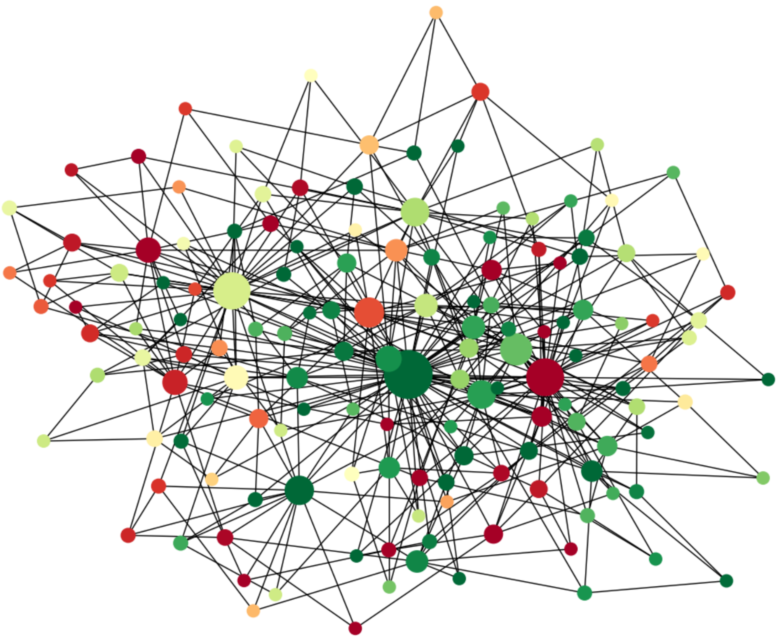
Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Engineering
Department of Mechanical and Industrial Engineering

Sigrid Bratsberg

Modelling Behaviour Change Dynamics on Social Networks towards Climate Change Mitigation

Master's thesis in Engineering and ICT
Supervisor: Astrid S. de Wijn, David Andersson and Andrew Ringsmuth
June 2019



An example of a clustered scale-free network

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Summary

English
This thesis is concerned with the spread of altruism in a population facing the problem of dangerous climate change. The population is divided into cooperators and defectors as in social dilemmas, and put into a network to simulate the spatial structure of relations. The use of a network is due to findings describing that the structure of people in a population influences the spread of opinions and may significantly change the result. The simple dynamics are made with both a continuous and discrete option of cooperating and put into a model. The results from the simulations show that we could have a resulting population of all defectors, all cooperators or equilibrium of both strategies depending on the different parameters. People of strong opinions could be used in the continuous cases to induce a change in the overall population until all agree, and external impacts could force both types of models to change the final state. Moreover, the use of networks introduces the phenomenon of local grouping, where people of the same strategy tend to be more stable together.

Norsk
Denne oppgaven handler om spredningen av altruisme i en befolkning som står overfor utfordringen med farlige klimaendringer. Befolkningen er delt inn i tilhengere og motstandere basert på holdninger, og satt inn i et sosialt nettverk for å simulere romlig struktur. Bruken av et nettverk skyldes funn som beskriver at strukturen av mennesker i en befolkning påvirker spredningen av meninger og kan endre resultatet vesentlig. Den enkle dynamikken er laget med både et kontinuerlig og diskret alternativ for samarbeid. Simuleringer er gjennomført basert på en samensatt modell. Resultatene viser at vi kan få en resulterende befolkning av kun motstandere, kun tilhengere eller en likevekt mellom begge strategiene, avhengig av de forskjellige parametrene. Personer med sterke meninger kan brukes i de kontinuerlige tilfellene for å fasilitere en endring i befolkningen til alle er enige, og en ekstern påvirkning kan føre til endring i sluttilstanden for både kontinuerlig og diskret dynamikk. I tillegg finner man samlinger av mennesker som er enige, slik at grupper med samme strategi viser seg å være mer stabile.

Preface

This project is written as a part of a five year master's degree in Engineering and ICT with specialisation in Product Design and Materials. The thesis has been carried out throughout the spring of 2019, being based on the project thesis written during the previous autumn semester. It is written on behalf of the Department of Mechanical and Industrial Engineering at the Norwegian University of Science and Technology (NTNU).

I would like to thank my supervisor Astrid S. de Wijn for her support, brilliant understanding and good advises helping me to steer in the right direction. I would also like to thank David Andersson for good discussions and always being patient with my questions. Lastly, I would like to thank Andrew Ringsmuth for introducing topics far from my previous knowledge and helping to find relevant literature.

NTNU, 07.06.2019
Sigrid Bratsberg

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Abbreviations

C	=	Continuous
CPR	=	Common Pool Resource
D	=	Discrete
SD	=	Standard Deviation

Chapter 1

Introduction

1.1 Background and motivation

The world today is affected by several big challenges and among one of the greatest is the risk of dangerous climate change. People of all nations share this challenge, and no single nation can solve the problem alone on behalf of the rest [1]. This calls for cooperation and altruistic behaviour among people. Fortunately, humans do not always act purely in their own best interests. Many are willing to sacrifice something for the greater good, even if there is no obvious immediate material reward. Such problems where the impact of a single person's behaviour is negligible in the big picture, but if everyone acts as most beneficial for them it would be harmful in the long term, are called social dilemmas [2].

1.2 The Problem

Understanding the spreading mechanisms of human behaviour and opinion is key to finding the necessary tools to mitigate the effects of climate change. In this thesis, the main focus will be on the spread of different strategies in a human population, in order to find out what is necessary for a successful model of cooperation in a community. To be able to build this model, two fields of science are combined, namely network theory and evolutionary game theory with focus on cooperation for sustainable development, in addition to several models used to simulate populations under different circumstances.

1.3 Project Scope

1.3.1 Objectives

This project will be looking into the human behaviour in closed and simplified populations, where the populations are divided into those who act altruistically and those who do not. The literature study will involve complex models and experiments used to explain different aspects of human behaviour and dynamics in a system. This includes general network theory, studies of spreading phenomena in networks and studies based on evolutionary game theory. The objectives of this thesis are to find which factors are important when building a network model of collective human behaviour. Based on these findings, a model is to be built by an iterative and exploratory method. Further, the model will be used for simulations with different factors presented in the literature study. Special interest is given to the spread of behaviour based on some people with higher influence.

1.3.2 Research Questions

The thesis tries to answer the following questions:
RQ1: Which factors are important to build a network model to represent human interactions with regards to climate change?
RQ2: What do models of human sustainability behaviour teach us about convincing society to behave altruistically?
RQ3: What is the impact of polarised opinions, and is there a difference in dynamics between binary and continuous opinions?
RQ4: How does the population react to the influence of some people of strong opinions?

1.3.3 Limitations

In order to keep this project within a reasonable scope, there have not been any attempts of gathering data concerning real networks or climate change attitudes, and comparing the model to these. The project is exploratory in the manner and aims to qualitatively, rather than quantitatively, describe the collective development of climate attitudes. Moreover, this project is not concerned about modelling how much the climate will change, and sustainability is only used as a motivation for people to choose between different attitudes and behaviours.

1.4 Thesis Structure

The thesis consists of both theory describing networks and models of cooperation, and this is split into two different chapters. The network science can be regarded as the tools used in order to build a network and analyse its properties, while the chapter about cooperation games is used to build the basis for the top level analysis and the bottom level interactions. The theory part aims to answer the first research question. Further, these two parts of the theory are combined into a proposed model in the fourth chapter. The model is used to simulate, and the results of the simulations will hence be presented. Finally, the implications of the simulations is discussed.

Chapter 2

Network Theory

In order to discuss and use networks as a basis for the models studied in this thesis, a summary of relevant network science is given in this section. Most of the following aspects are based on the theory from Barabási et al. [3], and the curious reader is encouraged to look up a more detailed explanation in his book, which could also be found online [4].

2.1 Basics

A network is described by a finite set of N nodes and L links between the nodes. The links can be given as a list of the links and the nodes they link together such that $w_{ij} = 1$ if nodes n_i and n_j are linked together and zero otherwise. In a weighted network, the links may be given a weight describing the bond strength between two nodes, such that the value of w_{ij} may be any number. The links can be defined as undirected or directed, meaning that the connection is either symmetrical or not. In this project, we will mainly be concerned with undirected links.

2.2 Social Networks

A social network is a network where the nodes represent individuals, and the links represent a relationship between these individuals. Examples of what links might represent includes (and is not limited to) friendships, communication, Facebook friends and co-working. Humans tend to seek together in communities, and within communities, the number of other people each is regularly interacting with is limited. Dunbar found this figure to be somewhere around 150 people [5], and is also found in village and tribe sizes and can thus be assumed to be a regular community size.

2.3 Network Properties

To be able to characterise and understand the behaviour in networks, some basic terminology is needed. In this section, some properties of networks will be briefly explained based on Barabási et al. [3].

2.3.1 Degree

The number of links connected to a node n_i is denoted as the degree k_i. Note that this is different for a directed network. The average degree in an undirected network is

$$\langle k \rangle = \tfrac{2L}{N}.$$

Networks can have many different degree distributions and this will be more discussed later.

2.3.2 Connectedness and Path Lengths

The nodes n_i and n_j are called connected if there is a way to get from one to another by traversing the links between. A set of links connecting two nodes is called a path. This is used to find the distance $d_{i,j}$, also called geodesic, between two nodes, which is the shortest path between the nodes and is counted as the number of links traversed. A network is connected if there is a path between any two nodes. The average path length $\langle d \rangle$ in a network is the average of the distances between all nodes gives as

$$\langle d \rangle = \frac{1}{N(N-1)} \sum_{i \neq j} d_{i,j}.$$

For social interaction networks, this number is found to be surprisingly small, giving rise to the popular theory of ”six degrees of separation”. This theory proposes that all people on earth are on average six acquaintances away from each other. Barabási et al. argues that such a low value might be realistic for social networks. The existence of random links binding together nodes of completely different parts of the network is the reason why some networks show the ”small world property” [6], such that the network has an average path length that scales with lgN.

2.3.3 Clustering

A property found in several types of network, especially social networks, is clustering [7]. If node n_i and n_j are linked, and n_j and n_k are linked, then there is a higher probability that n_i and n_k are linked to each other as well, forming a triangle (triad) [6]. The clustering coefficient measures the degree of clustering. It is given as ”in physical terms, C is the probability, averaged over the network, that two of your friends will be friends also of one another” by Newman and Park [8, p. 3]. A typical value for C in social networks is found to be around 0.5 [7].

2.3.4 Assortativity

In social networks, popular people tend to be friends with other popular people. This of property of degree correlation is called assortativity. Degree correlation is present if there is a systematic difference between the high-degree low-degree node links and what would be expected if it was random. A network where high-degree nodes are mainly connected to low-degree nodes is called disassortative.

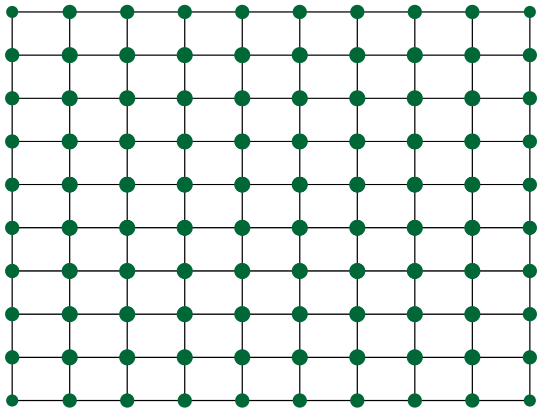


Figure 2.1: A 2D lattice with N=100

2.4 Network Structure

Networks can be described in terms of network structure and network types. Some networks with a particular set of properties have been given a name. In this section, four types of networks are described.

2.4.1 Regular Lattice

A simple network type is a regular network where all the nodes have the same number of connections and are structured regularly. An example of this is a regular 2D lattice network, also called a grid network, as seen in figure 2.1. In this type, all nodes have the same degree $k = 4$, except the nodes along the edges. This edge effect, usually called finite-size effect, can be avoided by using periodic boundary conditions. This gives a high degree assortativity, as the nodes connect to nodes of the same degree. The neighbourhood of a grid could be either the von Neumann neighbourhood with the nodes directly to the sides and above and below, or the Moore neighbourhood also including the diagonal nodes. In a 2D lattice network we would find a high average distance, as there are no ”shortcuts” across the network. Further, in a grid with von Neumann neighbourhood $\langle k \rangle \approx 4$, the clustering coefficient would be zero, as there are no triads formed. This is not the case with the Moore neighbourhood with $\langle k \rangle \approx 8$.

2.4.2 Random Networks

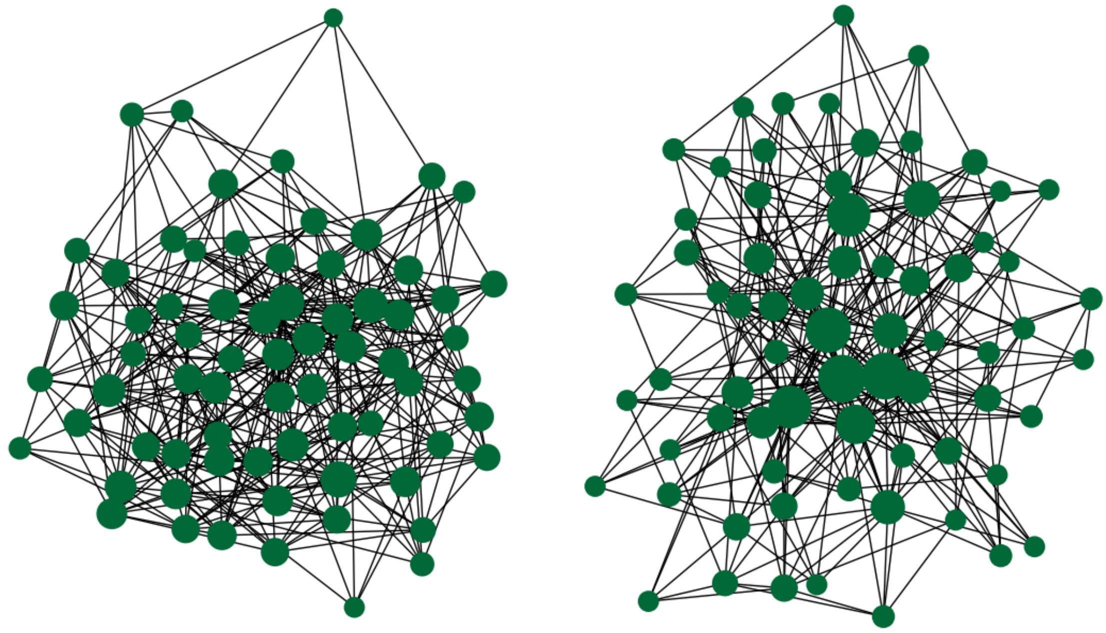
A network where any two nodes are linked to each other with probability p is called a random network. As the likelihood of linking is the same for all nodes, random networks have a dense degree distribution with a peak at $\langle k \rangle$. An example network is visualised in Figure 2.2a, where the node size indicates the degree of the specific node. Random networks are expected to have a low average distance [6], low clustering coefficient and an assortativity number of around zero[9].

2.4.3 Scale-Free Networks

Barabási et al. have described a type of network characterised by a power-law degree distribution of the nodes, given by the probability p_k of a given degree k as follows:

$$p_k = Ck^{-\gamma}.$$

In the given equation, C is a constant, and γ is the degree exponent, often between 2 and 3. This they named the Scale-Free Network as networks with $\gamma < 3$ can show arbitrarily large degrees. Note that in scale-free networks, there exist some hubs, being very well-connected nodes as a result of preferential attachment. The high degree is visible in figure 2.2b by the size of the largest nodes. A good example of a scale-free network is the internet, where the most popular websites are linked to from a lot of other websites, while the vast majority only have only a few links in comparison. Further, it has been argued that many types of social networks take the shape of scale-free networks, such as the actor network[3, 10]. This network consists of all the actors in the IMDB database, and two actors are linked if they have played in the same movie.



(a) Random network **(b)** Clustered Scale-Free Network

Figure 2.2: Example of two networks with N=70. The relative node size indicates the degree where larger nodes have a higher degree.

2.4.4 Clustered Scale-Free Networks

In a random network or scale-free network, the expected clustering coefficient is close to zero as links are made randomly, and not in any pattern. However, as it is not rare in social networks to have common friends, Holme and Kim proposed a model to build networks with a tunable clustering coefficient [10]. The network model combines the scale-free property with clustering by a probability p. This makes for a better approximation of a social network, with a low average distance and high clustering coefficient. The property this network type fails to reproduce is the degree assortativity that could be found in social networks.

2.5 Communities in Networks

In several networks and especially important for social networks is the principle of community structure. Intuitively a community is a group of people with a higher interaction density with each other than with others outside the community. This could be, for example, a person's family, friends or colleagues. Note that this is usually a larger group than a cluster, which by definition is a triadic closure. The community structure detected by an algorithm is also called partition as it divides the network into smaller parts.

2.5.1 Modularity

In order to find these communities, one would have to find a measure to calculate the quality of a partition. A measure designed for this purpose is modularity [3]. Given a randomly wired network, one would not expect any particular structure in the links. Then, by comparing the density of links within a given community partition and the expected density in a randomly wired network, one finds the relative link density called the modularity. The mathematical definition of the modularity is given as

$$M = \sum_{c=1}^{n_c} \Big[\frac{L_c}{L} - \big(\frac{k_c}{2L}\big)^2 \Big]$$

where L_c and k_c are the number of links in the community and the total degree of the nodes in the community respectively. The value of the modularity in a network is in the range [-1, 1] where a higher value is better, but all positive values indicate some community structure. Zero means that the whole network is in one community, and a negative value is obtained by assigning each node to its own community. The Louvain algorithm uses modularity maximisation to find the best partition [3]. Note that there are several partitions usually giving about the same modularity, so the Louvain algorithm optimises and chooses a good partition from the high modularity plateau.

Studying social networks, one could find communities in, for example, a network of mobile calls. An important finding in this graph is that most links between individuals in the same community have a higher weight than the links between individuals of different communities [3].

2.5.2 Complex Contagion

Spreading phenomena such as the one to be studied in this thesis can be assumed to fall within the category of complex contagion as described by Barabási et al.. The difference between simple contagion and complex contagion lies within the difficulty of the spreading process. With simple contagion such as diseases, it is often enough to be in contact with someone to contract it. In comparison, it is usually important to be repeatedly exposed to an idea, behaviour or attitude to adopt it, making this spreading process more demanding. In both cases, the existence of communities affects how the spreading materialises. The community structure may make it easier for the idea to spread as the clustered ties between nodes makes for repeated influences within the community. However, there is also a fair chance of the spreading being captured within the community, never reaching other communities.

Chapter 3

Theoretical Background

Several methods have been proposed to model and predict how people behave and affect their surrounding environment and the people in it. In the case of climate change, it is generally thought that the lack of support in the public is due to lack of information. However, studies show that it is not the case and that people's opinion is more tightly knit to the attitude of their peers [11]. In order to build a model to study this, several models and theories are explained in this section.

3.1 Tipping points and social norms

As climate change has become a subject of public discussion, people start to identify as a part of a social movement. This phenomenon is confirmed in the case of climate deniers (often also called sceptics), who are shown to display some feeling of group identity [12]. The separation between believers and deniers might further polarise the attitude of people creating echo chambers [13]. However, in social processes, the behaviour of people is affected by the expectation of behaviour and attitude of others [14]. This leads to different dynamics of behaviour change if it is visible for others, and to behaviour conformity in groups. Local clusters of cooperating people may emerge, and if the behaviour is sufficiently visible and easy to adopt, it might spread in the society. If a specific behavioural choice becomes more attractive the more widespread it is, it can be characterised as a virtuous or vicious cycle. The tipping point is the point where the cycle turns from one of the types to the other [14]. An example of such behaviour is choosing to drive an electric car. More people owning and driving such cars would lead to more charging stations and thus making it more attractive to own one as illustrated in fig 3.1. If a critical number of adopters is reached, the cycle may continue working without the influences of external factors.

However, the fear of stigmatisation, is in some cases, causing people to withhold their beliefs around climate change [15]. This goes both ways: some feel anxious about not living up the standards of green behaviour while others do not want to be labelled, for example, a ”tree hugger”. This could prevent the successful spread of behaviour so that the tipping point never is reached.

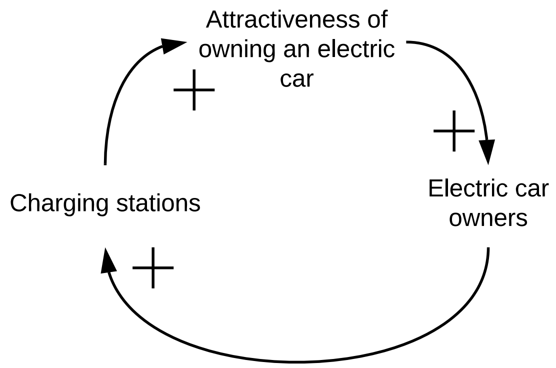


Figure 3.1: A causality diagram of a virtuous cycle of electric cars and charging stations, where more electric cars leads to the building of more charging stations, which increases the attractiveness of owning an electric car.

3.2 Game Theoretic Approaches

A common way to model and predict the outcome of a choice under a set of conditions is by using game theory. Usually, the group of people making the choice is called players, and the strategies are either cooperating or defecting. Depending on the choice made, the players typically receive some payoff or utility, such that the optimal and rational decision can be calculated. This is done by finding the Nash equilibrium, which is the alternative with the highest payoff no matter what the other player chooses. Evolutionary game theory uses repeated choices to analyse the development of different strategies, and the optimal decision may vary over time, as the players respond to the other players' choices. In public goods games, a set of players can choose between donating to a common pot or keeping their money by themselves. The pot is then multiplied by some factor greater than one and then divided evenly between all the players. The Nash equilibrium shows that the most beneficial for every single player is to donate nothing.

3.3 Common Pool Resource Models

Communities can share a limited (renewable) resource where all the individuals choose how much of the resource they extract. Since the resource is open to everyone, it is called a common pool resource (CPR) and may be, for example, a natural resource such as water. Those who extract a sustainable amount or less from the CPR are called cooperators. On the other hand, the agents may maximise their benefit in self-interest and extract more of the resource than sustainable. These agents are called defectors or free riders since they do not care about the best for the community as a whole or decide that the benefit of the resource is more important than society. This kind of situation is called a social dilemma [2]. If all members of the community maximised their individual benefit by claiming as much of the resource as possible, the community as a whole would be at risk of emptying the resource beyond repair. The phenomenon where a society — for the individual's benefit — extract more of the resource than sustainable until they trespass the resource's capacity is called the "Tragedy of the Commons" [16]. Models studying the cooperation in these kinds of cases are usually built based on evolutionary game theory, using a utility function to study which choice the people would make by repeated playing.[1]

[1]This section is rewritten based on my project thesis.

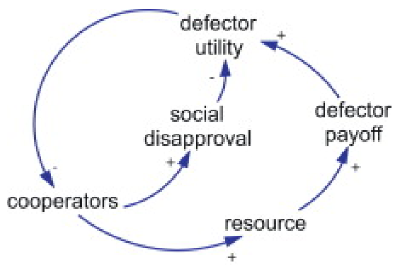


Figure 3.2: The causal loop of the TSL model from [18]

Milinski et al. define the case of dangerous global temperature rise as a collective-risk social dilemma with the following characteristics: "(i) people have to make decisions repeatedly before the outcome is evident, (ii) investments are lost (i.e., no refunds), (iii) the effective value of the public good (in this case, the prevention of dangerous climate change) is unknown, and (iv) the remaining private good is at stake with a certain probability if the target sum is not collected" [17, p. 1].

Three types of solutions are commonly mentioned for social dilemmas: motivational, strategic and structural. The first goes into the personal values and choices, the second changes the utilities of the people based on for example norms, and the third changes the structures such that it is not a social dilemma anymore [2].

3.3.1 The TSL-model

A much-studied model of cooperation within a community with a CPR is the TSL model (Tavoni, Schlüter, Levin) [18]. As the agents harvest the resource, some control mechanism may be added to the system to prevent the community from over-harvesting. One such mechanism proposed by Tavoni et al. is ostracism. That means that those who follow the general norm of being altruistic and staying at the sustainable level will exclude the defectors from social benefits and help from society. By use of a utility function, the model weights off the benefits of extracting the resource to the disadvantage of being ostracised from the community, as seen in figure 3.2. The pluses and minuses in the diagram refer to the positive and negative causality [19], such that more defector payoff gives more defector utility. This is balanced by the dynamics of more social disapproval, giving less defector utility. Moreover, as an evolutionary process of imitation is conducted, the two strategies are spread by comparing two randomly chosen agents utility and deciding to continue with the highest. The development of the two strategies in a society can follow different paths. The results may end up with a monomorphic population where all the agents share the same strategy or with a dimorphic population. Tavoni et al. [18] found that both results may emerge in the model depending on the different parameter values, especially the resource inflow.[2]

3.3.2 The TSL model in networks

One assumption in the original TSL-model is that the members in the society are well-mixed. As the dynamics in the community emerge as a result of the interactions between the members, one could argue that the structure of the relationships is essential. Hence the resulting state of the community is not dependent on only the total fraction of cooperators and the state of the

[2] This section is rewritten based on my project thesis.

resource, but also the norms around individuals. The TSL-model is further developed to account for this by Sugiarto et al. [20] by including a network structure of the agents and modified to have local discrete updating of the variables. They used asynchronous pairwise comparison such that an agent may update his strategy if the chosen neighbour's strategy is more beneficial. The more resources available for the population, the more people decide to defect, while scarce resources motivate people to cooperate. A result of this set up is that the regime shift from mainly cooperating to mainly defecting and back is characterised by hysteresis - the change is delayed compared to the change in the parameter. This is seen as they vary the amount of resource available, and the fraction of cooperators will gradually change until one point where a sudden jump is observed. The higher the degree of the nodes, the more evident is the hysteresis. From this, we get an additional result that is not evident in the case of a uniformly interacting population as in the TSL-model.[3]

Another important finding is that the average degree in the network is affecting the cooperation [21]. As the average degree is reduced, the system is less similar to a well-mixed population, and the effects of the social sanctioning are only applying to the defectors coupled to cooperators. Hence sub-communities of only defectors may emerge and survive stably.

3.3.3 Rewiring in Networks

As human relationships develop, new friendships are created and old may cease to exist. This could be replicated in networks by rewiring the links between nodes after a set of rules. Min et al. [22] conducted a study based on the TSL-model with this taken into account. With a certain probability based on the difference of utility between two agents, the chosen agent may dismiss the friendship and create another friendship, but only if the agent is a cooperator. This leads to the possibility of disconnected network components evolving, such as a one-strategy component. If the component is a pure defector one, it cancels the effects of ostracism for the agents in the component. This could possibly be leading to a stable dimorphic regime for the whole system, meaning that both strategies exist in equilibrium.

3.4 Social Media Echo Chambers

Social media offers a platform for people to discuss matters across a lot of different subjects. A study has been done on the subject of climate change on Twitter by Williams et al.. They collected data from tweets and the accompanying users' follower network based on five hashtags associated with climate change discussion both among climate change activists and the climate change deniers. The results show significant homophily for the follower networks and the retweet networks, meaning that people tend to follow and retweet others with the same opinion as themselves, possibly creating echo chambers.

Furthermore, the more active user showed a tendency to be the ones with the more polarised views. The study did not spread out over time, so the change in attitudes could not be studied. However, they found that those exposed to diverse opinions had a higher chance of being more neutral in their expressed views. The network of user mentions within tweets was the network with most mixing between activists and deniers.

[3]This section is rewritten based on my project thesis.

3.4.1 Opinion Leaders in Social Media

It has been argued that information is interpreted and spread from media to the public by some highly engaged people called opinion leaders [23]. These individuals are often well-connected in social networks, enforcing influence on both friends, family and colleagues[13]. Social media allows for extensive publications of individuals' views, making them hubs of opinion, where the number of followers can be a measure of possible influence. Nisbet and Kotcher suggest that the existence of opinion leaders could be used actively to promote climate change awareness, by making these spread information and views among their peers.

3.5 The Spread of Behaviour in Different Networks

Different network types show various properties, and it would be interesting to know which features are best to promote behaviour change. An experiment was designed for this purpose by Centola by studying the spread of health-related behaviour on constructed online social networks. This was done by giving people an anonymous profile on a constructed health site. Friendships were simulated by connecting the users to "health buddies". The linking of the health buddies followed either a clustered regular network topology or a random network. Then one person acted as an insurgent by starting the spread of a specific health forum and inviting their buddies to join the forum. The results indicate that the clustered network shows both faster and ultimately more successful spread of the forum. This is accounted to the fact that clustering generates redundant invites and the visibility of each of the buddies joining the forum. Those who received two or more invites were significantly more likely to join, and more invites correlated with several re-visits to the forum. However, this is not a very costly change in behaviour and may not be fully comparable to more complex behaviour adaptions. Studies of insurgents (called zealots) by Kashisaz et al. [25] show that a high level of random links makes for easier spread to a monomorphic regime with simple contagion.

3.6 Thresholds in Network Structures

In game theory, mathematical models are used to predict the strategy of a set of players under certain conditions. A class of such games is threshold games. Chwe [26] described the threshold as the number of neighbours in a network needed to agree on a choice before the agents decide to adopt the behaviour. This example is relevant for describing how attitudes or behaviours spread through a network of people as it becomes more popular. Note that people's choice does not depend on some external factor, such as resources or monetary incentives. Instead, the sheer number of people is enough to change people's behaviour, and it is thus called a cooperation game. The case studied by Chwe [26] assumes there are some people with a predisposed inclination towards following the proposed action, called insurgents. These may be able to spread the opinion to a sufficient part of the network so that the action takes place. The case analysed how many needs to be interested in going to a manifestation before it could be realised. The main result from this study is that the placement of these insurgents in the network is crucial for the successful spread in the network: they need to be dispersed in the network, but not spread so much that they lose their influence on their neighbours. This shows how the structure of the connections may change the result one would get by assuming everyone in a big pool. [4]

[4] This section was rewritten based on my project thesis.

3.6.1 Two-action Games on Networks

Let us have a look at games on networks with two possible actions as described by Jackson and Zenou [27]. The option between to do something or not gives rise to the name two-action games. This could be used with two classes of games called games of complements and games of substitutes. The first one is a game where the more of the neighbours of an agent chooses to do the action then it is more likely that the agent will do the same. The other is the opposite, that is the more of the neighbours do it, the less likely it is that the agent does the same. In both cases this could be used in threshold games and in a class of those called ”semi-anonymous” by Jackson and Zenou [27] as the agent only knows the actions of its closest neighbours and do not care about their individual choice, only how many chooses what. In these games, the threshold could be defined as a threshold given a degree $t(d)$. In a game of strategic complements, the agent would choose to do the action if $t(d)$ or more of the neighbours choose to do it and otherwise not.

Conversely, in a game of strategic substitutes, if $t(d)$ or more neighbours do it, the agent would choose to not do it and vice versa. A good example of such a threshold is the majority of the neighbours, being $t(d) = 0.5 \cdot d$. Another interesting proposition by Jackson and Zenou [27] is to use a continuous range of actions where the agents will adjust according to the neighbours, instead of two clearly defined actions.[5]

3.7 Linking Human Behaviour and Climate Models

There is done a lot of research on how climate change is developing and how much the global average temperature will rise. One such model is the Climate Rapid Overview and Decision Support model (called C-ROADS). By coupling the carbon model of the C-ROADS model and social models of risk perception and how this affects behaviour by the theory of planned behaviour [28], the resulting model gives a complete feedback loop as seen in figure 3.3 [29]. The idea of this coupled model is that extreme weather events, caused by the climate change, is perceived by human beings who keep this in memory and evaluate the risk such that it affects their attitude toward the environment. Other factors that might impact change in behaviour include the perceived social acceptance of changing behaviour and the perceived difficulty of such practice. If people then are convinced to conduct their actions in a different way it might change their emission of greenhouse gases, which again affects global climate change and the frequency of extreme weather events. [6]

Recent studies, on the other hand, suggest that this link between extreme weather events and change in behaviour is not that clear [30]. It is found some evidence suggesting that people believe more in global warming in warm periods or days. However, it is found to have limited effect on people’s behaviour in the long-term. This argues for the need to study how behaviour change based on other factors.

[5] This section is rewritten based on my project thesis.
[6] This section is rewritten based on my project thesis.

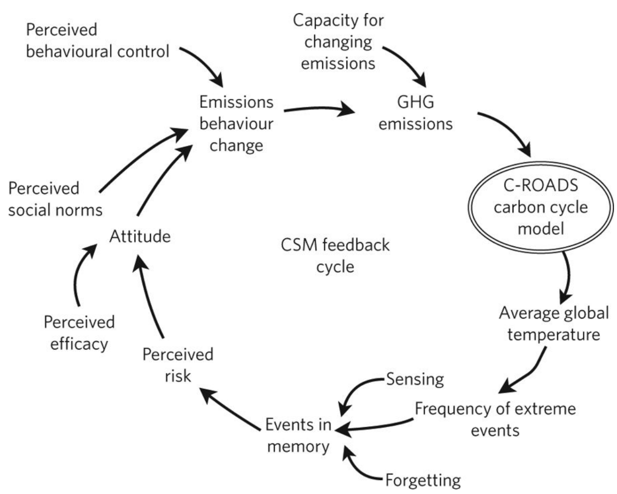


Figure 3.3: The causal loop from the model of Beckage et al. [29]

Chapter 4

The Model

In order to study the spread of climate change attitudes and corresponding behaviour in a population, a model of social connections is to be built. There are many ways to create this model, but to keep it comprehensible and within the project scope, a minimalist model is proposed.

4.1 Model Elements

Several different elements have been proposed to build the model. The most important are listed here:

- A population of individuals
- Degree distribution of social connections in the network
- A resource, possibly relative to a critical threshold
- An external control mechanism, such as publicly available information or policies
- Different kinds of interactions between the agents and the external control mechanism(s)

The general idea is to make create dynamics as seen in the causal loop in Figure 4.1. This simple feedback loop makes more cooperators increase the probability of interacting with a cooperator, which would increase the rate of people becoming cooperators. The rate of people becoming cooperators may also be affected by the exogenous factors of a sustainability-aware political climate and the utility and comfort of defecting. In the following sections, the different model elements and its structure will be discussed and chosen for further use in the model.

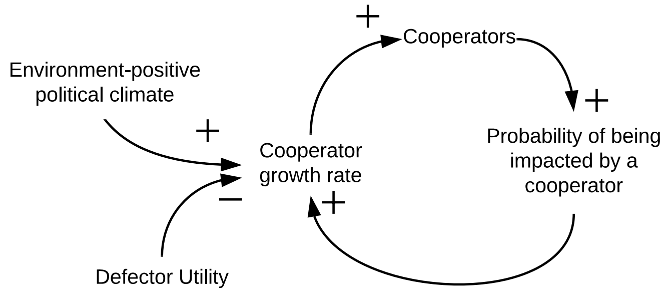


Figure 4.1: The causal loop of the proposed dynamics for the model.

4.2 The Population

The system we want to study is composed of autonomous decision-making entities, and therefore, we have chosen to use agent-based modelling as described by Bonabeau [31]. This implies that the number of cooperators and defectors at all times have to be integer numbers. The population is assumed fixed at a constant number of individuals N. Every person is represented by an agent n_i with the possibility of interaction with other agents.

The state of every agent n_i's attitude a_i towards the environment is set to be between two values: here chosen to be {-1, 1}. The use of attitude as the name of the variable is inspired from the model of Beckage et al. linking human behaviour to climate models, and the attitude is assumed to be directly linked to the impact of the behaviour the agent is expressing. The general idea is that values above zero indicate a cooperating strategy in the sense of doing sustainable actions protecting the environment from global warming. The values from zero down to minus one represents the defectors, those who do nothing to help to limit the harm on the environment or choosing to prioritise a comfortable lifestyle. There are two different versions of the model studied, one where the attitude is discrete (D) as a two-action model at {-1, 1} and one with continuous(C) values [-1, 1]. The perceived difficulty of adopting a different behaviour, as in the model of Beckage et al. [29], is included as a stubbornness weight $w_i \in [-1, 1]$ on all agents. This symbol is chosen as it can be regarded as the link strength from an agent towards itself.

The agents all need a state of the attitude before the simulations start, and several different distributions are proposed:

1. The discrete model starts with half the population being cooperators and half defectors. The continuous model starts with a Gaussian (normal) distribution around zero (neutral attitudes). This means that both model types have an average at zero.
2. A skewed population such that the average is not equal to zero. This can be realised by moving the normal distribution to one of the sides for C models and by increasing the fraction of one of the strategies for D models.
3. Some people already have a strong predisposed attitude so that they can be the insurgents convincing others to become cooperators or defectors as well, as seen in the models by Chwe, Kashisaz et al..

4.3 Network structure

The agents are linked to each other in a network structure. The network types studied are

- Grid network
- Random network (Erdős-Rényi model [3])
- Clustered Scale Free Network (Holme-Kim model [10])

as described in Section 2.4. The different network types show different characteristics as described in the theory section, and are thus used to see the impact of the different structures. In general, the clustered scale-free network might be regarded as the most realistic approximation of a social network. The random network is used as a control network, and the grid network shows good assortativity and modularity properties, even though it has much longer distances than realistic. The different network structures give rise to very different degree distributions. In the grid network, all non-edge nodes have the same degree; the random network have a binomial distribution with average $\langle k \rangle$ and the scale-free clustered network have a power-law distribution [3]. As the nearest neighbours in a grid are restricted to either a von Neumann neighbourhood or a Moore neighbourhood, we choose that the average degree for the other network types has to be $\langle k \rangle \approx 4$ or $\langle k \rangle \approx 8$ to be comparable.

4.3.1 Network Links

The link between two connected agents n_i and n_j is describing the friendship, being a generalisation of a static(not changing over time) communication network. As not all friendships are the same, the friendship strength is described by a weight w_{ij} indicating the frequency of interaction and valuation of friendship. In a realistic model, the links could be directed and also the weight depending on the direction of the friendship, but in this simplified model, we assume it to be symmetric in an undirected network. The value of the weights is in the range from zero to one. In the model, a truncated normal distribution is used to choose the weight in for the links $\mathcal{N}(0.5, 0.15^2)$. The value of the standard deviation $\sigma = 0.15$ is chosen since it makes 99.7% of the weights fall within the range of 0.05-0.95 before truncating.

In a realistic case, the network would be dynamic in size and structure, meaning that agents would have the option to change friends over time, as studied in the TSL-model with rewiring in Section 3.3.3. In this model, the network is assumed static in both number, friendship weights and in the likelihood of communication order. The latter means that at all time-steps it is equally likely that an agent interacts with any of its neighbours.

4.4 The Resource

Some of the models previously looked at, such as the TSL-model, have a specific ecological resource used as a parameter to analyse the development in the community. The ecological resource, in this case, may be modelled as the amount of greenhouse gas left for the world to emit before reaching the carrying capacity. Another option is using remaining $\mathrm{CO_2}$ quota per year as the resource, even though this could be trespassed. This could be used with a threshold according to the UN 1.5 degree goal, such that the cooperating strategy would be beneficial if the goal is accomplished. However, as most people do not perceive their direct impact on the climate [30, 32] we have decided to start by a model where the resource is not taken into account. Instead, here the defector utility U is set to be a constant; in other words, it does not depend on the amount of any resource left. This is possible as the greenhouse gas emission knows no physical upper limit; there is only a limit to the estimated sustainable levels. Resource scarcity was found to be one the motivations for agents to be cooperators in the TSL-model. With unlimited resources, as it might feel like in our case, other motivations must be considered.

4.5 The External Impact

Today most governments and local control units try to mitigate the effects of climate change by some regulations and other incentives to make it easier for people to choose to be environmentally friendly. This corresponds to structural changes that can be done in a social dilemma to make it less of a dilemma for individuals. To account for this, we have chosen to add an external factor which represents the political climate and how the following policies try to make people behave, called P. For both this factor and the utility U, values above zero indicate climate change mitigating and values under are incentives to non-sustainable behaviour. As suggested in the list of model elements, the external factor may also be other sources such as information regarding climate change. As the information available varies with which information sources people use, this could be set to be a stochastic variable. For example, when someone watches a movie about climate change being a hoax, the variable will be negative for them at that point. On the other side, someone else may read an article about how to be more climate-friendly, and the variable will then be positive. As media tend to make provocative news stories, we assume that this stochastic event $e_k \in \mathcal{U}(0, 0.25^2)$ is uniformly distributed with average at zero.

4.6 Agent Interactions

Other models used to study similar cases have been built based on game theoretic approaches. Despite this and the similarity to a social dilemma, we have chosen not to use a game theoretic approach. This is because the payoff for the different choices is not evident in a model without a resource (or public goods), and it can thus not be modelled as a CPR game. Furthermore, it is not evident which other game types this could be modelled as and thus the rules of the game is unknown. Milinski et al. [17] already said that the payoff for each player in global climate change is unknown and it might differ greatly from player to player. In addition, humans do not always act rationally by maximising their utility. Instead, one of the general solutions of social dilemmas is employed, as the political climate aims to reduce the dilemma by acting as a structural solution.

The dynamics of the interactions follow the same pattern as in the model of Sugiarto et al. [20] with pairwise asynchronous comparisons. For every time-step, an agent n_i with attitude a_i is chosen randomly to reconsider their strategy by being impacted by friend n_j with attitude a_j. As the impacted agent is chosen first and then the influencer(alter) is chosen among the first order neighbours, well connected (high degree) agents are expected to be chosen to spread their opinion more often.

The consideration is based on the following equation

$$x_{ij} = w_i \cdot a_i + P + U + w_{ij} \cdot a_j, \tag{4.1}$$

where P and U are the constants political climate and defector utility. These could be simplified to just one constant, but are kept to visualise better that they could impact the agents in both directions. The x is just the name of an intermediate result that will be used in the next step where the dynamics are divided for the discrete and continuous models. The weights w_i and w_{ij} refers to the agent n_i's belief in their own opinion and the neighbour's opinion.

4.6.1 Discrete Interactions

In the discrete model we use inequality

$$x_{ij} + e_k > 0 \tag{4.2}$$

to test if the agent n_i should be a cooperator or not, compared to a threshold of zero. The term e_k refers to a random element drawn from a probability distribution to account for random events people experience that affect their choices, such as reading articles or other impulses as discussed earlier. If the left-hand side of the inequality is less than or equal to zero, the agent will be a defector.

If both agents have the same opinion, naturally nothing will happen. If they have differing opinions, two different inequalities can be made if we simplify it by excluding the random term.

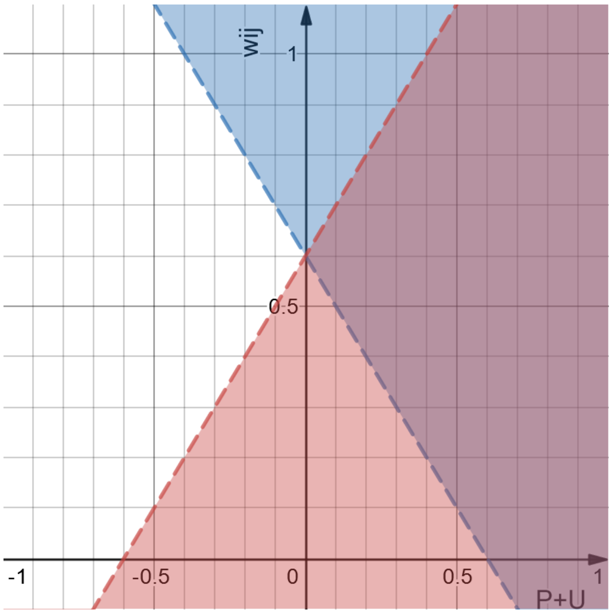


Figure 4.2: The different areas of impact for combinations of parameters with w_i of 0.6.

In the first we have n_i is a cooperator and is interacting with a defector such that n_i decides to be a cooperator following the inequality

$$1 \cdot w_i + (P+U) + w_{ij} \cdot (-1) > 0 \tag{4.3}$$
$$w_i + (P+U) - w_{ij} > 0 \tag{4.4}$$
$$w_i + (P+U) > w_{ij} \implies \text{Cooperator.} \tag{4.5}$$

Analogously we have the case where the agent n_i is a defector and is impacted by a neighbour n_j being a cooperator given by the following derived inequality:

$$-w_i + (P+U) > -w_{ij} \implies \text{Cooperator.} \tag{4.6}$$

These linear functions from inequalities 4.6 and 4.5 could be plotted as seen in figure 4.2 as the blue line and area and the red line and area respectively. Along the x-axis we can find the (P + U) parameter, that is the sum of the political climate and the defector utility. Along the y-axis is the link strength w_{ij}, and the intersection between the inequalities and the y-axis is determined by the stubbornness $w_i = 0.6$ in this example. The red line and area is the parameter combination making the agent stay a cooperator in 4.6, while the blue is the parameter combinations making the defecting agent in 4.5 become a cooperator. That means that the double-shaded region is the parameter combination always giving cooperators, the white area will always give defectors, the single-shaded red region will make the agents never change opinion, and the only-blue region makes the agents change opinion back and forth every time.

4.6.2 Continuous Interactions

In the continuous model, the test is turned into a gradual change based on the probability of the outcome of the test mentioned earlier as follows

$$\Delta a_i = |a_i - a_j| \cdot (P(x_{ij} + e_k > 0)(1 - a_i) - P(x_{ij} + e_k < 0)(1 + a_i)). \tag{4.7}$$

The first term gives the absolute value of the difference between the two agents, such that the impact is small if they already have the same opinion. The absolute value is used, as this is only meant to give the amplitude of the change, and not the direction. Then in the second term, it yields an impact based on the probability of the test, and the current value of the agent. If the agent is fully cooperating, it will not be impacted by the result of the first probability, and conversely, if the agent is fully defecting, it will not be impacted by the second probability. If the agent has a value in between, it might be impacted based on which probabilities are larger than zero.

Chapter 5

Results

The model is set up and programmed in Python v3.7.2 with the use of the package Networkx v2.2 [33]. This package is used for the creation and manipulation of complex networks and graphs. Plotting is done with the library MatplotLib [34]. The main part of the code used can be found in Appendix E.

5.1 Network Properties

The first thing to check is that the different network types display properties as expected. To do this, Networkx is used to calculate the assortativity, average path length, clustering coefficient and modularity for 100 networks and averaged. This is done for the network types grid, random and clustered scale-free with a network of 144 nodes and $\langle k \rangle \approx 4$ or $\langle k \rangle \approx 8$. The results are summed in tables 5.1 and 5.2. The results in table 5.1 show that the clustered scale-free network show the best properties in terms of the previously stated realistic values of low average path length, clustering of 0.5, and high modularity and assortativity. The property most off for this network type is the assortativity which shows disassortative tendencies, but as we do not have a realistic number to compare to it is not studied further. For $\langle k \rangle \approx 8$, on the other hand, the grid network shows the properties most realistic for social networks, except for the high average distance, so this degree is what we will use further.

Network Type	Avg. path length	Assortativity	Clustering	Modularity
Random	3.75	-0.03	0.03	0.53
Clustered Scale Free	3.27	-0.18	0.42	0.59
Grid	8.00	0.59	0.0	0.71

Table 5.1: Properties for different networks with N = 144 and $\langle k \rangle \approx 4$

Network Type	Avg. path length	Assortativity	Clustering	Modularity
Random	2.61	-0.02	0.05	0.34
Clustered Scale Free	2.53	-0.15	0.3	0.38
Grid	6.61	0.31	0.49	0.64

Table 5.2: Properties for different networks with N = 144 and $\langle k \rangle \approx 8$

5.2 Communities and Symmetric Influences

This section is explaining how the different aspects of the models work together.

5.2.1 Community Structure

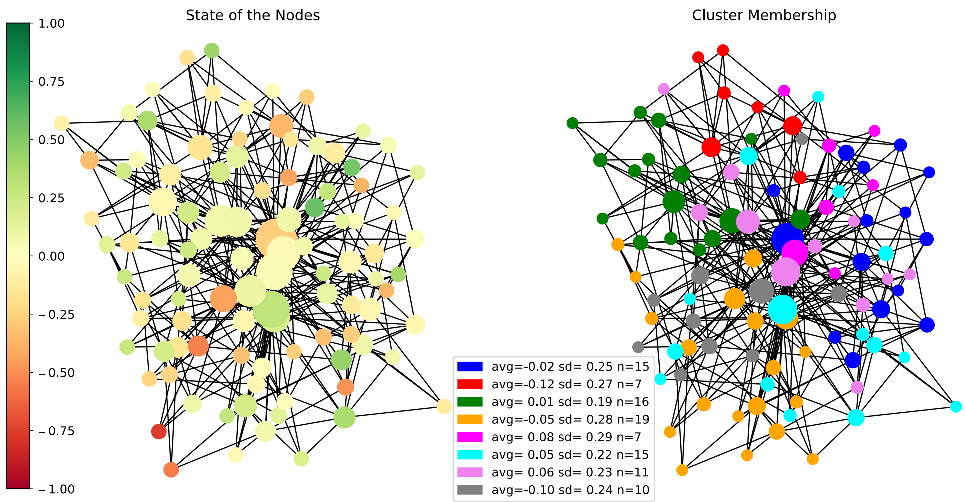
Given the different network structures, it would be interesting to see how a network is divided into different communities based on the modularity and how this affects the resulting attitudes. This is realised based on a python package [35] implementing the Louvain algorithm explained in section 2.5. Let the initial values in the continuous model be Gaussian distributed with $\mathcal{N}(0, 0.25^2)$, and the Political Climate P and Defector Utility U be equally large but opposite, and thus $P + U = 0$. A clustered scale-free network with N=100 and $\langle k \rangle \approx 8$ is shown in Figure 5.1. Note that the plotting of the node's positions is based purely on the topology of the network and not the attitude of the agents. The attitude of the agents is added later as the colour of the nodes, where green indicate a cooperator, red a defector and shades of yellow are neutral values as seen on the colour bar. The left part of 5.1a shows the initial states of the nodes. As expected, most nodes are found in shades of yellow, as they are in a neutral state.

The right part shows the resulting community structures from the Louvain partition algorithm, where colour indicates which community the agents belong to. In the legend, the resulting average state (avg) of the nodes in each community is given, along with the standard deviation of the average states and the number of nodes. The 100 nodes are divided into eight different communities, of 7 to 19 nodes in each community. Partitions of grid networks show approximately the same number of communities as random and clustered scale-free networks. However, the calculated modularity for different network types show some variability, as seen in tables 5.1 and 5.2. These values are all in the range of realistic values, for example, networks as calculated by Newman and Girvan [36], where they have found values of 0.3-0.7.

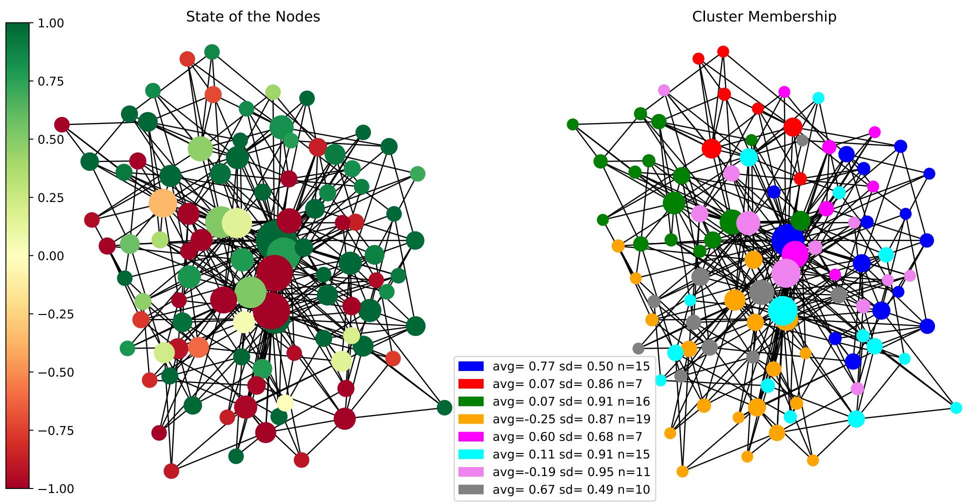
The simulation of the example network is run for 1000 time steps, and Figure 5.1b show the resulting network. There is a significant change in states for the nodes. Reading from the legend of the community structure, one can observe that there is much difference between the average state in the communities. The blue community has a considerable bias towards the cooperating strategy with average state avg = 0.77 (sd=0.50), whereas the orange community, on the other hand, is slightly defecting with avg = -0.250 (sd =0.87).

5.2.2 Example of Development On Grid

In order to understand how the nodes affect each other and how this develops in a network, an example grid network is studied. The regularity of the grid makes for the most natural visual inspection since spatial embedding in 2 dimensions is obvious. A grid network with $\langle k \rangle = 8$ is shown in Figure 5.2 for six different time steps from the beginning to time step t=1500. In the first snapshot 5.2a the agents show mostly neutral states. A few nodes are displaying more extreme views, especially a defector on the bottom row. In the second snapshot, some of the agents with previously stronger views have even stronger opinions, and several agents have adopted a stronger viewpoint. This spreading is most prominent in neighbours of the nodes showing stronger views in the first snapshot, such as the bottom row defector. In the third snapshot 5.2c the network shows some signs of attitude-clusters forming, where the agents of strong opinions have spread this to their neighbours and groups of more radicalised opinions

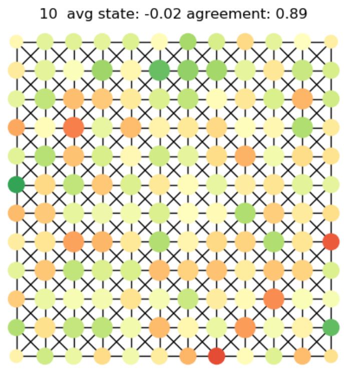


(a) State of the network before simulations

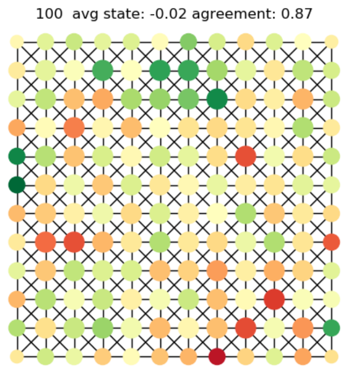


(b) State of the network after 1000 time steps

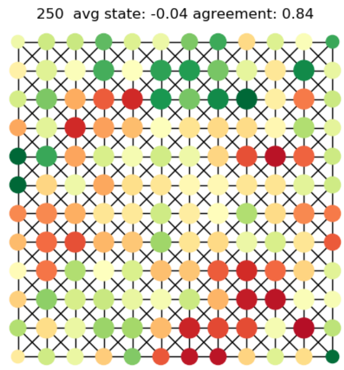
Figure 5.1: Example of a clustered scale-free network with N=100 (a) before and (after) a simulation of 1000 time steps. Left is a representation of the nodes with the value of the states corresponding to the colour as seen in the colour-bar. The right shows community affiliation, with the corresponding average state, standard deviation and number of nodes in the legend. This is equal at all times as community structure is dependent on network topology and not attitudes.



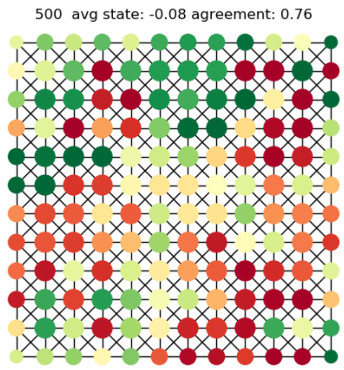
(a) t = 10



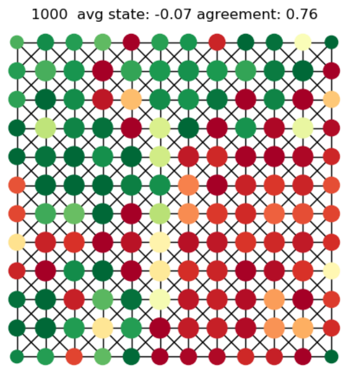
(b) t = 100



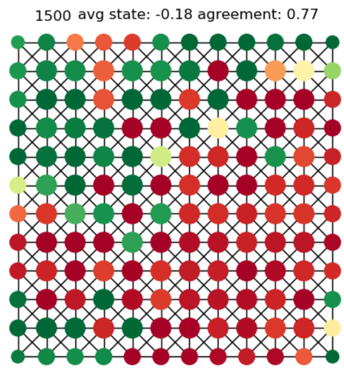
(c) t = 250



(d) t = 500



(e) t = 1000



(f) t = 1500

Figure 5.2: An example of a grid network with N=144. The sub-figures show snapshots at six different time steps in a simulation with 1500 time steps.

exist among the neutral agents. In the fourth and fifth snapshots, the neutral nodes are choosing sides and most of the agents are now strongly cooperators or defectors. In the last snapshot at t=1500 in Figure 5.2f, there are two main groups visible with the big defector group in the bottom right area, and a strong cooperator group in the top left area. This is the first snapshot where the average agreement has started to increase, as the process of radicalising leads to less agreement. Some nodes are showing neutral values in the interface between the two groups.

The general tendency through the development has been that the first nodes to be radicalised, also further impact the nodes around them, creating groups with stronger opinion. In this sense, a high number of neighbours can both increase the size of the boundary between two different groups, but also expose the agents to more influence. This might have an impact on the development of the example network, as the agents first radicalised was mainly along the edges, and the network may be subject to finite-size effects. An example of the development on a grid with discrete attitudes can be seen in the appendix A.

5.2.3 Development over Time

The symmetric model could be studied over more time. Let the initial values in the continuous model (C) be Gaussian distributed with $N(0, 0.25^2)$, and the Political Climate P and Defector Utility U be equal as in the models above. With this configuration, all forces are non-partisan and we would expect no side to be dominating on average. In the discrete two-action models

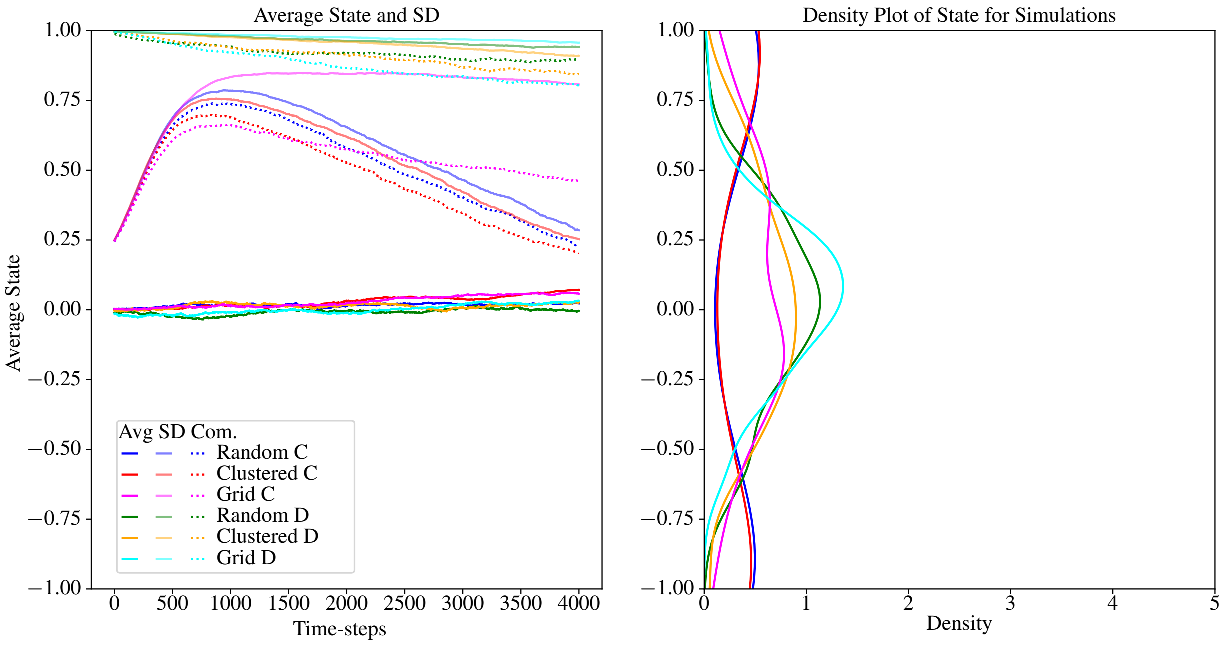


Figure 5.3: The development of the average state for different model types. The low-intensity lines show the associated average standard deviation within networks (solid) and within the communities (dotted).The right plot show the density of average resulting state for the 100 realisations for each model. The y-axis shows the average state to correspond with the left plot as a cross-section.

(D), the initial distribution is of approximately 50% cooperators. Simulating for 4000 time-steps shows how the networks are susceptible to possible influences.

Figure 5.3 visualises averages of 100 realisations of the three different network structures: random, clustered scale-free and grid network for both model types. The Figure shows clustered scale-free C in red and D in orange, random C in blue and D in green, grid C in pink and D in cyan. Most models using two-action dynamics visualises the fraction of cooperators, but since this is unfit in a model with continuous attitude values, we have chosen to rather visualise the average state of the attitudes on the y-axis, and the time steps on the x-axis. As this is the average of 100 simulations, the lines show the average of the average of the states within the simulation. Further, in order to make a comparison with the standard deviation as simple as possible, the average standard deviation is plotted directly into the same plot. Note that this is plotted directly, and not relative to the average state. As we can see from the solid line of high intensity in the Figure, the average state is staying at an even level around zero for all types. The low-intensity lines show the average of the standard deviation within each simulation network. For the continuous simulations, we see that the standard deviation first is increasing until a point at approximately t=600. After this first stage, the SD stabilises, and especially for the random and clustered scale-free networks, it decreases as the agents within the simulations reach some degree of consensus.

The dotted lines indicate the average of the standard deviation within the communities within the networks as following

$$SD_c = \langle\langle SD(a_c)\rangle_{network}\rangle_{simulations},$$

where a_c refers to the attitudes in the community. At the maximum standard deviation, the SD within the communities seem to decrease slightly before the SD within the networks as a whole. In the continuous grid model, the SD within the communities seems significantly smaller

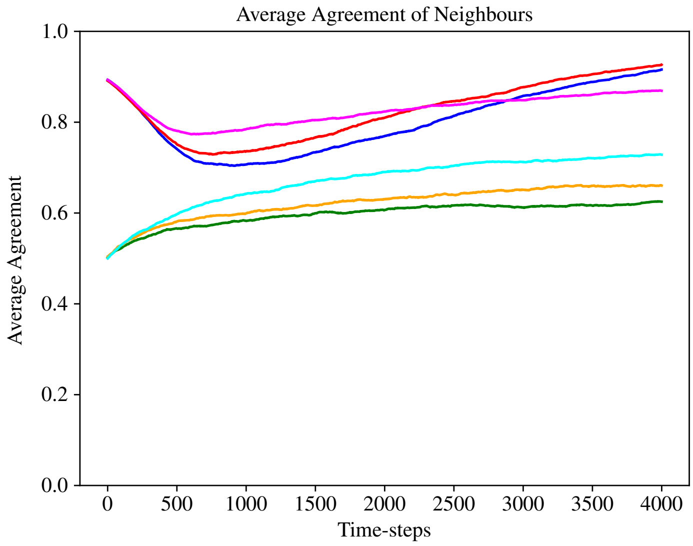


Figure 5.4: The average agreement among closest neighbours for the simulations in Figure 5.3.

than the whole networks, indicating that the communities settle on one of the strategies while the network has different strategies present. For the discrete models, one can observe a slight decrease in SD, but still high such that the simulations give a dimorphic population.

The right part of the plot shows a density plot of the average final state for the 100 simulations. This means that for the cyan discrete grid simulations, most networks end up with both strategies equally present, but with some networks with skewed final states. The clustered scale-free (C) and the random (C) networks show a different behaviour from the rest, as already seen in the standard deviation in the left plot. Most of the simulations in these cases end up as mostly cooperating or mostly defecting after 4000 time-steps, indicating that in time they will reach a monomorphic regime. This is probably the result of small noise giving one of the sides a slight advantage in the beginning, that further spreads throughout the network. As the average of the average state in the left plot is equal to zero, it indicates that it is equally many networks are ending up as defecting and cooperating.

Agreement of First Order Neighbours

In order to study the spreading effects, we have plotted the network-average of the average agreement $\langle A \rangle$ between the first order neighbours and every agent. For discrete models, it is simply the average number of agreeing neighbours. The continuous agreement among all nearest neighbours (n.n.) is calculated using the following formula for every agent n_i and averaged:

$$A_i = 1 - \frac{1}{2}|a_i - \langle a \rangle_{n.n.i}|. \tag{5.1}$$

In Figure 5.4, the average agreement is plotted, so that a value of 1 indicates that all the closest neighbours agree, and a value of 0 indicates that all the closest neighbours disagree. As this is an average for the whole network, it is not expected to fall under 0.5 due to triad closures in the network, even though it could be 0 for a specific node and its closest neighbours. A grid network with degree 4 could, in theory, be solved with a two-colouring algorithm such that 0 agreement could be reached, but this is not studied here.

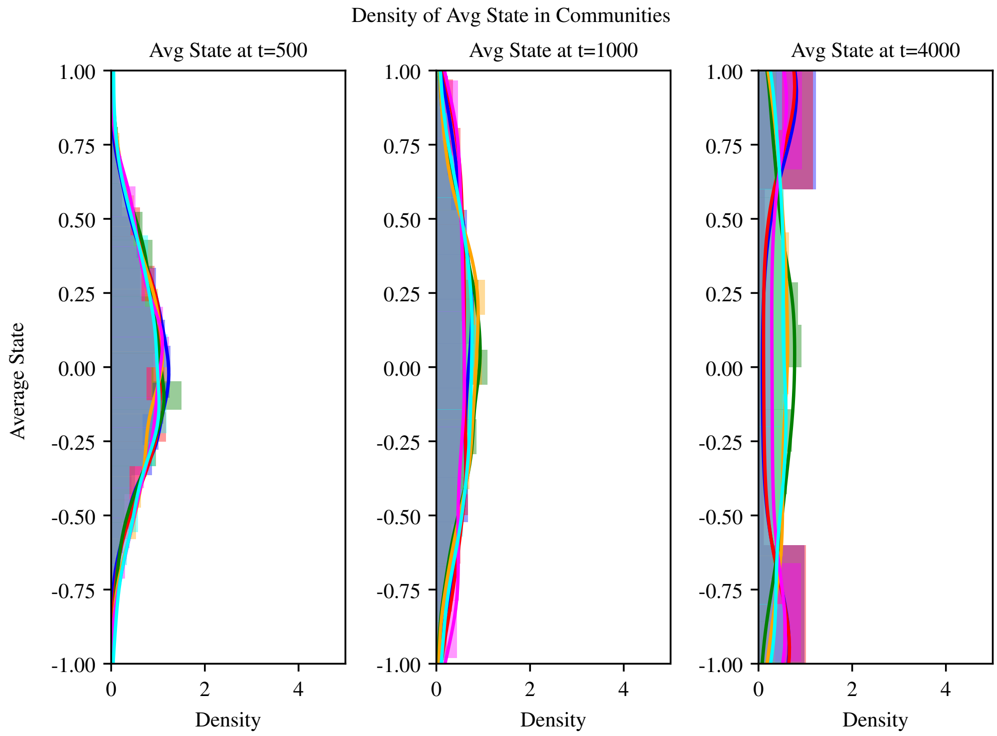


Figure 5.5: The density plot and histogram of average state within communities for the simulations at time step t=500, t=1000 and t=4000 5.3.

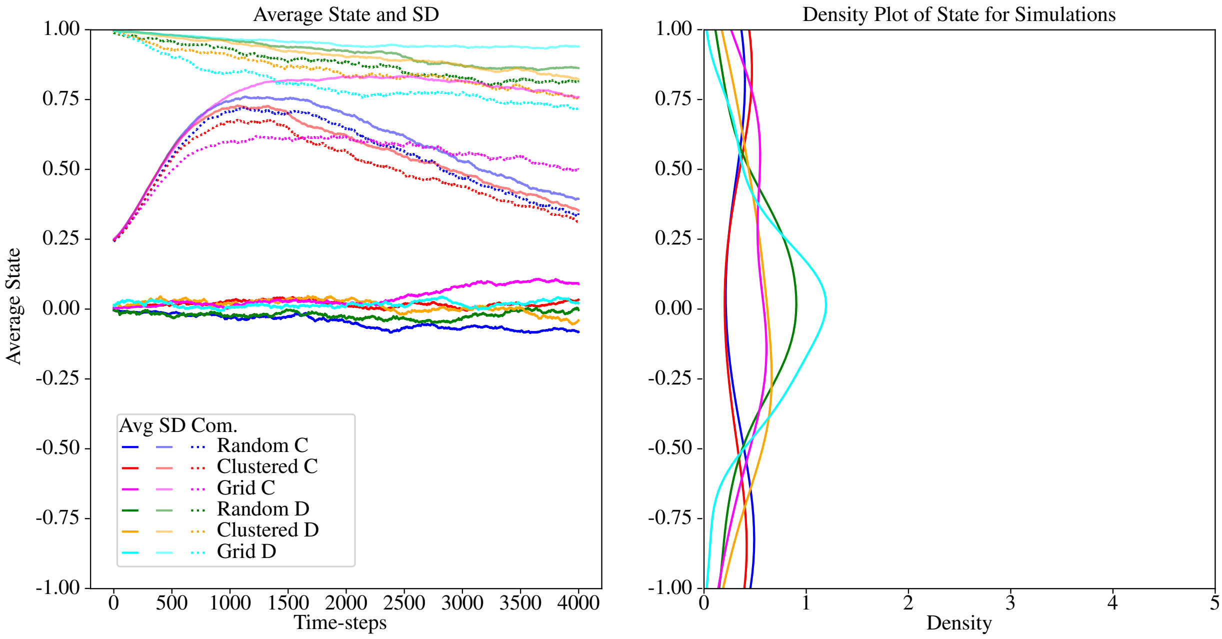
In the continuous models, it shows a high initial agreement as the opinions are normally distributed around neutral values, while the discrete case begins on approximately 0.5 as half the population start with each opinion. The continuous models first decrease the average agreement, as the model goes through a stage of radicalising, as already seen in the standard deviation. Then, for all network types and both discrete and continuous models, we can observe a steady increase in agreement among the closest friends up to a certain point. This indicates that the model stabilises as the agents form local groups of the same opinion.

Average State in Communities

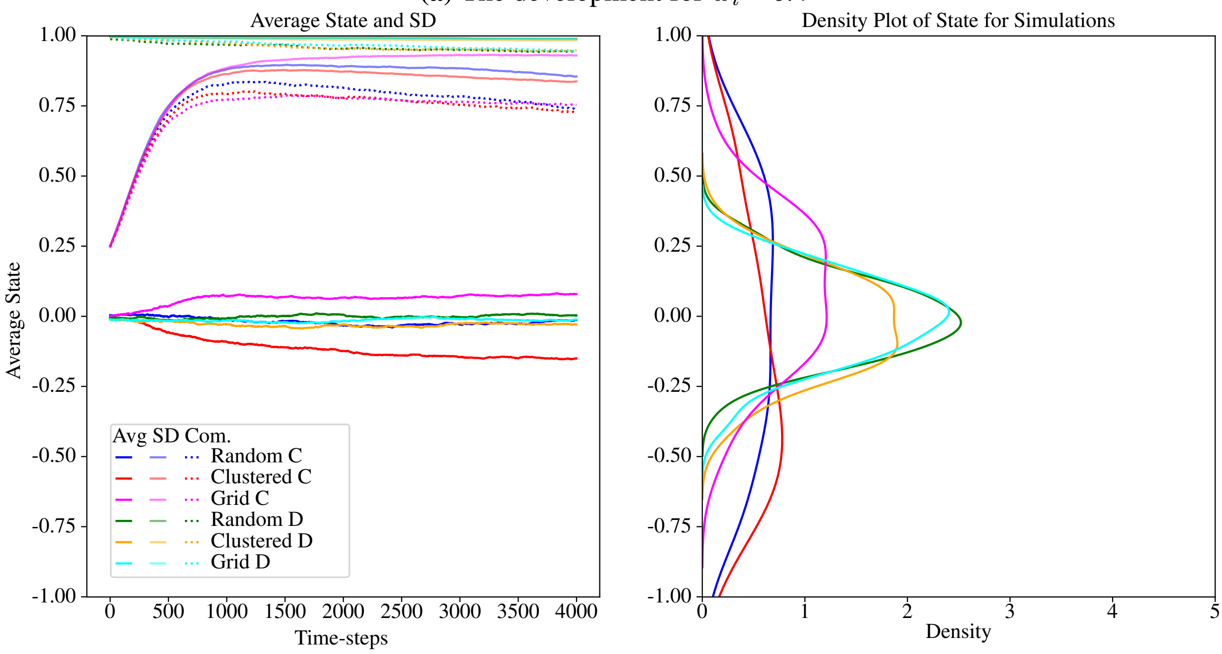
One last check of the simulations is conducted to see the development within the communities over time. Figure 5.5 show the density plot and histogram of the average state within the communities for the above simulation for time steps t=500, t=1000 and t=4000. In the beginning, it shows something close to a normal distribution as most communities either have both strategies present, or neutral strategies. At t=1000, the communities show average states from all over the range of average attitudes, some already settling in the radicalised areas. The last part shows the states at t=4000. It is not very clear from the figure, but at this point, all the continuous models show clear agreement within the communities, being either far to the cooperating side or the defecting side. Note that this also is true for the continuous grid model, even though the networks as wholes show mixed states. The discrete models, on the other hand, show more diversity, having communities with the average state at all the different state values, but with a slight preference for an even mix as there has been less change from the original values.

5.2.4 Effect of Stubbornness on Simulations

The stubbornness w_i decides how much of an agent's existing opinion should be taken into account when calculating the specific influences x_{ij}. It would be interesting to see how this value affects the balanced models. The basis used in the above examples is w_i is 0.6, so the value is changed to $w_i = 0.4$ and $w_i = 0.8$. As the expectancy value of the friendship weight $w_{ij} = 0.5$, the agents in the first case will, on average, value the opinion of the neighbour more than its own current attitude. The two cases are simulated and averaged over 50 simulations each, for 4000 time-steps, and the trend of the results can be seen in Figure 5.6. The first subfigure 5.6a shows $w_i = 0.4$, where the standard deviation increases as the network polarises. After t=1000, the models show less standard deviation as one of the strategies gets an advantage and the network chooses a side. Note that more C networks are ending with an average state of neutral values than the previous case, as seen from the density plot. In the corresponding subplot 5.6b for $w = 0.8$ the development of the standard deviation shows steep growth at the beginning for the continuous models, meaning that the agents polarise, and then the standard deviation stabilise at a high value. The standard deviation for the discrete models stays high at all times. The density plots show that all the model types end up with the majority of simulations in a dimorphic regime.



(a) The development for w_i = 0.4



(b) The development for w_i = 0.8

Figure 5.6: The development of the average state in the models for two different values of stubbornness.

5.3 Simulations with Insurgents

How much influence can one person exert on the population as a whole? To test the theory of Nisbet and Kotcher [23] with inspiration from Chwe [26] we have decided to add one insurgent to see if this would affect the development in the network. Let the initial values in the continuous model be Gaussian distributed with $N(0, 0.25^2)$, except for one insurgent with an initial value of a cooperator at 1 with a stubbornness of $w_i = 1$, so that the insurgent stays at the same side of the range as it starts. This insurgent is chosen to be the largest hub in the network in the case of random networks and clustered scale-free networks, and a central node in the case of a grid network. With the radicalised attitude, the insurgent may inspire behaviour change to spread in the network. A simulation of 144 nodes within each network (50 networks for each type), with $\langle k \rangle = 8$, is run for 4000 time-steps. The resulting development can be seen in Figure 5.7. The general shape of the resulting development follows the baseline of the balanced case. However, the continuous clustered scale-free and random networks show a much higher degree of spreading, to the extent that the clustered networks are almost approaching a monomorphic regime, and would probably do that in the long term. As the influences are most efficient in the case of clustered networks, we suspect this to be due to the large degree of the hub. The discrete models and the continuous grid network does not show the same level of influence from the insurgent. Yet, there is some effect from the insurgent, though working much slower.

The simulation is redone with the insurgent picked from the nodes with an average degree, instead of the node with highest degree, and the result can be seen in fig 5.8. In this plot, we can see the same dynamics, but the random and the clustered scale-free (C) network seem much more even in their influences. This indicates that the degree of the insurgent is important and that with the current dynamics, random links seem more critical than the redundancy of clustered influences proposed by Williams et al..

5.3.1 Two opposing insurgents

Given a network with the same conditions as above, just with two insurgents of opposing attitudes, the expected outcome would be a balanced average state. Running the simulation gives this, but with a tendency to make polarised communities.

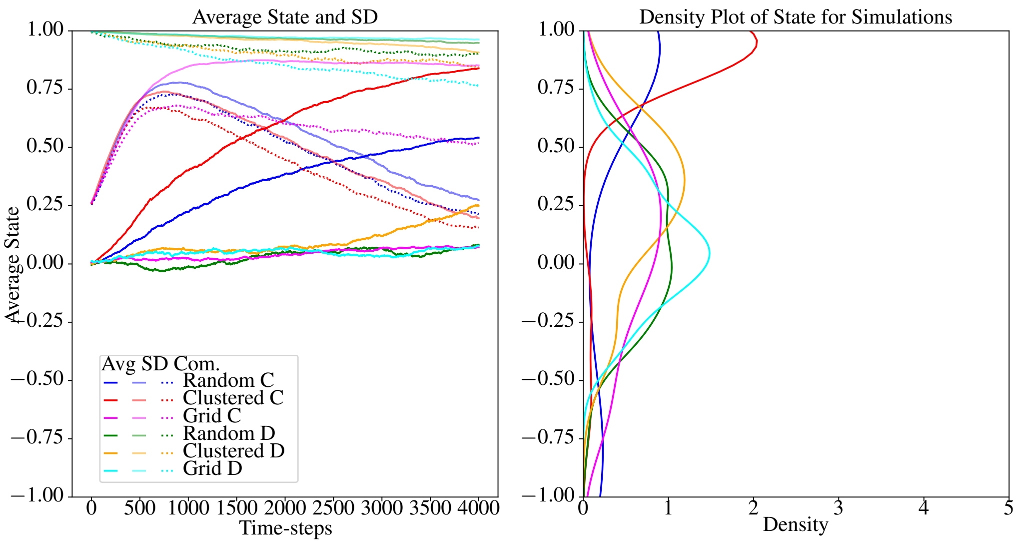


Figure 5.7: The development of the average state for different model types. The low-intensity lines show the associated average standard deviation within networks (solid) and within the communities (dotted). The right plot show the density of final average state for each model.

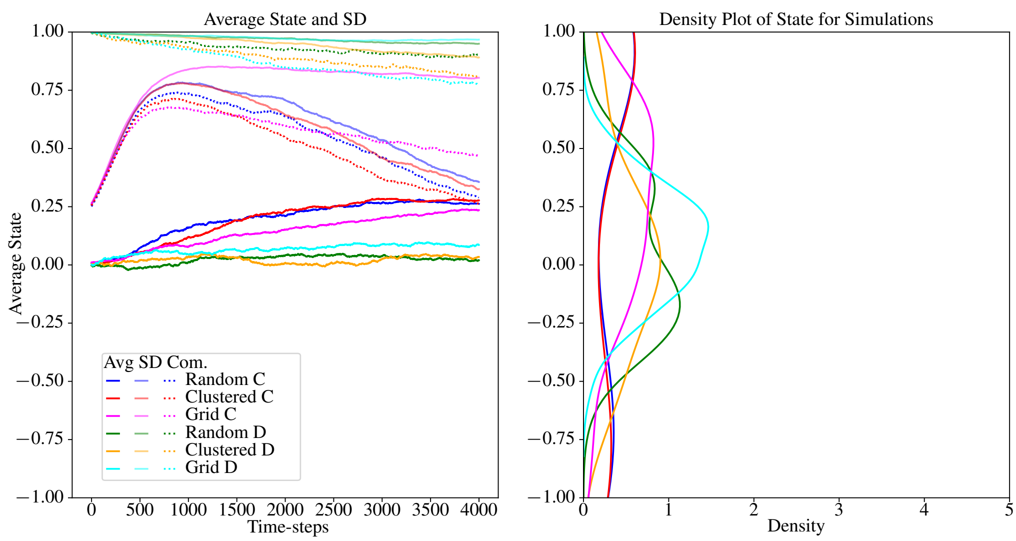


Figure 5.8: The development of the average state for models with an insurgent with degree = $\langle k \rangle$.

5.4 Uneven Initial Conditions

Given the current state of the human population, it would be unrealistic to assume that everyone has an initial neutral attitude about climate change. To account for this, several initial states are tried to check the change in development. Let the initial distribution be $\mathcal{N}(-0.1, 0.25^2)$ instead of distributed around zero. For the discrete cases, a number of agents corresponding to the change in average is chosen to be defectors such that the average state is equal to -0.1 for all models. Figure 5.9 shows the average of 50 runs of the simulation. The addition of a small skewness towards defecting strategy is most prominent in the continuous clustered case, which already is shown to be most sensitive to changes. Both random and grid (C) also show the same tendencies, while the discrete cases show slower development. The grid has an intriguing development of the standard deviation showing that both strategies are prominent in the networks, but the communities choose an internal strategy.

5.4.1 Development of Different Initial Condition

The case above tests the development of the initial condition for one specific value, and to confirm the pattern, several initial conditions are tested. In Figure 5.10 the development for simulations with initial state a) -0.2, b) -0.4, c) -0.6 and d) -0.8 is presented. Figure 5.10a shows simulations with initial average state at -0.2. The continuous cases all show that the average state of the agents in the network go towards defecting, while the discrete also show some change, but much slower. The standard deviation shows a much faster radicalising than with $avg = -0.1$, such that the standard deviation barely increases before converging towards zero. The density plot shows that the continuous models go towards all defecting, except some outliers in the grid models. All of the given examples of continuous models in Figure 5.10 show that the initial value is efficient for the radicalisation towards a monomorphic strategy decision.

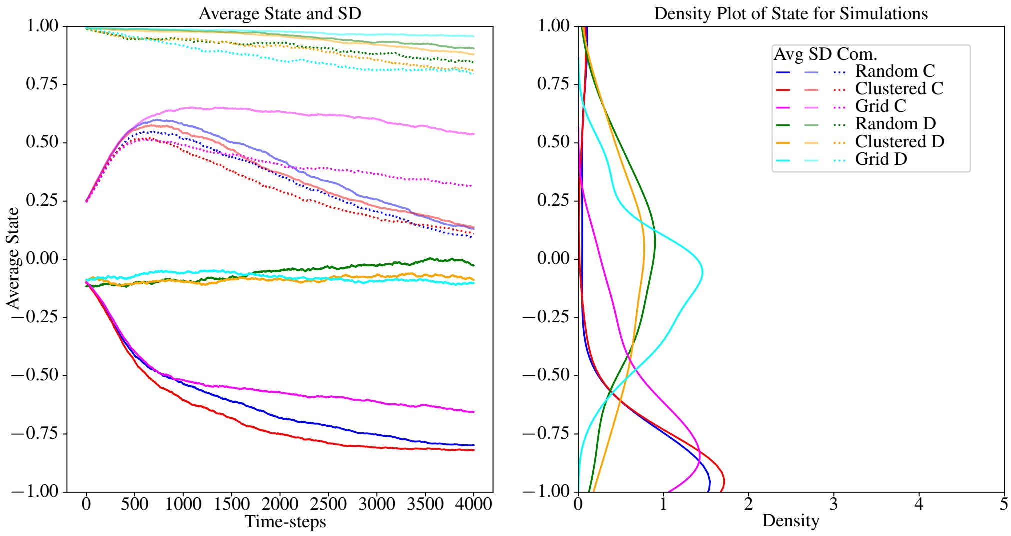
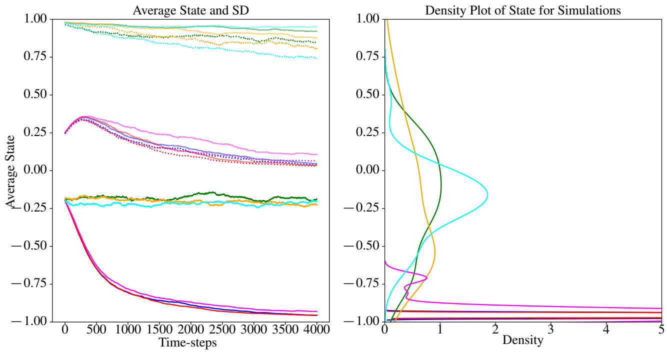
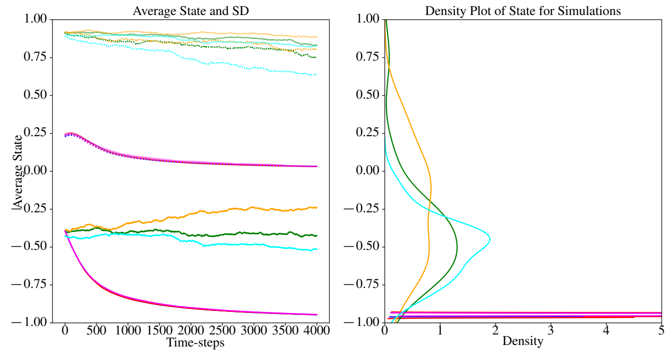


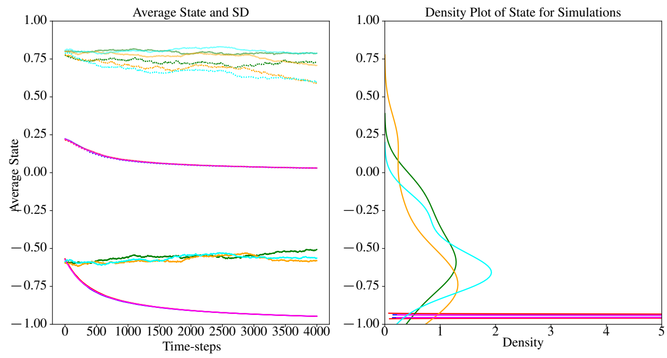
Figure 5.9: The development of the average state for initial average state = -0.1



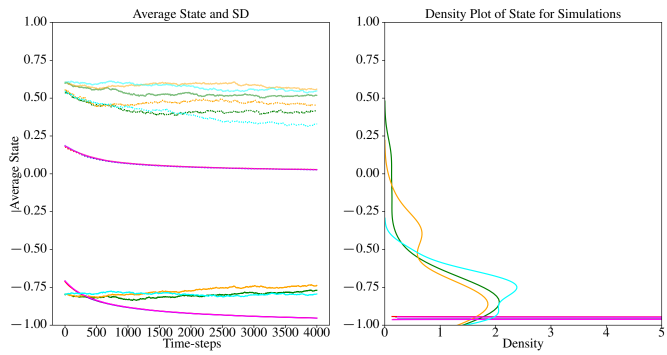
(a) avg state = -0.2



(b) avg state = -0.4



(c) avg state = -0.6



(d) avg state = -0.8

Figure 5.10: The development of the average state for different initial average states.

5.5 Uneven Initial Condition with One Insurgent

Let the initial condition start with the average state at -0.1, except for one hub-insurgent with a cooperating strategy. Letting the simulation run for 4000 time-steps generates the development as seen in Figure 5.11. The graph shows that the insurgent is most influential in the clustered scale-free network in both (C and D) models. Both these show an average state on the cooperative side. The other continuous models, on the other hand, show that the initial bias is more influential than the single insurgent. The discrete grid model and the random model do not seem to be much affected at all.

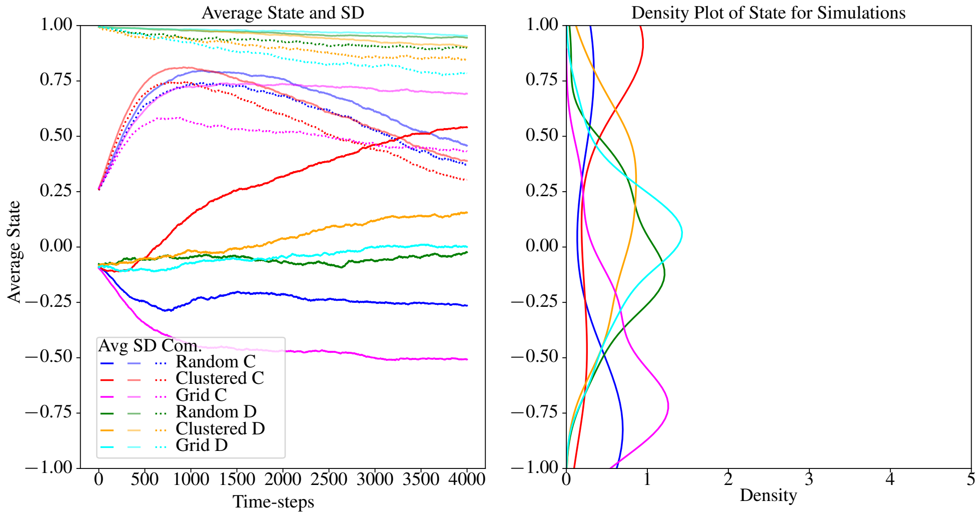


Figure 5.11: The development of the state for models with skewed initial conditions and one cooperator insurgent.

5.6 Uneven Initial Conditions with Political Forces

Let the initial distribution be $\mathcal{N}(-0.1, 0.25^2)$ for C models and equally skewed for D models. The political climate is set to be stronger than the defector utility such that $P + U = 0.05$. The plot in Figure 5.12 show the development of the average state over time for the simulations. In the continuous models, we can see that the initial skewed distribution is impacting the network fast by making the network more defecting. By studying the standard deviation, we see that in this stage, it is sharply increasing for the continuous models until most are radicalised and the standard deviation high. The discrete models do not show much impact of the initial conditions, as seen earlier, and reacts directly to the political climate. After some time (approximately t=500), the models change into a stage where all the models show a decrease in standard deviation. All the models seem to be overtaken by the political climate, and the agents are slowly turned towards a cooperating strategy. The development of first being impacted by the initially skewed average state and then being overtaken by the political climate leads to a bump in the development of the average state. The same set up is tried with networks with average degree $\langle k \rangle \approx 4$, and with larger network size. The result of these is found in the Appendices B and C.1. Both cases show similar development.

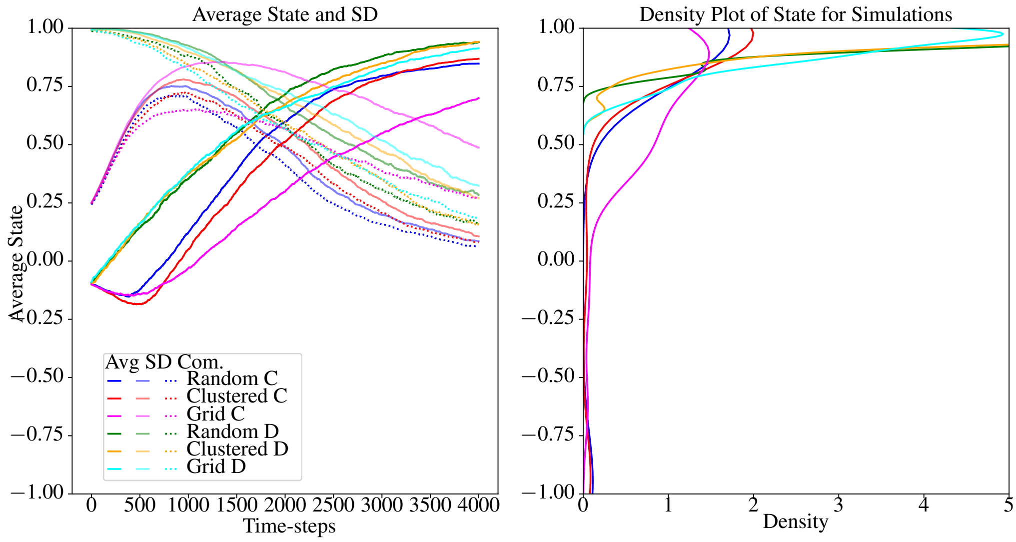


Figure 5.12: The development of the state for models with political forces and skewed initial conditions.

5.6.1 Polarised Initial Conditions

The same configuration is tried, except for an initial state that is polarised for both continuous and discrete models and average over 20 trials. This means that the continuous model also has agents that are either totally defectors or cooperators in the beginning, even though the dynamics are continuous. The result is illustrated in Figure 5.13. The average state is similar for all the models, and the bump seen in the previous case has disappeared. This means that the bump is a result of the normal distribution of initial values and the related first stage of radicalising. Studying the standard deviation shows that once again, the continuous models converge towards agreement faster than the discrete models, and the communities converge slightly before the networks as wholes.

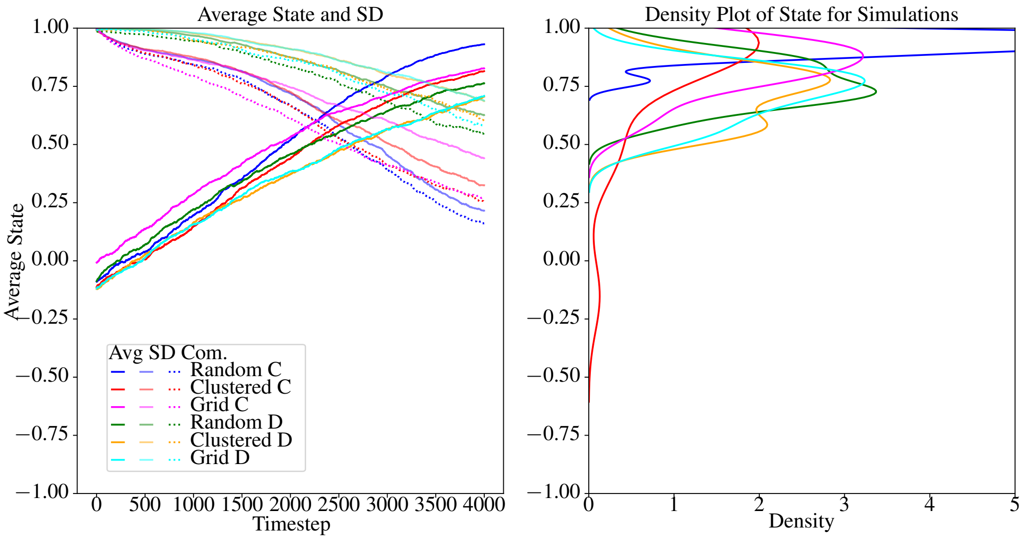
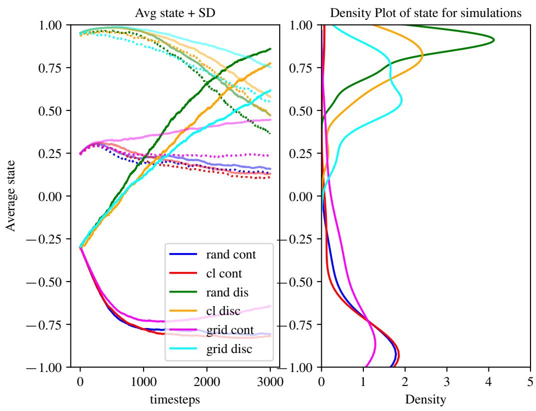


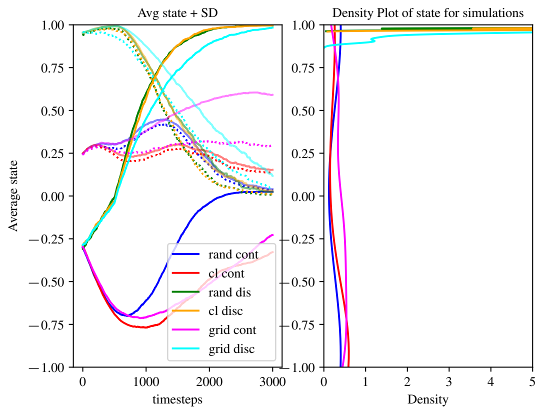
Figure 5.13: The development of the average state for models with political forces of $P+U = 0.05$ and skewed and polarised initial conditions.

5.7 Changing the Political Climate

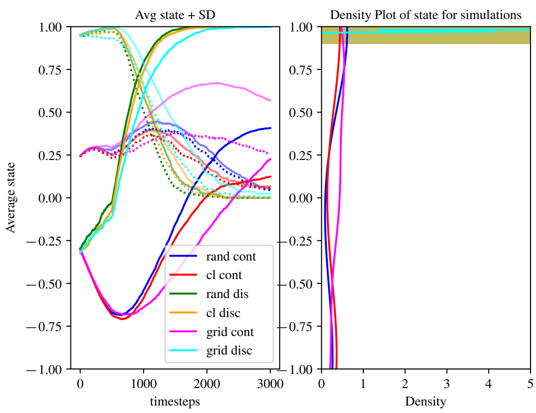
To further understand the impact of the political climate in combination with a skewed initial distribution, the political climate is changed approximately at the boundary between the stages, at t=500. This is done after the continuous models start with normal distributed initial values of -0.3 and the discrete models the polarised of the same average, and a political climate such that $P + U = 0.05$. The first sub-figure 5.14a shows how the development degenerates for a continued value of $P + U = 0.05$. The continuous models show that the initial values are most important, and the little political climate manages to some extent to stop the total defection of the populations compared to the development seen in section 5.4.1. The next two figures 5.14b and 5.14c show the development where the political climate is increased to $P + U = 0.15$ and $P + U = 0.25$. Looking at the average state, we can see that the average is balanced at a value higher than the first. The density plot on the right, however, shows that the C networks may end up with a final state in either the defecting or cooperating area, but the defecting states are most prominent for the former, while the latter gives networks in both ends of the range. Because the final standard deviation is minimal for all except the C grid, the networks will not switch from one strategy to the other. A long-time plot of the development with parameters as in Figure 5.14c can be seen in the appendix D. For the third value $P + U = 0.25$ 5.14c we see a sharp change in development for the discrete cases as the political climate starts affecting the interactions. The last Figure 5.14d visualises the development for $P + U = 0.35$ and in this case almost all networks go towards a cooperating regime. The standard deviations for the C models show a sharp bump in the stage after the political climate has changed, as the dynamics are much less dependent on the agreement among friends than the impact from the political climate. The box in the density plot of the last two sub-figures indicate that all the simulations end up with the same value, so that the histogram is plotted instead.



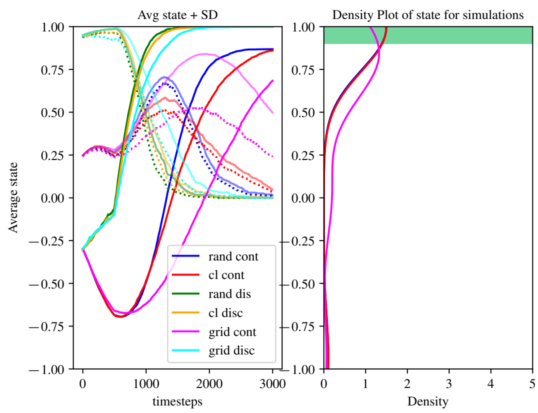
(a) $P + U = 0.05$



(b) $P + U = 0.15$



(c) $P + U = 0.25$



(d) $P + U = 0.35$

Figure 5.14: The development for different political climates starting after the first stage of development, for initial average state of -0.3.

Chapter 6

Discussion

6.1 Network Structure

In the theory part, three properties of social networks were described, namely clustering, assortativity (degree correlation) and average shortest path length. These properties can be found in different types of networks, but none of the types discussed in this thesis have been able to reproduce all of them at the same time, as seen in the results. Therefore, several network types have been used throughout the simulations, so that the results can be seen in relation to the properties. The two types most fit to describe social interaction networks are clustered scale-free networks and grid networks.

6.2 Evaluation and Interpretation of the Results

6.2.1 Community Structures

The existence of communities is an inherent property in networks, and especially important in social networks. A 2D lattice (grid) model does not intuitively for the human eye show any communities, but this is due to the symmetry of the linking. By analysis of the modularity, we find that it does give high modularity since all nodes are connected to only nodes nearby. This means that in a given partition, the edge nodes in a community does show high affiliation to its community, even though it is well-connected to the other neighbours as well. The clustered scale-free networks show disappointingly low modularity in comparison to the grid networks. This means that clustering does not automatically lead to natural community structures. During the simulations, all the networks display a gap between the average standard deviation of the state within the networks and the average standard deviation of the state within the communities. The gap indicates that the communities agree more than the total network. Throughout the simulations, the grid networks display the largest gap between community SD and network SD. The combination of high modularity and long average path length may explain why this is happening, by making well-defined communities where all nodes are in near proximity and impact each other accordingly.

6.2.2 Comparison Between the Discrete and Continuous Models

Most cooperation games divide the population into cooperators and defectors, but as this makes for a coarse simplification of reality, this thesis has also included continuous dynamics. In all simulations involving the development of the average state of the attitude in the community, both types of dynamics has been shown. The general trend is that the discrete model shows less sensitivity to changes in the agents' internal states. For example, the introduction of a slight bias in the initial distribution of the attitudes made a substantial impact on the continuous models, while barely any at all for the discrete models. This will be discussed more in the following sections. Another point where the two models differ significantly is the standard deviation. The models that start with equally many defectors and cooperators do have a standard deviation close to 1. As there are only two options for the attitude in the discrete model, the standard deviation gives a measure of how much one of the strategies is leading over the other on average. For the continuous model, on the contrary, a low standard deviation may also suggest that the agents agree on a neutral values, as seen initially.

6.2.3 The Effect of Continuous Normal-Distributed Initial Opinions

The continuous normal distributed model shows much change in the first time steps, as the neutral states seem less stable than the more radicalised states. When initialising the models with Gaussian distributed neutral states for all agents, it displays signs of being in an unstable dimorphic regime as the final states are diverging, except for the grid model. One reason for this radicalisation in the model is that the continuous dynamics lets the interacting agent n_i change his or her opinion even though the other agent n_j have a less extreme attitude. This is also the reason why the continuous model shows a drop in average agreement among neighbours in the polarising stage. The corresponding dynamics in real life would be when two people discuss something, and are somewhat agreeing, but through discussion, one of them adopts an even stronger opinion. However, since the continuous model gives an attitude increment relative to the difference of the attitudes of the interacting nodes, a more polarised pair of nodes would give a higher impact if the probability of the specific impact gives a state change. Intuitively, in reality, those who decide to adopt a substantial degree of a strategy do so by an active choice, and sometimes by ideology, and one would thus think that they would be hard to convince to change side. This is actually found by the small chance of a change of side with sufficiently high stubbornness. Yet, it should perhaps have been added some adjustments to make the neutral states more stable, as most people are not as radicalised as the results from this model. Alternatively, a proposed reason for this discrepancy is that the real population might still be in a phase of radicalisation.

The continuous models show much more sensitivity to both skewed initial states and the existence of insurgents, than the discrete models. This could be a sign that the successful strategy to change the opinion of a whole group of people could be achieved by changing it a little bit at the time, which would demand less effort than a sudden change at once for the agents. It might be what is necessary to make a virtuous cycle, as the continuous models show much more similarity to this phenomenon. This can be seen, for example, as a vicious cycle by the effect where an initial average state of -0.2 in time leads the whole network into a defecting regime. The discrete models, on the other hand, does not show much change at all.

6.2.4 Difference for the Discrete Models

The discrete models show, in general, little sensitivity to the state of the agents' attitude. In all cases with initially biased attitudes, the discrete model shows balanced development as earlier. Instead, the interaction is very much affected by the political climate. By analysis of the standard deviation in the different results, all discrete models show that the agents do, in fact, tend somewhat towards agreement. Yet, the continuous models show both faster and more successful consensus, except for the cases with strong political climate. The discrete models do, to some extent, show the same properties as the continuous models do in the second stage, where the agents have been radicalised. Assuming that the radicalising stage makes for an approximation of the group-identity forming stage of reality, the discrete model could perhaps be appropriate to use in models analysing polarised groups. What can be seen from studies of snapshots of such development, is that the change in opinion is happening by the borders of the local groups of agreement. This is reasonable, as groups of people talking to each other makes a norm and may expand this norm.

6.2.5 Stubbornness

The study of the effect of stubbornness has been limited. Since the value represents how much the agents value their current opinion in comparison to the neighbour's, it ends up being a factor radicalising the agents towards their initial tendency for sufficiently high values of w_i in the C model. This means that the populations enter stable dimorphic regimes, where there is little to no change. For very low values, the agents show less stubbornness and change side much more often. This makes it possible for mixed regimes to survive longer than in the base case, as is seen in the results (Section 5.2.4) by the high but decreasing standard deviation. The value used in the rest of the simulations $w_i = 0.6$ is slightly larger than the expected value of the friendship weights $w_{ij} \in \mathcal{N}(0.5, 0.15^2)$, such that the agents change opinion sometimes, but stay at the same or radicalise more often. Realistically, the agents should perhaps have been given a distribution of different values of stubbornness, as people show different levels of commitment to their opinions.

6.2.6 Insurgents

Insurgents may act as opinion leaders in the network when they are given a strong initial opinion, but the connectedness of the agents seems to be important. The first simulations with the insurgent being the largest hub in the network show that the clustered scale-free networks are much more affected by the insurgent than the other network types. However, when the insurgent is an agent with an average number of neighbours, the development was almost similar for the clustered scale-free networks and the random networks, both for continuous and discrete models. The main similarity between these two types is the long links (and thus short average distance), connecting different parts of the network. This shows that the interaction dynamics are not as similar to complex contagion as originally assumed, where clustering is shown to facilitate the spread of a behaviour. The higher level of success for the hub in the clustered scale-free network is also found in the discrete model, but disappears when the insurgent is an averagely connected node. This is not surprising, as in the discrete model, the insurgent would then be approximately the same as all the other agents.

Moreover, this spread might be less successful if the inter-community links were weaker, as claimed by Barabási et al.. This is not tested here, as all friendship weights are drawn from a normal distribution. For the continuous model there is a pattern: the insurgents act like strong forces in a specific direction in the beginning, such that the initially susceptible agents around the insurgent fast build a group of radicalised agents. This gives the insurgents' strategy a head-start when the second stage begins, and the boundaries between the different groups are essential, as this is where the persuasion happens. When the insurgent is a hub, it might be chosen to impact agents often, as many agents connect to it. The result from this might be several groups of cooperators from an early stage. It is not unrealistic to assume that some of the well-connected people in real life would have a strong opinion, and thus might facilitate the agreement in a group of susceptible people.

If popular people also show a higher level of influence on their friends, this could be useful to impact more substantial amounts of people. One drawback could be that these people also might be subject to a lot of influences themselves, which could make the nature of their expressed views more neutral. This possibility is avoided in the simulations by setting the insurgents' stubbornness to a high value. The existence of several opposing insurgent hubs is naturally causing people to radicalise, both in our model and in reality [13]. In reality, people might seek towards the insurgents they agree with, such that echo chambers are made. This is not accounted for in the simulations carried out in this project.

6.2.7 Political Climate

The effect of a sustainable political climate has been shown to be a steady growth in the number of cooperators. As already discussed, the political climate exerts the fastest and most successful influence on the discrete models, while the continuous models are gradually impacted in the second stage. The effect on the latter is notably due to the influence of the initial conditions, where the radicalisation is stronger than the impact of the political climate, so that the political climate does not affect the development much until the first stage is done. The simulations show that when the policies are added after the stage of radicalisation, it can still make a difference if it is strong enough, as seen in the results. However, we have also seen that the continuous models might already be heavily influenced by a bias towards defection, such that it is uncertain which final state the populations will end up in. In terms of social dilemmas, the addition of policies would mean that by decreasing the dilemma it gives more people the needed incentive to decide to cooperate, or to defect to a smaller degree. As the political climate is working as a constant force on everyone in this model, it would not be possible to say what types of impact would be most efficient.

6.3 Symmetry and Lack of Payoff

The interaction dynamics show straightforward properties, being symmetric and not inherently favouring defecting and cooperating differently. As the model does not apply any version of utility function or payoff, it is also not possible to calculate the optimised decision under any conditions. Therefore it is not possible to use standard game-theoretic analytical approaches to find Nash equilibria. This was an active choice as people do not always act rationally. However, assuming that the dynamics is symmetric might be unrealistic, and has no evident support in the theory presented in the literature study.

6.4 Weaknesses

6.4.1 Realistic Social Networks

It has proven challenging to find good characteristics of friendship and communication networks based on real life. Most studies gathering communication data, that have been seen in this project, are based on online or mobile phone call networks [3, 13]. The study of energy-conservation conversations [15] used ego-networks(a person and whom he or she talks to) as it is demanding to map the full inter-communication of the person's alters. A weakness of the model built in this project is the unsuccessful construction of an artificial but realistic social network topology. For a more realistic analysis of a whole social network, it might be most efficient to get a data set of online social networks, even though online social networks might be different from real life interactions.

6.4.2 Attitude and Exerted Behaviour

This thesis has been considering behaviour change based on the simple assumption that behaviour is directly linked to attitude. However, studies in Canada have found a significant gap between the intention and behaviour of people regarding sustainability [37]. The simplification of attitude-induced behaviour change might therefore be a significant weakness of the model when compared to reality. In order to make a more robust model, one should perhaps include some more psychological foundation of this link, or choose to focus solely on either attitude or behaviour.

6.5 Further Work

With the presence of today's social media and globalisation, it is possible to reach out to many people with little effort. Given that celebrities and people achieving 15-minutes-of-fame used their influence to impact people's opinion, would it have a long term impact on the development of people's decision? This could be further tested by introducing will-full hubs for shorter periods of time. To account for social media's one-way influences, the network could be changed into a directed one, since this is how large online sites such as Twitter and Instagram is working.

An interesting thing to try for further simulations is to test how many insurgents need to work in a network when the defector utility is stronger than the political climate, and the initial distribution is heavily biased towards the defecting strategy. This could replicate the situation in societies where the government is denying the risks of climate change, and the population's opinion is initially opting for the individual benefits.

The choices of people are not solely dependent on one-to-one conversations. It would give a widened understanding also to test how the agents in the model would behave under the influence of groups of friends, such as in majority games. Some few simplified tries to replicate such dynamics have been carried out in the initial stages of this project but was not included in this as it needed further development to show proper results. The discrete case could be implemented with a majority threshold game, while the continuous case gives more options for possible approaches for implementation.

This model could be changed to include the dynamics of traditional social dilemmas in evolutionary cooperation games by finding some way of using a payoff that can be maximised. It is not certain that this would add anything to the project, or teach us more about how populations would behave, but it would give another set of approaches for analysis of the dynamics.

Chapter 7

Conclusion

As the world is faced with the challenge of dangerous climate change, the population is divided between those who actively believe and cooperate to reduce the impact of human behaviour and those who do not. The decision of whether to cooperate or not can be said be to be a social dilemma, and people choose between their personal comfort and long term sustainable development. To model this, we have employed agent-based modelling on different types of networks reproducing properties of social networks. The model includes a simple form of interaction, but without a maximisation of any payoff for the agents. The models with continuous levels of cooperation and defection are shown to have the most dynamic development, with local groups of agents agreeing first and eventually whole networks reaching a stable agreement, especially when facilitated by insurgents. However, as these are found to be sensitive to small changes, both defector regimes and cooperator regimes are found as the result of the simulations, even with the same initial conditions. The discrete models, on the other hand, shows much less variability as it is more demanding to change the opinion of the agents. In this case, the most influential source of change is found to be an external impact.

This makes for the conclusion that in order to make a whole population agree on cooperating, it might be more successful to let people change their behaviour gradually, with the help from influences from popular insurgents and a favourable political climate.

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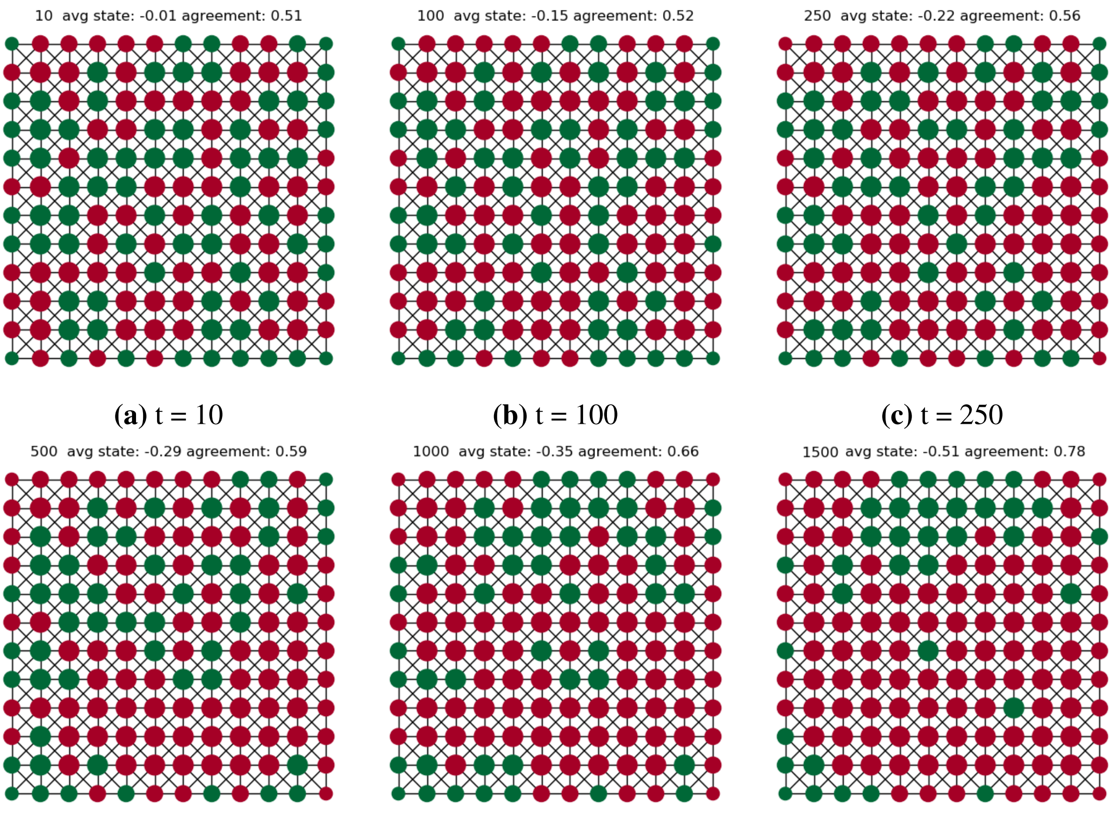
Appendices

Appendix A

Example of Discrete Development

The snapshots in the figure show that the agents change opinion from being randomly spread out to more organised groups. The change happens gradually through all the different time-steps.



(a) t = 10 **(b)** t = 100 **(c)** t = 250

(d) t = 500 **(e)** t = 1000 **(f)** t = 1500

Figure A.1: An example of a grid network with N=144 and discrete attitudes. The sub-figures show snapshots at six different time steps in a simulation with 1500 time steps.

Appendix B

Political Climate and Another Degree

A plot showing the development of the average state for 50 simulations of the different network types with $\langle k \rangle \approx 4$. The simulation is run 4000 time steps, and show that the development is approximately the same as the development for networks with $\langle k \rangle \approx 8$, just a bit slower.

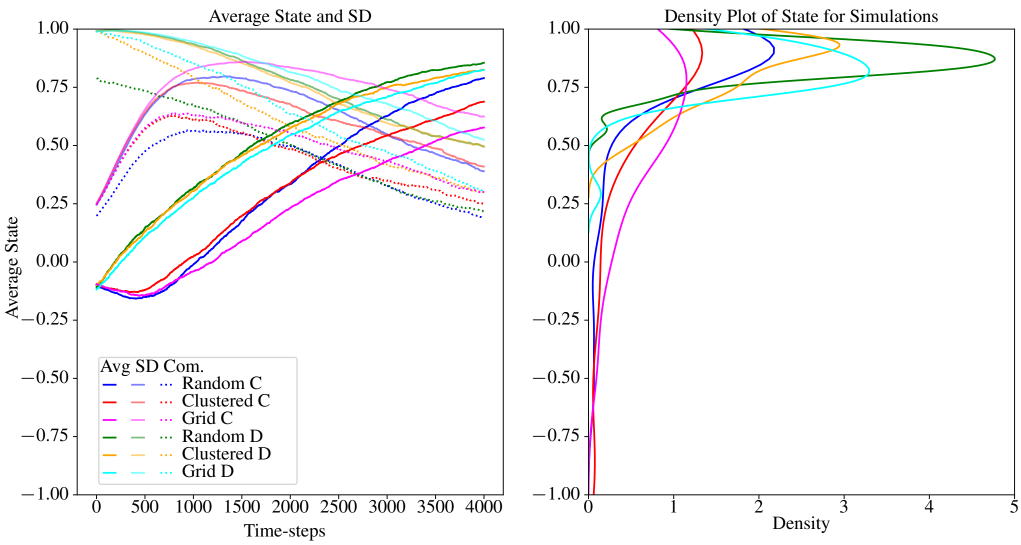


Figure B.1: The development of the average state for models with political forces of $P + U = 0.05$ and skewed initial conditions, for networks with $\langle k \rangle \approx 4$.

Appendix C

Political Forces in Larger Networks

The case with political forces being slightly higher than the defector utility such that $P + U = 0.05$ is retried in a larger network N = 256. The initial average state is -0.1 and the system is simulated for 10 000 time steps. The resulting figure can be seen in C.1. It is very similar compared to the result of the original network size, except that the increase in network size makes for slower development. This is not surprising, as it would take more time to interact through all the agents at least once.

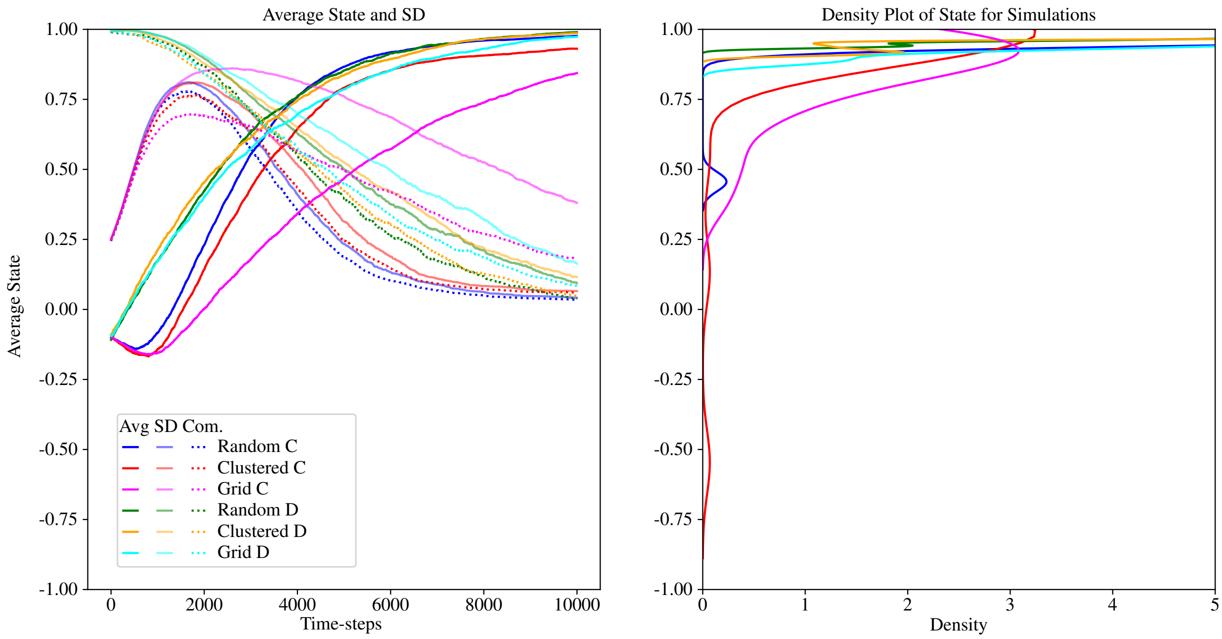


Figure C.1: The development of the average state for models with political forces of $P + U = 0.05$ and skewed initial conditions with network N=256.

Appendix D

Changing the Political Climate

This plot is showing the development of the average state for 30 simulations of the different network types. The simulation is run 8000 time-steps, and shows that the average state does not change any more when the standard deviation within the networks reaches zero. That means that of the 30 networks, a majority of the C have reached an average final state in the cooperating area, while some networks have been convinced by the defecting strategy.

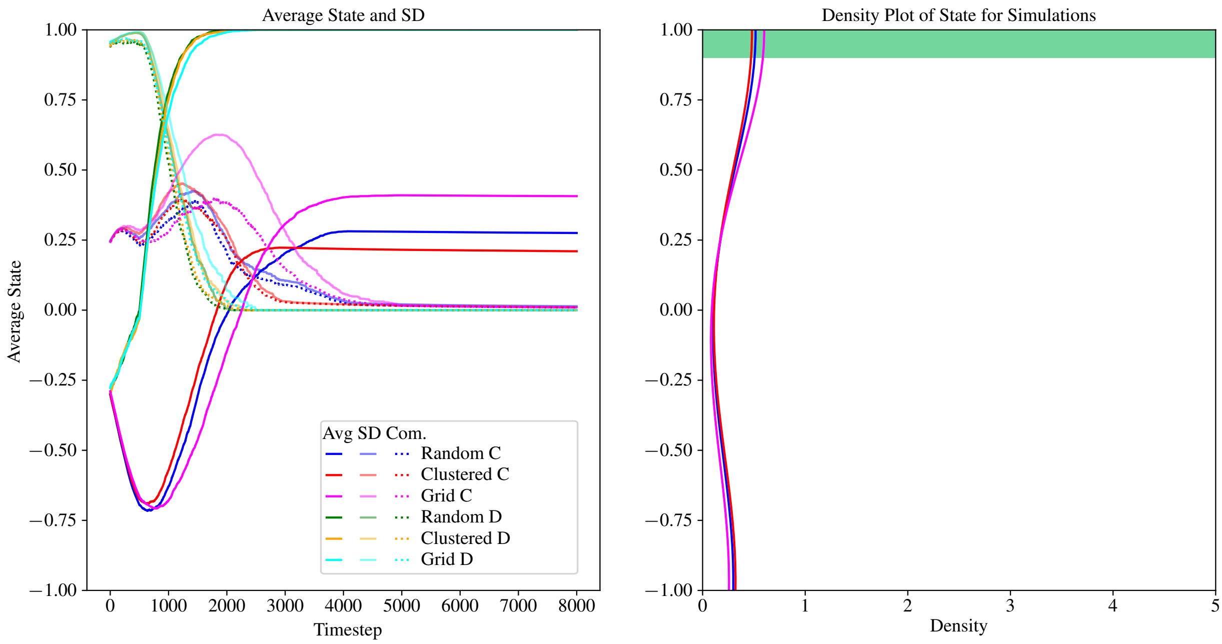


Figure D.1: The development of the average state for models with political forces of $P + U = 0.05$ and skewed initial conditions. The political climate is changed at t=500 so that $P + U = 0.25$.

Appendix E

Python Code

```
import numpy as np
import random
import matplotlib.pyplot as plt
import matplotlib.colors as col
import seaborn as sns
from statistics import stdev, mean
import imageio
import networkx as nx
from scipy.stats import truncnorm
import os
import community
from operator import itemgetter
import heapq
from IPython.display import Image
import matplotlib.patches as mpatches

def get_truncated_normal(mean=0, sd=1, low=0, upp=10):
  return truncnorm(
    (low - mean) / sd, (upp - mean) / sd, loc=mean, scale=sd)

#Constants and Variables

states = [1, -1] #1 being cooperating, -1 being defecting
defectorUtility = -0.20
politicalClimate= 0.25
newPoliticalClimate = 0.25
selfWeight = 0.6
d = 4 #degree
s = 100 #number of simulations
k=4000 #timesteps
continuous = True
skew =-0.3
initSD = 0.25
```

```
mypalette = ["blue","red","green", "orange", "magenta","cyan","
    violet", "grey", "yellow"]
randomness = 0.25

args = {"defectorUtility" : defectorUtility,
    "politicalClimate" : politicalClimate,
    "selfWeight": selfWeight, "d":d,
    "s": s, "k" : k, "continuous" : continuous, "type" : "cl",
    "skew": skew, "initSD": initSD, "newPoliticalClimate":
    newPoliticalClimate}

def simulate(i, newArgs):
  setArgs(newArgs)
  global args
  X = get_truncated_normal(0.5, 0.15, 0, 1)
  S = get_truncated_normal(args["skew"], args["initSD"], -1, 1)
  ind = None

  if(args["type"] == "cl"):
    model =ClusteredPowerlawModel(144, args["d"], skew=args["
    skew"], X=X, S=S)
  elif(args["type"] == "sf"):
    model = ScaleFreeModel(144, args["d"], skew=args["skew"], X
    =X, S=S)
  elif(args["type"] == "grid"):
    ind = [64]
    if(args["d"]>2): doubleDegree = True
    else:doubleDegree = False
    model = GridModel(12, skew=args["skew"], doubleDegree =
    doubleDegree, X=X, S=S)
  elif(args["type"] == "rand"):
    model = RandomModel(144, args["d"], skew=args["skew"], X=X,
     S=S)
  else:
    model = RandomModel(144, args["d"], X=X, S=S)

  model.addInfluencers(newArgs["influencers"], index=ind, hub=
    True)
  res = model.runSim(args["k"], clusters=True)
  return model

#Helper
def setArgs(newArgs):
  global args
  for arg, value in newArgs.items():
    args[arg] = value

```

```

def getRandomExpo():
  x = np.random.exponential(scale=0.6667)-1
  if(x>1): return 1
  elif (x< -1): return -1
  return x

class Agent:
  def __init__(self, state, selfWeight):
    self.state = state
    self.selfWeight = selfWeight

  def consider(self, neighbour, neighboursWeight, politicalClimate):
    if(self.selfWeight >= 1): return
    global args
    weight = self.state*self.selfWeight + politicalClimate + args["defectorUtility"] + neighboursWeight*neighbour.state

    if(args["continuous"]):
      p1 = (randomness+weight)*(1/(2*randomness))

      if(p1 <0): p1 = 0
      if(p1 > 1): p1=1

      delta = (1/2)*(-self.state+1)*(p1) - ((1/2)*(self.state+1))*(1-p1)
      increment = 2*delta*abs(self.state-neighbour.state)

      self.state += increment
      if(self.state > 1):
         self.state = states[0]
      elif(self.state <-1):
        self.state = states[1]
    else:
      if(weight + random.uniform(-randomness, randomness) > 0):
        self.state = states[0]
      else:
        self.state = states[1]

  def setState(self, newState):
    if(newState >= states[1] and newState <= states[0]):
      self.state = newState
    else:
      print("Error␣state␣outside␣state␣range:␣", newState)


```

```
class Model:
  def __init__(self, X = None, S=None):
    global args
    self.graph = nx.Graph()
    self.politicalClimate = args["politicalClimate"]
    self.ratio = []
    self.states = []
    self.statesds = []
    self.pos = []
    self.X = X
    self.S = S
    self.clusteravg = []
    self.clusterSD = []
    self.NbAgreeingFriends = []
    self.avgNbAgreeingList = []
    self.partition = None

  def interact(self):
    nodeIndex = random.randint(0, len(self.graph) - 1)
    node = self.graph.nodes[nodeIndex]['agent']
    neighbours = list(self.graph.adj[nodeIndex].keys())
    if(len(neighbours) == 0):
      return nodeIndex

    chosenNeighbourIndex = neighbours[random.randint(0, len(neighbours)-1)]
    chosenNeighbour = self.graph.nodes[chosenNeighbourIndex]['agent']
    weight = self.graph[nodeIndex][chosenNeighbourIndex]['weight']

    node.consider(chosenNeighbour, weight, self.politicalClimate)
    return nodeIndex


  def findNbAgreeingFriends(self, nodeIdx = None):
    global args
    nbs = []

    if(args["continuous"]):
      for nodeIdx in self.graph.nodes:
        state = self.graph.nodes[nodeIdx]['agent'].state
        neighbours = list(self.graph.adj[nodeIdx])
        neighStates = [self.graph.nodes[n]['agent'].state for n in neighbours ]
        if(len(neighbours) == 0):
```

```
          nbs.append(0)
          continue
        x = 1-abs((mean(neighStates)-state))/2
        nbs.append(x)
    else:
      for nodeIdx in self.graph.nodes:
        state = self.graph.nodes[nodeIdx]['agent'].state
        neighbours = list(self.graph.adj[nodeIdx])
        neighs = 0
        if(len(neighbours) == 0):
          nbs.append(0)
          continue
        for neighbourIdx in neighbours:
          if(state == self.graph.nodes[neighbourIdx]['agent'].state): neighs+=1
        nbs.append(neighs/len(neighbours))
    self.NbAgreeingFriends= nbs
    return nbs

  def updateAvgNbAgreeingFriends(self, nodeIndex):
    neighbours = list(self.graph.adj[nodeIndex].keys())
    if(len(neighbours) == 0):
      return self.avgNbAgreeingList[-1]
    nodeState = self.graph.nodes[nodeIndex]['agent'].state


    if(args["continuous"]):
      neighStates = [self.graph.nodes[n]['agent'].state for n in neighbours ]
      x = 1-abs((mean(neighStates)-nodeState))/2
      self.NbAgreeingFriends[nodeIndex] = x
      for node in neighbours:
        nodeState = self.graph.nodes[node]['agent'].state
        neighneigh = list(self.graph.adj[node])
        neighStates = [self.graph.nodes[n]['agent'].state for n in neighneigh ]
        x = 1-abs((mean(neighStates)-nodeState))/2
        self.NbAgreeingFriends[node] = x
    else:
      neighbours.append(nodeIndex)

      for n in neighbours:
        neighneighs = list(self.graph.adj[n])
        neighs = 0
        nState = self.graph.nodes[n]['agent'].state
        if(len(neighneighs) == 0):
          self.NbAgreeingFriends[n] = (0)
```

```
          continue
        for neighbourIdx in neighneighs:
          if(nState == self.graph.nodes[neighbourIdx]['agent'].state): neighs+=1
        self.NbAgreeingFriends[n] = neighs/len(neighneighs)

    return mean(self.NbAgreeingFriends)

  def addInfluencers(self, number = 0, index = None, hub = True):
    if(number == 0):
      return
    if(index == None):
      degrees = nx.degree(self.graph)
      if(hub):
        largest = heapq.nlargest(number, degrees, key=itemgetter(1))
        index = [t[0] for t in largest]

      else:
        index = [p[0] for p in degrees if p[1] == d*2]
        if(len(index) == 0 or len(index) < number ):
          extra = [p[0] for p in degrees if p[1] == d*2-1]
          index = index + extra
    for i in range(number):
      self.graph.node[index[i]]['agent'].setState(states[i % 2])
      self.graph.node[index[i]]['agent'].selfWeight = 1



  def countCooperatorRatio(self):
    count = 0
    for node in self.graph:
      if self.graph.nodes[node]['agent'].state > 0:
        count+=1
    return count/len(self.graph)

  def getAvgState(self):
    states = []
    for node in self.graph:
      states.append(self.graph.nodes[node]['agent'].state)
    statearray = np.array(states)
    avg = statearray.mean(axis=0)
    sd = statearray.std()
    return (avg, sd)

```

```
  def getFriendshipWeight(self):
    #weigth = random.uniform(0.1, 0.9)
    #global X
    weigth = self.X.rvs(1)
    return weigth[0]

  def getInitialState(self):
    global args
    if(args['continuous'] != True):
      state = states[random.randint(0,1)]
    else:
      #state = random.uniform(-1, 1)
      state = self.S.rvs(1)[0]
    return state

  def runSim(self, k, drawModel = False, gifname=None, clusters
 =False):
    if(self.partition ==None):
      self.partition = community.best_partition(self.graph)

    if(drawModel):
      draw_model(self)

    filenames = []

    #create list of number of agreeing friends
    self.findNbAgreeingFriends()
    self.avgNbAgreeingList.append(mean(self.NbAgreeingFriends))


    for i in range(k):

      nodeIndex = self.interact()
      ratio = self.countCooperatorRatio()
      self.ratio.append(ratio)
      (state, sd) = self.getAvgState()
      self.states.append(state)
      self.statesds.append(sd)
      avgFriends = self.updateAvgNbAgreeingFriends(nodeIndex)

      self.avgNbAgreeingList.append(avgFriends)

      global args
      if(i ==500 and (args["newPoliticalClimate"] != args["
 politicalClimate"])):
        self.politicalClimate = args["newPoliticalClimate"]

```

```
      if(clusters):
        (s, sds, size) = findAvgStateInClusters(self, self.partition)
        self.clusterSD.append(sds)
        self.clusteravg.append(s)

      if(gifname != None and (i % 1 == 0)):
        draw_model(self, True, i, extraTitle = f' avg_state: {self.states[-1]:1.2f} agreement: {self.avgNbAgreeingList[-1]:1.2f}')
        filenames.append("plot" + str(i) +".png")

    if(gifname != None):
      images = []
      for filename in filenames:
        images.append(imageio.imread(filename))
      # to get nice number of frames per second 0.08167
      imageio.mimsave("network" +gifname+ ".gif", images, duration=0.08167)

    (avgs, sds, sizes) = findAvgStateInClusters(self, self.partition)
    self.clusteravg.append(avgs)

    return self.ratio

  def populateModel(self, n, skew = 0):
    global args
    for n in range (n):
      agent1 = Agent(self.getInitialState(), args["selfWeight"])
      self.graph.node[n]['agent'] = agent1
    edges = self.graph.edges()
    for e in edges:
      weight=self.getFriendshipWeight()
      self.graph[e[0]][e[1]]['weight'] = weight

    if(skew != 0 and not args["continuous"] ):
      num = round(abs(skew)*len(self.graph.nodes))
      indexes = random.sample(range(len(self.graph.nodes)), num)
      for i in indexes:
        self.graph.node[i]['agent'].state = states[1]
    self.pos = nx.spring_layout(self.graph)

class GridModel(Model):
  def __init__(self, n, skew=0, doubleDegree=False, **kwargs):
```

```
    super().__init__(**kwargs)
    global args
    for i in range(n):
      for j in range (n):
        agent1 = Agent(self.getInitialState(), args["selfWeight"])
        self.graph.add_node(i*n+j, agent=agent1, pos=(i, j))
        self.pos.append((i, j))
        if(i!=0):
          weight = self.getFriendshipWeight()
          self.graph.add_edge(i*n+j, (i-1)*n+j, weight = weight)
        if(j!=0):
          weight = self.getFriendshipWeight()
          self.graph.add_edge(i*n+j, i*n+j-1, weight = weight)
    if(doubleDegree):
      for i in range(n):
        for j in range(n):
          if(i!=0 and j!=0 ):
            weight = self.getFriendshipWeight()
            self.graph.add_edge(i*n+j, (i-1)*n+j-1, weight = weight)
          if(i!=0 and j!=(n-1)):
            weight = self.getFriendshipWeight()
            self.graph.add_edge(i*n+j, (i-1)*n+j+1, weight = weight)
    if(skew != 0 and not args["continuous"] ):
        num = round(abs(skew)*len(self.graph.nodes))
        indexes = random.sample(range(len(self.graph.nodes)), num)
        for i in indexes:
          self.graph.nodes[i]['agent'].state = states[1]

class ScaleFreeModel(Model):
  def __init__(self, n, m, skew= 0, **kwargs):
    super().__init__(**kwargs)

    self.graph = nx.barabasi_albert_graph(n, m)
    self.populateModel(n, skew)

class ClusteredPowerlawModel(Model):
  def __init__(self, n, m, skew = 0, **kwargs):
    super().__init__(**kwargs)

    self.graph = nx.powerlaw_cluster_graph(n, m, 0.5)
    self.populateModel(n, skew)

```

```
class RandomModel(Model):
  def __init__(self, n, m, skew= 0, **kwargs):
    #m is avg degree/2
    super().__init__(**kwargs)
    p = 2*m/(n-1)

    self.graph =nx.erdos_renyi_graph(n, p)
    self.populateModel(n, skew)

import dill

def saveModels(models, filename):
  with open(filename, 'wb') as f:
    dill.dump(models, f)

def loadModels(filename):
  with open(filename, 'rb') as f:
    models = dill.load(f)
  return models

def findClusters(model):
  partition = community.best_partition(model.graph)
  return partition


def findAvgStateInClusters(model, part):
  states = [[] for i in range(len(set(part.values())))]

  for n, v in part.items():
    states[v].append(model.graph.node[n]['agent'].state)
  clusters = []
  sd = []
  clsize = []
  for c in range(len(states)):
    clusters.append(mean(states[c]))
    clsize.append(len(states[c]))
    if(len(states[c])>1):
      sd.append(stdev(states[c]))
    else:
      sd.append(0)
  return (clusters, sd, clsize)

def findAvgSDinClusters(model, part):
  states = [[] for i in range(len(set(part.values())))]
  for n, v in part.items():
    states[v].append(model.graph.node[n]['agent'].state)

```

```
  sd = []
  for c in range(len(states)):
    if(len(states[c])>1):
      sd.append(stdev(states[c]))
    else:
      sd.append(0)
  return sd

#-------- drawing functions ---------

def drawClusteredModel(model):
  if(model.partition==None):
    partition = findClusters(model)
  else:
    partition = model.partition

  for k, v in partition.items():
    model.graph.node[k]["louvain-val"] = v
  degrees = nx.degree(model.graph)

  edge_col = []
  for node in model.graph.nodes():
    edge_col.append(mypalette[model.graph.node[node]["louvain-val"] % 9 ])
  plt.figure(figsize=(16,16))
  plt.subplot(1, 2, 2, title="Cluster Membership")
  nx.draw(model.graph, model.pos, node_size=[d[1] * 20 for d in degrees], node_color =edge_col)
  (clusters, sd, clsize) = findAvgStateInClusters(model, part=partition)
  text = [f'x={clusters[c]:5.2f} sd={sd[c]:5.2f} n={clsize[c]}' for c in range(len(clusters))]

  ax = plt.gca()
  handles = [mpatches.Patch(color=mypalette[c], label=text[c]) for c in range(len(text))]
  ax.legend(handles=handles)

  draw_model(model)

def draw_model(model, save=False, filenumber = None, outline=None, partition=None, extraTitle=""):

  plt.figure(figsize=(4, 4))
  #plt.subplot(1, 2, 1, title="State of the Nodes")
  color_map = []
  intensities = []
```

```

  for node in model.graph:
    if model.graph.nodes[node]['agent'].state > 0:
      color_map.append((3/255,164/255,94/255, model.graph.nodes[node]['agent'].state))
      intensities.append(model.graph.nodes[node]['agent'].state)
    else:
      color_map.append((247/255,121/255,109/255, -1*model.graph.nodes[node]['agent'].state ))
      intensities.append(model.graph.nodes[node]['agent'].state)
  degrees = nx.degree(model.graph)
  #plt.subplot(121)
  nx.draw(model.graph, model.pos, node_size=[d[1] * 30 for d in degrees], linewidths=2, node_color =intensities, cmap=plt.cm.RdYlGn, vmin=-1, vmax=1 )
  #plt.colorbar(mcp)

  if(outline !=None):
    ax = plt.gca()
    ax.collections[0].set_edgecolor(outline)
    (clusters, sd, clsize) = findAvgStateInClusters(model, part = partition)
    text = [f'x={clusters[c]:5.2f} sd={sd[c]:5.2f} n={clsize[c]}' for c in range(len(clusters))]

    handles = [mpatches.Patch(color=mypalette[c], label=text[c]) for c in range(len(text))]
    ax.legend(handles=handles)
    plt.title("Snapshot of network with states and clusters")

  if(save):
    plt.title(str(filenumber)+extraTitle)
    plt.savefig("plot" + str(filenumber) +".png", bbox_inches="tight")
    plt.close('all')


def drawAvgState(models, avg =False, pltNr=1, title="", clusterSD = False):
  plt.xlabel("Timesteps")
  plt.ylabel("Average State")
  plt.subplot(1, 2, 1, title="Average State and SD")

  if(not avg):
    plt.ylim((-1, 1))
```

```
    for i in range(len(models)):
      plt.plot(models[i].states)
      plt.plot(models[i].statesds, alpha=0.5)
      if(clusterSD):
        sds = np.array(models[i].clusterSD)
        avgsd = sds.mean(axis=1)
        plt.plot(avgsd, linestyle=":")
  else:
    states = []
    sds = []
    plt.ylim((-1, 1))
    for i in range(len(models)):
      states.append(models[i].states)
      sds.append(models[i].statesds)
    array = np.array(states)
    avg = array.mean(axis=0)
    std = np.array(sds).mean(axis=0)
    p1 = plt.plot(avg, color=mypalette[pltNr-1], label=" ")
    p2 = plt.plot(std, color=col.to_rgba(mypalette[pltNr-1], 0.5), label=" ")

    if(clusterSD):
      avgSds = []
      for mod in models:
        array = np.array(mod.clusterSD)
        avgSd = array.mean(axis=1)
        avgSds.append(avgSd)
      array = np.array(avgSds)
      avgAvgSd = array.mean(axis=0)
      plt.plot(avgAvgSd, color=mypalette[pltNr-1], linestyle=":", label=title)

    return (p1, p2)

def drawCrossSection(models, pltNr = 1):
  values = []

  for model in models:
    values.append(model.states[-1])
  plt.subplot(1, 2, 2, title="Density Plot of State for Simulations")
  ax = plt.gca()
  #ax.set_xscale('log')
  plt.xlim((0, 5))
  plt.ylim((-1, 1))
  plt.xlabel('Density')
  try:
```

```
    sns.distplot(values, hist=True, kde=True, color = mypalette[pltNr-1], vertical=True)
  except:
    sns.distplot(values, hist=True, kde=False, color = mypalette[pltNr-1], vertical=True)

def drawClustersizes(models, pltNr = 1):
  sizes = []
  for model in models:
    part = findClusters(model)
    (avg, sd, size) = findAvgStateInClusters(model, part)
    for s in size:
      sizes.append(s)
  plt.subplot(1, 3, 3, title="Density Plot of clustersize simulations")
  plt.xlabel("Clustersize")
  sns.distplot(sizes, hist=True, kde=True, color = mypalette[pltNr-1])

def drawConvergence(variables, modelsList, pltNr = 1):
  endState = []
  for models in modelsList:
    values = []
    for model in models:
      values.append(model.states[-1])
    endState.append(mean(values))
  plt.subplot(1,2,2)
  plt.xlim((-1, 1))
  plt.ylim((-1, 1))
  plt.scatter(variables, endState, color=mypalette[pltNr-1])

def drawClusterState(models, pltNr = 1, step=-1, subplot=1):
  plt.title("Density of Avg State in Communities")
  if(step < 0):
    plt.subplot(1, 3, 3, title="Avg State after Simulation")
    states = []
    for i in range(len(models)):
      for c in models[i].clusteravg[-1]:
        states.append(c)
  else:
    plt.subplot(1, 3, subplot, title="Avg State at t="+ str(step))
    states = []
    for i in range(len(models)):
      for c in models[i].clusteravg[step]:
        states.append(c)
  ax = plt.gca()
```

```
  #ax.set_xscale('log')
  plt.xlim((0, 5))
  plt.ylim((-1, 1))
  plt.xlabel('Density')
  plt.ylabel('State')
  try:
    sns.distplot(states, hist=True, kde=True, color = mypalette[pltNr-1], vertical=True)
  except:
    sns.distplot(states, hist=True, kde=False, color = mypalette[pltNr-1], vertical=True)

def drawAvgNumberOfAgreeingFriends(models, pltNr = 1):
  avgNbAgreeingFriends = [model.avgNbAgreeingList for model in models]
  avgAvg = np.array(avgNbAgreeingFriends).mean(axis=0)
  plt.title("Average␣Agreement␣of␣Neighbours")
  plt.ylim((0, 1))
  plt.xlabel("Timesteps")
  plt.ylabel("Agreement")
  plt.plot(avgAvg, color=mypalette[pltNr-1])
```

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