Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Department of Industrial Economics and Technology Management

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# Financial Strategies for Repowering and Life-extension of Wind Turbines: A Real Options Approach

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Kunnskap for en bedre verden

# Abstract

The market for wind power is booming, driven by environmental concerns and rapidly increasing global energy demand. As installed wind power capacity increase, however, so does the amount of aging capacity. Because of structural safety concerns, wind turbines can only be operated within their designed technical lifetime, which is about 25 years. After that point, action need to be taken in order to keep the wind farm operative, such as repowering or life-extension. Since a lot of turbines are reaching the end of their technical lifetime, the choice between these strategies will become an increasingly important issue. To address this, we apply a real options approach to examine financial strategies for repowering and life-extension of wind farms.

First, we develop a tractable single-factor model to gain general insights into the investment problem. In particular, we analyse how the option values in general depend on the underlying factors. From this analysis we find that the repowering and life-extension option values vary at different rates when the underlying parameters are changed. Specifically, we find that the option values are increasing at different rates with the volatility of the profit flow, initial price level and the repowering scaling factor. Also, the option values are decreasing at different rates with the cost of capital, capacity efficiency decline and investment cost. These results show that the relative attractiveness between the options can change when the values of the underlying factors are changed.

Second, we develop a flexible multi-factor model to evaluate and gain insights into specific wind power projects. This model incorporates the stochastic nature of electricity prices, wind speeds and technological innovation, and can also evaluate project-specific characteristics such as technological restrictions and power purchase agreements. In order to determine the optimal investment strategies for end-of-life options with the multi-factor model, we utilize a numerical algorithm based on the least squares Monte Carlo approach.

To showcase the applicability and flexibility of the model, it is applied to a numerical case study of a generic Norwegian wind farm. From analysis of this case study, we find that the optimal decision for a wind farm operator strongly depends on how the market conditions develop throughout the lifetime of the wind farm. For favourable early market conditions, such as high electricity prices and production, the option to repower is preferable and will be exercised. For moderate early market conditions, the operator prefers to wait and see which way the market develops; while for unfavourable market conditions, the operator will wait for the life-extension option to become available and exercise it optimally during the remaining lifetime of the wind farm. Finally, the inclusion of contractual power purchase agreements in the case study shows that increasing the fraction of power production sold through long-term agreements reduces the electricity price risk exposure and incentivises earlier investments in both repowering and life-extension.

# Sammendrag

Markedet for vindkraft er i kraftig fremdrift, drevet av hensyn til miljøet og raskt økende global energietterspørsel. Men i tråd med at installert vindkraftkapasitet øker, øker også mengden aldrende kapasitet. På grunn av hensyn til strukturell sikkerhet kan vindturbiner kun brukes innenfor sin tekniske levetid, som er på rundt 25 år. Etter dette kreves aktive tiltak for at vindparken skal være operativ, slik som repowering og life-extension. Siden mange turbiner vil nærme seg slutten av sin tekniske levetid, vil valg og prioritering av disse tiltakene bli viktigere og viktigere over tid. For å adressere dette, anvender vi et realopsjonsrammeverk for å undersøke finansielle strategier for repowering og life-extension av vindparker.

Først, utvikler vi en letthåndterlig enfaktormodell for å oppnå generell innsikt i investeringsproblemet. Spesifikt, analyserer vi hvordan opsjonsverdien generelt avhenger av de underliggende faktorene. Fra denne analysen finner vi at opsjonsverdiene for repowering og life-extension varierer med ulike rater når de underliggende parametrene endres. Opsjonsverdiene øker med forskjellig rate når volatiliteten i profittstrømmen, startnivået for prisen eller skaleringsfaktoren for repowering øker; og opsjonsverdiene synker med varierende rate når avkastningskravet, nedgangsraten for kapasitets-effektiviteten eller investeringskostnadene øker. Disse resultatene tilsier at den relative attraktiviteten mellom opsjonene kan endres når de underliggende faktorene endres.

Deretter, utvikler vi en fleksibel multifaktormodell for å evaluere spesifikke vindkraft-prosjekter. Modellen inkorporerer de stokastiske egenskapene til strømpriser, vindhastighet og teknologisk innovasjon, og kan vurdere prosjektspesifikke karakteristikker slik som teknologiske restriksjoner og langsiktige kraftavtaler. For å vurdere optimale investeringsstrategier for end-of-life vindkraftopsjoner med multifaktormodellen, benytter vi en numerisk algoritme basert på least squares Monte Carlo-metoden.

For å synliggjøre hvor anvendbar og fleksibel modellen er, anvender vi den på en numerisk casestudie av en generisk norsk vindpark. Ved å analysere dette casestudiet finner vi at den optimale beslutningen for vindparkoperatøren er sterkt avhengig av hvordan markedsforholdene utvikler seg gjennom vindparkens levetid. For gunstige markedsforhold tidlig i levetiden, eksempelvis høy strømpris og kraftproduksjon, er repoweringopsjonen foretrukket og vil bli realisert. For moderate markedsforhold tidlig i levetiden, vil operatøren foretrekke å vente for og se hvilken retning markedet utvikler seg. For ugunstige markedsforhold tidlig i levetiden, vil operatøren foretrekke å vente til life-extensionopsjonen blir tilgjengelig og realisere den optimalt innen den resterende levetiden til vindparken. Det siste funnet er at inkludering av langsiktige kraftavtaler vil redusere vindparkens eksponering mot elektrisitetsprisrisiko, noe som gir insentiv for tidligere investering i både repowering og life-extension.

# Preface

This thesis is submitted as the concluding part of our Master of Science degrees in Industrial Economics and Technology Management, with specialization in Financial Engineering, at the Norwegian University of Science and Technology.

We would like to express our profound gratitude to our supervisors, Maria Lavrutich and Roel Nagy, for providing us with valuable feedback and guidance during our work with this thesis. Further, we would like to thank Stein-Erik Fleten for the useful input on model calibration procedures, and Lars Hegnes Sendstad for the guidance on modelling technological uncertainty.

Trondheim, June 10, 2019

Kjetil Flatland

Mats Thunestvedt Hove

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# Nomenclature and abbreviations

**Indices**

$L$: Lifetime-extension.

$R$: Repowering.

$i$: General index representing either repowering or lifetime-extension.

**Variables**

$g$: Gross operating profit margin.

$G$: Net operating profit margin.

$K_L$: Life-extension capacity efficiency scaling factor.

$K_R$: Repowering capacity efficiency scaling factor.

$I_R$: Cost of repowering, i.e. the cost of investing in a new turbine less the income from recycling the old turbine.

$I_L$: Cost of lifetime-extension.

$k$: Turbine capacity factor.

$\Delta t_L$: Length of life-extension, i.e. how much the technical lifetime of the turbine is extended.

$r$: Cost of capital.

$\gamma$: Rate of capacity efficiency decay.

$Q_t$: Capacity efficiency of the turbine at time $t$.

$V_R, V_L$: Repowering and life-extension option value respectively.

$\Omega_R, \Omega_L$: Value of immediate investment in repowering and lifetime-extension respectively.

**Variables exclusive to the single-factor model**

$\alpha_{gbm}$: Expected growth of the GBM operating profit margin.

$\sigma_{gbm}$: Volatility of the the GBM operating profit margin.

$G_R, G_L$: Investment threshold for the repowering option and the lifetime-extension option, respectively.

**Variables exclusive to the multi-factor model**

$X_0$: Log of the current electricity price.

$\alpha_P$: Mean-reversion level factor of the electricity log price.

$\kappa_P$: Mean-reversion speed of the electricity log price.

$\rho$: Correlation between the electricity log price and the weather factor.

$\kappa_U$: Mean-reversion speed of the weather factor.

$a, b$: Parameters of the Weibull distribution.

$w_0, w_1, w_2$: Turbine speed parameters.

$\Delta$: Time from previous technological innovation until the arrival rate changes.

$\lambda_1, \lambda_2$: Expected rate of technological innovation for $t < \Delta$ and $t \geq \Delta$ respectively.

$K$: Number of simulation paths.

$m$: Order of Laguerre polynomials.

$\Delta t$: Time step for simulation.

**Abbreviations**

GBM: Geometric Brownian motion.

NPV: Net present value.

O&M: Operation and maintenance.

PPA: Power purchase agreement.

# Introduction

The market for wind energy is booming and in rapid growth, driven by the desire to meet future energy demand in a sustainable way. A limitation on the speed of this growth is however that wind turbines have an expected lifetime of 20 – 25 years, after which operation must be stopped due to structural safety concerns. Therefore, the growing market for wind power carries with it the issue of how to properly handle a growing number of aging turbines. This has, however, already become a pressing concern, as 28 % of Europe's installed wind power capacity will be 15 years or older by 2020. Hence, financial strategies that assess the optimal utilization of end-of-life options for wind turbines will be of great importance in the years to come, and become increasingly crucial as the market for wind energy continues to grow.

When a wind turbine is approaching the end of its life, the operator has three options available: repowering, life-extension and decommissioning. The objective of this thesis is to analyse these options from a financial perspective. More specifically, we aim to find financial strategies for repowering and life-extension of wind farms that an operator can use to optimally allocate his assets. To this end, we focus on evaluating the option values, expected exercise time and how the optimal behaviour depends on the underlying factors.

To account for the irreversibility, uncertainty and flexibility present in the investment problem, we employ a real options approach, which entails evaluating the wind power end-of-life options as American-style call options. In order to evaluate the end-of-life options in a thorough and comprehensive manner, we develop two distinct models from the real options framework, a single-factor model and a multi-factor model. For the single-factor model, it is assumed that the underlying uncertainty in the operating profit margin can be captured by a single stochastic process. This allows for analytical, closed-form solutions that are tractable and well suited to provide general insights into the investment problem. Particularly, we disentangle the relationship between the option values and their underlying factors and analyse how the option values respond to changes in the parameter values.

For the multi-factor model, the profit flow is modelled through its underlying risk factors by allowing the price of electricity and wind speed to follow separate, correlated stochastic processes. Technological uncertainty is also included, by simulating the arrival of technological innovations through an inhomogeneous Poisson jump-process. While no longer analytically solveable, the increased flexibility of the multi-factor model makes it possible to tailor it to highly project specific conditions, thereby providing an excellent decision-making tool for specific wind projects. To solve the multi-factor model, the least squares Monte Carlo approach of Longstaff and Schwartz (2001) is employed, which is a flexible numerical solution scheme. In order to demonstrate the applicability of the model, we apply it to a case study of a generic Norwegian wind farm and analyse the insights provided.

Within the existing literature, there are several studies that conduct quantitative analysis on wind turbine end-of-life options. Himpler and Madlener (2012) study the economics and optimal timing of repowering for the case of Danish wind farms. They do so by applying a two-factor real options model, where revenues and investment costs are considered uncertain and modelled through separate geometric Brownian motions. From this, they find that uncertain revenues are a major hinder to further development of repowering in Denmark, and that the selling price of the used turbine is a minor factor in the repowering decision. The study is, however, limited to repowering and does not consider life-extension or decommissioning. It also imposes limitations on the underlying uncertainty of the revenue flows, as they are modelled through a single stochastic process. Mauritzen (2014) apply a Cox regression model to Danish wind farms and find that a combination of land scarcity and Danish subsidy policies makes turbines in better locations more likely to be repowered due to the higher alternative cost.

Santos-Alamillos et al. (2017) utilizes a mean-variance portfolio optimization approach to explore varying wind power repowering scenarios in Spain. They find that repowering can contribute to more efficient wind farm portfolios by reducing fluctuations in aggregate power supply and increasing productivity. Hou et al. (2017) develops an optimization tool for repowering offshore wind turbines with the objective of minimizing the levelized cost of energy. Castro-Santos et al. (2016) determines the main costs and feasibility of repowering through calculation of the project net present value (NPV), and conclude that repowering is a financially viable option. However, neither Santos-Alamillos et al. (2017), Hou et al. (2017) or Castro-Santos (2016) consider the optimal timing of the investment. Santos-Alamillos et al. (2017) and Hou et al. (2017) provide tools to optimize the investments after the decision to repower is made. Castro-Santos et al. (2016) apply a now-or-never decision tool to decide whether repowering is viable or not, thereby disregarding the value of flexibility in the investment timing as well as the underlying uncertainty in the profit flow.

There are also qualitative studies that aim to shed light on different aspects of end-of-life options. Ziegler et al. (2018) reviews current state-of-the-art methodology for lifetime-extension of onshore wind turbines in Germany, Spain, Denmark and the UK, and find that a significant market for end-of-life solutions will develop in the coming years. del Río, Calvo Silvosa and Iglesias Gómez (2011) makes a qualitative analysis of instruments and design options that support repowering of onshore wind turbines, with the aim of finding an optimal policy design for incentivising repowering of wind farms. They find that all instruments have their advantages and drawbacks, but feed-in-tariffs seem to be particularly appropriate to incentivise repowering.

Our thesis makes its contribution by addressing some of the limitations in the existing literature. First, we expand the study of Himpler and Madlener (2012) by evaluating both repowering and life-extension. Second, we develop both a single-factor model and a multi-factor model in order to address both general and case specific questions related to the end-of-life options. The single-factor model provides general insights into the investment decision of wind farm operators, and how the underlying factors affect the dynamics of the option values. From this analysis we find that the repowering and life-extension option values vary at different rates when the underlying parameters are changed; which indicate that the specific underlying parameters will determine how attractive the two options are relative to one another, and that parameter changes will shift this balance.

The multi-factor model, which can be calibrated to specific wind projects and impose technological and legal restrictions in a flexible manner, evaluates the option values, optimal exercise timing and which scenarios that make one option preferable to another. We thereby supplement the existing end-of-life options literature by proposing models that evaluates all the viable options for an operator, while accounting for practical aspects and individual conditions of the wind farm operation in a flexible manner. By applying the model to a numerical case study, we find that the optimal decision for the wind farm operator strongly depends on how the market conditions develop throughout the lifetime of the wind farm. For favourable early market conditions, the option to repower is preferable and will be exercised. For moderate early market conditions, the operator prefers to wait and see which way the market develops; while for unfavourable market conditions, the operator will wait for the life-extension option to become available and exercise it optimally during the remaining lifetime of the wind farm. Finally, the inclusion of contractual power purchase agreements in the case study shows that increasing the fraction of power production sold through long-term agreements reduces the electricity price risk exposure and incentivises earlier investment in both options. By reducing the exposure to market conditions, it also makes the result of preferring the repowering option for favourable early market conditions less prevailing.

The rest of this thesis is organized as follows. In Chapter 1, the business case for wind power end-of-life options is considered and it is argued for why this is a topic of growing economic interest. In Chapter 2, an investment decision analysis is conducted, with the objective of identifying an appropriate investment decision tool and evaluating different modelling approaches. In Chapter 3, the single-factor and multi-factor real options models are developed, and the single-factor model is analysed to provide some general insights into the investment problem. In Chapter 4, a numerical case study for a generic Norwegian wind farm is presented and the multi-factor model is calibrated to evaluate it. In Chapter 5, the results from the numerical case are presented, along with a discussion of the implications and insights these provide. Finally, in Chapters 6 and 7, the conclusions of the thesis are presented and further research is discussed.

# 1 – The business case for wind power end-of-life options

The world's energy consumption is expected to dramatically increase in the coming decades[1]. Simultaneously, growing environmental concerns put restrictions on the use of fossil fuels and nuclear power. The Paris agreement, which is a political agreement signed by virtually every country on earth, limits the carbon emissions of each country in an effort to mitigate the damaging effects of global warming[2]. To meet the increasing demand of energy, while simultaneously keeping the political commitment of the Paris Agreement, renewable energy will have to play an increasingly important role in the years to come. From Figure 1.1, it can be observed that Bloomberg (2018) expects the energy mix to consist of more than 64 % renewables by 2050, with wind and solar playing a dominant role.

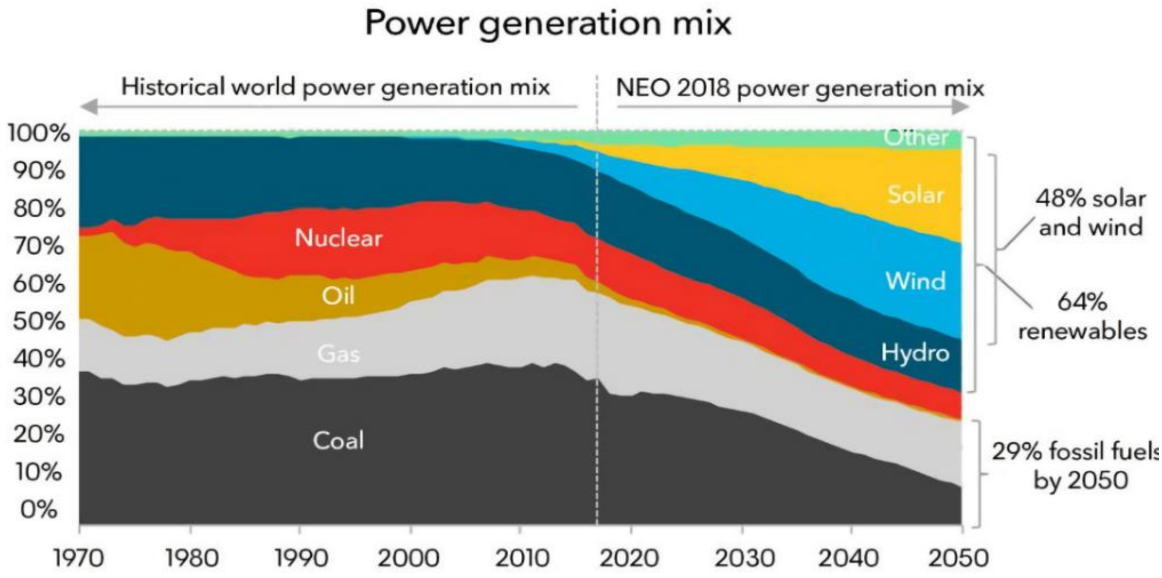


*Figure 1.1: Forecast for the power mix towards 2050. Source: Bloomberg*

The development of the energy industry already in progress shows a strong global commitment to reach this scenario. From Figure 1.2 it can be observed that wind power has become a massive global industry, with more than $1 trillion invested in the last 15 years alone. The trend of development shows that yearly wind power investments are rapidly increasing, along with other forms of renewable energy investments.

Determined to do its part in mitigating global climate change, the European Union (EU) has set a combined renewable energy production target for all its member states at 20% by 2020, and 32% by 2030[3]. By 2050, Bloomberg (2018) estimates the EU will reach almost 90% renewables, with wind and solar power expected to be the most important contributors. WindEurope (2017) estimates that by 2030, wind power will cover between 21.6 – 37.6 % of the EU's electricity demand, with between 147 – 351 billion EUR invested

1 https://www.eia.gov/todayinenergy/detail.php?id=32912

2 https://unfccc.int/process-and-meetings/the-paris-agreement/what-is-the-paris-agreement

3 https://ec.europa.eu/energy/en/topics/renewable-energy

in building 96 – 237 GW of new wind power capacity. Therefore, we can conclude that the market for wind power is booming and will continue to do so in the coming decades.

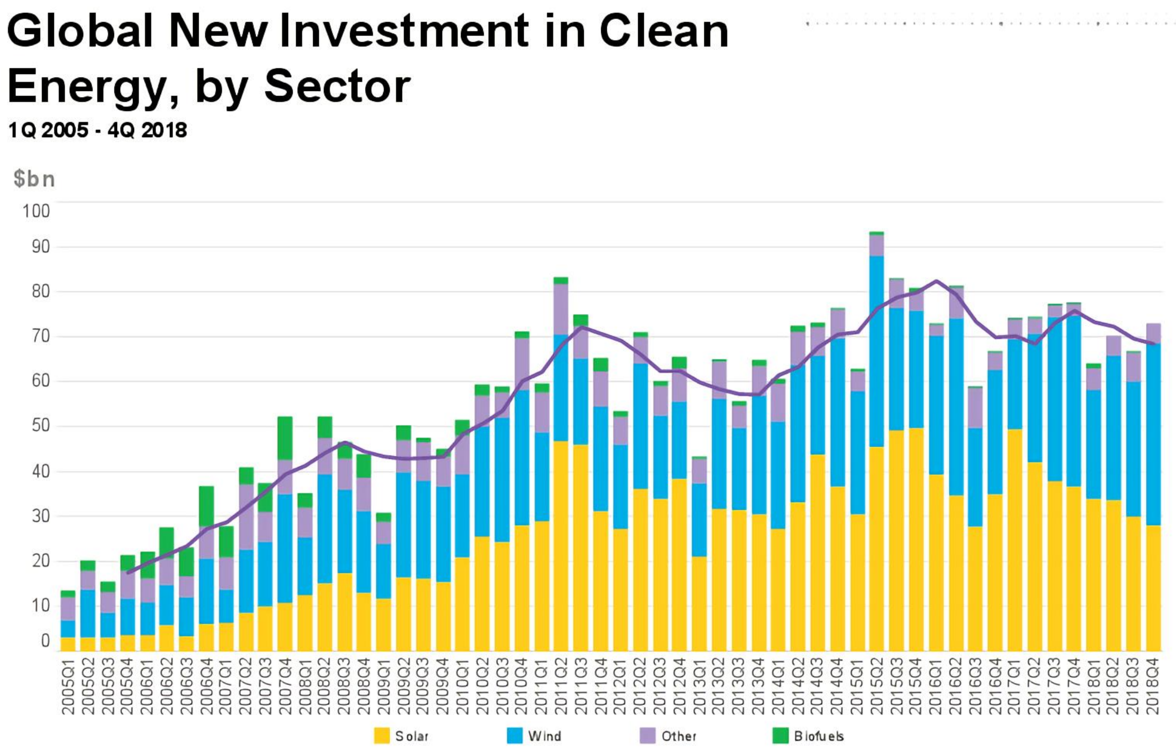


*Figure 1.2: Global new investment in renewable energy. Source: Bloomberg*

An important concern that naturally arise as this development continues, however, is how to deal with aging wind farms. The expected lifetime of a wind turbine is in the range of 20 – 25 years[4], hence as the world continues to install new wind power capacity in line with the prognosis from Figure 1.2, there will be an increasing amount of aging wind capacity to handle and a rapidly growing market for good end-of-life solutions for wind turbines.

This is, however, not only a concern for the future; the need for proper handling of aging turbines is already a pressing issue. In 2016, 12 % of the installed capacity in Europe were 15 years or older, and by 2020, this share will have increased to 28 % (Ziegler et al., 2018). In effect, many wind farms will reach the end of their life in the near future, and the demand for end-of-life evaluation will become increasingly crucial as installed capacity continues to increase.

[4] https://www.renewablesfirst.co.uk/windpower/windpower-learning-centre/how-long-do-wind-turbines-installations-last/

For wind farms nearing the end of their expected lifetime, there are three available options:

***Decommissioning*** is to dismantle the turbines and recycling the materials, in effect shutting down the operation of the wind farm.

***Life-extension*** is to extend the technical lifetime of the wind farm and is done by restoring or upgrading components of the turbines.

***Repowering*** is to decommission the old turbines and replace them with new and usually better turbines.

Figure 1.3 illustrates the decision tree a wind farm operator is facing at the end of the turbine lifetime, and in the following paragraphs each of the three options will be elaborated upon.

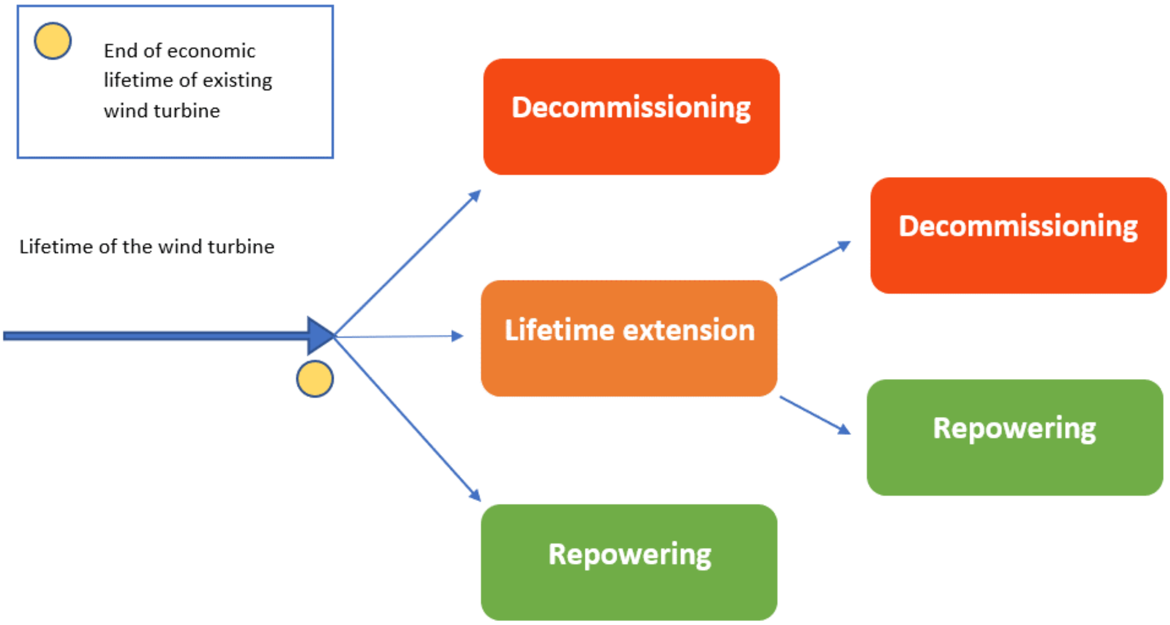


*Figure 1.3: Decision tree facing a wind farm operator*

Note that Figure 1.3 depicts the decision tree for the currently operating wind farm only. For each time the wind farm is repowered, the operator will face a new, identical decision tree. It should also be mentioned that while the wind farm operator makes his decision at the end of the economic lifetime of the wind farm, the decision can at most be postponed until the end of the technical lifetime of the wind farm. This is a quite important distinction, as the technical lifetime of a wind farm is how long it can operate before it is at risk of technical failure, while the economic lifetime is how long it is profitable to keep in operation considering alternative costs.

Although all three options are practically available, decommissioning before the end of the technical lifetime of a wind park is generally not economically viable. This is because the operating profit of a wind turbine will almost always be positive, and the initial investment is close to entirely irreversible after operation has started. For wind power the energy input is free, therefore, the operational costs are mainly related to maintenance (EWEA, 2009). It is generally also more profitable to repower a wind farm at the end of its technical lifetime rather than to decommission. This is because the location has already been deemed economically feasible for wind power production, and technological advancement during the lifetime of the wind park will allow the repowered wind farm to utilize the wind of the location even better and therefore be more profitable. Hence, decommissioning is generally not considered to be an attractive option. There are however circumstances that make decommissioning necessary, although they are usually not motivated by financial optimization. The wind farm operator might lack the necessary capital to repower or life-extend his wind farm at the end of the technical lifetime, or the government might be unwilling to grant the necessary concession to use the land for this purpose. The operator might also have been granted concession to use a location better suited for wind power, and lacks the capital for two simultaneous wind projects.

When assessing the life-extension option there are several aspects that need to be considered. As the design lifetime of different components varies, it is common to renew only one or a few components at a time. Also, the design lifetime of a component can vary quite significantly from its real lifetime, which depends on project specific factors such as local wind speeds, turbulence levels and wake effects from nearby turbines (Ziegler et al., 2018). It is therefore important to utilize site-specific data measured over time to estimate the remaining structural lifetime of components when determining how to optimally life-extend a turbine. Note that optimal life-extension of a turbine is not just a matter of financial evaluation, as there are strict legal restrictions to ensure structural safety (DNVGL, 2016). In the years post life-extension it is also required to subject the wind park to regular inspections to ensure that structural integrity has not been compromised, and if found to be the case, the turbine will be shut down until further life- extended (MegaVind, 2016).

In addition to allowing the turbine to operate and generate revenues longer, life-extension can also have the added benefit of significantly improving the performance of the turbine and reducing the levelized cost of electricity. It is also relatively inexpensive compared to repowering[5]. Note that recent developments to monitoring systems have made predicting the remaining lifetime of individual turbine components more accurate and less expensive[6], hence improving the viability of life-extension. Since decommissioning is often not an economically viable end-of-life option, as was discussed above, we will for modelling purposes assume that life-extension is always followed by repowering. Note that this does not entail disregarding decommissioning entirely from the modelling, as choosing not to exercise the other end-of-life options in effect is decommissioning.

Repowering is typically done at the end of the economic lifetime of a wind farm, when the opportunity cost of not repowering is estimated to be greater than the expected present value of future operating profit flows from the current wind farm. There can also be factors unrelated to the economics of the specific wind project that motivate repowering, for instance improving the stability of the power grid by repowering to a turbine that has more reliable output. Because the operating profit of aging wind farms is almost always positive,

[5] https://www.spicatech.dk/about-us/press/is-lifetime-extension-of-your-ageing-turbine-the-right-solution/

[6] https://www.ge.com/renewableenergy/digital-solutions/digital-wind-farm

scarcity of land is a necessary condition for repowering to be an economically viable option. If viable land is not a scarce resource, it would always be more beneficial to keep the old wind farm operating until the end of its technical lifetime and invest in a new wind farm elsewhere rather than repower. In effect, without land scarcity, the option to repower would become an option with only one relevant exercise time – at the end of the technical lifetime of the wind farm.

Scarcity of land arises because land suitable for producing wind power is a limited resource. Broadly speaking, there are four factors that affect the attractiveness of wind power production at a location. First, political regulation limits where wind farms can be installed. Because of noise and visual pollution, wind farms need to be kept away from residential areas, nature attractions and other locations of public interest. Second, the local wind conditions determine the potential for wind power production at the location. Naturally, higher average wind speeds at a location is favourable as it allows higher power production. Third, the cost of installing new grid infrastructure for transmission of power from the wind farm will vary across locations. In some areas the infrastructure is already good enough to handle the power from wind farms, while in other locations additional transmission capacity will have to be built at a significant investment cost. Fourth, the price of power varies for different areas, both between countries and often also within countries. As power markets become more integrated this is a factor of decreasing importance, but it is still an important consideration for many projects. The combination of these four factors makes the land suitable for wind power production limited, which implies significant value in optimal resource utilization.

From the business case for wind power end-of-life options, we have formulated several research questions that we aim to answer throughout this thesis. We have separated the questions into two categories, general and project-specific, as we need to tailor a method specifically suitable for addressing each category.

**General questions:**

- Which factors affect the end-of-life options?
- How do underlying factors affect the option values and the investment decision?
- How does the option value of repowering change relative to the option value of life-extension when the underlying parameters change?

For an operator to make an informed and optimal decision regarding end-of-life options for his wind farm, knowing which underlying factors that affect this decision and how is of vital importance. From the foundation of these underlying factors, we can then disentangle the investment problem into separate parts, quantify them and combine them into a decision tool. By analysing how changes in the underlying parameters affect the option values, we can build intuition around the investment problem behaviour and identify general scenarios for which one option is preferable to another.

**Project-specific questions:**

- What is the value of having the option to repower or life-extend a specific wind farm?
- How long should a specific wind farm operator be expected to wait before exercising an end-of-life option for his wind farm?
- In which scenarios are one of the options preferable to the other?
- How does long-term power purchase agreements (PPAs) affect the option values and investment decisions for a specific wind farm?

The project-specific questions are of interest because they are more closely related to the actual decision making. Answering the general questions provides a general understanding of the end-of-life option behaviour and its relationship with underlying factors, while answering the project-specific questions provide insights and recommendations tailored to specific, real wind projects. This insight should allow the wind farm operator to determine the optimal end-of-life option decision for his wind farm, and how much value these options can provide him when utilized. Quantified option values can also be useful for potential wind power investors to aid them in properly assessing the value of the asset they are considering investing in. Finally, by knowing how the option values for a specific wind farm are affected by PPAs, the operator can determine whether such an agreement would be optimal for his wind farm.

# 2 – Investment decision analysis

In this chapter, we first discuss appropriate investment decision tools to help us quantify the investment problem, and to determine the value and investment strategies for wind power end-of-life options. Then, we develop an understanding of what drives the profitability of a wind turbine during its lifetime. More specifically, we identify the factors that contribute to the operational profit flow of a wind turbine and how they change over time.

## 2.1 – Finding the investment decision tool

Choosing the appropriate decision-making tool depends on the problem at hand and the objectives one wishes to achieve. Consequently, we need to know the strengths and weaknesses of the possible decision tools at our disposal and evaluate how these will impact what we are analysing. One possible approach is to use a cash flow analysis where each cash flow element is estimated based on available information and assumptions about the future. From the cash-flows one can find the profit flows for each period, which can be discounted with an appropriate risk-adjusted cost of capital to find the net present value (NPV) of the project. This type of analysis has the advantages of being simple to perform and providing results that are easy to interpret. However, it also has some clear drawbacks. According to Dixit and Pindyck (1994), NPV performs poorly when the following characteristics are present: Uncertainty about future profits, investment costs are fully or partially irreversible (sunk) and the investment timing is flexible. In the NPV case, everything is based on predefined parameter values which does not take uncertainty into account. Also, NPV studies do not allow for flexibility in investment timing - it's a now or never decision tool.

Depending on the research questions and the problem at hand, NPV can still provide some useful insights. Campoccia et al. (2009) utilize NPV-analysis in order to compare different political energy support schemes in European countries, with the main goal of putting into evidence the main differences in the support policies adopted. Thus, the goal is not to give exact representation of the problem, but rather showcase main characteristics. Chong et al. (2011) use NPV-analysis to assess the economic feasibility of an innovative wind-solar hybrid renewable energy system. In this case, NPV is used to give indications to whether the project is even possible, and not to give advice on the final investment decision.

When uncertainty, irreversibility and managerial flexibility are present in an investment problem, Dixit and Pindyck (1994) argue for the implementation of real options models. This modelling approach was first introduced by Myers (1977) and the goal was to address the shortcomings of traditional capital budgeting approaches, such as the NPV. Real options can be considered an extension of the financial option theory and treats investment opportunities and real assets in much the same way as financial call or put options. In contrast to the "now-or-never" proposition implicit in traditional NPV-analysis, the real option method seeks to gain value by deferring an irreversible investment expenditure, and in such take managerial flexibility into account. The option pricing theory of Black and Scholes (1973) was therefore applied to the valuation of "real", non-financial assets with learning and flexibility. Trigeorgis and Mason (1987) refer to the investment project value with managerial flexibility as "expanded" or "strategic" NPV, which is the sum of the traditional NPV and the value of managerial flexibility. Hence, the real options model is a valuable tool that more accurately estimates the value of uncertain and flexible projects than traditional capital budgeting methods (Herath and Park, 1999; Benninga and Tolkowsky, 2002; Newton et al., 2004).

Real options models can incorporate uncertainty and managerial flexibility, thereby gproviding a more realistic representation of many actual investment problems. However, there are also some weaknesses with this approach. The two most evident being the complexity of calculations and the fact that many financial practitioners are unfamiliar with the approach. Horn et al. (2015) find that only 6% of the 1500 largest companies in Norway, Sweden and Denmark (500 from each country) make use of real options models. Of the non-users, 70% reported that they were unfamiliar with real options concepts and techniques, while the remaining 30% argued that the complexity was the primary hinder for implementation. This discrepancy between how favourable academics and practitioners view real options models indicate that the focus of continued research should be not only on developing better models, but on convincing practitioners of their viability and reducing the knowledge gap needed to apply them to real problems. A motivation for this thesis has therefore been to make the modelling transparent and easy to implement. Horn et al. (2015) do however also find the percentage of real options practitioners in the energy sector to be 24%, the highest of any specific industry and four times the business community average. It can therefore be inferred that the characteristics of the energy industry makes real options modelling a more attractive tool.

Having discussed potential decision tools more generally, we now return our attention to financial strategies for repowering and life-extension of wind farms. According to Martínez-Ceseña and Mutale (2011) and Santos et al. (2014), investments in renewable energy are highly irreversible and subject to several uncertainty factors. Consequently, they argue that real options modelling is the most suitable tool for renewable energy valuation. According to Kozlova (2017), there is a growing number of papers that use real options modelling to evaluate renewable energy related projects. The reason for this development is found to be the model’s ability to incorporate uncertainty and managerial flexibility when assessing irreversible investment cost. As a result, we believe a real options model will be the most suitable tool to assess a wind farm’s end-of-life options.

In the rest of this chapter we discuss the components of the wind farm profit flow in more detail. In Chapter 2.2, we look at how the profit flow can be modelled. Then, in Chapter 2.3, we present general characteristics of the price of energy and discuss how the electricity price can be modelled. Finally, in Chapter 2.4, we provide background information needed to understand how a wind farm produces energy and discuss how production can be modelled.

## 2.2 – Modelling approach for the profit flow

Since uncertainty is an important element of the profit flow of renewable energy projects, we consider different possibilities for how it can be modelled. One alternative is to allow the profit flow to follow a single stochastic process that captures all the underlying uncertainty, and as a result get a single-factor real options model. This is done by Kitzing et al. (2017) where the operational gross margin per unit capacity follows a geometric Brownian motion (GBM). The uncertainty in the profit flow can also be described by a single stochastic process by assuming there is only one component of the profit flow that contributes to the uncertainty. This is done by several papers, for example Fleten and Maribu (2004) who assess investment timing and capacity choice of small-scale wind turbines where electricity is the only uncertainty factor following a GBM. Another possibility is to let the production and electricity price follow separate, uncorrelated GBMs and combine them into a single GBM by utilizing that the product of two uncorrelated GBMs will itself follow a GBM (Dixit and Pindyck, 1994). By doing so, it is possible to reformulate the problem into a single-factor model and solve the problem using a standard approach.

Letting the operating profit flow follow a single stochastic process makes it possible to derive analytical, tractable results while still accounting for the general characteristics of the underlying uncertainty of the investment decision. This can be advantageous because it allows for general sensitivity analysis of how different parameters affect the profitability of the project and to develop a sense of the behaviour of the investment problem. A single-factor model can therefore be a valuable tool to provide general results and to build intuition behind the problem. A drawback is, however, that whenever the investment problem is affected by several underlying factors that contribute to the total uncertainty, some valuable information might be lost by combining them into a single factor.

Another viable alternative is therefore to allow the profit flow to be driven by multiple underlying stochastic processes and thereby design a multi-factor model. A natural way of doing so, is to separate the profit flow into its primary components and let them follow separate stochastic processes that capture their individual characteristics. This approach can make it possible to capture more of the underlying uncertainty that affect the profit flow, thereby making the model better aligned with the physical conditions of the real world. A drawback of this approach is however increased complexity, and that results are hard to generalize beyond the specific project in consideration.

Motivated by finding answers to the research questions posed in Chapter 1, we find it appropriate to develop both a tractable single-factor model and a flexible multi-factor model. The single-factor model can provide insights into the general behaviour of the end-of-life options and can, due to its tractability, make it easier to disentangle the effects of the underlying parameters in the model. To be able to derive closed-form, tractable solutions for the single-factor model, it will be necessary to make some simplifying assumptions. However, in order to be able to evaluate specific projects, these assumptions might need to be relaxed. Hence, we also develop a more flexible multi-factor model that can be calibrated to specific wind projects and thereby evaluate them in a more accurate and realistic fashion. This allows us to capture project specific variations such as PPAs, and some non-financial restrictions related technological and legal aspects of the wind farm operation.

In order to build a multi-factor model, we need to split the profit flow into its primary components through separate stochastic processes. Therefore, in the next two subchapters, we describe general characteristics and behaviour of the price of energy and wind power production, and discuss how they can be modelled.

## 2.3 – Price of energy

The first primary component of the operating profit flow is the price of energy. As the revenues of a wind turbine are generated by producing energy and selling it, the price at which the energy is sold is of major importance. Wind turbines generate energy in the form of electric power, which is sold either through a market or through long-term contracts. The price can also be subject to political subsidies, most often designed to make renewable power production more attractive. In this chapter we discuss these sources of income for the wind farm in more detail.

### 2.3.1 – The market price of electricity

Electricity share many of the same features as other commodities such as gold and copper. It is homogeneous, in that it cannot be easily distinguished where or how it is produced, and it is traded in global markets. However, as described in Burger, Graeber and Schindlmayr (2014), electricity differs from other commodities on several accounts. First and foremost, electricity cannot be stored easily or inexpensively on a large scale in the same manner that for instance gold can. Batteries can be utilized to store some energy, but they come at a significant cost. For this reason, most of the energy produced must be used immediately when it is generated, and consequently, supply and demand of electricity must always match at any given time across the grid. Second, electricity exhibit transportation constraints. This is because transportation of electricity causes loss of energy, making it economically infeasible to transport energy over very long distances. Third, the demand for electricity is highly inelastic, which results from the fact that electricity is a basic need in many households. Fourth, electricity prices exhibit some seasonal effects that are caused by demand varying throughout the year. In the Nordics, the heating demand is much higher during winters due to the cold weather, which leads to significantly higher demand for electricity during winter than summer. In other parts of the world, the opposite can be observed as high summer temperatures drive up the electricity demand for cooling. Fifth, the behaviour of the market is heavily dependent on the country or region-specific composition of the power plant portfolio. That is, the market behaviour depends on which energy sources provide market supply.

In order to better understand how prices are settled in the market, we consider the day-ahead Nord Pool market as an example. On the supply side there are a wide range of technologies competing for the demand. In a typical country, the supply will be covered by renewables, nuclear power, hydro power, natural gas and other fossil fuels. The supply creates a stepwise function, as illustrated in Figure 2.1, and is determined by the marginal cost of production for each supplier. From the figure we can observe that the bids from nuclear and wind power enter the supply curve at the lowest level, due to their low marginal costs, followed by combined heat and power plants; while condensing plants and gas turbines have the highest marginal costs of power production. The electricity spot price is finally determined by matching supply and demand. Consequently, the power plant portfolio that contributes to the supply side will play a major role in the final market price. If more of the power was produced from the low marginal cost power plants, the price level would be moved downwards. The steepness of the demand curve should also be noted, which illustrates that the demand is inelastic.

Another important observation is that in periods of increased production, the electricity price goes down. When the wind is blowing strongly, the production of wind power goes up, and consequently the price likely goes down. Similarly, when production decrease, the prices goes up. Therefore, the electricity price will be negatively correlated with the production.

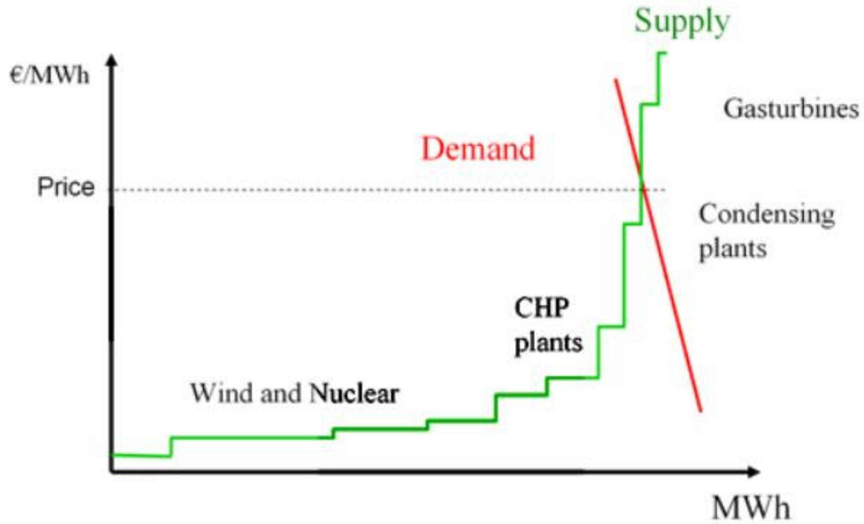


*Figure 2.1: Example of annual supply and demand for the Nordic power system.*
*Source: Windenergythefacts*

### *2.3.1.1 – Modelling approach for the price of electricity*

With the general characteristics of electricity in mind, we turn our consideration to how the prices behave and can be modelled. The analysis done on electricity prices in this thesis will be focused on the Nord Pool Market, as this is where we most easily can retrieve usable data. According to Seifert and Uhrig-Homburg (2007) it can be shown through investigation of historical spot prices on the Nord Pool market that the prices are characterised by high volatility, strong mean-reversion and frequent jumps. The presence of mean-reversion can be validated by performing statistical tests, such as the variance ratio test and ADF test (Palchykov and Vardøy, 2018). In addition, Lucia and Schwartz (2002) analyse the Nordic Exchange's spot, futures and forward prices and conclude that seasonality effects can be observed and are of crucial importance in explaining the shape of the futures and forward curves.

For these reasons, Schwartz (1997), Schwartz and Smith (2000), Lucia and Schwartz (2002) and Pilipovic (2007), argue for the inclusion of mean-reverting behaviour, long term uncertainty and seasonality when modelling electricity prices. Also, Kaminski (1997), Clewlow and Strickland (1999), Deng (2000) and Seifert and Uhrig-Homburg (2007) emphasizes the importance of including jump components in the electricity price models. A model proposed by Benth, Kallsen and Meyer-Brandis (2007) manages to capture mean-reverting behaviour, jumps and seasonality through season-dependent jumps and could be utilized to capture the aforementioned characteristics.

However, these articles have the objective of modelling electricity prices as accurately as possible. Depending on the problem at hand, it may be both advantageous and reasonable to make simplifying assumptions about the price process.  The choice of price process comes with a tradeoff between accuracy and complexity. By including more of the known characteristics, the process is better able to replicate realistic behavior. However, the added complexity may not provide any additional insights into the problem at hand if short-term variations are not of interest. For long-term investment decisions, it is possible to exclude the effects of seasonality and jumps to only focus on long-term behavior. The justification of doing so is that short-term variations have relatively little impact on long-term decisions (Fleten et al., 2007). For modelling long-term behavior of electricity prices, an Ornstein-Uhlenbeck process that takes mean-reversion into account or a more simple GBM are both viable options. Of these two, the Ornstein-Uhlenbeck is the most flexible one and be calibrated to mimic a GBM by letting the mean-reversion speed be low.

### 2.3.2 – Power purchase agreements

As is evident from the previous discussion, the electricity price is subject to considerable uncertainty, making it hard to predict what the price will be in the future. As a result, the power producer will be facing considerable price risk, and thereby uncertainty in future profits. A method for power producers to reduce the exposure to this risk, is by entering long-term contracts known as power purchase agreements (PPA). In fact, it is estimated that approximately 55 % of all the electricity produced in the Nordic market is sold through PPAs or through bilateral electricity trade (trading that takes place outside the power exchange)[7]. However, the spot and forward prices have considerable impact on the prices agreed on in such contracts. In Figure 2.2 we illustrate the dynamics of PPA contracts.

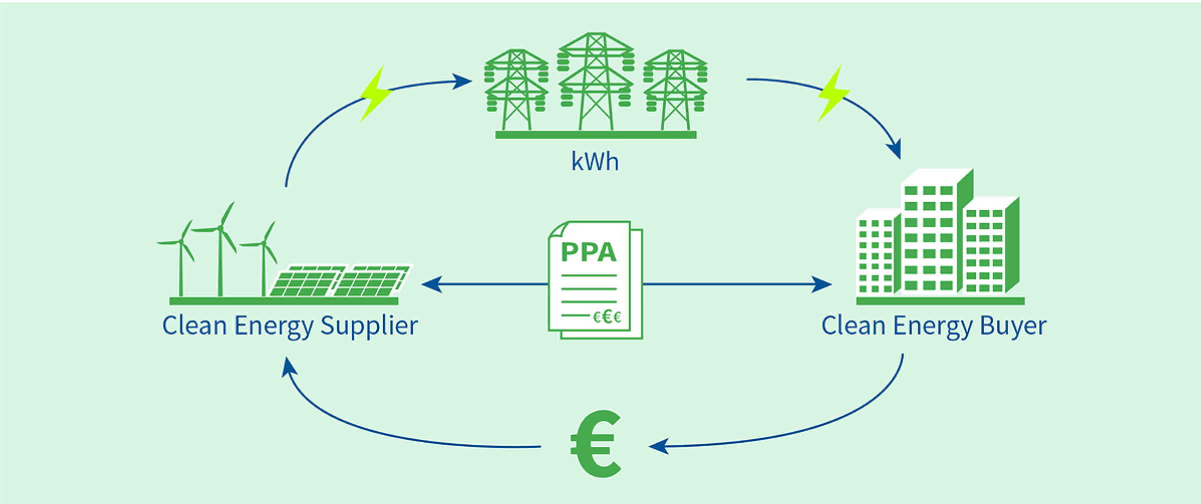


*Figure 2.2: Diagram showing the dynamics of a PPA. Source: Enel*

A PPA is a contract between two parties: A seller (the party producing energy) and a buyer (the party purchasing and often using the energy). In the contract, the two parties can specify everything related to the energy transaction. This includes the amount of energy, at what price and at what time the energy is to be delivered[8]. Through PPAs, the power producer can reduce some of the electricity price risk exposure and hedge some of the future revenues. In doing so, the energy producer will also be able secure debt financing at lower interest rates as the risk of default is diminished by the PPA.

[7] https://www.wind-energy-the-facts.org/power-markets.html

[8] https://en.wikipedia.org/wiki/Power_purchase_agreement

PPAs have, in addition to reducing price risk for the power producer, other positive ramifications. In many cases the buyer is a large energy consumer, i.e. an industrial company that utilizes electricity as a key input in their production, implying that the buyer itself is exposed to significant electricity price risk. Therefore, both parties can have large hedging benefits from settling on a common price. PPAs have also become a popular contract form for businesses that want to reduce their carbon footprint. By entering agreements with renewable energy producers, companies can make a visible commitment to sustainability, and at the same time manage volatile energy costs. For this reason, the volume of corporate renewable PPAs almost tripled from 2015 to 2016[9].

Most importantly, since PPAs reduce the exposure to electricity price risk, it can contribute to making renewable energy projects viable to particularly risk-averse investors such as governments. This was the case for the Fosen Vind project, currently the largest wind farm in Norway, where Hydro bought close to a third of the expected future power production from the project[10]. While it is certain that the agreement made the profit flow of Fosen Vind less risky and was beneficial to the realization of the project, the exact contractual specifications remain secret to the public. The exact impact of the agreement on the investment decision is therefore not possible to ascertain. This lack of public insight is a common denominator for PPAs, as the specific content of these contracts is sensitive information due to competition. Therefore, we include PPAs in our model in order to better understand the impact the agreements have on the profitability of wind farms and the viability of end-of-life options.

### 2.3.3 – Subsidy schemes

In order to reach political targets for renewable energy and to stimulate faster development, many governments have introduced different subsidy schemes that incentivise power producers to invest in green energy. The subsidy schemes vary from country to country, but they are all designed to either increase revenues or to lower costs. In Europe, the most common schemes are tradeable green certificates (TGC), feed-in tariffs (FIT) and feed-in premiums (FIP).

In Norway, the subsidy scheme utilized is the TGC. With TGCs, the producers receive, once approved for the support scheme, a certain number of certificates based on their production targets which they can sell on the certificate market. Thereby, renewable energy producers receive the income from selling their certificates on top of the income from the energy they sell. The utility companies are on their end obligated to purchase the certificates sold on the market, and to levy the cost onto the end consumers when selling the energy. In effect, the cost of the certificates is carried by the consumers[11].

The Norwegian market for TGCs is currently shared between Norway and Sweden. Renewable power producers can receive certificates for up to 15 years, with the number of certificates being decided based on the amount of energy produced in the allocation

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[9] http://resource-event.eu/new-to-ppas/

[10] https://www.adressa.no/nyheter/okonomi/2016/02/23/Hydro-kj%C3%B8per-str%C3%B8m-fra-Fosen-turbinene-12192768.ece

[11] http://www.vindportalen.no/Vindportalen-informasjonssiden-om-vindkraft/OEkonomi/Stoetteordninger-for-vindkraft/Elsertifikater

period. However, this is about to change in the coming years. With increasing technological development and decreasing capital costs, renewable power projects are becoming more profitable. The governments in Norway and Sweden have therefore decided to end the support system with a deadline coming up soon. In order to be eligible for support in Norway, new wind farm installations need to be connected to the grid before the end of year 2021. In addition, those who start operation after 2020 will only be able to receive support until 2035[12].

Because subsidy schemes are soon to be removed in Norway, and the general international trend is following in the same direction due to increasingly profitability, we do not explicitly include political subsidy schemes in our model. For further references to studies that explicitly considers subsidies, see Eryilmaz and Homans (2016) and Boomsma and Linnerud (2015) who investigate how uncertainty in renewable energy policy affects investment decisions, Monjas-Barroso and Balibrea-Iniesta (2013) who consider which of the countries Denmark, Finland and Portugal that has the strongest public incentive for wind energy investments, and Boomsma, Meade and Fleten (2012) and Kitzing et. al. (2017) who look at how different support schemes affect the investment decision. It should, however, be noted that our multi-factor model is flexible enough to incorporate deterministic subsidy schemes such as feed-in tariffs and premiums with very minor adjustments, and stochastic subsidy schemes with only somewhat more extensive alterations, if this should be desirable. Intuitively, with the tendency of existing subsidies being retracted, one would expect repowering to become significantly more valuable relative to life-extension close to the retraction deadline. The reason being that repowering would make the wind farm eligible for the subsidy scheme, if performed before the deadline, while life-extension would not.

## 2.4 – Production of energy

The second major component of the operating profit flow is the production of energy. To identify the factors that determine how much power is produced at any given time, it is essential to understand technological and physical relations that govern how wind power is generated. In addition, we investigate how the performance of wind turbines change over time as well as the historical and future development of wind power technology. Finally, we consider the behaviour of wind, as the driving force behind wind power production, and identify suitable approaches to modelling it.

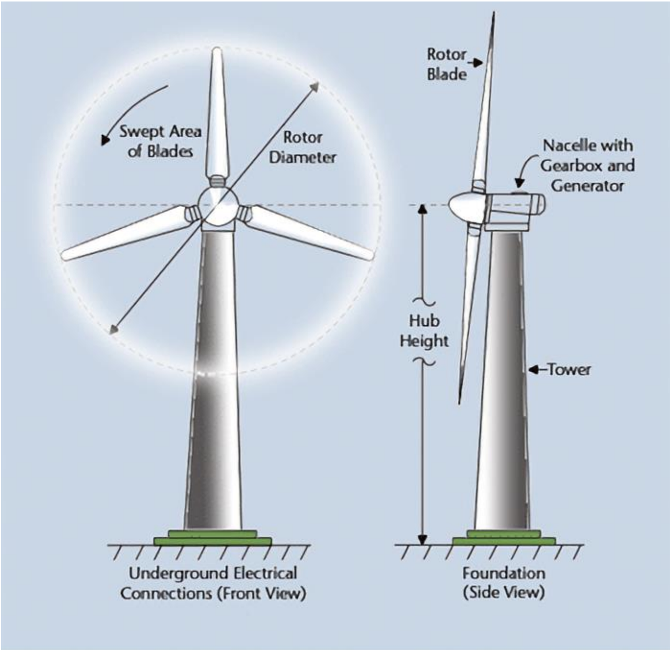
### 2.4.1 – Wind power technology

This subchapter is structured as follows. In 2.4.1.1, we describe the main components of a wind turbine and introduce important equations related to the power generation. In 2.4.1.2, we investigate how the performance of wind turbines change over the operational lifetime. Finally, in 2.4.1.3 we address technological development and discuss different ways of modelling technological innovation.

[12]http://www.vindportalen.no/Vindportalen-informasjonssiden-om-vindkraft/OEkonomi/Stoetteordninger-for-vindkraft/Elsertifikater

### *2.4.1.1 – Wind turbines and power generation*

A wind turbine consists of a nacelle, mounted atop a tower, and a hub with three blades that is connected to the nacelle. Inside the nacelle there is a gearbox, a generator and some other minor components. Lift power generated by the wind causes the turbine blades to rotate, which in turn causes the generator inside the nacelle to rotate and generate electrical power.



Drawing of the rotor and blades of a wind turbine, courtesy of ESN

*Figure 2.3: Main components and dimensions of a wind turbine*

The power generated from a wind turbine can be calculated as (Dixon and Hall, 2014)

$$h = \frac{1}{2} C_p \rho \varepsilon_g A w^3, \tag{2.1}$$

where $C_p$ is the turbine power coefficient, $\rho$ is the air density, $A$ is the cross-sectional area swept by the turbine blades, $\varepsilon_g$ is the generator efficiency, and $w$ is the wind speed. The power coefficient of the turbine gives the ratio of the mechanical power extracted by the turbine relative to the wind power passing through the cross-section of the turbine blades. The maximum theoretical power coefficient, known as the Betz limit, is 59.3% (Dixon and Hall, 2014). Note that the Betz limit is calculated assuming no frictional losses, which is likely not possible in reality.

The *rated capacity* is the maximum power output a turbine is designed to achieve, and will typically be available for a fairly large range of wind speeds. If the wind speed drops below this interval, the power produced will drop rapidly. The lowest wind speed for which a turbine can produce power, is known as the *cut-in speed*. The highest speed for which a turbine can produce power, before the turbine will be shut down to avoid damage, is known as the *cut-off speed*. Because the turbine will not operate at its rated capacity all the time, the *capacity factor,* which is the ratio of average power output over the rated capacity, is used to indicate actual power production. See Figure 2.4 below for an illustration of how the power output varies with wind speed.

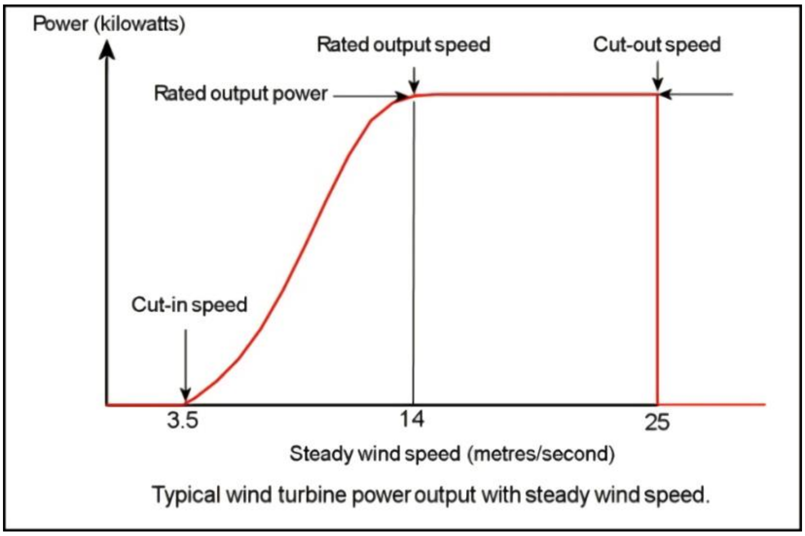



*Figure 2.4: Example of power output of a wind turbine for different wind speeds. Source: Windpowerprogram*

By including the capacity factor, the energy generated yearly by a wind turbine can be expressed as

$$E = h * t * k = \frac{1}{2} C_p \rho \varepsilon_g A w^3 t k, \tag{2.2}$$

where $t$ is the number of hours per year and $k$ is the capacity factor. Having a high capacity factor is important for the wind farm operator because it allows more energy to be produced, and production to be more consistent. Thus, when designing a wind turbine, it is important to ensure two things: high power output for design conditions and a high capacity factor, which enables longer operation at rated capacity.

#### *2.4.1.2 – Performance of wind turbines over time*

Wind turbines are usually delivered by their producers with a promised lifetime of 20-25 years[13], which is what the technical lifetime of the turbine is expected to be. In practise, this means that the turbine is expected to perform within a set of structural specifications for the designed lifetime, and the turbine producer will be obligated through contractual agreements to compensate the operator for any deviations from these specifications. This

[13] https://www.renewablesfirst.co.uk/windpower/windpower-learning-centre/how-long-do-wind-turbines-installations-last/

insurance protects the operator from the financial burden of most irregular and unexpected performance decline.

The operator is, however, not protected against efficiency decline due to natural wear from use. A study conducted by Staffell and Green (2014) on 282 wind farms in the United Kingdom found that the capacity factor, also known as the load factor, decreased significantly as the turbines aged. By adjusting for local wind conditions, the study estimated that the capacity factor decline caused by aging alone was at around 1.6 % per year. A more detailed representation of the findings is presented in Figure 2.5 below.

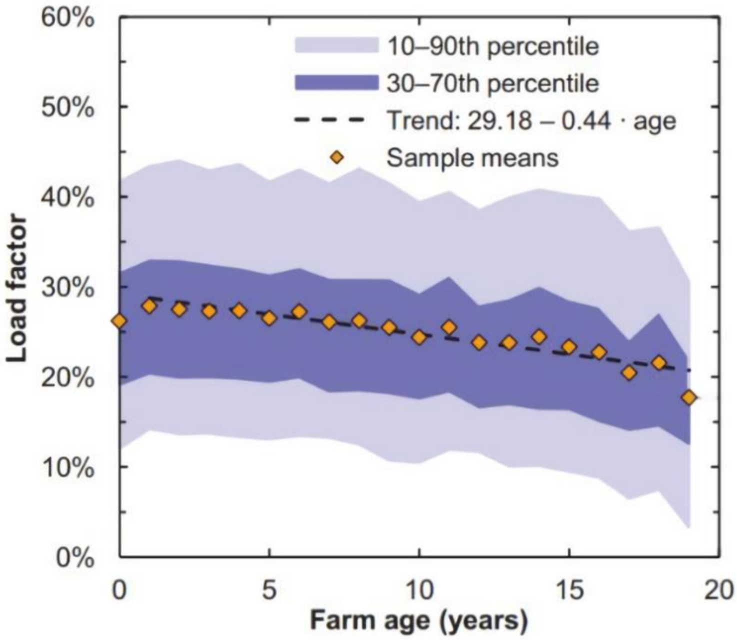



*Figure 2.5: Distribution of load factors for UK wind farms as they aged. Source: Staffel and Green (2014)*

By performing regular maintenance and replacing minor components, some of the performance decline can be slowed down, but not completely reversed. We therefore distinguish between reversible and irreversible capacity decline. Staffell and Green (2014) argue that if the capacity factor decreases irreversibly with age, the wind farms will produce lower cumulative output and, therefore, have increased levelized cost of electricity. If the decline becomes too large, it could be profitable to prematurely replace the turbines with newer models, implying that the economic lifetime could be significantly shorter than the technical lifetime[14].

The operating and maintenance costs (O&M costs) are also expected to increase as the turbine ages, primarily to counteract reversible capacity decline, hence the costs can be expected to increase while the revenues decline. Note that although Staffell and Green (2014) studied wind farms in the UK, we can expect very similar behaviour for any other turbines as most commercial turbine models operate in the same way and are exposed to the same forces and sources of strain and wear.

[14] Recall that the technical lifetime of a turbine is how long it can operate before it is at risk of technical failure, while the economic lifetime is how long it is profitable to keep it in operation.

### *2.4.1.3 – Technological development*

With the power equation, $h = \frac{1}{2} C_p \rho \varepsilon_g A w^3$ in mind, it seems natural that the development of wind turbines is going in the direction of becoming bigger and better. The swept area is $A = \pi D$, hence doubling the diameter of a turbine will increase the power by a factor of 6.28. In addition, as wind speed increases significantly with height, a taller turbine will experience higher average wind speed. Because $P \propto V^3$, a doubling of the average wind speed will increase the power production by a factor of 8. Improving the efficiency has in contrast relatively limited potential, as there is only a first-order dependence between power and efficiency, and state-of-the-art turbines are already fairly close to the absolute limit of the Betz factor.

Looking back to the early 1990s, new turbines had a diameter and hub height of about 35m[15]. By the early 2000s, the diameter of new turbines had increased to 80m and the hub height to 70m[16]. By 2009, the diameter had increased to 107 m and the hub height to 80 m[17]. Today, the largest turbine in the world is the Vesta V164 at a massive 164m diameter and 118m hub height[18]. Currently in the pipeline is the GE Haliade X[19], at an immense 220 m diameter and 150 m hub height, expected to be fully developed and deployed within 3-5 years; and also in progress are the Sandia SNL-100 and the UpWind (Dalhaug, 2018), both expected to dwarf the Haliade X when completed. The development can clearly be seen from Figure 2.6, where the global state-of-the-art turbines over time are visualised.

Increasing the power output of turbines also makes it possible to generate the same amount of power with fewer turbines, which significantly reduces the investment, operating and maintenance cost per unit of power produced over the turbine lifetime. Increased scales of economy are also a significant benefit with larger turbines[20]. Since 2009, the price per kWh of installed wind power has decreased by 65%, and optimistic forecasts expect a further 50% drop by 2030[21]. In Figure 2.7, the capital expenditures for wind turbines are plotted for the last 20 years, showing a visible declining trend. There is however a caveat for specific wind farms when turbines become larger, which is that the necessary spacing between turbines to avoid interference from wake effects increases with the rotor diameter (Schwanz, Henke and Chouhy Leborgne, 2012). For an existing wind farm with fixed area, the potential output increase through repowering will therefore be somewhat diminished by the fact that it will have to reduce the number of turbines to accommodate for their larger size.

[15] https://www.4coffshore.com/windfarms/vindeby-denmark-dk06.html

[16] https://www.4coffshore.com/windfarms/horns-rev-1-denmark-dk03.html

[17] https://www.4coffshore.com/windfarms/inner-dowsing-united-kingdom-uk11.html

[18] http://www.mhivestasoffshore.com/v164-8-0-mw-testing-programme-to-be-ramped-up-with-installation-of-two-additional-onshore-turbines-in-denmark/

[19] https://www.ge.com/renewableenergy/wind-energy/offshore-wind/haliade-x-offshore-turbine

[20] https://www.renewableenergyworld.com/ugc/articles/2017/05/08/is-bigger-best-report--part-1-limits-to-scale-in-wind.html

[21] https://www.vox.com/energy-and-environment/2017/8/30/16224582/wind-solar-exceed-expectations-again

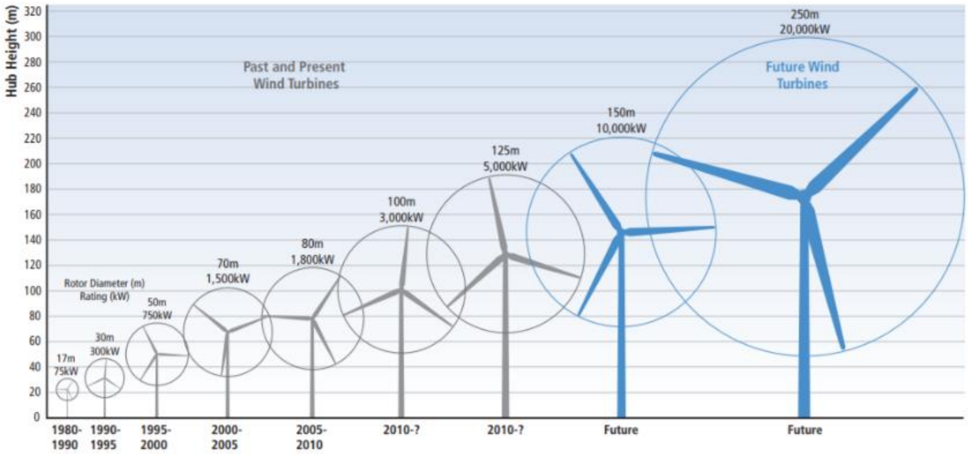



*Figure 2.6: Development of wind turbines over time. Source: Wiser and Yang (2010)*


*Figure 2.7: Declining trend for wind turbine capital expenditures over time. Source: IRENA (2018)*

In addition to producing more power, an objective of wind turbine improvement is also to produce power more reliably. Since the capacity factor indicates the average power output compared to the maximum possible output, it provides an indication of the consistency of the power production and is thereby a good measure of reliability. The average capacity factor for new turbines has increased from 25.4% in 1998 - 2001, to 32.1% in 2004 - 2011, to 42.5% in 2014 - 2015[22]. Today, many new turbines reach well above 50% capacity factor, with the current record held by the Norwegian energy producer Equinor's

[22] https://www.vox.com/energy-and-environment/2018/3/8/17084158/wind-turbine-power-energy-blades

offshore Hywind turbines at 65%[23]. From the wind turbine energy equation, $E = \frac{1}{2} C_p \rho \varepsilon_g A w^3 k$, it can be observed that the energy production depends linearly on the capacity factor. Hence, improved capacity factor will significantly increase the energy produced, and thereby the revenues generated. A less obvious, but also quite significant benefit of this development, is that more reliable power production reduces the need for backup power. This can significantly reduce costs and makes wind power a more attractive part of the energy mix.

The technological development of wind turbines can affect the investment decision in at least two ways. First, the alternative cost of wind farms currently in operation increases when better turbines becomes available, because the new turbines would be able to produce more power on the same land. The assumption of land scarcity is implicit in this argument. In other words, valuable land can be utilized better with newer turbines, which creates an incentive for replacing existing turbines. Second, as technology develops and globally installed capacity increases, wind turbine producers become better positioned to enjoy economies of scale[24]. This leads to lower investment costs, which provides incentive for earlier investment. The rate of technological development is therefore an important factor impacting the end-of-life investment decision and should be included in financial models in an appropriate manner.

#### 2.4.1.3.1 – Modelling approach for technological innovation

When modelling technological innovation for a specific industry, the goal is typically to replicate the probability of new technology arrival in the coming period. This is commonly done by introducing a stochastic jump process, with an arrival rate reflecting the probability of new technology arrival, see for instance Farzin, Huisman and Kort (1998). Hoppe (2002) conducts a survey on existing literature on the timing of new technology adoption, and finds it appropriate to separate it into two broad categories: literature concerned with the modelling of arrival rates and value of new technology (see McCardle (1985) and Vettas (1998)), and literature concerned with the strategic interaction in the product market (see Hendricks (1992) and Riordan (1992)). In our thesis, the relevance of technological uncertainty is through the stochastic nature of new technology arrivals, as the value of the new technology is evaluated directly through the real options model.

The arrival rate of the stochastic jump process can either be considered constant for the entire modelling period (Farzin, Huisman and Kort, 1998), have different levels for different regimes (Hagspiel, Huisman and Nunes, 2015) or be time-varying (Easley et.al., 2007). In reality, the expected arrival rate will be time-dependent, because as time pass circumstances involving the development changes and more information becomes available. The inclusion of time-dependency does, however, come at a significant cost of added complexity, and unless it can be quantified in a reliable manner it will not provide any meaningful contributions to the accuracy of results. In Easley et al. (2007), time-varying arrival rates are used in the context of financial trading, where short-term variations in investor information cause time-varying arrivals of stock trades. When

[23] https://www.equinor.com/en/news/15feb2018-world-class-performance.html

[24] https://www.renewableenergyworld.com/ugc/articles/2017/05/08/is-bigger-best-report--part-1-limits-to-scale-in-wind.html

considering stock trading, short-term variations are very significant. However, for long-term investment problems such as in renewable energy projects, short-term variations even out in the long-run. Therefore, time-varying arrival rates may be unnecessarily complex for wind power end-of-life option analysis.

Assuming a constant arrival rate, on the other hand, provides mathematical tractability and can be used to model long-term behaviour of technology innovation. However, as already mentioned, the arrival rate is by nature time-varying. For instance, the arrival rate may change following very recent innovations or breakthroughs in the development process. It can therefore be useful to consider a middle ground between constant arrival rate and time-varying rates, which can be achieved by allowing the arrival rate to change discretely for different regimes. This allows for some flexibility in the modelling of technological innovation, while also keeping complexity relatively modest.

### 2.4.2 – Wind speed

From the power equation of a wind turbine, it can be observed that the wind speed is the only parameter with exhibiting stochastic behaviour. The other equation parameters will either be specified through the turbine design conditions, or can be measured at the specific wind farm location. For instance, the swept area is determined by the diameter of the turbine blades, which is given from design conditions, and the local air density at the wind farm can be found by measurement. Thus, to investigate the uncertainty of the energy production, we focus our attention on wind speeds.

The phenomenon of wind is, very generally explained, caused by differences in air pressure. More precisely, when there exist differences in pressure, air accelerates from higher to lower pressure. At medium and high latitudes, the wind speed is also affected by the rotation of the earth (Nfaoui, 2012). Local wind speeds are also strongly affected by the topography of the area, as hitting obstacles will generate friction that slows it down. Observations of wind show that wind speeds are non-steady and fluctuate significantly both throughout a day and across different days[25]. This can be seen from Figure 2.8, which shows power output from one of the world's biggest wind farms, Jiuquan, on a typical day, and Figure 2.9 which shows a typical daily average wind speed curve over a month.

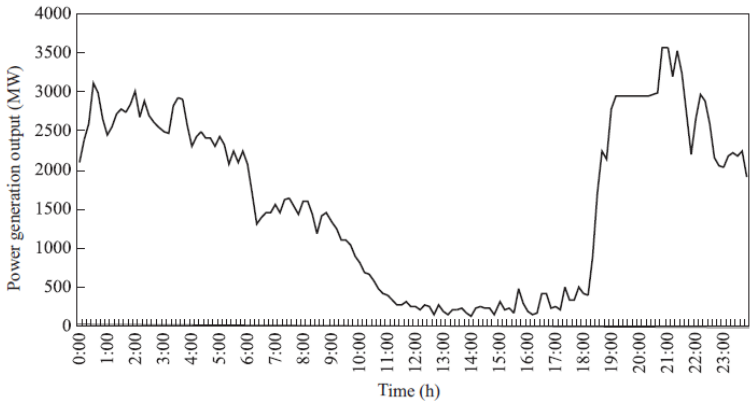



*Figure 2.8: Curve of typical daily distribution of wind power generation in Jiuquan. Source: Li and Zhi (2016)*

[25] http://www.wind-power-program.com/wind_statistics.htm

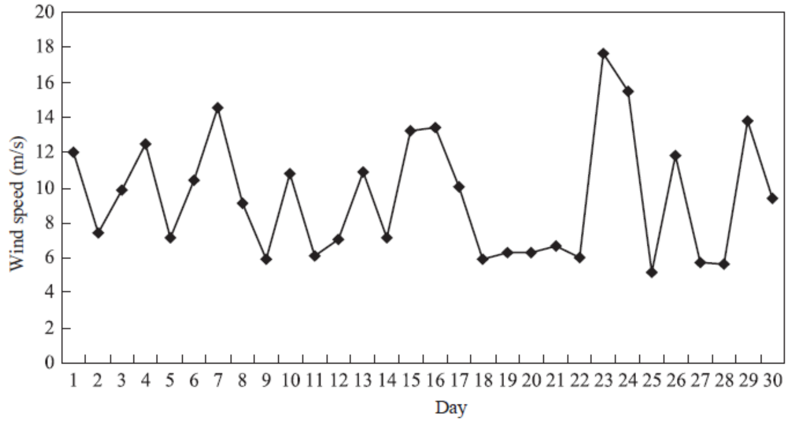



*Figure 2.9: Typical daily average wind speed curve over a month. Source: Li and Zhi (2016)*

These characteristics make the prediction of future wind speeds challenging. Thus, to estimate the power delivered from a wind turbine through its power curve it is necessary to know the probability distribution of the wind speed. Despite that the distribution of wind speeds is highly location specific, the distributional shapes from different locations share many properties. Therefore, our objective is to find one or more distributions that adequately describes the general behaviour of wind speeds.

#### ***2.4.2.1 – Modelling wind speed***

Studies on the distribution of wind speeds show that there are several distributions that could fit historical wind speed data. Pobocikova, Sedliackova and Michalkova (2017) demonstrate that the Weibull, Gamma and lognormal distributions provide satisfactory representation of the wind speed data at an airport in Slovakia at a significance level of 0.05. However, there are some significant differences between them, and the study concludes that the Weibull distribution gives the best fit to the available data. Garcia et al. (1998) reaches the same conclusion when testing the Weibull and lognormal distribution on 20 locations in Navarre. Hennessey (1977), Justus et al. (1978), Conradsen, Nielsen and Prahm (1984) and Pang, Forster and Troutt (2001) also show that the Weibull distribution in general provides a good, but not perfect, fit to wind speed data. An example of how the Weibull distribution fits wind speed data is shown in Figure 2.10.

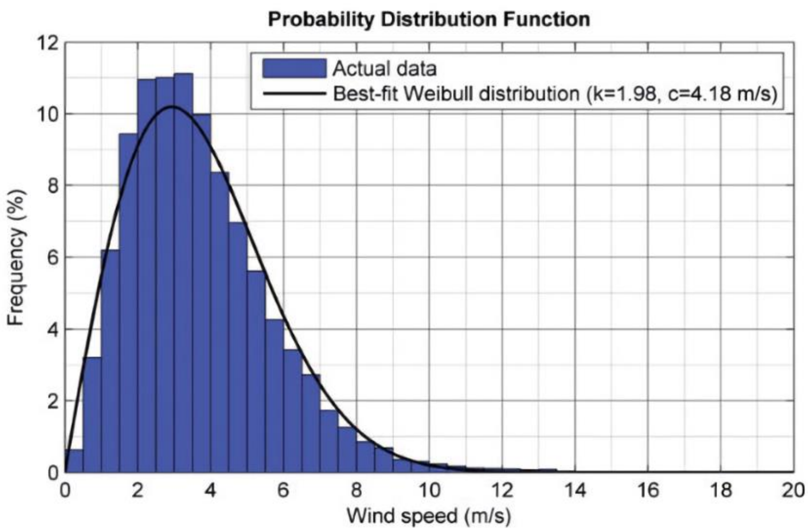



*Figure 2.10: Fit of Weibull distribution for a given set of wind speed data. Source: Ricci et al. (2014)*

Even though many papers argue in favor of the Weibull distribution, some studies have found the Gamma and lognormal distributions to better fit to the wind speed data at certain locations. One example of this is Akyuz and GamGam (2017). There are also studies conducted on specific locations that find no distribution to be consistently superior to others. Zhou et al. (2010) tests the Weibull, Rayleigh, gamma, lognormal and inverse Gaussian distribution on wind speed data at five locations in North Dakota and find that no particular distribution outperforms the others at all locations. Hence, the goodness of fit of a particular distribution must be considered to be location specific, and the choice of distribution should be based on the wind speed data available.

# 3 - Real options modelling

In this chapter, we present and develop the two real options models we use to address the research questions posed in Chapter 1. These models are, as was discussed in Chapter 2, a tractable single-factor model used for general analysis and a flexible multi-factor model used for case-specific analysis. We begin in Chapter 3.1 by developing the general concepts that apply to both models. Then, in Chapter 3.2, we develop the single-factor model and present some general insights derived from it. Finally, in Chapters 3.3 and 3.4, we develop the multi-factor model and discuss how to apply it through a numerical solution algorithm known as least squares Monte Carlo.

## 3.1 – General modelling concepts

In the following subchapter we present a general description of our real options modelling problem and formalise its major fundamental concepts. We begin by formulating the investment decision as an optimal stopping problem. Then, we define the operating profit margin as a measure for the turbine profitability, and finally introduce the capacity efficiency that provides a useful adjustment to the profit margin.

### 3.1.1 – The optimal stopping problem

We consider a profit-maximizing wind power producer, that is operating a single wind farm. The wind farm can consist of one or more wind turbines. The producer receives cash flows continuously from operating his wind farm and has the option to make a single irreversible investment in either repowering or life-extension at some time $t$. As decommissioning is very rarely economically motivated, as was discussed in Chapter 1, we have chosen to omit this option from explicit consideration. Consequently, after performing life-extension, the wind farm will have to be repowered.

We can view this situation as an optimal stopping problem, where the producer wants to find the optimal time, $\tau$, to exercise his options. In general, an optimal stopping problem will divide the state space into two distinct regions, the continuation region and the stopping region, separated by an optimal investment threshold. By expressing the continuation value as a partial differential equation, the investment threshold and option value can then be derived by using the stopping value as a boundary condition. This relies on the observation that the value of continuation and the value of immediate investment, i.e. stopping value, must be equal at the optimal threshold. At each point in time, the producer will have to compare the expected value of keeping his current wind farm in operation for another period and the value of exercising his best option immediately and receiving the expected stopping value $\Omega_t$. Note that $\Omega_t$ will be either repowering or life-extension depending on which is expected to be most profitable at time $t$. Mathematically, we can express this situation with a recursive optimization algorithm known as the Bellman equation. The general equation will then take the form

$$rV_t = \max_{\tau} \left\{ \mathrm{G}_t + \frac{1}{\mathrm{dt}} E[dV] \right\}, \tag{3.1}$$

with the boundary condition

$$V_\tau = \Omega_\tau$$

Here $V_t = V(G_t)$ is the value of wind farm at time $t$, and $G_t$ is the operating profit margin at time $t$, which will be explained in detail in the next subchapter.

### 3.1.2 – The operating profit margin

The operating profit margin is a good measure of the profitability of a turbine, as it measures the revenue generated less the cost of production at any given time. Therefore, we start by defining the gross operating profit margin, $g_t$, as the instantaneous revenue flow we receive from operation. More specifically, the gross operating profit margin is the product of the electricity price and the instantaneous power production of the turbine at full capacity efficiency.

As discussed in Chapter 2.4.1, the performance of wind tubines tend to decline over time while O&M costs tend to increase. To capture this effect, we introduce the turbine capacity efficiency $Q_t$ and assume it to follow an exponentially declining process on the form

$$Q_t = Q_0 e^{-\gamma t}, \tag{3.2}$$

where $Q_0$ is the initial level and $\gamma$ is the capacity decline factor per time unit. Note that $\gamma$ can be allowed to vary over time or change depending on the turbine's state of operation. The capacity efficiency is not related to the traditional measures for turbine efficiency, but is rather a measure of current power output relative to the rated power ouput of the turbine. By multiplying this with the gross operating profit margin, we can then define the net operating profit margin as

$$G_t = g_t Q_0 e^{-\gamma t} \tag{3.3}$$

The gross operating profit margin can then, by calibrating $\gamma$, capture both declining performance and increasing O&M costs over time. It can also, by adjusting $Q_0$, account for the initial cost of operation for the specific wind farm in consideration.

After repowering, we assume the initial level of the capacity efficiency to increase as it is measured relative to the rated power output of the turbine currently in operation. In practice this means that the capacity efficiency after repowering can be significantly above 100%, to reflect the technological improvements of the repowered turbine, both in terms of higher output and better performance. Similarly, the capacity efficiency can also increase post life-extension, as life-extension might improve the performance of components and partially reverse performance decline suffered by the turbine over its operational lifetime. By denoting the scaling of capacity efficiency post repowering as $K_R$ and the capacity efficiency post life-extension as $K_L$, the capacity efficiency for the three different states can now be expressed as

$$Q_t = Q_0 e^{-\gamma t}, \text{ original capacity efficiency.}$$

$$Q_t^R = K_R Q_0 e^{-\gamma t}, \text{ capacity efficiency after repowering.}$$

$$Q_t^L = K_L Q_0 e^{-\gamma t}, \text{ capacity efficiency after lifetime-extension.}$$

## 3.2 – Developing the single-factor model

Having considered the general modelling concepts, a single-factor model can now be developed to address the general research questions from the end of Chapter 1 and provide some general insights into how the end-of-life option values depend on their underlying factors. We begin the chapter by making some model-specific assumptions and discuss why they are made and how realistic they are. Then, we formulate the optimal stopping problem based on these assumptions and go on to solve the stopping problem to derive a closed-form, single-factor model. Finally, we utilize the single-factor model to analyse the investment problem and present the general results.

### 3.2.1 – Underlying assumptions

To be able to derive an analytical model with closed-form solutions, we first make some simplifying assumptions. While these assumptions might be too rigid for a specific wind project, we believe them to be justified for a more general analysis of wind turbine end-of-life option behaviour.

The first assumption is that the gross operating profit margin can be considered to follow a geometric Brownian motion on the form

$$dg_t = \alpha_{gbm} g_t dt + \sigma_{gbm} g_t dZ_t \tag{3.4}$$

This way of modelling the operating profit margin of a wind turbine is in line with the method of Kitzing et al. (2017). Their argument is that modelling the operating profit margin of a wind turbine with a GBM seems reasonable as a first approximation due to the long-term horizon of wind power projects. This argument in turn is based on a result from Dixit and Pindyck (1994), which states that the product of two processes following GBM will also be a GBM. In other words, if the electricity price and the instantaneous power production are GBMs, then so is the gross operating margin. Hence, the assumption we make is essentially that both the spot price of electricity and the wind speed follows GBM, or in other words that both are lognormally distributed. As discussed in Chapters 2.2.1 and 2.4.2.1, this can for many situations be a reasonable assumption.

With this assumption in place, we can then show that the net operating profit margin $G_t$ will evolve according to the following GBM (see Appendix A2.1 for proof)

$$dG_t = \left(\alpha_{gbm} - \gamma\right) G_t dt + \sigma_{gbm} G_t dZ_t \tag{3.5}$$

By expressing $G_t$ in this manner, with the capacity efficiency enveloped into the stochastic process, direct time dependence is avoided. This is essential to be able to derive an analytical and tractable solution.

The second assumption we make is that the technical lifetime of both the turbine currently in operation and the repowered turbine is infinite. Considering that an option will have to be exercised within the technical lifetime of the turbine for the solution to be non-trivial, and that the repowered turbine is expected to operate for around 25 years following the exercise time, this assumption is likely to have limited effect on the results. The magnitude of the option values might be overestimated to some degree, but the model dynamics and option behaviour should not be significantly affected. Therefore, making this assumption seems reasonable considering the objectives of the model, and it allows the model to avoid direct time-dependence.

Finally, we assume that the model parameters are deterministic. This includes $K_R$, $K_L$, $\gamma$, $\alpha_{gbm}$ and $\sigma_{gbm}$. Considering $K_R$ to be constant is equivalent to assuming that technological innovation is deterministic. While necessary to provide an analytical solution, it comes at the cost of being unable to analyse the impact of technological uncertainty on the option values. It can however still evaluate how technological development, i.e. increasing values of $K_R$, will affect the general behavior of the options. For $K_L$ and $\gamma$, time-variation can be considered highly project specific, and thus for a more generalised analysis they might as well be assumed constant. While both parameters, $\alpha_{gbm}$ and $\sigma_{gbm}$, related to our stochastic profit flow are likely to exhibit some short-term variation, in particular $\sigma_{gbm}$ because the underlying electricity price can be expected to display some form of time-dependence, clustering and leverage effects (Goto and Karolyi, 2004), the long time horizon of our investment problem should make these effects insignificant (Fleten et al., 2007).

### 3.2.2 – Formulating the optimal stopping problem

Making use of our assumptions, we can now formulate our optimal stopping problem as

$$V = \max_{\tau}\left\{\underbrace{E[\int_{t=0}^{\tau} G_t e^{-rt}\,dt]}_{Value\ of\ operating} + max\left\{\underbrace{E[\int_{t=\tau}^{\tau+\Delta t_L} G_t e^{\gamma\tau} K_L e^{-rt} dt - I_L e^{-r\tau}]}_{Value\ of\ life-extension} + \underbrace{E[\int_{t=\tau+\Delta t}^{\infty} G_t e^{\gamma\Delta t} K_R e^{-rt} dt - I_R e^{-r(\tau+\Delta t_L)}]}_{Value\ of\ repowering\ after\ life-extension},\ \underbrace{E[\int_{t=\tau}^{\infty} G_t e^{\gamma\tau} K_R e^{-rt} dt - I_R e^{-r\tau}]}_{Value\ of\ repowering}\right\}\right\} \tag{3.6}$$

To find an analytical solution, we need to separate this into two subproblems. Let $V_R$ denote the value of the option to repower, and $V_L$ denote the value of the option to life-extend. We can then consider the options $V_R$ and $V_L$ separately to find an analytical solution for each, and then compare these solutions afterwards. While doing this separation is an analytical necessity, it also allows us to consider how underlying factors affect the two option values separately. Our optimal stopping problem is now reformulated to

$$V_R = \max_{\tau}\left\{\underbrace{E\left[\int_{t=0}^{\tau} G_t e^{-rt}\,dt\right]}_{Value\ of\ operating} + E\left[\underbrace{\int_{t=\tau}^{\infty} G_t e^{\gamma\tau} K_R e^{-rt} dt - I_R e^{-r\tau}}_{Value\ of\ repowering}\right]\right\} \tag{3.7}$$

$$V_L = \max_{\tau}\left\{\begin{matrix}\underbrace{E\left[\int_{t=0}^{\tau} G_t e^{-rt}\,dt\right]}_{Value\ of\ operating} + \underbrace{E\left[\int_{t=\tau}^{\tau+\Delta t_L} G_t e^{\gamma\tau} K_L e^{-rt} dt - I_L e^{-r\tau}\right]}_{Value\ of\ lifetime-extension} \\ + E\left[\underbrace{\int_{t=\tau+\Delta t}^{\infty} G_t e^{\gamma\Delta t_L} K_R e^{-rt} dt - I_R e^{-r(\tau+\Delta t_L)}}_{Value\ of\ repowering\ after\ lifetime-extension}\right]\end{matrix}\right\} \tag{3.8}$$

Note that because we incorporated the capacity efficiency into the stochastic process of $G_t$, we need to introduce the terms $e^{\gamma\tau}$ and $e^{\gamma\Delta t_L}$ to ensure that the performance decline of the current turbine does not affect the performance of the repowered turbine.

In the following subchapters, we derive the two option values $V_R$ and $V_L$ from the optimal stopping problem. The derivations can be found in full detail in appendix A2.2 and A2.3.

### 3.2.3 – Solving the optimal stopping problem

By inserting the expression for $G_t$ into equations 3.7 and 3.8, and setting $\tau = 0$, we find the stopping values for repowering and life-extension, respectively, to be

$$\Omega_{\mathrm{R}} = E\left[\int_0^{\infty} K_R G_t e^{-rt} dt - I \mid G_0 = G\right] = \frac{K_R Q_0 G}{r - (\alpha_{gbm} - \gamma)} - I_R, \tag{3.9}$$

and

$$\Omega_L = E\left[\int_0^{\Delta t} G_t K_L e^{-rt} dt - I_L \mid G_0 = G\right] + E\left[\int_{\Delta t}^{\infty} G_t e^{\gamma\Delta t} K_R e^{-rt} dt - I_R e^{-r\Delta t} \mid G_0 = G\right]$$

$$= \frac{GQ_0}{r-(\alpha_{gbm}-\gamma)}\left(K_L\left(1 - e^{-\left(r-(\alpha_{gbm}-\gamma)\right)\Delta t}\right) + K_R e^{-(r-\alpha)\Delta t}\right) - \left(I_L + I_R e^{-r\Delta t}\right) \tag{3.10}$$

The value of continuation can be found from the Bellman equation as

$$rV_i(G_t) = \mathrm{G}_t + \frac{1}{dt} E[dV_i] \tag{3.11}$$

Note that $i$ is a general index representing either repowering (R) or life-extension (L).

Applying Ito's lemma to expand $dV_i$ provides

$$dV_i = \frac{\partial V_i}{\partial G}\left((\alpha_{gbm} - \gamma) G_t dt + \sigma_{gbm} G_t dz\right) + \frac{1}{2}\frac{\partial^2 V_i}{\partial G^2}\sigma_{gbm}^2 G_t^2 dt, \tag{3.12}$$

and inserting the expression for $dV_i$ into equation 3.12 results in the ordinary differential equation (ODE)

$$rV_i(G) = G_t + (\alpha_{gbm} - \gamma) G_t \frac{\partial V_i}{\partial G} + \frac{1}{2}\sigma_{gbm}^2 G_t^2 \frac{\partial^2 V_i}{\partial G^2} \tag{3.13}$$

By using the stopping values as boundary conditions, this ODE can be solved to construct the single-factor model, which is presented in Figure 3.1.

**The life-extension option**

$$V_L = \left(\frac{G}{G_L}\right)^{\beta} \left(\frac{Q_0 G_L}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t} - 1\right) - \left(I_L + I_R e^{-r\Delta t}\right)\right),$$

with

$$G_L = \frac{\beta}{(\beta - 1)} \frac{r^*(I_L + I_R e^{-r\Delta t})}{Q_0(K_L(1 - e^{-r^*\Delta t}) + K_R e^{(\gamma - r^*)\Delta t} - 1)}$$

$$\beta = \left(\frac{1}{2} - \frac{(\alpha_{gbm} - \gamma)}{\sigma_{gbm}^2}\right) + \sqrt{\left(\frac{(\alpha_{gbm} - \gamma)}{\sigma_{gbm}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{gbm}^2}} > 1$$

$$r^* = \mathrm{r} - (\alpha_{\mathrm{gbm}} - \gamma)$$

**The repowering option**

$$V_R = \left(\frac{G}{G_R}\right)^{\beta} \left(\frac{Q_0 G_R (K_R - 1)}{r^*} - I_R\right),$$

with

$$G_R = \frac{\beta}{(\beta - 1)} \frac{r^* I_R}{Q_0(K_R - 1)}$$

$$\beta = \left(\frac{1}{2} - \frac{(\alpha_{gbm} - \gamma)}{\sigma_{gbm}^2}\right) + \sqrt{\left(\frac{(\alpha_{gbm} - \gamma)}{\sigma_{gbm}^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_{gbm}^2}} > 1$$

$$r^* = \mathrm{r} - (\alpha_{\mathrm{gbm}} - \gamma)$$

*Figure 3.1: Summary of the single-factor model*

To provide some intuition behind the model, a brief interpretation of the repowering value expression is provided. The lifetime-extension expression has a very similar interpretation; hence we do not delve into it specifically.

The term $\frac{G_R(K_R-1)}{r-(\alpha_{gbm}-\gamma)} - I_R = \left(\frac{G_R K_R}{r-(\alpha_{gbm}-\gamma)} - I_R\right) - \frac{G_R}{r-(\alpha_{gbm}-\gamma)}$ is the value of paying $I_R$ to receive a perpetual cash flow $K_R G_R$, minus the value of a perpetual cash flow $G_R$. Both with a discount rate $r$ and a growth rate $\alpha_{gbm} - \gamma$. Hence, we can interpret this as the value gained by repowering at the optimal threshold $G = G_R$, minus the lost cash flow from decommissioning the old turbine at the same threshold. The term $\left(\frac{G}{G_R}\right)^{\beta}$ can be interpreted as a stochastic discount factor, that discounts the payoff the firm receives by repowering at $G = G_R$ to the current time. Hence, the total expression can be interpreted as the value of paying a strike price of $I_R - G_R/r^*$ to receive the value of the underlying asset $K_R G_R/r^*$, discounted with an appropriate factor.

### 3.2.4 – Insights from the single-factor model

With the single-factor model presented in Figure 3.1., we can now address the general research questions posed at the end of chapter 1. These three questions are:

- Which factors affect the end-of-life options?
- How do underlying factors affect the option values and the investment decision?
- How does the option value of repowering change relative to the option value of life-extension when the underlying parameters change?

To provide robust answers, we rely primarily on the mathematics of the model to analyse how the option values depend on the underlying factors. In addition, we provide some qualitative reasoning for the economics behind the results and visualise them graphically. In doing so, we hope to build a stronger intuition for the option value behavior that can back up the mathematical understanding. Note that for simplicity in the mathematical expressions, we utilize $\eta_i$ to denote terms of the expression that are independent of the parameter currently under consideration.

***General insight 1 -*** *The option values are decreasing in $r$ at different rates.*

The repowering investment threshold can be reformulated as $G_R = \eta r^*$, where $r^*$ is linearly increasing in $r$. Hence, it can be observed that $G_R$ is linearly increasing in $r$. By inserting this expression into the repowering option value, it can be reformulated as $V_R = \frac{\eta_1}{(r^*)^{\beta+1}} - \frac{\eta_2}{(r^*)^{\beta}}$. Recalling that $\beta > 1$, the option value must then be decreasing in $r$.

The life-extension investment threshold $G_L$ can be reformulated as $G_L = \frac{r^*(\eta_1+\eta_2 e^{-r\Delta t})}{\eta_3+\eta_4 e^{-r^*\Delta t})}$, which is non-linearly increasing in $r$. The life-extension option value can then be reformulated as $V_L = \frac{\eta_5}{(G_L)^{\beta}}\left(\left(\frac{G_L}{r^*}\eta_4 e^{-r^*\Delta t} + \eta_3\right) - \left(\eta_1 + \eta_2 e^{-r\Delta t}\right)\right)$, which is decreasing in $r$ at a different rate than the repowering option value. It might not be immediately obvious from the mathematical expression that the option value is decreasing, but the trend is quite obvious from the plot presented in Figure 3.2 below. The intuition behind these results is that increasing $r$ will make future cash flows from the repowered or life-extended turbine less valuable relative to the cash flows received from continued operation which occur earlier. It also makes the repowering and life-extension investment costs greater relative to the cash flows from the repowered or life-extended turbine received later in the future, which favours postponing the option investment.

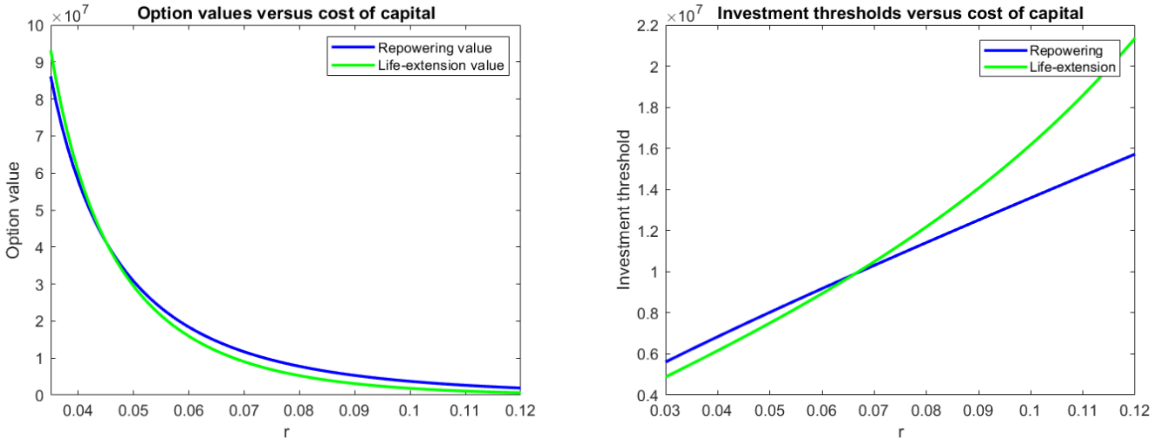



*Figure 3.2: Option values and investment thresholds for different costs of capital*

**General insight 2 –** *The option values are increasing in σ at different rates.*

It can be observed that for both the option values and thresholds, the volatility only enters the expressions through $\beta$. While $\beta$'s dependence on volatility might not be straightforward to interpret from its expression, Dixit and Pindyck (1994) show that $\beta$ on such a functional form will be decreasing in $\sigma$. The repowering threshold can be reformulated as $G_R = \frac{\beta}{\beta-1}\eta_1$ and the life-extension threshold as $G_L = \frac{\beta}{\beta-1}\eta_2$. It can then be observed that both investment thresholds are increasing in $\sigma$, and at different rates for $\eta_1 \neq \eta_2$. The repowering option value can be reformulated as $V_R = \left(\frac{G}{G_R}\right)^{\beta}(G_R\eta_4 - \eta_3)$ and the life-extension option value as $V_L = \left(\frac{G}{G_L}\right)^{\beta}(G_L\eta_5 - \eta_6)$. Recalling that the investment threshold should be above the current value of the profit margin for our investment decision, or the option would already be in the stopping region, it must hold that $G/G_R < 1$ and $G/G_L < 1$. The term $(G/G_i)^{\beta}$ is therefore increasing for increasing $\sigma$, and thereby are both option values increasing in $\sigma$. It can further be observed that the option values will be increasing in $\sigma$ at different rates when $(G_R\eta_4 - \eta_3)G_R^{-\beta} \neq (G_L\eta_5 - \eta_6)G_L^{-\beta}$, which implies that their value will change relative to one another if this condition holds. These results are in line with the findings of Dixit and Pindyck (1994), who argue that increasing volatility will result in both increased option value and reduced probability of exercise simultaneously.

The intuition behind these results is that higher volatility makes future cash flows more uncertain, hence waiting for new information becomes more valuable. The flexibility of being able to choose when or if to exercise an option therefore has increased value when the volatility of the operational profit flow is higher. The results are visualised in Figure 3.3.

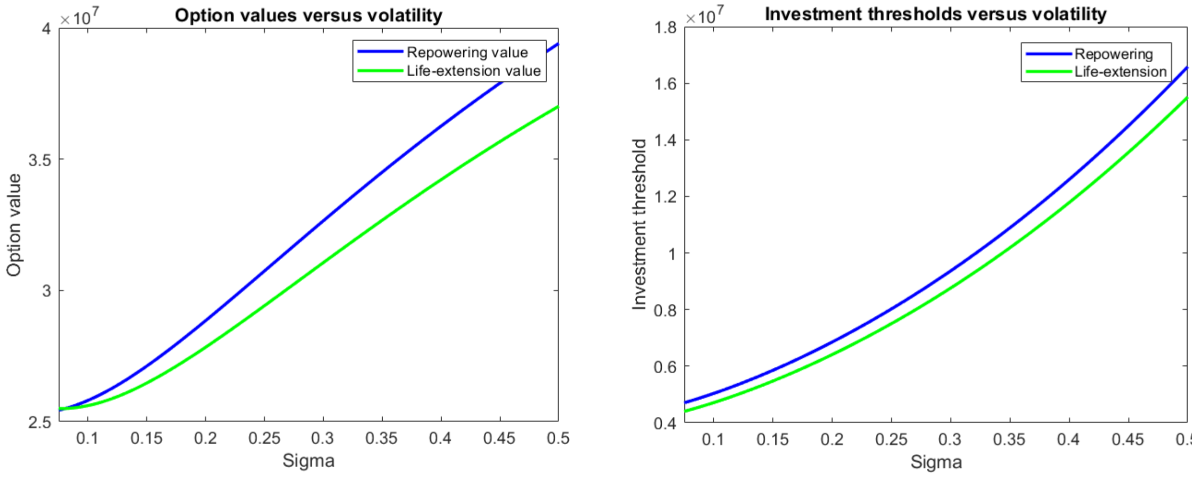



*Figure 3.3: Option values and investment thresholds for different values of the operational profit flow volatility*

**General insight 3 –** *The option values are decreasing in $\gamma$ at different rates.*

The repowering investment threshold can be reformulated as $G_R = \eta r^*$, where $r^*$ is linearly increasing in $\gamma$. Hence, it can be observed that $G_R$ is linearly increasing in $\gamma$. By inserting this expression into the repowering option value, it can be reformulated as

$V_R = \frac{\eta_1}{(r^*)^{\beta+1}} - \frac{\eta_2}{(r^*)^{\beta}}$. Recalling that $\beta > 1$, the option value must then be increasing in $\gamma$.

The life-extension investment threshold $G_L$ can be reformulated as

$G_L = \frac{r^*\eta_1}{\eta_2 - \eta_3 e^{-r^*\Delta t}}$, which can both increase and decrease nonlinearly in $\gamma$ depending on the values of $\eta_1$, $\eta_2$ and $\eta_3$. The life-extension option value can then be reformulated as

$V_L = \frac{\eta_4}{(G_L)^\beta}\left(\left(\frac{G_L}{r^*}\eta_3 e^{-r^*\Delta t} + \eta_2\right) - \eta_1\right)$, which again provides no straightforward dependence on $\gamma$. To supplement our mathematical understanding, we therefore plot the option values and find the behaviour displayed in Figure 3.4 to hold for realistic parameters. The result is therefore that the life-extension option value is decreasing in $\gamma$, and at a different rate than the repowering option depending on the parameter values.

The intuition behind these results is that increasing $\gamma$ makes the turbine capacity efficiency drop faster over time, thereby making both the currently operating turbine and the repowered turbine less valuable. The repowered turbine is, however, affected more severely by this decline than the current turbine, as the repowered turbine has a greater initial capacity efficiency. In combination with the cost of repowering being independent of $\gamma$, thereby making the cost larger relative to the value gained by exercising the option, the repowering option becomes less attractive. The life-extension option is less affected by this; as it allows for postponed repowering and continued operation of the current turbine at a point where the absolute value of the capacity efficiency already has dropped quite far, making the relative effect of $\gamma$ less significant. The resulting effect is therefore that the life-extension option value is decreasing in $\gamma$ at a lower rate than the repowering option value.

An important point to make regarding this result, is that it is contingent upon both the currently operating turbine and the repowered turbine experiencing the same $\gamma$. If this was not the case, and the $\gamma$ of the currently operating turbine were to increase relative to the repowered turbine, the option values would increase. While not visualised, this can be observed from the mathematical expressions of the model. The intuition is that repowering (and life-extension followed by repowering) becomes more valuable because the currently operating turbine becomes less valuable relative to the repowered turbine.

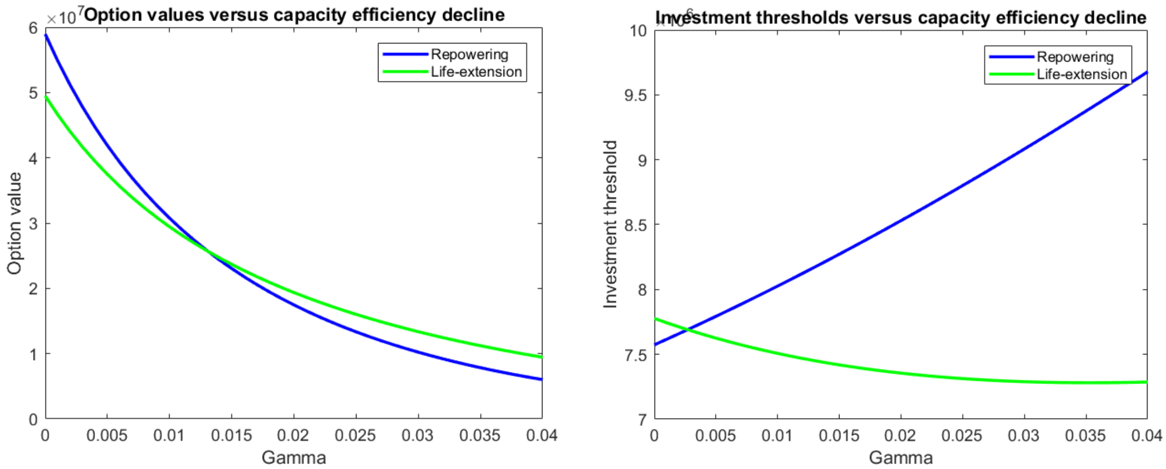



*Figure 3.4: Option values and investment thresholds for different values of the capacity efficiency decline*

It can be remarked from Figure 3.4 that while both option values are decreasing in $\gamma$, the repowering threshold is increasing while the life-extension threshold is decreasing. This might seem surprising at first, but recall that the investment threshold is primarily an indicator of how likely the option is to be exercised. For the life-extension option, one can therefore reason that while an increase in $\gamma$ makes the probability of hitting the investment threshold higher, it also makes the value of exercising the option lower, and this second effect is dominating.

**General insight 4 –** *The option values are increasing in $P_0$ at different rates.*

It can be observed that the current price of electricity $P_0$ only enters the option value expressions through $G$, which is linearly increasing in $P_0$. The investment thresholds are independent of $P_0$, which is natural considering the investment problem formulation. The repowering option value can be reformulated as $V_R = G^{\beta}\eta_1$ and the life-extension option value as $V_L = G^{\beta}\eta_2$. It can then be observed that both option values are increasing in $P_0$, and at different rates when $\eta_1 \neq \eta_2$. Assuming this condition holds, this implies that the relative value of the two options will change depending on the current market conditions.

The intuition behind these results is that both repowering and life-extension increases the capacity efficiency of the wind turbine, and thereby increases the wind power output, which results in better utilization of the high electricity price. The results are visualised in Figure 3.5 below.

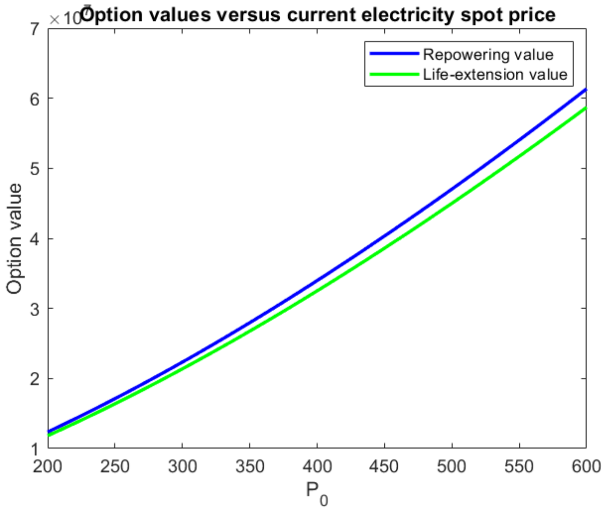



*Figure 3.5: Option values for different values of the current electricity price*

**General insight 5 –** *The option values are increasing in $K_R$ at different rates.*

The repowering investment threshold can be reformulated as $G_R = \frac{\eta_1}{K_R - 1}$, which is decreasing in $K_R$. Inserting this expression into the repowering option value and reformulating it then yields $V_R = \left(\frac{1}{G_R}\right)^{\beta} (\eta_2 - \eta_3)$, which is increasing in $K_R$. The life-extension investment threshold can be reformulated as $G_L = \frac{\eta_4}{\eta_5 + K_R e^{\eta_6}}$, which is decreasing in $K_R$ at a different rate than the repowering threshold when $\frac{\eta_1}{K_R - 1} \neq \frac{\eta_4}{\eta_5 + K_R e^{\eta_6}}$. The life-extension option value can then be reformulated as $V_L = \left(\frac{1}{G_L}\right)^{\beta} (G_L K_L \eta_7 + \eta_8)$, which is increasing in $K_R$ at a different rate than the repowering option value when $\eta_2 - \eta_3 \neq G_L K_L \eta_7 + \eta_8$.

The intuition behind this result is that increasing $K_R$ represents greater technological development, which makes the output difference between the currently operating turbine and the turbine you could repower to larger. The cost of repowering will naturally also increase as the new turbines get bigger and more sophisticated, but the increased operational cash flows increase at a higher rate. The reason why the life-extension option behaves in a very similar manner as the repowering option in regard to changes in $K_R$ can be attributed entirely to the fact that the life-extension option includes the value of repowering post life-extension. Hence, as $K_R$ becomes large the life-extension option value is dominated by the postponed repowering value. The results are visualised in Figure 3.6.

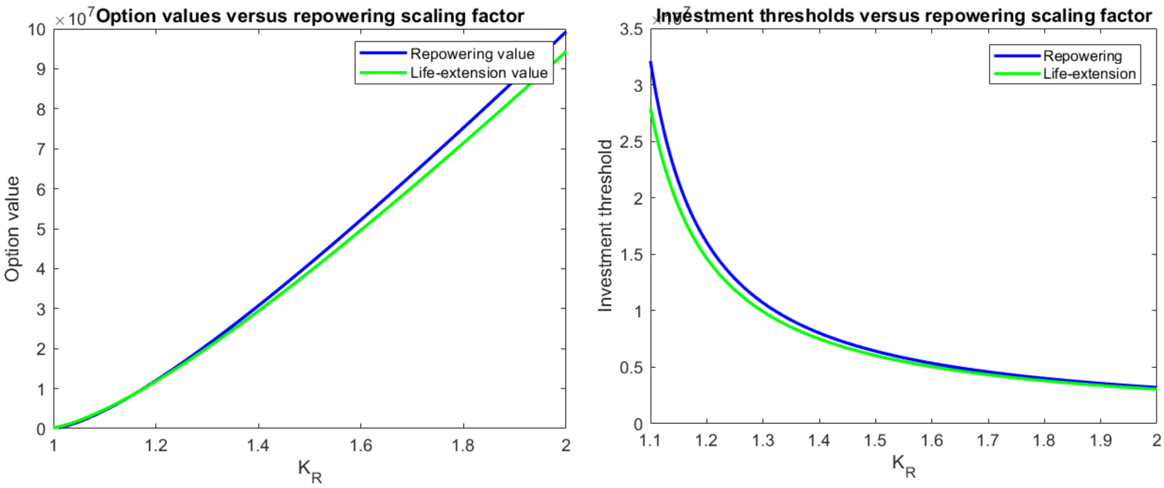



*Figure 3.6: Option values and investment thresholds for different values of the repowering scaling factor*

**General insight 6 –** *The repowering option value is decreasing in $I_R$, and the life-extension option value is decreasing in both $I_R$ and $I_L$.*

The repowering investment threshold can be reformulated as $G_R = \eta_1 I_R$, hence the threshold is increasing in $I_R$. Inserting this into the repowering option value and reformulating it yields $V_R = \eta_2(I_R)^{1-\beta}$, which is decreasing in $I_R$ because $1-\beta < 0$. The life-extension investment threshold can be reformulated as $G_L = \eta_3 + \eta_4 I_R$ or $G_L = \eta_5 + \eta_6 I_L$, hence the threshold is increasing in both $I_R$ and $I_L$. Inserting these expressions into the life-extension option value and reformulating then yields $V_L = \eta_7(\eta_3 + \eta_4 I_R)^{1-\beta} + \eta_8 + \eta_9 I_R$ or $\eta_7(\eta_5 + \eta_6 I_L)^{1-\beta} + \eta_{10} + \eta_{11} I_L$, hence the life-extension option value is decreasing in both $I_R$ and $I_L$. These insights on the investment costs of $I_R$ and $I_L$ are visualized in Figure 3.7 and 3.8.

The intuition behind these results is that when the cost of investment increases while the benefit from investment remains the same, the options become more expensive to exercise while the underlying value is unchanged, thereby reducing the option value. The life-extension option value is negatively affected by increases in both investment costs because the strike price of the life-extension option is a weighted sum of the two investment costs on the form $S = I_L + I_R e^{-r\Delta t}$.

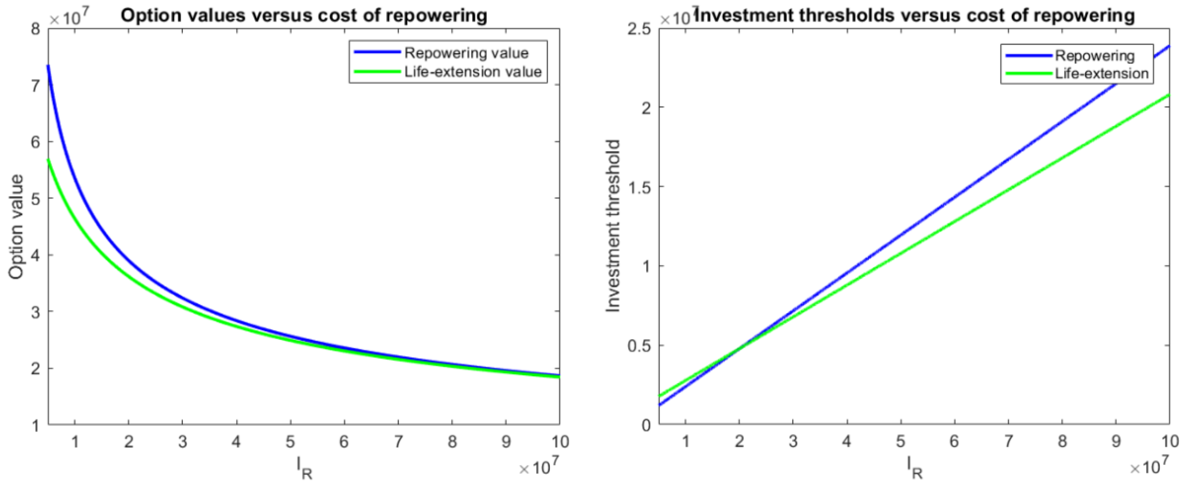



*Figure 3.7: Option values and investment thresholds for different costs of repowering*

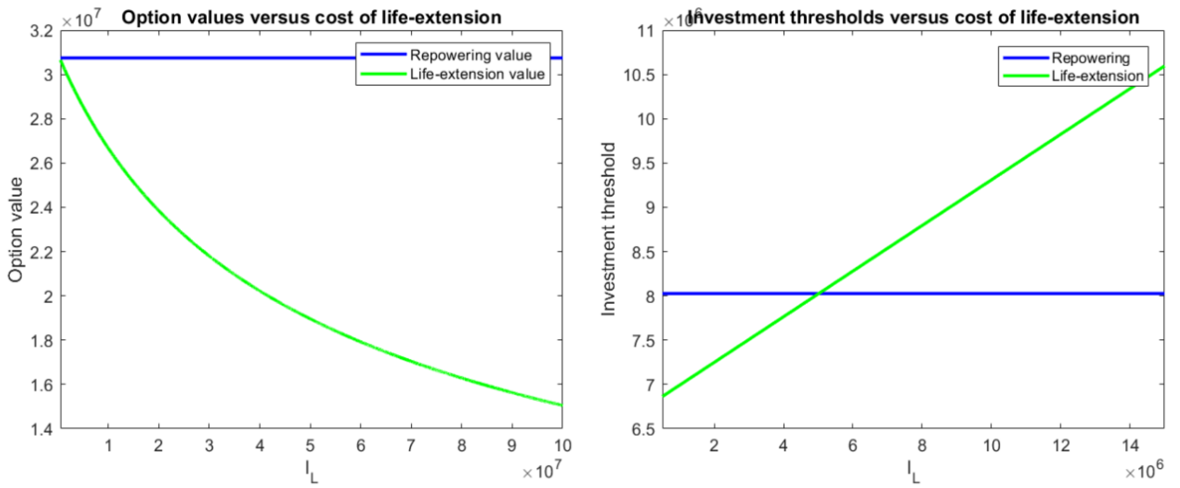



*Figure 3.8: Option values and investment thresholds for different costs of life-extension*

## 3.3 – Developing the multi-factor model

Having addressed the general research questions from Chapter 1 through our single-factor model, we can now turn our attention to the project-specific research questions. To appropriately address these, we develop a more flexible multi-factor model that can be calibrated to a specific project. We begin the chapter by discussing how to model the underlying stochastic factors in our investment problem, which are the price of electricity, power output and technological innovation. From this, we can redefine the operating profit margin and then reformulate the optimal stopping problem. Finally, we estimate the stopping values for our optimal stopping problem and discuss how to implement the numerical procedure of least squares Monte Carlo to estimate the option value and expected timing of exercise.

### 3.3.1 – Modelling the electricity price

Recall from Chapter 2.3.1.1 that considering the long time horizon of the end-of-life wind power investment problem, short-term characteristics such as jumps and seasonality can be expected to have little impact. We therefore limit the scope of our focus to modelling of the long-term behavior of the electricity price. Since the Ornstein-Uhlenbeck both captures mean-reversion and is flexible enough to mimic the behavior of a GBM if that should be desirable, we choose to proceed with an Ornstein-Uhlenbeck process.

Let the logarithm of the electricity price follow the process

$$dX_t = \kappa_P \left( \alpha - \frac{\sigma^2}{4\kappa_P} - X_t \right) dt + \sigma dZ_P, \tag{3.14}$$

which implies the electricity spot price can be expressed as $P_t = e^{X_t}$.

Modelling the price in this manner allows for mean-reversion and captures some of the skewness in the price. It also makes the implicit assumption that prices are always positive, which is reasonable considering that empirical data show that power prices very rarely are negative. In addition, as storage technology continues to improve and renewable energy becomes better integrated into the power grid, the likelihood of negative electricity prices will become even smaller in the future.

### 3.3.2 – Modelling the power output

The main challenge of modelling the power output is to find an appropriate distribution for capturing the characteristics of the wind speed. From the discussion in Chapter 2.4.2 it is evident that several distributions could provide satisfactory results, and that in specific cases some distributions are superior to others. Hence, the most appropriate distribution should be chosen on a case-by-case basis by evaluating the measured wind data from the site. Evaluating the wind distribution data from our numerical case, we find the Weibull distribution to provide the best fit. Hence, we want to design a stochastic process that ensures the wind speeds are Weibull distributed. By applying a procedure from Ernstsen and Boomsma (2018), a variable that is approximately Weibull distribution can be defined by utilizing a standard normal variable and a mathematical transformation. Note that this procedure is applicable to any choice of wind distribution, hence the expressions we derive for the multi-factor model are not Weibull-specific. By replacing the cumulative Weibull distribution with the cumulative distribution of your choice in the expressions, they will hold for that distribution.

Assume that the wind speed is driven by some underlying stochastic weather factor $U_t$, and let it follow an Ornstein-Uhlenbeck process on the form

$$dU_t = -\kappa_U U_t dt + \sqrt{2\kappa_U} dZ_U, \tag{3.15}$$

where the dynamics of the process are chosen such that $U_t$ is normally distributed. We assume $Z_P$ and $Z_U$ to be correlated with a correlation coefficient $\rho$, to reflect that electricity prices tend to drop when there is much wind power available and vice versa.

The cumulative distribution function for a Weibull distribution is

$$F(w) = 1 - e^{-\left(\frac{w}{a}\right)^b} \quad \text{for } w \geq 0 \text{ and } F(w) = 0 \text{ for } w < 0,$$

where $b$ is a shape parameter and $a$ is a scale parameter. The inverse cumulative distribution is then given by

$$F^{-1}(x) = \xi(-\ln(1-x))^{1/\beta} \text{ for } 0 \leq x < 1$$

The Ornstein-Uhlenbeck process for the weather factor is what is known as $\rho$-mixing (Forman and Sørensen, 2008). It then follows from Theorem 4.2 from Bradley (1985) that for all transformations of $\rho$-mixing processes, for which the autocorrelation is defined, it is bounded by a function that decays exponentially. To obtain the appropriate limiting process we let $\Phi(\cdot)$ denote the standard normal distribution function and $\Phi_t(\cdot)$ denote the distribution function of $U_t$. Also let $F(\cdot)$ denote the desired cumulative distribution function for a random variable, and assume that the density has support given by the interval $I \subseteq \mathbb{R}$ such that the distribution function is strictly increasing on the interior of $I$. With this assumption, the quantile function $F^{-1}(\cdot)$ is the inverse of $F(w)$ for $w \epsilon I$. Following the procedure of Ernstsen and Boomsma (2018), if the wind speed, $W_t$, then is defined as the random variable $W_t = F^{-1}(\Phi(U_t))$, the fact that $U_t$ converges in distribution to the standard normal distribution when time goes to infinity can be utilized to get

$$\mathbb{P}(W_t \leq w) = \mathbb{P}\left(U_t \leq \Phi^{-1}(F(w))\right) = \Phi_t\left(\Phi^{-1}(F(w))\right) \rightarrow F(w) \; for \; t \rightarrow \infty$$

Thus, the cumulative distribution function of wind speeds is given by the desired $F(\cdot)$ in the limit. We can then define the wind speed as

$$W_t = F^{-1}(\Phi(U_t)), \tag{3.16}$$

where $F(\cdot)$ is the cumulative distribution function of a Weibull distribution. With this, we have ensured the modelled wind speeds are approximately Weibull distributed, with non-normal, mean-reverting behaviour and exponentially decaying autocorrelation. Note again that from the results above, $F(\cdot)$ can be chosen to be whichever distribution is most suitable for the specific wind speed data under consideration.

Considering the power output for different wind speeds presented in Chapter 2.4.1, a piecewise linear power output function, $h$, can be expressed on the form

$$h(w) = \begin{cases} 0 & if\ w < w_0\ or\ w > w_2 \\ h_{01}(w) & if\ w_0 < w < w_1 \\ h_{12} & if\ w_1 < w < w_2 \end{cases}$$

where $h_{12}$ is the rated capacity and $h_{01}(w) = \frac{1}{2} C_p \rho \varepsilon_g A w^3$.

This function can more conveniently be expressed as a piece-wise linear function of the underlying weather factor $U_t$ as

$$h(f(u)) = \begin{cases} 0 & if\ u < u_0\ or\ u > u_2 \\ h_{12}(u - u_0)/(u_1 - u_0) & if\ u_0 < u < u_1, \\ h_{12} & if\ u_1 < u < u_2 \end{cases} \tag{3.17}$$

where $u_0 = \Phi^{-1}\big(F(w_0)\big), u_1 = \Phi^{-1}\big(F(w_1)\big)$ and $u_2 = \Phi^{-1}\big(F(w_2)\big)$.

Note that for $u_0 < u < u_1$ we have made the simplifying assumption that the power output will increase linearly from 0 to $h_{12}$. While this is not entirely realistic, it is necessary to simplify the derivations of the expected values.

### 3.3.3 – Modelling technological innovation

As discussed in Chapter 2.4.1.3.1, it is common to model technological innovation as a stochastic jump process. Motivated by including technological uncertainty in the multi-factor model to make it more realistic and flexible, we therefore assume the repowering scaling factor to follow a stochastic jump process on the form

$$K_R(t) = 1 + (K_r - 1)\theta_t, \tag{3.18}$$

where $K_r$ represents the scaling of the capacity efficiency factor of the repowered turbine, i.e. the size of the technological innovation. Recall from Chapter 2.4.1.3.1 that we also consider it beneficial to allow the arrival rate of innovation to change across different regimes. For our investment problem, the time elapsed since the previous innovation occurred can be considered a reasonable indicator for regime change, as developing larger and more sophisticated turbines requires solving a series of complex engineering problems that often rely on the solution of previous problems. In line with the method of Hagspiel, Huisman and Nunes (2015), we define $\theta_t$ as a non-stationary Poisson process with an arrival rate $\lambda_t$ that depends on the time since the previous technological innovation arrived. Denote the time elapsed since the previous technological innovation, $\Gamma_t$, and assume the sequence of inter-renewal times $\{\Gamma_t\}$ consists of independently and identically distributed random variables. Right after a technology arrival, i.e. a turbine with better performance being made commercially available, the arrival rate is equal to $\lambda_1 \geq 0$. If no arrival should happen for a time period of length $\Delta$, the arrival rate changes to $\lambda_2 \geq 0$. The arrival rate can now be formally stated as

$$\lambda_t(\omega) = \lambda_1 1_{\{\Gamma<\Delta\}} + \lambda_2 1_{\{\Gamma\geq\Delta\}}, \tag{3.19}$$

where $1_{\{A\}}$ is equal to one if $A$ is true and zero otherwise. The time between consecutive technology arrivals, hereby denoted by $Y$, is now given by

$$Y \sim Y_1 1_{\{\Gamma<\Delta\}} + Y_2 1_{\{\Gamma\geq\Delta\}},$$

with

$$Y_1 \sim Exp(\lambda_1), \quad Y_2 \sim Exp(\lambda_2).$$

Assuming $\Delta$ to be constant, Hagspiel, Huisman and Nunes (2015) now show that the first two moments of $Y$ can be expressed as

$$E[Y] = \frac{1}{\lambda_1}\left(1 - \mathrm{e}^{-\lambda_1\Delta}(1 + \lambda_1\Delta)\right) + \mathrm{e}^{-\lambda_1\Delta}\left(\frac{1}{\lambda_2} + \Delta\right)$$

$$Var[Y] = \frac{1}{\lambda_1^2}\left(1 + \frac{1}{\lambda_2^2}e^{-2\Delta\lambda_1}(\lambda_1 - \lambda_2)\lambda_2 + \frac{1}{\lambda_2^2}e^{-2\Delta\lambda_1}(\lambda_1 - \lambda_2)\lambda_1(-1 + 2e^{\Delta\lambda_1}(1 + \Delta\lambda_2)\right)$$

For the special case of $\lambda_1 = 0$, the expressions simplify to

$$E[Y] = \Delta + \frac{1}{\lambda_2}$$

$$Var[Y] = \frac{1}{\lambda_2^2}$$

Using these expressions, we can calibrate the arrival rates $\lambda_1$ and $\lambda_2$ based on real data. Finally, we can now define our stochastic jump as

$$\theta_t = \begin{cases} 0 & with\,probability\;\; 1 - \lambda_t \\ 1 & with\,probability\;\; \lambda_t \end{cases} \tag{3.20}$$

In theory we could utilize the same procedure to make $K_L$ stochastic, in the same manner as $K_R$. However, $K_L$ represents the scaling of the capacity efficiency factor after replacing components of the turbine, and has little to do with technological innovation. Therefore, we do not consider it to be stochastic in nature. Consequently, we define a deterministic, time-dependent process for it on the form

$$K_L(t) = 1 - (1 - K_l)(1 - e^{-\gamma t}) \tag{3.21}$$

Note the difference in subscripts between $K_L$ and $K_l$. Defining the process in this way, $K_l$ represents the fraction of the performance decay experienced by the turbine in the time interval $[0, t]$ that can be reversed by life-extending it.

### 3.3.4 – Redefining the operating profit margin

Using our recently defined stochastic processes for the electricity spot price and the weather factor, the gross instantaneous operating profit margin, $g_t$, can now be expressed as

$$g_t = P_t h(w_t) = e^{X_t} h(f(U_t)) \tag{3.22}$$

For such a process, Ernstsen and Boomsma (2018) show that the expected value can be found from equation 3.23 below. Note that $\varphi$ denotes the standard normal probability density function and $\Phi$ denotes the standard normal cumulative density function.

$$\begin{aligned} E[g_t \mid X_s, U_s] = \ & e^{X_s \exp\left(-\kappa_p (t-s)\right) + \alpha_t} \left[ h_{12}\left(\Phi(\tilde{u}_2) - \Phi(\tilde{u}_1)\right) + \frac{h_{12}}{\tilde{u}_1 - \tilde{u}_0}\left(\varphi(\tilde{u}_0) - \varphi(\tilde{u}_1)\right) \right. \\ & \left. - \frac{\tilde{u}_0 h_{12}}{\tilde{u}_1 - \tilde{u}_0}\left(\Phi(\tilde{u}_1) - \Phi(\tilde{u}_0)\right) \right], \end{aligned} \tag{3.23}$$

with

$$\tilde{u}_i = u_i + (u_i - U_s)\frac{e^{-\kappa_U (t-s)}}{1 - e^{-2\kappa_U (t-s)}} - \rho\sigma \frac{\sqrt{2\kappa_U}}{(\kappa_U + \kappa_P)} \frac{1 - e^{-\kappa_U (t-s)}}{1 - e^{-2\kappa_U (t-s)}}$$

$$\alpha_t = (\alpha - \epsilon_\sigma)\left(1 - e^{-\kappa_P (t-s)}\right)$$

$$\epsilon_\sigma = \frac{\sigma^2}{4} e^{-\kappa_P (t-s)}$$

Introducing the exponentially declining capacity efficiency $Q_t = Q_0 e^{-\gamma t}$ and setting the starting time to zero, i.e. $s = 0$, the net operating profit margin can be expressed as

$$G_t = Q_t g_t = e^{-\gamma t} e^{X_t} h(f(U_t)) \tag{3.24}$$

The expected value can then be expressed as

$$\begin{aligned} E[G_t \mid X_0, U_0] = Q_0 & e^{X_s \exp(-\kappa_p t) + \alpha_t - \gamma t} \left[ h_{12}\left(\Phi(\tilde{u}_2) - \Phi(\tilde{u}_1)\right) + \frac{h_{12}}{\tilde{u}_1 - \tilde{u}_0}\left(\varphi(\tilde{u}_0) - \varphi(\tilde{u}_1)\right) \right. \\ & \left. - \frac{\tilde{u}_0 h_{12}}{\tilde{u}_1 - \tilde{u}_0}\left(\Phi(\tilde{u}_1) - \Phi(\tilde{u}_0)\right) \right] \end{aligned} \tag{3.25}$$

From this, we can take a step further and introduce PPAs to our model. Let $\Upsilon$ be the fraction of the total power production that is sold at the spot price, and $(1-\Upsilon)$ the fraction sold through a PPA at a fixed price $\bar{P}$. The net operating profit margin can then be expressed as

$$\tilde{G}_t = Q_0 e^{-\gamma t}\big((1-\Upsilon)\bar{P}h(w_t) + \Upsilon P_t h(w_t)\big)$$

$$= Q_0 e^{-\gamma t}\Big((1-\Upsilon)\bar{P}h\big(f(U_t)\big) + \Upsilon e^{X_t} h\big(f(U_t)\big)\Big) \tag{3.26}$$

Taking the expectation of this expression, we find that

$$E\big[\tilde{G}_t \mid X_0, U_0\big] = Q_0 e^{-\gamma t}\Big((1-\Upsilon)\bar{P}E\big[h\big(f(U_t)\big) \mid U_0\big] + \Upsilon E[g_t \mid X_0, U_0]\Big) \tag{3.27}$$

with

$$E\big[h\big(f(U_t)\big)\big] = h_{12}\left(\Phi\left(\frac{u_2-\mu_u}{\sigma_u}\right) - \Phi\left(\frac{u_1-\mu_u}{\sigma_u}\right)\right) + \frac{h_{12}\sigma_u}{\sqrt{2\pi}(u_1-u_0)}\left(e^{-\frac{(u_0-\mu_u)^2}{2\sigma_u^2}} - e^{-\frac{(u_1-\mu_u)^2}{2\sigma_u^2}}\right)$$
$$+ \frac{(\mu_u-u_0)h_{12}}{u_1-u_0}\left(\Phi\left(\frac{u_1-\mu_u}{\sigma_u}\right) - \Phi\left(\frac{u_0-\mu_u}{\sigma_u}\right)\right), \tag{3.28}$$

where

$$\mu_u = U_0 e^{-k_u t},$$

$$\sigma_u = \sqrt{1-e^{-2k_u t}}.$$

Detailed derivation of equation 3.28 above can be found in Appendix A3. Given this new expression for the operating profit margin and its expectation, we can now move on to redefine the optimal stopping problem. In the next subchapter we formulate our optimal stopping problem, before we move on to consider different possible solution schemes.

### 3.3.5 – Reformulating the optimal stopping problem

Making use of our new modelling assumptions from the previous subchapters, and setting $Q_0$ to 1 for simplicity, we can now reformulate the optimal stopping problem from equation 3.6 in the following manner

$$V = \max_{\tau} \left\{ \underbrace{E[\int_{t=0}^{\tau} \tilde{G}_t e^{-rt}\, dt]}_{Value\ of\ operating} + max \left\{ \underbrace{E[\int_{t=\tau}^{T_L} \tilde{G}_t K_L(t) e^{-rt} dt - I_L e^{-r\tau}]}_{Value\ of\ life-extension} + \right.\right.$$

$$\left.\left. \underbrace{E[\int_{t=T_L}^{T_L+25} e^{\gamma T_L} \tilde{G}_t K_R(t) e^{-rt} dt - I_R e^{-r(T_L+\tau)}]}_{Value\ of\ repowering\ after\ life-extension},\ \underbrace{E[\int_{t=\tau}^{25} \tilde{G}_t K_R(t) e^{-rt} dt - I_R e^{-r\tau}]}_{Value\ of\ repowering} \right\}\right\}, \tag{3.29}$$

with $T_L = (25 - \tau_L) + \Delta t_L$, where $\tau_L$ denotes the time the life-extension option is exercised.

From this formulation, the stopping values for repowering and life-extension, respectively, are found to be

$$\Omega_R = K_R \int_0^{25} e^{-rt} E\left[\tilde{G}_t \mid X_{\tau_R}, U_{\tau_R}\right] dt - I_R \tag{3.30}$$

$$\Omega_L = \left[K_L \int_0^{T_L} E[\tilde{G}_t \mid X_{\tau_L}, U_{\tau_L}] e^{-rt} dt - I_L\right]$$

$$+ \left[K_R \int_{T_L}^{T_L+25} e^{\gamma T_L} E[\tilde{G}_t | X_{T_L}, U_{T_L}] e^{-rt} dt - I_R e^{-rT_L}\right] \tag{3.31}$$

Note that when we calculate the stopping value, the values for $K_R$ and $K_L$ will not be time-dependent as their level is set at the time of stopping. As there exists no analytical solutions for the integrals in the stopping values, these will have to be solved numerically for every set of input values $(t, X_t, U_t)$.

In contrast to the single-factor model, the introduction of multiple stochastic factors and time-dependent variables implies that an analytical solution is no longer available. An appropriate numerical solution scheme must therefore be applied to find an approximate solution. To find the most appropriate numerical solution method for our model, we begin the next subchapter by evaluating different relevant alternatives. Then, after making a choice of solution scheme, we provide a detailed description of how this scheme can be applied to our model through an algorithm.

## 3.4 - The numerical procedure

To solve the problem presented in Chapter 3.3, several numerical methods can be applied. The most commonly used methods include the binomial lattice approach (see Cox, Ross and Rubinstein (1979)), the finite difference method (see Brennan and Schwartz (1978)) and Monte Carlo simulation techniques, which was first developed by Boyle (1977). The binomial lattice approach and finite difference method both have the disadvantage of being poor at handling problems with many stochastic processes. In the binomial lattice method, the number of nodes grows exponentially with the number of stochastic processes, which makes applying it to multi-factor models computationally challenging. For the finite difference method, a maximum of two or three stochastic processes can be included in the problem, because higher dimensional partial differential equations cannot be obtained. Longstaff and Schwarz (2001) argue that simulation methods provide a promising alternative to traditional finite difference and binomial techniques; as they can be readily applied to options that depend on multiple factors, and value derivatives with both path-dependent and American-exercise features. They are also well suited to parallel computing, which allows for significant gains in computational speed and efficiency. Simulation is also highly flexible, for instance allowing state variables to follow general stochastic proccesses such as jump diffusions, as in Merton (1976) and Cox and Ross (1976), non-Markovian processes, as in Heath, Jarrow and Morton (1976), and even general semimartingales, as in Harrison and Pliska (1981). The Monte Carlo simulation method in particular is well suited to handle multiple stochastic processes, as the computational time complexity only increases linearly with the number of processes included. We therefore consider Monte Carlo simulation to be the most appropriate method for solving our multi-factor model.

In the literature, there are several nuances to the Monte Carlo simulation approach. The traditional method presented by Boyle (1977) was forward-looking and could therefore only be used to value options with fixed exercise time, i.e. European-style options. Therefore, several authors have suggested ways of adjusting this approach to be able to value American-style options where exercising is possible at any time within a given timeframe. Tilley (1993) suggests an algorithm that resembles the lattice method in finding the continuation value of holding the option. However, this method is unable to capture the complexities involving a multi-factor model. Carriere (1996) proposes a backwards induction algorithm and shows that the pricing of an American-style option is equivalent to calculating a series of conditional expectations. However, Stentoft (2004) argues that the conditional expectation is challenging to compute, making Tilley's (1993) model less suitable for multi-factor models. Broadie and Glasserman (1997) uses a combined simulation and decision tree approach to create a confidence interval for the option value that converges to the true price. However, Grant et al. (1997) show that the data quantity increases exponentially for each time step. Thus, for problems with many possible exercise times, this model becomes infeasible to solve.

Longstaff and Schwarz (2001) pioneered a relatively simple algorithm for valuing American-style options based on Monte Carlo simulation. The algorithm is called least squares Monte Carlo (LSM), a name derived from its use of cross-sectional least squares regression to estimate the continuation value of an option at a given possible exercise point. The algorithm is simple to apply, computationally inexpensive and can be implemented for problems with many stochastic factors. Also, the method is flexible in the sense that one can control the behavior in each decision node, e.g. through restricting possible exercise points to some nodes. It has also become popular within the real options literature (Kozlova, 2017), something Rodrigues and Armada (2006) credit to its simplicity. Based on these advantages, we find the algorithm suitable to apply to our real options model. In the next subchapter, we delve into the specifics of how the algorithm is applied to our model.

### 3.4.1 - The LSM-algorithm

Returning our attention to the optimal stopping problem presented in Chapter 3.3.5, we can now apply the LSM algorithm in order to determine the option value. To do so, we start by discretizing the time interval until maturity, $[0, T]$, into N intervals, so the length of each interval is $\Delta t = \frac{T}{N}$. The stochastic variables $(X, U)$ are simulated with $K$ paths, and combined to $K$ paths of simulated realizations. Applying this terminology, we can now discretize the expected development of our stochastic variables to get

$$dX_t = \kappa_P \left( \alpha - \frac{\sigma^2}{4\kappa_P} - X_t \right) \Delta t + \sigma\sqrt{\Delta t}\, W_{X,t}, \qquad W_{X,t} \sim N(0,1) \tag{3.32}$$

$$dU_t = -\kappa_U U_t \Delta t + \sqrt{2\kappa_U \Delta t}\, W_{U,t}, \qquad W_{U,t} \sim N(0,1) \tag{3.33}$$

For each of the $K$ paths we can now simulate the development of the stochastic factors by utilizing equations 3.32 and 3.33 above and generating random standard normal numbers. At each time and for each path we generate two random standard normal numbers $r_1$ and $r_2$, and use these to calculate our stochastic increments as

$$W_{X,t} = r_1 \quad ; \quad W_{U,t} = \rho r_1 + \sqrt{1 - \rho^2}\, r_2 \tag{3.34}$$

The transformation above, which is an application of the Cholesky decomposition, ensures that our stochastic increments in addition to being standard normally distributed also share the correlation $\rho$ of the electricity price and the weather factor.

Dividing our optimal stopping problem into a finite number of sub-problems, we can express the option value at each point in time with a Bellman equation as

$$V(t_n, X_{t_n}, U_{t_n}) = \max\left(\Omega(t_n, X_{t_n}, U_{t_n}), e^{-r\Delta t} E\left[V(t_{n+1}, X_{t_{n+1}}, U_{t_{n+1}})\right]\right), \tag{3.35}$$

where the exercise value $\Omega(t_n, X_{t_n}, U_{t_n})$ is whichever is highest of the repowering and life-extension value in the given node. Note that this value also includes the cost of exercising the option.

In the final time step $T$, which is the expected remaining technical lifetime of the turbine, the continuation value is zero and the optimal decision is to repower or life-extend the turbine if the expected value of doing so is positive. The value of exercise can be estimated from the stopping value expressions utilizing a numerical solution scheme, and we will utilize a globally adaptive quadrature which is a native function in Matlab. In all previous time steps, the optimal decision is determined by comparing the value of continuation with the value of immediate exercise. It is in this area the LSM algorithm makes its major contribution, as it provides a method for estimating the value of continuation. This is done by regressing subsequently realized cash flows and assuming that the expectation function conditional on information available at each step can be expressed as a linear combination of a set of basis functions denoted $\Psi_i(t, X_t, U_t)$, where $i = 1,2, \dots, m$ is the order of the basis functions. By denoting the regression coefficients $\alpha_i$, we can then express the continuation value as

$$f(t, X_t, U_t) = \sum_{i=1}^{m} \alpha_i \Psi_i(t, X_t, U_t) \tag{3.36}$$

Moren and Navas (2003) find that the LSM-algorithm is robust to the choice of basis function. In addition, Palchykov and Vardøy (2018) find benefits to using Laguerre polynomials, hence we choose to utilize this functional form in our conditional expectation regression. The Laguerre polynomial is a polynomial function with the general form

$$L_n(X) = e^{-\frac{X}{2}} \frac{e^X}{n!} \frac{d^n}{dX^n} (X^n e^{-X}) \tag{3.37}$$

As recommended by Longstaff and Schwartz (2001), we use both our stochastic parameters and their cross product in the conditional expectation function, to get the functional form

$$\begin{aligned} f(t, X_t, U_t) = &\sum_{i=1}^{m} a_i \, e^{-\frac{X}{2}} \frac{e^X}{i!} \frac{d^i}{dX^i} \left(X^i e^{-X}\right) + \sum_{j=1}^{m} a_j \, e^{-\frac{U}{2}} \frac{e^U}{j!} \frac{d^j}{dU^j} \left(U^j e^{-U}\right) \\ &+ \sum_{k=1}^{m} a_k \, e^{-\frac{XU}{2}} \frac{e^X U}{k!} \frac{d^k}{d(XU)^k} \left((XU)^k e^{-XU}\right) \end{aligned} \tag{3.38}$$

The continuation value expressed in equation 3.39 above is however only the expected value of postponing the exercise of your option another period. For our wind turbine, as with other assets that provide cash flows prior to options being exercised, it is also necessary to include the cash flows received by waiting another period. The operational profit flow from operating a wind turbine for a single period, can be found as

$$\pi_{\Delta t} = \int_0^{\Delta t} G_t\,(x_T\,, u_T) e^{-rt} dt = \int_0^{\Delta t} e^{-\gamma t} g_T \, e^{-rt} dt$$

$$= g_T \int_0^{\Delta t} e^{-(r+\gamma)t} dt = \frac{g_T}{r+\gamma} \left(1 - e^{-(r+\gamma)\Delta t}\right), \tag{3.39}$$

where for relatively small time-steps

$$g_T = \Upsilon e^{-\gamma T} e^{x_T} h(u_T) + (1 - \Upsilon) e^{-\gamma T} \bar{P} h(u_T) \tag{3.40}$$

However, because $h(u_T)$ relies on a single realization of the weather factor that is assumed representative for the entire period $\Delta t$, it will become increasingly inaccurate as the length of $\Delta t$ increases. Therefore, if the frequency of the time-steps used in the calculations are not sufficiently high to properly capture the short-term variations of the wind speed, it is better to use an expected average for the power output. By denoting the capacity factor of the specific wind turbine $k_t$, this can be expressed as

$$g_T = \Upsilon e^{-\gamma T} e^{x_T} h_{12} k_t + (1-\Upsilon) e^{-\gamma T} \bar{P} h_{12} k_t \quad (3.41)$$

Note that $[g_T] = NOK/h$, and it should have the time scale as $\Delta t$. Hence, if e.g. $[\Delta t] =$ years, $g_T$ should be multiplied with 8670 h/year. Combining the operation profit flow with equation 3.39 above, we can now express the total expected continuation value as

$$F(t, X_t, U_t) = \sum_{i=1}^{m} \alpha_i \Psi_i(t, X_t, U_t) + \pi_{\Delta t}(X_t, U_t) \quad (3.42)$$

By starting at the final time step and proceeding recursively through each step until the first, we can for each path find an optimal exercise time $\tau_k$ and a corresponding optimal exercise value $V\left(\tau_k, X_{\tau_k}, U_{\tau_k}\right)$. The option value is then found by taking the average of the discounted exercise values for all simulation paths, which can be expressed as

$$V(t_0, X_0, U_0) = \frac{\sum_{k=1}^{K} V\left(\tau_k, X_{\tau_k}, U_{\tau_k}\right) e^{-r(\tau_k - t_0)}}{K} \quad (3.43)$$

Using the exercise times $\tau_k$, we can also construct a logical matrix $M$ where each index $(k, t)$ will take the value 1 if $t \mid k = \tau_k$ and 0 otherwise. The expected time of exercise can then be found as a weighted sum of the exercise times on the form

$$E_{t,exercise} = \frac{\sum_{t=1}^{T/\Delta t} \sum_{k=1}^{K} M(k,t) * t}{K} \quad (3.44)$$

and the probability of the option being exercised within a certain time $t_1$ can be estimated as

$$P_{t_1}^{*} = \frac{\sum_{t=1}^{t_1/\Delta t} \sum_{k=1}^{K} M(k,t)}{K} \quad (3.45)$$

The accuracy and computational time complexity of the LSM algorithm depends on the number of simulated paths $K$, the number of time steps $T/\Delta t$ and the number of basis functions $M$. Stentoft (2004) shows that the estimated option value from the LSM algorithm converges to its true value as $K$ and $M$ tends to infinity, although this is not computationally feasible to simulate. Therefore, one must find a compromise that yields satisfactory results both in terms of accuracy and computational time.

# 4 - Model calibration and parameterization

Choosing if and when to undertake investment in end-of-life options, in addition to deciding which option to invest in, is a key decision for any wind power producer. This decision is largely determined by the input parameters used in the model, hence it is essential to choose parameters on a case-by-case basis to best capture the characteristics of each specific project. In the numerical case we present in this thesis, we consider a single generic Norwegian wind farm with homogenous turbines. The model is calibrated to this case, and parameters are chosen by using a combination of market data, industry reports and scientific literature. It should be emphasized that neither parameter estimates, nor the calibration methods can be applied indiscriminately to other wind power projects, and should be evaluated for each specific project. In the following subchapters we first calibrate the risk factors, followed by estimation of financial, turbine, end-of-life and simulation parameters. Finally, we discuss some case-specific assumptions. The parameter values used in the baseline case are summarised in Table 4.1 below.

| Parameter | Description | Category | Base case value |
|---|---|---|---|
| $X_0$ | Log of current electricity price | Electricity | 5.923 NOK/MWh |
| $\alpha_P$ | Mean reversion level factor for the electricity log price | Electricity | 5.765 NOK/MWh |
| $\sigma_P$ | Annual volatility for the electricity log price | Electricity | 17.27% |
| $\kappa_P$ | Mean reversion speed for the electricity log price | Electricity | 0.1858 |
| $\rho$ | Correlation between the electricity log price and weather factor | Electricity | -0.1503 |
| $\kappa_U$ | Mean reversion speed for the weather factor | Wind | 0.00165 |
| $a$ | Weibull distribution shape parameter | Wind | 10.03 |
| $b$ | Weibull distribution scale parameter | Wind | 1.96 |
| $r$ | Cost of capital | Finance | 5% |
| $I_R$ | Cost of repowering | Finance | 30 MNOK |
| $I_L$ | Cost of life-extension | Finance | 3 MNOK |
| $T$ | Expected technical lifetime of turbine | Turbine | 25 years |
| $h_{12}$ | Rated power output of turbine | Turbine | 3 MW |
| $k$ | Capacity factor of turbine | Turbine | 45% |
| $\gamma$ | Annual capacity efficiency decline | Turbine | 1.6% |
| $w_0$ | Turbine cut-in speed | Turbine | 4 m/s |
| $w_1$ | Turbine rated speed | Turbine | 10 m/s |
| $w_2$ | Turbine cut-off speed | Turbine | 25 m/s |
| $K_R$ | Repowering capacity efficiency factor / scaling factor | Repowering | 1.25 |
| $\Delta$ | Time from previous innovation until arrival rate change | Repowering | 10 years |
| $\lambda_1$ | Innovation arrival rate for t < Δ | Repowering | 0 |
| $\lambda_2$ | Innovation arrival rate for t >= Δ | Repowering | 0.05 |
| $K_L$ | Life-extension capacity efficiency factor | Life-extension | 0.25 |
| $\Delta t_L$ | Length of life-extension | Life-extension | 5 years |
| $K$ | Number of simulation paths | Simulation | 5000 |
| $m$ | Order of Laguerre polynomials | Simulation | 7 |
| $\Delta t$ | Time step | Simulation | 0.25 years |

Table 4.1: Baseline case parameter values

## 4.1 – Price of electricity

Recall that the natural logarithm of the electricity price $X_t$, follows the Ornstein-Uhlenbeck process

$$dX_t = \kappa_P \left(\alpha_P - \frac{\sigma^2}{4\kappa_P} - X_t\right) dt + \sigma_p dZ_P$$

In line with the method of Dixit and Pindyck (1994) we can integrate this process and reformulate it to a regression model on the form

$$x_t - x_{t-1} = a + bx_{t-1} + \varepsilon_t, \tag{4.1}$$

and apply it to a set of discrete price data. In doing so, the volatility of the model can be estimated as

$$\hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{\frac{2\log(1+\hat{b})}{\left(1+\hat{b}\right)^2 - 1}} \tag{4.2}$$

Considering that our model is evaluating a long-term investment decision, we would ideally like to calibrate it to a long-term prognosis from a reliable source. A drawback with the regression model presented above is however that it requires relatively high frequency of data for accuracy, while prognoses tend to provide very few data points. Therefore, we instead utilize historical spot data for the central Norwegian power market from Nord Pool, and estimate the volatility $\sigma_P$ to be 17.27% annually. The remaining two parameters, $\kappa_P$ and $\alpha_P$, are however better suited to be calibrated to a prognosis. This is because in contrast to the volatility that captures rapid, short-term variations, these two parameters are primarily related to the long-term trend of the process and can therefore be estimated from lower frequency data. Note that while we assume constant volatility for our numerical case, the model can easily be implemented with dynamic volatility as well.

Using a 22-year prognosis from Statnett, the system operator of the Norwegian power system, we set the mean-reversion level of the process, $\alpha_P - \frac{\sigma^2}{4\kappa_P}$, to be 5.725 NOK/MWh. The prognosis is presented in Figure 4.1. Note that as the figure presents electricity prices in €/MWh, so we need to use the €/NOK exchange rate to convert the values to NOK/MWh and take the natural logarithm for our parameter estimates. From the prognosis we also set the initial level of our process, $X_0$, to 5.923 NOK/MWh. Having determined values for volatility, initial level and mean-reversion level, we can now set the mean-reversion speed $\kappa_P$ to follow the trend of the prognosis. From Dixit and Pindyck (1994) we find the expected value of an Ornstein-Uhlenbeck at a given time $t$ to be

$$E[X_t \mid X_0] = X_0 e^{-\kappa_P t} + \left(\alpha_P - \frac{\sigma^2}{4\kappa_P}\right)(1 - e^{-\kappa_P t}) \tag{4.3}$$

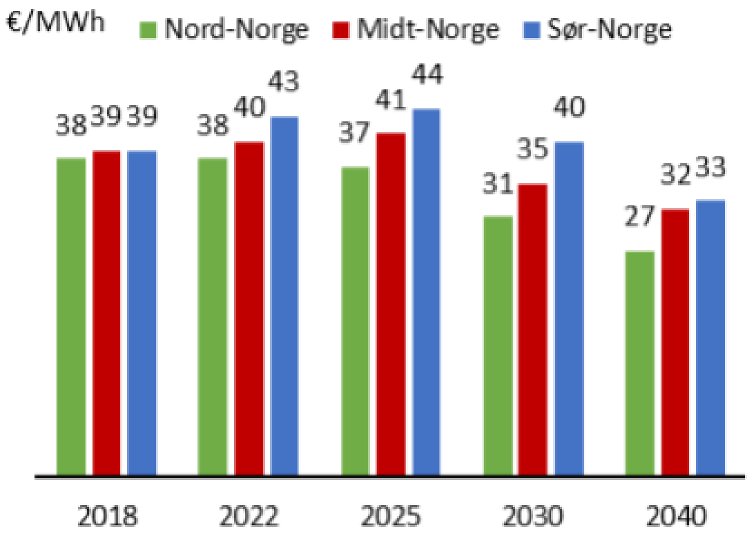



*Figure 4.1: 22-year prognosis of electricity prices. Source: Statnett*

In line with the prognosis we expect the process to reach its mean-reversion level for the first time in 2040, and can thereby solve the equation above iteratively to find $\kappa_P = 0.1858$. From the expression for the mean-reversion level we can then determine $\alpha_P$ to be 5.765 NOK/MWh. While it should be noted that multiple values of $\kappa_P$ satisfy the above equation depending on the tolerance level used in the convergence criterion for the iterative solution process, we consider the chosen value to represents a decent approximation of the drift. This is visualised in Figure 4.2 below.

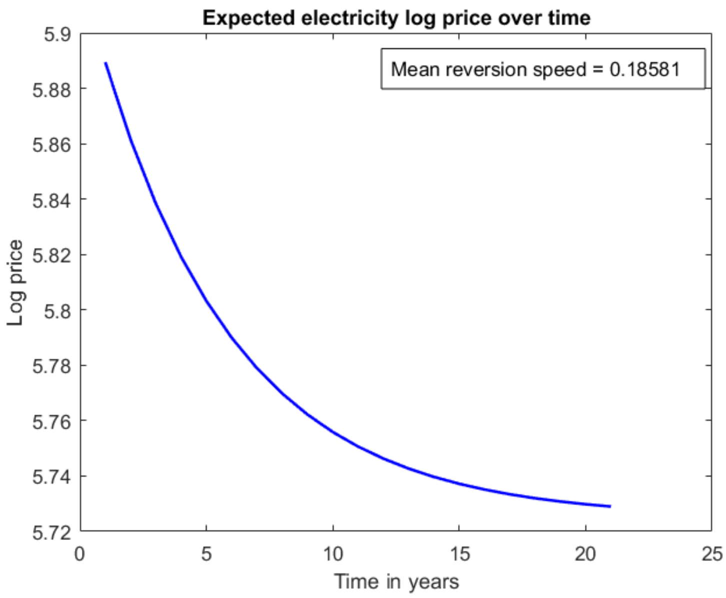



*Figure 4.2: Expected electricity log price over time from the Ornstein-Uhlenbeck process*

The correlation $\rho$ between the price of electricity and the weather factor depends to a large degree on how much of the total energy mix in an area consists of wind power. The larger the market share of wind power, the larger is the dependence of the total energy supply on the weather and thereby the correlation between weather and prices. The estimate of this parameter is, therefore, dependent on area-specific data, and ideally also requires high-frequency data of wind speeds and electricity prices. Unfortunately, companies are not very forthcoming with such wind data for competitive reasons, and meteorological data, if available, are measured at too low altitude above the ground to be applied to wind turbines. In the absence of appropriate data for estimation, we therefore make due with the value of $\rho = -0.1503$ from Ernstsen and Boomsma (2018), that was estimated for a model with similar risk factors as ours. Intuitively, this seems to be a sensible value as the correlation should be negative and relatively modest considering that wind power only makes up a relatively small part of the energy mix in Norway. It should however be noted that because wind power is a significantly larger part of the energy mix in Denmark than in Norway, utilizing this estimate for a Norwegian wind farm is likely to provide a more conservative option value.

The PPA-price $\bar{P}$ is determined by negotiation between the two contract parties and will therefore not only depend on specific market conditions, but also the negotiating power of the two parties involved. While PPA contracts are relatively common, the publicly available data on them are limited as companies tend to withhold price information for competitive reasons[26]. A general guideline is that the contract should be fairly priced[27], but its interpretation in a specific situation seems quite subjective. Nonetheless, in an effort to apply this principle, we choose to utilize the prognosis from Figure 4.1. as a risk-neutral long-term proxy for the price of electricity and calibrate $\bar{P}$ to this as a vector of yearly values. However, as was discussed in Chapter 2.3.2, the PPA allows the producer to reduce the price risk of his project and thereby acquire debt financing at lower interest rates. We therefore consider it fair that the producer pays a modest risk premium of 2% to the buyer of the contract, and include this in our estimate.

## 4.2 – Weather factor

Recall that the weather factor follows the Ornstein-Uhlenbeck process given by

$$dU_t = -\kappa_U U_t dt + \sqrt{2\kappa_U} dZ_U$$

We can thereby apply the regression model from equation 4.1 to a discrete data set. Utilizing wind speed time series from the Norwegian area Måkaknuten, measured at 100m altitude above ground level, we can run the regression analysis. In doing so, we estimate the weather factor volatility to be 5.74% and from this we find the mean-reversion speed to be $\kappa_U = 0.00165$.

As discussed in Chapter 2.3.2, the most common way to approximate the wind speed distribution in a model is through a Weibull distribution function. Nielsen (2011) consider

[26] https://e24.no/energi/norsk-hydro/hydro-inngaar-vindkraftavtale-med-verdens-lengste-varighet/24396381

[27] https://aleasoft.com/ppa-opportunity-agents-risk-management-european-electricity-market/

and compare the methods of moments estimation, maximum likelihood estimation and rank regression to estimate the parameters of a two-parameter Weibull distribution. His findings are that for a sufficiently large data set, no method is significantly more accurate than another. We therefore choose to apply maximum likelihood estimation to our data set, resulting in a Weibull scale parameter of $a = 10.03$ and a Weibull shape parameter of $b = 1.96$. For transparency, the historical realized distribution of wind speeds for Måkaknuten is presented in Table 4.2 below.

| *Yearly average wind speed at 100m height* | | | |
|---|---|---|---|
| Number of observations | % of total | Interval | Unit: |
| 1314 | 0.77% | 0 - 1 | m/s |
| 4235 | 2.48% | 1 - 2 | m/s |
| 7198 | 4.22% | 2 - 3 | m/s |
| 10364 | 6.08% | 3 - 4 | m/s |
| 13056 | 7.65% | 4 - 5 | m/s |
| 14992 | 8.79% | 5 - 6 | m/s |
| 15320 | 8.98% | 6 - 7 | m/s |
| 14771 | 8.66% | 7 - 8 | m/s |
| 13353 | 7.83% | 8 - 9 | m/s |
| 12163 | 7.13% | 9 - 10 | m/s |
| 10823 | 6.34% | 10 - 11 | m/s |
| 9605 | 5.63% | 11 - 12 | m/s |
| 8193 | 4.80% | 12 - 13 | m/s |
| 7321 | 4.29% | 13 - 14 | m/s |
| 6092 | 3.57% | 14 - 15 | m/s |
| 5208 | 3.05% | 15 - 16 | m/s |
| 4184 | 2.45% | 16 - 17 | m/s |
| 3313 | 1.94% | 17 - 18 | m/s |
| 2486 | 1.46% | 18 - 19 | m/s |
| 6595 | 3.87% | > 19 | m/s |

*Table 4.2: Realised wind speed distribution for Måkaknuten at 100m altitude above ground level*

## 4.3 – Technological innovation

Recall that for technological innovation we have the stochastic jump model

$$K_R(t) = 1 + (K_R - 1)\theta_t,$$

where $\theta_t$ follows a non-stationary Poission process with an arrival rate $\lambda_t$. Denoting the time since the previous innovation $\Delta$, $\lambda_t$ can be expressed as

$$\lambda_t(\omega) = \lambda_1 1_{\{\Gamma<\Delta\}} + \lambda_2 1_{\{\Gamma\geq\Delta\}}$$

The technological innovation parameters $\lambda_1, \lambda_2$ represent the rates at which we expect advances in wind turbine technology to be made available, and $\Delta$ the time before regime-change. Notice that this is not just a matter of new technology being invented, it must also be made commercially available and economically viable. After a new turbine model has been designed, it might need to clear extensive legal hurdles before it can be sold. The company producing it might also need time to improve its production and logistics processes and reduce costs before it becomes economically viable for buyers.

While historical data for wind power technology the last 20-30 years indicate a very swift and exponential growth, this development is expected to significantly slow down in the future (Ziegler et al., 2018). In part this is caused by size restrictions, as the newest turbines already are so massive it is making transport and installation difficult. While these restrictions are less pressing offshore, these turbines introduce other complications such as buoyancy and waves that must be adequately addressed when designing new turbines. Considering these factors, and looking at a combination of historical data and prognoses, we estimate the expected time between one technological innovation being made commercially available and the next to be around 15 years, with $\lambda_1 = 0$ and $\Delta = 10$ years. Recalling from Chapter 2.4.1.3 that wind turbine innovation is primarily in the direction of bigger and taller, it seems reasonable to assume it will take at least a fairly significant amount of time to find solutions to all the added technological complexities.

## 4.4 – Financial parameters

The cost of capital $r$ for a wind power project is naturally a function of the risk characteristics of that specific project. This among others includes price risk, production risk and operational risk. In addition, the financing structure of the project will significantly impact $r$ as equity financing carries significantly higher risk than debt financing does. The average cost of capital for wind power projects in OECD countries is according to IRENA (2018) at 7.5%. A recent empirical study from Steffen (2019), however, estimated the cost of capital in renewable projects in 46 different countries, and found the appropriate rate for countries such as Denmark and the US to be around 5 %. Considering Norway is fairly similar to these countries in terms of for instance political stability, and that Norway generally is considered to have low risk in its energy projects (Fossen, 2018), we consider it reasonable to set the cost of capital for our case at 5%.

According to a report by NewEnergyUpdate (2018) the average cost of repowering is 1 million euros per MW for repowering, and 100 000 euros per MW for life-extension. The cost of repowering in particular can however be significantly reduced by planning ahead for it when building the initial wind park. Good initial planning can allow much of the infrastructure for power transport from the initial project to be reused, and experience and business connections from the initial project can be conserved to be used in the repowering project to reduce costs in all aspects. Assuming for our numerical case that we are able to realize much of these cost reductions by good initial planning, and utilizing an exchange rate of roughly 10 NOK/Euro, we set the cost of repowering to 8 million NOK per MW and the cost of life-extension to 1 million NOK per MW.

## 4.5 – Turbine parameters

The rated power output of the wind park $h_{12}$ determines the scale of the investment problem in consideration and is a given input for any specific wind park. We set it to 3 MW to match one of the turbines in GE's product portfolio[28]. From the turbine models of the two largest global wind turbine producers, Vestas and GE, we can then find normal values for the speed parameters to be $w_0 = 4m/s$, $w_1 = 10\,m/s$ and $w_2 = 25\,m/s$. Wind

[28] https://www.ge.com/renewableenergy/wind-energy/onshore-wind/turbines/3mw-platform

turbines are usually optimized during design to operate at a specific location, hence we have chosen our speed parameters from within the normal range to fit our wind distribution.

According to a 2017 market report from the U.S. Department of Energy, the average capacity factor of newly installed wind power in the USA in the period 2014 to 2016 was 42%, up from 31.5% in the period 2004 - 2011[29]. Considering that the US primarily uses the same turbine suppliers as Norway, namely Vestas and GE, we assume this information is also fairly representative for Norway. Assuming this trend continues, and that our turbines are state-of-the-art, we therefore set our capacity factor $k$ to 45%. Building further on the assumption that our turbines are state-of-the-art, we can set the expected technical lifetime to $T = 25$. As was mentioned in Chapter 1.1, the expected lifetime of a wind turbine is is 20-25 years, hence this parameter value is on the higher end of the industry average.

The capacity decline $\gamma$ is highly project specific, as it can vary quite significantly both with turbine type and location. Different turbine types wear in different ways, and local weather conditions and topography cause large variations in the distribution of forces and chemical exposure to the turbine. Performance data for specific turbines over time is, however, yet another type of data companies tend not to share, hence we have to use a more general estimate as a proxy. Assuming the findings of Stafell and Green (2014) presented in Chapter 2.3.1.2 also are applicable to Norway, which seems reasonable considering that Norway and Britain use the same turbine suppliers, we set $\gamma$ to 1.6% annualy. Note that while we assume the state-of-the-art turbine in our numerical case will experience somewhat lower performance decline than an average turbine, the chosen value of $\gamma$ also incorporates increasing O&M costs.

## 4.6 – End-of-life option parameters

The repowering capacity scaling factor, $K_R$, represents the potential improvement to the wind farm's performance by repowering. As a wind farm has constraints on the available area, this is not just a matter of new technology being available. With increasing rotor diameter of a wind turbine, the wake effects become larger, hence a general rule of thumb is that there should be 3-10 diameters[30] between each turbine to avoid significant energy loss by wind shadowing from upstream turbines. In addition to predicting how much performance increase a newer turbine model will offer, this parameter should therefore also reflect the area restrictions of the park and how many of the new turbine types can be fit into the park. Taking this into consideration, and looking at a combination of historical data and prognoses for future development, we find $K_R = 1.25$ to be a reasonable value. For a wind park with the possibility of expanding to the surrounding area this value might be significantly higher, and for a wind park with very tight area restrictions it might be even lower.

[29] https://www.energy.gov/eere/wind/downloads/2017-wind-technologies-market-report

[30] https://www.planningni.gov.uk/de/index/policy/planning_statements/pps18/pps18_annex1/pps18_annex1_wind/pps18_annex1_technology/pps18_annex1_spacing.htm

The life-extension capacity scaling factor, $K_L$, represents the amount of capacity decline suffered by the turbine from the start of its operation that can be reversed by extending the lifetime of key components. Like $\gamma$, this parameter is highly project specific. Hence, the question of which components that should be life-extended and how much they can be improved depend significantly on turbine type, location and how it has been worn over its lifetime. While we have assumed that life-extension in total will only reverse capacity decline, it can, however, improve specific components past their original performance. A common example of this is the control system, that can be significantly improved by utilizing new software[31]. As little to no data seems to be publicly available on the nature of specific performance increase through life-extension, we set $K_L$ to 25% which intuitively seems to be within the realistic range.

The increase of the technical lifetime by life-extension $\Delta_{t_L}$ can be up to 20 years depending on the project specifics[32]. This is not only a matter of how long life-extension is technologically possible for the specific wind project, but also how long is economically viable. Increasing the length of life-extension naturally comes at a higher investment cost, and a higher opportunity cost by foregoing the option to repower, hence a financial model might be needed to find the optimal $\Delta_{t_L}$ for a specific project. Because we lack data on how the cost of life-extension depends on $\Delta_{t_L}$, we choose to set a fixed value of $\Delta_{t_L} = 5$ years for our numerical case, at the cost of $I_L = 1$ mill NOK per MW. Finding the optimal length of life-extension with our model is however simply a matter of estimating $I_L$ as a function of $\Delta_{t_L}$, and then comparing the stopping values for all realistic pairs $\left(\Delta_{t_L}, I_L(\Delta_{t_L})\right)$ in each node.

## 4.7 – Simulation parameters

As mentioned in Chapter 3.4.1, Stentoft (2004) find that the solution provided by the LSM algorithm converges to its true value as the number of simulation paths, $K$, and order of Laguerre polynomials, $m$, tends to infinity. As this is not practically feasible, as doing so would require an infinite amount of computational time, we must find a satisfactory balance between accuracy and computational time complexity. As the variance of Monte Carlo simulations decrease with the square root of the number of simulation paths[33], the number of simulations required to find a satisfactory threshold of accuracy can be quite high. In order to lower this threshold, we therefore introduce a variance reduction technique known as antithetic variates to our simulations. Using our baseline model parameters and conducting extensive numerical testing, we find $K = 5000$ and $m = 7$ to be the optimal parameter values for our case. Increasing the number of simulation paths or order of polynomials further does not yield significantly different results, hence we find this to be the lowest computational time complexity we can achieve without significantly sacrificing accuracy. Note that by choosing the order of polynomials to be 7, we get a total of 24 basis functions.

[31] https://www.spicatech.dk/about-us/press/is-lifetime-extension-of-your-ageing-turbine-the-right-solution/

[32] https://www.windpowerengineering.com/projects/policy/business-case-wind-farm-lifetime-extension/

[33] https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-90-computational-methods-in-aerospace-engineering-spring-2014/probabilistic-methods-and-optimization/error-estimates-for-the-monte-carlo-method/

Regarding the time step $\Delta t$, this is again a trade-off between computational time and realism. Ideally having a $\Delta t$ of 1 hour or less would be optimal, as it would allow the use of equation 3.40 instead of the approximation in equation 3.41. However, combining such a step length with reasonable numbers for $K$ and $m$ would result in more than a month of computational time. In applying the model to evaluate a real wind power project this might be a perfectly acceptable timeframe, but as we intend to conduct sensitivity analysis, this is not feasible in our case. Because we have to set a time step that is too high to utilize equation 3.40, we lose no additional accuracy by choosing a somewhat longer time step. Therefore, we set $\Delta t$ to 0.25 years, which we find provides a good balance between accuracy and computational time complexity.

## 4.8 – Case-specific assumptions

In addition to our parameter calibrations, we make some case-specific assumptions. These assumptions are motivated by a combination of making the case realistic, making the results from the sensitivity analysis more straightforward to interpret and reducing the computational time of our algorithm.

The first assumption we make, is that the wind park we are considering is at the very beginning of its operational life, i.e. $t_0 = 0$. The model can however be applied to any value of $t_0$, i.e. at any point in the lifetime of a wind park, hence this assumption is simply to make the results easier to interpret. By the same motivation, we choose to utilize the same $\gamma$ for the turbine both before and after exercising an end-of-life option. This seems reasonable to do, as $\gamma$ was calibrated from fairly general data. Note that this is not a necessary assumption to make in general, as the model can incorporate different values for $\gamma$ if that seems more appropriate for a specific project. It might for instance be relevant to allow gamma to change following repowering and life-extension, or assume it to have some dependence on time.

The next assumptions we make, are that repowering will only be available after 10 years of operation and life-extension only available after 20 years of operation. The motivation for both is a combination of imposing realistic technological constraints and reducing the computational time of our model. For repowering, we have chosen to model the technological innovation through a stochastic process where the arrival rate for $t < \Delta = 10$ is zero. Therefore, there will be no improved turbine available for $t < 10$ for any simulation path, and thereby no incentive to repower before this time. For life-extension, there exist technological and legal restrictions that can provide incentive to avoid life-extension too long before the expected technical lifetime of the turbine. As was discussed briefly in Chapter 1, the structural integrity of the wind turbine is prioritized over economic interests when considering life-extension (DNVGL, 2016).

Evaluating which components to life-extend is therefore not just simply a matter of optimizing the performance of the turbine, but of reducing the probability of structural failure to a point where the turbine will be legally allowed to operate past its design lifetime. The evaluation of remaining life will require an assessment of the site-specific component loads, based on the turbine's operational history and measured wind conditions, along with a probabilistic analysis contingent on the current structural condition and the impact of possible material degradation on the structure's fatigue-limit state

(MegaVind, 2016). For further discussion on the technical details and considerations of life-extension, see MegaVind (2016). It suffices to say that it is necessary to measure the fatigue of different components relatively late in the operational life, which combined with performance data and measurements over the turbine lifetime can be used to provide a somewhat accurate estimate of the probability of failure for different components and thereby decide which components should be life-extended. The design lifetime of components and estimations of future structural integrity and fatigue damage are also legally subjected to strict safety margins. For instance, assuming quite extreme wind conditions, such as the 90$^{th}$ percentile of wind turbulence, when assessing probability of failure. Hence, the actual lifetime of a component is often significantly longer than the design life or remaining life estimations, as it in reality experiences less extreme loads. Measuring and estimating the remaining lifetime of components closer to the expected design lifetime will therefore provide more accurate predictions, which in turn can reduce costs for life-extension by for instance allowing for less extensive upgrades or replacing fewer components. Therefore, we consider the restriction of not being able to life-extend until 20 years of operation to be a realistic addition to our case study. The choice of 20 years is however fairly subjective, so we include a brief discussion of the implications for our case of changing the restriction to 15 years.

The next assumption we make, is that the fraction of power sold through PPA contracts, $\Upsilon$, is constant both pre and post repowering. Depending on the contract specifics, the PPA-fraction might change over time and might have to be renegotiated when repowering. This depends on the contract for the specific project though, so assuming it to be constant for our specific case study is perfectly reasonable and makes the PPA-analysis easier to conduct and interpret. The model is however perfectly capable of including different values for $\Upsilon$ at different times and in different operational states, if that should be the case for the specific project under consideration.

Last, we assume that volatility for our stochastic risk factors are constant over time. Dynamic volatility can be included in the model without any major adjustments, but as discussed in Chapter 3.3.1, the long-term investment horizon of our numerical case makes the short-term fluctuations of volatility unimportant. It should also be noted that for such a long time horizon, even attempting to predict the short-term variations of volatility might be completely futile.

# 5 - Results

By implementing the multi-factor model in Matlab, and utilizing parameter values determined in Chapter 4, we analyse the end-of-life options for our numerical case in order to address the project-specific research questions posed at the end of Chapter 1. We begin the analysis by considering the base case option value and expected investment timing for each option, and compare them to determine the base case implications for an operator. Next, we perform sensitivity analysis to see how the option value and investment timing change when parameters diverge from their base case estimates. Finally, we analyse how including PPAs impacts the investment problem, and discuss the ramifications.

In order to make the analysis more extensive and informative, we have performed valuation on three separate end-of-life option scenarios. These scenarios are as follows:

i. The option to repower: The wind farm operator only considers repowering.
ii. The option to life-extend: The wind farm operator only considers life-extension. Note that because we assume decommissioning is not financially motivated, as was discussed in Chapter 1.2, the life-extension option we consider for our case will always be followed by repowering at the end of the extended lifetime.
iii. The option to either repower or life-extend: The wind farm operator considers both the option to repower and the option to life-extend, and can choose the most valuable one at any point in time. This implies that if one option is exercised, the other is relinquished. In figures and tables that follow, we refer to having both options available as the double option.

## 5.1 – Base case results and analysis

For the base case we have estimated the option value and the expected exercise time for the three options mentioned above. The results are summarized in Table 5.1. From the table it can be observed that all the options are expected to be exercised before the end of the technical lifetime of the wind turbine. The option to perform life-extension has the lowest option value and the latest expected exercise time, which can be explained by the technological restriction imposed on the possible exercise times for this option. If the operator could perform life-extension before the last 5 years of the expected technical life, the option would become more valuable and the expected exercise time would be reduced. This has been confirmed by running a simulation with a less strict technological restriction imposed on the life-extension option that allowed for life-extension in the last 10 years of operation. It can further be observed that repowering has significantly higher option value than the life-extension option and earlier expected exercise. Therefore, if the operator could only have one of the two options available, the option to repower would be preferable. This effect is largely a result of strong market conditions making early exercise valuable, and since repowering is available earlier than life-extension it becomes more valuable for the operator. Note that this observation is a result of comparing the options after they have been evaluated separately, and not by comparing them simultaneously, which is what we do with the double option.

| | Option value $[MNOK]$ | Expected exercise time $[years]$ |
|---|---|---|
| **Repowering** | 34.256 | 18.73 |
| **Life-extension** | 28.666 | 21.46 |
| **Double option** | 35.156 | 18.55 |

*Table 5.1: Option value and expected exercise time for the base case*

Having both options available, i.e. the double option, can be observed to provide both the highest value and earliest expected exercise time for the operator. This implies that even though repowering is overall more valuable than life-extension to the operator, there are possible market conditions for which life-extension is most valuable. If this was not the case, and repowering was exercised before life-extension in every simulation path, the double option would have the same value and expected exercise time as the option to repower.

By investigating the simulation paths where different options are exercised, we can make some interesting observations. In scenarios where the market conditions are very favourable early in the operational lifetime of the turbine, in particular the price of electricity is high, the option to repower will be preferred because it allows the operator to utilize the favourable market conditions sooner. In scenarios where the market conditions early on are not favourable, the operator is better off by waiting to see how the market conditions develop. The longer the operator waits, the closer he gets to the point where the life-extension option becomes available, and thereby is more likely to perform life-extension. If the options have not been exercised before 20 years of operation, a situation is reached where life-extension dominates the option to repower. This is because life-extension allows the operator to keep the current wind farm in operation longer than what is possible with only repowering.

The behaviour of the options can also be analysed by looking at how the exercise times are distributed over the technical lifetime of the turbines.  Therefore, we have plotted the cumulative probability of option exercise for each option and indicated the expected exercise time with a dashed line. The plots for repowering, life-extension and the double option are shown in Figure 5.1, Figure, 5.2 and Figure 5.3, respectively.

From Figure 5.1, it can be observed that the option to repower will not be exercised during the first 10 years of operation. This is because new technology is not expected to arrive in this period, and thus it is not attractive for the operator to exercise the option. In the last 15 years of operation, where the probability of new technology arrival increases, the cumulative probability of option exercise increases almost linearly before it increases abruptly the last year. The sharp increase in exercise probability towards the end can be explained by the restriction imposed by the technical lifetime, as the turbine will no longer be operational past this point and the operator is therefore unable to postpone the exercise decision any further. In the simulations, there are scenarios where continued operation beyond the technical lifetime would be beneficial because the market conditions have not reached optimal levels for option exercise. However, operation past this point is a matter of structural safety and is not legal unless thorough inspections show that the probability of structural failure is below acceptable levels.

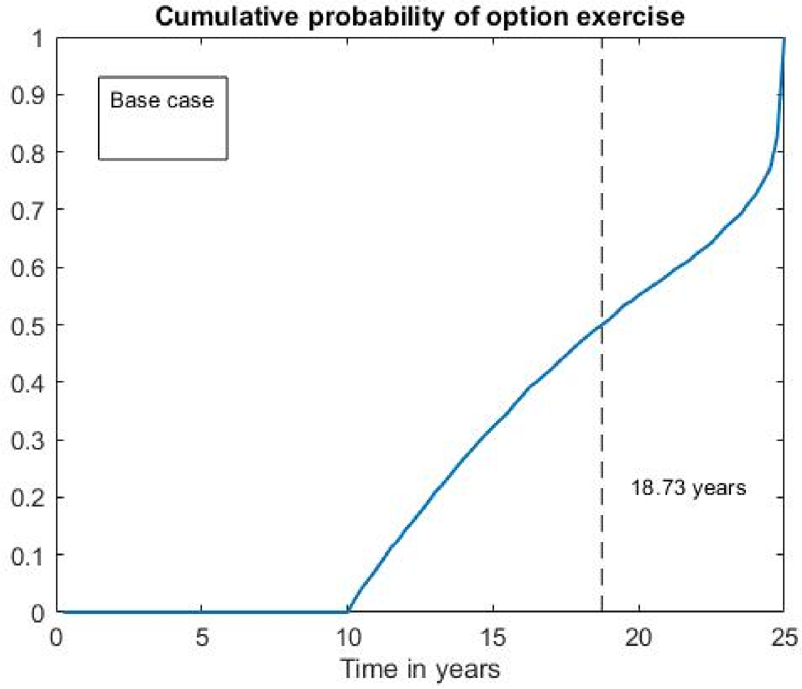



*Figure 5.1:Cumulative probability of exercising the option to repower*

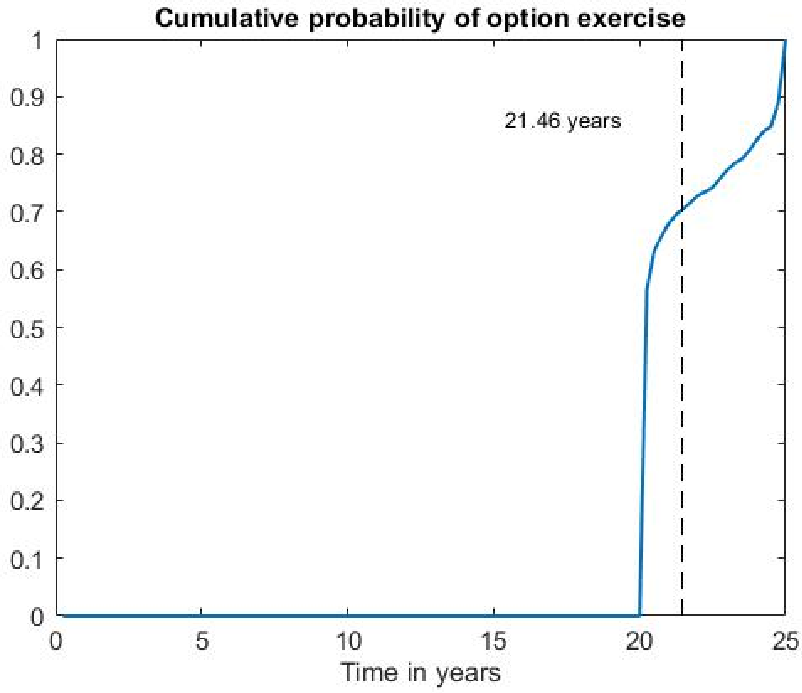



*Figure 5.2: Cumulative probability of exercising the life-extension option*

From Figure 5.2, the effects of the technical restriction imposed on the life-extension value is quite apparent. After 20 years, when the option to life-extend becomes available, a sharp increase in the probability of exercise can be observed. Thus, it is safe to conclude that the technical restriction has significant impact on the investment decision of the operator. While this technical restriction is a realistic aspect of the investment decision and should not be neglected, the choice of exactly 20 years is, as was mentioned in Chapter 4.8, somewhat subjective. For comparison we therefore conduct a separate simulation with the restriction set to 15 years, to see how the option value and expected exercises time is affected. The result of this is displayed in Figure 5.3. From the figure it can be observed that if life-extension is available earlier, the option value would increase significantly, and the expected exercise time would be lower. It can also be noted that the expected exercise time of the life-extension option is now lower than the expected exercise time of repowering. This implies that the relative favourableness of the two options has changed, and that the operator in this scenario would prefer life-extension over repowering for favourable market conditions early on. Considering this, it should be emphasised once again that both restrictions and parameter values are case-specific, hence results and analysis should be considered highly case-specific as well.

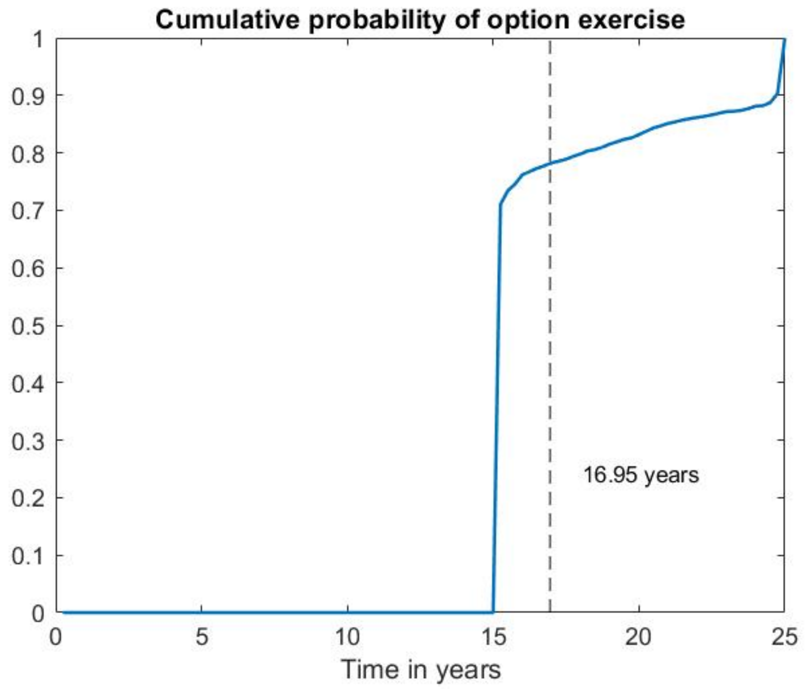



*Figure 5.3: Cumulative probability of exercising life-extension option with relaxed restriction*

From Figure 5.4, it can be observed that having both options available provides the same shape, but shifted downwards, as the repowering option during the first 20 years of operation. However, after 20 years a jump occurs as the life-extension option becomes available. As discussed above, the operator is likely to wait for the option to life-extend to be available when the market conditions are less favourable. Waiting longer due to unfavourable market conditions, makes it more likely that life-extension will be the preferable option.

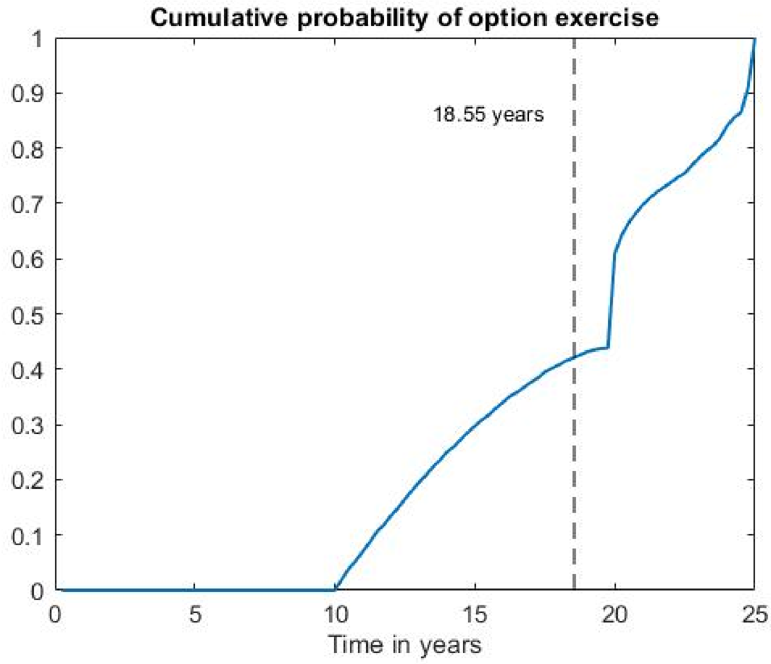



*Figure 5.4: Cumulative probability of exercising when having both options*

To further emphasize how the option to life-extend provides value when both options are available, the cumulative probability of all three options are visualised in the same graph in Figure 5.5. From the figure it can be observed that the probability of exercising the repowering option before 20 years is about 54%, however, when both options are available the probability of having exercised an option before 20 years is only 44%. This 10% reduction implies that there are less favourable market scenarios for which the operator is better off waiting for the life-extension option to become available.

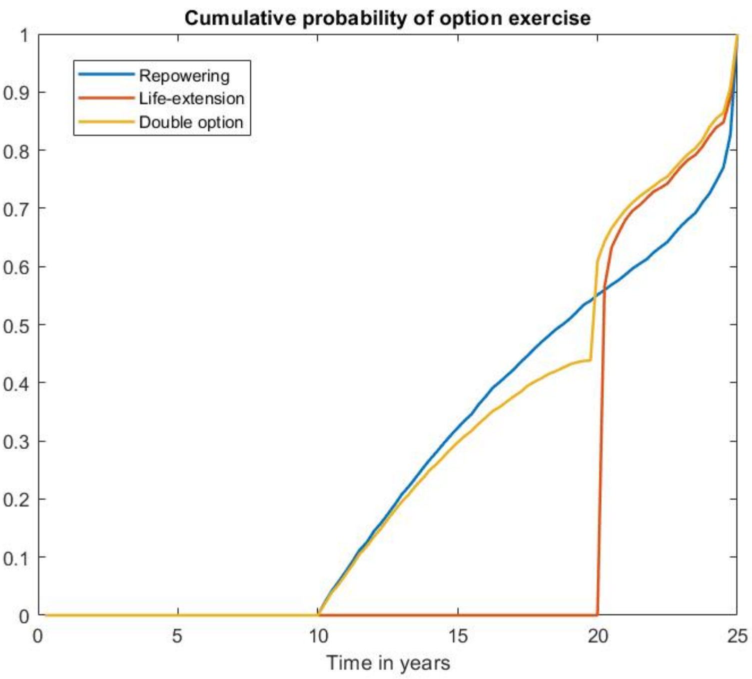



*Figure 5.5: Comparison of cumulative probability of exercising for all options*

From the above discussion, the option behaviour for our base case can be summed up in the following manner:

> *Favourable market conditions:* For favourable early market conditions, the option to repower is preferable and will be exercised.
>
> *Moderate market conditions:* For moderate early market conditions, the operator prefers waiting to find out which way the market develops. If the market conditions improve sufficiently before the 20-year mark, he will exercise the repowering option. If market conditions decline or fail to improve sufficiently before the 20-year mark, he will exercise the life-extension option.
>
> *Unfavourable market conditions:* For unfavourable early market conditions, that remain unfavourable, the operator will wait for the life-extension option to become available and exercise it optimally during the last 5 years of the technical lifetime.

These results are in line with the findings of Décamps, Mariotti and Villeneuve (2006) which shows that having two mutually exclusive investment opportunities increases the demand for information and provides an additional incentive for delayed investment. This is what we observe for moderate market conditions, where the operator waits to see which way the market develops. It should again be emphasized that these results are contingent on the assumptions made for the specific case and should therefore not be considered general findings.

## 5.2 – Sensitivity analysis

We perform sensitivity analysis in order to observe how changes in the parameter values affect the value and the expected exercise time of the options. The sensitivity analysis is conducted by changing one parameter at a time, while the other parameters are fixed at their base case value. Specifically, we consider how the option value and exercise time changes when a parameter is allowed to diverge $\pm 10\,\%$ and $\pm 20\,\%$ from its base case value. Key findings for each parameter are presented and addressed in an orderly manner, with a particular focus on identifying trends that affect the value of repowering relative to life-extension. Complete sensitivity tables can be found in Appendix A1. A final remark before we delve into the analysis is that because the LSM algorithm is an estimation method, the sensitivity results might be subject to some numerical error. In particular changes of small magnitude might provide inconsistent trends as a result, hence one should be careful not to draw strong conclusions based on weak trends.

### 5.2.1 – Logarithm of the initial electricity price level, $X_0$

The logarithm of the initial price level reflects the current market price of electricity. This is an important parameter to assess, because evaluation in any real options model is contingent upon the current market conditions. In Table 5.2, the sensitivity analysis of $X_0$ is presented. For all three options, it can be observed that the option values increase with $X_0$. As higher initial value of the electricity price means that the initial market conditions are more favourable, this makes intuitively sense. This result is also in line with what we expect from the general insights from the single-factor model.

| | | $X_0$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\,\%$ | $-10\,\%$ | *Base case* | $+10\,\%$ | $+20\,\%$ |
| ***Repowering*** | ***Relative change in option value*** | 32.650 (−4.69 %) | 33.459 (−2.33 %) | 34.256 (−) | 35.102 (+2.47 %) | 36.049 (+5.23 %) |
| | ***Expected exercise time [years]*** | 19.12 | 18.91 | 18.73 | 18.55 | 18.36 |
| ***Life − extension*** | ***Relative change in option value*** | 28.453 (−0.74 %) | 28.557 (−0.38 %) | 28.666 (−) | 28.769 (+0.36 %) | 28.868 (+0.70 %) |
| | ***Expected exercise time [years]*** | 21.47 | 21.46 | 21.46 | 21.45 | 21.45 |
| ***Double option*** | ***Relative change in option value*** | 33.664 (−4.24 %) | 34.366 (−2.25 %) | 35.156 (−) | 35.900 (+2.11 %) | 36.830 (+4.76 %) |
| | ***Expected exercise time [years]*** | 18.93 | 18.74 | 18.73 | 18.37 | 18.19 |

*Table 5.2: Option value and expected exercise for changes in logarithm of initial price level*

It can be observed for the repowering option and the double option that investment is expected sooner for increasing $X_0$. This is also in line with expectations, because if the initial market conditions are more favourable, the investment is likely to be justified sooner. The expected exercise time for the life-extension option, however, is unaffected by the changes in $X_0$. This is a natural consequence of the technical restriction imposed on the possible exercise time of the option. Since the option cannot be exercised the first 20 years of operation, the favourable initial market conditions do not have significant impact.

A final observation on $X_0$ is that the repowering option is more sensitive to changes than the life-extension option is, both in terms of option value and expected exercise time. This can be connected to the findings from the base case analysis, where favourable market conditions were found to motivate early exercise of the repowering option. Higher $X_0$ equates to more favourable initial market conditions. Hence, for increasing $X_0$ one will observe more repowering and less life-extension if the operator has both options available. For decreasing $X_0$, this is reversed, and one will observe less repowering and more life-extension.

### 5.2.2 – Cost of capital, $r$

From Table 5.3 it can be observed that the option value and expected time of exercise decreases with $r$ for all three options. Intuitively, it makes sense that the option values decrease with $r$, because the future expected cash flows are discounted at a higher rate and therefore becomes less valuable. This result is also consistent with the general insights from the single-factor model, where increasing $r$ was found to reduce option value. The trend of decreasing exercise time for increasing $r$ is, however, less intuitively obvious, and is related to the fact that all options must be exercised before or at the end of the technical lifetime. If the options were available in perpetuity, as was assumed for the single-factor model, the investment thresholds would increase with $r$ and thus postpone the expected exercise time. However, as the options must be exercised within a given timeframe, the operator is better off exercising the options earlier for increasing $r$ because the future profit flows will be discounted at a higher rate and, thus, be less valuable if he waits longer.

| | | $r$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\ \%$ | $-10\ \%$ | *Base case* | $+10\ \%$ | $+20\ \%$ |
| ***Repowering*** | ***Relative change in option value*** | 44.782 (+30.73 %) | 39.187 (+14.39 %) | 34.256 (−) | 29.974 (−12.50 %) | 26.157 (−23.64 %) |
| | ***Expected exercise time [years]*** | 19.39 | 18.98 | 18.73 | 18.53 | 18.44 |
| ***Life − extension*** | ***Relative change in option value*** | 40.650 (+41.81 %) | 34.230 (+19.41 %) | 28.666 (−) | 24.045 (−16.12 %) | 20.230 (−29.43 %) |
| | ***Expected exercise time [years]*** | 21.65 | 21.49 | 21.46 | 21.43 | 21.41 |
| ***Double option*** | ***Relative change in option value*** | 46.065 (+31.03 %) | 39.944 (+13.62 %) | 35.156 (−) | 30.683 (−12.72 %) | 26.469 (−24.63 %) |
| | ***Expected exercise time [years]*** | 19.42 | 19.06 | 18.55 | 18.31 | 18.30 |

*Table 5.3: Option value and expected exercise time for changes in the cost of capital*

Further considering the results of Table 5.3, some more interesting observations can be made. The life-extension option value is affected more by changes in $r$ than the repowering option value is, hence the cost of capital will change the value of repowering relative to life-extension. Also, looking at the relative changes in the double option value, it can be observed that it tends to lie between the relative changes of the repowering and life-extension option values. Considering this in combination with the fact that the expected exercise time of repowering falls with $r$ while while the expected exercise time of life-extension is unaffected, it can be reasoned that increasing $r$ will incur a shift in favour of repowering when both options are available. For decreasing $r$ the opposite occurs, and the expected exercise time of the double option can be observed to increase faster than for the repowering option suggesting a significant shift towards more life-extension exercise in the simulation paths.

### 5.2.3 – Capacity efficiency decline, $\gamma$

From Table 5.4 it can be observed that the option value is decreasing with $\gamma$ for all three options. This is a reasonable result because higher $\gamma$ means that the turbine is degrading faster and, therefore, becomes less valuable; which has a higher absolute impact on the repowered and life-extended turbine because of the higher initial capacity efficiency. This is also supported by the general insights from the single-factor model.

For the expected exercise time of the options, there are no discernible trends. For the life-extension option the expected exercise time remains the same for all changes in $\gamma$, while for the repowering option the expected exercise time increases slightly for both decreasing and increasing $\gamma$. Intuitively, it makes sense that only small changes in the expected exercise time can observed for changes in $\gamma$, because the same capacity efficiency decline is assumed for all states of operation. If the repowered or life-extended turbines were assumed to have a lower $\gamma$ than currently operation turbine, one would expect investment to come sooner because the incentive of switching to better performing turbine would be stronger.

| | | $\gamma$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\,\%$ | $-10\,\%$ | *Base case* | $+10\,\%$ | $+20\,\%$ |
| ***Repowering*** | ***Relative change in option value*** | 35.069 (+2.37 %) | 34.917 (+1.93 %) | 34.256 (−) | 33.150 (−3.23 %) | 32.512 (−5.09%) |
| | ***Expected exercise time [years]*** | 19.01 | 18.89 | 18.73 | 18.97 | 18.96 |
| ***Life − extension*** | ***Relative change in option value*** | 30.150 (+5.18 %) | 29.203 (+1.87 %) | 28.666 (−) | 27.954 (−2.48 %) | 27.262 (−4.90 %) |
| | ***Expected exercise time [years]*** | 21.46 | 21.51 | 21.46 | 21.46 | 21.46 |
| ***Double option*** | ***Relative change in option value*** | 36.121 (+2.74 %) | 35.820 (+1.89 %) | 35.156 (−) | 34.508 (−1.84 %) | 33.761 (−3.97 %) |
| | ***Expected exercise time [years]*** | 18.86 | 18.60 | 18.55 | 18.49 | 18.52 |

*Table 5.4: Option value and expected exercise time for changes in the capacity efficiency decline*

### 5.2.4 – Mean-reversion parameter, $\alpha$

From Table 5.5 it can be observed that all three option values are strongly increasing in $\alpha$. We can also observe a higher relative change from the base case for increasing $\alpha$ than for decreasing $\alpha$. These observations can be explained by the way the process of the electricity price is defined in the model. The price of electricity is modelled through its logarithm; hence the mean-reversion parameter is calibrated accordingly. In the option stopping values $\alpha$ enters through an exponential term, which is the option values are so sensitive to changes in $\alpha$. The observed skewness of the sensitivity is likely also a result of this exponential relationship, as $e^{1.1\alpha}$ provides a large change from $e^{\alpha}$ in absolute value than $e^{0.9\alpha}$ does.

| | | $\alpha$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\ \%$ | $-10\ \%$ | $Base\ case$ | $+10\ \%$ | $+20\ \%$ |
| $Repowering$ | $Relative\ change\ in\ option\ value$ | 2.4729 (−92.78 %) | 13.298 (−61.18 %) | 34.256 (−) | 71.460 (+108.61 %) | 134.50 (+292.63 %) |
| | $Expected\ exercise\ time\ [years]$ | 22.97 | 20.32 | 18.73 | 17.96 | 17.92 |
| $Life-extension$ | $Relative\ change\ in\ option\ value$ | 3.8877 (−86.44 %) | 12.631 (−55.94 %) | 28.666 (−) | 56.879 (+98.42 %) | 106.95 (+273.09 %) |
| | $Expected\ exercise\ time\ [years]$ | 21.96 | 21.66 | 21.46 | 21.42 | 21.41 |
| $Double$ option | $Relative\ change\ in\ option\ value$ | 4.0805 (−88.39 %) | 14.491 (−58.78 %) | 35.156 (−) | 72.269 (+105.57 %) | 135.67 (+285.91 %) |
| | $Expected\ exercise\ time\ [years]$ | 21.59 | 20.00 | 18.55 | 17.80 | 17.77 |

*Table 5.5: Option value and expected exercise time for changes in the mean-reversion parameter*

The expected exercise time can be observed to be decreasing with $\alpha$ for all three options. This is reasonable as $\alpha$ is by far the most significant parameter in determining the mean-reversion level of the electricity price. If this level increases, future prospects will be more favourable, thereby motivating earlier exercise to take advantage of this. Note, however, that the decrease in exercise time is almost the same when $\alpha$ is increased by 10 % and 20 %. This can largely be explained by the restrictions on the possible repowering and life-extension times that were discussed in Chapter 4.8, which in effect impose limitations on the sensitivity of the expected exercise time.

By comparing all the options with each other, another interesting observation can be made. The repowering option is most sensitive to changes in $\alpha$, and the double option has relative changes that lie between the repowering and life-extension option. Hence, increasing $\alpha$ makes repowering more attractive relative to life-extension, while decreasing $\alpha$ has the opposite effect. This can be connected to the findings from the base case, where favourable market conditions resulted in earlier exercise of repowering. With high $\alpha$, favourable market conditions become more likely. Hence, if a wind farm operator expects an increase in the mean-reversion level of the electricity price, he would be more inclined to exercise the repowering option relative to the life-extension option. If the operator expects a decrease in the mean-reversion level of the electricity price, the reverse will be the case.

### 5.2.5 – Volatility of the electricity price, $\sigma$

From Table 5.6 it can be observed that the expected investment time increases with $\sigma$. This finding is line with both the insights from the single-factor model and general real option studies such as Dixit and Pindyck (1994). When the future becomes more uncertain, waiting for more information becomes more valuable. Thus, an operator exposed to more uncertain electricity prices is more likely to postpone exercising his options.

| | | $\sigma$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\,\%$ | $-10\,\%$ | ***Base case*** | $+10\,\%$ | $+20\,\%$ |
| ***Repowering*** | ***Relative change in option value*** | 34.292 (+0.11 %) | 34.296 (+0.12 %) | 34.256 (−) | 34.259 (+0.01 %) | 34.161 (−0.28 %) |
| | ***Expected exercise time*** [*years*] | 18.48 | 18.58 | 18.73 | 18.87 | 19.12 |
| ***Life − extension*** | ***Relative change in option value*** | 28.940 (+0.96 %) | 28.560 (−0.36 %) | 28.666 (−) | 28.632 (−0.12 %) | 28.451 (−0.88 %) |
| | ***Expected exercise time*** [*years*] | 21.24 | 21.43 | 21.46 | 21.54 | 21.65 |
| ***Double option*** | ***Relative change in option value*** | 35.214 (+0.16 %) | 34.710 (−1.27 %) | 35.156 (−) | 34.842 (−0.89 %) | 34.916 (−0.68 %) |
| | ***Expected exercise time*** [*years*] | 18.28 | 18.65 | 18.55 | 18.84 | 18.99 |

*Table 5.6: Option value and expected exercise time for changes in the electricity price volatility*

Following the general insights from the single-factor model and the discussion of Dixit and Pindyck (1994) we would also expect the option value to increase with $\sigma$. However, as can be observed from Table 5.6, this is not the case for our numerical case. Instead, we find that changes in $\sigma$ has little to no effect on the option values. From the cumulative probability plots presented in Chapter 5.1, it can be observed that the cumulative probability of option exercise within the technical lifetime is 100% for all options in our numerical case. The options are thereby so profitable in our numerical case that deciding not to exercise at any time is not optimal. This removes a significant part of the flexibility value of the options, as having the option not to exercise at the end of the technical lifetime is essentially worthless. The restriction from the technical lifetime also removes a part of the flexibility value of the options, as the currently operating turbine becomes worthless at the end of the technical lifetime and thereby forces exercise. In combination, this is likely why our results differ from those of Dixit and Pindyck (1994), as they consider a perpetual option that can be exercised at any time. It should also be noted that unlike Dixit and Pindyck (1994) who assumes their underlying asset values follows a GBM where all the volatility is encapsulated in $\sigma$, the $\sigma$ of our model is only the volatility of one of the three underlying risk factors and the mean-reversion speed of the electricity price $\kappa_P$ limits the impact of the price volatility as it draws prices towards the mean-reversion level.

### 5.2.6 – Correlation between electricity log price and weather factor, $\rho$

No discernible trends can be observed for any of the three options in response to changes in $\rho$. For both the option values and expected time of investment, there are only very small changes. For this reason, the sensitivity table for $\rho$ has been excluded from this chapter, and instead we refer to Appendix A1 where the complete sensitivity tables for all parameters can be found.

The fact that no clear trends can be observed for changes in $\rho$ does however have some implications for our investment problem. Since the option value and expected investment time is almost unaffected by the correlation between the electricity price and the weather factor, it might seem reasonable to exclude it from the model. Its inclusion was inspired by Ernstsen and Boomsma (2018), who found that negative correlation between power production and electricity prices resulted in lower value of generation and reduced investment incentives. However, the key difference that should be noted is that Ernstsen and Boomsma utilized hourly data. In contrast, we utilize annualized data with quarterly time steps in our numerical case. Hence, if we had used a shorter time-step for the simulations, the correlation would likely have a larger impact.

### 5.2.7 – The repowering capacity factor, $K_r$

From Table 5.7 it can be observed that all three option values are increasing with $K_r$. This is in line with the general insights from the single-factor model, and makes intuitively sense because the repowering capacity factor reflects the improvement available by repowering.

| | | $K_r$ | | | | |
|---|---|---|---|---|---|---|
| | | − 20 % | −10 % | Base case | + 10 % | + 20 % |
| **Repowering** | **Relative change in option value** | 30.132 (−12.04 %) | 30.521 (−10.90 %) | 34.256 (−) | 38.115 (+11.27 %) | 41.953 (+22.47 %) |
| | **Expected exercise time [years]** | 18.12 | 19.13 | 18.73 | 18.37 | 18.08 |
| **Life − extension** | **Relative change in option value** | 25.848 (−9.83 %) | 26.854 (−6.32 %) | 28.666 (−) | 30.233 (+5.47 %) | 31.612 (+10.28 %) |
| | **Expected exercise time [years]** | 21.23 | 21.55 | 21.46 | 21.49 | 21.56 |
| **Double option** | **Relative change in option value** | 30.405 (−13.51 %) | 31.363 (−10.79 %) | 35.156 (−) | 38.359 (+9.11 %) | 42.005 (19.48 %) |
| | **Expected exercise time [years]** | 18.89 | 19.31 | 18.55 | 18.30 | 18.24 |

*Table 5.7: Option value and expected exercise for changes in the repowering capacity factor*

It can further be observed that the repowering option value is more sensitive to changes in $K_r$ than the life-extension option value is. Hence, increasing $K_r$ will make repowering more attractive relative to life-extension, making early exercise of the repowering option more likely relative to early exercise of the life-extension option when the double option is available. For decreasing $K_r$, this behaviour is reversed.

The expected exercise time of the options display non-monotonic behaviour in $K_r$. For increasing $K_r$, the expected exercise time for the repowering and double option decreases, while it for the life-extension option increases. When $K_r$ is reduced 10 % from its base case value, the expected investment time increases for all options; however when $K_r$ is reduced 20 % from its base case value, the expected investment time increases for all option relative to their 10 % value. In addition, when $K_r$ is reduced 20 % from its base case value, the expected exercise time of the repowering and the life-extension options decreases relative to the base case, while the expected exercise time for the double option increases. While interesting, this make the interpretation of the expected investment time challenging, and developing a more comprehensive picture of the situation would likely require extensive simulations for a broader span of $K_r$ values. It would, however, be interesting to do so in order to determine if there are contradicting effects that govern how the expected exercise time responds to changes in $K_r$, and what the physical interpretation of these effects might be.

## 5.2.8 – The life-extension efficiency decline reversal factor, $K_l$

From table 5.8, no discernible trend can be observed for any of the option values in response to changes in $K_l$. There is also no strong trend for the expected exercise times. For the life-extension option, there seems to be a weak trend toward increasing expected exercise time for increasing $K_l$, but the relative changes are so small that it must be considered insignificant when accounting for the possibility of numerical error. This is a surprising result at first glance, as $K_l$ represents the amount of efficiency loss that can be reversed by performing life-extension. Hence, we would expect the life-extension option value to be significantly increasing with $K_l$. However, small changes in $K_l$ affect the investment decision minimally for our numerical case. One possible explanation for this result might be that a large part of the value from the life-extension option comes from the repowering that follows after the extension. If so, the primary value added by life-extension is the flexibility to postpone repowering further when market conditions are unfavourable.

| | | $K_l$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\,\%$ | $-10\,\%$ | *Base case* | $+10\,\%$ | $+20\,\%$ |
| *Repowering* | *Relative change in option value* | − | − | − | − | − |
| | *Expected exercise time [years]* | − | − | − | − | − |
| *Life − extension* | *Relative change in option value* | 28.229 (−1.52 %) | 28.537 (−0.45 %) | 28.666 (−) | 28.607 (−0.21 %) | 28.651 (−0.05 %) |
| | *Expected exercise time [years]* | 21.52 | 21.47 | 21.46 | 21.50 | 21.58 |
| *Double option* | *Relative change in option value* | 35.133 (−0.07 %) | 35.158 (+0.01 %) | 35.156 (−) | 35.196 (+0.11 %) | 35.200 (+0.13 %) |
| | *Expected exercise time [years]* | 18.49 | 18.51 | 18.55 | 18.58 | 18.60 |

*Table 5.8: Option value and expected exercise for changes in the life-extension efficiency decline reversal factor*

## 5.2.9 – Mean-reversion speed of the electricity price, $\kappa_P$

From Table 5.9 it can be observed a weak trend of the expected exercise time decreasing with $\kappa_P$. This makes sense intuitively, because increasing mean-reversion speed for the electricity price implies less variation in the price, thereby making the influence of the price volatility less prominent. As previously discussed, less uncertainty about the future reduces the incentive to wait and motivates earlier exercise.

For the option values, no discernible trend is observable. This is quite expected, both because the mean-reversion speed for our numerical case is quite low in absolute terms and because the Ornstein-Uhlenbeck process of the electricity log price is fairly insensitive to small absolute changes in the mean-reversion speed. Hence, for changes to the mean reversion speed to have any significant impact on the option values, we would expect to need significantly larger changes in absolute value than the small changes incurred by the relative changes utilized in the sensitivity analysis. Even so, we can argue qualitatively that we would expect increasing $\kappa_P$ to decrease the option value for our numerical case.

| | | $\kappa_P$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\ \%$ | $-10\ \%$ | *Base case* | $+10\ \%$ | $+20\ \%$ |
| **Repowering** | *Relative change in option value* | 34.281 (+0.07 %) | 34.284 (+0.08 %) | 34.256 (−) | 34.279 (+0.07 %) | 33.791 (−1.36 %) |
| | *Expected exercise time [years]* | 19.04 | 19.00 | 18.73 | 18.61 | 18.74 |
| **Life − extension** | *Relative change in option value* | 28.312 (−1.24 %) | 28.405 (−0.91 %) | 28.666 (−) | 28.520 (−0.51 %) | 28.832 (+0.58 %) |
| | *Expected exercise time [years]* | 21.66 | 21.58 | 21.46 | 21.46 | 21.33 |
| **Double option** | *Relative change in option value* | 35.266 (+0.31 %) | 35.191 (+0.10 %) | 35.156 (−) | 35.147 (−0.03 %) | 35.152 (−0.01 %) |
| | *Expected exercise time [years]* | 18.78 | 18.66 | 18.55 | 18.45 | 18.35 |

*Table 5.9: Option value and expected exercise for changes in the electricity mean-reversion speed*

This is because the mean-reversion level of the price is significantly below the current price level, thus increasing the reversion speed would cause the expected price to drop quicker and early market conditions to become less favourable, thereby reducing the expected value of option exercise.

### 5.2.10 – Mean-reversion speed of the weather factor, $\kappa_U$

| | | $\kappa_U$ | | | | |
|---|---|---|---|---|---|---|
| | | $-20\ \%$ | $-10\ \%$ | *Base case* | $+10\ \%$ | $+20\ \%$ |
| **Repowering** | *Relative change in option value* | 34.574 (+0.93 %) | 34.438 (+0.53 %) | 34.256 (−) | 34.124 (−0.39 %) | 34.009 (−0.72 %) |
| | *Expected exercise time [years]* | 18.67 | 18.69 | 18.73 | 18.77 | 18.79 |
| **Life − extension** | *Relative change in option value* | 28.834 (+0.59 %) | 28.646 (−0.07 %) | 28.666 (−) | 28.309 (−1.25 %) | 28.335 (−1.15 %) |
| | *Expected exercise time [years]* | 21.44 | 21.48 | 21.46 | 21.54 | 21.52 |
| **Double option** | *Relative change in option value* | 35.476 (+0.91 %) | 34.960 (−0.56 %) | 35.156 (−) | 34.589 (−1.61 %) | 34.420 (−2.09 %) |
| | *Expected exercise time [years]* | 18.47 | 18.70 | 18.55 | 18.80 | 18.83 |

*Table 5.10: Option value and expected exercise for changes in the weather factor mean-reversion speed*

From Table 5.10 it can be observed a weak trend towards decreasing repowering option value and later expected exercise time with increasing $\kappa_U$. For the two other options no such trend is discernible. We therefore reason that $\kappa_U$ has very little effect on the option value and investment timing for our numerical case, which can be explained by the low base case reversion speed. However, it should again be emphasized that both the parameter estimation and the numerical procedure provide sources of numerical error. We are therefore careful not to draw any strong conclusion when only weak trends are observable. Intuitively, we would expect increasing $\kappa_U$ to decrease the option values. This is based on the same type of argument as for the mean-reversion level of the electricity price; the mean-reversion level is not a favourable level. The mean-reversion level of the weather factor is 0, which translates into a wind speed of about 8.3 m/s for our Weibull parameters, which is below the rated speed of the turbine we consider. Hence, a higher mean-reversion speed makes it more unlikely for the turbine to operate at rated capacity, which makes it less profitable.

## 5.3 – PPA analysis

One of the objectives of this thesis is to figure out how PPAs affect the end-of-life investment decision for a specific wind farm. To analyse this, we have run simulations where we have allowed $\Upsilon$, the the fraction of energy that is sold through PPAs, to vary.

Recall that $\Upsilon = 0$ equates to all energy being sold through PPAs, while $\Upsilon = 1$ equates to all energy being sold through the spot market (which is the baseline case). Thus, decreasing $\Upsilon$ means increasing fraction of PPA. The simulations results are presented in Table 5.11 below.

| | | $\Upsilon$ | | | | |
|---|---|---|---|---|---|---|
| | | $\mathbf{1.0}$ $(\boldsymbol{Base\ case})$ | $\mathbf{0.75}$ | $\mathbf{0.5}$ | $\mathbf{0.25}$ | $\mathbf{0.0}$ |
| $\boldsymbol{Repowering}$ | $\boldsymbol{Option\ value}$ $[\boldsymbol{MNOK}]$ | 34.256 | 33.215 (−3.04 %) | 32.379 (−5.48 %) | 31.616 (−7.71 %) | 30.917 (−9.75 %) |
| | $\boldsymbol{Expected\ exercise\ time}$ $[\boldsymbol{years}]$ | 18.73 | 18.54 | 18.22 | 17.99 | 17.89 |
| $\boldsymbol{Life-extension}$ | $\boldsymbol{Option\ value}$ $[\boldsymbol{MNOK}]$ | 28.666 | 28.188 (−1.67 %) | 27.784 (−3.08 %) | 27.030 (−5.71 %) | 26.551 (−7.38 %) |
| | $\boldsymbol{Expected\ exercise\ time}$ $[\boldsymbol{years}]$ | 21.46 | 21.20 | 21.10 | 21.07 | 21.06 |
| $\boldsymbol{Double\ option}$ | $\boldsymbol{Option\ value}$ $[\boldsymbol{MNOK}]$ | 35.156 | 33.864 (−3.67 %) | 33.248 (−5.43 %) | 32.295 (−8.14 %) | 31.454 (−10.53 %) |
| | $\boldsymbol{Expected\ exercise\ time}$ $[\boldsymbol{years}]$ | 18.55 | 18.53 | 18.09 | 18.08 | 18.07 |

*Table 5.11: Option values and expected exercise time for different PPA-fractions*

From the table it can be observed that the option values are decreasing with $\Upsilon$, which is most likely caused by the risk premium the operator pays for the contract. By increasing the fraction of energy sold through PPAs, the total amount of risk premiums paid increases and, thereby, the expected option values are reduced. Recall from Chapter 2.3.2 that PPAs reduce the price risk and thereby reduce the cost of capital for the wind farm. Based on the sensitivity analysis on $r$ presented in Chapter 5.2.2, we can conclude that this will have a positive impact on the option values. This effect is, however, not reflected in the table above, as $r$ has been kept constant for all PPA fractions. An appropriate way to include the dependence of $r$ on $\Upsilon$ in the model, would be to express $r$ as a linear function on the form

$$r = r_f + r_p + \Upsilon r_{el},$$

where $r_{el}$ is the risk-premium related to the electricity price risk and $r_p$ is the risk-premium related to the remaining risk of the project. We have, however, chosen not to include this in our PPA analysis for two reasons. The first is that since we observe that $\Upsilon$ and $r$ have competing effects on the options, it would make it difficult to disentangle the effect of each parameter on the result when considered together. To perform meaningful sensitivity analysis, we therefore need to consider $\Upsilon$ and $r$ in isolation. The second reason is that even if we did consider both simultaneously, the results would heavily depend on being able to quantify how much of the risk in the specific case we consider originates from the price exposure and how much originates from other sources. To do so would be both difficult and likely even more unreliable than regular parameter estimation.

It can be observed that the expected time of investment is reduced when $\Upsilon$ increases, which is in line with intuition as increasing $\Upsilon$ reduces the uncertainty of future profit flows and thereby reduces the incentive to wait. This is also consistent with the results found for $\sigma$ in Chapter 5.2.5, which seems reasonable as both reduced $\sigma$ and increased $\Upsilon$ diminishes the electricity price risk. As a result, the wind farm operator is incentivised to exercise his end-of-life options sooner if part of future energy production can be sold through PPA contracts.

Finally, it can be observed that the repowering option is more sensitive to changes in $\Upsilon$ than life-extension is. Thus, life-extension becomes more valuable relative to repowering as $\Upsilon$ decreases for our case. A further observation is that decreasing $\Upsilon$ will make the operator more likely to perform life-extension when both options are available. This is can be reasoned from the fact that the double option experiences the highest negative changes for decreasing $\Upsilon$. Since life-extension has the lowest option value, this implies that the PPA makes life-extension more likely relative to repowering. This effect would have been further strengthened if $r$ had been adjusted for the reduction in price risk, as decreasing cost of capital was found to cause shift towards life-extension in Chapter 5.2.2. A final remark is that since the price risk exposure decreases with $\Upsilon$, the exposure to favourable market conditions is reduced. Hence, the result of exercising the repowering option for favourable early market conditions in the base case becomes less prevailing when $\Upsilon$ is increased.

# 6 - Conclusions

This thesis develops two real options models, a tractable single-factor model and a flexible multi-factor model, to evaluate investment in end-of-life solutions for aging wind farms. Our primary focus is on assessing optimal financial strategies and providing tools to aid wind farm operators in optimally allocating their assets. The need for proper assessment of end-of-life solutions will become increasingly important as the market for wind power continues to grow, and how well these solutions are handled will play an important role in the speed of this growth. By evaluating both repowering and life-extension, and by including uncertainty in electricity prices, production of energy, and technological innovation, this thesis expands on the currently existing literature. Our single-factor model provides general insights into the behaviour of the investment problem, while our multi-factor model allows for evaluating specific wind farms with non-financial restrictions and PPAs.

The results from the single-factor model show that the repowering and life-extension option values are decreasing at different rates with the cost of capital, capacity efficiency decline and investment cost. The option values are also increasing at different rates with the volatility of the profit flow, initial price level and the repowering scaling factor. Consequently, the specific underlying parameters will determine how attractive the two options are relative to one another, and parameter changes will shift this balance.

To a large extent, the case-specific results from the multi-factor model coincide with what we expect from the general results of the single-factor model. For the case study considered, we find the value of having the option to repower or life-extend the wind farm to be 34.25 MNOK and 28.67 MNOK, respectively. For a wind farm operator with a brand new wind farm, the option to repower is expected to be exercised after 18.73 years, while the option to life-extend is expected to be exercised after 21.46 years. If the operator has both options available, the value of this double option is found to be 35.16 MNOK and the expected exercise time to be 18.15 years. We also find that all options will be exercised before or at the end of the technical lifetime; which implies that the repowering and life-extension options are very attractive, and as assumed, decommissioning is not economically viable for the case.

From conducting sensitivity analysis on the case study, we find that the optimal decision for the wind farm operator depends on how the market conditions develop throughout the lifetime of the wind farm. For favourable early market conditions, such as high electricity prices and production, the option to repower is preferable and will be exercised. For moderate early market conditions, the operator prefers to wait and see which way the market develops; while for unfavourable market conditions, such as low electricity prices and production, the operator will wait for the life-extension option to become available and exercise it optimally within the remaining lifetime of the wind farm. Finally, we find that the option values are decreasing with the fraction of energy sold through PPA contracts. Simultaneously, the operator is expected to exercise his options earlier, as the PPA reduces the exposure to electricity price risk. In addition, less exposure to market conditions makes the result of preferring the repowering option for favourable early market conditions less prevailing. All these findings are, however, contingent on the case-specific assumptions and conditions, and cannot readily be generalized.

# 7 - Discussion

Before delving into the broader context of our results and interesting directions for further research of our investment topic, we feel it appropriate to make some remarks on our methodology and how it could be improved upon for future applications of the model. For several of the parameters considered in the sensitivity analysis of our numerical case, there were weak or undiscernible trends. This might have been a result of only including four divergence points from the base case. Conducting a more comprehensive sensitivity analysis with a larger range of variation from the base case, before regressing the results to evaluate the statistical significance of trends, would likely provide a more accurate picture of the option behaviour. The computational time required to do so would however have made this infeasible considering the limited time available for our thesis, but for future applications of our model it should be remarked that this can be beneficial to provide more comprehensive results. Even though we deliberately omitted political subsidy schemes, as was discussed in Chapter 2.3.3, we can make a qualitative argument for how its inclusion could have affected the results. With the tendency of existing subsidies being retracted, one would expect repowering to become significantly more valuable relative to life-extension close to the retraction deadline. The reason why being that repowering would make the wind farm eligible for the subsidy scheme, if performed before the deadline, while life-extension would not. For the Norwegian TCG scheme, this deadline is set at the end of 2021, hence if the Norwegian wind farm we considered in our numerical case was already relatively close to its expected lifetime this could have had interesting implications.

Considering a broader context than the wind farm operator, our modelling results can also have interesting implications for other stakeholders in wind power. Such stakeholders include for instance policy makers, capital investors and competing power producers. A policy maker might for instance, motivated by improving the stability of the national power grid by having a larger amount of more reliable and technologically advanced wind turbines in the power mix, utilize the modelling results to devise new policies that make repowering more attractive relative to life-extension for wind farm operators. Capital investors might naturally use the models to estimate how the option values affect the expected value of wind projects they consider investing in, and competing power producers might use the modelling results to better predict how wind farms will affect their future competitive strength.

Next, considering interesting extensions to our model and scientific study, the first that comes to mind is applying the multi-factor model to an offshore wind farm case study. This case study might provide useful insights into the end-of-life investment decisions for offshore wind farms, and could also make an interesting comparison for an onshore case. A more involved extension would be to model the technological innovation in a more elaborate way, as capturing the real stochastic characteristics of innovation through a model is a very extensive and complicated topic. Considering that the end-of-life option values and behaviour rely significantly on technological innovation, this could provide some very interesting results. Yet another extension would be to incorporate decommissioning more explicitly through the inclusion of some stochastic restriction, and attempt to estimate the probability of this restriction being imposed and how this affects the end-of-life options. Because the decision to decommission usually is based on some form of restriction, such as lack of capital or legal concession, it would be interesting to model decommissioning as a distinct option if one is able to predict these restrictions.

As discussed in Chapter 1, wind power is expected to become one of the dominant energy sources in the future global energy markets. A natural consideration that then arise is how a larger amount of wind power in the energy mix affects the profitability of individual wind farms and their end-of-life options. It can be reasoned qualitatively that this increase in market share will significantly increase the negative correlation between the wind speed and price of electricity, as a larger part of the market supply depends on the wind speed. From this, it can be reasoned that individual wind farms will face reducing revenues as their production peaks to a large degree aligns with an increasing number of other wind farms. An interesting extension to our model would therefore be the application of game theory, to determine how competition from other wind farms, and other power producers in general, affect the investment problem. See Huberts et al. (2015) for a review of how game theory is considered in the real options literature.

# Literature


Akyuz, H. and Gamgam, H. (2017). Statistical analysis of wind speed data with Weibull, lognormal and gamma distributions.

Benninga, S., Tolkowsky, E. (2002). Real options—an introduction and an application to R&D valuation. The Engineering Economist, 47 (2), 151–168.

Benth, F., Kallsen, J. and Meyer-Brandis, T. (2007). A Non-Gaussian Ornstein–Uhlenbeck Process for Electricity Spot Price Modeling and Derivatives Pricing. *Applied Mathematical Finance*, 14(2), pp.153-169.

Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), pp.637-654.

Bloomberg (2018). *New Energy Outlook 2018*. [online] Bloomberg. Available at: https://about.bnef.com/new-energy-outlook/#toc-download

Boomsma, T. and Linnerud, K. (2015). Market and policy risk under different renewable electricity support schemes. *Energy*, 89, pp.435-448.

Boomsma, T., Meade, N. and Fleten, S. (2012). Renewable energy investments under different support schemes: A real options approach. *European Journal of Operational Research*, 220(1), pp.225-237.

Boyle, P. (1977). Options: A Monte Carlo approach. *Journal of Financial Economics*, 4(3), pp.323-338.

Bradley, R. (1985). Basic properties of strong mixing conditions. *Air Force Office of Scientific Research*.

Brennan, M. and Schwartz, E. (1978). Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis. *The Journal of Financial and Quantitative Analysis*, 13(3), p.461.

Burger, M., Graeber, B. and Schindlmayr, G. (2014). *Managing energy risk*. Chichester: Wiley.

Campoccia, A., Dusonchet, L., Telaretti, E. and Zizzo, G. (2009). Comparative analysis of different supporting measures for the production of electrical energy by solar PV and Wind systems: Four representative European cases. *Solar Energy*, 83(3), pp.287-297.

Castro-Santos, L., Vizoso, A., Camacho, E. and Piegiari, L. (2016). Costs and feasibility of repowering wind farms. *Energy Sources, Part B: Economics, Planning, and Policy*, 11(10), pp.974-981.

Chong, W., Naghavi, M., Poh, S., Mahlia, T. and Pan, K. (2011). Techno-economic analysis of a wind–solar hybrid renewable energy system with rainwater collection feature for urban high-rise application. *Applied Energy*, 88(11), pp.4067-4077.

Clewlow, L. and Strickland, C. (1999). Valuing Energy Options in a One Factor Model Fitted to Forward Prices. *SSRN Electronic Journal*.

Conradsen, K., Nielsen, L. and Prahm, L. (1984). Review of Weibull Statistics for Estimation of Wind Speed Distributions. *Journal of Climate and Applied Meteorology*, 23(8), pp.1173-1183.

Cox. J., J. E. Ingersoll. and S. A. Ross. 1985, 'An Intertemporal General Equilibrium Model of Asset Prices," Econometrica, 53, 363-384.

Cox, J., Ross, S. and Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3), pp.229-263.

Dahlhaug, O. (2018). *Wind Turbines*. Lecture slides from the course TEP4195 – Turbo machinery, NTNU.

Décamps, J., Mariotti, T. and Villeneuve, S. (2006). Irreversible investment in alternative projects. *Economic Theory*, 28(2), pp.425-448.

del Río, P., Calvo Silvosa, A. and Iglesias Gómez, G. (2011). Policies and design elements for the repowering of wind farms: A qualitative analysis of different options. *Energy Policy*, 39(4), pp.1897-1908.

Deng, S. (2000). Stochastic models of energy commodity prices and their application: Mean-reversion with jumps and spikes.

Dixit, A. and Pindyck, R. (1994). *Investment under uncertainty*. Princeton University Press.

DNVGL (2016). *Lifetime extension of wind turbines*. [online] Available at: https://rules.dnvgl.com/docs/pdf/DNVGL/ST/2016-03/DNVGL-ST-0262.pdf

Easley, D., Engle, R., O'Hara, M. and Wu, L. (2007). Time-Varying Arrival Rates of Informed and Uninformed Trades. *Journal of Financial Econometrics*, 6(2), pp.171-207.

Ernstsen, R. and Boomsma, T. (2018). Valuation of power plants. European Journal of Operational Research, 266(3), pp.1153-1174.

Eryilmaz, D. and Homans, F. (2016). How does uncertainty in renewable energy policy affect decisions to invest in wind energy?. *The Electricity Journal*, 29(3), pp.64-71.

EWEA (2009). *The economics of wind energy*. [online] Available at: http://www.ewea.org/fileadmin/files/library/publications/reports/Economics_of_Wind_Energy.pdf [Accessed 30 Mar. 2019].

Farzin, Y., Huisman, K. and Kort, P. (1998). Optimal timing of technology adoption. *Journal of Economic Dynamics and Control*, 22(5), pp.779-799.

Fleten, S. and Maribu, K. (2004). Investment timing and capacity choice for small-scale wind power under uncertainty. *Lawrence Berkeley National Laboratory*.

Fleten, S., Maribu, K. and Wangensteen, I. (2007). Optimal investment strategies in decentralized renewable power generation under uncertainty. Energy, 32(5).

Forman, J. and Sørensen, M. (2008). The Pearson Diffusions: A Class of Statistically Tractable Diffusion Processes. *Scandinavian Journal of Statistics*, 35(3), pp.438-465.

Fossen, Ø. (2019). *Technology and finance drive wind power technology in Norway*.

Garcia, A., Torres, J., Prieto, E. and de Francisco, A. (1998). Fitting wind speed distributions: a case study. *Solar Energy*, 62(2), pp.139-144.

Goto, M. and Karolyi, G. (2004). Understanding Electricity Price Volatility Within and Across Markets. *SSRN Electronic Journal*.

Hagspiel, V., Huisman, K. and Nunes, C. (2015). Optimal technology adoption when the arrival rate of new technologies changes. European Journal of Operational Research, 243(3), pp.897-911.

Harrison, J. hl., and S. K. Pliska, 1981. "Martingales and Stochastic Integrals in the Theory of Continuous Trading", Stochastic Processes and their Applications, 11, 261-271.

Heath. D., R. Jarrow, and A. Merton. 1992. "Bond Pricing and the Term Structure of Interest Rates," Econometrica, 60, 77-106.

Hendricks, K. (1992). Reputation in the adoption of a new technology. *International Journal of Industrial Organization*, 10, pp.663-667.

Hennessey, J. (1977). Some Aspects of Wind Power Statistics. *Journal of Applied Meteorology*, 16(2), pp.119-128.

Herath, H., Park, C., 1999. Economic analysis of R&D projects: an options approach. The Engineering Economist, 44 (1), 1–35.

Himpler, S. and Madlener, R. (2012). Repowering of Wind Turbines: Economics and Optimal Timing. *SSRN Electronic Journal*.

Hoppe, H. (2002). The Timing of New Technology Adoption: Theoretical Models and Empirical Evidence. *The Manchester School*, 70(1), pp.56-76.

Horn, A., Kjærland, F., Molnár, P. and Steen, B. (2015). The use of real option theory in Scandinavia's largest companies. International Review of Financial Analysis, 41, pp.74-81.

Hou, P., Enevoldsen, P., Hu, W., Chen, C. and Chen, Z. (2017). Offshore wind farm repowering optimization. *Applied Energy*, 208, pp.834-844.

Huberts, N., Huisman, K., Kort, P. and Lavrutich, M. (2015). Capacity Choice in (Strategic) Real Options Models: A Survey. Dynamic Games and Applications, 5(4), pp.424-439.

IRENA (2018). Renewable Power Generation Costs in 2017. [online] Available at: https://www.irena.org/-/media/Files/IRENA/Agency/Publication/2018/Jan/IRENA_2017_Power_Costs_2018.pdf [Accessed 16 May 2019]

Justus, C., Hargraves, W., Mikhail, A. and Graber, D. (1978). Methods for Estimating Wind Speed Frequency Distributions. *Journal of Applied Meteorology*, 17(3), pp.350-353.

Kaminski, V. (1997). The challenge of pricing and risk managing electricity derivatives. *The US Power Market - Risk Publications*.

Kitzing, L., Juul, N., Drud, M. and Boomsma, T. (2017). A real options approach to analyse wind energy investments under different support schemes. Applied Energy, 188, pp.83-96.

Kozlova, M. (2017). Real option valuation in renewable energy literature: Research focus, trends and design. Renewable and Sustainable Energy Reviews, 80, pp.180-196.

Longstaff, F. and Schwartz, E. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. Review of Financial Studies, 14(1), pp.113-147.

Lucia, J. and Schwartz, E. (2002). Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research*, 5(1), pp.5-50.

Martínez-Ceseña, E. and Mutale, J. (2011). Application of an advanced real options approach for renewable energy generation projects planning. Renewable and Sustainable Energy Reviews, 15(4), pp.2087-2094.

Mauritzen, J. (2014). *Scrapping a Wind Turbine: Policy Changes, Scrapping Incentives and Why Wind Turbines in Good Locations Get Scrapped First*.

McCardle, K. (1985). Information Acquisition and the Adoption of New Technology. *Management Science*, 31(11), pp.1372-1389.

MegaVind (2016). *Strategy for Extending the Useful Lifetime of a Wind Turbine*. [online] Available at: http://www.vindmoellegodkendelse.dk/media/1163/strategy-for-extending-the-useful-lifetime-of-a-wind-turbine.pdf

Merton. R. C.. 1976. "Option Pricing When Underlying Stock Returns Are Discontinuous." Journal of Financial Economics, 3. 125-144.

Monjas-Barroso, M. and Balibrea-Iniesta, J. (2013). Valuation of projects for power generation with renewable energy: A comparative study based on real regulatory options. *Energy Policy*, 55, pp.335-352.

Moreno, M. and Navas, J. (2003). *Review of Derivatives Research*, 6(2), pp.107-128.

Myers, S. (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2), pp.147-175.

NewEnergyUpdate (2018). *Wind Turbine Lifetime extension: An owners guide*. [online] Available at: http://1.newenergyupdate.com/LP=18362 [Accessed 16 May 2019].

Newton, D., Paxson, D., Widdicks, M. (2004). Real R&D options. International Journal of Management Reviews, 5–6 (2), 113–130

Nielsen, Mark A., "Parameter Estimation for the Two-Parameter Weibull Distribution" (2011). All Theses and Dissertations. 2509.

Palchykov, P. and Vardøy, E. (2018). *Technology-driven Investment in Aluminum Smelters: Real Options Valuation Under Uncertainty*. MSc. NTNU.

Pang, W., Forster, J. and Troutt, M. (2001). Estimation of Wind Speed Distribution Using Markov Chain Monte Carlo Techniques. *Journal of Applied Meteorology*, 40(8), pp.1476-1484.

Pilipović, D. (2007). *Energy risk*. Maidenhead: McGraw-Hill.

Pobočíková, I., Sedliačková, Z. and Michalková, M. (2017). Application of Four Probability Distributions for Wind Speed Modeling. *Procedia Engineering*, 192, pp.713-718.

Renewable Energy World (2017). *Is Bigger Best*. [online] Available at: https://www.renewableenergyworld.com/ugc/articles/2017/05/08/is-bigger-best-report--part-1-limits-to-scale-in-wind.html.

Renewables First - The Hydro and Wind Company. (2018). How long does a wind turbine last? - Renewables First. [online] Available at: http://www.renewablesfirst.co.uk/windpower/windpower-learning-centre/how-long-dowind-turbines-installations-last/

Ricci, R., Vitali, D. and Montelpare, S. (2014). An innovative wind–solar hybrid street light: development and early testing of a prototype. *International Journal of Low-Carbon Technologies*, 10(4), pp.420-429.

Riordan, M. (1992). Regulation and Preemptive Technology Adoption. *The RAND Journal of Economics*, 23(3), p.334.

Rodrigues, A. and Rocha Armada, M. (2006). The Valuation of Real Options with the Least Squares Monte Carlo Simulation Method.

Santos-Alamillos, F., Thomaidis, N., Usaola-García, J., Ruiz-Arias, J. and Pozo-Vázquez, D. (2017). Exploring the mean-variance portfolio optimization approach for planning wind repowering actions in Spain. *Renewable Energy*, 106, pp.335-342.

Santos, L., Soares, I., Mendes, C. and Ferreira, P. (2014). Real Options versus Traditional Methods to assess Renewable Energy Projects. Renewable Energy, 68, pp.588-594.

Schwartz, E. and Smith, J. (2000). Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science*, 46(7), pp.893-911.

Schwartz, E. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance*, 52(3), pp.923-973.

Seifert, J. and Uhrig-Homburg, M. (2007). Modelling jumps in electricity prices: theory and empirical evidence. *Review of Derivatives Research*, 10(1), pp.59-85.

Steffen, B. (2019). Estimating the Cost of Capital for Renewable Energy Projects. *SSRN Electronic Journal*.

Stentoft, L. (2004). Assessing the Least Squares Monte-Carlo Approach to American Option Valuation. Review of Derivatives Research, 7(2), pp.129-168.

Trigeorgis, L., Mason, S. (1987). Valuing managerial flexibility and strategy in resource allocation. Midland Corporate Finance Journal, J. 5 (1), 14–21.

US Department of Energy (2018). *2017 Wind Technologies Market Report*. [online] Available at: https://www.energy.gov/eere/wind/downloads/2017-wind-technologies-market-report [Accessed 13 May 2019].

Vettas, N. (1998). Demand and Supply in New Markets: Diffusion with Bilateral Learning. *The RAND Journal of Economics*, 29(1), p.215.

WindEurope (2017a). *Wind Energy in Europe, Scenarios for 2030*. [online] Available at: https://windeurope.org/about-wind/reports/wind-energy-in-europe-scenarios-for-2030/ [Accessed 14 May 2019].

WindEurope (2017b). *WindEurope Annual Statistics 2017*. [online] WindEurope. Available at: https://windeurope.org/wp-content/uploads/files/about-wind/statistics/WindEurope-Annual-Statistics-2017.pdf [Accessed 14 Mar. 2018].

Zhou, J., Erdem, E., Li, G. and Shi, J. (2010). Comprehensive evaluation of wind speed distribution models: A case study for North Dakota sites. *Energy Conversion and Management*, 51(7), pp.1449-1458.

Ziegler, L., Gonzalez, E., Rubert, T., Smolka, U. and Melero, J. (2018). Lifetime extension of onshore wind turbines: A review covering Germany, Spain, Denmark, and the UK. Renewable and Sustainable Energy Reviews, 82, pp.1261-1271.

# Appendix

## Appendix A1 – Complete sensitivity tables

| Parameter \ Change from base case | | $-20\%$ | $-10\%$ | Bae case | $+10\%$ | $+20\%$ |
|---|---|---|---|---|---|---|
| $\boldsymbol{X_0}$ | $[MNOK]$ | 32.650 ($-4.69\ \%$) | 33.459 ($-2.33\ \%$) | 34.256 | 35.102 ($+2.47\ \%$) | 36.049 ($+5.23\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 19.12 | 18.91 | 18.73 | 18.55 | 18.36 |
| $\boldsymbol{r}$ | $[MNOK]$ | 44.782 ($+30.73\ \%$) | 39.187 ($+14.39\ \%$) | 34.256 | 29.974 ($-12.50\ \%$) | 26.157 ($-23.64\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 19.39 | 18.98 | 18.73 | 18.53 | 18.44 |
| $\boldsymbol{\gamma}$ | $[MNOK]$ | 35.069 ($+2.37\ \%$) | 34.917 ($+1.93\ \%$) | 34.256 | 33.150 ($-3.23\ \%$) | 32.512 ($-5.09\%$) |
| | [$Expected\ exercise\ time$, in years] | 19.01 | 18.89 | 18.73 | 18.97 | 18.96 |
| $\boldsymbol{\alpha}$ | $[MNOK]$ | 2.4729 ($-92.78\ \%$) | 13.298 ($-61.18\ \%$) | 34.256 | 71.460 ($+108.61\ \%$) | 134.50 ($+292.63\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 22.97 | 20.32 | 18.73 | 17.96 | 17.92 |
| $\boldsymbol{\sigma}$ | $[MNOK]$ | 34.292 ($+0.11\ \%$) | 34.296 ($+0.12\ \%$) | 34.256 | 34.259 ($+0.01\ \%$) | 34.161 ($-0.28\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 18.48 | 18.58 | 18.73 | 18.87 | 19.12 |
| $\boldsymbol{\rho}$ | $[MNOK]$ | 34.287 ($+0.09\ \%$) | 34.298 ($+0.12\ \%$) | 34.256 | 34.227 ($-0.08\ \%$) | 34.148 ($-0.32\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 18.83 | 18.74 | 18.73 | 18.72 | 18.78 |
| $\boldsymbol{K_r}$ | $[MNOK]$ | 30.132 ($-12.04\ \%$) | 30.521 ($-10.90\ \%$) | 34.256 | 38.115 ($11.27\ \%$) | 41.953 ($+22.47\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 18.12 | 19.13 | 18.73 | 18.37 | 18.08 |
| $\boldsymbol{\kappa_P}$ | $[MNOK]$ | 34.281 ($+0.07\ \%$) | 34.284 ($+0.08\ \%$) | 34.256 | 34.279 ($+0.07\ \%$) | 33.791 ($-1.36\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 19.04 | 19.00 | 18.73 | 18.61 | 18.74 |
| $\boldsymbol{\kappa_U}$ | $[MNOK]$ | 34.574 ($+0.93\ \%$) | 34.438 ($+0.53\ \%$) | 34.256 | 34.124 ($-0.39\ \%$) | 34.009 ($-0.72\ \%$) |
| | [$Expected\ exercise\ time$, in years] | 18.67 | 18.69 | 18.73 | 18.77 | 18.79 |

Table A.1: Sensitivity analysis of model input parameters for the option to **repower**

| Parameter \ Change from base case | $-20\%$ [MNOK] | $-10\%$ [MNOK] | Base case [MNOK] | $+10\%$ [MNOK] | $+20\%$ [MNOK] |
|---|---|---|---|---|---|
| $X_0$ $[MNOK]$ | 28.453 $(-0.74\ \%)$ | 28.557 $(-0.38\ \%)$ | 28.666 | 28.769 $(+0.36\ \%)$ | 28.868 $(+0.70\ \%)$ |
| $X_0$ $[Expected\ exercise$ $time$, in years] | 21.47 | 21.46 | 21.46 | 21.45 | 21.45 |
| $r$ $[MNOK]$ | 40.650 $(+41.81\ \%)$ | 34.230 $(+19.41\ \%)$ | 28.666 | 24.045 $(-16.12\ \%)$ | 20.230 $(-29.43\ \%)$ |
| $r$ $[Expected\ exercise\ time, in\ years]$ | 21.65 | 21.49 | 21.46 | 21.43 | 21.41 |
| $\gamma$ $[MNOK]$ | 30.150 $(+5.18\ \%)$ | 29.203 $(+1.87\ \%)$ | 28.666 | 27.954 $(-2.48\ \%)$ | 27.262 $(-4.90\ \%)$ |
| $\gamma$ $[Expected\ exercise\ time, in\ years]$ | 21.46 | 21.51 | 21.46 | 21.46 | 21.46 |
| $\alpha$ $[MNOK]$ | 3.8877 $(-86.44\ \%)$ | 12.631 $(-55.94\ \%)$ | 28.666 | 56.879 $(+98.42\ \%)$ | 106.95 $(+273.09\ \%)$ |
| $\alpha$ $[Expected\ exercise\ time, in\ years]$ | 21.96 | 21.66 | 21.46 | 21.42 | 21.41 |
| $\sigma$ $[MNOK]$ | 28.940 $(+0.96\ \%)$ | 28.560 $(-0.36\ \%)$ | 28.666 | 28.632 $(-0.12\ \%)$ | 28.451 $(-0.88\ \%)$ |
| $\sigma$ $[Expected\ exercise\ time, in\ years]$ | 21.24 | 21.43 | 21.46 | 21.54 | 21.65 |
| $\rho$ $[MNOK]$ | 28.686 $(+0.07\ \%)$ | 28.499 $(-0.58\ \%)$ | 28.666 | 28.692 $(+0.09\ \%)$ | 28.581 $(-0.30\ \%)$ |
| $\rho$ $[Expected\ exercise\ time, in\ years]$ | 21.48 | 21.52 | 21.46 | 21.47 | 21.46 |
| $K_r$ $[MNOK]$ | 25.848 $(-9.83\ \%)$ | 26.854 $(-6.32\ \%)$ | 28.666 | 30.233 $(+5.47\ \%)$ | 31.612 $(+10.28\ \%)$ |
| $K_r$ $[Expected\ exercise\ time, in\ years]$ | 21.23 | 21.55 | 21.46 | 21.49 | 21.56 |
| $K_l$ $[MNOK]$ | 28.229 $(-1.52\ \%)$ | 28.537 $(-0.45\ \%)$ | 28.666 | 28.607 $(-0.21\ \%)$ | 28.651 $(-0.05\ \%)$ |
| $K_l$ $[Expected\ exercise\ time, in\ years]$ | 21.52 | 21.47 | 21.46 | 21.50 | 21.58 |
| $\kappa_P$ $[MNOK]$ | 28.312 $(-1.24\ \%)$ | 28.405 $(-0.91\ \%)$ | 28.666 | 28.520 $(-0.51\ \%)$ | 28.832 $(+0.58\ \%)$ |
| $\kappa_P$ $[Expected\ exercise\ time, in\ years]$ | 21.66 | 21.58 | 21.46 | 21.46 | 21.33 |
| $\kappa_U$ $[MNOK]$ | 28.834 $(+0.59\ \%)$ | 28.646 $(-0.07\ \%)$ | 28.666 | 28.309 $(-1.25\ \%)$ | 28.335 $(-1.15\ \%)$ |
| $\kappa_U$ $[Expected\ exercise\ time, in\ years]$ | 21.44 | 21.48 | 21.46 | 21.54 | 21.52 |

Table A.2: Sensitivity analysis of model input parameters for the option to perform **life-extension**

| Change from base case / Parameter | | $-20\%$ | $-10\%$ | Base case | $+10\%$ | $+20\%$ |
|---|---|---|---|---|---|---|
| $\boldsymbol{X_0}$ | $[MNOK]$ | 33.664 (−4.24 %) | 34.366 (−2.25 %) | 35.156 | 35.900 (+2.11 %) | 36.830 (+4.76 %) |
| | [*Expected exercise time*, in years] | 18.93 | 18.74 | 18.73 | 18.37 | 18.19 |
| $\boldsymbol{r}$ | $[MNOK]$ | 46.065 (+31.03 %) | 39.944 (+13.62 %) | 35.156 | 30.683 (−12.72 %) | 26.469 (−24.63 %) |
| | [*Expected exercise time, in years*] | 19.42 | 19.06 | 18.55 | 18.31 | 18.30 |
| $\boldsymbol{\gamma}$ | $[MNOK]$ | 36.121 (+2.74 %) | 35.820 (+1.89 %) | 35.156 | 34.508 (−1.84 %) | 33.761 (−3.97 %) |
| | [*Expected exercise time*, in years] | 18.86 | 18.60 | 18.55 | 18.49 | 18.52 |
| $\boldsymbol{\alpha}$ | $[MNOK]$ | 4.0805 (−88.39 %) | 14.491 (−58.78 %) | 35.156 | 72.269 (+105.57 %) | 135.67 (+285.91 %) |
| | [*Expected exercise time*, in years] | 21.59 | 20.00 | 18.55 | 17.80 | 17.77 |
| $\boldsymbol{\sigma}$ | $[MNOK]$ | 35.214 (+0.16 %) | 34.710 (−1.27 %) | 35.156 | 34.842 (−0.89 %) | 34.916 (−0.68 %) |
| | [*Expected exercise time*, in years] | 18.28 | 18.65 | 18.55 | 18.84 | 18.99 |
| $\boldsymbol{\rho}$ | $[MNOK]$ | 35.254 (+0.28 %) | 34.831 (−0.92 %) | 35.156 | 34.711 (−1.27 %) | 34.773 (−1.09 %) |
| | [*Expected exercise time*, in years] | 18.55 | 18.75 | 18.55 | 18.75 | 18.75 |
| $\boldsymbol{K_r}$ | $[MNOK]$ | 30.405 (−13.51 %) | 31.363 (−10.79 %) | 35.156 | 38.359 (+9.11 %) | 42.005 (19.48 %) |
| | [*Expected exercise time*, in years] | 18.89 | 19.31 | 18.55 | 18.30 | 18.24 |
| $\boldsymbol{K_l}$ | $[MNOK]$ | 35.133 (−0.07 %) | 35.158 (+0.01 %) | 35.156 | 35.196 (+0.11 %) | 35.200 (+0.13 %) |
| | [*Expected exercise time*, in years] | 18.49 | 18.51 | 18.55 | 18.58 | 18.60 |
| $\boldsymbol{\kappa_P}$ | $[MNOK]$ | 35.266 (+0.31 %) | 35.191 (+0.10 %) | 35.156 | 35.147 (−0.03 %) | 35.152 (−0.01 %) |
| | [*Expected exercise time*, in years] | 18.78 | 18.66 | 18.55 | 18.45 | 18.35 |
| $\boldsymbol{\kappa_U}$ | $[MNOK]$ | 35.476 (+0.91 %) | 34.960 (−0.56 %) | 35.156 | 34.589 (−1.61 %) | 34.420 (−2.09 %) |
| | [*Expected exercise time*, in years] | 18.47 | 18.70 | 18.55 | 18.80 | 18.83 |

Table A.3: Sensitivity analysis for the double option to **repower or perform life-extension**

## Appendix A2 – Derivations for the single-factor model

### A2.1 - The net operating profit margin

The net operating profit margin can be expressed as

$$G_t = g_t e^{-\gamma t},$$

where $g_t$ follows geometric Brownian motion on the form

$$dg_t = \alpha g_t dt + \sigma g_t dZ.$$

The partial derivatives of $G_t$ can then be found as

$$\frac{\partial G}{\partial t} = -\gamma g_t e^{-\gamma t}, \ \frac{\partial G}{\partial g} = e^{-\gamma t} \text{ and } \frac{\partial^2 G}{\partial g^2} = 0.$$

Using Ito's lemma to expand $dG_t$ yields

$$dG_t = \frac{\partial G}{\partial t} dt + \frac{\partial G}{\partial g} dg + \frac{1}{2}\frac{\partial^2 G}{\partial g^2}(dg)^2.$$

Inserting the expression for $dg_t$ and the partial derivatives of $G_t$ results in

$$dG_t = -\gamma g_t e^{-\gamma t} dt + e^{-\gamma t}(\alpha g_t dt + \sigma g_t dZ) = (\alpha - \gamma) g_t e^{-\gamma t} dt + \sigma g_t e^{-\gamma t} dZ$$

$$= (\alpha - \gamma) G_t dt + \sigma G_t dZ \ \blacksquare$$

### A2.2 – The repowering option value

For the repowering option we can define the optimal stopping problem

$$V_R = \underbrace{\max}_{\tau} \left\{ \underbrace{E\left[\int_{t=0}^{\tau} G_t e^{-rt}\, dt\right]}_{Value\ of\ operating} + E\left[\underbrace{\int_{t=\tau}^{\infty} G_t e^{-\gamma \tau} K_R e^{-r(t-\tau)} dt - I_R e^{-r\tau}}_{Value\ of\ repowering}\right] \right\}$$

Note that the value of operating is simply the expected present value of the net operational profit margin from the current time until time $\tau$; while the value of repowering is the expected present value of the net operational profit margin of the repowered turbine from time $\tau$ until infinity, minus present value of the investment cost of repowering.

The value of instant repowering, i.e. the stopping value, is then given by

$$\Omega_R = E\left[\int_0^{\infty} K_R G_t e^{-rt} dt - I_R \mid G_0 = G\right] = K_R G \int_0^{\infty} e^{-(r-(\alpha-\gamma))t} dt - I_R$$

$$= \frac{K_R G}{r-(\alpha-\gamma)} - I_R \blacksquare,$$

where we have used the fact that

$$E[G_t \mid G_0 = G] = Ge^{(\alpha-\gamma)t},$$

and set $Q_0$ to 1. In the continuation region the Bellman equation applies, which can be expressed as

$$rV_R(G_t) = \mathrm{G}_t + \lim_{dt \to 0} \frac{1}{dt} E[dV_R]$$

Applying Ito's lemma to expand $dV_R$ results in

$$dV_R = \frac{\partial V_R}{\partial G} dG_t + \frac{1}{2} \frac{\partial^2 V_R}{\partial G^2} (dG_t)^2.$$

Using the fact that $dG_t = (\alpha-\gamma)G_t dt + \sigma G_t dz$, combined with the approximations

$$dt^2 \approx 0,\ dt * dz \sim (dt)^{\frac{3}{2}} \approx 0 \text{ and } (dz)^2 = dt,$$

gives

$$(dG_t)^2 = \sigma^2 G_t^2 dt.$$

Note that we assume that expressions with $dt$ to the power of more than unity is zero, because these terms go to zero faster than $dt$ when $dt$ goes to zero.

Inserting this into the expression for $dV_R$ gives

$$dV_R = \frac{\partial V_R}{\partial G}\big((\alpha-\gamma)G_t dt + \sigma G_t dz\big) + \frac{1}{2}\frac{\partial^2 V_R}{\partial G^2}\sigma^2 G_t^2 dt.$$

Inserting this into the Bellman equation, and noting that $E[dz] = 0$, results in an ODE on the form

$$rV(G) = G_t + (\alpha-\gamma)G_t \frac{\partial V}{\partial G} + \frac{1}{2}\sigma^2 G_t^2 \frac{\partial^2 V}{\partial G^2}$$

$$=> \frac{1}{2}\sigma^2 G_t^2 V''(G) + (\alpha-\gamma)G_t V'(G) - rV(G) = -G_t$$

This differential equation can be solved by guessing a functional form of the solution to the homogeneous and the inhomogeneous equation. Consider first the homogeneous equation

$$\tfrac{1}{2}\sigma^2 G_t^2 V''(G) + (\alpha-\gamma)G_t V'(G) - rV(G) = 0,$$

and guess the functional form

$$V_h(G) = AG^{\beta} \Rightarrow V_h'(G) = \beta A G^{\beta-1} \Rightarrow V_h''(G) = \beta(\beta-1)AG^{\beta-2}$$

Inserting this into the homogeneous equation yields

$$\frac{1}{2}\sigma^2 G_t^2\big(\beta(\beta-1)AG^{\beta-2}\big) + (\alpha-\gamma)G_t\big(\beta A G^{\beta-1}\big) - \rho A G^{\beta} = 0$$

$$\Rightarrow G^{\beta}A\left(\frac{1}{2}\sigma^2\beta(\beta-1) + (\alpha-\gamma)\beta - r\right) = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2\beta(\beta-1) + (\alpha-\gamma)\beta - r = 0 \Rightarrow \beta^2 + \left(\frac{2(\alpha-\gamma)}{\sigma^2} - 1\right)\beta - \frac{2r}{\sigma^2} = 0$$

$$\Rightarrow \beta_{1,2} = \left(\frac{1}{2} - \frac{(\alpha-\gamma)}{\sigma^2}\right) \pm \sqrt{\left(\frac{(\alpha-\gamma)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

$V_h(G) = A_1 G^{\beta_1} + A_2 G^{\beta_2}$ is therefore a homogeneous solution. From this expression, it can be observed that $\beta_1 > 1\ and\ \beta_2 < 0$.

Making a practical observation, one can infer that the option to repower is worthless if the operational profit flow is zero, hence $V_R \to 0$ for $g_t \to 0$. This is simply a property of the GBM of $G_t$. Because $dG_t$ depends on $G_t$, $G_t$ will stay at zero if it reaches zero, hence the option would be worthless at this point.

Since $\beta_2 < 0$, $A_2$ will have to be zero. If this was not the case, the option value function would become very large as $G_t$ went to zero. Hence,

$$V_h(G) = A_1 G^{\beta_1} = AG^{\beta},$$

where

$$\beta = \left(\frac{1}{2} - \frac{(\alpha-\gamma)}{\sigma^2}\right) + \sqrt{\left(\frac{(\alpha-\gamma)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

Next, a particular solution is needed. Considering the inhomogeneous equation

$$\frac{1}{2}\sigma^2 G_t^2 V''(G) + (\alpha-\gamma)G_t V'(G) - rV(G) = -G_t, \qquad (*)$$

guessing the functional form

$$V_p(G) = bG \Rightarrow V_p'(G) = b \Rightarrow V_p''(G) = 0,$$

and inserting into $(*)$ yields

$$(\alpha-\gamma)G_t b-\rho b G_t=-G_t => \ (\alpha-\gamma)b-rb=-1$$

$$=> b(\alpha-\gamma-r)=-1 => b=\frac{1}{r-(\alpha-\gamma)}$$

Hence,

$$V_p(G)=\frac{G}{r-(\alpha-\gamma)}$$

Finally, by combining the particular solution with the homogeneous solution the total solution can be expressed as

$$V_R^{tot}(G)=V_h(G)+V_p(G)=AG^{\beta}+\frac{G}{r-(\alpha-\gamma)}$$

where $\beta$ is given by

$$\beta=\left(\frac{1}{2}-\frac{(\alpha-\gamma)}{\sigma^2}\right)+\sqrt{\left(\frac{(\alpha-\gamma)}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}$$

To determine the optimal investment threshold, $G_R$, and the option value parameter, $A$, boundary conditions must be applied. By using the stopping value, the boundary conditions for this second order differential equation can be expressed as

$$V_R^{tot}(G_R)=\ \Omega_R$$

$$\Rightarrow AG^{\beta}+\ \frac{G}{r-(\alpha-\gamma)}=\ \frac{K_R G}{r-(\alpha-\gamma)}-I_R \qquad \textit{(Value-matching)}$$

and

$$\frac{\partial V_R^{tot}}{\partial G}=\frac{\partial \Omega_R}{\partial G}$$

$$\Rightarrow \beta AG^{\beta-1}+\frac{1}{r-(\alpha-\gamma)}=\ \frac{K_R}{r-(\alpha-\gamma)} \qquad \text{(Smooth-pasting)}$$

Note that the value-matching condition and the smooth-pasting condition are found by using the fact that the value of continuation and stopping, and their derivates, are equal at the boundary between the continuation and stopping region, that is at $G=G_R$.

Applying the first boundary condition results in

$$V(G_R)=\ AG_R^{\beta}+\frac{G_R}{r-(\alpha-\gamma)}=\ \frac{K_R G_R}{r-(\alpha-\gamma)}-I_R$$

$$=> \; A = G_R^{-\beta}\left(\frac{G_R}{r-(\alpha-\gamma)}(K_R-1)-I_R\right) \text{ or } AG_R^{\beta-1} = \frac{K_R-1}{r-(\alpha-\gamma)} - \frac{I_R}{G_R}$$

Applying the third boundary condition results in

$$V'(G_R) = \beta A G_R^{\beta-1} + \frac{1}{r-(\alpha-\gamma)} = \frac{K_R}{r-(\alpha-\gamma)}$$

$$=> \; \beta\left(\frac{K_R-1}{r-(\alpha-\gamma)} - \frac{I_R}{G_R}\right) + \frac{1}{r-(\alpha-\gamma)} = \frac{K_R}{r-(\alpha-\gamma)} => -\frac{\beta I_R}{G_R} = \frac{K_R-1}{r-(\alpha-\gamma)} - \frac{\beta(K_R-1)}{r-(\alpha-\gamma)}$$

$$=> \; G_R = \frac{\beta}{(\beta-1)} * \frac{I_R\big(r-(\alpha-\gamma)\big)}{K_R-1}$$

For simplicity define the adjusted discount rate $r^* = r-(\alpha-\gamma)$, and the threshold can then be expressed as

$$G_R = \frac{\beta}{(\beta-1)} * \frac{I_R r^*}{K_R-1} \blacksquare$$

The option value can then be expressed with respect to the threshold value by inserting the expression for A into the option value function to find

$$V_R^{tot}(G) = AG^{\beta} + \frac{G}{r^*} = G_R^{-\beta}\left(\frac{G_R}{r^*}(K_R-1)-I_R\right)G^{\beta} + \frac{G}{r^*}$$

$$= \left(\frac{G}{G_R}\right)^{\beta}\left(\frac{G_R(K_R-1)}{r^*} - I_R\right) + \frac{G}{r^*}$$

This expression is however the expected value of the wind farm including the repowering option. To isolate the value of the repowering option, one can subtract the term $\frac{G}{r^*}$, which is the expected value of operating the current wind farm in perpetuity. In doing so, the option value is expressed as

$$V_R = \left(\frac{G}{G_R}\right)^{\beta}\left(\frac{G_R(K_R-1)}{r^*} - I_R\right) \blacksquare$$

Note that this option value only holds for the continuation region. As soon as the threshold is hit, the option value will be equal to the stopping value.

### A2.3 – The life-extension option value

For the lifetime-extension option we can define the optimal stopping problem

$$V_{\mathrm{L}} = \underbrace{\max}_{\tau} \left\{ \begin{array}{c} \underbrace{E\left[\int_{t=0}^{\tau} G_t e^{-rt}\,dt\right]}_{Value\ of\ operating} + \underbrace{E\left[\int_{t=\tau}^{\tau+\Delta t} G_t e^{\gamma\tau} K_L e^{-r(t-\tau)} dt - I_L e^{-r\tau}\right]}_{Value\ of\ lifetime-extension} \\ + E\left[\underbrace{\int_{t=\tau+\Delta t}^{\infty} G_t e^{\gamma\Delta t} K_R e^{-rt} dt - I_R e^{-r(\tau+\Delta t)}}_{Value\ of\ repowering\ after\ lifetime-extension}\right] \end{array} \right\}$$

The value of instantaneous investment, i.e. the stopping value, for $V_L$ is then

$$\Omega_L = E\left[\int_0^{\Delta t} G_t K_L e^{-rt} dt - I_L \mid G_0 = G\right] + E\left[\int_{\Delta t}^{\infty} G_t e^{\gamma\Delta t} K_R e^{-rt} dt - I_R e^{-r\Delta t} \mid G_0 = G\right]$$

$$= K_L G \int_0^{\Delta t} e^{-(r-(\alpha-\gamma))t} dt - I_L + K_R G e^{\gamma\Delta t} \int_{\Delta t}^{\infty} e^{-(r-(\alpha-\gamma))t} dt - I_R e^{-r\Delta t},$$

let $r^* = r - (\alpha - \gamma)$, which gives

$$\Omega_L = K_L G \int_0^{\Delta t} e^{-r^* t} dt - I_L + K_R G e^{\gamma\Delta t} \int_{\Delta t}^{\infty} e^{-r^* t} dt - I_R e^{-r\Delta t}$$

$$= K_L G \left[-\frac{1}{r^*} e^{-r^* t}\right]_0^{\Delta t} - I_L + K_R G e^{\gamma\Delta t} \left[-\frac{1}{r^*} e^{-r^* t}\right]_{\Delta t}^{\infty} - I_R e^{-r\Delta t}$$

$$= \frac{K_L G}{r^*}\left(1 - e^{-r^*\Delta t}\right) - I_L + \frac{K_R G e^{\gamma\Delta t}}{r^*} e^{-r^*\Delta t} - I_R e^{-r\Delta t}$$

$$= \frac{G}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t}\right) - \left(I_L + I_R e^{-r\Delta t}\right)$$

In the continuation region, the situation is the same as for the repowering option. Hence, the general solution to the partial differential equation still holds. Combining this with the new stopping value, the boundary conditions for the lifetime-extension option canbe expressed as

$$AG^{\beta} + \frac{G}{r^*} = \frac{G}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t}\right) - \left(I_L + I_R e^{-r\Delta t}\right)$$

*(Value-matching)*

$$\beta A G^{\beta-1} + \frac{1}{r^*} = \frac{1}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t}\right)$$

*(Smooth pasting)*

Applying the value-matching condition gives

$$A{G_L}^{\beta} + \frac{G_L}{r^*} = \frac{K_L G_L}{r^*}\left(1 - e^{-r^*\Delta t}\right) - I_L + \frac{K_R G e^{(\gamma - r^*)\Delta t}}{r^*} - I_R e^{-r\Delta t}$$

$$=> AG_L^{\beta-1} = \frac{K_L}{r^*}\left(1 - e^{-r^*\Delta t}\right) - \frac{I_L}{G_L} + \frac{K_R e^{(\gamma - r^*)\Delta t}}{r^*} e^{-r^*\Delta t} - \frac{I_R e^{-r\Delta t}}{G_L} - \frac{1}{r^*} \quad (**)$$

Applying the smooth-pasting condition then gives

$$\beta AG_L^{\beta-1} + \frac{1}{r^*} = \frac{K_L}{r^*}\left(1 - e^{-r^*\Delta t}\right) + \frac{K_R e^{(\gamma - r^*)\Delta t}}{r^*}$$

$$\underbrace{=>}_{(**)} \beta\left(\frac{K_L}{r^*}\left(1 - e^{-r^*\Delta t}\right) - \frac{I_L}{G_L} + \frac{K_R e^{(\gamma - r^*)\Delta t}}{r^*} - \frac{I_R e^{-r\Delta t}}{G_L} - \frac{1}{r^*}\right) + \frac{1}{r^*} = \frac{K_L}{r^*}\left(1 - e^{-r^*\Delta t}\right) + \frac{K_R e^{(\gamma - r^*)\Delta t}}{r^*}$$

$$=> \frac{(\beta - 1)}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t} - 1\right) = \frac{\beta}{G_L}\left(I_L + I_R e^{-r\Delta t}\right)$$

$$=> G^* = \frac{\beta}{(\beta - 1)} \frac{r^*\left(I_L + I_R e^{-r\Delta t}\right)}{\left(K_L(1 - e^{-r^*\Delta t}) + K_R e^{(\gamma - r^*)\Delta t} - 1\right)}$$

$$=> V_L^{tot}(G) = \left(\frac{G}{G_L}\right)^{\beta}\left(\frac{G_L}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t} - 1\right) - \left(I_L + I_R e^{-r\Delta t}\right)\right) + \frac{G}{r^*},$$

which is the total value of the wind farm. By subtracting the term $\frac{G}{r^*}$, which is the expected value of operating the current turbine in perpetuity, the life-extension option value can be isolated as

$$V_L(G) = \left(\frac{G}{G_L}\right)^{\beta}\left(\frac{G_L}{r^*}\left(K_L\left(1 - e^{-r^*\Delta t}\right) + K_R e^{(\gamma - r^*)\Delta t} - 1\right) - \left(I_L + I_R e^{-r\Delta t}\right)\right) \blacksquare$$

# Appendix A3 – Derivations for the multi-factor model

## A3.1 – Expected value of wind power production

The expected value of the wind farm power output can be expressed as

$$E\left[h\left(f(U_t)\right) \,\middle|\, U_0\right]$$

Recall that the weather factor $U_t$ follows an Ornstein-Uhlenbeck process on the form

$$dU_t = -\kappa_U U_t dt + \sqrt{2\kappa_U} dZ_U,$$

hence the first two moments can be expressed as

$$E[U_t \,|U_0] = U_0 e^{-\kappa_U t}$$

$$Var[U_t \mid U_0] = 1 - e^{-2\kappa_U t}.$$

Utilizing lemma 4.B.1 from Ernstsen and Boomsma (2018), and defining a general function for $h$ on the interval $(u_0, u_1)$ on the form

$$h(u) = (au + b)|_{u \,\epsilon\, (u_0, u_1)},$$

we can then express

$$E[h(U)] = (a\xi + b)\left(\Phi\left(\frac{u_1 - \xi}{\sigma_u}\right) - \Phi\left(\frac{u_0 - \xi}{\sigma_u}\right)\right) + \frac{a\sigma_u}{\sqrt{2\pi}}\left(e^{-\frac{(u_0-\xi)^2}{2\sigma_u^2}} - e^{-\frac{(u_1-\xi)^2}{2\sigma_u^2}}\right),$$

where $\xi = \mu_u$.

Recall that the turbine power output follows the process

$$h(f(u)) = \begin{cases} 0 & if\ u < u_0\ or\ u > u_2 \\ h_{12}(u - u_0)/(u_1 - u_0) & if\ u_0 < u < u_1, \\ h_{12} & if\ u_1 < u < u_2 \end{cases}$$

We must therefore use the process described above to determine $E[h(U)]$ for each of the four intervals $(u < u_0)$, $(u_0, u_1)$, $(u_1, u_2)$ and $(u > u_2)$. However, as $h\left(f(u)\right) = 0$ on $(u < u_0)$ and $(u > u_2)$, the expected value of $E[h(U)]$ on these intervals can immediately be observed to be zero. Combining the expressions for $E[h(U)]$ on each interval by utilizing lemma 4.B.3 from Ernstsen and Boomsma, we find that

$$E\left[h\big(f(U)\big)\right] = h_{12}\left(\Phi\left(\frac{u_2-\xi}{\sigma_u}\right) - \Phi\left(\frac{u_1-\xi}{\sigma_u}\right)\right) + \frac{\sigma_u h_{12}}{\sqrt{2\pi}(u_1-u_0)}\left(e^{-\frac{(u_0-\xi)^2}{2\sigma_u^2}} - e^{-\frac{(u_1-\xi)^2}{2\sigma_u^2}}\right)$$
$$+ \frac{(\xi-u_0)h_{12}}{u_1-u_0}\left(\Phi\left(\frac{u_1-\xi}{\sigma_u}\right) - \Phi\left(\frac{u_0-\xi}{\sigma_u}\right)\right),$$

where $\xi = U_0 e^{-\kappa_U t}$ and $\sigma_U = \sqrt{1-e^{-2\kappa_U t}}$.

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