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Applying an integrated real options approach to oil field development:

Valuing new information through appraisal under market and reservoir risk

Inti J. Castro

Partik Singh

Industrial Economics and Technology Management

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Supervisor: Verena Hagspiel

Co-supervisor: Semyon Fedorov

Norwegian University of Science and Technology

Department of Industrial Economics and Technology Management

Abstract

This thesis studies the investment problem of an oil company that faces the question of whether and when to invest in development of an oil field and, whether to invest in appraisal and how to determine optimal production. For years, the traditional discounted cash flow (DCF) method has been the predominant approach used by firms and investors to make capital budgeting decisions. However, the approach fails to exploit the managerial flexibilities inherent in field development projects. To exploit these flexibilities we apply an integrated real options approach that allows us to account for market and reservoir risk. The former is modelled by two stochastic price processes, as a geometric Brownian motion (GBM) and as an Ornstein Uhlenbeck process. The latter is modelled by an optimization model that derives the optimal production plan. We analyze a stylized case designed in cooperation with industry professionals that resembles real aspects of petroleum projects. We evaluate the option to develop an oil field and the option to obtain more information through an appraisal program, and find the optimal oil field value and investment strategy.

Our study show four main findings; (i) There is considerable value in accounting for the managerial flexibility of deferral and investment in appraisal compared to the traditional DCF method;(ii) The value of obtaining new information may be significant; (iii) The firm will adjust its production based on the oil price and be willing to make a larger investment if the price condition is promising; Finally (iv) The valuation where we assume oil prices to follow a mean reverting process results in lower values compared to when we assume oil prices to follow a GBM process.

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Sammendrag

Denne oppgaven studerer investeringsproblemet til et oljeselskap som må ta stilling til når og om det skal utvikle et oljefelt, om de skal investere i et avgrensningsprogram og hvordan de skal bestemme optimal produksjon for oljefeltet. I lengre tid har den tradisjonelle diskonterte kontantstrømsmetode (DCF) vært den dominerende fremgangsmåten blant selskaper og investorer ved investeringsbeslutninger. Dette er til tross for at metoden ikke kan utnytte de operasjonelle fleksibilitetene som er tilstede i et oljeutviklingsprosjekt. For å dra nytte av disse, anvender vi en realopsjonsanalyse som lar oss inkludere både markeds- og reservoar risiko i modelleringen. Førsnevnte modelleres ved bruk av to stokastiske prisprosesser, som en geometrisk Brownian motion (GBM) og som en Ornstein Uhlenbeck prosess (OU). I oppgaven analyserer vi et fiktivt case utarbeidet i samarbeid med fagfolk i industrien med mange likhetstrekk til ekte petrolemsprosjekter. Vi evaluerer opsjonen til å utvikle et oljefelt og opsjonen til å innhente mer informasjon gjennom et avgrensningsprogram, og finner oljefeltets optimale verdi og tilhørende investeringsstrategi.

Vår studie viser fire funn; (i) Det er betraktelig verdi i å ta hensyn til fleksibilitetene av å kunne utsette investeringen, og muligheten til å investere i et avgrensningsprogram, sammenlignet med den diskonterte kontantstrømsmetode; (ii) Verdien av å innhente mer informasjon om reservoaret kan være signifikant; (iii) Selskapet vil justere sin produksjon basert på oljeprisen og vil være villig til å foreta større investeringer dersom nåværende pris er lovende; Avslutningsvis, (iv) Verdien av oljefeltet er mindre når vi antar at oljeprisen følger en prosess med reverseringseffekt (OU) sammenlignet med når vi antar at oljeprisen følger en GBM prosess.

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Preface

This thesis is written as a the concluding part of a Master of Science in Industrial Economics and technology Management at the Norwegian University of Science and Technology(NTNU).

Gratitude is extended to our supervisor Prof. Verena Hagspiel for proposing such an interesting topic to work with and for continuous support through the semester. We thank co-supervisor Semyon Fedorov for his feedback and helpful discussions. We also would like to thank Prof. Milan Stanko at the Department of Geoscience and Petroleum (IGP) for guidance with the reservoir modelling. Finally, we would like to thank Thomas Lerdahl, VP Reservoir and Production Technology at OKEA, for introduction to the process of field development and for providing data to work with.

Trondheim, 09 June 2019.

Inti J. Castro　　　　Partik Singh

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Chapter 1

Introduction

This master thesis studies the development decision of an oil field on the Norwegian continental shelf. The decision maker faces the question of whether and when to invest in development of a field and how it should produce optimally. Field development projects are characterized by great uncertainties, but also feature managerial flexibility. We therefore also account for the opportunity to obtain more information about the reservoir by investing in appraisal at an additional cost. Furthermore, development projects are characterized by large initial investments, high operating expenses and great uncertainty surrounding oil prices and reservoir size resulting in large variation in project economics. To exploit the inherent flexibilities in field development projects one can pay close attention to certain key factors of the project to model the value of the flexibilities and determine the optimal investment strategy.

For years, the traditional discounted cash flow (DCF) method has been the way firms and investors make capital budgeting decisions. The method is simple and intuitive to understand, making it an easy choice for decision makers to base their decisions on. However, its simplicity comes at a cost, i.e. the method assumes conditions that are not consistent with reality and thus the method may lead to sub optimal decisions. One of the most structural issues with the method is its deterministic perspective of reality. The DCF assumes all uncertainty to be resolved once the now-or-never investment decision has been made, and does not consider that the decision maker may have the choice to change or alter his decisions if new information is revealed in the future. Such assumptions are not realistic. The DCF method is thus incapable to account for the value of managerial flexibility. As fields are becoming economically marginal, the need for better valuation methods, that are able to capture the value of flexibilities in a project, is needed. These flexibilities are commonly referred to as "real options" to highlight the similarity to financial options (Brandão et al. [2005]). Even though real options theory is appealing from a theoretical perspective, and have been so since Stewart Meyers coined it in 1977, their use in practice have been limited [1]

[1] Horn et al. [2015] and Ryan and Ryan [2002]

Authors have argued that the disconnect between academic and practical use of real options as a capital budgeting method is due to the fact that many of the models presented in academia are too stylized to capture the complexities of real world problems (Guedes and Santos [2016]). On the other hand, the use of real options analysis (ROA) to capture the complexities in projects quickly becomes too mathematical for decision makers to use with relative ease.

We attempt to deal with these challenges by constructing a ROA for a valuation of an oil field development, where we develop a model that makes a reasonable trade-off between computational ease and the ability to capture crucial characteristics in an oil field development project. In addition, we contribute to the research field by making this model user friendly as we use input that are economic figures already required of the decision maker by regulation processes by the Norwegian petroleum Directorate (NPD).

We analyze how one can look at field development projects through an integrated real options approach first presented by Smith and McCardle [1999]. Specifically, they considered reservoir uncertainty and market uncertainty. We extend Smith and McCardle [1999] by; (1) Extending the integrated approach to also consider the decision maker's expectations of reservoir size and the updating of these through an optional appraisal program, (2) By determining optimal production profiles for the given expectations about reservoir size, also taking into account the varying operational and capital expenditures. To account for market risk, we model future oil prices as stochastic using both a geometric Brownian motion(GBM) process and an Ornstein-Uhlenbeck(OU) process to see how option values are affected by oil price development assumptions.

Our study shows four main findings: (i) There is considerable value in accounting for the managerial flexibility of deferral and investment in appraisal compared to the traditional DCF method, (ii) The value of obtaining new information may be significant, (iii) The firm will adjust its production based on the oil price and be willing to make a larger investment if the price condition is promising and finally (iv) The valuation where we assume prices to follow a mean reverting process results in a lower valuation compared to when we assume prices to follow a GBM process.

The thesis is structured in the following manner: In Chapter 2 we present a brief overview of the Norwegian petroleum industry and how the NPD regulates the activity on the Norwegian continental shelf. In Chapter 3 we present how we define flexibility and discuss the flexibilities we account for in the valuation. We also explain different valuation methods in the literature, explain their differences and motivate for the use of ROA. Chapter 4 contains a thorough review of the literature, both for ROA in the petroleum industry and for ROA as a field in general. In Chapter 5 a discussion regarding stochastic price modelling is presented followed by the model description in Chapter 6. The results and comparative statics and the conclusion is presented in Chapter 7 and Chapter 8, respectively.

Chapter 2

Introduction to the Norwegian petroleum industry

In the following chapter we give an introduction to the Norwegian petroleum industry. We look at the framework used to regulate petroleum activities and finally we look at the field development process on the Norwegian continental shelf (NCS). The information presented in this chapter is publicly available and gathered from the Ministry of Petroleum and Energy (MPE) and Norwegian Petroleum Directorate (NPD).

2.1 Petroleum activities on the Norwegian continental shelf

Approvals and permits from the Norwegian Petroleum Directorate(NPD), an advisory body of the Ministry of Petroleum and Energy (MPE), are necessary in all phases of the petroleum activities. This includes everything from the award of exploration and production licences in connection with acquisition of seismic data and exploration drilling, to plans for development and operation, and plans for field cessation.

The continental shelf is divided into smaller parts called blocks. The authorities carry out economic and social evaluation and potentially open up new blocks for activity. Licenses are awarded by the MPE. The production license is valid for an initial period, known as the exploration phase. This phase may last for up to 10 years, during which the license holder has to initiate geological/geophysical, seismic tests and/or exploration drilling. If the block is proven to have petroleum deposits, the licensees can enter an expansion period for the development and operation. Before the development an appraisal period is common where the goal is to increase certainty regarding the size of the deposits. If the results are negative, they relinquish the license. Given that the companies in the license are considered capable of carrying out development and operation by the authorities, they can submit a formal document, a Plan for Development and Operations (PDO) to be approved by the MPE for further development. There are strict regulations as to the content of this document, and the details are discussed below. If the ministry grants the development, the applicants can start the development and operation of the oil field. Cessation of the petroleum activities are also regulated by the MPE and licensees are required to submit a decommissioning plan ahead of cessation.

2.2 Field development on the Norwegian continental shelf

As mentioned earlier, the PDO is a formal document submitted to the Ministry. When key milestones are reached in the petroleum project, the companies are eligible to submit a PDO. Following is a description of the process to be granted permit to develop a field on the NCS.

The PDO describes the licensees evaluations of the petroleum deposits and what consequences the planned development will have. A PDO thus account for the entire development concept. The authorities have defined key milestones of the development phase. These are:

- Concretisation Decision - BOK: The licensees identify at least one technically and financially feasible concept that lays the foundation for further studies and leads to concept selection.
- Decision to Continue - BOV: The licensees decide to continue studies for one concept that leads to a Decision to Implement.
- Decision to Implement - BOG: The licensees make an investment decision that results in submission of a PDO.

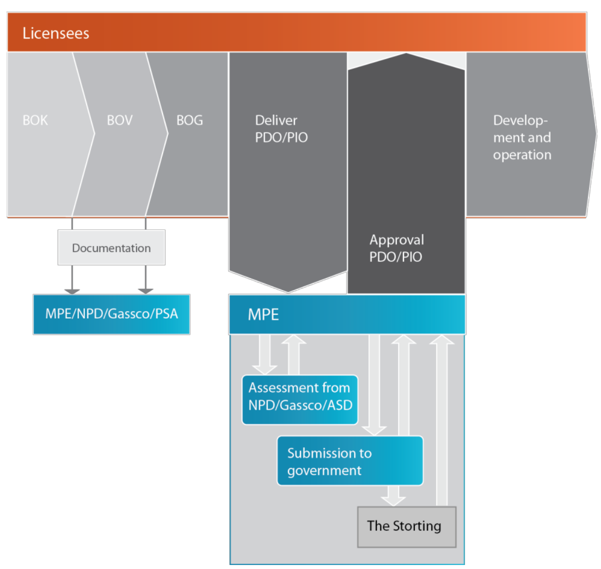


Figure 2.1: Overview of key milestones and processes for an oil project on the NCS. [Source: Norwegian-Petroleum]

Figure 2.1 summarizes the entire process. During the first three milestones, the companies and the MPE have a continuous dialog regarding the technical, financial and social aspects of the project. The Petroleum Saftey Authority (PSA) is also a part of the project. If natural gasses are present in the reserves, the state owned company, Gassco is also involved in the process. After the submission of the PDO, the MPE along with the other government institutions decide whether the PDO should be approved. Upon approval, the development and operation of the field can finally begin.

2.3 Staged investment in the petroleum industry

The milestones above are based on results from different stages in the process of developing a field. Petroleum production is a multi-stage process which includes sequential investment decisions, which can generally be divided in exploration, development and extraction. In this section the different stages are discussed along with the criteria for moving to the next stage.

In the first stage, i.e. exploration, a company acquires permits to explore a tract of land and conduct geological, geophysical and seismic surveys. Even if the surveys indicate petroleum reserves in the land, this has to be confirmed. The confirmation is done by a wildcat (exploratory) well. If the wildcat confirms hydrocarbons, appraisal wells are drilled to precisely determine the characteristics of the reservoir. The samples collected from these wells give important information about the size, quality, composition and architecture of the reservoir. The exploration stage is characterized by high initial investment as drilling wells is costly.

After the appraisal wells have further confirmed the presence of hydrocarbons and given sufficient information about the deposits, the next step, i.e. field development, can start. This phase consists of further investigation of the reservoir. The aim of this phase is to provide a technical and commercially feasible option for field development, and additional wells might be required. Their purpose is to reduce the uncertainty surrounding the reservoirs, and provide a better mapping of the reserve is achieved. In this phase the engineers also consider every practical aspect of the entire project moving forward. This includes logistics, power, safety and much more. Finally, the engineers choose the best suited field concept based on the gathered information. If it is decided to move forward from the development phase, the implementation of the plans from the development phase can start and subsequently the extraction of the deposits.

Chapter 3

Flexibility

The term flexibility is frequently used in the field of corporate resource allocation, especially in projects where decisions are made sequentially, as in petroleum projects. In sequential decision making the issue of flexibility arises when a decision maker must choose between a position today that accommodates changes in the future against one that leaves little or no room for changes. When the future state of a situation is uncertain, a reversible, or flexible position is usually preferred in order to act as uncertainty is resolved over time. The decision maker thus has the opportunity to change his mind when he receives new information, essentially limiting risk of early commitment when the future is unknown. There is of course a trade-off to be made regarding early investment benefits and being able to capitalize on possible favorable opportunities in the future. We discuss flexibility in the context of investments in petroleum projects.

There are two distinct set of issues associated with flexibility: The first concerns the definition and modelling of flexibility and the other concerns how to value flexibility. In this chapter, we aim to formalize and explain how we define flexibility and what kinds of flexibilities that are most relevant in petroleum projects. Finally, we discuss methods to value the flexibility.

3.1 Definition of flexibility

The literature has frequently addressed the notion of flexibility and provided several definitions of the term. Benjaafar et al. [1995] defines flexibility as "the property of an action that describes the degree of future decision making freedom this action leaves once it is implemented." Mason et al. [1995] state that the "flexibility of a project is nothing more (or less) than a description of the options made available to management as part of the project". These definitions are in line with the common view of what flexibility is, but note that in the first definition there is no restriction on what "future" is, and in the second there is no limit for how long the "options made available" are valid. Lund [1997] provides the following definition: "Flexibility is the possibility to make future decisions after an action is taken". He discusses the term "possibility" and acknowledges the difficulty in quantifying the "possibility". He next gives attention to the word "decisions" and explains how it includes a spectrum of actions available to the decision maker, but without a more distinct description of the decision, the comparison between them is of no value. We do however choose the definition provided by Lund [1997], and similarly to him define how to compare flexibilities in order to subsequently quantify the flexibility.

3.2 The value of flexibility

3.2.1 Comparing different flexibilities

The need to quantify flexibility raises the question of how to compare flexibilities in order to choose the action that gives the highest value. An overly simplified solution would be to assign the initial action with the highest number of available actions left as the most valuable. This solution would be ill-founded, as the available action left could have different values. Consider Table 3.1 taken from Lund [1997] for a sequential two staged investment decision with three possible initial actions A1, A2 and A3.

Table 3.1: The table shows the initial and second period decision available to the decision maker in a two staged investment.

Action in first period	Numbers of actions available in second period	Actions available in second period
A1	2	B1, B3
A2	3	B1, B3, B4
A3	2	B1, B2

In Table 3.1 both the number of actions available and the actions themselves are listed. If we only consider the number of actions available after the initial actions are taken to make up the value of a decision, we would conclude that A2 is the better one (It has three actions available in the second period compared to two with other actions in period one). However, this might not be true as every action in period two may not need to be worth the same, meaning that B1, B2 and B3 need not be equally valuable. The value in flexibility is not necessarily in how many actions one has available, but rather in the gain from being able to choose an action. This means that an action in the first period that only has one action available in the second period may still be more valuable than an action in the first period that has many actions available in the second period.

3.2.2 Important types of flexibility in petroleum projects

In the following we will discuss different option types, i.e. different types of flexibilities relevant in the petroleum industry. We consider only managerial flexibilities to the scope of this research as we only want to consider decisions at the project level. We therefore do not discuss flexibility in capital structure i.e.

financial flexibility. The managerial flexibility is associated with the operating flexibilities management has during a project, and includes everything from start till termination of the project.

3.2.2.1 The Flexibility to Defer Investments

The flexibility to defer an investment refers to the flexibility when the decision maker has the possibility to decide whether to investment immediately, or to postpone the decision. In a general sense, it can also be looked upon as the flexibility to decide if the investment should be made at all. Koller [2010] claims that the option to defer is equivalent to a call option on a stock. This way of valuing flexibility in investment opportunities are commonly referred to as real options to highlight the similarity to financial options (Brandão et al. [2005]). Koller [2010] compares the stock to an undeveloped oil reserve, and the call option as a lease on that reserve. The development cost is then equivalent to the strike price. As long as the option is alive, i.e the lease is valid, the decision maker can choose to develop the field by paying the development cost.

McDonald and Siegel [1986] and Paddock et al. [1988] found that the option of waiting to invest could have significant value. For projects with large initial investments, this flexibility can present a significantly large value. The deferral would be valuable if the decision maker is able to get additional valuable information regarding the reservoir or the financial situation surrounding the project. A common example here is to wait and observe the oil price development. The development is usually modelled through stylized stochastic models. Reservoir specific information can also be valuable. An oil field development requires large initial investment, as the capital expenditure is usually large compared to the rest of the costs associated with the project. Consider for instance a new technology that can improve the current appraisal technology available. This technology is however still being developed and requires the firm to postpone its appraisal activities. It can then potentially save the firm unnecessary expenditures if the firm can use the technology to more precisely determine the size of the reservoir. If the amount saved by appraisal is greater than the opportunity cost and the cost of the new technology, the option to defer has value.

The main driver of value is the uncertainty in both the reservoir and the financial situation in the market. Like its financial counter-part, the call option, the real option's value increases with increased uncertainty. Figure 3.1 from Lund [1997] illustrates how the value of a call option increases with increasing volatility in the oil price. The straight line represents the value without flexibility, and is the result from a simple NPV analysis. As the figure illustrates, this valuation is lower than the curves, which represent the value the project with flexibility and different volatilities, with $\sigma_1 < \sigma_2$. In section 3.3 we will discuss methods for evaluating projects with flexibility.

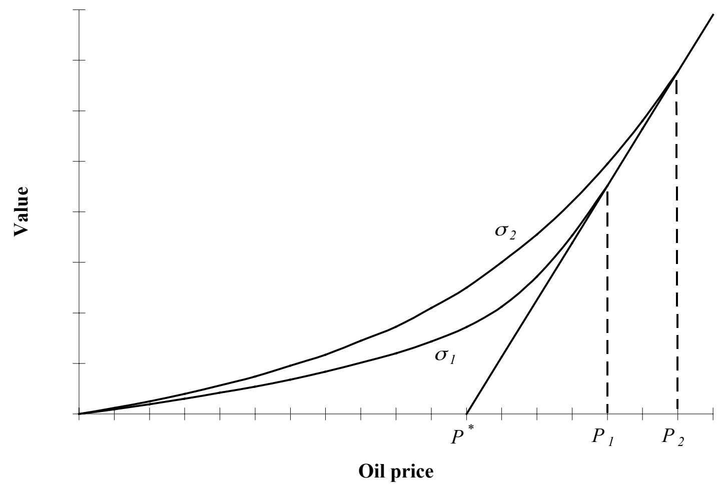


Figure 3.1: Value of project with/without the flexibility to defer investment. The straight line starting from P* shows the value if there is no flexibility. The curves represent the values with flexibility and variance σ_1 and σ_2 where $\sigma_1 < \sigma_2$. [Source: Lund [1997]]

3.2.2.2 The Flexibility to Abandon

The flexibility to abandon is the flexibility to terminate a project when it does not perform well enough. Usually economic factors are the base for a termination. The option to abandon a project is equivalent to a put option on a stock (Koller [2010]). If we consider an oil field as the project, the expected resale value of the project is then equivalent to the strike price. As the liquidation value puts a lower bound on the project, a project with the option to abandon is more valuable than a project without.

In theory, the abandonment of a field development project can take place at any time. However, local authorities might have laws and regulations regarding the termination of such projects which may restrict the firm's ability to terminate. For instance, Norwegian authorities require a predetermined program for the development project. The predetermined plan may turn out not to be optimal, but deviation from the plan may not be feasible. This is mainly the case for the Norwegian continental shelf where the NPD[1] imposes strict regulations on all activity on the shelf (NPD [2019]).

As with other options, the value of the abandonment option decreases with reduced uncertainty. This means that as time passes and one learns more about the project's cash flows, the value of being able to terminate the project diminishes. In addition, the capital expenditures of an oil field development is a substantial part of the total cost, resulting in the exercise price increasing significantly after the initial investment. Consequently making the option less likely to be exercised, and thus less valuable(Lund [1997]). For field development projects abandonment usually corresponds to shutting down production and disman-

[1] Norwegian Petroleum Directorate

tle infrastructure. However, in some cases the equipment from the field, or the entire field can be sold to other parties. In this case the option's value is determined by the future oil price, decommissioning costs and the salvage value of the equipment, which are all uncertain (Jafarizadeh and Bratvold [2012]).

3.2.2.3 The Flexibility to Expand/Contract Capacity

The flexibility to expand a project is equivalent to a financial call option on a stock. Conversely, the flexibility to contract an option is conceptually equivalent to a put option on the stock. If the project performs well/poorly, the decision maker has the ability, but not the obligation to make an additional follow-up investment and expand/contract the project. The cost of expanding or future spending saved by using the option is equivalent to its exercise price (Koller [2010]).

Real world projects should be engineered to accommodate options to expand/contract if necessary. Capacity in petroleum projects can refer to various factors illustrated in Figure 3.2. Here the three axes represent different types of expansion/contractions where the level of expansion/contractions can be changed by varying along the axes.

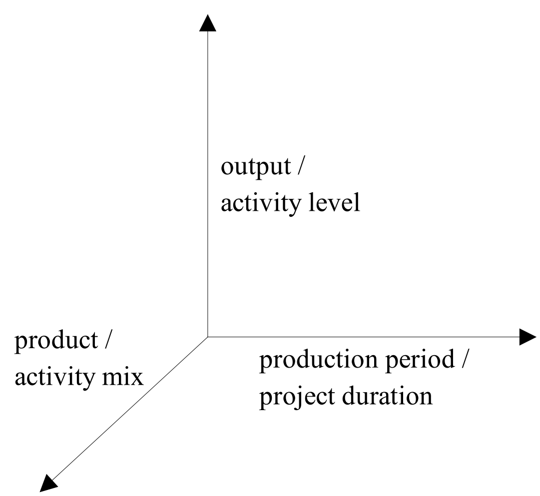


Figure 3.2: The figure illustrates expansion/contraction possibilities for a project. [Source:Lund [1997]]

For petroleum projects, it can be argued that the option to change the product mix is not really an option, as the product is fixed through physical properties of the reservoir (Lund [1997]). The capacity expansion/contraction of the output level from the field during operations refers to the ability to produce oil at a higher/lower production rate. A main reservoir specific uncertainty is the size of the reservoir. Therefore firms should build production facilities that take the production rate into consideration, and let the decision maker adjust it according to new information. The third axis, project duration, can be seen as dependent on the activity level decisions, especially in the production period. If the decision maker wishes to extend/shorten the duration, he will decrease/increase the production rate. A higher production rate can be of value if the current financial situation in the oil market is favourable.

3.2.2.4 Choosing options to include

We have till now discussed several flexibilities that may be included in an options analysis. However, not every option is found in every investment problem, and the challenge is if we have successfully identified the option inherent in the specific project at hand. This means that there is no list of options we have to include in all projects as the list is dependent on the details of each project, which vary with each project. We discuss how this may be done in practice in chapter 6.1

3.3 Methods for Valuing Flexibilities

Before we begin the discussion of different methods to value flexibilities inherent in petroleum projects, we need to properly motivate for the departure from the traditional valuation approaches. We therefore begin this section by bringing forth the main criticism of the NPV approach and argue that new approaches are needed. We follow this with alternative approaches that have gained much interest in the capital asset allocation field, and that are able to capture the value of the embedded options in real world projects. We discuss two methods for contingent valuation, namely Real Options Analysis and Decision Analysis.

3.3.1 Traditional approach

The traditional approach to capital budgeting is commonly termed Discounted Cash Flows (DCF) or static NPV approach. The terms are used interchangeably in the text. Although criticized, it is still widely used in valuation of exploration & production projects due to its simplicity (Willigers et al. [2017]). One of the strengths of the method is that it is fairly simple to apply. First, one calculates the present value of the expected stream of profits that the project will generate. Next, one calculates the present value of the stream of expenditures needed in the future. Finally, the net present value (NPV) is obtained by subtracting the latter to the prior giving us the present value of the project. To determine whether an investment should be made, a simple rule is used; If the resulting NPV is greater than zero, the method suggests that the investment should be made, otherwise not. Formally the NPV can be represented as:

$$NPV = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} - I_0 \tag{3.1}$$

where

CF_t: Net Cash flow at time t

r: Discount rate

I_0: Initial investment cost

T: Time of final cash flow

However, the simplicity of the method comes at the cost of assumptions about the project that have to be made. These assumptions fail to capture and reflect the characteristics of real-world projects. First, the investment is assumed to be of a "now-or-never" type meaning that the decision maker has to determine whether to invest today, using only information available today (Copeland and Antikarov [2001]). Second, the cash flows of the project are assumed to be deterministic and known at the time of investment. The discount rate is assumed constant for the whole lifetime of the project and usually determined by using the weighted average cost of capital (WACC) (Willigers et al. [2017]). Furthermore, the decision maker cannot change his decision after the initial investment has been made meaning that the decision maker cannot respond to information that is revealed during the project by changing his decision.

There are several issues with the assumptions that have been pointed out. For instance, Neely III and de Neufville [2001] states that the method is conceptually wrong because it assumes that initiation entails a complete commitment to the deterministic cash flows. In real world projects information is usually revealed as the project runs and managers adapt to this information. For example, if a firm is to develop an oil field and prices were to rise, the firm might be able to develop more aggressively and expand production capacity. On the contrary, if prices were to fall below expectation the firm might be able to scale back or even terminate the project reducing future losses. Furthermore, (Neely III and de Neufville [2001]) state that " (the method is wrong) mechanically by assuming a single cash flow, they also assume that the value of this average cash flow equals the average value of a range of cash flows". The discount rate has also been a subject of discussion. The rate determined by the WACC is based on the firm's correlation to market fluctuations. Thus, if the project risk characteristics are not the same as the company's, this discount rate will be wrong (Hahn [2005]). Furthermore, a constant discount rate implies that the risk is constant throughout the project lifetime. When uncertainty stems from exogenous factors, Neely III [1998] argues that constant discount rates fail to reflect this. This is the case for field development projects where much of the uncertainty stems from the oil price. Ramirez [2002] goes even further and states that risk changes dynamically through the project lifetime and that the discount factor should reflect this. Thus, using a constant discount rate one cannot reflect the risk in a project.

Summarized, the traditional approach is not capable of capturing the flexibility that is inherent in real world projects. Furthermore, applying the approach may lead to undervalued investment opportunities, sub-optimal decisions and underinvestment (Trigeorgis [1993]). We will return to the presented issues when we discuss decision analysis and the real option approach in the following sections.

3.3.2 Decision analysis and the Real options approach

The traditional approach described above may be simple to understand, but it lacks many practical considerations. One of the biggest concern analysts have is that the static NPV method is not able to evaluate projects with significant managerial flexibilities. To deal with these kind of problems one has to consider approaches that are able to account for and model such flexibilities. There are two major approaches that do so: Decision analysis and Real option analysis. Both methods acknowledge and account for uncertainty about the future and incorporate managerial flexibility. The difference between the two methods is that they are based on different theoretical foundation. In the following sections the two approaches will be explained.

3.3.2.1 Decision Analysis

The decision analysis approach involves a structured representation of the uncertainties and future decisions available to the decision maker (Smith and McCardle [1998],Trigeorgis [1995]). Typically in this framework the decision problem is solved by constructing either a decision tree, a dynamic program or an influence diagram that describes the uncertainties, alternatives and the sequence of decisions surrounding the project (Smith and McCardle [1998]). However, only decision trees and dynamic programming are commonly used. The values for the cash flows, the discount rate and probabilities that are used are based entirely on the decision maker's beliefs regarding the project given the information that is available to him at the time (Trigeorgis [1995]). The decision maker's preference for the cash flows are captured by both the discount rate and a utility function unless the decision maker is assumed to be risk-neutral. The expected value of the project is then found by applying a "roll back" procedure, for instance by a dynamic programming procedure, working backwards through the sequence of decisions obtaining the optimal value and the optimal decision strategy (Smith and McCardle [1998]). Worth noting is that if a utility function is included, the value of the project is actually the expected utility of the project and not the present value (Trigeorgis [1995]). The implication of this is that the calculated project value may not be the same as the market value of the project because the market may value risk differently.

Decision tree analysis (DTA) A decision tree is a sequence of decisions and chance nodes, ending on a terminal node. A decision node indicates a point where the decision maker faces a decision. The branches out of a decision node represents the decisions available to the decision maker (Ramirez [2002]).

Figure 3.3 illustrates how one can structure the decision in an oil field development project. The specific example in this particular tree initially considers whether the field should be abandoned or developed. Next, it illustrates the decision maker's beliefs about the size of the reservoir. The decision maker would typically assign (subjective) probabilities to the different size outcomes, and thus structure his expectations about future outcomes of uncertain events. The same will be done for cost and prices, which are usually also considered uncertain in oil field development projects.

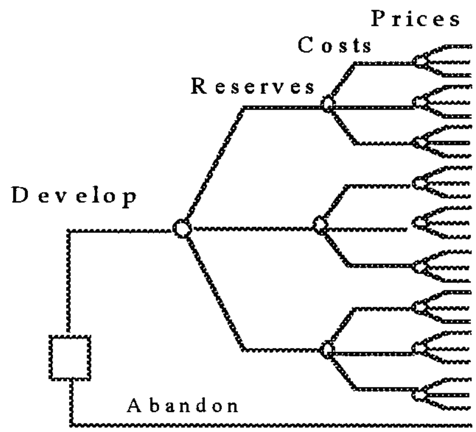


Figure 3.3: The figure illustrates how decisions and possible future outcomes can be organized in a tree structure. [Source: Smith and McCardle [1999]]

The solution procedure in DTA is as follows; First compute the terminal node values. The next step is to apply the "roll-back" procedure; Starting at the terminal nodes one works backwards finding the optimal decision at each decision node by choosing the decision that gives the highest expected value (de Neufville [1990]). This procedure is repeated all the way to the initial node resulting in an optimal value and the associated decision strategy. An advantage of this approach is that one obtains a strategy for every state and time in the tree, which may be beneficial if the decision maker chooses to defer investment, rather then to assume that the chosen strategy will remain unchanged even if new information arrives. Then he would still have an optimal strategy to follow, regardless of what node he is at. The second and maybe the most important advantage is that the tree representation is simple to understand and gives an intuitive representation of the problem. However, for problems with multiple decisions and several uncertainties the intuitiveness of the tree structure may be reduced and more importantly the calculations become computationally more demanding.

Dynamic programming Dynamic programming (DP) decomposes the decision problem by diving it into smaller subproblems separating each decision point. The theoretical foundation for this approach is the Bellman's principle of optimality stating that "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". The Bellman equation which stems from this principle (Equation. 3.2) decomposes the series of decisions into N sub problems, each subproblem consisting of an immediate payoff π and a continuation value. The continuation value consists of the expected value of all subsequent decisions at stages $n+1, ...N$. The bellman equation is given by:

$$V_n(i) \;=\; \max_a \left[\pi(i,a) + \frac{1}{1+r} \sum_j p_{ij}(a) V_{n+1}(j) \right] \quad \forall i, n = 1, ..., N-1 \tag{3.2}$$

and

$$V_N(i) = \max_a [\pi(i,a)] \quad \forall i \tag{3.3}$$

Where

$V_n(i)$: Maximum expected value in state i at stage n

$\pi(i,a)$: The immediate payoff obtained by taking action a in state i

r: The discount rate per period

p_{ij}: The probability of going from state i to state j given action a

N: The total number of decision stages

In the same way as in the DTA approach the optimal value is found by working backwards from the last stage, $n = N$, to the initial stage, $n = 0$. At $n = N$ the Bellman equation is reduced to Equation 3.3. Here the optimal decision is the one that maximizes the immediate payoff. For all other stages the optimal decision a is the one that maximizes the sum of the immediate payoff π and the continuation value. Note that at each stage where $n < N$ the continuation value is easily obtained as the optimal decision and associated value are already obtained in the previous stage, $n + 1$.

Several authors have applied dynamic programming to development projects (Bjørstad et al. [1989], Dixit and Pindyck [1994] and Lund [1997]). An advantage of the approach is the simple mathematical formulation we get of the problem. A second advantage is that one only needs to find the current optimal decision as the subsequent optimal decisions are assumed in the continuation value, which is computationally favorable (Lund [1997]). However, a common critique against this method is the "curse of dimensionality". In short, when the number of decision variables increases, the problem size becomes challenging because the number of continuation values to calculate increases (Lund [1997]). In the same way, increasing the possible number of outcomes of each decision will lead to the same challenge.

The Decision Analysis (DA) approach deals with incorporating flexibility which the NPV model is not capable of. Even though the DA deals with one of the issues with the NPV method, the issue of determining the correct discount rate remains. As discussed above, the decision maker's preference for cash flows are reflected in the discount rate, meaning it is a subjective rate. When dealing with incorporating flexibility, another issue arises; how should the probabilities for different outcomes be determined? The DA approach leaves this up to the decision maker, which may result in different probability distributions among decision makers for the same projects. Next, we look at modelling of flexibility and valuation of risky cash flows from an option theory perspective which attempts to resolve some of these issues.

3.3.2.2 Option theory approach

The term real options refers to the application of option pricing to value real world projects and was coined by Stewart Myers in 1977 (Borison [2005]). Similar to the DA approach, real option analysis (ROA) is able to capture the flexibility and uncertainty inherent in real world projects. However, they differ fundamentally in that rather than relying on subjective input to valuation as in DA, ROA instead relies on market information to determine the project value (Zhang [2010]). An important outcome of this is that the value obtained by applying ROA is the market value of the project (Dixit and Pindyck [1994]), i.e the price that it would have if traded in the securities market. Additionally, by using market information the approach overcomes the challenge in DA and DCF of determining an appropriate discount rate. The theoretical foundation stems from financial theory which we discuss in the next section before we move on to explain the underlying assumptions of ROA and how the approach is applied to value projects.

Financial options By definition, a financial option is the right, but not the obligation, to buy or sell a financial asset at a pre-determined later date for a pre-determined price. The option to *buy* is called a *call option* and the counterpart, the option to *sell*, is called a *put option*. The time at which the decision to exercise the options has to take place is either at a fixed date in the future, *European style options*, or within a specified time period, *American style options*. The price needed to pay in order to receive the asset is called the strike price. An option is a derivative, meaning that its price is derived from the value of an underlying asset.

Black and Scholes [1973] presented a closed form formula to price European options. This concept is one of the most important concepts in modern financial theory presenting the basis for the majority of the option pricing models used (Bungartz et al. [2012]). The formula for an European call option can be stated as

$$\mathrm{C(S,t)} = \mathrm{N(d_1)S} - \mathrm{N(d_2)Ke}^{-rt} \tag{3.4}$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{3.5}$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{3.6}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \mathrm{e}^{-\frac{1}{2}z^2} dz \tag{3.7}$$

where

$C(S,t)$: Call premium

S : Current stock price

t : Time until option expiration

K: Option strike price

r : Risk-free intrest rate

$N(x)$: Cumulative standard normal distribution

σ : Standard deviation

Real options Real options are the application of financial options theory to value investment decisions on real assets. A real option is "the right, but not the obligation to take an action (e.g., deferring, expanding, contracting or abandoning a project or investment) at a predetermined cost (the exercise price), for a determined period of time (the life of the option)" (Ramirez [2002]). Even though the pricing concept is borrowed from financial options pricing models, the underlying assumptions are not identical. Lütolf-Carroll and Pirnes [2009] explain the differences in the assumptions: A financial option is a derivative, meaning that its value depend on the uncertainty of its underlying financial security. For financial options, the underlying and the option is normally traded on regulated exchanges. The idea behind real options is capital budgetting, i.e. managerial decisions related to illiquid assets. These assets are usually research and development, natural resource extractions, intellectual property or other non-financial assets, and are typically not traded on exchanges. It follows that assumptions behind the Black & Scholes option pricing formula do not apply to real options. The pricing formula stated in Equation 3.4 assumes that the uncertainty in the option stems from only one source, the volatility in the underlying financial security, and on a single decision date. Real options, however, can have multiple sources of uncertainty, both financial and non-financial.

As mentioned real options rely on the use of market information to value the project at hand and the classic real options approach makes some important assumptions about the market. First it is assumed that the markets are complete in the sense that it is possible to replicate the projects risk characteristics by finding a traded asset or by creating a replicating portfolio (Dixit and Pindyck [1994]). In the case of an oil field development projects the oil price is commonly assumed to be the driver for uncertainty and the chosen assets underlying (Dixit and Pindyck [1994], Schwartz [1997] and Paddock et al. [1988]). We will refer to this as the complete market assumption. Furthermore, the value of the replicating portfolio (a traded asset or several assets) is assumed to vary stochastically (Borison [2005]). The implication of these assumptions are important. As the risk can be spanned in the market it is possible to create a replicating portfolio and value this by a no-arbitrage argument[2]. The value of this portfolio is then the market value of the project. Conceptually the approach seems promising. By using market information the project value derived is the market price of the project if it were traded, thus there is no need to make assumptions about the decision makers' preferences for the cash flows. Furthermore, as mentioned the discount rate is not determined subjectively. Instead it is determined by the return of the portfolio. As the risk can be spanned it is possible to create a risk free portfolio, which in turn is discounted by the risk-free rate. There are several ways of applying this procedure to investment problems. We focus on contingent claim analysis (CCA) and binomial trees (BT). The first relies on solving differential equations in continuous time, while the latter is a numerical approach relying on the use of decision trees similar to the DTA approach.

[2]For there to be no arbitrage all portfolios with the same risk must earn the same return

Contingent claim analysis (CCA) In this approach, one begins by creating a risk-free replicating portfolio by holding (going long) the option to invest and shorting n units of the project (or the underlying which is perfectly correlated with the project) (Dixit and Pindyck [1994]). Second, as the portfolio is risk-free, its return is equated to the risk free rate. Third, applying Ito calculus and assuming a stochastic process for the underlying, a differential equation for the portfolio's stochastic development is derived. Finally, two boundary conditions that specifies the conditions that must hold at the optimal time of investment are applied to obtain both the option value of the project and the optimal time to invest. For a thorough explanation of the solution procedure the reader is referred to Dixit and Pindyck [1994].

The procedure explained above, conceptually seems straightforward. However, there is no closed form mathematical solution available to the project at hand. For each project, differential equations in continuous time must be established and solved. Furthermore, the boundary conditions are unique to the project at hand and must be specified for each project. Thus a simpler solution procedure seems desirable. A decision tree approach is an alternative approach that avoids continuous time calculations and the problem of establishing boundary conditions. We discuss this approach in the next section.

Binomial Tree approach The binomial approach transforms the decision problem from a continuous time problem to a numerical one by discretizing the price process. Its simplicity is achieved by the strong assumption that the price of the underlying asset at any time only may change to an up or a down value (McDonald [2013]) . This approach to option pricing was first developed by Cox, Ross and Rubinstein as a procedure to value options with early exercise (McDonald [2013]). The approach is, similar to CCA, based on the replicating portfolio and can be explained as follows:

Denote the value of the replicating portfolio by V, assume a stochastic process for the underlying asset and discretize it making the assumption mentioned above. This is illustrated Figure 3.4 where V in the next period may only take the value of V_u or V_d.

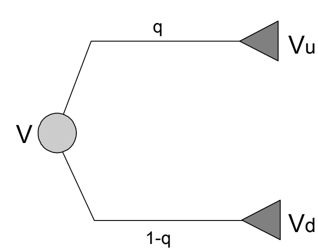


Figure 3.4: An example of a two period binomial tree illustrating the portfolio value, V for an up (V_u) and a down move (V_d). Adapted from Brandão et al. [2005].

Now, instead of using a subjectivediscount rate one assumes that the firm is risk neutral, calling for the use of the risk-free rate. Then by Equation 3.8 we derive the "risk neutral" probability denoted by q in Figure 3.4. These are the probabilities for an up or down move given that the firm is risk neutral. Thus one avoids determining the probabilities and discount rates subjectively. This can be done because the resulting project values are correct not just in a risk-neutral world, but in the real world as well (Hull [2009]).

$$V = \frac{qV_u + (1-q)V_d}{1+r_f} \tag{3.8}$$

Note now that if the project is valued using risk neutral probabilities we end up with the initial project value (V). The solution procedure to a real options problem is as follows; First, develop the binomial tree for the decision problem assuming a price process and calculate the project value (V) at each node. An example of a developed tree is illustrated in Figure 3.5.

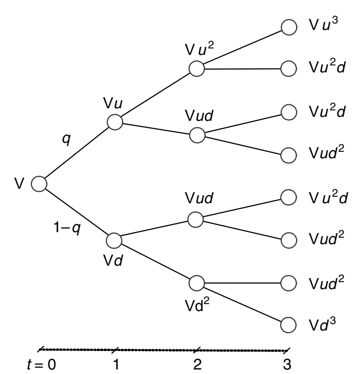


Figure 3.5: An example of a multiperiod binomial tree illustrating the value development of the replicating portfolio (V). Adapted from Brandão et al. [2005].

Then, starting at the terminal nodes (at time T) and work back to time t=0 applying the following decision rules:

$$E[\pi_i] = \begin{cases} max(V_t^i - I, 0) & t = T \\ max(V_t^i - I, q_u * E_t[\pi_{t+1}^u] + q_d * E_t[\pi_{t+1}^d]) & t < T, \end{cases} \tag{3.9}$$

Equation 3.9 states that at time T, i.e at expiration time of the option, the option is only exercised if the project value $(V_T^i)^3$ is greater than the investment cost (I). That is, the option value denoted by π is

[3] i refers to a specific node at a specific time. For instance, at time T, $i = 0$ refers to the upper node, $i = 1$ to the node below that etc.

worth either 0 or the surplus of $V_T^i - I$. Moving from the right to left in the tree the decision rule is to exercise now if the value of doing so is higher than the expected value of waiting another period discounted at the risk free rate. Note that since one works backwards the expected value is already calculated. This decision rule is applied all the way to the initial node and the value of the option at time t=0 is thus obtained.

Chapter 4

Literature review

4.1 Real option approaches used in the literature

In the following chapter, different applications of the real options methodology in the literature regarding capital budgeting are presented, before moving on to the different technical modelling assumptions in the literature. We base our presentation of the different approaches and their classification on Borison [2005] and discuss the view of proponents and critics of the different approaches. We consider the article to give a wide overview of different approaches in the literature and an adequate place to start the discussion on real options application. We also extend and add to the classification provided by Borison [2005]

The method considered as the starting point of real options analysis is classified as the "classic" approach by Borison [2005]. The assumptions of this modelling approach are the same as for financial option pricing. One assumes that the returns and risk of a real investment opportunity can be replicated by a traded portfolio. This portfolio is then priced based on a no-arbitrage argument. We will refer to these assumptions as the classic assumptions. The portfolio is then scaled according to the investment opportunity at hand and valued. The use of a replicating portfolio is considered the standard in real options application (Borison [2005], Rigopoulos [2015]).

In 1977 Stewart Myers coined the term "real options" and is recognized as the first to see option-like-features in real investment opportunities. Myers' work is mainly conceptual, while Tourinho [1979] is recognized as the first to use a real options approach to value natural resources. In his paper an oil reserve is considered analogous to holding a real option on the underlying resource, whose exercise price is the cost to develop it and extract the commodity.

Fundamentally, the difficulty with the classic approach is the justification for the existence of a replicating portfolio that exactly matches the risk of a real corporate investment. Borison [2005] argues that there has been given little attention to the covariance between real and financial assets, so that there is no theoretical principle for a replicating portfolio. He further argues that there is little empirical evidence to support a replicating portfolio theory. It is difficult to accept that e.g. a particular field development project would be highly correlated with a traded financial instrument. Dixit and Pindyck [1994] also acknowledge the strict requirement that the stochastic component of the return on the asset(real investment opportunity), has to be exactly replicated by the stochastic component of the return on some traded asset (or dynamic portfolio of traded assets). They suggest instead that markets only need to be sufficiently complete, meaning that at least in principle one could find a replicating portfolio.

The difficulty of finding an exactly replicating portfolio led to an alternative approach that Borison [2005] names the subjective approach. Essentially this approach make the same assumptions as the classic approach. One assumes a complete market, but the approach also includes assumptions about the probability distribution of the project's cash flows. This way one does not attempt to find the replicating portfolio, but uses subjective estimates to decide the volatility of the underlying investment. Then standard option tools are applied to value the investment. Luehrman [1998a] argues that the framework of the subjective approach "bridges the gap between the practicalities of real-world capital projects and the higher mathematics associated with formal option-pricing theory". The primary difficulty with this approach is the inconsistency between relying on a replicating portfolio, which should be found in the market, but using subjective inputs in the model. Borison [2005] exemplifies this by considering to value a traded asset and states that "it seems extremely unwise to rely on subjective assessments, rather than on direct market data (to do so)". At best the analysis would be restricted to a qualitative one. Luehrman [1997] claim that for generalists who have businesses to manage, absolute truth is not crucial, but getting closer to the truth is good enough. Additionally, a challenge is how to determine the value of the subjective inputs, perhaps most importantly the discount rate. Brandão et al. [2005] refer to Triantis and Borison [2001] and claim that Leuhrman's approach has generally been considered too simplistic.

The demanding requirement of a complete market is at the center of discussion when it comes to choosing an approach. Another strand of literature step even further away from standard option pricing theory than the subjective approach does. The Marketed Asset Disclaimer (MAD) approach assumes that there is no perfectly correlated twin security for the project at hand. Copeland and Antikarov [2001] argue that one "can use the project itself (without flexibility) as an estimate of the price it would have if it were a security traded in the open market." Borison [2005] criticizes the approach for its ignorance of the possibility that there might exist a replicating portfolio or that some elements of the investment might have market equivalents.

Borison [2005] argues that the approaches discussed above are all examples of "one-size-fits-all", meaning that proponents claim that any type of corporate investment can be valued by them. The revised classic approach differentiates between projects, and acknowledges that there are two types of risk, private and market risk. Here hedgeable risk is referred to as market risk and all other are termed private risk. The approach suggests the use of the classic approach wherever the assumptions of the approach are met, and use the subjective approach otherwise. The revised classic approach leaves it up to the decision maker to decide the dominating risk in the project and choose an approach accordingly. Amram and Kulatilaka [2000] support the idea of choosing the approach that is best suitable to capture the risk that drives the investment. Challenges with this approach is that the revised approach can only account for either private or market risk, all based on what the decision maker considers to be the dominating risk in the project.

Smith and Nau [1995] describe an approach they refer to as the integrated option pricing approach. The approach assumes that real world investment problems are neither purely market driven nor driven entirely by private risk. Smith and Nau [1995] claim that in contrast to the revised approach, the integrated approach is designed to handle both risks at the same time. Smith and McCardle [1998] explain that the basic idea of the approach is to use option pricing to value risk that can be hedged by trading existing assets and decision analysis to value risk that cannot be hedged. This means that they use market information to the extent that it can explain the uncertainty of the project, and use subjective judgement on the portion of uncertainty that the market cannot value. Brandão et al. [2005] argue that under ideal conditions investment problems can be divided into private and market risk, but that this is usually not the case in real life. As a practical matter, it may be difficult to find a replicating portfolio for all market risks in a project. Thus, they suggest using the MAD assumptions from Copeland and Antikarov [2001], and use the investment without flexibility as the underlying. Kretzschmar and Moles [2006] claim that Copeland and Antikarov [2001] miss the basic point of the real option approach: identifying and characterizing the underlying asset, which they avoid by using the project itself as the underlying. Borison [2005] concludes that "... (the integrated) approach is consistent, relevant and reasonably accurate." He further claims that the integrated approach is based on the most accurate and consistent theoretical and empirical foundation, and will result in the most accurate and credible result.

4.2 Real options in the petroleum literature

In the next phase of the literature review we focus on literature aiming to solve challenges related to petroleum projects. Contrary to the review above, we also focus on the technical aspects specific to the petroleum industry.

The literature usually divides petroleum project in phases. Even though authors focus on different sets of phases, the most common division is exploration, development, extraction and abandonment. Paddock et al. [1988] value an offshore petroleum lease using option theory. They view the possibility to explore, develop and extract as options. The decision maker has a finite time to exercise the option by paying the cost of entering the different phases. The time corresponds to the length of the license acquired. If the option is exercised, the license is converted into a production license, otherwise the lease must be returned to the government. This can be done at any time and is incorporated as the option of abandonment. Paddock et al. [1988] consider two sources of uncertainty in their model, namely production rate and the value of a unit of developed reserve. The production rate is modelled by a declining exponential function, and they assume the value of the developed reserve to follow a geometric Brownian motion process (GBM). They argue that this is a reasonable assumption as historical commodity prices suggest that the GBM distribution is able to model the prices well. They further use contingent claims analysis along with the assumption of a complete market and no arbitrage arguments to find the optimal timing for investment.

Similar to Paddock et al. [1988], Smith and McCardle [1999] also view the project to develop an oil field as a call option. The project is considered to be marginal at current prices, but may have considerable value if prices were to rise in the future. In addition to including the option to develop and abandon the project as in Paddock et al. [1988] , Smith and McCardle [1999] also account for the option to expand the production once the decision to develop has been made. The reasoning for adding the extra option is two-folded; first the option helps the managers to account for decision flexibility. Second, they argue that the Decision theory suggests that adding options can only increase the project value because of the added flexibility, and that the added value of capacity expansion is significant. However, Smith and McCardle [1999] claim that managers rarely incorporate these options thoroughly resulting in lower valuation result compared to when managerial flexibility is accounted for.

Furthermore, Smith and McCardle [1999] also discuss the number of variables that should be included in their model, as the computational difficulty increases significantly when the number of variables increases. They discuss three alternative methods to solve the decision tree. They consider two assumptions regarding price movements, namely the GBM and the OU process. First, they reduce the number of uncertain parameters and decisions modelled. They then apply a decision tree analysis approach to the problem. In the second model of Smith and McCardle [1999] a dynamic programming approach is applied to get a better understanding of the initial development decision. Additionally, the costs of waiting and abandonment are specified. Finally, they look at a simulation approach and argue that given a specific decision policy it becomes easy to find the mean and distribution of the cash flows, but that it is difficult to find the optimal decision strategy. The mean reverting model suggests that one should consider whether the project is profitable at the long-term average prices, while short-term prices are not particularly important. Smith and McCardle [1999] argue that this does not mean that the value of flexibility in its entirety is eliminated, as other flexibilities still hold value.

Cortazar and Schwartz [1997] and Dixit and Pindyck [1994] assume oil prices to follow a mean reverting stochastic process. For simplification they assume deterministic reserves, development investment and production unit cost. Cortazar and Schwartz [1997] also argue that a simple GBM can trigger a too late investment as it neglects mean reversion, in line with the arguments of Smith and McCardle [1999]. Cortazar and Schwartz [1997] apply a three stage contingent claim analysis assuming oil prices to follow a one factor mean reverting Brownian motion process. Furthermore, they use no arbitrage arguments to determine the functional relationship that must exist between both assets so that an investor cannot earn a profit without assuming risk as explained in (Dixit and Pindyck [1994]) and find that a significant fraction of the oil field value may be provided by the flexibility of delaying development investment. Dias and Rocha [1999] present a mean reverting Brownian motion model where they allow jumps in the price modelled by a Poisson process. This allows for modelling of dramatic news, wars etc that will affect the financial markets. They argue that this approach makes more economic sense than previous approaches used in the real options literature. They find that the option value is higher with this price process compared to modeling the price as a GBM.

Smith and McCardle [1998], focus on using an integrated approach on valuation and management of oil properties. They argue that DTA usually ignores the market opportunities for hedging. The paper presents two models where the authors extend the foregoing model by adding options to the next. They argue that this enables them to better capture the value of the options inherent in the project. In the models the price is assumed to follow a GBM while the technical risk is accounted for by assuming the production rate to also follow a GBM process. In the first model they look at only having the option to abandon. In the second, they add the option to drill more wells. They additionally suggest the extension to account for the option to suspend the production. In contrast to their earlier work, they now also assume the production rate to be stochastically distributed as well as the oil price. The idea of the integrated procedure is to use option pricing methods to value risk that can be hedged by trading existing securities, and use decision analysis procedures to value risks that cannot be hedged by trading. Smith and McCardle [1998] conclude that by using an integrated approach one produces results that are consistent with those that would be produced given a detailed model of the securities markets.

Lund [1997] applies a stochastic dynamic programming model for a petroleum project evaluation under uncertainty. The focus of the paper is to add value through flexibility. The model handles both financial and technical risk by assuming oil price, well rate and reservoir volume as variables. The oil price is modelled by a GBM process, which is approximated by a Cox, Ross, Rubenstein binomial tree. The reservoir volume is modelled by a priori probabilities with three possible outcomes, while the well rates are modelled by uniform a priori probabilities with two possible outcomes. The transition between rates are modelled by Markov processes. These variables in particular are chosen as Lund [1997] argue that they have the most significant impact on the production profile and the cash flow. In the particular case studied by Lund [1997], he concludes that today's common valuation methods, i.e. the DCF approach usually overestimates the value of the project. When compared to other projects, the method might then give sub-optimal investment decisions. The value of flexibility, that the options approach is able to capture, is significant and he argues that especially the value of capacity flexibility is substantial.

Cortazar et al. [2003] and Armstrong et al. [2004] also considers both market and private risk. Cortazar et al. [2003] argue that natural resource exploration investments are contingent on price and geological-technical uncertainty and develop a model where these factors collapse to a one-factor model. The consideration of technical and financial risk resemble the approach taken by Smith and McCardle [1998]. Furthermore, Cortazar et al. [2003] include the option that the exploration investment schedule may be stopped and/or resumed at any moment. In addition, when exploration is completed, the decision maker has the flexibility to postpone investment. When the facility is running, the decision maker also has the option to close/ re-open production. Armstrong et al. [2004] also consider technical uncertainty and value new technical information using Bayesian probabilities. Lima and Suslick [2006] consider additional sources of uncertainty associated with reserve development. More specifically capital and operational expenditures and production levels. Lima and Suslick [2006] show that the traditional assumption of using oil price volatility as a proxy for the project volatility may not give realistic results as most projects do not have a linear relationship between long term average oil price and operational expenditures.

Guedes and Santos [2016] argue that real option valuation is only valuable if a clinical approach is made, i.e. the analysis is tailor-made to capture the unique characteristics of individual projects. They model the oil price uncertainty both as a GBM process and as the mean reversion model developed by Cortazar and Schwartz [1997]. Next they use a binomial and trinomial tree to approximate the GBM process and mean reverting process (MRP), respectively. They argue for the use of a GBM process by its computational ease and it being relatively good at modelling oil prices. The use of the MRP is reasoned by GBM's lack of capturing commodity prices' tendency to revert back to a long-term mean. However, in order to remain tractable and intuitive, assumptions and simplifications have to be made and that is a challenge as they argue that real projects are often complex and have multiple sources of uncertainties.

Chapter 5

Oil Price Modelling

5.1 Oil Price Process Modelling

Some authors (Paddock et al. [1988], Brennan and Schwartz [1985], McDonald and Siegel [1985]) have argued that future oil prices can be modelled by follow geometric Brownian motion process, while other authors (Laughton and Jacoby [1993] and Schwartz [1997], Pindyck [1999]) argue that the oil price have mean reverting properties, and should be modelled by assuming a mean reverting price process. The controversy is unresolved in the literature and researchers are yet to reach consensus. The goal of this chapter is to discuss different approaches to model the behaviour of future oil prices. We will focus on stochastic oil price process modelling approaches. We will also elaborate on our decision regarding the chosen approach for our analysis.

5.2 Geometric Brownian motion

The geometric Brownian motion (GBM) is a continuous-time stochastic process where the variable follows a Brownian motion with drift. The assumption of geometric Brownian motion is widely made in finance to model future stock prices, and especially in the Black-Scholes option pricing model. Mathematically the GBM process can be stated as:

$$dP_t = \alpha P_t dt + \sigma P_t dz_t \tag{5.1}$$

where α is the drift rate, P_t is the oil price, σ is the volatility and dz_t, the increment of a standard Brownian motion.[1]Both the standard deviation, σ, and the expected drift rate, $E(\alpha)$, are assumed constant over the time horizon. Note that this implies a higher price variability with time.

[1]is a normally distributed variable with $dz_t \sim \mathcal{N}(0,\, 1)$.

One particular desirable property of this stochastic process is that there exists a closed form solution to many asset valuation problems if the assumption of GBM is made for the variable (Postali and Picchetti [2006]). Furthermore, the geometric Brownian motion process is lognormally distributed, resulting in only positive values for the randomly varying quantity. For oil prices (and any other asset for that sake) negative prices are undesirable for price modelling purposes. Another desirable property is that given X_t follows Equation 5.1, the expected future value of the oil price is easy to find. It is equal to:

$$E[P(t)] = P_0 e^{\alpha t} \tag{5.2}$$

5.2.1 Criticism and Motivation for GBM

For the GBM to make sense as a price process one main assumption has to be made. In the earlier academic finance literature, a widely accepted assumption is that of an efficient market. It was believed that the securities markets were extremely efficient in reflecting information about stocks and the market as a whole. This view results in no analysis being able to predict future prices and thus achieve greater returns than those that could be achieved holding a random portfolio with comparable risk. The efficient market assumption implies that all the information is incorporated in the current price of a security, and that all subsequent prices are the result of random departures of the previous ones. (Malkiel [2003]). This assumption is essential for the use of a GBM process to model future oil prices. This implies that the current price contains all existing information about the movement of future oil prices.

One implication of using a random walk as a price modelling process is that the price never moves towards a long term equilibrium. A major critique against the use of GBM is that commodity prices have been observed to converge towards a mean value (Pindyck [1999], Postali and Picchetti [2006],Baker et al. [1998]), which suggests a process that incorporates a mean reversion component to be more appropriate for oil price modelling. However, proponents of the GBM argue that the approach is still a valuable assumption to model future oil prices. Firstly, Pindyck [1999] observe that the mean reverting rate is very low, implying a long half-life[2]. His argument is based on the fact that the unit root test[3] can only be rejected for relatively long time periods, in particular 127 years [4]. Thus, he argues that for investment decision purposes one might as well treat the price as a random walk. Furthermore, he also found that the equilibrium price might fluctuate over time. He also argues that "for irreversible investment decisions for which energy prices are the key stochastic state variables, the GBM assumption is unlikely to lead to large errors in the optimal investment rule".

[2]The expected time it takes for the price to reach the intermediate value of the current price and the long term mean

[3]A unit root test determines whether a time series variable is non-stationary and possesses a unit root. The null hypothesis is generally defined as the presence of a unit root and the alternative hypothesis is either stationarity, trend stationarity or explosive root depending on the test used. Source: Wikipedia contributors [2018]

[4]Postali and Picchetti [2006] found it to be 148 years

Secondly, Dixit [1992] points out, and Postali and Picchetti [2006] repeat, that GBM shows a particular advantage over any mean reverting process, which is that GBM better accommodate for the effects of uncertainty and irreversibility in the optimal decision to invest. Postali and Picchetti [2006] states that "In the extent that sunk costs in the development are relevant only because there is uncertainty about future conditions ... this process(GBM) allows to build the worst scenario to investors who take their decision (among the described processes), which can be more suitable for a risk averse agent." This becomes apparent as the GBM usually result in delaying the project (Cortazar and Schwartz [1997] and Smith and McCardle [1999]) if it is possible to wait for more favourable oil prices, resulting in higher trigger prices.

In summary, several properties allow the GBM to be a reasonable candidate for the price process chosen in a real options analysis. These properties are mainly that the GBM is argued to provide computational ease and allows for closed form solution for many problems. Additionally, its simplifying assumption of neglecting mean reversion is considered adequate, and finally it is considered to accommodate sufficiently for the affects of uncertainty and irreversibility. There are however skeptics who do not agree with the neglection of mean revering prices. We next present the Ornstein-Uhlenbeck process, a mean reverting process.

5.3 The Ornstein-Uhlenbeck process

The Ornstein-Uhlenbeck (OU) process can be considered as a modification of a random walk where there is a tendency to move towards a mean value. The attraction towards this value is the greatest when furthest away from the mean value. The instant change in value for the oil price following an Ornstein-Uhlenbeck process, P_t, is presented mathematically as:

$$dP_t = \eta(\overline{p} - P_t)dt + \sigma dZ, \tag{5.3}$$

where the mean value is denoted by $\overline{p}$, the mean reversion speed as η, the value of the variable at time t as P_t and the constant volatility as σ. The expected value of the oil price at any given future time t is given by:

$$E[p_t] = \overline{p} + (p_o - \overline{p})e^{-\eta t} \tag{5.4}$$

where p_o being the current oil price, $\overline{p}$ is the equilibrium price and η is the mean reversion speed.

5.3.1 Criticism and Motivation for Ornstein-Uhlenbeck process

The logical basis for the use of a mean reverting process stems from microeconomics: when prices are below their long term mean level, the demand for this product tends to increase, while the supply tends to decrease. When the price is above the long term mean value, the demand will decrease, resulting in prices decreasing as well. This is due the fact that as prices decreases, the consumption of the commodity will increase. However, the low prices will result in producers postponing the decision to invest and/or close less efficient units. This in turn will lead to a reduction in supply. Figure 5.1 illustrates how the price P_o will behave if it was above the long term equilibrium price $\overline{P}$. Proponents of mean reversion in commodity prices frequently use these type of arguments (Bastian-Pinto et al. [2010], Schwartz and Smith [2000], Dias [2004]). There are several empirical studies that support the claim of microeconomic forces in the market, and thereby a mean reversion of commodities prices (Bessembinder et al. [1995], Pindyck and Rubinfeld [1998], Pindyck [1999])

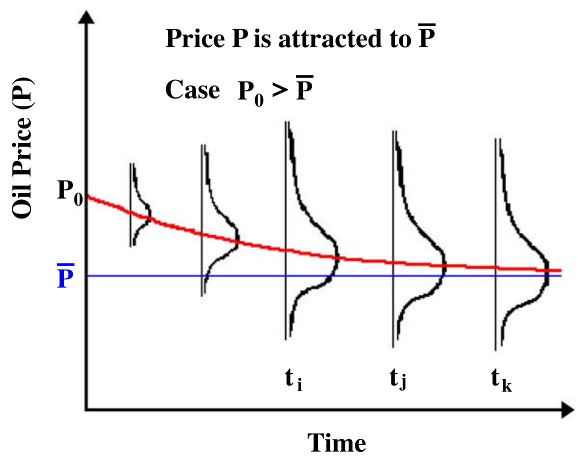


Figure 5.1: The movement of a mean reverting price process, where the long term equilibrium price $\overline{P}$ is assumed constant and below the current price, P_0. [Source: Dias [2004]]

One downside of assuming a mean reverting price is that computationally it is more difficult to deal with. To account for the mean reversion, both the reversion speed, η, and the equilibrium price $\overline{P}$, have to be estimated, adding additional complexity. Assumptions regarding constant η and $\overline{P}$ can be made to deal with these complexities. Along with the basis of mean reversion and the above mentioned assumptions, the simplest form of a mean reversion process, the Ornstein-Uhlenbeck process(Bastian-Pinto et al. [2010]), is considered for this analysis. The process is thus preferred because of its relative ease to understand and handle computationally. However, this approach present some shortages, as constant η's and $\overline{P}$'s are assumptions that may hold for shorter periods, but with increasing time horizons they are doubtful.

5.4 Final remarks

Most studies are suggestive rather then conclusive on the matter of price processes (Postali and Picchetti [2006] and Canarella et al. [2013]). As the profitability of oil development project relies heavily on oil prices, which are uncertain, the GBM appear to the best choice due to its ability to account for worst case scenarios. However, as discussed it neglects a commonly agreed upon characteristics of oil price, mean reversion. The affects of both mean reversion and random walk on the project profitability is considered interesting, and therefore both processes are examined.

Chapter 6

Model Description and Solution Procedure

6.1 The problem

In this section we present a model to evaluate an oil field development project. We discuss the flexibilities we consider inherent in the investment opportunity, as well the factors considered uncertain in detail below. The main uncertainties are considered to be the oil prices and the size of the reservoir. We analyze a stylized case which resembles many real aspects of an actual petroleum project and is drafted with help from industry experts and industry professionals. We start by describing the decision problem that the decision maker faces. Then we move on to the solution procedure to find the optimal investment strategy before discussing some main components of the analysis such as reservoir modelling and Bayesian probability updating.

We consider the following problem: An oil company is considering if and when it should develop a an oil field, and how it should produce optimally. Additionally, it has an opportunity to invest in an appraisal program. The appraisal program is defined as a process for data acquisition that serves to reduce reservoir size uncertainty.

The situation is illustrated in the decision tree in Figure 6.1. At time t=0 the decision maker has the choice to invest in development of an oil field, invest in an appraisal program or delay the decision to invest in development. We assume that the decision maker has a lease to develop the field that is active for the next five years. We also assume that the option to invest in appraisal is only available a t=0, i.e. it is a now-or-never decision. If the decision maker decides to invest in appraisal, it is assumed that the results from the appraisal arrives immediately. Additionally, there is an investment cost associated with the appraisal. We furthermore assume that the decision maker has some prior knowledge about the reservoir size, but there is high uncertainty, see Figure 6.2, and this is considered the a priori scenario. For each scenario we assume that the reservoir can be one of seven sizes as illustrated by the horizontal axis in the graph in Figure 6.2. The reservoir size uncertainty is reduced through appraisal. Specifically, the output from the appraisal is one of three scenarios, Result 1, Result 2 and Result 3, as illustrated in the Figure 6.1. . Result 1(Grey line) indicates that the base case reservoir size is the most likely, Result 2 (Red line) indicates the low case reservoir size as most likely and Result 3(Blue line) indicates the high reservoir size as most likely. Note that the probability distribution changes given appraisal results, leading to distributions centering around the most likely case (indicated by the appraisal) as illustrated in Figure 6.2.

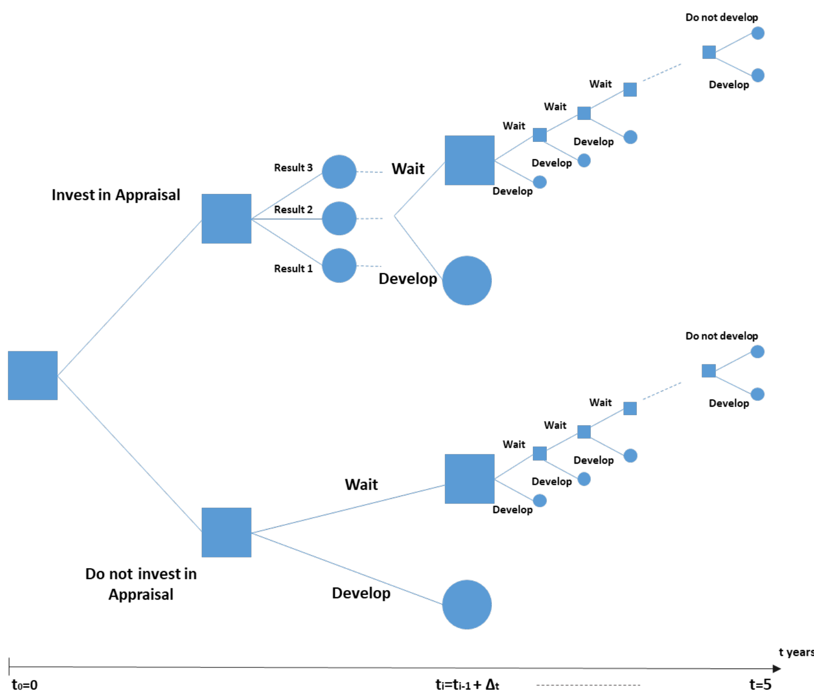


Figure 6.1: The figure illustrates the decision problem that the oil company faces.

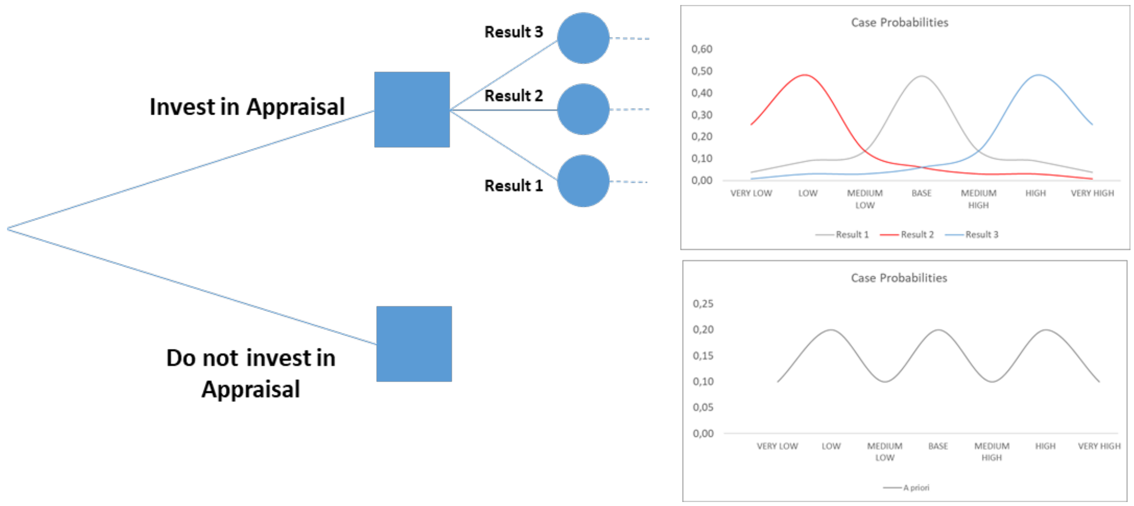


Figure 6.2: The figure shows the probability distribution of the reservoir size before and after appraisal.

Finally, the option to delay development is available for 5 years, until t=5. At t=5, the decision maker has to either decide to develop the field or return the lease to the government. During the period of five years, the decision maker can decide to invest now or to wait another period, i.e. the decision to invest or defer is reconsidered at every time step. This is illustrated in Figure 6.1. At t=5, the decision maker has the choice to develop now, or loose the possibility to do so in the future.

There are several reasons for why the decision maker might not want to develop the field right away. On the one hand, he has the option to delay investment and wait for a more favorable market condition, i.e. higher oil prices. On the other hand there is also a risk of lower prices in the future, meaning that one might loose revenues by postponing development and should perhaps produce as much as possible today, or even refrain from developing. We consider oil prices to be one of the biggest sources of uncertainty, and in order to account for this we assume that oil prices develop stochastically over time. To account for different characteristics observed in commodity prices, we consider two different stochastic processes to model the oil prices. Our choices are explained and motivated in Chapter 5.

Furthermore, we also account for reservoir uncertainty. We find the optimal production profiles given the decision maker's expectations of the reservoir size and model how additional information, received through the appraisal process, will affect the production plan. We then apply a real options approach to value this project, and in the following sections we elaborate on the methods and assumptions made in order to do so.

6.2 Solution Procedure

The solution procedure for the problem presented above starts with the general approach to the real options problem. We apply the integrated approach to solve the problem, as presented by Smith and Mc-

Cardle [1999] and Smith and Nau [1995]. In the authors' opinion Smith and McCardle [1999] and Smith and Nau [1995] point out a significant fact; oil field development projects are characterized by two fundamentally different risks, and these should not be modelled in the same way[1]. The authors' considers this approach to be based on the most accurate and consistent theoretical foundation, in line with claims made by Borison [2005]. For the case presented above, we acknowledge the difference between market and private risk, and hence manage them differently. For risk that can be hedged in the market, i.e. the market risk, we use the real options' classic assumptions. For un-hedgeable risk, i.e private risk, we use a decision analysis approach[2]. Furthermore, we simplify the continuous modelling of oil price development by using a binomial tree approach to discretize the price process[3].

To manage the uncertainty related to the reservoir size, we start with the initial expectations the decision maker has of the size of the reservoir. Next, the decision maker is given the choice to conduct an appraisal, which will reduce his reservoir size uncertainty. If the decision maker choose to conduct appraisal, the probability of the different outcomes [4] change, i.e. the decision maker's certainty regarding the possible size outcomes changes. For both choices the optimization model is applied, and optimal production profiles are generated. Such management of reservoir uncertainty is, to the authors' knowledge, rarely seen in the real options literature, and is considered as one of this thesis' main contributions to the field of capital budgeting for petroleum projects.

The production profiles generated (for a given initial oil price) are then used as a input to the options model where we also account for uncertainty related to the oil price. The production profiles generated for the initial value, are used for each node where the decision maker finds it profitable to invest. This simplification is made to prevent an optimization of the production profile at each node(i.e. for every price) in the price tree. This enables us to handle the model computationally and keeps the model tractable. The authors consider this to be the first step towards an extensive valuation model and acknowledge the limitation this assumption set. We discuss how the conceptual work for an extension on this is already considered and how the solution can be implemented in Chapter 8

The option model is built up in the following way: First we set up the decision tree as illustrated by Figure 6.3. In the figure each node is a decision node where the decision maker must decide whether to invest or wait another period. There are N time periods with j+1 nodes at each time step j. At j=0 the time is t=0, while at j=N the time is t=5, the expiration time of the option to develop the field. This means that j governs the time illustrated by the horizontal axis, while y refers to a specific node for a given time step j illustrated by the vertical arrow. Also note that at j=0 the decision maker has to decide whether to conduct appraisal or not. The price is governed by the given price process and for an initial price P_0 , it

[1] For a thorough explanation of the approach the reader is referred to Chapter 4

[2] For a thorough explanation of the approach the reader is referred to Chapter 3.3.

[3] The discretization and motivation for this approach is found in Chapter 7

[4] We model the Bayesian updating of probabilities in section 6.4

will either increase or decrease with a factor u or d, respectively as seen by the vertical axis in Figure 6.3. The magnitude of these factors are dependent on the price process chosen, and we explain how they are determined in section 6.5. The current price at j=0 (t=0) is given by P_0 and the price at any later node is given by P_j^y.

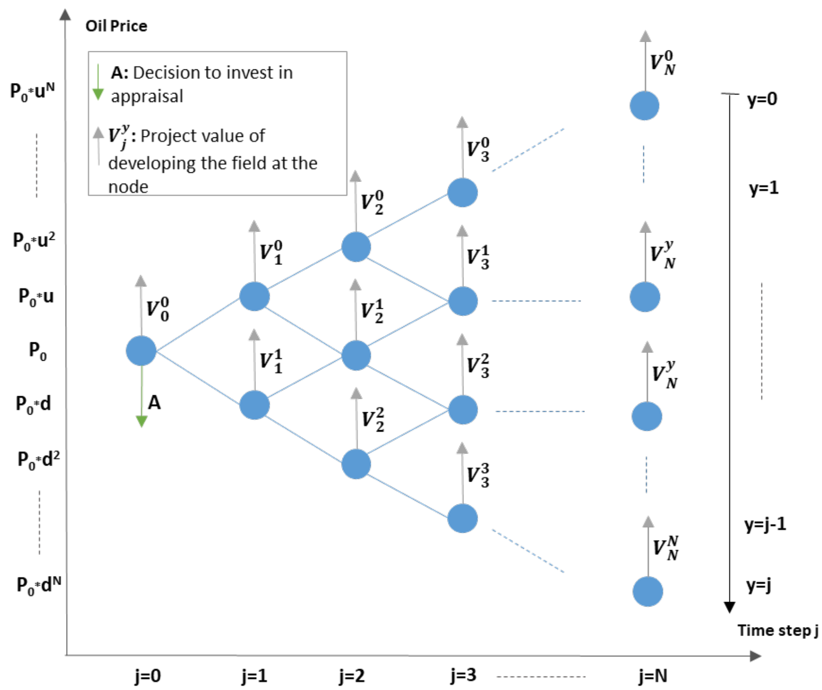


Figure 6.3: The figure illustrates the solution procedure of the model. At each node the decision maker may exercise the option to develop the field. At the initial node he may also invest in appraisal.

At each node, V_j^y represents the expected value of developing the field at time step j for node y. The value of V_j^y is given by:

$$E_j^y[V_j^y] = \sum_{t_p=T_d+1}^{T_p} \frac{E_j^y[R_{t_p}] - E_j^y[C_{t_p}^0]}{(1+r)^{t_p}} - I_D - \frac{I_A}{(1+r)^{T_p}} \tag{6.1}$$

where $E_j^y[V_j^y]$ is the expected project value at time step j for node y, T_p is the project duration[5], t_p is the time after development of the field, which takes T_d years, $E_j^y[C_{t_p}^o]$ is the expected operational expenses, I_A is the abandonment cost and I_D is the development cost given by:

$$I_D = \sum_{t_p=1}^{T_d} \frac{E_j^y[C_{t_p}^c]}{(1+r)^{t_p}} \tag{6.2}$$

[5]The abandonment rule for the option model is to cease production the first time we have negative cash flows. The time up to this point is defined as the project duration.

where $E_j^y[C_{t_p}^c]$ are the expected capital expenditures related to the development and T_d is the total development time which we assume to be 2 years. Furthermore, $E_j^y[R_t]$ is the expected revenues at time t given by:

$$E_j^y[R_{t_p}] = E_j^y[P_{t_p}]q_{t_p} \tag{6.3}$$

where $E_j^y[P_{t_p}]$ is the expected oil price at time t_p given development at time step j at node y[6] and q_{t_p} is the production rate at time t_p. Figure 6.4 illustrates the timeline of the project once the decision to develop has been made.

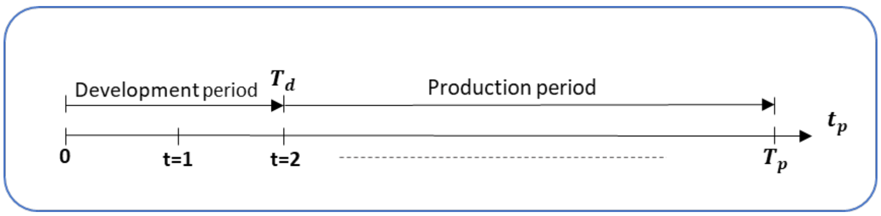


Figure 6.4: Project duration time line when the decision to develop is made.

Now, the options approach is applied. Starting at the terminal nodes ($j = N$) the decision maker must choose between either developing the field or delivering the lease back to the government.

For all other nodes, $j < N$, we define $E[\pi_{j+1}^u]$ and $E[\pi_{j+1}^d]$ as the expected option value for an up and down move in the next time period, respectively. The decision maker must choose between developing the field now or waiting another time period and compares the value of exercising now, V_j^y and the expected option value of waiting another period.

The decision rules discussed above are summarized as:

$$E[\pi_j^y] = \begin{cases} max(V_j^y - I_D, 0) & j = N, \\ max(V_j^y - I_D, p_u E_j^y[\pi_{j+1}^u] + p_d E_j^y[\pi_{j+1}^d]) & 0 < j < N, \end{cases} \tag{6.4}$$

where the option value in present time is π_0.

As mentioned initially there are 7 possible outcomes of the reservoir size in each scenario. The individual production profiles for all cases are generated simultaneously taking into account all the possible outcomes[7]. Then the option model finds the expected value of the scenario by probability weighting all the option values computed for each case. This is then the option value for that particular scenario. Thus to

[6]After the decision to invest has been made, the oil price is assumed to be equal to the expected values of the respective price processes. For the expressions, the reader is referred to Chapter 5.

[7]The optimization model is explained in section 6.3

account for this the approach explained above is repeated for each of the 7 cases of the reservoir size in a scenario, and the expected value for a given scenario is then given by:

$$E[W_s] = \sum_{c_i=1}^{7} p_{c_i} E_{c_i}[\pi_0] \tag{6.5}$$

where s is the given scenario and c_i indicates case i. There are four scenarios, where $E[W_0]$ is the value without appraisal, $E[W_1]$ is the value with appraisal given Result 1 from appraisal, $E[W_2]$ is the value with appraisal given Result 2 from appraisal and $E[W_3]$ is the value with appraisal given Result 3 from the appraisal.

To compare the value of appraisal and the development without such an investment, we find the expected value for the 3 scenarios we achieve by conducting appraisal[8] and compare it to the option value generated by using a priori information about the reservoir:

$$W = max[W_0, p_{Result1}W_1 + p_{Result2}W_2 + p_{Result3}W_3 - I_{Appraisal}] \tag{6.6}$$

where $p_{Result1}$, $p_{Result2}$ and $p_{Result3}$ are the probability of the appraisal giving Result 1, Result 2 and Result 3, respectively and $I_{Appraisal}$ is the appraisal cost. The investment strategy leading to the highest NPV result is then chosen. We discuss our findings in Chapter 7

6.3 Reservoir Modelling - Optimizing the production profiles

In this section we present a reservoir model to handle the optimization of production profiles, which we consider the first part of reservoir risk modelling. The second part related to obtaining more information is discussed in the next section.

An important question to answer is: How should the firm plan its production to accommodate for reservoir size uncertainty? Building production platforms are costly and irreversible. Building a large platform increases the maximum level of production and is beneficial if the reservoir size is high, but may be costly and crucial to the profitability of the project if the reservoir size actually is low. Therefore, being able to plan the production optimally is desirable. Previous work in the real options literature that consider both market and technical risk have handled the latter by letting production rates follow a stochastic process such as Smith and McCardle [1998] or by considering the reservoir size as stochastic as Guedes and Santos [2016]. The optimization model that we present determines the production rates so they jointly maximize the Expected Net Present Value of all the 7 cases in a scenario. Thus we account for reservoir uncertainty and acknowledge that production rates is something that the firm may decide and something they should try to optimize. Furthermore, the NPV is a function of the production rates, operational- and capital

[8] Result 1, Result 2 and Result 3 are all possible outcomes from the appraisal program.

expenditures, number of wells and the oil price. In the optimization model the oil price is considered deterministic, the number of wells is considered as a free variable, the operational expenditures considered as a function of the production rate and the capital expenditures modeled as a function of the production rate and number of wells. In this way we offer a model that has a logical connection where increasing capacity, building a larger platform, and/or increasing the number of wells, resulting in higher capital expenditures, but allows for a higher production rate. Similarly, the operational expenditures increase if the production increases.

We consider the following maximization problem:

$$Max \sum_{i=1}^{I} (p_i NPV_i) \tag{6.7}$$

subject to

$$q_{it} \leq q_{pl}, \quad i \in I, \quad t \in T, \tag{6.8}$$

$$q_{pl} \leq q_{pp,it}, \quad i \in I, \quad t \in T, \tag{6.9}$$

$$q_{pl} \leq N_w K_w, \tag{6.10}$$

$$\sum_{t=1}^{T} q_{it} \leq N_i, \quad i \in I, \tag{6.11}$$

$$q_{pp,it} = N_w(-m^{ref} N_{p,it} + q_{pp0} M_i), \tag{6.12}$$

$$q_{pp,it}, q_{pl}, q_{it}, \geq 0 \tag{6.13}$$

$$N_w \in \{0, 1, ..., \mathbb{R}\} \tag{6.14}$$

where

i: Indicator for case i, $\quad i \in I, \quad I = 7,$

t: Time t, $\quad t \in T, \quad T = 16,$

p_i=Probability of case i, $\quad i \in I$

$q_{pp,it}$=Production potential for case i at time t

q_{pl}=Plateau rate

q_{ti}= Production rate for case i at time t

q_{pp0}= Production potential from reference case

M_i= Reservoir size multiplier for case i

N_i= Reservoir size for case i

N_w=Number of wells

K_w=Production capacity per well

m^{ref}=Decline parameter for reference case

$N_{p,it}$=Cumulative production for case i at time t

The objective function, Equation 6.7, jointly maximizes the ENPV for a scenario by multiplying the probability of case i, p_i with the NPV of case i, NPV_i. The expression for NPV_i is stated as:

$$NPV_i = \sum_{t=1}^{T} \frac{P_0 q_{it} - C_t^c(q_{it}) - C_t^o(q_p, N_w)}{(1+r)^t} \tag{6.15}$$

where P_0 is the initial oil price, C_t^c is the capital expenditure (capex) at time t and C_t^o is the operational expenditures (opex) at time t. The capex are considered to include the drilling expenditures (drillex) and are calculated in the following manner:

$$C_t^c = A^c + B^c q_{pl} + N_w C_w \tag{6.16}$$

where A^c and B^c are regression constants from a specific case assumed to be representative for the stylized project we consider and q_{pl} is the plateau rate of the field, i.e the highest production rate determined by the optimization. The plateau rate defines the size of the platform, which is the same for all cases as the firm must decide the size of the platform before the reservoir size is known. The drillex is represented by the cost per well, C_w , times the number of wells, N_w. We do not differentiate between different production wells and make the simplifying assumption that the cost of all wells are equal. In addition, the costs per well also include the cost of maintaining them. These are simplifying assumptions that allows us to easily manage the cost variables and give a tractable model computational wise. The assumptions made are subject to further discussion, but are considered out of scope for this thesis.

The main types of operating costs are related to the maintenance of platforms and wells and the costs of day-to-day operation of the facilities. The cost related to maintenance of wells are assumed to be calculated in the opex. Furthermore, we assume that all labour cost, machine maintenance and maintenance on other equipment is included in the opex. The opex is given by:

$$C_t^o = A^o + B^o q_{it} \tag{6.17}$$

Where A^o and B^o are regression constants from a specific case assumed to be representative for the case we analyze. Furthermore, the costs are assumed to vary with the production level reflecting that the costs mentioned above increases when production increases.

Now that all variables have been explained we turn back to the NPV. As can be seen from Figure 6.15 the value increases if the production level increases. However, a higher level of production requires a larger facility and more wells. Additionally, the opex also increases. Thus the proposed way of defining the NPV reflects the fact that there is no free money to be made and also shows the importance of optimizing the values for the production rates.

Below the constraints of the optimization model are explained.

Constraint 6.8 states that the production rates (q_{it}) are restricted to take values lower or equal to the plateau rate (q_{pl}), which is the maximum production level at any time t. The plateau rate is the same for all cases as the firm must decide the size of the platform before the reservoir size is known.

Constraint 6.9 restricts the plateau rate (q_{pl}) to take values below or equal to the production potential ($q_{pp,it}$)[9] for case i at time t. Thus the production rates are indirectly restricted to take values less or equal to the production potential.

The plateau rate also depends on the number of wells chosen. This is expressed in Constraint 6.10 where q_{pl} is restricted to take values less or equal to the product of number of wells (N_w) and the well capacity of each well (K_w).

Next, Constraint 6.11 ensures that the cumulative production in case i is less than the reservoir size for case i.

Constraint 6.12 states how the values of the production potential for case i at time t ($q_{pp,it}$) are determined. The production potential of a field represents the maximum production that is possible given the number of wells and technical characteristics such as reservoir pressure and porosity[10]. q_{pp0} represents the initial production potential value for a reference case which we assume to have the same characteristics as the field we analyze. We scale this value based on the size of the reservoir for case i relative to the reference case with M_i, which is a size multiplier. Furthermore, the production potential decreases over time due to wearness on the wells and decreases more rapidly if production is higher. Furthermore, the decline parameter (m^{ref}) and the cumulative production ($N_{p,it}$) determines the rate at which the production potential decreases. The decline parameter value is taken from the reference case and assumed to be representative for the case at hand.

As the reader may have noted, the production rates (q_{it}) are restricted by both q_{pl} and $q_{pp,it}$. The difference between them deserves some explanation. The former can be thought of as the maximum production determined by the optimization. The latter can be thought of as the theoretical limit of production rates of the field at any time given by Constraint 6.12. Figure 6.5 illustrates the difference. In the figure the green line presents the production potential which decreases with time, while the blue line represents the production rate (field rate) as time passes. Initially the production rate is limited by the plateau rate, but eventually it is limited by the production potential of the field. The constraint in place ensures that the optimization models the production rates according to these restrictions.

[9] See further down for an explanation of the parameter

[10] The reader is referred to Stanko [2019] for a more detailed explanation

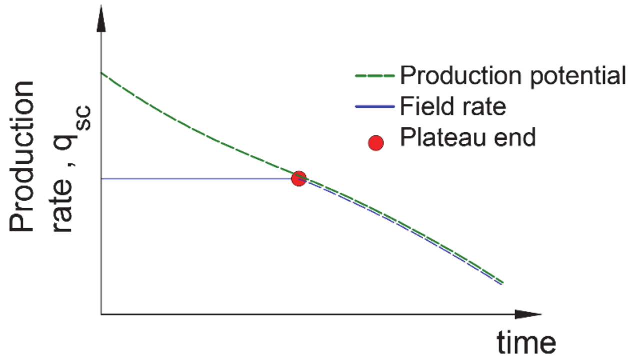


Figure 6.5: The figure illustrates a typical production potential of a field and the relationship between q_{it}, $q_{pp,it}$ and q_{pl}. Source: [Stanko [2019]]

Finally the two last constraints, 6.13 and 6.14, forces q_{it}, $q_{pp,it}$ and q_{pl} to take positive values and N_w to take integer values.

Summarized we have presented an optimization model to determine the optimal production rates that jointly maximizes the ENPV when there is uncertainty about the reservoir size. The model accounts for the important trade off between increasing capacity, resulting in higher revenues and high irreversible initial investments and increased operation expenditures.

6.4 Updating the probabilities

The goal of this section is to illustrate how the probability distributions for the reservoir are updated if the appraisal program is conducted. The situation can be illustrated by the decision tree in Figure 6.6, which shows the chronological sequence of information revelation. The goal is to derive the missing probabilities, i.e the probability distributions of the appraisal results and the distribution given that we have conducted appraisal and obtained a result.

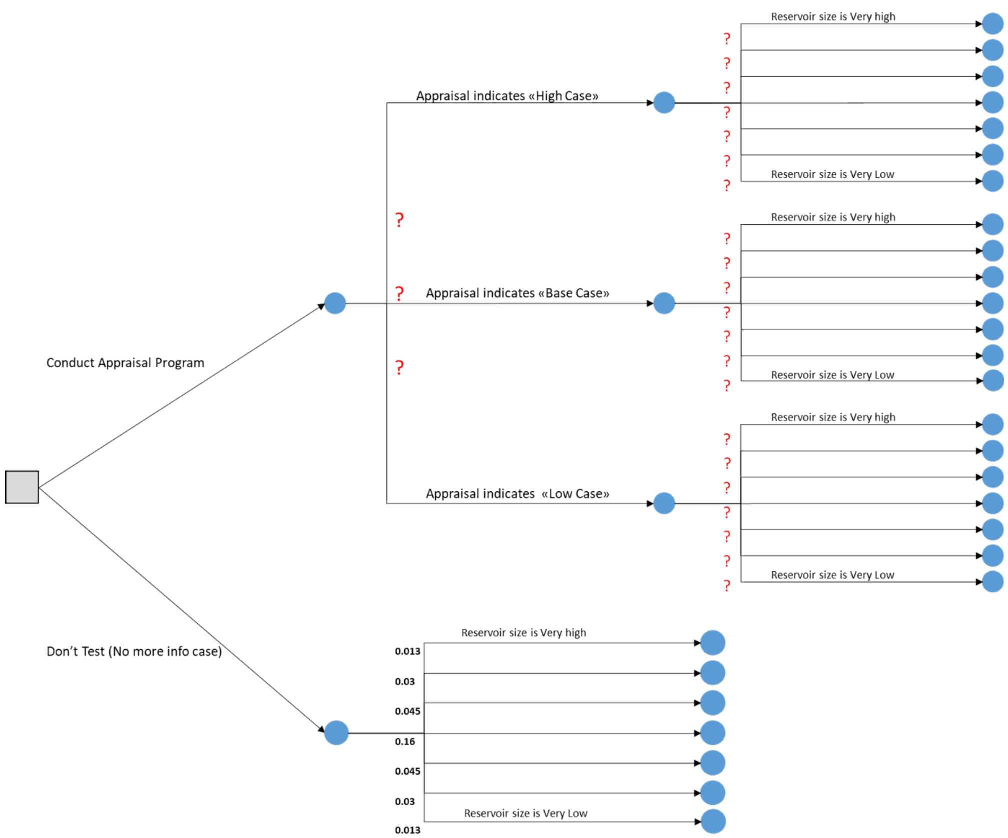


Figure 6.6: Chronological Probability Tree

To do this we use the approach suggested by Coopersmith and Cunningham [2002]. The approach is based on Bayesian statistics and allows the decision maker to update the probabilities as he learns more about the reliability of the appraisal program. In short the approach involves:

1. Conducting a "Reliability Interview"
2. Developing the Reliability tree
3. Using Bayesian statistics to obtain the desired probabilities

The Reliability Interview "expresses the reliability of the information in terms of the true state of nature of the actual uncertainty" (Coopersmith and Cunningham [2002]). The probabilities are the decision maker's subjective belief and are obtained by an interview. Specifically, one asks what the decision maker's belief are about the probability that the information will indicate a specific event given that the event actually happened. Applied to our case it is the probability that the appraisal result indicates that the reservoir size is case i given that the reservoir size actually is case i. This can be formally stated as:

$$P(R_i|C_i) \tag{6.18}$$

where

R_i: Appraisal result indicate size i

C_i: Reservoir size is actually case i

Next, the reliability tree is developed based on the results from the previous step as illustrated by Figure 6.7. In the figure the probabilities on the nodes connected to the source nodes are the a priori probabilities for a given reservoir size. Moving to the right, the next set of probabilities are the result from the interview. That is, given that that the reservoir size is actually C_i what is the probability that the appraisal program will indicate either the high, base or low case as most probable. One may now calculate the joint probability of a realized reservoir size and an appraisal result by using Equation6.19. These are the end node probabilities. Note that the tree is not in a chronological order with respect to when uncertainty is revealed.

$$P(C_i \cap R_i) = P(R_i|C_i)P(C_i) \tag{6.19}$$

Figure 6.7: Reliability tree developed

In the final step we use Bayesian probabilities to "flip" the reliability tree and obtain the probability distribution given that our appraisal result indicates a reservoir size. These values are derived by using Equation 6.21. Note that we have used the relation in Equation 6.20

$$P(R_i \cap C_i) = P(C_i \cap R_i) \tag{6.20}$$

$$P(C_i|R_i) = \frac{P(R_i \cap C_i)}{P(R_i)} \tag{6.21}$$

The resulting chronological tree is illustrated by Figure 6.8 and the probabilities before and after appraisal are summarised in Table 6.1.

Table 6.1: In the table the a priori probabilities and the derived posteriori probabilities after appraisal are presented

		A posteriori		A priori
Case	Indicates Low	Indicates Base	Inidcates High	
Very Low	0.037	0.256	0.008	0.100
Low	0.090	0.481	0.030	0.200
Medium Low	0.134	0.135	0.030	0.100
Base	0.478	0.060	0.060	0.200
Medium High	0.134	0.030	0.135	0.100
High	0.090	0.030	0.481	0.200
Very High	0.037	0.008	0.256	0.100
$P(R_i)$	0.34	0.33	0.33	

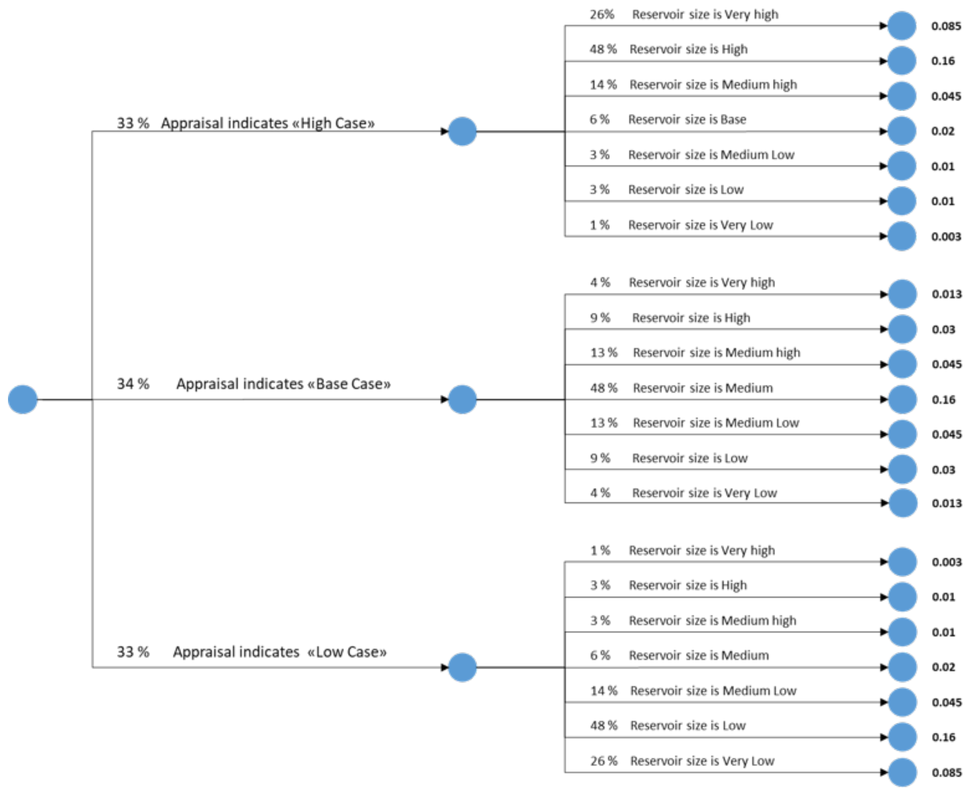


Figure 6.8: Chronological tree developed

The results show that the appraisal changes the distribution from a situation with high uncertainty about the reservoir size to a situation where the probability distributions are centered towards the reservoir size that the program indicates as most likely. By conducting appraisal, more information about the reservoir is obtained and the uncertainty about the most likely outcome is reduced. The decision maker may then take advantage of this by adjusting his production plan for the field, possibly resulting in a higher profit gain. However, the results also show high uncertainty about the outcome of the appraisal($P(R_i)$), i.e which case that it will indicate as most probable.

Finally, the reader may have noted that the resulting probability distributions posteriori depend both on the initial belief about the distribution and the decision maker's certainty towards the appraisal program indicating the actual reservoir size. The benefit of this is that the decision maker may update his probabilities reflecting the fact that over time he may learn more about the accuracy of the appraisal program and the initial exploration results which is the source for the a priori distribution.

6.5 Parametrization of the price processes

In Chapter 5 we discussed how we would use geometric Brownian motion and an Ornstein-Uhlenbeck process to model the oil price. In order to use these processes, certain parameters need to be determined. Some of these are estimated using various estimation techniques, and others by logical arguments with based in financial theory and/or the current financial condition. Next, we discuss how we set these parameters for the two oil price processes below. However, it is not the scope of this thesis to make an extensive discussion of the determination of the different initial parameters. We do however describe how we choose to calculate the different prices through a discretization method. Table 7.1 summarizes the estimation results.

6.5.1 Geometric Brownian motion (GBM)

6.5.1.1 Discretizing the price process

In order to generate numerical values for the project value with relative computational ease, we apply a discretization of the continuous geometric Brownian motion. For this purpose we use the Cox, Ross-Rubenstein binomial tree. For a given initial oil price, the up and down movements of the price at each node can be stated as:

$$
\begin{aligned}
u &= e^{\sigma\sqrt{\Delta t}} \\
d &= e^{-\sigma\sqrt{\Delta t}}.
\end{aligned}
\tag{6.22}
$$

An example of a binomial tree which governs the oil price at each time step is illustrated in Figure 6.9. The process is divided into N time steps, with length Δt. The discretization is chosen to simplify calculations, while the same theoretical foundation and arguments of a GBM apply to both the continuous and discrete version, as the discrete version converges to the continuous solution when Δt → 0. Thus the simplification does not compromise the accuracy of the price modeling for small step sizes.

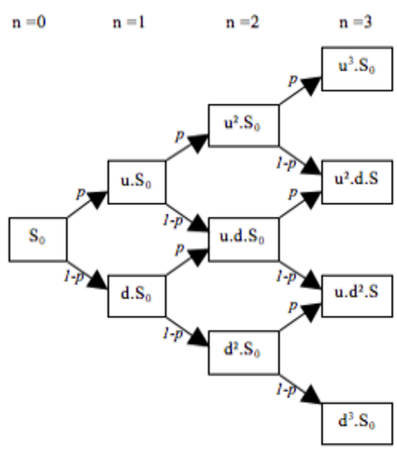


Figure 6.9: The Cox, Ross and Rubinstein binomial tree. [Source:Wikipedia: Binomial options pricing model [2019]]

Worth noting is that because of equal up and down movements the resulting tree is recombining, making calculations easier and computational demand less requiring.

Furthermore, at each node in the tree there is a probability associated with an up move , denoted by p_u, and a down move, denoted by p_d. The expressions for these are given by:

$$p_u = \frac{e^{(r-\delta_t)\Delta t} - d}{u - d}$$
$$p_d = 1 - p_u \tag{6.23}$$

where r is the risk free rate and δ_t is the convenience yield. Note that the probabilities are constant through time, meaning that the probability for an up move/ down move is the same regardless of the previous move. Similar to Guedes and Santos [2016], we assume that for the risk neutral process the convenience yield for oil is equal to the risk free rate resulting in a risk neutral drift for the oil price equal to zero.

6.5.1.2 *Risk free rate,* r

The project lifespan is assumed to be 16 years. Consequently, we try to find a government bond representing the risk free rate for such a period. We assume the 10 year government bond to be representative for the time horizon for our specific case [11]. The rate is stated to be equal to be 1.88%

6.5.1.3 *Volatility Estimation,* σ_{GBM}

The volatility for the GBM-process is estimated using the annualized standard deviation of the returns mathematically presented by:

$$\sigma_{GBM} = \sqrt{\frac{\sum_{i=1}^{n}(x_t - \overline{x})^2}{n-1}}\sqrt{52} \tag{6.24}$$

where x_t is the continuously compounded weekly return. The data used to estimate the volatility is the European Brent spot prices from May 1987 to November 2018 [12], and the annualized volatility was estimated to be equal to 30.13%.

6.5.2 Ornstein-Uhlenbeck process (OU)

6.5.2.1 *Discretizing the price process*

We use the discrete approximation of the OU-process proposed by Nelson and Ramaswamy [1990]. This approximation is a binomial tree and is applied mainly due to the computational ease of discrete functions. The process is discretized into a sequence of n periods of duration Δt where the prices move either up or down. The magnitude of the jumps are the same as in the GBM discretization, thus resulting in a recombining tree identical to the one in Figure 6.9. The probabilities associated with an up or down move

[11] NorgesBank [2019] only has bonds up to 10 years. We therefore find it appropriate to use this value
[12] Source: EIA

however differ in that they are not constant over time. They can be expressed as:

$$p_u = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\eta \ln(\frac{\overline{P}}{P_t})}{\sigma}\sqrt{\Delta t} & \text{if } 0 \leq p_u \leq 1, \\ 0 & \text{if } p_u < 0, \\ 1 & \text{if } p_u > 1, \end{cases} \tag{6.25}$$

As one can see, the probabilities are *censored* so that they always lie between 0 and 1. This is due to probabilities being invalid if one of the following conditions are met:

$$\ln(\overline{P}/P_t)\sqrt{\Delta t} > \sigma, \quad then \quad p_{up} > 1 \tag{6.26}$$

$$\ln(\overline{P}/P_t)\sqrt{\Delta t} < -\sigma, \quad then \quad p_{up} < 0 \tag{6.27}$$

It may be argued that such an approach is suboptimal because of truncated values [13]. One way to avoid this problem is by choosing a relatively small step size, i.e Δt, preventing Equation 6.26 and Equation 6.27 to give probabilities above and below 1 and 0, respectively. Another alternative is to use an uncensored version, such as the one presented by Bastian-Pinto et al. [2010]. However such alternatives are cumbersome and complex to implement.

For the estimation of the equilibrium value, $\overline{x}$, the mean reversion speed, η, and the volatility for the OU-process, σ_{MRP}, we follow the methods presented in Guedes and Santos [2016].

6.5.2.2 Mean Reversion parameters

We apply the regression:

$$x_t - x_{t-1} = a + bx_{t-1} + \epsilon_t \tag{6.28}$$

on the weekly European Brent spot price from May 1987 to November 2018[14] to find the estimates for the regression constants a and b. These were found to be $\hat{a}$=0.012 and $\hat{b}$=-0.003. The mean reversion speed,$\hat{\eta}$, is then equal to

$$\hat{\eta} = -\ln 1 + \hat{b} \tag{6.29}$$

and found to be $\hat{\eta}$=0.0141. The relatively small value of $\hat{\eta}$ implies that the data (slowly) reverts to a mean value.

[13]Values above 1 and values below 0 will be truncated to 1 and 0 respectively, in accordance to Eq. 6.25

[14]Source: EIA

Using equation 6.28, we also find the long term average value that the oil price reverts to by:

$$\overline{x} = -\hat{a}/\hat{b} \tag{6.30}$$

$\overline{x}$ is then estimated to be 55.39$

6.5.2.3 Volatility Estimation, σ_{MRP}

The volatility for the mean reversion process is estimated by the following equation:

$$\hat{\sigma} = \hat{\sigma}_{\epsilon}\sqrt{\frac{ln(1+\hat{b})}{(1+\hat{b})^2 - 1}} \tag{6.31}$$

where σ_{ϵ} is the standard error of the regression. The volatility is found to be σ_{MRP}=21.33%

6.5.3 Appraisal cost

For this analysis we set the appraisal cost to be 150 MNOK. This value was determined by the help of industry experts at NTNU and industry professionals.

Chapter 7

Results and Comparative Analysis

In the following chapter we present the results from the real options analysis (ROA) of the development project presented in Chapter 6. For comparative reasons, we also present the value of the project calculated by the static NPV approach[1]. The parameters used in the two price process are shown in Table 7.1. The rest of the initial parameters are shown in Table 7.2. How these were estimated was explained in Chapter 6.

Table 7.1: The table summarize the parameters related to the two price processes. The data used is European Brent Spot prices from May 1987 to November 2018

Ornstein Uhlenbeck		geometric Brownian motion	
Parameters	**Values**	**Parameters**	**Values**
Mean Reversion Speed, $\hat{\eta}$	0.0141	σ_{GBM}	30,13 %
$\hat{a}$	0.012		
$\hat{b}$	-0.003		
Mean value, $\overline{x}$	55.39 $		
σ_{MRP}	21.33 %		

[1]We use this term instead of Discounted cash flow (DCF) to emphasize that it is a static approach as explained in section 3.3.1.

Table 7.2: The table shows the values of initial parameters in the option model along with their symbol, numeric value and unit of measure.

Parameter	**Symbol**	**Value**	**Unit**
Option model parameters			
Initial Oil Price	P_0	70	USD
Risk Free Rate	r	1.88	%
Convenience Yield	δ	1.88	%
FX	-	7.5	NOK/USD
Cost parameters			
Capex Regression constant 1	A^c	0	MNOK
Capex Regression constant 2	B^c	1,284	MNOK
Opex Regression constant 1	A^o	606	MNOK
Opex Regression constant 2	B^o	46	MNOK
Abex	I_A	80	MUSD
Appraisal Cost	$I_{appraisal}$	20	MUSD
Reservoir parameters			
Cost per Well	C_w	563	MNOK
Size Multipliers for case i	M_i	0.4, 0.6, 0.8, 1, 1.2, 1.5, 1.7	-

7.1 Static NPV and Options Analysis with Deferral

In chapter 3.3 we presented the static NPV approach. To compare its value to an options analysis, we compute the static NPV value for the project. Assuming a current oil price of 70$ and a discount rate equal to 7% [2] we find the NPV of the project to be equal to 657 MNOK. For the static NPV approach one can conclude that the project is profitable, and the decision maker would undertake the investment. The results are shown in Table 7.3.

Table 7.3: The table shows the result from the static NPV approach with both the opportunity to appraise and the standard case without appraisal.

Results in MNOK

	standard case	w/ appraisal
Static NPV	657	5,532

[2] As determined by Regjeringen.no for similar projects

Assuming the same current oil price (70 $) for the ROA, a risk free rate of 1.88 %, a time step of 0.0033 years [3] and accounting for the option to defer development, we find an expected project value of 14,596 MNOK when we consider oil prices to follow a GBM and 13,260 MNOK when we consider oil prices to follow a mean reverting OU-process. The results are summarized in Table 7.4. The difference can be explained by prices being bounded for the mean reverting process and boundless for the GBM[4]. For a longer project horizon one would therefore expect a greater difference between these values. However, the project is considered profitable for both price processes.

The difference in the value for the two approaches desserves some attention. Should the reservoir size be lower than expected or the current financial situation change to the worse, the decision maker can choose to refrain from investing. This partly explains the higher valuation for the ROA compared to the NPV approach. Additionally, in the options analysis the decision maker will at each decision point compares the value of investing right away and the expected value of deferring investment. If the expected value of waiting is higher than immediate investment, he may choose to wait. Thus the approach accounts for future price conditions in the valuation and managerial flexibility, while the NVP only consider current conditions. Furthermore, the option valuation uses the risk free rate while the NPV approach relies on a subjective discount factor. For the option valuation we use a discount rate of 1.88 % while for the NPV we have assumed a 7% discount rate[5], which in turn affects the present value of the project. Combined these factors explain the difference between the value of the project from the two methods.

Table 7.4: The table shows the option values for the different options accounted for under the two stochastic oil price processes considered.

Results in MNOK

	Brownian Motion price Model		Mean-reverting price Model	
	w/ deferral	w/ deferral + appraisal	w/ deferral	w/ deferral + appraisal
Expected Value	14,596	16,145	13,260	14,867

7.2 The option to appraise

To value the option to appraise, we maintain the same initial assumptions regarding price and discount rate, and find that the project value increases to 16,145 MNOK when we assume prices to follow a GBM

[3]The time step between each time period as explained in section . The low time step avoids truncating probabilities for the OU process for reasonable oil prices

[4]The GBM being a random walk will have the same probabilities of up and down moves at each node, giving it the possibility to grow boundlessly. The probabilities in the mean reverting process will however depend on each node, and drag the current price towards the equilibrium price.

[5]As determined by Regjeringen.no for similar projects

process and to 14,867 MNOK when we assume prices to follow a OU-process. This is an increase of 10.6% and 12.1% for the two price processes, respectively, compared to only adding the option to defer the development. Table 7.4 shows the valuation results. Note that this is also the value of appraisal, i.e how much the firm would be willing to invest in appraisal. The difference in expected values of the project from the case with appraisal to the one without appraisal show that the ability to update the decision makers beliefs regarding the reservoir size is of substantial value, for both price processes. The explanation lies in the decision maker's ability to adapt to the new information and plan his production according to the new information. One might expect the value calculated assuming prices to follow GBM to be even greater compared to when we assume prices to follow the OU-process as an up move for the GBM has higher probability. However, this may be explained by the short expiration time for the option (opportunity to develop) of 5 years and the relatively low mean reversion speed. For a longer time horizon one might see the discrepancy between GBM and mean reversion to a greater degree because of the boundless price growth expected from a random walk and the mean reversion of the OU process.

The value of adding appraisal is also calculated using the static NPV approach, as seen in Table 7.3. We then find the value of the project to be 5,532 MNOK. Despite the higher project value compared to the NPV value without appraisal, the value is still however substantially lower than the ones found when we consider prices to follow stochastic processes and adding the option to appraise. This is explained by the deterministic view of the static NPV where one commits to appraise at time t=0. For the option approach however, the firm compares the value of the project with and without the added appraisal option and only invests if the value of the new information is higher than the cost of obtaining it, i.e the appraisal cost.

7.3 Comparative Analysis

In this section we compare the option value of deferral with the option value of combining deferral and appraisal. The latter option will be referred to as the appraisal option from this point on.

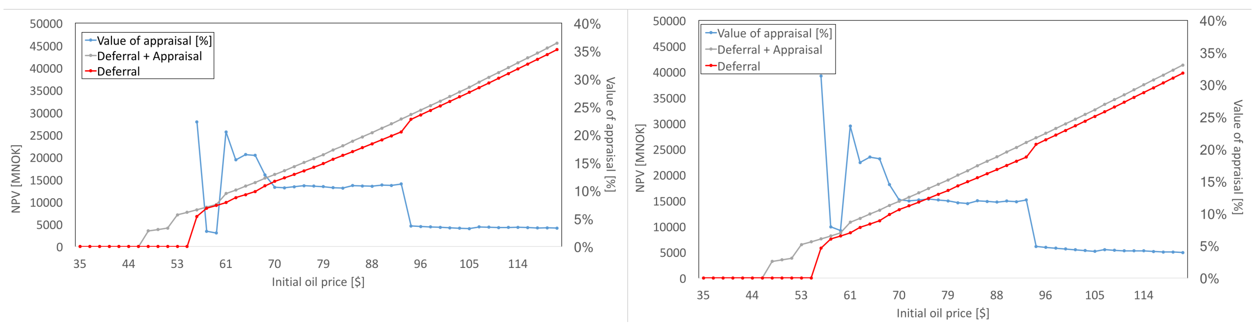
7.3.1 Initial Oil Price

The price tree generated for the option model depends heavily on the initial oil price assumed, which in turn governs the option price. We therefore find it interesting to vary this initial assumption to analyze its effect on the result. The initial oil prices are varied and new production profiles are generated for new initial values, before new option values are computed. Figure 7.1a shows how the results behave for changes in the initial oil price assuming the oil prices follow a GBM process. The red and grey line show the option values for the deferral option and the appraisal option, respectively. Additionally, the blue line shows the percentage increase in option value by adding appraisal. We refer to this value as the value of appraisal[6].

[6]Note that we distinguish between the option value of appraisal and the value of appraisal. The former is the option value , while the latter is the additional value added by appraisal.

We see that the option value increases as prices increase. Furthermore, we also see that the options has no value for initial prices below \$47.25 and \$56, respectively. From \$47.25 the appraisal option has value as seen by the grey line in Figure 7.1a. This is due to the project being profitable at \$47.25 for appraisal Result 3 (High case most probable). Given a result from the appraisal the decision maker is better able to plan for the case that the appraisal indicates as most likely. On the other hand he will not develop the field if appraisal gives Result 1 (Base case most probable) or Result 2 (Low case most probable). This illustrates the flexibility that the appraisal option offers. The grey line showing the option value of appraisal then steadily increases as Result 1 (Base Case most probable) also becomes profitable with increasing prices.

When prices reach \$56, we see from the red line that the a priori scenario (the scenario with only the option to defer) also becomes profitable. At this point the value of appraisal is relatively large. This is explained by the fact that at \$56 the appraisal option has already been positive for several lower initial oil prices and been increasing in value, however, the deferral option only first becomes valuable at \$56. Because the relative difference between the first time deferral becomes valuable compared to the option value of appraisal at that point, the value of appraisal is substantially large initially.



(a) Oil prices assumed to follow a GBM process

(b) Oil prices assumed to follow an OU process

Figure 7.1: The figures shows the effect on the option to develop when the initial oil prices varies stochastically as (a) GBM process and (b) OU-process

For oil prices between \$56 and \$59, one can see that the value added by appraisal drops substantially when the option to defer increases in value. This makes sense as relatively more of the option value stems from the option to defer compared to the option to appraise, for increasing initial oil prices. This is due to the relative gain from being able to produce at optimal production rates decrease as the initial price increases, resulting in the lower value for appraisal.

A surprising result is the sharp increase in the value of appraisal (blue line) at an initial oil price around \$59. The sharp increase may at first seem inconsistent with intuition. However, the reason for the increase is that it also becomes profitable to produce for Result 2 (Low Case most probable), causing a jump in the expected option value of appraisal.

From the same blue line, we also see drops between $61 and $70. The drop at an initial oil price of $67 can be explained by the optmization increasing production rates for the a priori scenario as it is profitable to do so. The drop in the value of appraisal at $70 is the consequence of yet another increase in production levels for the a priori case. For initial prices between $70 and $91 we see a steady decrease in value of appraisal as the option to defer gains value with higher initial oil prices.

Finally, when initial oil price reaches $91 another drop in the value of appraisal can be observed. The reason this time is that we plan for an additional well for the a priori case, resulting in higher production levels and thereby resulting in appraisal being less valuable compared to when we have lower initial oil prices. This can also be observed by the jump in the value of deferral (red line) in Figure 7.1a at $91. We expect to see such drops after additional increments in the number of wells chosen, followed by a steady decrease in the value of appraisal until it diminishes.

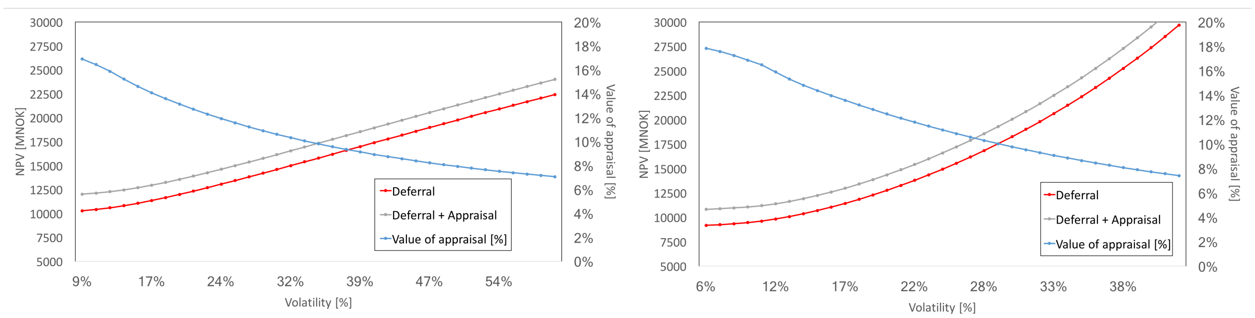
When we assume prices to vary as a mean reverting process, see Figure 7.1b, we see the same exact trends as we see in Figure 7.1a. We do however see the value added by appraisal to consistently be slightly higher when we assume prices to follow a mean reverting process compared to when we assume prices to follow a GBM process. An explanation lies in the pessimistic view of OU-process when the equilibrium price is lower than the initial price. Because of this future view, the ability to plan and produce at optimal production levels holds more value when one assumes prices to follow an OU-process for low initial oil prices. Additionally, by assuming an OU-process we consistently get a lower value for the project compared to a GBM process, in line with expectations.

Another interesting finding is the lowest value for which the project will be profitable for the two options considered. Assuming the price to follow a GBM process we find the lowest value for which the project is profitable to be $56 for the option to defer and $47.25 when we add the option to appraise. These are the first points at which the grey and red lines are positive in Figure 7.1a. The lower price point for profitability for the latter case is a consequence of the appraisal result. When the decision maker conducts an appraisal he is better able to plan for the case that the appraisal indicates as most likely. Should the appraisal indicate the high case as most probable, the decision maker will be encouraged to develop at lower prices, resulting in positive option values. However, a priori the probabilities are high for base, low and high outcomes resulting in the decision maker planing less for a specific case and more for the whole range of outcomes[7]. Assuming prices to follow an OU-process we find the first point of profitability to be the exact same as when we assume prices to follow a GBM process. This is due to the assumption made for how the production profiles are generated, i.e. as they only depend on the initial oil price. We explain the computation of production profiles in Section 6.3 and discuss them further in Chapter 8.

[7]The updating of probabilities was explained in detail in section 6.4

7.3.2 Volatility

In Chapter 3 we discussed how increased uncertainty generally leads to a greater option value as the option holder is able to limit the downside while profiting from the increased upside potential. Now, we conduct a sensitivity analysis on the volatility where we compute the option values for volatilities 70% below the initial value and up to 100% above the initial value. As the the initial volatility for the price processes are different the horizontal axis in Figure 7.2a and 7.2b have different values. Figure 7.2a shows how the model assuming prices to follow a GBM process reacts to changes in volatility. As the figure illustrates, for increasing volatility the value of the project increases, both when we account for the option to defer, and when we account for the option to appraise. The blue line shows the added value of appraisal declining as a consequence of increased volatility. At first this might seem counter intuitive, especially considering the argument that increased volatility results in higher option values. The negative trend in value of appraisal is interpreted as a consequence of the optimal production profiles becoming sub-optimal as the price used to generate them becomes increasingly volatile. On the other hand, when the prices are less volatile, production profiles are more in line with the initial price used to determine them, resulting in an increase in appraisal value for lower volatilities. The option value on the project will however increase as a consequence of increased value in the deferral option for more volatile prices.



(a) Oil prices assumed to follow a GBM process

(b) Oil prices assumed to follow an OU process

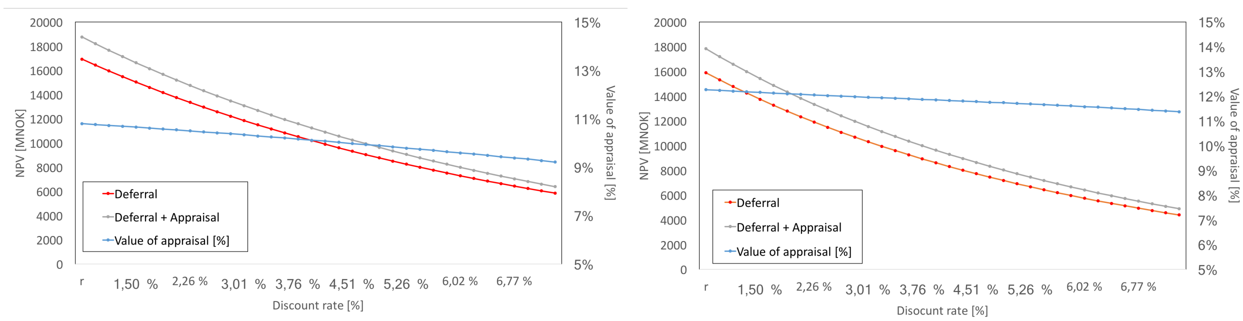
Figure 7.2: The figures show the effect on the decision to develop when varying oil price volatility for price processes modelled stochastically as (a) GBM process and (b) OU-process.

These observations are also consistent with the findings when we assume the oil price to follow a mean reverting process. The argumentation for the behaviour is similar to the one presented for the price assumption of GBM above.

We do however find in Figure 7.2b that the value of appraisal for mean reverting prices is consistently higher compared to the value of appraisal when we assume prices to follow a GBM process. The explanation for this is given by the characteristics of the OU-process itself. For an initial oil price close to the equilibrium price, the mean reverting process will predict future prices to be closer to the initial price compared to when we assume prices to follow GBM. Future oil prices being close to the initial oil price will result in the production profiles generated to be more accurate and thus result in a higher value of appraisal.

7.3.3 Discount Rate

We use the risk free rate as the discount factor for the option model consistent with option theory[8]. From Figure 7.3 it is evident that the option value for the project, regardless of the options we include and the price process assumed, decreases as the discount rate increases. This is consistent with intuition; as discount rates increase, future revenues hold less value at present time. Furthermore, if delays in the arrival time of information from the appraisal had been modelled, we would expect the discount rate to have a greater impact on the value of appraisal. This is due to the flexibility of observing price changes in the time period we are delayed. We discuss this further in Chapter 8.



(a) Oil prices assumed to follow a GBM process

(b) Oil prices assumed to follow an OU process

Figure 7.3: The figures show the effect on decision to develop when varying discount rates and modelling prices stochastically as (a) GBM process and (b) OU-process.

[8]This assumption is explained in section 3.3.2.2

Chapter 8

Conclusion and Model Extensions

This thesis studies the investment problem of an oil company that faces the question of whether and when to invest in development of an oil field and how it should produce optimally. The profit of the firm is highly reliant on the oil price and the reservoir size of the field. The former is modelled by two stochastic price processes, as a geometric Brownian motion and as an Ornstein Uhlenbeck process. The latter is modelled by an optimization model that derives the optimal production plan. Additionally, we consider the case where the firm has an opportunity to obtain more information by investing in appraisal at an additional investment cost.

Our results show four main findings: (i) There is considerable value in accounting for the managerial flexibility to defer investment, which the traditional Discounted Cash Flow (DCF) approach is not able to account for. This is because the DCF approach neglects the stochastic feature of the oil price and the firm's possibility to wait and account for this by investing if price conditions become profitable in the future. Furthermore, we confirm previous studies (Dixit and Pindyck [1994], Fleten et al. [2011]) that show that the value of the option increases for higher uncertainty.

(ii) The value of obtaining more information about the reservoir size may be significant. A priori the firm faces high uncertainty about the reservoir size, but undertaking additional appraisal reduces this uncertainty. Specifically, our results indicate that the firm should invest for lower prices when adding the option to appraise and that the value of appraisal is higher for lower initial oil prices. This makes sense as the appraisal program reduces uncertainty and allows the firm to optimize the production based on the outcome of the appraisal, thus allowing for better planning for low oil prices. Furthermore, our results indicate that for high prices the appraisal value diminishes compared to the overall value because the field is profitable regardless of the reservoir size.

(iii) The firm will adjust its production based on the oil price and be willing to make a larger investment if the price condition is promising. However, our results also indicate that the firm only will increase its capacity, i.e invest in more wells, for relatively large increments of the oil price because of the high investment costs associated with capacity expansion. Additionally, we find that when prices are between $47 - $ 56, the firm will only produce if it has the appraisal option. This is because one is able to specifically plan for the case where the appraisal indicates a large reservoir. For production to be profitable at such low prices, this specific planning is necessary. Thus our results indicate that the combination of being able to get new information and adjust production plans accordingly may change the firms view of the profitability of the project.

(iv) Finally, in line with the option theory and other studies (e.g Guedes and Santos [2016]) we find that modeling the price as a mean reverting process results in a lower value for investment opportunities than by modelling the prices as a random walk. This is due to the mean reverting process' tendency to revert to the long term equilibrium price, which in our case is lower than the current price. Our results indicate this tendency for both the deferral and the appraisal options.

The model in our thesis is based on the integrated real options approach first presented by Smith and McCardle [1998]. We extended the approach by determining the optimal production profiles by an optimization model. This allowed us to consider how an oil company can plan its production optimally when there is uncertainty about the reservoir size. Furthermore, we also modeled arrival of new information through Bayesian updating. This allowed us to model the flexibility of obtaining more information about the reservoir size in a logical and statistical framework. Finally, we applied a numerical real options approach to solve the model. The real options approach allowed us to model managerial flexibility and to derive the optimal decision of whether and when to invest, and whether to conduct appraisal. To the degree of incorporating a more realistic reservoir model in a real options framework, our model is a first step towards a fully realistic model. The model proposed optimizes profiles for initial oil prices, but in real life the decision maker would want to optimize the production profiles in accordance to future price developments. Furthermore, the arrival of information from the appraisal was assumed to be immediate, which is another simplifying assumption.

This concludes our findings in the thesis. Now we propose some model extensions that in the authors' opinion will either improve the modeling of accounted flexibilities, or extend the model to account for other flexibilities that may be present in oil field development projects.

Optimize the production profiles at the price which development takes place

For simplicity the optimization of the production profiles were based on the initial oil price. However, as we discussed in the comparative analysis section the profiles will likely change if the initial price changes. Thus, a natural extension is to run the optimization at each decision point. This is straightforward to implement, but in the authors' opinion requires the use of a more robust optimization tool than the Excel solver, which was applied in this thesis. The reason for this choice was to offer a simple and user friendly interface in a commonly known software. However, the authors' would argue that in order to create a more realistic model, one has to use more complex software and be willing to trade away some of the user friendliness of Excel.

Account for stochastic price movements after the decision to develop has been made

In the model the price is assumed to be equal to the expected value of the stochastic process after the decision to develop has been made. Thus we do not account for future changes in oil price after the development decision has been made. By modeling the price as stochastic after the decision to develop has been made, it would enable the firm to decide how long it should produce based on future prices and not only based on the current oil price. This could in turn lead to better decision making and add more value to the project. Conceptually, the implementation is straightforward; At each decision point in the decision tree a separate tree modeling the value of developing instantly would have to be implemented to model the future oil prices. For a recombining n-period binomial tree the number of decision nodes is given by $(n+1)(n+2)/2$. With the additional modeling required the number of nodes to calculate would increase by $(n_p+1)(n_p+2)/2$ where n_p is the number of periods chosen by the modeller. With a project duration of 16 years and using the same time step as assumed in our case, the number of nodes would increase from 1 127 251 to 12 654 452 nodes[1]. Thus the computational cost could be significant.

Adding the Option to Expand

In the implementation it was assumed that once the firm decides to investment it did not have any possibility of increasing the production capacity. However, if prices were to rise the option to expand production may add value (Smith and McCardle [1998]). Additionally, it may be the case where the reservoir size is bigger than anticipated making this option desirable. An interesting extension is therefore to allow for the option to increase the number of wells at any time after development. By doing so the firm would need to undertake an additional investment, but would only do so if the expected gains for an additional well is larger than the cost of it.

Delay in arrival time of information

As explained in Chapter 6, we assume that the information from appraisal arrives immediately after the decision to conduct appraisal is made. In reality an appraisal program may take up to several years to conduct. Moreover, in this delay period prices may change substantially. In addition, when the development of the project is delayed we also need to account for discounting effects which may affect the value of appraisal. These factors may potentially lead to economically better investment strategies if accounted for, and are thus desirable to include in the project valuation.

[1] Remember that the current model only considers 5 year of price development, but the extension considers 16 years of price development at each node in the current tree.

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Appendix A

Appendix

A.1 Historical weekly European Brent oil spot prices

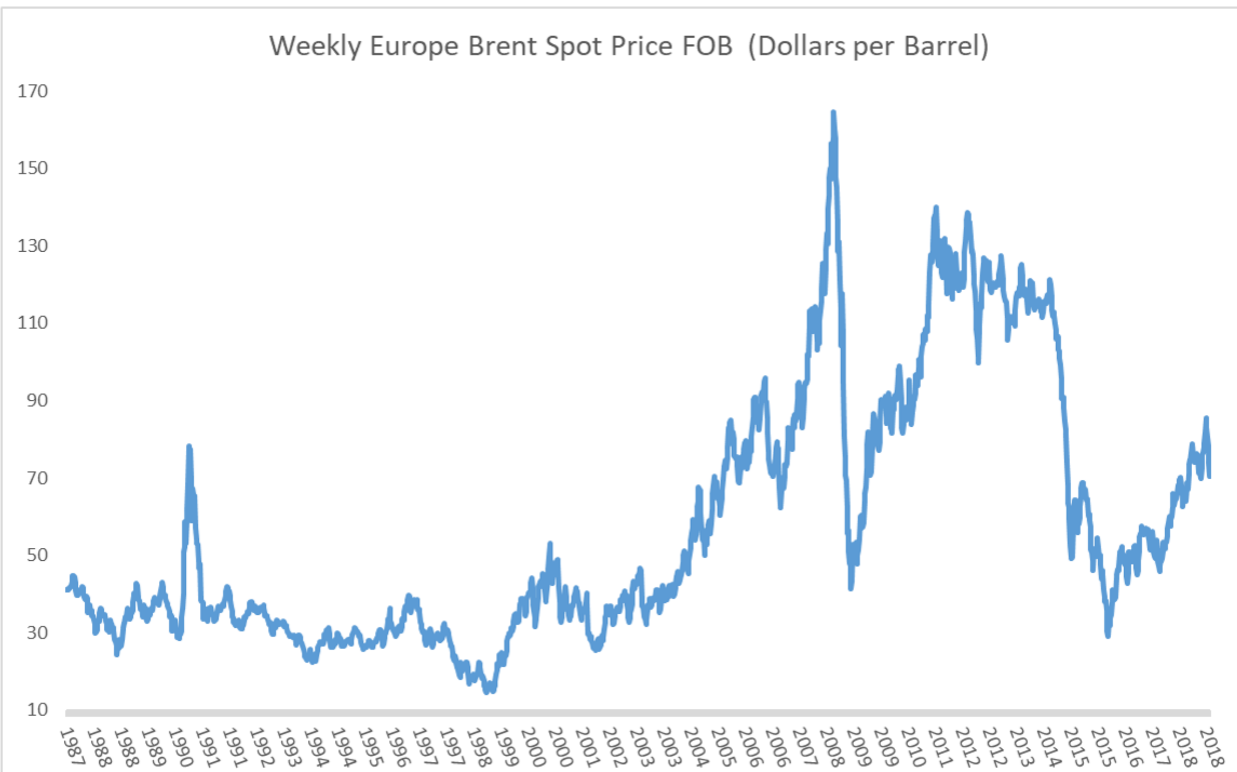

Figure A.1: Weekly European Brent Spot prices for the period 1987-2018 in 2018 prices

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