash \backslash$ Fedem" + " $\backslash \backslash$ M_full.res")
dict $=$ ModRed.readMappingFileFedem(self._resourcesFolder + " $\backslash \backslash$ beam.nas" +
$\backslash$ Fedem" + "<br>MEQN.res")
eNodes $=[1,2,3,4]$
iNodes $=[\mathrm{i}$ for i in range $(5,106)]$
M_full $=$ ModRed.partitionMatrix (M_full, eNodes, iNodes, dict, 6)
gravVec $=$ ModRed.calculateGravityVector(M_full, H, 4, 101, 6)
print (gravVec)
for row in range ( 0 , gravVec. RowCount):
for col in range ( 0 , gravVec.ColumnCount):
self.assertAlmostEqual (gravVec[row, col ], correct [row, col ], places=5)
def test_calculateGravityVectorReduced_beam ( self) :
with open( self . _resourcesFolder +" $\backslash \backslash$ beam.nas" $+" \backslash \backslash$ Fedem" $+" \backslash \backslash$ gravVec.p") as file:

```
        correct = pickle.load( file )
```

```
    M_red = ModRed.readMMFMatrix(self._resourcesFolder + "\\\mathrm{ beam.nas" + "\\\Ansys" +}
    "\\M_red.mmf", "sub")
        gravVec = ModRed.calculateGravityVectorReduced(M_red, 4, 2)
        diffs = []
        for row in range (0, gravVec.RowCount):
        for col in range (0, gravVec.ColumnCount):
        diff = correct [row, col] - gravVec[row, col]
        diffs . append(diff )
        if (diff > 1.0):
                self.fail ('Too large diff")
    def test_calculateGravityVectorReduced_twoQUAD4(self):
    with open( self . resourcesFolder + "\\twoQUAD4" + "\\\Fedem" + "\\\gravVec.p")
    as file:
            correct = pickle.load( file )
        CST = ModRed.readMMFMatrix(self.resourcesFolder + "\\\twoQUAD4" + "\\\Ansys"
    + "\\\CST.mmf", "full")
        NOR = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" + "\\Ansys"
    + "\\\NOR.mmf", 'full")
    H = CST.Append(NOR) # H = [CST NOR]
    M_red = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" +
    "\\Ansys" + "\\M_red.mmf", "sub")
    gravVec = ModRed.calculateGravityVectorReduced(M_red, 2, 2)
    print (gravVec)
    diffs = []
    for row in range(0, gravVec.RowCount):
        for col in range (0, gravVec.ColumnCount):
            diff = correct [row, col] - gravVec[row, col]
            diffs . append(diff )
    print (diffs )
    if (max(diffs)) > 20:
        self . fail ("Too large diff")
        def test_calculateGravityVectorReduced_plate (self):
    correct = ModRed.readFedemVector(self. resourcesFolder + "\\plate .nas" +
        "\\\Fedem" + "\\gravVec.res")
    M_red = ModRed.readMMFMatrix(self._resourcesFolder + "\\plate.nas" + "\\Ansys"
    + "\\M_red.mmf", "sub")
    gravVec = ModRed.calculateGravityVectorReduced(M_red, 4, 2)
    print (gravVec)
    diffs = []
    for row in range (0, gravVec.RowCount):
```

```
        for col in range(0, gravVec.ColumnCount):
        diff = correct [row, col] - gravVec[row, col]
        diffs .append(diff )
        print ( diffs )
        print (max(diffs ))
        if (max(diffs)) > 20:
        self . fail ('Too large diff")
def test_calculateGravityVector_Ansys_twoQUAD4( self ):
        with open( self . _resourcesFolder + "\\\twoQUAD4" + "\\\Fedem" + "\\\gravVec.p")
        as file:
            correct = pickle.load( file )
        M_full = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" +
        "\\\Ansys" + "\\\M_full_lumped.mmf", "full")
        CST = ModRed.readMMFMatrix(self._resourcesFolder + "\\twoQUAD4" + "\\\Ansys"
    + "\\\ST.mmf", "full")
        NOR = ModRed.readMMFMatrix(self.resourcesFolder + "\\twoQUAD4" + "\\Ansys"
    + "\\\NOR.mmf", 'full")
    H = CST.Append(NOR) # H = [CST NOR]
    dict = ModRed.readMappingFile(self. _resourcesFolder + "\\twoQUAD4" +
    "\\\Ansys" + "\\M_full_lumped.mapping")
    eNodes = [1, 3]
    iNodes = [2, 4, 5, 6]
    M_full = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict , 6)
    gravVec = ModRed.calculateGravityVector(M_full, H, 2, 4, 6)
    print (" calculated: " + str (gravVec))
    print (" correct: " + str ( correct ))
    for row in range(0, gravVec.RowCount):
        for col in range(0, gravVec.ColumnCount):
            self .assertAlmostEqual(gravVec[row, col ], correct [row, col ], places=1)
        def test_node2dof ( self ):
    dofDict = {
        "UX": 1,
        "UY": 2,
        "UZ": 3,
        "ROTX": 4,
        "ROTY": 5,
        "ROTZ":}
    }
    node = 1
    dofText = "UX"
    correctDof = 1
```

    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
    node \(=1\)
    dofText = "UY"
    correctDof \(=2\)
    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
    node \(=1\)
    dofText = "UZ"
    correctDof \(=3\)
    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
    node \(=1\)
    dofText = "ROTX"
    correctDof \(=4\)
    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
    node \(=2\)
    dofText = "UY"
    correctDof \(=8\)
    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
    node \(=3\)
    dofText = "UZ"
    correctDof \(=15\)
    dof \(=\) ModRed.node2dof(node, dofDict, 6, dofText)
    self . assertEqual (dof, correctDof)
        def test_calculateGravityVector_Fedem_twoQUAD4(self):
            with open( self . resourcesFolder +" \(\backslash \backslash\) twoQUAD4" + " \(\backslash \backslash\) Fedem" \(+" \backslash \backslash\) gravVec.p")
        as file:
            correct \(=\) pickle. \(\operatorname{load}(\) file \()\)
        M_full \(=\) ModRed.readFedemMatrix(self._resourcesFolder + " \(\backslash \backslash\) twoQUAD4" +
        \(" \backslash \backslash\) Fedem" + " \(\backslash \backslash\) M_full.res")
            ident \(=\) la.Double.SparseMatrix. CreateIdentity (12)
            B = ModRed.readFedemVector(self. _resourcesFolder + " \(\backslash\) twoQUAD4" + " \(\backslash \backslash\) Fedem"
        \(+" \backslash \backslash\) BMAT.res")
            nullMat \(=\) la.Double.SparseMatrix \((12,2)\)
            phi \(=\) ModRed.readFedemVector(self. _resourcesFolder + " \(\backslash \backslash\) twoQUAD4" +
        " \(\backslash \backslash\) Fedem" + " \(\backslash \backslash\) phi.res")
            CST \(=\) la.Double. \(\operatorname{SparseMatrix~}(36,12)\)
            CST.SetSubMatrix ( \(0,12,0,12\), ident)
            CST.SetSubMatrix (12, 24, 0, 12, B)
    \(\operatorname{NOR}=\) la.Double. \(\operatorname{SparseMatrix}(36,2)\)
    NOR.SetSubMatrix ( \(0,12,0,2\), nullMat)
    NOR.SetSubMatrix(12, 24, 0, 2, phi)
    \(\mathrm{H}=\mathrm{CST}\). Append(NOR) \(\quad \# \mathrm{H}=[\mathrm{CST}\) NOR]
    dict \(=\) ModRed.readMappingFileFedem(self._resourcesFolder + " \(\backslash \backslash\) twoQUAD4" +
    " \(\backslash \backslash\) Fedem" + " \(\backslash \backslash\) MEQN.res")
    eNodes \(=[1,3]\)
    iNodes \(=[2,4,5,6]\)
    M_full = ModRed.partitionMatrix (M_full, eNodes, iNodes, dict , 6)
    gravVec \(=\) ModRed.calculateGravityVector(M_full, H, 2, 4, 6)
    print (gravVec)
    for row in range ( 0, gravVec.RowCount):
        for col in range ( 0 , gravVec.ColumnCount):
            self .assertAlmostEqual(gravVec[row, col ], correct [row, col ], places=4)
        def test_calculateGravityVector_Fedem_unsortedENodes ( self ):
    Gravity vector with eNodes \(=[22,28,6,44]\)
    Using the beam.nas model
    correct \(=\) ModRed.readFedemVector(self. _resourcesFolder + " \(\backslash \backslash\) beam.nas" +
    \(" \ \backslash\) Fedem" + " \(\backslash \backslash\) unsrtENodes" + " \(\backslash \backslash\) gravVec.res")
    B = ModRed.readFedemVector(self. _resourcesFolder + " \(\backslash \backslash\) beam.nas" + " \(\backslash \backslash\) Fedem" +
    " \(\backslash\) unsrtENodes" + " \(\backslash \backslash\) Bmat.res")
    ident \(=\) la.Double.SparseMatrix(24). CreateIdentity (24)
    CST = la. Double.SparseMatrix \((630,24)\)
    CST.SetSubMatrix ( \(0,24,0,24\), ident)
    CST.SetSubMatrix (24, 606, 0, 24, B)
    phi \(=\) ModRed.readFedemVector(self. resourcesFolder + " \(\backslash \backslash\) beam.nas" + " \(\backslash \backslash\) Fedem"
    \(+" \backslash\) unsrtENodes" +"\\phi.res")
    nullMat = la. Double.SparseMatrix \((24,2)\)
    NOR = la.Double.SparseMatrix (630, 2)
    NOR.SetSubMatrix(0,24, 0, 2, nullMat)
    NOR.SetSubMatrix(24, 606, 0, 2, phi)
    \(\mathrm{H}=\mathrm{CST}\).Append(NOR) \(\quad \# \mathrm{H}=[\mathrm{CST}\) NOR]
    \# Building M_full:
    M_full = ModRed.readFedemMatrix(self._resourcesFolder + " \(\backslash \backslash\) beam.nas" +
    " \(\backslash \backslash\) Fedem" + " \(\backslash \backslash\) unsrtENodes" + "\\M_full.res")
    dict \(=\) ModRed.readMappingFileFedem(self._resourcesFolder + " \(\backslash \backslash\) beam.nas" +
    \(" \ \backslash\) Fedem" + " \(\backslash \backslash\) unsrtENodes" + "\\MEQN.res")
    eNodes \(=[22,28,6,44]\)
    ```
    iNodes = [i for i in range (1, 106)]
    for e in eNodes:
        iNodes.remove(e)
    M_full = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict , 6)
    gravVec = ModRed.calculateGravityVector(M_full, H, 4, 101, 6)
    print (gravVec)
        diffs = []
    for row in range (0, gravVec.RowCount):
        for col in range (0, gravVec.ColumnCount):
        diff = correct [row, col] - gravVec[row, col]
        diffs .append(diff )
    print (diffs )
    print ("max:" + str (max(diffs)))
    for row in range (0, gravVec.RowCount):
        for col in range(0, gravVec.ColumnCount):
        self .assertAlmostEqual(gravVec[row, col ], correct [row, col ], places=5)
    def test_calculateGravityVector_Ansys_beam ( self ):
    with open( self . resourcesFolder + "\\\beam.nas" + "\\\Fedem" + "\\gravVec.p") as
    file:
        correct = pickle.load( file )
        CST = ModRed.readMMFMatrix(self._resourcesFolder + "\\\beam.nas" + "\\\Ansys" +
        \CST.mmf", "full")
        NOR = ModRed.readMMFMatrix(self._resourcesFolder + "\\beam.nas" + "\\\Ansys" +
        \NOR.mmf", "full")
        H = CST.Append(NOR) # H = [CST NOR]
    # Building M_full:
    M_full = ModRed.readMMFMatrix(self._resourcesFolder + "\\\beam.nas" + "\\\Ansys"
    + "\\M_full_lumped.mmf", "full")
    dict = ModRed.readMappingFile(self. resourcesFolder + "\\\beam.nas" + "\\\Ansys"
    + "\\M_full_lumped.mapping")
    eNodes = [1, 2, 3, 4]
    iNodes = [i for i in range (5, 106)]
    M_full = ModRed.partitionMatrix(M_full, eNodes, iNodes, dict , 6)
    gravVec = ModRed.calculateGravityVector(M_full, H, 4, 101, 6)
    diffs = []
    for row in range (0, gravVec.RowCount):
        for col in range(0, gravVec.ColumnCount):
            diff = correct [row, col] - gravVec[row, col]
            diffs .append(diff )
    print (diffs )
    print ("max:" + str (max(diffs )))
```

```
            for row in range(0, gravVec.RowCount):
            for col in range(0, gravVec.ColumnCount):
                self .assertAlmostEqual(gravVec[row, col ], correct [row, col ], places=1)
    def test_readMappingFile ( self ):
        path = self . _resourcesFolder + "\\twoQUAD4" + "\\Ansys" + "\\M_full.mapping"
        dict = ModRed.readMappingFile(path) # dict[dof] = equation
        self . assertEqual ( dict [1], 13)
        self . assertEqual ( dict [2], 14)
        self . assertEqual ( dict [7], 19)
        self . assertEqual (dict [13], 31)
        self . assertEqual (dict [14], 32)
        self . assertEqual ( dict [18], 36)
if __name_- == "__main__":
        unittest .main()
```


[^0]:    ${ }^{1}$ Image courtesy of [7].

[^1]:    ${ }^{2}$ Image courtesy of [12]

[^2]:    ${ }^{1}$ Performed in test_results.py, in method test_quadraticSumDifference_gravityVector_beam_Fedem

[^3]:    def test_partitionMatrix_difference_twoQUAD4 ( self ) :

