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The effect of passive index tracking funds on asset return correlations

How mutual fund flows and ETF trading volumes decrease benefits of diversification through increased asset return correlations of index constituents

Master's thesis in Industrial Economics and Technology Management
Supervisor: Alexei A. Gaivoronski
June 2019

Master's thesis

NTNU
Norwegian University of Science and Technology
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Kunnskap for en bedre verden

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Abstract

This thesis investigates the effect of investments in index tracking mutual funds and exchange traded funds (ETFs) on the correlations of returns of index constituents in the U.S. A monthly multiple linear regression model is used to show that flows of funds in and out of mutual funds have a positive and significant effect on asset return correlations at a 5% significance level within the large, mid and small cap indices (S&P 500, S&P 400 and S&P 600, respectively). This has not previously been shown. The linear regression also shows that ETF trading volumes have a positive and significant effect on asset correlations within the large, mid and small cap indices at a 1% significance level. Asset return correlations are shown to increase in bad economic times, which is modeled using a Markov switching model. In addition, an intradaily correlation model is used to show that a lagged effect of ETF trading volumes on asset correlations is present and significant at high frequencies. This effect can be used in correlation forecasting at a five minute frequency.

Sammendrag

Denne oppgaven studerer effekten av investeringer i tradisjonelle indeksfond og ETF-er på korrelasjoner mellom underliggende aksjer i amerikanske indekser. En månedlig regresjonsmodell brukes til å vise at kapitalstrøm til og fra tradisjonelle indeksfond har en positiv og signifikant effekt på korrelasjonene mellom aksjer på et signfikansnivå på 5%, innenfor indekser som følger aksjer med henholdsvis stor, middels og lav markedsverdi (S&P 500, S&P 400 og S&P 600). Dette har ikke tidligere blitt vist. Den lineære regresjonsmodellen viser også at ETF-handelsvolumer har en positiv og signifikant effekt på aksjekorrelasjoner innen S&P 500, S&P 400 og S&P 600 på et signifikansnivå på 1%. Det vises at aksjekorrelasjoner øker i dårlige økonomiske tider, modellert ved bruk av en Markov switching-modell. Videre brukes en høyfrekvent korrelasjonsmodell til å vise at ETF-handelsvolumer har en signifikant forsinket effekt på aksjekorrelasjoner. Denne effekten kan brukes til å prognosere aksjekorrelasjoner med en frekvens på fem minutter.

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Chapter 1

Introduction

Passive investing has been on the rise in recent years. The share of total U.S. stock fund assets in passive funds has increased from 11.7% in 2000 to 42.0% in 2016.[35] An important part of passive investments is index tracking funds, which aim to follow a market index such as the S&P 500, and where asset weights are based on market capitalizations (cap). Two important types of index tracking funds exist, mutual funds and exchange traded funds (ETFs). The main difference between these are that ETFs can be traded in real time on an exchange between customers, while investors in mutual funds must invest with the provider directly. Advocates of passive investments boast about low management fees and that investing in a combination of the market index and the risk free asset gives the highest possible Sharpe ratio, as stated by the CAPM model of Treynor (1961).[36] However, research indicates that passive investing has some adverse effects on stock markets, for example increasing the overall correlation of stock returns.[34] The focus of this thesis is to investigate the effect of investments in passive funds on the correlations of asset returns of index constituents, and to investigate if this effect can be utilized to improve correlation forecasting.

Several papers have researched the effect of index tracking ETFs on asset return correlations, including Leippold, Su and Ziegler (2016), Staer and Sottile (2018) and Da and Shive (2013).[21][33][10] However, no prior research has been done into the effect of index tracking mutual funds, and this is a main contribution of this thesis to the literature. As of January 2018, 3.3 trillion USD was placed in passive mutual funds, compared to 3.4 trillion USD in index tracking ETFs, meaning that further understanding of the effects of the mutual funds is of great importance.[1] Another main contribution of this thesis is that it is the first to utilize the effect of index tracking ETFs in forecasting future correlations.

Hereafter two main models are employed, a monthly multiple linear regression model and an intradaily correlation forecasting model. The monthly model regresses the weighted average correlation of index constituents on ETF trading activity, mutual fund flows, the likelihood of being in a bear market (stock market downturn) and the VIX. The intradaily correlation model utilizes the ETF trading activity directly in correlation forecasting using a direct conditional correlation (DCC) model. The main motivation of the intradaily model is that forecasted correlations can be used in financial modelling. The motivations behind both models will be explained in more detail in chapter 4.

In addition to investigating the effects of mutual fund flows, the monthly regression model has four additional and significant insights. First, the model is used to investigate the effects on other indices than just the S&P 500, specifically the S&P 400 mid cap index and the S&P 600 small cap index. This was in order to have a more robust investigation into a wider range of assets, not only the 500 largest assets which most of the literature focuses on. Secondly, an investigation is conducted into the effects of several subsets of index tracking mutual funds and ETFs, based on what they track. The main classifications here are large cap, mid cap and small cap. The motivation was to investigate not only the relations between trading of ETFs of a certain classification and the corresponding index correlations, but also cross-relations such as the effect of large cap ETF trading on mid cap asset correlations. Thirdly, the monthly regression model successfully models the known effect that asset correlations increase in bad economic times by using the inferred probability of being in a bear market calculated with a Markov switching model. This is an important result in its own right, as previous papers have modeled this effect by including several macroeconomic variables, with varying success. Furthermore, by reducing the number of variables required to include the effect the model has increased statistical power as it is more parsimonious. Hence this assists in the main focus of the model, which is investigating the effects of passive fund activity. Finally, the monthly regression model expands upon the results of previous papers on the effect of index tracking ETFs on asset return correlations such as Leippold, Su and Ziegler (2016), Staer and Sottile (2018) and Da and Shive (2013) by showing similar results over longer and more recent time periods than originally used.[21] [33] [10]

The inclusion of the Markov switching model improves the research on asset return correlations in its own right by reducing the number of variables required to model the effect of bad economic times. It also helps in the investigation of the effect of ETF trading and fund flows, which is one of the main goals of this thesis, by making the model more parsimonious.

The intradaily correlation model contributes to the literature in three main ways.

It models the effects from trading of index tracking ETFs on asset correlation forecasts, which is, as previously mentioned, a novel approach in the literature. Secondly, it supports the results of Staer and Sottile (2018) that the effects of ETF trading are both present and strong at intradaily frequencies, while using more recent data and a different model.[33] Furthermore, this thesis further develops the direct conditional correlation model with exogenous variables (DCCX) of Vargas (2008), by showing its usefulness in a new application that fits well with the model's restrictions.[37]

The main results of this thesis will now be summarized. The monthly linear regression model shows that asset return correlations for the S&P 500 constituents is increasing with absolute fund flows into index tracking mutual funds in the period January 2005 to September 2018. The same is true for constituents of the S&P 400 mid cap index and the S&P 600 small cap index. Furthermore a t-test confirms that this effect is statistically significant at a 5% significance level. ETF activity is also shown to have a positive and significant influence for all three indices used.

When splitting fund activity variables, trading of large cap ETFs was shown to have a significant effect on correlations of all investigated indices, indicating a spillover effect due to correlation between small cap asset prices and the general market. Additionally, small cap ETF trading significantly increases correlations in the small cap S&P 600 index. The only split ETF trading variable that does not affect correlations in its corresponding index is the mid cap ETF volumes. However, this may be due to multicollinearity amongst the ETF trading variables. Similar to the results for small cap ETF trading, fund flows of small cap mutual funds have a significant and positive effect on correlations of the small cap index. However, both the large cap and mid cap mutual fund flows have an insignificant effect on their corresponding indices. There is even stronger evidence of multicollinearity amongst the fund flow variables.

The intradaily correlation forecasting model of this thesis shows that adding ETF trading activity as an exogenous variable to the DCC model gives significantly improved covariance estimates than the normal DCC model as measured by log likelihood. A likelihood-ratio test showed that this difference is significant with a p-value of order of magnitude 10^{-12}. The relative performance of these different covariance estimates is also tested when applied to Markowitz portfolio optimization. It is shown that the portfolio base on the DCCX estimates give on average lower squared returns, however this difference is not shown to be statistically significant at a 5% significance level.

Chapter 2

Literature review

Traditional finance theory states that stock returns are correlated due to corresponding asset fundamentals as well as macroeconomic factors. However, with the rising share of investments in passive index tracking funds, more research has been conducted regarding its effects in the financial markets. More specifically, there is relevant research regarding the impact of index tracking ETF trading volumes on asset return correlations. Most previous research has focused on low frequency models, such as monthly models, in order to include macroeconomic explanatory variables. There is also some limited research into these effects on intradaily frequencies.

The literature review begins with explaining the implications of ETF trading on asset correlations and market inefficiency. Next, several studies on the effects of ETF trading volume, especially on asset correlations, were reviewed. Several researchers have found that an increase in trading volume from index tracking ETFs leads to increased asset return correlations. In particular, Leippold, Su and Ziegler (2016), Staer and Sottile (2018) and Da and Shive (2013) touch specifically on the topic of ETF trading volume effects on the correlation of the underlying assets in the tracked index, and these papers were therefore studied comprehensively.[21] [33] [10]

Asset correlations are often assumed to be dependent on being in a bear market or downturn in the business cycle, and it is therefore necessary to include one or more explanatory variables for economic downturns. This is often done by including several macroeconomic variables, but as this thesis shows this can also be done with the use of Markov switching models. No previous paper studying the effect of ETF trading volumes on asset correlations use Markov switching models to model the likelihood of being in a bear market. This literature review will therefore elaborate on other studies explaining the use of these Markov switching models. The literature usually focuses on two types of Markov switching models, either the Hamilton (1998) fixed transition probability model which is used in this thesis or the time varying transition probability (TVTP) extension.[17][18]

One hypothesis of this thesis is that the relationship between ETF trading and asset correlations can be used to forecast asset correlations at short time intervals. To explore whether this is possible, various forecasting models and a method for comparing forecasts are introduced. The dynamic conditional correlation (DCC) model of Engle (2002) is introduced as a correlation forecast model.[12] It is then explained how Vargas (2008) expanded the DCC model to account for exogenous variables in the DCCX model.[37] Then, a method by Engle and Colacito (2006) for comparing conditional correlation forecasts by using mean-variance optimization is reviewed.[13]

Finally, the literature review is summed up with this thesis' contributions to the literature.

2.1 Implications of ETF trading and increased asset return correlations

According to Sullivan and Xiong (2012), ETF trading accounted for about a third of all trading in the U.S.[34] This share has only increased since then. Sullivan and Xiong (2012) conclude with the observation that correlations increase with an increase in passive ETF trading, and thus the overall market efficiency and diversification benefits decline:

> This increased level of trading associated with passive investing, however, comes with important consequences. It means an increased trading commonality among index constituents through the interactions of market participants. Such trading commonality then gives way to a rise in systematic fluctuations in overall demand, which, in turn, leads to a fundamental impact on the overall market and investors' portfolios. In short, the growth in trading of passively managed equity indices corresponds to a rise in systematic market risk. From this finding, one can infer that the ability of investors to diversify risk by holding an otherwise well-diversified U.S. equity portfolio has markedly decreased in recent decades. As this research has demonstrated, U.S. equity portfolios have become less diversified in recent years; returns for all subsets have become more correlated, leaving no areas for in-

> vestors to improve diversification and thus mitigate risk. Put another way, investors' equity portfolios are increasingly moving in lockstep with swings in the overall market. All equity investing, indexed or otherwise, is thus plainly a more risky prospect for investors.[34]

The negative effect on diversification is confirmed by Israeli, Lee and Sridharan (2017).[19] As hypothesized by the authors, trading costs rose with the share of ETF investments, and the stock began to move more in line with its sector and with the overall market, and less in line with its own earnings. In addition, fewer analysts covered the stock as ETF ownership rose. In addition, Bleiberg, Priest and Pearl (2017) state that the implications might provide efficiency questions for the stock markets as a whole:

> [...], there would be far reaching implications. At the broadest level, it would lead to questions about the ability of the stock market to serve as an effective allocator of capital in the economy. While the stock market itself does not provide the initial start-up capital to new companies, the valuation of publicly traded stocks is often used as a benchmark by the venture capital and private equity investors who do provide that capital. If stock prices became less efficient at reflecting fundamental company information, those early stage investors would have an inaccurate view of which companies deserve capital, and at what cost.[6]

Bleiberg, Priest and Pearl (2017) also sum up their discussion with a quote from Michael Mauboussin of Credit Suisse, explaining the possible implications of trading in passive funds and increased asset correlations:

> Markets tend to be informationally efficient when investors use heterogeneous decision rules. This is the wisdom of crowds. The loss of diversity as the result of converging decision rules creates fragility in the market and the possibility of prices departing substantially from value. This is the madness of crowds.[25]

Evidently, the effect of increased ETF trading increases asset return correlations, which in turn decreases overall market efficiency and diversification benefits. It is therefore interesting and important to research these effects further.

2.2 Asset correlation models and trading volumes in passive funds

There are several papers studying the effect of trading volume in passive index tracking ETFs on the correlation of the stocks in the underlying index. Three papers in particular touch specifically on the topic of how passive index tracking ETF investments affect stock return correlations, and will be important for the understanding of the research topic.

First, Leippold, Su and Ziegler (2016) investigate the trading activity in index futures and ETFs on equity return correlations both theoretically and empirically.[21] They explain how arbitrageurs create market equilibrium in the ETF and futures markets, and consequently that demand shocks to those derivatives should theoretically increase stock return correlations. Furthermore, they hypothesize that there should be a spillover effect from demand shocks to the derivatives to asset return correlations outside of the underlying index. They empirically investigate the relationship between demand shocks in futures and ETFs on stock return correlations at an aggregate level on the S&P 500 index.

The aggregate correlation measure used follows from Pollet and Wilson (2010) and is denoted $\hat{\rho}_t$.[27] They calculate the S&P 500 value weighted average monthly correlation using daily stock returns. Their main explanatory variable for the regression is a so-called trading ratio, defined as the trading volume of index futures or ETFs divided by the trading volume of S&P 500. This definition of ETF trading ratio is used throughout this thesis, as ETF trading ratio is an exogenous variable in the monthly correlation model presented in section 4.1. The trading ratio constitutes a proxy for demand shocks that hit only the index derivatives or ETF market. The average correlations are regressed on the demand shock proxies, more specifically the ARIMA residuals of the different trading ratios for futures and ETFs ($r_{Legacy,t}$, $r_{Emini,t}$ and $r_{ETF,t}$). To account for other possible determinants of correlation, they include control variables from the stock and bond markets as well as several macroeconomic variables. The regression also includes three lags of the dependent variable ($\hat{\rho}_{t-1}$, $\hat{\rho}_{t-2}$ and $\hat{\rho}_{t-3}$) due to the autocorrelation structure. Thus, the resulting regression is

$$\begin{aligned}\hat{\rho}_t = {} & \beta_0 + \beta_1 SPVol_t + \beta_2 CurrentReturn_t + \beta_3 \Delta 3MonthTbill_t \\ & + \beta_4 CreditSpread_t + \beta_5 \Delta IndProd_t + \beta_6 Inflation_t + \beta_7 VIX_t \\ & + \beta_8 EPU_t + \sum_{n=1}^{3} \beta_{8+i}\hat{\rho}_{t-i} + \gamma_1 r_{Legacy,t} + \gamma_2 r_{Emini,t} + \gamma_3 r_{ETF,t} + \epsilon_t\end{aligned} \tag{2.1}$$

Here $SPVol_t$ is the realized volatility, $CurrentReturn_t$ is the current month return of the S&P 500 index and VIX_t is the CBOE Volatility Index. According to Solnik (1995) and Ang and Chen (2002), correlations increase in volatile markets and bear markets. [22] [3] The VIX is included as a direct measure of market expectations of near-term volatility from S&P 500 index option prices, which has been found to be a better predictor of future volatility than historical volatility by Jiang and Tian (2005). [20] The Treasury bill rate $\Delta 3MonthTbill_t$, the credit spread $CreditSpread_t$ and three macroeconomic variables are included to include the effect of increased correlations during bear markets. Specifically, the three macroeconomic variables included are industry production $IndProd_t$, inflation $Inflation_t$ and economic policy uncertainty EPU_t. Economic policy uncertainty is measured using a proxy defined by Baker, Bloom and Davis (2016). [4]

Leippold, Su and Ziegler (2016) test three hypotheses based on their theoretical model on the regression in equation 2.1:

- H1: ETF and futures trading activity affects the correlations of S&P 500 stocks.
- H2: ETF and futures trading activity affects the correlations of non-S&P 500 stocks.
- H3: ETF trading activity has a stronger impact on correlations than futures trading activity.

All three of the hypotheses are confirmed, and the conclusion is that index trading activity, a proxy for demand shocks, can explain a large part of the time variation in stock return correlations. [21]

Staer and Sottile (2018) contribute to the literature by focusing on daily and intradaily correlation caused by trading in ETFs. [33] Their work builds on arbitrage theory where arbitrageurs can buy (sell) the underlying ETF basket and sell (buy) the ETF shares at times when the value of underlying assets deviates sufficiently from the ETF share price. At market close both positions are closed through a mechanism called an "in-kind" transaction, in which authorized market participants exchange the underlying assets with the fund provider for the equivalent ETF shares. The arbitrageur thus gains the difference in values minus the transaction costs as profit.

Staer and Sottile (2018) create a stock-level indicator called "equivalent volume" (EV) as an alternative proxy of the amount of asset trading volumes that comes from ETF trading. [33] However, this approach constrains their model to investigate correlations between each individual asset and the total index. As the monthly regression model of this thesis investigates the effect on weighted average correlations, in the same fashion as Leippold, Su and Ziegler (2016), EV is not applicable and will therefore not be explained further.[21]

Staer and Sottile (2018) have two approaches to estimating daily co-movement. First, they employ dynamic conditional correlations (DCC) from the multivariate volatility model family on daily stock returns.[12] Their second approach uses five minute intradaily returns in order to calculate short-horizon correlations based on the Pearson correlation.[33] The correlation estimates are then used as the dependent variable in a regression, with equivalent volume as the explanatory variable as well as correlation lags and return control variables.

Their results show that the use of intradaily rather than daily data has more power in testing correlations between stocks, which they discuss could be due to the high-frequency nature of the arbitrage trading. They find a strong, positive relation between EV and the correlations of a stock's returns with those of the ETF's other constituents. Using the daily DCC model, they find that a 1% increase in EV is associated with a 0.02% increase in daily correlation. Due to the positive skew and high kurtosis of the equivalent volume distribution, a 4% increase in daily correlations is not uncommon. However, with intradaily samples and the Pearson correlation estimator, the magnitude of the relation between equivalent volume and correlations is 13 times stronger than for the daily DCC model. The study is robust to a variety of control variables and estimation procedures.

As the relation between equivalent volume and correlations was much stronger on intradaily data, Staer and Sottile (2018) speculates that studying intradaily ETF trading activity relative to the underlying stocks will contribute further insights into the pricing of the arbitrage-linked securities and the potential implications for market participants. This is what inspired this thesis' study of the relation between stock correlations and ETF trading activity using intradaily data.[33]

Da and Shive (2013) also investigate the relation between ETF activity and return correlations of the underlying stocks in the U.S. market.[10] They use different measurements of ETF activity than the previously discussed papers, and also conclude that that ETFs may reduce diversification. By also studying the S&P 400 mid cap index, they show that the correlation effect from ETF activity is stronger among small and illiquid stocks and during market turbulence. There has also been research into other markets than just the U.S. market. For example, Greenwood and Sosner (2007) research the effect of trading volumes in ETFs on excess correlations within the Nikkei 225 index in Japan, with findings similar to the previously discussed papers.[16] This thesis also expands the focus of the research by studying other indices than the S&P 500 in the U.S. market.

As seen, there is active research on the topic of increased asset return correlations due to increases in ETF investments. One main deficiency is the lack of passive, index tracking mutual funds as an explanatory variable for the increased asset return correlations. Mutual funds constitute a large share of total investments, and are therefore important to investigate further along with improvements to the ETF trading volume studies. While mutual funds can not affect correlations in the same way as ETFs because they are not traded in real-time and there are therefore no in-kind transactions, a hypothesis of this thesis is that mutual fund flows can increase correlations due to the fund provider buying and selling the entire underlying basket. This is explained in more detail in section 4.1.2.

2.3 Markov switching models for likelihood of bear market

As stated in section 2.2, asset return correlations are shown to increase when in a bear market or economic downturn. It is therefore important to model the likelihood of being in such a state, through the use of one or more explanatory variables.

The three papers discussed in section 2.2 use different macroeconomic variables to act as a proxy for economic downturns, since this has been observed to be one of the main drivers of increased asset correlations. This effect has for example been discovered by Preis et al. (2012).[28] However, the number of variables included to model downturns is often high, and might cause multicollinearity. For example Leippold, Su and Ziegler (2016) include the current month's S&P 500 return, the three-month Treasury bill, the credit spread and variables for industry production, inflation and economic policy uncertainty.[21] An alternative option to modelling the likelihood of being in a bear market that has never been used in this field of research is to use a Markov switching model. Introducing the Markov switching model would in this case reduce the number of exogenous variables used to model economic downturns from five to one, making the model more parsimonious and thus increasing statistical power.

The Markov switching model is introduced by Goldfeld and Quandt (1973), but was popularized by Hamilton (1989).[15][17] Hamilton (1989) extended the Markov-switching framework to an autoregressive process, and provided an iterative filter that produced both the model likelihood function and filtered regime probabilities. According to Hamilton (1989), the econometrician is presumed not to observe regime shifts directly but must instead draw probabilistic inference about whether and when they may have occurred based on the observed behavior of the series. The paper presents an algorithm for drawing such probabilistic inference in the form of a non-linear iterative filter. The filter also permits estimation of population parameters using maximum likelihood estimation. Markov switching models result in inferred probabilities of being in one regime or another, which in this case can be seen as being in downturn or not.

Hamilton (1989) applies the switching model to the inferred probability of being in an economic recession using growth national product (GNP) data.[17] In addition to Hamilton (1989), numerous academic papers have been written, mostly about applications of Markov switching models used to model recession, business cycles or the dynamics of other macroeconomic or financial time series. Several have applied the model to the probability of being in a bear market, by using stock market returns as the input data. Schaller and Norden (1997) for instance showed that this gives very strong results.[29] The literature usually focus on either the Hamilton (1989) fixed transition probability model or the time varying transition probabilities (TVTP) extension.[18]

A Markov switching model therefore offers an optional explanatory variable representing the likelihood of being in a bear market or a downturn in the business cycle. The inclusion of Markov switching probabilities helps the study of asset correlation effects in two ways. First, Markov switching probabilities as an exogenous variable for a bear market has been shown to have a large and significant effect on asset return correlations in this thesis. By explaining this effect with the use of one variable instead of many, the risk of multicollinearity is reduced and it is easier to interpret the regression results. Second, simplifying the regression model helps isolate the ETF trading and fund flow effects on asset return correlations, improving the investigation into these effects. Again, these effects are the most central to this thesis. A Markov switching model is used in the monthly regression model, which is fully specified in section 4.1.

2.4 Models for forecasting asset correlations

The papers reviewed in section 2.2 all used various volatility models in order to construct a dependent variable in a regression where index tracking fund volumes were used as an explanatory variable. A novel approach proposed by this thesis is to use ETF trading volumes directly in creating correlation forecasts. The benefit of this approach is that improved correlation forecasts can be utilized in other areas of finance, such as portfolio optimization. In contrast the monthly regression model can not be used in the same way because correlations are calculated exogenously in a prior stage. This section therefore reviews papers on various correlation models. Specifically, the DCC model is reviewed in order to act as a baseline, and the DCCX model is reviewed in order to allow ETF trading ratio to be used directly in forecasting as an explanatory variable.

2.4.1 The DCC model

Engle (2002) introduced the Dynamic Conditional Correlation (DCC) model as a way to estimate correlation matrices for multivariate models.[12] The DCC model has a clear computational advantage over multivariate GARCH models in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated. Engle's DCC model is formulated as the statistical specification in equations 2.2.

$$
\begin{aligned}
\mathbf{y}_t &= \boldsymbol{\mu}_t + \mathbf{r}_t \\
\mathbf{r}_t &\sim N(0, \mathbf{H}_t) \\
\mathbf{H}_t &= \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \\
\mathbf{D}_t^2 &= \text{diag}\{\boldsymbol{\omega}\} + \text{diag}\{\boldsymbol{\kappa}\} \circ \mathbf{r}_{t-1}\mathbf{r}_{t-1}' + \text{diag}\{\boldsymbol{\lambda}\} \circ \mathbf{D}_{t-1}^2 \\
\boldsymbol{\epsilon}_t &= \mathbf{D}_t^{-1}\mathbf{r}_t \\
\mathbf{Q}_t &= \overline{\mathbf{Q}} \circ (\boldsymbol{\iota}\boldsymbol{\iota}' - \mathbf{A} - \mathbf{B}) + \mathbf{A} \circ \boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \mathbf{B} \circ \mathbf{Q}_{t-1} \\
\mathbf{Q}_t^* &= \text{diag}\{\mathbf{Q}_t\} \\
\mathbf{R}_t &= \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}
\end{aligned}
\tag{2.2}
$$

Here $\mathbf{r}_t$ are stock returns minus the mean, $\mathbf{R}_t$ is a correlation matrix containing the conditional correlations, $\mathbf{H}_t$ is the covariance matrix, $\boldsymbol{\iota}$ is a vector of ones, $\overline{\mathbf{Q}} = \frac{1}{T}\sum_t \boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t'$, and $\mathbf{A}$, $\mathbf{B}$, $\boldsymbol{\omega}$, $\boldsymbol{\kappa}$, $\boldsymbol{\lambda}$ are parameters to be estimated. The $\circ$ symbol represents Hadamard-multiplication, which is entry-wise matrix multiplication. Setting the parameter matrices $\mathbf{A}$ and $\mathbf{B}$ to scalars α and β simplifies the model to the scalar DCC model. The fully specified model in its scalar form is given in equation 4.12. The diag$\{\}$ operator applied to a vector creates a matrix with its values on the diagonal and zeros elsewhere, and applied to a matrix it sets non-diagonal elements to zero.

Since all covariance matrices are by definition positive definite as portfolio variance has to be positive for any vector of portfolio weights $\mathbf{w}$, all correlation matrices $\mathbf{R}_t$ have to be positive definite, as well as having ones on the diagonal. It can be shown that since $\mathbf{R}_t$ is given by $\mathbf{Q}_t$ as in equation 2.2, all $\mathbf{Q}_t$ have to be positive definite. One can also show that when $\mathbf{Q}_t$ is positive definite and $\mathbf{R}_t$ is specified as in equation 2.2, all diagonal entries of the $\mathbf{R}_t$ matrix are unity.

The equation for $\mathbf{D}_t^2$ in equation 2.2 is simply the univariate GARCH process. A large advantage of the DCC model is that the model parameters can be estimated in two steps, making it significantly less computationally heavy. Specifically, one first calculates the conditional variances by estimating $\boldsymbol{\omega}$, $\boldsymbol{\kappa}$ and $\boldsymbol{\lambda}$ in the univariate GARCH model. Secondly, one estimates the parameters of $\mathbf{Q}_t$, $\mathbf{A}$ and $\mathbf{B}$, using normalized residuals calculated with the conditional variances from the univariate GARCH model. In both steps the estimation uses log likelihood maximization. The estimation procedure is explained in depth in section 2.5, along with the estimation procedure for the DCCX model.

Engle (2002) compares several correlation estimators with the proposed DCC model in simulations where the true correlation structure is known. It was found that the DCC models had lower mean absolute error in the correlation estimates than alternative models, which were the historical average of the last 100 days, the RiskMetrics model with parameter $\lambda = 0.94$, and the orthogonal GARCH model.[12]

The DCC model will be used in the intradaily correlation model, as explained further in 4.2.1, functioning as a benchmark model for the DCCX model that will include the ETF trading ratio as an explanatory variable. This is further explained in 2.4.2.

2.4.2 The DCCX model

In a paper discussing the effects of foreign exchange and equity returns on DCC, Vargas (2008) introduces a novel DCC model which incorporates exogenous variables that affect the conditional covariance.[37] This DCCX model is identical to the DCC model in equation 2.2, except for the expression for $\mathbf{Q}_t$ which is given in equation 2.3.

$$\begin{aligned} \mathbf{Q}_t = (&\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{K}\boldsymbol{\gamma}'\overline{\mathbf{x}}) \\ &+ \mathbf{A}'(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}')\mathbf{A} + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} + \mathbf{K}\boldsymbol{\gamma}'\mathbf{x}_{t-1} \end{aligned} \tag{2.3}$$

$\mathbf{A}$, $\mathbf{B}$ and $\boldsymbol{\gamma}$ are parameters to be estimated. $\boldsymbol{\gamma}$ is a $k \times 1$ vector where k is the number of included exogenous variables. In the case of only one exogenous variable, $\boldsymbol{\gamma}$, $\overline{\mathbf{x}}$ and $\mathbf{x}_t$ reduce to scalars. Again, setting parameter matrices $\mathbf{A}$ and $\mathbf{B}$ to scalars α ad β reduces the model to scalar DCCX. Furthermore, the average values are calculated as in equations 2.4.

$$\hat{\overline{\mathbf{Q}}} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t', \quad \hat{\overline{\mathbf{x}}} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t \tag{2.4}$$

$\mathbf{K}$ is either the identity matrix or a matrix of ones. Vargas (2008) shows that setting $\mathbf{K}$ as a matrix of ones, as opposed to the identity matrix, makes the model more dynamic as it allows the exogenous variable to affect non-diagonal elements of the $\mathbf{Q}_t$ matrix.[37] It is then specified that $\gamma_k \in [0, 1]$ and that the exogenous variable has to be strictly positive due to positive definiteness constraints. This is a fairly limiting constraint, as it does not allow for the exogenous variable $\mathbf{x}_{t-1}$ to have a negative impact on the conditional covariance $\mathbf{Q}_t$.

The DCCX model will be used with ETF trading as an explanatory variable for the intradaily correlation model as explained further in section 4.2.2. This is a new application of the DCCX model, and thus contributes to the literature on the model.

2.5 Estimation of model parameters

In this section the estimation of the parameters in the DCC model and the DCCX model will be presented as outlined in the literature. Schopen (2012) discusses in depth methods for estimating the DCCX in the model proposed by Vargas (2008).[30] [37] As previously mentioned, Engle (2002) showed that one of the main advantages of the DCC model is the ability to estimate the parameters in two steps, called a Quasi-Maximum Likelihood (QML) estimation procedure.[12] In this two step procedure the univariate GARCH parameters are first estimated, before the multivariate correlation parameters are estimated in a second step. This also holds true for the DCCX model. However, an important difference between the models is that the DCCX estimation must have a non-linear constraint. This will be discussed in more detail later in this section. The likelihood function for both models is shown in equation 2.5, where the univariate GARCH parameters and multivariate correlation parameters are called $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, respectively.

$$L(\boldsymbol{\phi}, \boldsymbol{\theta}|\mathbf{r}) = \prod_{t=1}^{T} \left[(\sqrt{2\pi})^{-N} |\mathbf{H}_t|^{-\frac{1}{2}} \exp\big(-\frac{1}{2} \mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{r}_t \big) \right] \tag{2.5}$$

Taking the log of equation 2.5 and using substitution yields the log-likelihood function in equation 2.6.

$$\begin{aligned} l(\boldsymbol{\phi}, \boldsymbol{\theta}|\mathbf{r}) &= -\frac{1}{2} \sum_{t=1}^{T} (N \log(2\pi) + \log|\mathbf{H}_t| + \mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{r}_t) \\ &= -\frac{1}{2} \sum_{t=1}^{T} (N \log(2\pi) + \log|\mathbf{D}_t \mathbf{R}_t \mathbf{D}_t| + \mathbf{r}_t' \mathbf{D}_t^{-1} \mathbf{R}_t^{-1} \mathbf{D}_t^{-1} \mathbf{r}_t) \\ &= -\frac{1}{2} \sum_{t=1}^{T} (N \log(2\pi) + 2\log|\mathbf{D}_t| + \log|\mathbf{R}_t| + \boldsymbol{\epsilon}_t' \mathbf{R}_t^{-1} \boldsymbol{\epsilon}_t) \end{aligned} \tag{2.6}$$

The log-likelihood function in equation 2.6 is then split into two components, in compliance with the QML estimation procedure. The split log-likelihood function is given in equation 2.7, where the volatility term and the correlation term are as stated in equations 2.8 and 2.9, respectively.

$$l(\boldsymbol{\phi}, \boldsymbol{\theta}) = l_{volatility}(\boldsymbol{\theta}) + l_{correlation}(\boldsymbol{\phi}, \boldsymbol{\theta}) \tag{2.7}$$

$$l_{volatility}(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}(N\log(2\pi) + 2\log|\mathbf{D}_t| + \mathbf{r}_t'\mathbf{D}_t^{-1}\mathbf{D}_t^{-1}\mathbf{r}_t) \tag{2.8}$$

$$\begin{aligned} l_{correlation}(\boldsymbol{\phi}, \boldsymbol{\theta}) &= -\frac{1}{2}\sum_{t=1}^{T}(\log|\mathbf{R}_t| + \boldsymbol{\epsilon}_t'\mathbf{R}_t^{-1}\boldsymbol{\epsilon}_t - \boldsymbol{\epsilon}_t'\boldsymbol{\epsilon}_t) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\left(\log|\mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}| + \boldsymbol{\epsilon}_t'(\mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1})^{-1}\boldsymbol{\epsilon}_t\right) \end{aligned} \tag{2.9}$$

Finally, the log-likelihood is maximized in two steps, shown in equation 2.10. The fact that the estimated parameters of the first step are assumed as given in the second step is why it is called quasi-maximum likelihood and not standard maximum likelihood. However, Engle (2002), who proposed this method, showed its consistency and asymptotic normality.[12]

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg\max\{l_{volatility}(\boldsymbol{\theta}|\mathbf{r})\} \\ \hat{\boldsymbol{\phi}} &= \arg\max\{l_{correlation}(\boldsymbol{\phi}|\hat{\boldsymbol{\theta}}, \mathbf{r})\} \end{aligned} \tag{2.10}$$

As noted earlier in this section the DCCX estimation is more complex than the DCC estimation due to a non-linear constraint. The DCC model can have the requirement that $\mathbf{Q}_t$ is positive definite satisfied by simply adding a linear inequality constraint to the likelihood maximization problem, specifically $\alpha + \beta < 1$ for the scalar DCC model. However, this is not possible for the more complex DCCX model. Since a matrix is positive definite if and only if its smallest eigenvalue is positive, Schopen (2012) discusses different approaches to the optimization problem with the non-linear constraint that the minimum eigenvalue of $\mathbf{Q}_t$ is positive.[30] Specifically, two approaches are proposed. One is sequential quadratic programming methods that solves the Karush-Kuhn-Tucker equations, and the second method is applying a penalty function for constraints that are near or beyond the boundary. In this thesis the former is used.

The model parameter estimation techniques described above will be used for both the intradaily DCC and DCCX models, as explained further in section 4.2.

2.6 Testing and valuing dynamic correlation for asset allocation

This thesis compares the performance of different dynamic correlation models given real world data. As one can not know the actual underlying correlation structure of asset returns, one must use various statistical tests when comparing different models. One way to compare two correlation estimates is a likelihood ratio test, which can be used to compare the statistical significance of different models. Another way of testing that is used by several studies and based upon the work of Diebold and Mariano (1995), is comparing the performance of Markowitz optimized asset portfolios calculated using the different covariance estimates.[11] This method has a clear economic basis. However, most such tests are joint tests of forecasted returns and covariances. As Chopra and Ziemba (1993) have shown, correctly estimated expected returns are ten times more important than correctly estimated variances, and correlations are even less important.[8] Engle and Colacito (2006) propose a test to compare the relative performance of alternative methods of dynamic covariance modeling, which isolates the effect of covariance information from expected returns.[13]

The test is based on the classical asset allocation problem of minimizing portfolio variance given a required rate of return for each period t, with the inclusion of a risk free asset and allowing short positions. Once weights have been constructed for the whole forecast period, a time series of the difference in squared returns is created, shown in equation 2.11. Hereafter, squared returns is defined as returns minus the mean return, squared.

$$u_t = ((\mathbf{w}_t^1)'(\mathbf{r}_t - \overline{\mathbf{r}}))^2 - ((\mathbf{w}_t^2)'(\mathbf{r}_t - \overline{\mathbf{r}}))^2 \tag{2.11}$$

Here, $\mathbf{w}_t^j$ is the vector of portfolio weights for covariance model j at time t. The standard Diebold-Mariano test would test the null hypothesis that the mean of $\boldsymbol{u}$ is zero.[11] This would be done by regressing u_t on a constant using a Newey-West covariance matrix in order to account for heteroscedasticity, autocorrelation and non-normality. In order to make the test more powerful, Engle and Colacito (2006) further adjust for heteroscedasticity by creating a second test time series v_t by dividing u_t by an estimate of its standard deviation.[13] The estimate used is the geometric average of the two standard deviation estimates resulting from each covariance matrix estimate $\mathbf{H}_t^1$ and $\mathbf{H}_t^2$, such that v_t is given by equation 2.12.

$$v_t = u_t[2(\boldsymbol{\mu}'(\mathbf{H}_t^1)^{-1}\boldsymbol{\mu})(\boldsymbol{\mu}'(\mathbf{H}_t^2)^{-1}\boldsymbol{\mu})]^{1/2} \tag{2.12}$$

Both u_t and v_t are regressed on a constant using generalized method of moments with heteroscedasticity and autocorrelation consistent (HAC) covariance matrix in order to get two normally distributed test statistics. Under the null hypothesis, the mean of both $\boldsymbol{u}$ and $\boldsymbol{v}$ are zero. As $\boldsymbol{v}$ is the improved test statistic as it is more robust to heteroscedasticity and autocorrelation, it will be used in the t-tests where the intradaily correlation models are compared in section 4.2.4.

2.7 Contributions to the literature

As observed in previous literature, this thesis further strengthens the hypothesis that trading volumes in ETFs increase correlations between the underlying stocks held by the ETFs. This has been shown in numerous studies over the lifetime of ETFs, and especially after gaining a larger share of trading volumes in the U.S. However, no previous paper has studied correlation effects from fund flows into or out of passive index mutual funds. A reason for this could be that market data for mutual fund flows is not easily retrieved. The data used in this thesis for example was provided by Eikon, which is proprietary and had several faults which required manual adjustments, as explained in section 3.1.2. However, the addition of fund flow data provides an important contribution to correlation analysis due to mutual funds' large share of asset values. Fund flows for index-tracking mutual funds are distinguished from index-tracking ETFs because of the nature of investing in the two instruments. ETFs can easily be traded intradaily between customers with little or no delay on purchases. With mutual funds, investors inject or withdraw capital directly to or from the fund provider, which then purchases or sells shares of the underlying assets. This process usually takes one or several days.

Previous studies have modelled downturns by numerous macroeconomic variables, such as GDP growth and inflation. Instead, this thesis utilizes a bear market probability variable by modelling business cycle downturns using a Markov switching model, as explained in section 2.3. Instead of including several explanatory variables trying to explain downturns together, a single downturn variable decreases risk multicollinearity and increases statistical power as it makes the model more parsimonious. This is an important result in itself, and the more parsimonious model also assists in the research into the effects of fund flows and trading, which is the main focus of this thesis.

The intradaily correlation model of this thesis shows similar results regarding the intradaily effects of ETF trading on stock correlations to those of Staer and Sottile (2018).[33] However, a different model is used and it is applied to a different data set. Staer and Sottile (2018) use a correlation estimate as the dependent variable in a regression where an ETF trading parameter is an explanatory variable, while the model in this thesis directly utilizes an ETF trading parameter in the estimation of correlations. This is the most important contribution of the intradaily correlation model of this thesis to the literature. As the lagged variable was shown to be statistically significant in this thesis' model, it is possible to use the results to improve correlation forecasting models. This thesis further adds to the literature of comparing correlation forecasts, by applying the model presented by Engle and Colacito (2006).[13] It was shown that for the dataset used in this paper, it is not possible to significantly lower the realized volatility of a mean-variance optimized portfolio by including ETF trading ratio in the correlation forecasting model.

Lastly, this thesis contributes to the literature on the DCCX model created by Vargas (2008).[37] In spite of the advantages of DCCX which include more flexibility due to the ability to take into account the effect of various exogenous variables, the use of the model in financial literature has been fairly modest. This thesis is the first to employ the DCCX model to test the effect of ETF trading ratio on asset correlations, presenting a new area of application for the model. In fact, this area is an excellent application for the model due to it adhering to the constraints of the model, namely that the exogenous variable and its effect on correlations must be positive.

Chapter 3

Data

In this thesis data was used from two sources. Publicly available data was sourced from Yahoo! Finance, while proprietary data was sourced from the Thomson Reuters Eikon Database. Specifically, Thomson Reuters was used to download fund flows from the largest American index tracking mutual funds, as well as high frequency intradaily price and volume data for stocks and ETFs.

The U.S. equities market was chosen because it has one of the worlds largest index tracking fund market shares. The majority of previous studies on the subject have researched the American market, and especially the S&P 500 index.

3.1 Data for the monthly regression model

3.1.1 Choice of period and frequency

Monthly data starting in January 2005 and ending September 2018 is included in the model. 2005 was chosen as a starting point because this is around the time when index tracking ETFs started to gain a significant market share in the U.S. Passive index tracking mutual funds already had a significant share at this point, so this time frame is appropriate for this variable as well. This gives a total of 165 data points for each variable in the regression model, 12 months a year for 13 years and 9 months in 2018.

A monthly frequency was chosen because fund flow data is only available with a monthly frequency for the majority of the index tracking mutual funds in the Eikon database. ETF data was available at higher frequencies, but the fund flow data constrained the model to be implemented with monthly data. As stated in section 2.2, Leippold, Su and Ziegler (2016) also base their model on monthly data, although they use ETF and futures returns which both have daily data available.[21] However, their choice of the monthly frequency might be constrained by some of their macroeconomic variables, as for example industry production. They also disregard variables such as GDP due to the data only being available at a quarterly frequency.

3.1.2 Fund flow data

In order to get data on fund flows to American index tracking mutual funds, data from 50 different funds was aggregated. The index tracking mutual funds included are the 25 largest American mutual funds as well as all index tracking mutual funds owned by the 10 largest mutual fund companies, measured by Retail Net Assets (RNA). The index tracking mutual funds included track either the large cap, mid cap or small cap stock categories, or combinations of them.

As of September 2018, the total net asset values (NAV) of the funds included add up to 1.6 trillion USD. The total NAV of passive U.S. index tracking mutual funds is approximately 3.3 trillion USD as of January 2018. Hence, the funds included cover approximately 47% of all American index tracking mutual funds by total NAV.

For October 2007, the Eikon data for the fund Vanguard 500 Index Fund Admiral Shares (VFIAX), which is currently the largest index tracking mutual fund by market capitalization, had a negative fund flow equivalent to 30% of its total NAV. This fund is in a family of three funds, VFIAX, VFINX and VIFSX. The latter, Vanguard 500 Index Fund Signal Shares, was founded in late 2006. VIFSX was closed in 2014, and therefore Eikon does not have fund flow data available for this fund. SEC filings for 2007 show that while VFIAX and VFINX had large negative fund flows that year, VIFSX had positive fund flows of equal size. It was therefore concluded that the large negative spike in October 2007 likely was a transfer of shares from the old funds to the new one, for which data is not available, and not an actual flow out of Vanguard, and therefore the fund flows for those two funds were set to zero for that month.

Another instance where an adjustment had to be made was for VSMPX, which is in the same family of funds as VITSX. The data for the fund VSMPX for April 2015 was missing, while VITSX had a large negative fund flow for this month. The difference in NAV between February and March 2015 was used as a proxy for the fund flow for VSMPX in this month. This approximation was made on the basis that the fund VITSX had a part of it split into the newly founded VSMPX in April 2015. With this added data point, fund flows related to movements between the two funds cancel each other out for this month in the aggregated time series.

The total fund flows used in the model are shown in figure 3.1.

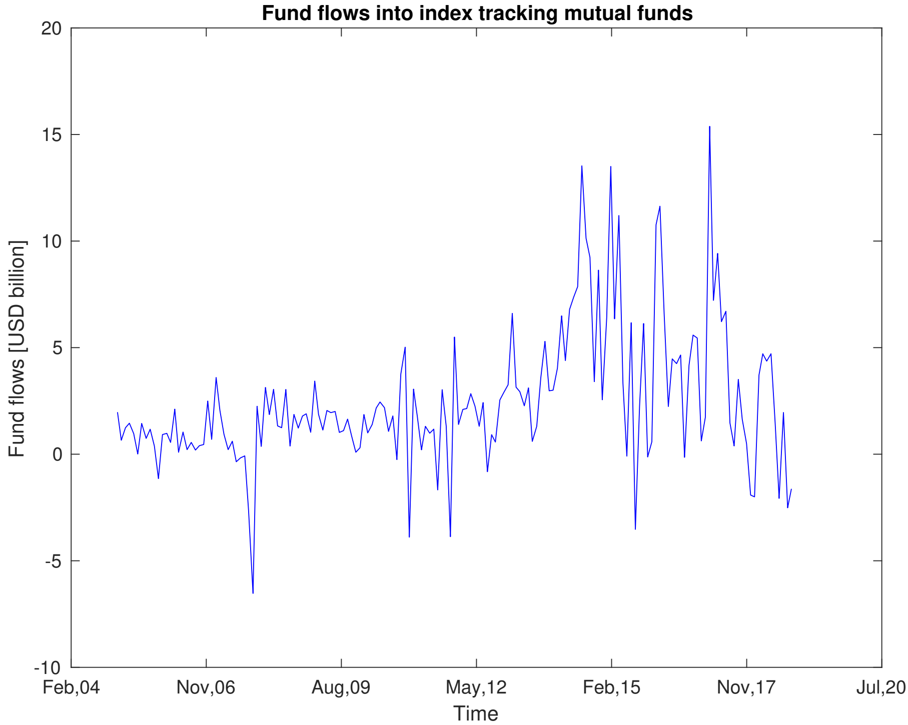

Figure 3.1: Fund flows into the included U.S. index tracking mutual funds over time. The volatility of monthly fund flows into index tracking mutual funds is significantly higher in the period 2013-2018 than earlier.

3.1.3 Trading volumes

Trading volume data for equities and ETFs was required in order to calculate the ETF trading ratio. This data was provided by Yahoo! Finance with daily frequencies. As Yahoo! only provides trading volumes in number of shares, this thesis approximated the average daily trading price as the average of daily high and low prices, in order to estimate the total daily trading volume in USD. Daily high and low prices were also downloaded from Yahoo!. This was then aggregated to get monthly volumes in USD. This procedure was done for all included ETFs as well as all stocks within the S&P 500 large cap index, the S&P 400 mid cap index, and the S&P 600 small cap index. The ETF and S&P 500 trading volumes are shown in figure 3.2.

The ETFs included were all American index tracking ETFs with more than 5 billion USD in NAV as of October 24^{th} 2018.[9] This gave a total of 66 ETFs, which constitutes the majority of American index tracking ETFs by NAV.

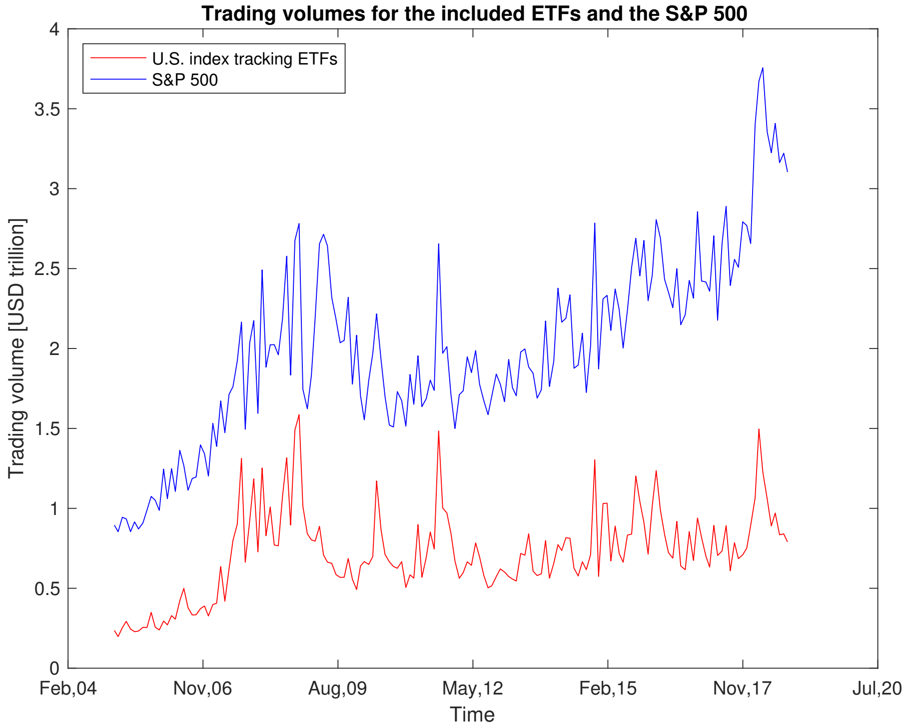

Figure 3.2: Monthly trading volumes for the included U.S. index tracking ETFs and for the S&P 500 over time. The trading volume of S&P 500 stocks has grown more than the trading volume of index tracking ETFs during the last 10 years.

3.1.4 Fund classification

An alternative specification of the model required splitting funds into groups based on classifications. Each ETF and index tracking mutual fund was classified as either a large, mid or small cap fund, or a combination of the three. The classification was done based on the holdings of each fund. To split the fund flows and trading volumes for funds that were put in multiple size categories, a flat distribution was assumed. For example the trading volumes from the ETF iShares S&P 1500 Index Fund (ITOT), which holds assets ranging from large to small cap, was evenly distributed between the three categories.

3.1.5 Closing prices

Daily closing prices for the S&P 500, S&P 400 and S&P 600 indices were downloaded from Yahoo!. This data was adjusted for both dividends and splits. Daily closing prices were used instead of monthly as for all the other time series because the monthly correlation of returns is the variable of interest, and realized covariance using daily log returns was used to calculate this. The S&P 500 index was chosen to act as a proxy for the large cap stock market in the U.S. Similarly the S&P 400 acted as a proxy for the mid cap market and the S&P 600 for the small cap market. These indices are created by Standard & Poor's in order to be representative for their respective segments, which is why they are chosen.

3.1.6 Other data for the monthly regression model

In order to calculate weighted average correlation, market capitalization data was downloaded from Eikon with monthly frequency from each of the companies listed in the S&P 500, S&P 400 and S&P 600 indices as of October 10th 2018. The data was downloaded such that the date corresponded to the Eikon fund flow data, or the first available date before the corresponding date from the Eikon dataset.

Closing prices for the CBOE Volatility Index (VIX) were downloaded from Yahoo! Finance. As with market capitalization, each month uses the VIX closing price corresponding to the month end date from the Eikon fund flow data, or the first available date before the corresponding date from the Eikon dataset. Quotes for the GSPC index were downloaded in the same manner, for use in the Markov switching model.

3.2 Data for the intradaily DCCX model

The intradaily DCCX model required three types of data: intradaily asset returns for S&P 500 assets, intradaily trading volumes for S&P 500 assets and intradaily trading volumes for ETFs. The data was downloaded from the Thomson Reuters Eikon database for the three months that make up Q1 2019, in other words January, February and March 2019.

3.2.1 Choice of frequency and period

As discussed in chapter 4, the intradaily model works best with the highest frequency data available. The Thomson Reuters Eikon database had minute data available. A problem encountered with minute interval data was that some assets were often not traded at all during a minute, leaving the closing price unaltered. This corresponds to zero log return for multiple assets, therefore using this interval would lead to artificially high correlations. Therefore the data, for both closing prices and volumes, were aggregated to a five minute interval, removing this problem almost entirely (there were extremely few instances of S&P 500 assets not being traded at all during a five minute interval).

The Eikon database only had three months of data at a frequency of one minute, which when aggregated to five minute data translated into 4,697 data points after removing overnight periods. The 90% (4,227) first of these were used in model parameter estimation, and the remaining 10% (470) were used for out of sample performance testing.

3.2.2 Adjustments made to the intradaily dataset

As in the monthly correlation model, an average of the period high and low price was multiplied with the volume in order to estimate the volume in dollars for the relevant period. Since the period was much shorter this is a better approximation than in the previous model. Data was downloaded for all 505 S&P 500 tickers except two, as well as the same 66 ETFs used in the monthly regression model. The two tickers for Twenty-First Century Fox were removed due to a demerger which was completed on the 19th of March. On this date 74% of the company was sold to Disney. The remaining 26% stayed public, however under new tickers. Since the change was fairly radical, a decision was made to remove the company entirely from the dataset.

Available minute data was aggregated to get five-minute data. In the case of asset closing prices the closing price of the last minute where an asset was traded was used as the closing price for the five minute interval. In the case of volumes, the volumes of all minutes within the five minute range where the asset was traded was aggregated for a total period volume.

Finally, overnight returns were removed from the dataset as is standard in the literature for intradaily datasets, as for example stated in Andersen and Bollerslev (1997).[2] Unlike the monthly data it is not necessary to adjust for dividends and splits, because dividend payments and splits are done outside of trading hours and thus only affect overnight returns.

Chapter 4

Methodology

This chapter specifies all models used in this thesis, which can be split into two main categories. Firstly, the monthly correlation regression models are defined in section 4.1. Secondly, the intradaily correlation forecasting models are defined in section 4.2.

There are two reasons in particular that a forecasting model is chosen at intradaily frequency while a regression model is chosen at monthly frequency. First, theory states that there should be a lagged effect of fund trading on correlation at short time intervals, which makes forecasting feasible. However, this effect is expected to occur at far higher frequencies than monthly. Secondly, a regression model is better at treating multiple exogenous variables as there is a wider literature on statistical inferences for this model type, which was required for the monthly model.

4.1 Monthly correlation model methodology

To model the effect of passive investments on stock return correlations on a monthly basis, a linear regression model with autoregressive lags was used. This was separately done for correlations of large cap index stocks, mid cap index stocks and small cap index stocks. An alternative version of the linear regression model was also run for each index, hereafter called the split model, where the ETF trading ratios and mutual fund flows are split based on which indices they track. This approach allows for direct relations between fund activity and correlations of the underlying index as well as cross-relations such as the effect of large cap ETF trading ratio on mid cap stock correlations. Leippold, Su and Ziegler (2016) hypothesized that there should be a spillover effect from ETF trading onto correlations outside of the underlying assets, and the split model approach allows this thesis to investigate these effects further.[21]

In this chapter every part of the model will be discussed, including dependent variables, explanatory variables, autoregressive lags, as well as the full model specification. Additionally, several statistical tests applied to make inferences about the results are briefly explained.

4.1.1 Dependent variables

The model was run with three different dependent variables, which are the weighted average correlation of all stocks listed in the S&P 500 index, the S&P 400 index and the S&P 600 index. These three indices are benchmarks for the U.S. large cap, mid cap and small cap markets, respectively. The correlations $\rho_{ij,t}$ were approximated using realized covariance on all daily log returns within the month, a time series of approximately 20 values with no overlap between months. This is shown in equation 4.1, where t is the month and m is the number of days in that month. Each correlation was then weighted using the ratios of the market capitalization of the two stocks to the total market capitalization of the relevant index, and finally weighted correlations for all assets were added together to be used as a proxy for the total level of correlation in the index. This is shown in equation 4.2.

$$
\begin{aligned}
\sigma_{ij,t} &= \sum_{\tau=1}^{m} r_{i,t+\tau/m} r_{j,t+\tau/m} \quad \forall t \in T \\
\rho_{ij,t} &= \frac{\sigma_{ij,t}}{\sqrt{\sigma_{ii,t}\sigma_{jj,t}}}
\end{aligned}
\tag{4.1}
$$

$$
CORR_t = \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ijt} w_{it} w_{jt} \tag{4.2}
$$

Here w_{it} is the market capitalization of asset i divided by the total market capitalization of all index constituents in month t, and N is the number of stocks in the relevant index. $CORR_t$ is the weighted average correlation at time t. Leippold, Su and Ziegler (2016) use the same non-overlapping monthly value weighted correlation measure based on daily stock returns.[21] The resulting time series for each index is plotted in figure 4.1.

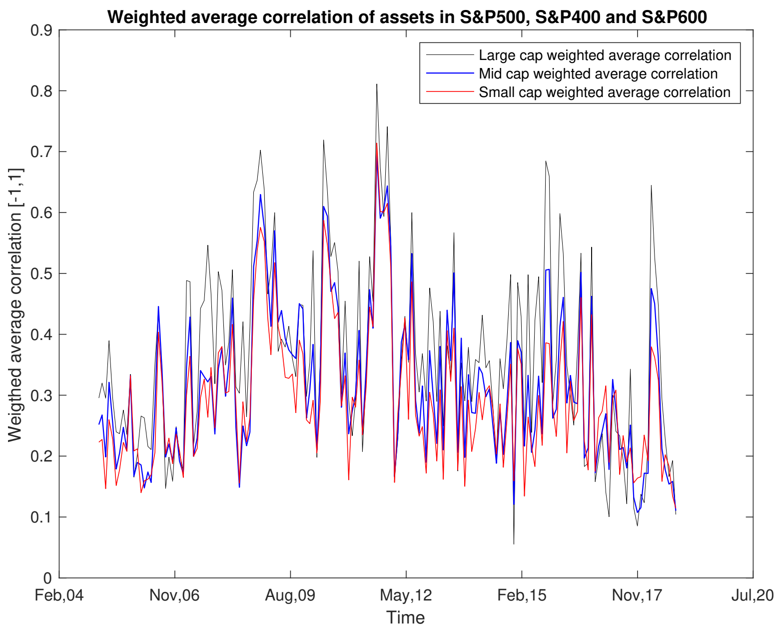

Figure 4.1: Weighted average correlation for each market, named $CORR_t$ in monthly regression model specification. Notice high degree of comovement between the different indices.

4.1.2 Explanatory variables

The basic aggregated model includes four exogenous explanatory variables. The first variable in the aggregated model is the ratio of trading volume of index tracking ETFs to the total trading volume of included assets, denoted $ETF_{ratio,t}$, given in equation 4.3. In the split model, both the numerator and the denominator is split into fund size category. This variable is denoted $ETF^{size}_{ratio,t}$ and is given in equation 4.4. For example, the mid cap ETF ratio variable $ETF^{mid}_{ratio,t}$ divides mid cap ETF trading volume by the total trading volume of the S&P 400. As explained in section 2.2 the ETF trading ratio variable is included to include the effects of arbitrage trading of ETFs through in-kind transactions on asset correlations.

$$ETF_{ratio,t} = \frac{\text{Total ETF USD trading volume}}{\text{Total USD trading volume of S\&P 500, S\&P 400 and S\&P 600}} \tag{4.3}$$

$$ETF^{size}_{ratio,t} = \frac{\text{ETF USD trading volume of category}}{\text{Total USD trading volume of corresponding index}} \tag{4.4}$$

The ETF ratio given in equation 4.3 is also used by Leippold, Su and Ziegler (2016), although they only used the trading volume of the S&P 500.[21] They explain that by dividing by the stock trading volumes, this ratio isolates demand shocks that occur exclusively to the ETFs. First differences of the trading ratio were used as an Augmented Dickey-Fuller (ADF) test revealed one unit root, implying non-stationarity. A second ADF test rejected the null hypothesis of further unit roots. Furthermore, ARMA residuals of the first differences (ARIMA) were used, in order to avoid spurious regression. Using the SBIC methodology for choosing ARIMA lags, which is explained in detail in section 4.1.3, yielded an ARIMA model order of $p = 0$ and $q = 1$, in other words a first order moving average process. Leippold, Su and Ziegler (2016) use the same ARIMA model order for the trading ratios, also based on the SBIC method.[21] Furthermore, all of the split ETF trading ratios were non-stationary just like the aggregated ratio, so ARIMA residuals were used in all cases.

The second explanatory variable is the total flow of funds into U.S. index tracking mutual funds, denoted FF_t. A hypothesis of this thesis is that fund flows in and out of index tracking mutual funds also increase asset correlations. Unlike ETFs, mutual funds are not traded in real time and therefore there is no arbitrage trading due to in-kind transactions to drive this relationship. However, when there are net positive or negative fund flows the mutual fund provider will within a certain amount of time buy or sell the entire underlying basket of assets. This should increase correlations as all the asset prices increase or decrease simultaneously. This time series was converted into absolute values, as one would expect both a positive and a negative total fund flow to increase correlations as they would trigger a collective acquisition or liquidation of assets, respectively. It was also divided by its maximum value in order to be of the same order of magnitude as the other explanatory variables.

For the split model, the fund flows were split into three variables based on which index the mutual funds track. Each index tracking mutual fund was classified as either a large, mid or small cap fund, or a combination of the three. The classification was done based on the holdings of each fund. The fund flows were then split between the categories accordingly, as described in section 3.1.4.

As discussed in the literature review, previous studies indicate that correlations increase in volatile and bear markets. Therefore two more exogenous explanatory variables were included in order to have these effects included in the model. As a proxy for market volatility the CBOE Volatility Index, denoted VIX_t, was included. An ADF test revealed that the VIX had a single unit root over the time period used, and therefore the first difference was used. Leippold, Su and Ziegler (2016) also include the VIX in order to model the expectation of volatility in the near future implied by S&P 500 index options.[21]

Leippold, Su and Ziegler (2016) model economic downturns with several macroeconomic variables, including the three-month Treasury bill rate, the credit default spread, industrial growth, inflation and economic policy uncertainty, as mentioned in section 2.2.[21] However, instead of modelling downturns using these macroeconomic variables, a Markov switching model was used to generate one variable for the inferred probability of being in a bear market, denoted $BEAR_t$. The variable is estimated from a Markov switching model based on two regimes, which can be represented as in equation 4.5, where the different regimes have different mean returns and volatilities.

$$\begin{aligned} r_t^{GSPC} &= \mu_1 + \epsilon_t \qquad \text{where} \quad \epsilon_t \sim N(0, \sigma_1^2) \\ r_t^{GSPC} &= \mu_2 + \epsilon_t \qquad \text{where} \quad \epsilon_t \sim N(0, \sigma_2^2) \end{aligned} \tag{4.5}$$

r_t^{GSPC} are the log returns of GSPC (the S&P 500 index) at time t, while μ_1 and μ_2 are the expected returns in the different regimes, or more specifically in a bull and a bear market state. σ_1^2 and σ_2^2 represent the volatilities in the different regimes. Introducing a state variable $S_t \in \{1, 2\}$, the two equations 4.5 can simply be written as in equation 4.6.

$$r_t^{GSPC} = \mu_{S_t} + \sigma_{S_t} u_t \qquad \text{where} \quad u_t \sim N(0, 1) \tag{4.6}$$

If the state variable S_t was observable, one could have treated the given model as a regression with dummy variables for times in the bear market state. However, since it is not observable in Hamilton's Markov switching model, the transition of states is stochastic. The dynamics of the switching process is driven by a transition probabilities of switching from on regime to another or staying in the current regime. An assumption made is that the Markov property is valid, namely that there is only serial dependence between adjacent states. The transition probabilities, which are assumed constant, are given by equations 4.7.

$$\begin{aligned} p_{11} &= Pr[S_t = 1 | S_{t-1} = 1] \qquad p_{12} = Pr[S_t = 2 | S_{t-1} = 1] = 1 - p_{11} \\ p_{22} &= Pr[S_t = 2 | S_{t-1} = 2] \qquad p_{21} = Pr[S_t = 1 | S_{t-1} = 2] = 1 - p_{22} \end{aligned} \tag{4.7}$$

Hence the unknown parameters of the model are $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p_{11}$ and p_{22}, which are estimated using maximum likelihood estimation. One step of the estimation procedure is calculating the inferred probability of the state variable being 2 (the bear market state), $P(S_t = 2 | \psi_t) \quad \forall t$, where ψ_t is all available information up to time t. This inferred probability is the variable of interest for the regression

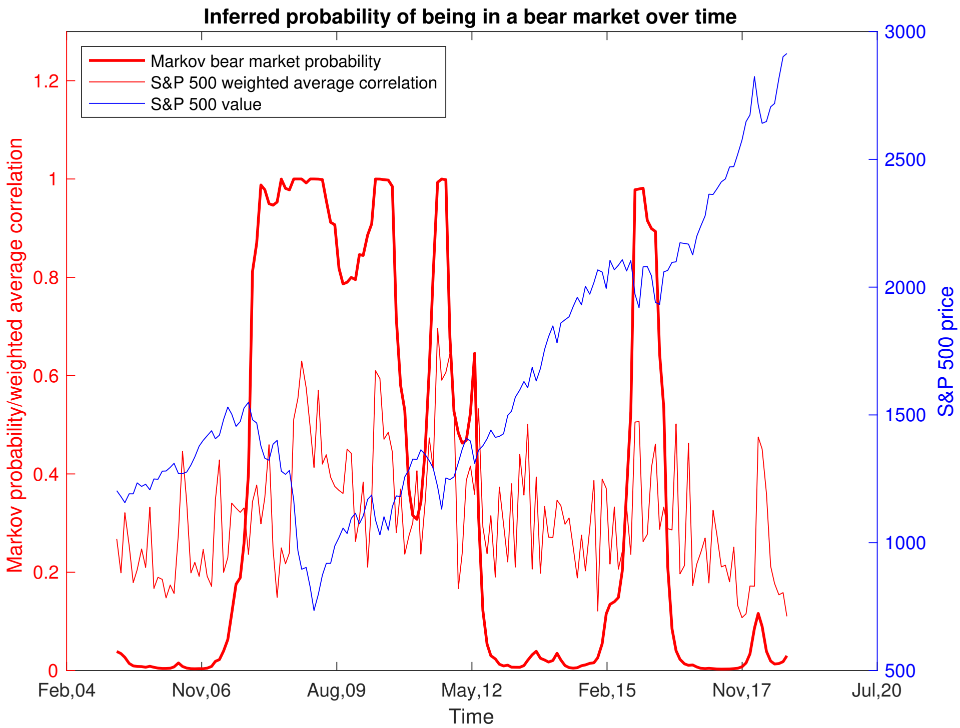

Figure 4.2: Inferred probability of being in a bear market, named $BEAR_t$ in monthly regression model specification. Plotted together with the S&P 500 price that it is based on, as well as the S&P 500 weighted average correlation. Notice high probability during Great Recession.

model used in this thesis, and is plotted in figure 4.2 together with the S&P 500 weighted average correlation and the S&P 500 value. An in-depth explanation of the maximum likelihood estimation procedure for the Markov switching model is given in appendix C.

4.1.3 Autoregressive lags

Autoregressive lags were used because the Breusch-Godfrey test rejected the null hypothesis of no autocorrelation for the basic monthly linear regression model with no autoregressive or moving average lags. Schwarz's Bayesian information criterion (SBIC) is a method of determining the number of ARMA lags to use in a model. It is performed by running the full ARMAX model, i.e. ARMA with exogenous variables, with different combinations of AR and MA lags (p and q, respectively). Finally one chooses the combination that minimizes the SBIC function given in equation 4.8, where $\hat{\sigma}^2$ is the residual variance, $k = p + q + 1$

and T is the number of observations.

$$SBIC = ln(\hat{\sigma}^2) + \frac{k}{T} ln(T) \tag{4.8}$$

For this model the result of the SBIC test was a model including three autoregressive lags and no moving average lags. AR and MA lags p and q in the range zero to ten were used for the different combinations in the SBIC procedure, giving $11^2 = 121$ different combinations. Multiple model versions were used with different dependent and explanatory variables as previously discussed. However, the SBIC optimization method was performed on the model with S&P 500 weighted average correlation as the dependent variable and without splitting fund flows or ETF trading volumes by fund category. This was in order to make the various monthly regressions more easily comparable. The SBIC criterion yielded the same results as Leippold, Su and Ziegler (2016), who also include three lags of the monthly dependent variable in the regression due to the autocorrelation structure.

4.1.4 Complete model specification

The complete specification of the monthly linear regression model without splitting fund flows and ETF trading volumes is shown in equation 4.9, and the model with split fund variables is shown in equation 4.10.

$$\begin{aligned} CORR_t = \alpha &+ \beta_{total}^{\Delta ETF} \Delta ETF_{ratio,t} + \beta_{total}^{ff} FF_t + \beta_{\Delta VIX} \Delta VIX_t \\ &+ \beta_{BEAR} BEAR_t + \sum_{i=1}^{3} \beta_{ARi} CORR_{t-i} + \epsilon_t \end{aligned} \tag{4.9}$$

$$\begin{aligned} CORR_t = \alpha &+ \beta_{large}^{\Delta ETF} \Delta ETF_{ratio,t}^{large} && (4.10) \\ &+ \beta_{mid}^{\Delta ETF} \Delta ETF_{ratio,t}^{mid} + \beta_{small}^{\Delta ETF} \Delta ETF_{ratio,t}^{small} \\ &+ \beta_{large}^{ff} FF_t^{large} + \beta_{mid}^{ff} FF_t^{mid} + \beta_{small}^{ff} FF_t^{small} && (4.11) \\ &+ \beta_{\Delta VIX} \Delta VIX_t + \beta_{BEAR} BEAR_t \\ &+ \sum_{i=1}^{3} \beta_{ARi} CORR_{t-i} + \epsilon_t \end{aligned}$$

Here $CORR_t$ is the weighted average correlation within each index (S&P 500, S&P 400 and S&P 600). FF_t is the fund flow, and in the split model the relevant index is in superscript. $\Delta ETF_{ratio,t}$ is the first difference of the ETF ratio, and in the split model the relevant index is in subscript. Furthermore ΔVIX_t is the first difference of the VIX index, $BEAR_t$ is the inferred probability of being in a bear market calculated using Markov transition probability, and $CORR_{t-i}$ is the i$^{\text{th}}$ lagged value of weighted average correlation. $\epsilon_t \sim N(0, \sigma^2)$ is the error term, which is assumed to be normally distributed with constant variance.

4.1.5 Statistical tests and inferences

Several statistical tests are run on the regression results to test parameter significance and if the multiple linear regression model assumptions are upheld.

A t-test is performed for each parameter coefficient to test if the coefficients are significantly different than zero. The null hypothesis is that the coefficient is zero. Similarly, an F-test is performed to test if the coefficients are jointly different from zero. The null hypothesis is that all coefficients are simultaneously equal to zero.

A Jarque-Bera (JB) test is performed to test the assumption that the error terms are normally distributed, which they are under the null hypothesis.

The assumption that the error terms have constant variance (homoscedasticity) is tested using White's test. The null hypothesis is homoscedasticity. White's test revealed the presence of heteroscedasticity in two of the six regressions, specifically the split and non-split small-cap index model. While the OLS estimators are still unbiased and consistent in the presence of heteroscedasticity, they no longer have the minimum variance. Therefore White's heteroscedasticity-consistent standard errors of White (1980) were used in the coefficient significance t-tests for these two regressions.

Furthermore, as previously mentioned, a Breusch-Godfrey (BG) test is performed to test for the presence of autocorrelation in the error terms. The error terms have no autocorrelation under the null hypothesis. Because of the presence of autocorrelation, autoregressive lags were included.

Model fit is compared using the $R^2_{adjusted} \in [0, 1]$ metric, where a higher value indicates better model fit. The adjusted R^2 was utilized in order to penalize for the inclusion of explanatory variables that have little impact on the model.

4.2 Intradaily correlation model methodology

A scalar DCCX model was used in order to model the intradaily correlation effects caused by index-tracking ETF trading volumes. Specifically, the five minute ETF trading ratio was included as an exogenous variable with one lag. The intradaily DCCX model was compared to the simpler intradaily DCC model which only uses lagged returns and correlation estimates. The parameters of the DCC and DCCX model were estimated using the first 90% of data points, hereafter called the in-sample-period. The models are compared with a likelihood-ratio (LR) test to test the significance of the exogenous variable coefficient.

Covariance forecasts for the DCC and DCCX models are generated for the final 10% out-of-sample period using the estimated parameters as well as $t-1$ returns and exogenous variable values, simulating the decision making situation of a daytrader. The covariance forecasts are then used to create Markowitz optimized portfolios that minimize volatility given the same required rate of return for the out-of-sample period and including the risk-free asset. Only the top 10 stocks in the S&P 500 by market cap are used due to computational constraints. In addition, for comparison, covariance forecasts based on the RiskMetrics model with a decay factor of 0.94 were included.

The market cap weighted portfolio with a risk-free asset is also included for comparison reasons. The risk-free asset is included such that it has the same expected rate of return as the other portfolios. This portfolio simulates a passive index tracking strategy for the top 10 S&P 500 assets. The total number of portfolios calculated is therefore four (DCC, DCCX, RiskMetrics and market cap weighted).

The realized returns and volatilities of the four portfolios are compared in the out-of-sample period. Additionally, the covariance forecasts for the DCC, DCCX and RiskMetrics models are compared more rigorously using the test proposed by Engle and Colacito (2006), as seen in section 2.6.[13] Note that while only the top 10 S&P 500 assets are used in the portfolio calculations, the trading ratio is calculated using the trading volumes for all stocks in the index as the ETFs trade the wider basket of assets.

The correlation forecasts are made with the hypothesis that differences between the ETF price and the underlying index price regularly occur. Staer and Sottile (2018) argue that the time between ETF demand or supply shocks and arbitrage induced trading likely is short, so the analysis should be done on high frequency data.[33] However, as explained in section 3.2.2, data availability constrains the frequency to five minute intervals. Within short time periods, ETF trading can cause a price increase or decrease in the ETF price compared to the price of the underlying index. Then, if the price difference between the ETF and the underlying index becomes larger than the transaction cost, arbitrage trading will begin, and the spread will decrease, i.e. the prices will start to converge. Specifically, the arbitrage trading is done by buying the undervalued asset and shorting the overvalued asset, and closing both positions using and in-kind transaction. When the underlying index moves, the constituents will move together, thus increasing correlations. An example of this process is shown in figure 4.3. As the figure illustrates, there should

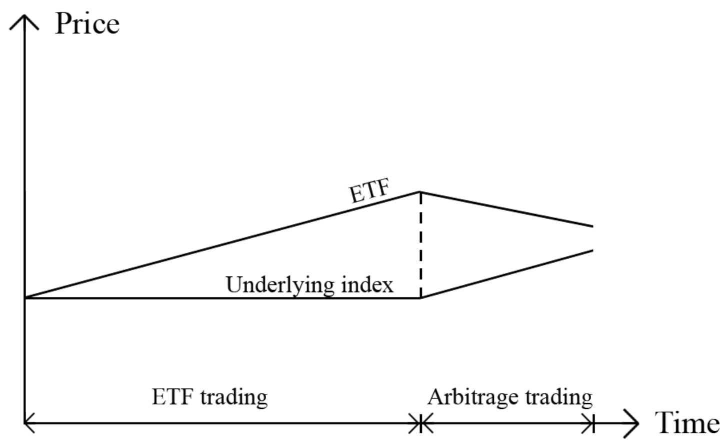

Figure 4.3: A theoretical example of ETF trading leading to increased correlations due to arbitrage trading. ETF trading creates a spread between the ETF price and the price of the underlying index. After some time the price difference is greater than the transaction cost. Then the arbitrage trading starts, and the ETF price and the underlying index price will start to converge.

be a lag between the ETF demand shock and the resulting increase in correlations, which could allow better forecasting of volatility with the inclusion of the ETF trading ratio as a lagged explanatory variable. Note that the high frequency nature of this effect makes the DCCX model infeasible for the monthly model, which is one reason why a regression model was used instead.

4.2.1 Intradaily DCC model

The DCC model introduced by Engle (2002) is based exclusively on the time series properties.[12] The conditional correlations in the DCC model follow a GARCH-type structure, and are influenced by past conditional correlations, current standardized returns and the long-term average of the conditional correlation. As shown in section 2.4.1, the DCC model in its scalar form is specified in equation 4.12.

$$
\begin{aligned}
\mathbf{y}_t &= \boldsymbol{\mu}_t + \mathbf{r}_t \\
\mathbf{r}_t &\sim N(0, \mathbf{H}_t) \\
\mathbf{H}_t &= \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \\
\mathbf{D}_t^2 &= \text{diag}\{\boldsymbol{\omega}\} + \text{diag}\{\boldsymbol{\kappa}\} \circ \mathbf{r}_{t-1}\mathbf{r}_{t-1}' + \text{diag}\{\boldsymbol{\lambda}\} \circ \mathbf{D}_{t-1}^2 \\
\boldsymbol{\epsilon}_t &= \mathbf{D}_t^{-1}\mathbf{r}_t \\
\mathbf{Q}_t &= \overline{\mathbf{Q}}(1-\alpha-\beta) + \alpha\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \beta\mathbf{Q}_{t-1} \\
\mathbf{Q}_t^* &= \text{diag}\{\mathbf{Q}_t\} \\
\mathbf{R}_t &= \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}
\end{aligned}
\tag{4.12}
$$

As mentioned in chapter 2, the equation for $\mathbf{D}_t^2$ specifies the univariate volatility process. Any univariate volatility model could be used. Here the chosen model is the standard GARCH(1,1). The equation for $\mathbf{Q}_t$ represents the DCC dynamics in its scalar form (which differentiates the specification in equation 4.12 from the non-scalar specification in equation 2.2), and includes one order of lagged residuals as well as one order of lagged conditional correlations.

As mentioned in section 2.5, the parameters of the DCC model are estimated using a QML estimation procedure. The likelihood function in equation 4.13 is maximized in two steps, where the first step is the estimation of the univariate GARCH parameters $\boldsymbol{\theta}$ and the second step is the multivariate correlation parameters $\boldsymbol{\phi}$. Equation 4.14 shows the two functions to be maximized, and equations 4.15 and 4.16 specify the decomposed log-likelihood function. These equations were derived in detail in section 2.5.

$$L(\boldsymbol{\phi}, \boldsymbol{\theta}|r) = \prod_{t=1}^{T} \Big[(\sqrt{2\pi})^{-N} |\mathbf{H}_t|^{-\frac{1}{2}} \exp\big(-\frac{1}{2} \mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{r}_t \big) \Big] \tag{4.13}$$

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg\max\{l_{volatility}(\boldsymbol{\theta}|\mathbf{r})\} \\ \hat{\boldsymbol{\phi}} &= \arg\max\{l_{correlation}(\boldsymbol{\phi}|\hat{\boldsymbol{\theta}}, \mathbf{r})\} \end{aligned} \tag{4.14}$$

$$l_{volatility}(\boldsymbol{\theta}) = -\frac{1}{2} \sum_{t=1}^{T} (N \log(2\pi) + 2\log|\mathbf{D}_t| + \mathbf{r}_t' \mathbf{D}_t^{-1} \mathbf{D}_t^{-1} \mathbf{r}_t) \tag{4.15}$$

$$\begin{aligned} l_{correlation}(\boldsymbol{\phi}, \boldsymbol{\theta}) &= -\frac{1}{2} \sum_{t=1}^{T} (\log|\mathbf{R}_t| + \boldsymbol{\epsilon}_t' \mathbf{R}_t^{-1} \boldsymbol{\epsilon}_t - \boldsymbol{\epsilon}_t' \boldsymbol{\epsilon}_t) \\ = -\frac{1}{2} \sum_{t=1}^{T} \big(\log|\mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}| &+ \boldsymbol{\epsilon}_t' (\mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1})^{-1} \boldsymbol{\epsilon}_t \big) \end{aligned} \tag{4.16}$$

Both the first step and the second step of the QML are maximized using an interior-point optimization algorithm for the DCC model. This can be done because they only have linear constraints. The requirement that $\mathbf{Q}_t$ is positive definite is fulfilled through the inclusion of the linear constraint $\alpha + \beta < 1$.

4.2.2 Intradaily DCCX model

The intradaily DCCX model is, as mentioned in section 2.4.2, an extension to the intradaily DCC model, and is chosen in order to include the ETF trading ratios as an exogenous explanatory variable. Again, the scalar DCCX model specification is identical to the scalar DCC model specification given in equation 4.12, except for that the time-varying conditional covariance evolves according to equation 4.17, as proposed by Vargas (2008).[37] As mentioned in section 4.2, this thesis hypothesizes and validates that the exogenous variable (ETF trading ratio) has a lagged effect on the dependent variable, which fits Vargas' model as seen in equation 4.17.

$$\mathbf{Q}_t = (\overline{\mathbf{Q}} - \alpha\overline{\mathbf{Q}} - \beta\overline{\mathbf{Q}} - \mathbf{K}\gamma\overline{x}) + \alpha(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}') + \beta\mathbf{Q}_{t-1} + \mathbf{K}\gamma x_{t-1} \tag{4.17}$$

In equation 4.17, $\overline{x}$ and $\overline{\mathbf{Q}}$ is estimated as $\hat{\overline{x}} = T^{-1} \sum_{t=1}^{T} x_t$ and $\hat{\overline{\mathbf{Q}}} = \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t'$, respectively. Note that γ, $\overline{x}$ and x_t are scalars since only one exogenous variable is included. As proposed by Vargas (2008), $\mathbf{K}$ is a matrix of ones and $\gamma \in [0, 1]$.[37] As discussed in chapter 2, this forces the exogenous variable to have a non-negative effect on correlations as the exogenous variables are required to be positive. However, this is a tolerable restriction in this case as several papers in the literature have shown that ETF trading ratio positively affects correlations. This is also the result shown by this thesis in the monthly regression model, see chapter 5. Furthermore, since the exogenous variable is the ratio of two positive numbers, the condition of positive exogenous variables is also fulfilled.

Like the DCC model, the parameters of the DCCX model were estimated using QML estimation as outlined in section 4.2.1. For the DCCX model however, the second step of the QML estimation, shown in equation 4.16, included the non-linear constraint that the minimum eigenvalue is greater than zero. This was as mentioned earlier in order to ensure positive definiteness of the $\mathbf{Q}_t$ matrix. Therefore a sequential quadratic programming (SQP) method, which focuses on solving the Karush-Kuhn-Tucker constraints, was used in the second step of estimation. The in-sample and out-of-sample correlation estimates from the DCCX model are displayed in appendix B.

4.2.3 Likelihood-ratio test

The DCC and DCCX models are compared with a likelihood-ratio (LR) test. The LR test compares the maximized log likelihoods of an unconstrained and a constrained model. For N stocks, the DCC model includes $3N$ GARCH parameters $\boldsymbol{\omega}, \boldsymbol{\kappa}$ and $\boldsymbol{\lambda}$, as well as the two conditional correlation parameters α and β. The DCCX model has all the same parameters, as well as the exogenous variable parameter γ. Thus, the DCC model is a constrained version of the DCCX model with $\gamma = 0$.

The log likelihood for the unrestricted and restricted model are denoted L_u and L_r, respectively. The LR test statistic is given by equation 4.18, where m is the number of restrictions, which is $m = 1$ in this case. The null hypothesis for the LR test in the DCC model versus the DCCX model case is that the exogenous variable coefficient $\gamma = 0$. Under the null hypothesis the test statistic is asymptotically $\chi^2(m)$-distributed as explained by Silvey (1975).[31]

$$LR = -2(L_r - L_u) \sim \chi^2(m) \tag{4.18}$$

4.2.4 Out-of-sample covariance forecast performance in portfolio optimization

The different volatility models are compared by evaluating the realized returns and variances of mean-variance optimized portfolios in out-of-sample data, and robustly testing for statistically better performance in portfolio optimization using the test proposed by Engle and Colacito (2006).[13] This is done in four steps.

First, the covariance forecasts are made for the DCC model and the DCCX model. Again, the model parameters α, β and γ are estimated using the in-sample data. As previously mentioned, the in-sample period constitutes the first 90% of observations, while the out-of-sample period constitutes the final 10% of observations. The covariance forecasts $\mathbf{H}_t$ are made with the parameter estimates from the in-sample period and with $t-1$ lagged data from the out-of-sample period for each time t, simulating the data available to a day-trader. For DCC and DCCX, the covariance forecasts $\mathbf{H}_t$ are derived from the relationship in equation 4.12, and the out-of-sample $\mathbf{Q}_t$ matrices are derived as in equation 4.19 and 4.20, respectively. Note that $\overline{\mathbf{Q}}$ is the average over the in-sample period. In addition, covariance forecasts are also made with the RiskMetrics volatility model for comparison. The RiskMetrics model is given in equation 4.21, where $\lambda = 0.94$, according to RiskMetrics' recommendation.

$$\mathbf{Q}_t = \overline{\mathbf{Q}}(1-\alpha-\beta) + \alpha\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} + \beta\mathbf{Q}_{t-1} \tag{4.19}$$

$$\mathbf{Q}_t = (\overline{\mathbf{Q}} - \alpha\overline{\mathbf{Q}} - \beta\overline{\mathbf{Q}} - \mathbf{K}\gamma\overline{x}) + \alpha(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}) + \beta\mathbf{Q}_{t-1} + \mathbf{K}\gamma x_{t-1} \tag{4.20}$$

$$h_{ij,t} = \lambda h_{ij,t-1} + (1-\lambda) r_{i,t-1} r_{j,t-1} \tag{4.21}$$

Second, the optimal portfolio weights for the different models are computed by performing Markowitz portfolio optimization described in equation 4.22, where the portfolio variance is minimized given a required rate of return. This is the standard portfolio optimization problem, where short positions are allowed and the risk-free asset is included. The risk-free asset return was set to zero due to the high frequency. Short positions are allowed as this is required by the test proposed by Engle and Colacito (2006).[13] The problem was first described by Markowitz

(1952) and expanded upon in Markowitz (1956).[23] [24]

$$\begin{aligned} \underset{\mathbf{w}_t}{\text{minimize}} \quad & \mathbf{w}_t'\mathbf{H}_t\mathbf{w}_t \\ \text{s.t.} \quad & \mathbf{w}_t'\boldsymbol{\mu}_t \geq \mu_0 \end{aligned} \tag{4.22}$$

Here $\boldsymbol{\mu}_t$ is the expected returns vector and is simply the arithmetic mean over the in-sample-period, and μ_0 is the required rate of return. According to portfolio optimization theory, all solutions to the problem in equation 4.22 will be a combination of the risk-free asset and the tangent portfolio when the risk-free asset is included. Hence, when the portfolios are used to compare covariance forecasts it is only necessary to run the optimization with one required rate of return, as the weights of risky assets of portfolios given other required rates of return will simply be a linear combination of those of the first portfolio. Therefore the required rate of return was arbitrarily set to the average of the maximum and minimum of the expected returns of the assets.

Third, the portfolio values in the forecasted out-of-sample period are plotted and compared. In addition to the portfolio weights calculated for the intradaily DCC model, the intradaily DCCX model and the RiskMetrics model, the standard market cap weighted portfolio weights are included for comparison purposes.

Finally, the test proposed by Engle and Colacito (2006) is performed in order to test if the covariance forecasts given by the intradaily DCCX model gives statistically significantly better portfolio performance than the DCC model.[13] Here portfolio performance is measured in lower squared returns. The theory behind the test was presented in section 2.6. The portfolio values are used to calculate u_t and v_t, given in equation 2.11 and 2.12, respectively. Then, equation 4.23 and 4.24 are estimated using generalized method of moments (GMM) with a vector heteroscedasticity and autocorrelation consistent (HAC) covariance matrix.

$$u_t = \beta_u + \epsilon_{u,t} \tag{4.23}$$

$$v_t = \beta_v + \epsilon_{v,t} \tag{4.24}$$

Given G_u and G_v, which are the robust variance estimates, we get the test statistics given in equation 4.25 and 4.26, where $\bar{u} = \frac{1}{T}\sum_{t=1}^{T} u_t$ and $\bar{v} = \frac{1}{T}\sum_{t=1}^{T} v_t$.

$$T^{1/2}G_u^{-1/2}(\bar{u} - \beta_u) \sim \text{Student-t}(\nu = T - 1) \tag{4.25}$$

$$T^{1/2}G_u^{-1/2}(\bar{v}-\beta_v) \sim \text{Student-t}(\nu = T-1) \tag{4.26}$$

Under the null hypothesis, both $\beta_u = 0$ and $\beta_v = 0$. When T is large, these test statistics approximate to equations 4.27 and 4.28

$$T^{1/2}G_u^{-1/2}\bar{u} \sim N(\beta_u, 1) \tag{4.27}$$

$$T^{1/2}G_v^{-1/2}\bar{v} \sim N(\beta_v, 1) \tag{4.28}$$

Chapter 5

Results and discussion

5.1 Results and discussion of the monthly regression model

The monthly regression model specified in section 4.1.4 yielded several important results. The total fund flow coefficient β^{ff}_{total} is positive and significant at the 5% significance level for all three aggregated models, meaning total absolute fund flow had a positive and significant impact on asset return correlation for constituents of all three indices. This result has not been shown before and is important in understanding the effects of passive investing. Furthermore, the total ETF trading ratio coefficient $\beta^{etf}_{\Delta total}$ is positive and significant at the 1% level for all three indices, meaning that the ETF share of trading volume drives correlation between asset returns for assets of all sizes. This contributes to the literature which mainly focuses on the S&P 500 index by also showing the effect for the S&P 400 and S&P 600 indices.

The split model results showed that large cap ETF trading ratio $\beta^{etf}_{\Delta large}$ is significant not only for stock return correlations in the large cap index, but also for small and mid cap indices. Additionally, both small cap ETF trading ratios and small cap mutual fund flows are shown to significantly increase return correlations in the S&P 600 small cap index. Even so, large and mid cap mutual fund flows and mid cap ETF trading ratio was not shown to have any effect on any of the indices. It is hard to determine how the explanatory power is distributed between large, mid and small cap ETF trading volumes and index tracking mutual fund flows due to multicollinearity likely being present among the time series

The bear market probability coefficient β_{BEAR} was positive and significant at a 1% significance level in all regressions. This is an important result as it shows the effectiveness of the Markov switching model at explaining the effect of bad economic times on asset return correlations.

5.1.1 Overview of calculation and regression results

Plots of all model variables are in appendix A. In addition, plots of the main explanatory variables, namely total fund flow and total ETF trading ratio, are included here along with the weighted average correlation of the S&P 500 index, in figures 5.1 and 5.2. The results of the monthly autoregressive linear regression models specified in section 4.1.4, in addition to important metrics like R^2 and p-values of tests of the linear regression model assumptions, can be seen in tables 5.1, 5.2 and 5.3. Each table includes the results of both the basic model (aggregated instead of split) and the alternative model with split fund flows and ETF trading volumes. Table 5.1 uses S&P 500 (large cap) weighted average correlation as the dependent variable, table 5.2 uses S&P 400 (mid cap) weighted average correlation, and table 5.3 uses S&P 600 (small cap) weighted average correlation.

S&P 500 (Large cap)	Aggregated model	Split model
β^{ff}_{total}	0.0768	
	*(0.0450)	
$\beta^{etf}_{\Delta total}$	1.9622	
	**(0.0000)	
β^{ff}_{large}		-0.0320
		(0.5331)
β^{ff}_{mid}		-0.2343
		(0.4196)
β^{ff}_{small}		0.4382
		(0.1254)
$\beta^{etf}_{\Delta large}$		2.2679
		**(0.0000)
$\beta^{etf}_{\Delta mid}$		0.1820
		(0.6705)
$\beta^{etf}_{\Delta small}$		0.0679
		(0.2147)
$\beta_{intercept}$	0.0519	0.0622
	(0.0694)	*(0.0414)
$\beta_{\Delta VIX}$	0.0024	0.0024
	(0.1666)	(0.1514)
β_{BEAR}	0.0704	0.00733
	**(0.0041)	**(0.0025)
β_{AR1}	0.4015	0.3716
	**(0.0000)	**(0.0000)
β_{AR2}	0.1851	0.1976
	**(0.0025)	**(0.0009)
β_{AR3}	0.1588	0.1427
	**(0.0037)	*(0.0103)
$R^2_{adjusted}$	0.6713	0.6960
	Regression test p-values	
F-test vs constant model	0.0000	0.0000
JB test (Non-normality)	0.2332	0.0741
White's test (Heteroscedasticity)	0.5664	0.9694
BG test (Autocorrelation)	0.3382	0.3453

Table 5.1: Estimated coefficients, R^2 and assumption test p-values. Estimated coefficients are given along with t-test p-values. p-values significant at a significance level of 5% are highlighted with one star, and those significant at a 1% significance level are highlighted with two stars. Variable names are explained in section 4.1.4.

S&P 400 (Mid cap)	Aggregated model	Split model
β^{ff}_{total}	0.0654	
	*(0.0422)	
$\beta^{etf}_{\Delta total}$	1.4750	
	**(0.0000)	
β^{ff}_{large}		-0.0092
		(0.8330)
β^{ff}_{mid}		-0.2083
		(0.4053)
β^{ff}_{small}		0.3537
		(0.1547)
$\beta^{etf}_{\Delta large}$		1.6526
		**(0.0000)
$\beta^{etf}_{\Delta mid}$		0.0369
		(0.9197)
$\beta^{etf}_{\Delta small}$		0.0668
		(0.1614)
$\beta_{intercept}$	0.0325	0.0452
	(0.1836)	(0.0899)
$\beta_{\Delta VIX}$	0.0021	0.0021
	(0.1542)	(0.1421)
β_{BEAR}	0.0572	0.0620
	**(0.0040)	**(0.0020)
β_{AR1}	0.4811	0.4457
	**(0.0000)	**(0.0000)
β_{AR2}	0.2268	0.2266
	**(0.0005)	**(0.0004)
β_{AR3}	0.0856	0.0735
	(0.1307)	(0.2054)
$R^2_{adjusted}$	0.6540	0.6702
	Regression test p-values	
F-test vs constant model	0.0000	0.0000
JB test (Non-normality)	0.5000	0.3748
White's test (Heteroscedasticity)	0.0820	0.3600
BG test (Autocorrelation)	0.1705	0.2264

Table 5.2: Estimated coefficients, R^2 and assumption test p-values. Estimated coefficients are given along with t-test p-values. p-values significant at a significance level of 5% are highlighted with one star, and those significant at a 1% significance level are highlighted with two stars. Variable names are explained in section 4.1.4.

S&P 600 (Small cap)	Aggregated model	Split model
β^{ff}_{total}	0.0416	
	*(0.0438)	
$\beta^{etf}_{\Delta total}$	1.2317	
	**(0.0000)	
β^{ff}_{large}		0.0103
		(0.3805)
β^{ff}_{mid}		-0.2548
		(0.0728)
β^{ff}_{small}		0.3287
		*(0.0267)
$\beta^{etf}_{\Delta large}$		1.2466
		**(0.0000)
$\beta^{etf}_{\Delta mid}$		0.1414
		(0.3564)
$\beta^{etf}_{\Delta small}$		0.0954
		*(0.0198)
$\beta_{intercept}$	0.0462	0.0556
	*(0.0148)	**(0.0084)
$\beta_{\Delta VIX}$	0.0012	0.0015
	(0.1974)	(0.1377)
β_{BEAR}	0.0553	0.0579
	**(0.0021)	**(0.0013)
β_{AR1}	0.4418	0.4260
	**(0.0000)	**(0.0000)
β_{AR2}	0.2389	0.2338
	**(0.0001)	**(0.0001)
β_{AR3}	0.0697	0.0517
	(0.1295)	(0.1974)
$R^2_{adjusted}$	0.6185	0.6344
	Regression test p-values	
F-test vs constant model	0.0000	0.0000
JB test (Non-normality)	0.2684	0.2831
White's test (Heteroscedasticity)	**0.0009	*0.0290
BG test (Autocorrelation)	0.1704	0.1906

Table 5.3: Estimated coefficients, R^2 and assumption test p-values. Estimated coefficients are given along with t-test p-values. p-values significant at a significance level of 5% are highlighted with one star, and those significant at a 1% significance level are highlighted with two stars. Variable names are explained in section 4.1.4. Due to presence of heteroscedasticity in both models heteroscedasticity-consistent standard errors are used in t-tests.

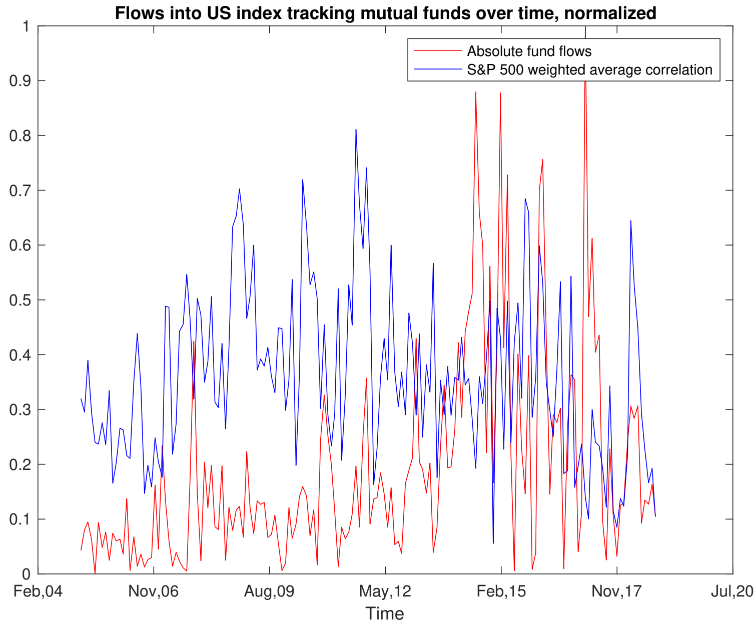

Figure 5.1: Absolute total fund flows of U.S. index tracking mutual funds and S&P 500 weighted average correlation

5.1.2 Interpretation of regression results

The most important results can be summed up as follows. Both the total mutual fund flows and ETF trading ratios generally increase asset correlations significantly at a 5% significance level or better. Furthermore, the split variables that significantly affect correlations are the large cap ETF trading ratio, the small cap ETF trading ratio and the small cap fund flows. The first significantly increases correlations in all indices, and the two latter both significantly increase correlations in the small cap index. However, the lack of more significant results could be due to the presence of multicollinearity in the time series. The inferred probability of a bear market given by the Markov switching model appears to be very successful in including the effect of bad economic times in this model. This variable increases correlations in regressions and is significant at a 1% significance level in all cases. The VIX is shown to have an insignificant effect at a 10% significance level for all models. The regression results will now be interpreted in more detail.

The total index tracking mutual fund flow is shown to significantly increase correlations between assets of the large cap index, as well as the mid and small cap indices, at a 5% significance level. In other words, β^{ff}_{total} is positive and significant at the 5% level in all three aggregated models. This means that fund flows to index

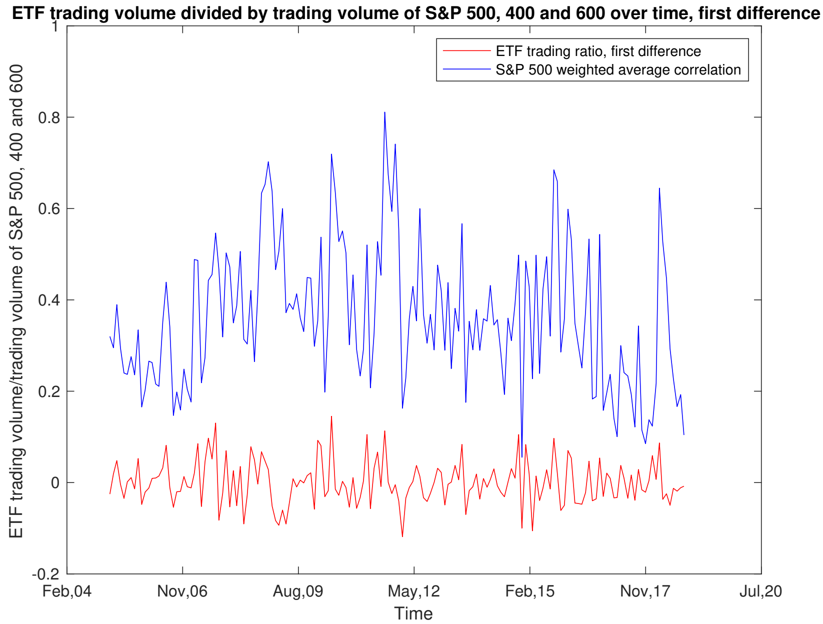

Figure 5.2: Total ETF trading ratio and S&P 500 weighed average correlations

mutual funds most likely have explanatory power on the weighted average correlation of the S&P 500, the S&P 400 and the S&P 600. This has not been proven before, and is a main result of this thesis. Since the mutual funds have such a large share of the U.S. equity market, the significance of effects on asset correlations from mutual fund flows could be large. Possible implications of increased asset correlations due to fund flows into index tracking mutual funds are described in section 5.1.4.

The ETF trading ratio coefficient $\beta^{etf}_{\Delta total}$ is significant at the 1% level in all aggregated models, implying that the ETF share of trading volume drives correlation between returns for stocks in all indices studied. These results supports the previous findings of Leippold, Su and Ziegler (2016), Da and Shive (2013) and Staer and Sottile (2018), among others.[21] [10] [33] It strengthens them by extending the time horizon and including more recent data. Da and Shive (2013) showed an effect on mid cap asset correlations using several measurements of ETF activity, none of which are the ETF trading ratio used in this thesis. Hence this thesis has shown for the first time that using the ETF trading ratio can model this effect in a parsimonious manner for mid cap assets, as well as small cap assets.

The coefficients of trading ratios of large cap ETFs $\beta^{etf}_{\Delta large}$ are significant at the 1% level in the split models for the large cap index correlations as well as the mid and small cap index correlations. A possible explanation is that non-S&P 500 stocks are on average positively correlated with the total market, which means that price fluctuations caused by index tracking fund trading are transferred to the correlated stocks outside of the traded indices. In other words, there may be a type of spillover effect onto correlations outside of the S&P 500. This resonates with the hypothesis of Leippold, Su and Ziegler (2016) that demand shocks to derivatives lead to increased asset return correlations both inside and outside of the underlying index.[21] As the S&P 500 market cap is around 80% of the total value of the U.S. stock market, these assets drive the overall market to a large extent which could be why the large cap ETF volumes have such a significant effect on all indices investigated.[32]

The small cap ETF trading volume is shown to significantly increase asset correlations in the S&P 600 small cap index, i.e. its coefficient $\beta^{etf}_{\Delta small}$ is positive and significant. This resonates with the results of Da and Shive (2013), who showed that ETF trading volumes had a larger effect on correlations for small and illiquid assets.[10] However, they observe the effect for the S&P 400 index and had not split ETFs into different categories. They had also used different measurements of ETF activity than this thesis. Therefore, this is the first time in the literature that there has been shown a link between trading volume of ETFs that track a small cap index and asset return correlations in the corresponding index's constituents. As one can see in figure 5.3, the small cap ETF trading ratio is more volatile than the mid cap ratio, which in turn is more volatile than the large cap ratio. This could be because smaller stocks are less liquid, so shocks to ETF trading have a larger effect on the trading ratio. Similarly, ETF trading shocks might have a larger impact on correlations of small cap stocks as they are less liquid and therefore more responsive to a demand shock.

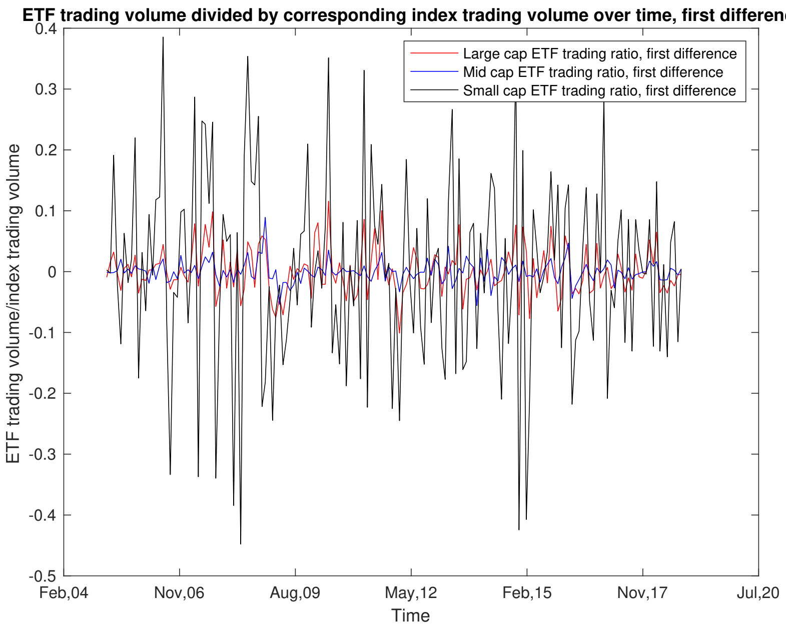

Figure 5.3: First differences of ETF trading ratios, split by classification

The coefficients for mid cap ETF trading volume $\beta^{etf}_{\Delta mid}$ is however not significant for any of the indices, not even the mid cap index. Note that this differs from the results of Da and Shive (2013), as they had not split the ETFs into categories.[10] Two possible explanations for why this is insignificant have been identified. One explanation could be that passive mid cap ETFs, like small cap ETFs, have much lower market shares than large cap passive ETFs. Additionally, mid cap assets are more liquid than small cap assets, so that asset prices are less reactive to ETF trading shocks, as can be seen in figure 5.3. In other words, mid cap ETFs might not have a large enough market share to significantly move the prices of the relatively liquid underlying assets simultaneously. Another explanation could be that multicollinearity between the large cap, mid cap and small cap ETF trading ratios reduce the significance of the mid cap and small cap ETF trading ratio variables. The correlation matrix for different ETF trading ratio time series is shown in table 5.4 reveals that multicollinearity between these variables likely is present.

	Large	Mid	Small
Large	1		
Mid	0.4790	1	
Small	0.5644	0.3173	1

Table 5.4: Correlation between subsections of first differences of ETF trading ratios

When mutual fund flows are split, the regression maintains a high R^2 value, but the explanatory fund flow variables have mostly insignificant coefficients. The exception is the small cap mutual fund flows, which has a positive and significant effect on correlations within the S&P 600 small cap index. This has never been shown before, and is similar to the result of this thesis that small cap ETF trading ratios also significantly increase correlations of the small cap index. As was the case for the split ETF trading ratios, the correlation matrix for fund flow variables reveals that multicollinearity between these variables is most likely present, as shown in table 5.5. This could explain why we do not see an effect from large and mid cap mutual fund flows on their respective index's average correlations.

	Large	Mid	Small
Large	1		
Mid	0.6715	1	
Small	0.6556	0.9616	1

Table 5.5: Correlation between fund flows of subsections of index tracking mutual funds

The estimated parameters of the Markov switching model specified in 4.1.2 are shown in tables 5.6 and 5.7. As one would expect, mean returns are lower and volatility is higher in the bear market state. The bear market probability coefficient β_{BEAR} is significant at the 1% level in all six regressions. This means that downturns are likely to increase average correlation of stocks, which supports the findings of Ang and Chen (2002) that correlations between U.S. stocks and the aggregate U.S. market are greater for downside moves than for upside moves.[3] This thesis clearly shows that the use of one downturn probability parameter based on a Markov switching model is a good indicator of the business cycle, and serves as a good alternative to the inclusion of several macroeconomic explanatory variables. Markov switching models have not previously been used in papers regarding the effect of investment in passive investment funds on asset return correlations. This is an important result in itself. It also helps in the main investigation of this thesis, which is the investigation into the effect of passive index funds on asset correlations, by making the regression model more parsimonious which leads to greater statistical power.

	μ	σ
Bull state	0.0112	0.0226
Bear state	-0.0055	0.0583
Total period	0.0055	0.0395

Table 5.6: Estimated mean and standard deviation of monthly returns in the bull market state, the bear market state and over the whole period

	Bull state	Bear state
Bull state	0.9631	0.0369
Bear state	0.0718	0.9282

Table 5.7: Estimated transition probabilities between bull and bear market states. Probability of transition from state in row to state in column.

The VIX coefficient β_{VIX} was shown to be insignificant at a 10% significance level for all models. The VIX parameter was included to adjust for effects shown in previous literature. Longin and Solnik (1995) found that correlation between U.S. stock prices rises in periods of high volatility, while Leipold, Su and Ziegler (2016) found the VIX to be significant for the S&P 500 at the 10% level, but not at the 5% level.[21][22] However, this thesis does not find evidence that market volatility leads to increased asset correlations.

Three AR lags were chosen in the model based on the Schwarz's Bayesian information criterion (SBIC) used on an ARMAX model. The three lags included were all significant on the 1% level for the aggregated model on the S&P 500. Furthermore, the first two lags are significant at the 1% level for all six regressions. Leippold, Su and Ziegler (2016) also included three monthly lags in their AR model for regressing the average correlation of U.S. index stocks on different explanatory variables.[21] In their model, lags one and three were found to be significant. Hence, both models show somewhat similar autocorrelation structures.

5.1.3 Test of underlying assumptions of monthly regression model

The standard monthly linear regression model assumes that the errors are normally distributed, the variance of the errors is constant (homoscedasticity) and the correlation between the error terms over time are zero (no autocorrelation). These assumptions were tested, with the results described below.

The Jarque-Bera test reveals that the null hypothesis that the regression residuals are normally distributed can not be rejected for any of the regressions.

The White's test reveals presence of heteroscedasticity for both the aggregated and split model for the S&P 600. A consequence of this is that for these regressions, the OLS estimators will still be unbiased and consistent, but might not have the minimum variance. To adjust for this, the t-tests of coefficient significance in these regressions were calculated using White's heteroscedasticity-consistent standard errors. This procedure was proposed by White (1980).[38]

The Breusch-Godfrey test reveals that the null hypothesis of no autocorrelation can not be rejected for any of the regressions. In other words, this indicates that there is no autocorrelation. Note that this was for the model including three autoregressive lags. The reason that the lags were added was that the model without lags failed the Breusch-Godfrey test.

5.1.4 Implications of results

This thesis has shown that fund flows to index tracking mutual funds significantly increases asset return correlations. The implication of this is that if the market share of passive mutual funds keep increasing, fund flows are likely to increase as well, leading to higher correlations in general and therefore lower diversification benefits. This is similar to the already known effects of index tracking ETF trading volumes. Note that the market share of passive mutual funds does not directly influence correlations, but fund flows to passive mutual funds likely does. However, one would expect the fund flows in and out of mutual funds to increase with fund market share.

Sullivan and Xiong (2012) found that growth in passively managed equity indices corresponds to a rise in systematic market risk.[34] The paper points out that investors' ability to diversify risk by holding a diversified equity portfolio is weakened as a consequence. Furthermore, they show that U.S. equity portfolios have become less diversified in the recent years, as returns for all subsets have become more correlated. Not surprisingly, the results leads to the conclusion that index tracking mutual funds contribute to this effect, in addition to index tracking ETFs. This thesis shows that this effect is present in mid and small cap assets as well as the more researched S&P 500 index. An implication of this is that negative effect on diversification extends beyond the S&P 500 to the broader market. This means that the systematic risk in broad market indices such as the Russel 2000 index and the Wilshire 5000 index could also increase, and not only due to their large cap constituents.

According to Anadu et al. (2018), approximately 3.3 trillion USD was placed in U.S. passive mutual funds and 3.4 trillion USD in U.S. index tracking ETFs as of

January 2018.[1] As the two fund categories are approximately the same size, the effects from index tracking mutual funds on total average correlations should be of comparable size to those of index tracking ETFs.

5.1.5 Possible improvements to the monthly regression model

Fund flows and ETF trading volumes are split into funds tracking large, mid and small cap stock indices in the model. Some funds follow combinations of these categories, and for these funds there was little data available about the proportions of asset allocations in different categories. In this thesis, a flat distribution has therefore been assumed. Two alternative ways to address this have been identified. The best would be a more accurate allocation of mutual fund flows to market subsections (large, mid and small cap), which would likely give the split model more explanatory power. The same is likely true for the allocation of ETF volumes to market subsections (large, mid and small cap). Alternatively, when investigating the weighted average correlation of an index one could exclude funds that do not exclusively track that index. For example one could exclusively use funds tracking the S&P 400 on the weighted average correlation of S&P 400 constituents. This would improve correctness of fund mapping, however it would significantly reduce the amount of ETF trading volume included in the model.

Trading volumes of index futures and E-mini futures are not included as explanatory variables in the model, although Leippold, Su and Ziegler (2016) found that demand shocks to ETFs and futures lead to stronger stock price comovement.[21] They also found that demand shocks to ETFs have a higher impact on stock return correlations than shocks to futures, which is the main reason why it is not included in the model. However, the model could possibly be improved by taking into account effects from demand shocks of futures.

A Markov switching model is used to generate one variable for the inferred probability of being in a bear market. This switching model uses constant transition probabilities to generate a probability estimate, but could be further refined by including time varying transition probabilities. Using this method Bazzi et al. (2017) were able to achieve an improvement in model fit compared to a model with constant transition probabilities in a model for the likelihood of U.S. production being in recession, stable or in a growth period.[5] However, as the probability of downturn is significant at the 1% level in all regressions, it appears to be a good measurement of the macroeconomic environment.

Three lags were included in all six regressions, although it was only proven optimal by the SBIC information criterion for the aggregate model on the S&P 500. This was done to make the results more comparable. However, one could argue that using the SBIC method to get the optimal number of lags for each regression would give a better model.

5.2 Results and discussion of the intradaily DCCX model

The intradaily DCCX model yielded several important and never before proved results. It was shown that ETF trading significantly increases asset correlations with a five minute lag. This is important mainly for two reasons. First, it shows that the effect from ETF trading on correlations on a monthly basis, previously proven by multiple papers, is also evident on a five-minute intradaily basis. Second, this strengthens the hypothesis of a lagged effect between ETF trading and the arbitrage trading that leads to the higher correlations, as explained in chapter 4. This lag allows the possibility of forecasting correlations, providing a real world use to the research done by previous papers.

The performance of the correlation forecasts presented so far were compared using Markowitz optimization. This thesis showed that a Markowitz portfolio based on the forecasted volatilities using the intradaily DCCX model had on average lower squared returns than one based on the standard intradaily DCC model. As the objective function in the Markowitz optimization is to minimize expected volatility given a required rate of return, this could indicate a better correlation forecast. However, the difference between the performance of the DCCX and DCC model was not significant when using the test proposed by Engle and Colacito (2006) at a 5% significance level.[13] Both the DCCX and DCC portfolios had significantly lower realized variances than the RiskMetrics portfolio.

5.2.1 Overview of calculation and estimation results

As explained in chapter 4, the first step of the DCC and DCCX model is estimating time-varying variances using univariate GARCH models. Table 5.8 shows the estimated parameters of the univariate models. The forth column of table 5.8 shows the long term volatility estimates, which is highest for AMZN and lowest for XOM.

Ticker	ω	κ	λ	$\frac{\omega}{1-\kappa-\lambda}$
MSFT	0.0010	0.1688	0.7727	0.0164
AAPL	0.0008	0.1433	0.8164	0.0192
AMZN	0.0010	0.1958	0.7752	0.0333
FB	0.0021	0.1687	0.7599	0.0287
BRKb	0.0006	0.1767	0.7827	0.0146
JNJ	0.0005	0.1982	0.7641	0.0135
GOOG	0.0017	0.1904	0.7262	0.0202
XOM	0.0009	0.1386	0.7905	0.0128
JPM	0.0009	0.1568	0.7874	0.0153
V	0.0008	0.1551	0.7853	0.0132

Table 5.8: Univariate GARCH model estimated parameters

The second step is estimating the parameters of the multivariate scalar intradaily DCC model and multivariate scalar intradaily DCCX model, respectively. The results of this, along with model significance level given by the likelihood-ratio test, are shown in table 5.9. Furthermore, in sample average correlations weighted on market cap for both the DCC and DCCX model are shown in figure 5.4.

Model	α	β	γ	Log likelihood	p-value (null: DCC)
DCC	0.0091	0.8993	0	4.2135e+04	1
DCCX	0.0116	0.7747	0.1928	4.2159e+04	2.399e-12

Table 5.9: Multivariate models' estimated parameters. Final column is the p-value against the null hypothesis that $\gamma = 0$, ie. the standard DCC model.

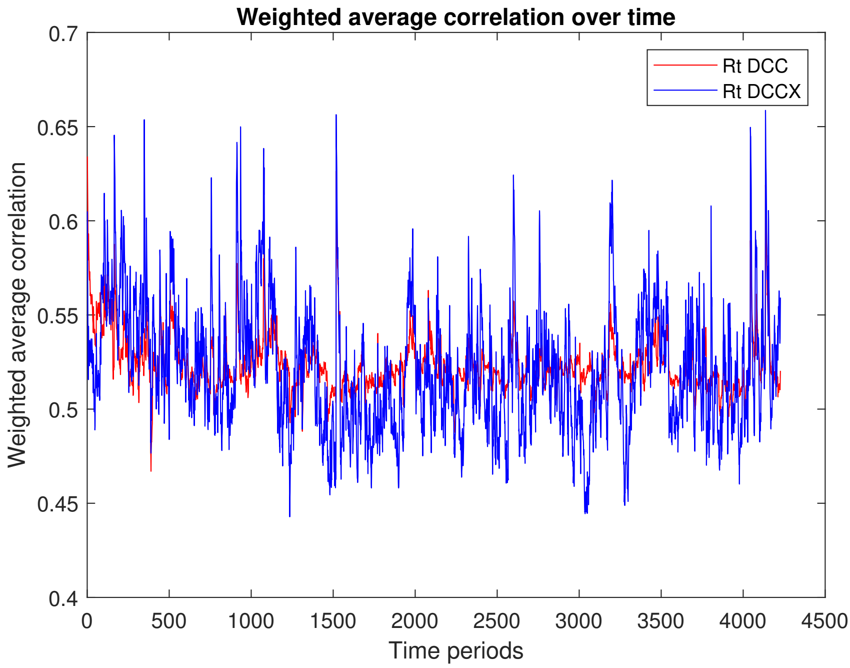

Figure 5.4: In sample weighted average correlation for the top 10 S&P 500 companies for the intradaily DCC and DCCX model

The likelihood-ratio test statistic, which is asymmetrically distributed as $\chi^2(1)$ under the null hypothesis that $\gamma = 0$, gives the model a p-value of 2.399e-12 versus the standard intradaily DCC model. The intradaily DCCX model including the ETF trading ratio as an explanatory variable is therefore statistically significant.

5.2.2 Test of model covariances using Markowitz portfolio optimization

In section 5.2.1 it was shown that the inclusion of the exogenous ETF trading variable yielded a DCCX model with significantly higher log-likelihood than the basic DCC model. However, this did not translate into statistically significant lower squared returns. Figure 5.6 shows the evolution of portfolio values in the out-of-sample testing period, and figure 5.5 shows the out-of-sample forecasted weighted average correlation at time t, based on information available at $t-1, \forall t$. As explained in section 4.2.4, the three portfolios based on volatility models were weighted using Markowitz optimization with a risk-free asset and allowing short positions. The relative weights of risky assets in the market cap weighted portfolio were based on market cap. However, a risk-free asset was included here as well in order for the required rate of return to be equal to the other portfolios. The testing period was the final 10% of the time periods, which is roughly six days.

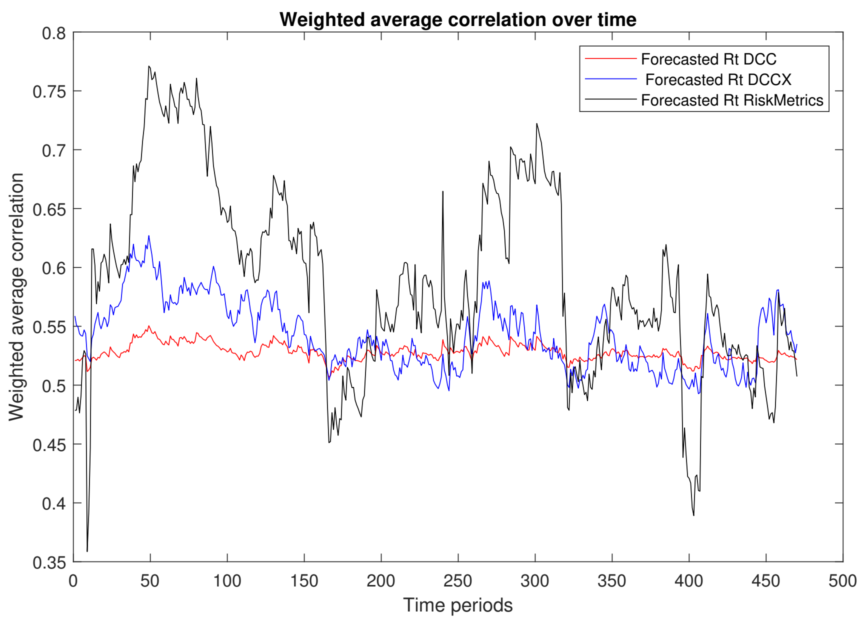

Figure 5.5: Out-of-sample forecasted weighted average correlation. Note that at every point in time t information available at $t-1$ is used.

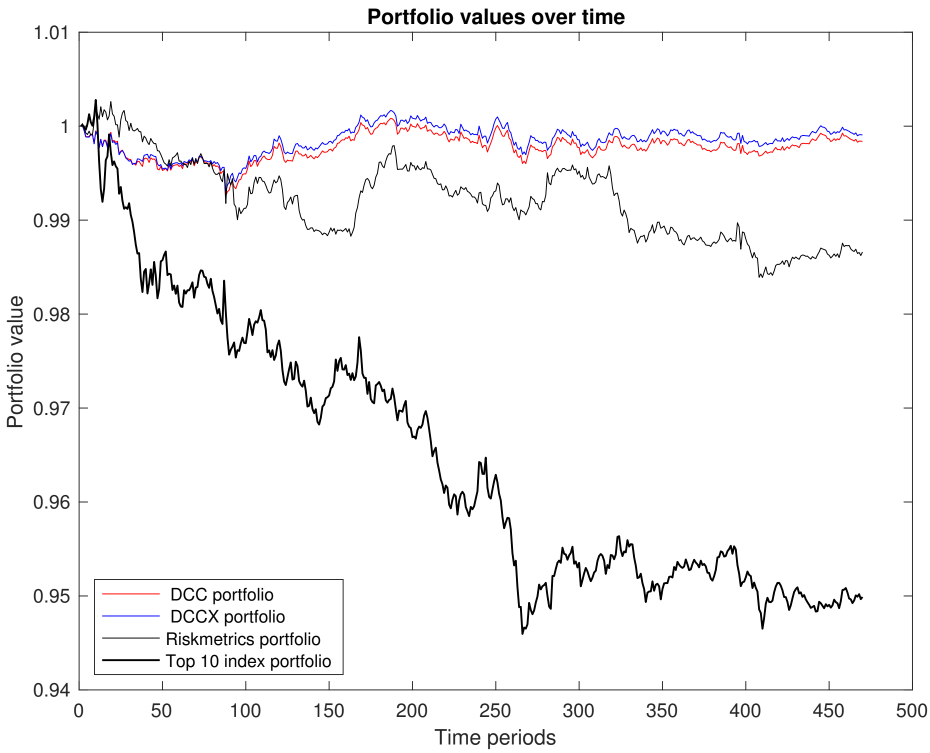

Figure 5.6: Portfolio values of four different portfolio strategies over time. All have the same required rate of return per period.

It is important to note that transaction costs are not included in the Markowitz optimization model. If they were not included the optimized portfolios might not have beaten the market, although the transaction cost free portfolios do so in figure 5.6. As one can see in figure 5.5, the correlations of the DCCX model and DCC model seem to be fairly similar after the 150th out-of-sample time period. This appears to be reflected in figure 5.6, as the returns are fairly similar indicating that the two models gave fairly similar portfolio weights after this point.

As the Markowitz algorithm is designed to minimize variance given an expected return requirement, the main result is the squared returns. The sum of squared returns (realized variance) and the normalized squared returns of each portfolio are shown in table 5.10. DCC had on average 1.8% higher squared returns than DCCX, meaning that Markowitz optimization using DCCX correlations on average gave better results. However, the Engle-Colacito test shown in equations 4.25 and 4.26 showed that this difference was not significant at a 5% significance level, when adjusting for heteroscedasticity, autocorrelation and non-normality. The p-values and t-statistics are shown in table 5.11. A positive t-statistic means that the volatility model of the column performs better than the volatility model of the row (lower squared returns). In other words, the t-statistics in table 5.11 show that the DCCX model gave insignificantly better results than the DCC model at the 5% significance level, while both significantly outperformed the simpler RiskMetrics model, as one might expect. Again, the RiskMetrics model was included for reference.

	DCCX	DCC	RiskMetrics	Market cap
Sum of squared returns	6.64e-05	6.76e-05	1.93e-04	5.56e-04
Normalized squared returns	100	101.8	290	838

Table 5.10: The sum of squared returns for the of out of sample period. All sums of squared returns are normalized to the lowest value (DCCX) in the second row.

	DCC	DCCX	RiskMetrics
DCC	-	0.0750	7.070e-09
		(1.7845)	**(-5.8975)
DCCX	0.0750	-	4.8752e-09
	(-1.7845)		**(-5.9631)
RiskMetrics	7.070e-09	4.8752e-09	-
	**(5.8975)	**(5.9631)	

Table 5.11: Assumption p-values with corresponding t-statistics in parenthesis for the Engle-Colactio test. A positive t-statistic means that the volatility model of the column performs better than the volatility model of the row (lower squared returns). t-statistics significant at a significance level of 5% are highlighted with one star, and those significant at a 1% significance level are highlighted with two stars.

As mentioned in section 2.6, the mean-variance results are more sensitive to variance estimates than correlation estimates, and expected return estimates are more important than both. The portfolio optimized on the RiskMetrics forecasts has different variance and covariance forecasts from DCC and DCCX, while DCC and DCCX have the same variance forecasts but different correlation forecasts. This may explain why RiskMetrics has significantly worse performance than the other two models, while DCC and DCCX have very similar performance, as seen in table 5.10.

5.2.3 Implications of results

The results of this high frequency intradaily volatility model are in accordance with previous literature on the low frequency effects of ETF trading on asset correlations as well as the monthly regression model of this thesis. The results are also in accordance with the results of Staer and Sottile (2018), which show that ETF trading has a strong and positive effect on correlations at high frequencies.[33]

However, the most important result of the intradaily DCCX model of this thesis is that there is evidence of a lagged effect between ETF trading and increased asset correlations. This paves way for a novel real world application, namely in forecasting asset covariances. Covariance forecasts are important as they have many different applications, such as portfolio optimization, Value at Risk calculations, hedging strategies and multi-asset derivative pricing. The inclusion of exogenous variables directly into the DCC model as proposed by Vargas (2008) is a novel approach in the research in the field of effects of index tracking ETF funds on asset correlations.[37] While the improved covariance forecasts did not lead to significantly lower squared returns of Markowitz-optimized portfolios compared with portfolios based on standard DCC model volatilities, it is possible that a further refined model, as will be discussed in section 5.2.4, will yield improved results. Additionally, as Engle and Colacito (2008) point out, correct expected returns are roughly ten times more important than correct expected variances, which again are twice as important as correlations, indicating the difficulty of significantly improving portfolios based only on correlation models.[13]

5.2.4 Possible improvements to the intradaily DCCX model

The main focus in this thesis' intradaily correlation estimates has been to isolate the effect of ETF trading ratio on correlations. There are improvements that could be done to the volatility and correlation estimates that have not been performed in this thesis, with the intent of making the model simple enough to be able to isolate the effects of ETF trading on asset comovement.

Several studies have argued that a negative shock to financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude, including Glostan, Jagannathan and Runkle (1993) and Nelson (1991).[14][26] In the case of equity returns, such asymmetries are typically attributed to leverage effects, whereby a fall in the value of a firm's stock causes the firm's debt to equity ratio to rise. One could therefore improve the univariate volatility models by utilizing more advanced models than the GARCH model, such as GJR-GARCH or EGARCH. Additionally, Engle and Capiello (2006) show a similar asymmetric effect in conditional correlations.[7] It would be possible to include an indicator variable to capture asymmetric effects in the DCC and DCCX models, as proposed by Engle and Capiello (2006) and Vargas (2008).[7] [37]

It would also be possible to include other exogenous variables than ETF trading ratio, as was done in the monthly correlation model. A natural choice could be a variable that models the state of the economy, like the Markov switching model did with monthly data. However, challenges arise as the intradaily data only covers approximately three months of trading, which means that the major macroeconomic factors are less likely to change significantly over the period. There are also issues related to data availability. One could also include an exogenous variable that captures the effect of volatility in the market on asset correlations, such as the VIX, although this did not yield significant results in the monthly linear regression model of this thesis.

Chapter 6

Conclusion

This thesis has investigated the effects of index tracking funds on asset return correlation of index constituents with two different models, a monthly linear regression model and an intradaily correlation forecasting model.

The monthly regression model was used to examine the effects of index tracking mutual fund flows and ETF trading volumes on the asset return correlations of index constituents of three different indices, the S&P 500 (large cap), the S&P 400 (mid cap) and S&P 600 (small cap). It has been shown that total absolute fund flows for index tracking mutual funds has a positive and significant effect on asset return correlations within the S&P 500, S&P 400 and S&P 600, something that no previous paper has shown. Additionally, this thesis has shown that the first differences of ETF trading ratios has a positive and significant effect on all three investigated indices. For the S&P 500 this was what was expected, building upon the research of Leippold, Su and Ziegler (2016)[21], and this thesis adds to this research by showing that it holds for the longer period of January 2005 to September 2018. Furthermore, it has not previously been demonstrated that the ETF trading ratios also have a positive and significant impact on the mid and small cap indices used, namely the S&P 400 and S&P 600, respectively.

By investigating not only the large cap assets in the S&P 500, it was revealed that the effects studied have much broader implications for the market than just the correlations of the 500 largest stocks in the U.S. This thesis also investigated how splitting mutual funds and ETFs into various categories based on which type of asset they track affected the model. Two important results came from this investigation. Firstly, the ETFs that track large cap stocks have high significance for all indices, signifying the importance of these funds. Secondly, the ETFs and mutual funds that track small cap stocks significantly increase correlations of the small cap index.

Additionally, previous research by Leippold, Su and Ziegler[21], Staer and Sottile (2018)[33] and Da and Shive (2013)[10] have used various macroeconomic indicators to explore the hypothesis that asset return correlations are higher, and thus the benefits of diversification less, in bad economic times. The results were at best inconsistently significant depending on which other variables were included. This thesis demonstrated that the inferred probability of being in a bear market, calculated using a Markov switching model, has a high degree of significance on asset return correlations of all three indices investigated. This is an important result in its own right. By making the regression model more parsimonious it also helps in the central investigation of this paper, which is the effect of passive index tracking funds on asset correlations.

The intradaily correlation model revealed that index tracking ETF trading increases stock return correlations with a five minute lag. This is important mainly because it strengthens the hypothesis of a lagged effect between the ETF trading and the arbitrage trading that induces the higher correlations, but also because the results support the findings of previous studies. This thesis is the first to use the ETF trading ratio directly in the estimation of correlations, as opposed to regressing correlations estimates on an ETF trading parameter as done by Leippold, Su and Ziegler (2016), Staer and Sottile (2018) Da and Shive (2013) on a monthly frequency and Staer and Sottile (2018) on a five minute frequency.[21] [33] [10] Consequently, the intradaily correlation model proposed is the first to provide a real world use of the effects shown in the previous literature, namely the use of ETF trading ratio as an exogenous variable to improve correlation forecasting. Furthermore, this thesis builds upon the work of Vargas (2008) who first proposed the DCCX model.[37] The DCCX model has gained limited traction. This thesis proposes a novel application of the model that fits well with the model's restrictions, namely that the exogenous variable must be positive and also that its effect on correlations must be positive, thus supporting the model's usefulness.

The coefficient of the exogenous variable (ETF trading ratio) in the intradaily correlation model was significantly different from zero with a p-value of 2.399e-12. This result is in accordance with that of Staer and Sottile (2018), who utilized a regression model at a five minute frequency and found the effect of ETF trading activity to be significant and large. However this thesis used more recent data and a different model.[33] When testing the performance of the DCCX covariance forecasts versus the DCC covariance forecasts in Markowitz optimization this thesis found that the DCCX portfolio on average had lower squared returns. However, the test proposed by Engle and Colacito (2006) showed that this difference was not significant at a 5% significance level.[13] It is still possible that the DCCX covariance forecasts give significantly improved results in other applications, such as Value at Risk calculations or risk parity calculations.

Passive index tracking funds were designed to take advantage of the benefits of diversification while avoiding expensive management fees. An unintended consequence is that they lessen the benefit they were designed to exploit.

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Appendix A

Regression time series plots

Figures A.1, A.2, A.3, A.4, A.5, A.6 and A.7 show the time series used in the regression model, where figure A.1 is the dependent variable and the rest are explanatory variables.

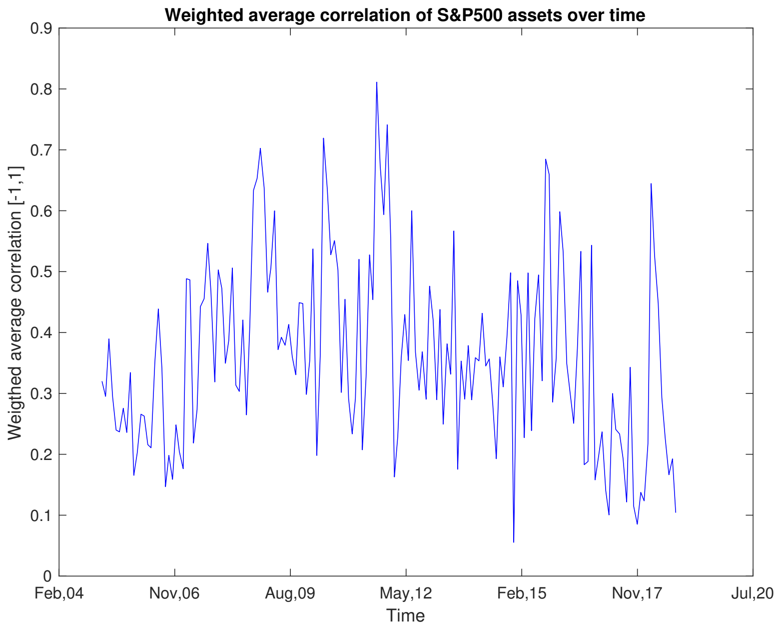

Figure A.1: Weighted average correlation of S&P 500 assets

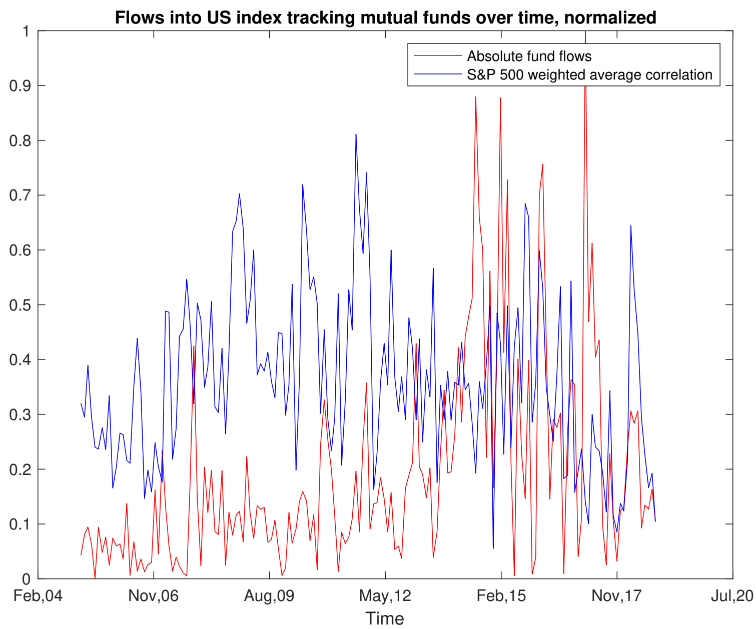

Figure A.2: Absolute total fund flows of U.S. index tracking mutual funds

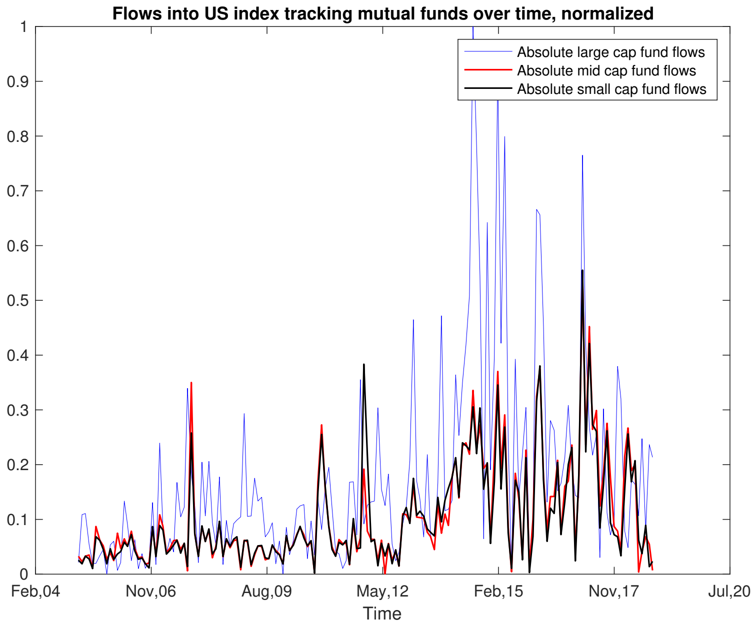

Figure A.3: Absolute fund flows of U.S. index tracking mutual funds, split by classification

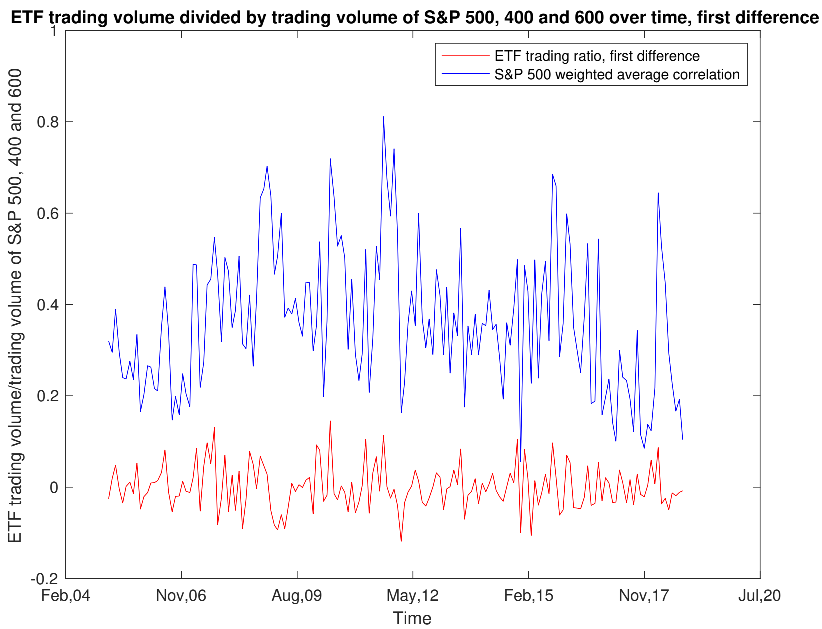

Figure A.4: First difference of ETF trading ratio

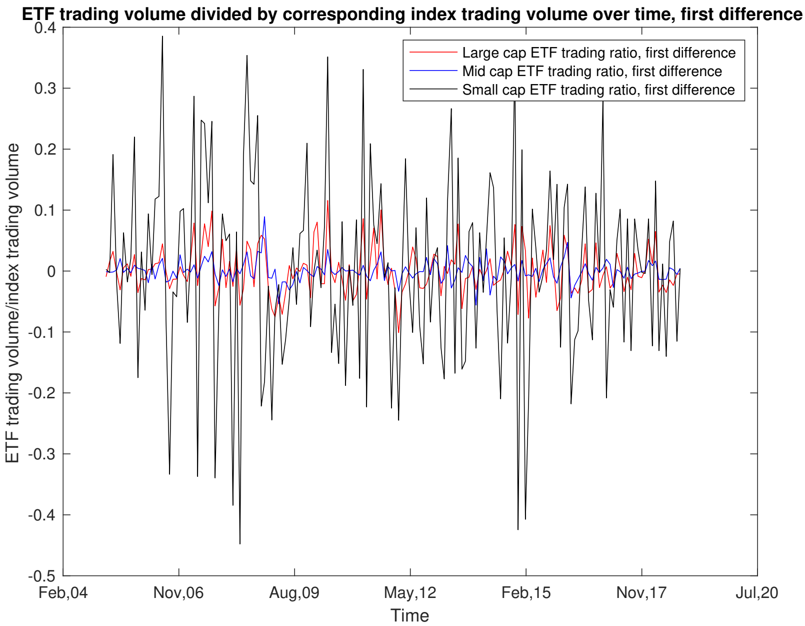

Figure A.5: First differences of ETF trading ratios, split by classification

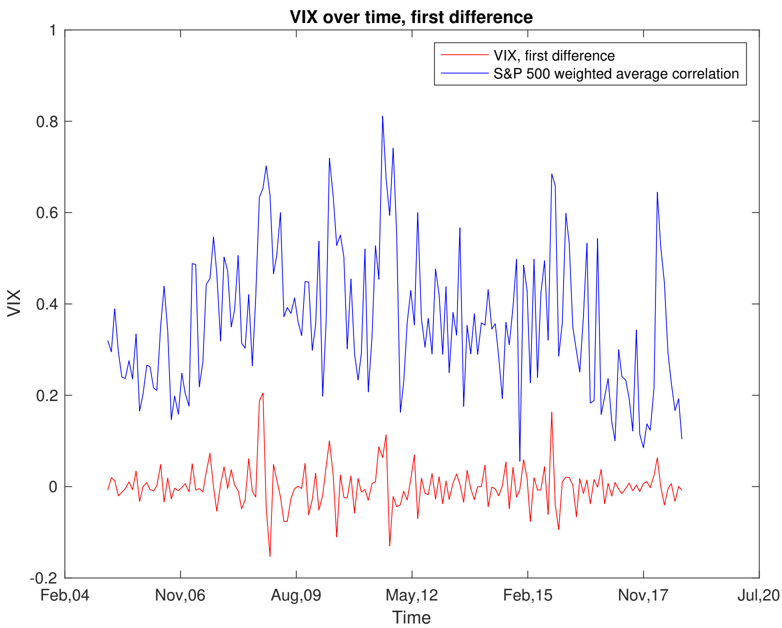

Figure A.6: CBOE Volatility index

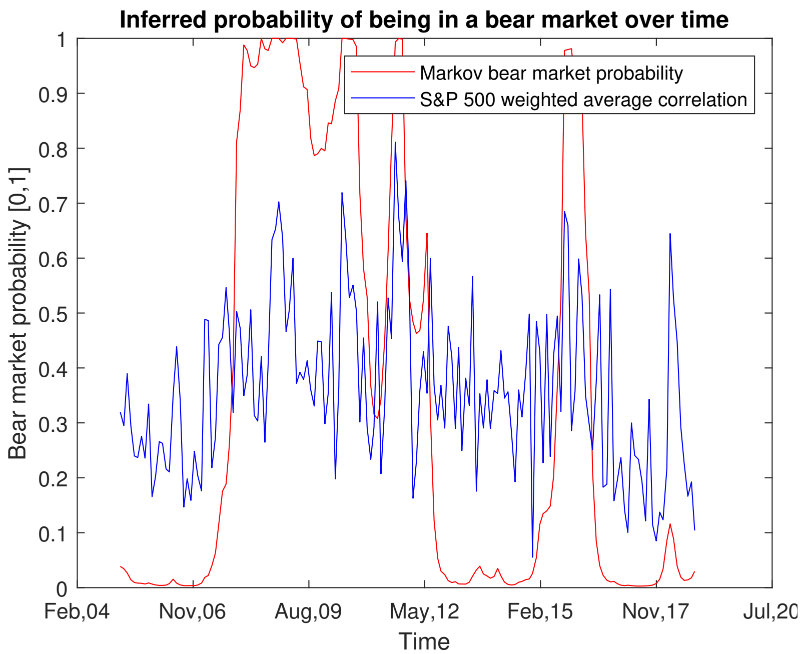

Figure A.7: Inferred probability of being in a bear market

Appendix B

Intradaily DCCX model correlation plots

The correlation time series between the top 10 stocks of the U.S. stock market from the intradaily DCCX model are plotted in figure B.1 and B.2. The plots include both the final 10% of in-sample correlations and the forecasted correlations in the out-of-sample period.

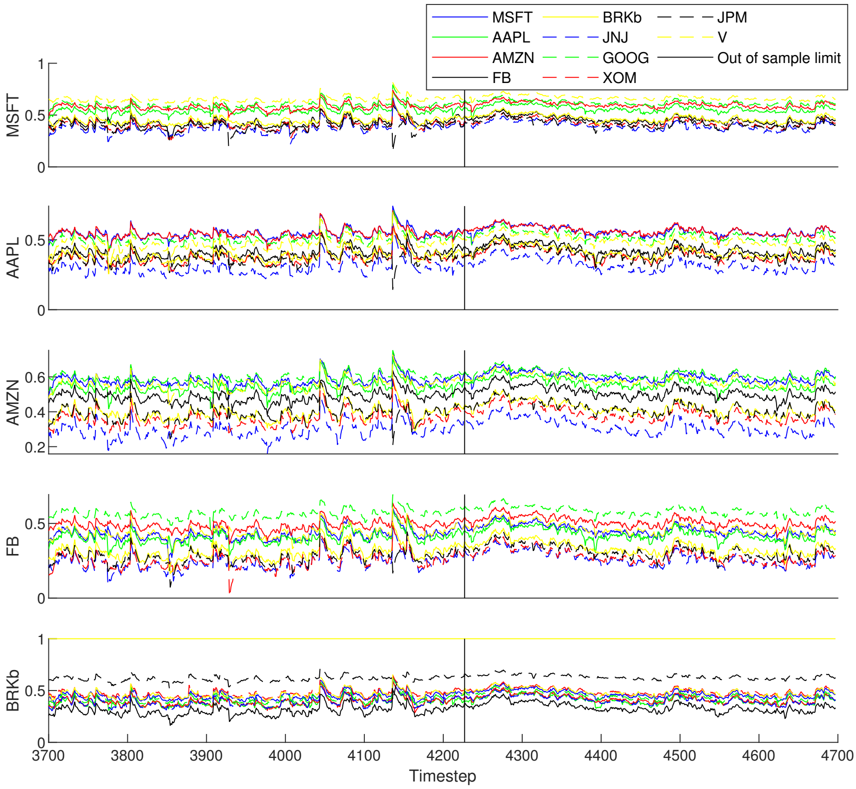

Figure B.1: Correlations between top 10 U.S. stocks over time

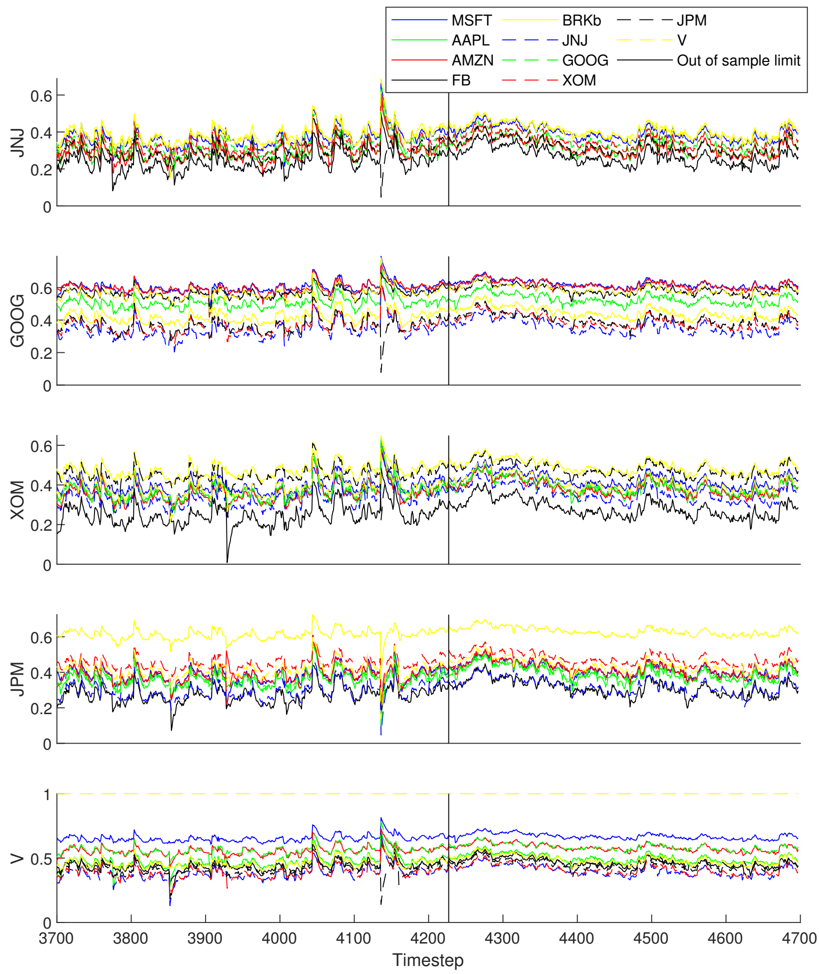

Figure B.2: Correlations between top 10 U.S. stocks over time

Appendix C

Markov switching model parameter estimation

As previously stated, the Markov switching model with two states can be stated as in equations C.1 and C.2, where the former states the formulation of the mean equation for the returns and the latter states that the state variable S_t follows a Markov process.

$$\begin{array}{lll} r_t = \mu_1 + \epsilon_t & \text{where} & \epsilon_t \sim N(0, \sigma_1^2) \\ r_t = \mu_2 + \epsilon_t & \text{where} & \epsilon_t \sim N(0, \sigma_2^2) \end{array} \tag{C.1}$$

$$\begin{array}{ll} p_{11} = Pr[S_t = 1 | S_{t-1} = 1] & p_{12} = Pr[S_t = 2 | S_{t-1} = 1] = 1 - p_{11} \\ p_{22} = Pr[S_t = 2 | S_{t-1} = 2] & p_{21} = Pr[S_t = 1 | S_{t-1} = 2] = 1 - p_{22} \end{array} \tag{C.2}$$

Here the model parameters are μ_1, μ_2, σ_1^2, σ_2^2, p_{11} and p_{22}, expressed jointly as $\boldsymbol{\theta}$. The likelihood function can be expressed as in C.3.

$$L(\boldsymbol{\theta}) = \prod_{t=1}^{T} f(y_t | \boldsymbol{\theta}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} e^{(-\frac{y_t - \mu_{S_t}}{2\sigma_{S_t}^2})} \tag{C.3}$$

If the state variable S_t was already known it would be simple to estimate μ_{S_t} and $\sigma_{S_t}^2$ by maximizing the log likelihood in the normal manner. However, since it is stochastic it must be estimated in a separate step. Taking logs and using the law of total probability yields:

$$\ln L(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} f(y_t|S_t = j, \boldsymbol{\theta}) P(S_t = j|\boldsymbol{\theta}) \tag{C.4}$$

$$f(y_t|S_t = j, \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} e^{(-\frac{y_t - \mu_{S_t}}{2\sigma_{S_t}^2})} \tag{C.5}$$

The density function $f(y_t|S_t, \boldsymbol{\theta})$ as seen in equation C.5 is simple to evaluate. For one iteration of the optimization algorithm (given one set of parameters $\boldsymbol{\theta}$), $P(S_t = j|\boldsymbol{\theta})$ can be calculated iteratively for all t and j. First, one must set $P(S_1 = 1|\boldsymbol{\theta})$ and $P(S_1 = 2|\boldsymbol{\theta})$ to a set of initial values. One could include these as a parameter in the maximum likelihood estimation, however Hamilton (1989) proposes to simply set them to the unconditional expectations (steady-state probabilities) as seen in equation C.6.[17] Next, $P(S_t = j|\boldsymbol{\theta})$ is iteratively calculated for $t \in [2, T]$, shown in equation C.7. Finally, the timeseries $P(S_t = j|\boldsymbol{\theta})$ is used to evaluate the log likelihood, given in equation C.4.

$$\begin{aligned} P(S_1 = 1) &= \tfrac{1-p_{11}}{2-p_{11}-p_{22}} \\ P(S_1 = 2) &= \tfrac{1-p_{22}}{2-p_{11}-p_{22}} \end{aligned} \tag{C.6}$$

$$\begin{aligned} P(S_t = 1|\boldsymbol{\theta}) &= \textstyle\sum_{i=1}^{2} p_{i1} P(S_{t-1} = i|\boldsymbol{\theta}) \\ P(S_t = 2|\boldsymbol{\theta}) &= \textstyle\sum_{i=1}^{2} p_{i2} P(S_{t-1} = i|\boldsymbol{\theta}) \end{aligned} \tag{C.7}$$

The procedure explained above to evaluate the log likelihood function can now be used in an optimization algorithm to estimation the model parameters $\boldsymbol{\theta}$, as in equation C.8

$$\boldsymbol{\theta} = \text{argmax} \sum_{t=1}^{T} \ln \sum_{j=1}^{2} f(y_t|S_t = j, \boldsymbol{\theta}) P(S_t = j|\boldsymbol{\theta}) \tag{C.8}$$

Note that while $\boldsymbol{\theta}$ is the set of decision variables, the timeseries $P(S_t = j)$ of the final iteration may also be of interest. For instance, that is what is used in the monthly linear regression model of this thesis.

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