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Predictive Maintenance and Battery Management for wireless sensors

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Predictive Maintenance and Battery Management for wireless sensors

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Reliability, Availability, Maintainability and Safety (RAMS)
Submission date: June 2019
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Preface

The master's thesis is a part of the requirement of two-year international in master program RAMS (Reliability, Availability, Maintainability, and Safety) at the Norwegian University of Science and Technology (NTNU) during the spring semester of 2019. The student, Sanjay Shah in collaboration with NTNU and EQUINOR, carries out the project.

The report will mainly focus on the Battery management for wireless sensors through the main objective is to develop the discharge model for batteries and replacement strategies for failed sensors used at different equipment in oil and gas plant.

The report is targeted to the students and researchers who are researching for the development of battery degradation models for monitoring performance of equipment, especially for the wireless sensor network system. The readers of the report should know condition monitoring and maintenance strategies.

Trondheim, 11 June -2019

Sanjay Shah

Acknowledgment

I want to express my sincere gratitude to my main supervisor Professor Anne Barros who provided useful sights, supports, and guidance while performing the project. She helped me to find out the necessary literature needed for the project and always stirred me in the right direction whenever I needed help. I am also equally grateful to my co-supervisor, Himanshu Srivastav, for providing me assistance and suggestions during the project.

I would also like to thank to contact person in Equinor ASA, Erling Lunde who provided the platform to work in this project and gave all the necessary information about the overview of the battery management system, which was supportive to understand the problems of the project.

Additional thanks to prof. Anne Barros and my close friends for providing me emotional supports throughout the project.

Finally, I would like to thank my parents and my siblings to providing support and encouragement throughout the work.

Summary

Wireless sensor networks (WSNs) becomes an emerging technology and has been adopted by several companies for data communication and processing. Many leading companies use wireless sensor's network for different purposes. Each sensor is powered by lithium thionyl chloride (LTC) batteries. LTC batteries are non-re-chargeable. Hence, as and when the battery gets discharged beyond the acceptable limits, it must be replaced by a new battery. This thesis aims to find an optimum replacement strategy that minimizes the replacement cost without compromising on the availability of WSNs.

A relevant literature review is performed to find out the critical concept of battery management and its discharge phenomena for a wireless sensor's network. Most of the available literature is only limited for rechargeable batteries but not for the LTC battery which we are searching. One of the research works shows that battery replacement and sensor replacement is identically the same in the context of cost. Based on this information, a literature search for this thesis is shifted from battery replacement to sensor replacement as a part of the maintenance operation.

Relevant literature review regarding failed sensor node replacement is carried out, and only three research articles are found. Those three articles are thoroughly summarised in this project. The first articles used the Markov decision process (MDP) and proposed a solution with the sensor replacement as the only maintenance operation. The second article suggests four different replacement strategies with a mathematical model whereas the last article describes the homogenous and heterogeneous nature of wireless sensor networks and performs a maintenance action by introducing multi-cluster management technique to replace the failed node.

In this thesis, a probabilistic model is developed to find the optimal policy for node replacement by comparing the expected long-run cost of each policies. Long run cost

includes replacement cost and performance loss rate in terms of cost. The optimal policy is the compromise between replacement cost and performance loss. A qualitative study is performed by setting up a hypothetical wireless sensor network comprised of ten sensor nodes with assumed parameters. An expected long-run cost for each policy is calculated, and the results are plotted. Finally, the sensitivity analysis is performed to check the variation of long-run cost by changing the value of different parameters.

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Acronyms

WSN	Wireless Sensor Network
LTC	Lithium Thionyl Chloride
RAMS	Reliability, Availability, Maintainability and Safety
NTNU	Norges Teknisk-naturvitenskapelige Universitet (Eng: Norwegian University of Science and Technology)
IR	Internal Resistance
DOD	Depth of Discharge
MDP	Markov Decision Process
SA	Service Agent
SN	Sensor Nodes
V	Voltage
mW	milliWatt
Ah	Ampere hour
Wh/Kg	Watt-hour per kilogram
mA	milliAmpere
cm	centimetre
mm	millimeter
g	gram
Cp	replacement cost
Mp	performance loss rate in terms of cost
Ep	Expected long-run cost

Chapter 1

Introduction

1.1 Background

Since the beginning of the third Millennium, Wireless Sensor Networks (WSNs) becomes an emerging technology and has been adopted by several companies for its different industrial purposes(Buratti et al., 2009). The usage is not only limited by factories and companies, but also used for various other application, for example, battlefield surveillance, traffic monitoring, health care services, environmental and habitat monitoring, and disaster risk management (Misra et al., 2010).

Each wireless sensor networks consists of several nodes, which is the crucial component for data sensing and processing for communication and requires energy to achieve the desired function (Nighot et al., 2014). Generally, the power is supplied to the sensor nodes with the help of batteries (Nighot et al., 2014). Many wireless sensors are being deployed for various purposes at different facilities in oil and gas industries, each powered by the Lithium battery. Lithium technology was first commercialized in 1991 (Blomgren, 2017) and was able to receive significant attention as a leading source of energy due to its high energy density, high operating voltage, sizeable operating temperature range, outstandingly long shelf life and low self-discharge rate (Jain et al., 1998).

Although very reliable battery is used for wireless sensor nodes to function well, the energy of the battery is discharged after a certain period, and overall performance of wireless sensor network is decreased as sensor node fails (Misra et al., 2010). Therefore, battery or sensor replacement becomes vital for this critical network. This maintenance activity is always associated with some economic loss since it requires human resources to perform a certain level of maintenance (Barroso et al., 2004).

Financial loss is not only linked with maintenance operation but also with the storage of battery. Batteries are being ordered and stored in large stocks. Keeping this inventory for a long time reduces the battery capacity through self-discharge phenomena (Cheng et al., 2016; Rodrigues et al., 2017). From last few years, battery management for sensor nodes is very emerging topic been able to create significant interest among researchers (Lajara et al., 2015).

1.2 Problem description

Equinor, the leading Oil, and Gas Company use wireless sensor networks for data communication and processing. Sensors are used at different locations on different facilities to measure noise level as an output. Each sensor operates differently and uses the same battery type.

There are three different types of sensor with average power consumption at the plant.

- Type 1: 9 mW - 10 units (2 batteries)
- Type 2: 4 mW - 30 units (1 battery)
- Type 3: 2 mW - 60 units (1 battery)

Some sensors used for long hours and some of them are less, which results in the variation in capacity and discharge curve. Degradation rate and a lifetime of batteries largely depend upon its usage since LTC battery is the primary battery and it is non-rechargeable. Sensors are replaced when the battery is completely discharged, each time either one sensor is replaced or several but not all of them. It means that the time duration for replacement is not fixed; it is on a random basis. There is no observed data available for the replacement of batteries. Statistically, one can assume that the replacement time follows the random distribution throughout the year, which is one of the major constraints of the proper battery management system. Therefore, replacement methods/strategies are something to consider. Another main constraint is to keep the batteries for a long time since the long storage time reduce the battery life, which is for LTC batteries estimated for 1.5 years (Batteries, 2018; Jain et al., 1998). Also, sensors are always connected with power, and they are being operated at different temperature ranges from (0-30) ˚ C. Therefore, batteries degradation is highly depending upon the temperature variation. Thus, lifetime analysis of batteries according to temperature variation is the major problem.

The constraints, as mentioned earlier, lead to some economic loss and require more workers to operate. Therefore, the company is interested in the improvement of their replacement strategy and workforce optimization to increase the overall economic efficiency. Hence, workforce optimization and possible maintenance strategies are essential. The policy can be chosen in a very holistic way, which not only helps to improve its performance technically but also economically. In the case of non-rechargeable batteries, batteries should be disposed of once it gets discharged, so one cannot go for the corrective maintenance. It requires periodic inspection and predictive maintenance could be the best way to meet the above criteria.

This thesis focuses on solving the maintenance problem by adopting an appropriate probabilistic modelling approach to find an optimum replacement strategy, which minimizes the replacement cost without compromising on the availability of WSNs.

1.3 Objectives

The main objective of this master thesis is to use the available information, develop a probabilistic model, and provide replacement strategies for the battery management for wireless sensor network. The goal can be addressed as the following listed tasks.

1. Study and present the characteristics of the battery in general and discuss its chemistry along with its discharge phenomenon.

2. Study and present the specific properties of the LTC battery and its discharge phenomenon. The factors (like temperature, voltage, usage of sensors) which affects the discharge of the battery
3. Literature survey for different types of wireless sensor networks and its maintenance and replacement strategies
4. To find out the methods, inputs, and experimental data or parameters to develop the model for managing failed battery in a wireless sensor network.
5. Verify how this model is used for solving the maintenance purpose and assessment of replacement cost and performance loss of the network.
6. Discuss the challenges, opportunities, and future works of the work done in the project.

1.4 Limitations

1. Previous research work shows the degradation phenomenon mainly for a rechargeable battery, and very fewer studies have been conducted for non-rechargeable battery (LTC battery). Therefore, the availability of only a few research articles shows that this area has not been explored so much.
2. Due to the limited accessibility of enough data and information, the establishment of the physical model seems to be tough. That is the reason we decided to move towards developing a probabilistic model.
3. During modelling, we assume that either replacing a battery or sensor node is identically the same. It might not resemble with the real scenario; the cost of the battery may not be the same as of sensor nodes.
4. The reliable data source is rare because the nature of the problem is new; Probable values are used for the analysis of the model. Unavailability of actual failure data is the biggest obstacle for approaching the exact solution to the problem.

5. Any experimental data do not support the input parameters used in the model, and the parameters considered for a qualitative study is hypothetical. This thesis does not target to solve any specific problem but to demonstrate the challenges and approach of probabilistic modelling to find out the maintenance strategy for managing failed sensor nodes.

1.5 Research Approach

Research approach includes a literature survey about the general operation of batteries, basic chemistry, and general characteristics of the battery. LTC Batteries performance and discharge phenomenon, including temperature's variation effect and lifetime consideration. Further, different literature is reviewed for the wireless sensor networks, including its maintenance approach and replacement strategies.

Most of the literature is search in oria.no, and different database are used like Scopus, Compendex, and Science direct. (Batteries, 2018; Cheng et al., 2016; Jain et al., 1998; Linden Reddy, 2002) are the primary source of literature to understand about the battery terminologies and its characteristics. These (Dutta et al.; Misra et al., 2010; Taboun et al., 2015) are the three main research article is reviewed and studied for the wireless sensor network, its types and nature of maintenance and different replacement strategies — these literature help to understand the various technology and terminologies related to batteries and wireless sensor networks.

1.6 Structure of the Report

The remaining chapters of the report will be structured in the following way.

- Chapter 2: Description of batteries

Chapter 2 includes a theory regarding most of the essential functions of batteries, which are being used in wireless sensors. It consists of a bit elaborated description on battery chemistries. Also, it provides theory regarding the aging of batteries, performance evaluating its efficiency and capacity change with temperature and discharge rate.

- Chapter 3: Lithium thionyl chloride batteries (Li/$SOCL_2$)

The characteristics of the LTC battery is described in this chapter, along with their chemistry. Also, the lifetime consideration and discharge phenomena of LTC battery are well explained.

- Chapter 4: Literature review for WSNs

This chapter includes some explicit description of the type of wireless sensor networks, different maintenance strategies, and replacement strategies for failed sensor node. Also, cost optimization technique with some mathematical model followed by the discussion and comparison of varying replacement strategies is presented.

- Chapter 5: Modelling of wireless sensor networks (WSNs)

This chapter presents a procedure and guidelines for modeling of wireless sensor networks. A detailed explanation of Markov Chains and transition matrix is shown, which further used in the estimation of long-run cost.

- Chapter 6: Numerical Analysis

This chapter includes the step-by-step calculation procedure to estimate the long-run cost and determine the optimal policy for a hypothetical network. Sensitivity analysis is performed at the end of this chapter.

- Chapter 7: Conclusion and further work

The most important part; discussion and conclusion of the findings and recommendations for additional works are presented in chapter 4.

Chapter 2

Description of batteries

This chapter consists of the background theory of the relevant topics, which includes:

- Types of batteries and its operation principle.
- Basic terminologies used in battery technology.
- What are the essential characteristics of batteries?
- What is the batteries lifetime and how it fails under certain conditions?
- What are the main influencing factors for failure?

A battery is a device that utilizes chemical energy present in the active materials and converts it into electrical energy through an electrochemical discharge reaction. It is composed of either one cell or many depending upon their usage and application (Linden Reddy, 2002; Spitzer Spitzer, 2000). The batteries are classified mainly into two categories:

1. Primary batteries:

These are the batteries which are not rechargeable or capable of quickly recharge electrically, and once it is being discharged must be replaced or discarded. Some examples of primary batteries are alkaline-manganese, carbon-zinc (dry cell), and mercury-zinc, silver-zinc, and lithium batteries. It is very convenient, lightweight, and usually very inexpensive. Primary batteries are well known for its useful shelf life and high energy density at different discharge rates (from low to moderate). Also, it is suitable for maintenance purpose and easy to handle (Linden Reddy, 2002).

2. Secondary batteries:

They are rechargeable batteries, and it can be recharge electrically through passing direct current through them once its charge is discharged. Examples of secondary batteries are nickel-cadmium, nickel-hydrogen, nickel-iron, nickel-metal hydride, lead-lead dioxide, silver-cadmium, silver-zinc, and lithium-ion. It is characterized by its high-power density, high discharge rate, and generally have outstanding performance even at low temperature (Linden Reddy, 2002; Spitzer Spitzer, 2000).

2.1 Basic operation of a battery

Generally, any battery consists of a metal anode, porous cathode, and a non-aqueous electrolyte. The general design and operation of the battery cell are illustrated in below figure 2-1, adopted from RWTH, Aachen University (Sauer, 2018). It consists of the following primary materials:

- Current collectors: An inner part of high electrical conductivity used to conduct current from or to an electrode during discharge. It must be chemically stable and display high conductivity.
- Active masses: The materials in the electrodes of the battery that which takes part in the electrochemical reactions. They are mostly porous substances with the large inner surface.
- Electrolyte: The medium that provides the ion transport mechanism between positive and negative electrodes of a cell. An ion conductor and an isolator for electrons. There are several kinds of electrolytes available, aqueous electrolytes, liquid non-aqueous electrolytes, and solid body electrolytes.
- Separator: An ionically permeable but electronically non-conductive material installed between anode and cathode of a cell. It prevents any electronic contact between them. It is used for the electrical isolation of the active masses against each other.

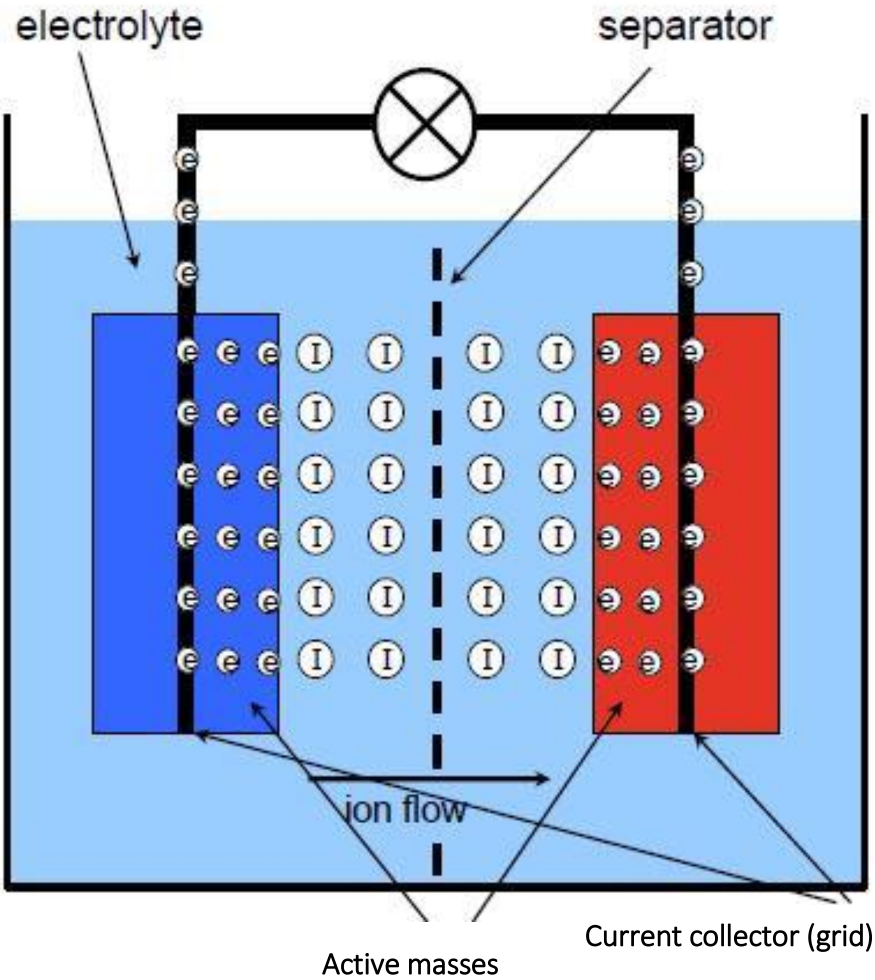

Figure 2-1: basic operation of a battery cell(Sauer, 2018).

2.2 Basic characteristics of batteries

There are several characteristics, which help to identify a battery. Batteries are rated in terms of their cell chemistry (figure 2-1), nominal voltage, and Ah (ampere hour) capacity. The voltage rating is based on the number of cells associated in series and the nominal voltage of the individual cell (2.0 V for lead acid, 1.2 V for nickel-cadmium and 3.6 for Lithium thionyl chloride) (Communications, 2005; Linden Reddy, 2002).

The Ah capacity available from a fully charged battery depends on its temperature, rate of discharge, and age. The nominal voltage of a cell is fixed by the electrochemical features of the active chemicals used in the cell, the so-called cell chemistry. The actual voltage is emerging at the terminals at any specific time, as with any cell, determined by the load current and the internal impedance of the cell and it varies with temperature, the state of charge (SOC) and with the age of the cell.

The figure 2-2 shows typical discharge curves for different batteries cells using a range of cell chemistries when discharged. Note that each cell chemistry has its characteristic nominal voltage and discharge curve. Some chemistries such as Lithium batteries have an almost flat discharge curve while others such as Lead-acid, have a noticeable slope(Communications, 2005; Linden Reddy, 2002; Spitzer Spitzer, 2000).

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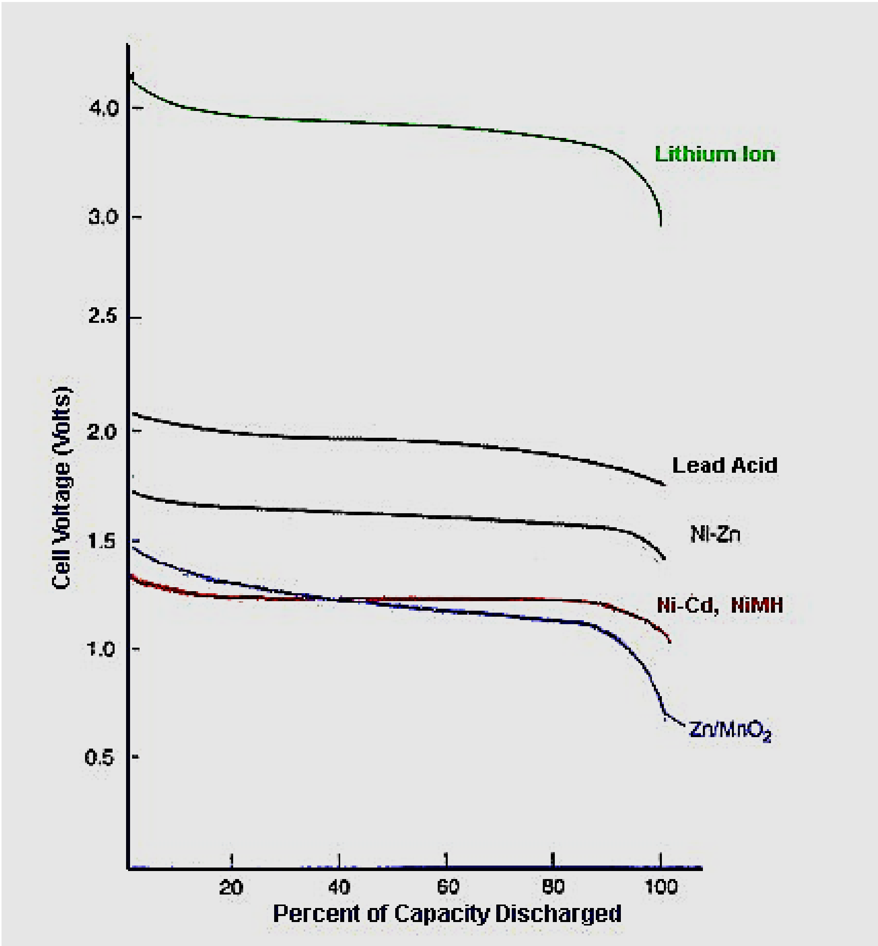

Figure 2-2: Discharge profile of battery systems((Communications, 2005)).

A flat discharge graph simplifies the design of the application in which the battery is used since the supply voltage stays constant all over the discharge cycle. A sloping curve enables the estimation of the State of Charge (SOC) of the battery; meanwhile, the cell voltage can be used as a measure of the residual charge in the cell. Modern

Lithium batteries have a very flat discharge curve, and other methods must be used to determine the State of Charge (Communications, 2005; Linden Reddy, 2002; Spitzer Spitzer, 2001).

Usually, most of the batteries are rated at room temperature (25°C), the C-rate (1-hour rate) and the beginning of life. Batteries, however, often are evaluated in terms of the end-of-life capacity, i.e., the minimum capacity before the battery is considered unserviceable. The state-of-charge (SOC) of a battery is the percentage of its available capacity relative to the fully charged capacity. By this definition, an ultimately charged battery has a state of charge (SOC) of 100%, and a battery with 20% of its capacity eliminated has a state-of-charge of 80%. For example, a battery evaluated at 30 Ah, but only capable of delivering 24 Ah when completely charged, will have a state-of-health of 24/30 * 100 = 80%. Thus, the state-of-health considers the loss of capacity as the battery ages (Communications, 2005; Spitzer Spitzer, 2001).

Chapter 3

Lithium thionyl chloride batteries

(Li/$SOCL_2$)

The users very well adopt the lithium thionyl chloride (Li/SOCL2) batteries for its high nominal cell voltage of 3.6 V and high energy densities. Li/$SOCL_2$ batteries have been fabricated in a diversity of dimensions and designs. The lowest capacity is as small as 420 mAh for water or coin cells and large up to 10,000 Ah for prismatic cells. The batteries of our interest are the LTC batteries (Tadrian batteries) of size D, and the generic name is SL-2780. It is also known as bobbin type cylindrical batteries (figure 3-1) due to its cylindrical configuration.

Further, the Tadiran batteries cell is designed with mechanical hermetic seal and safety vent. The sealing is designed between positive and negative cell terminals that ensure insulation and provides first-rate shelf and operational lives. Besides, a safety vent is incorporated with the hermetic seal to prevent from over or burst pressure of the cell. The specific energy for the LTC batteries of size D is 720 Wh/Kg, and energy density is 1270 Wh/dm^3. Some other properties of SL-2780 are mentioned in below table 2-1 (Batteries, 2018; Linden Reddy, 2002).

Table 3-1: General characteristics of bobbin type SL-2780 batteries (Batteries, 2018; Linden Reddy, 2002).

S.no	Characteristics	Values
1	Nominal Capacity	19.0 Ah
2	Nominal Current	4 mA
3	Maximum current for continuous discharge	340 mA
4	Pulse current capability	600mA
5	Anode surface area	45 cm2
6	Lithium content	5 g
7	Weight	93 gm
8	Diameter	32.9 mm
9	Height	61.5 mm
10	Volume	52.3 cm3

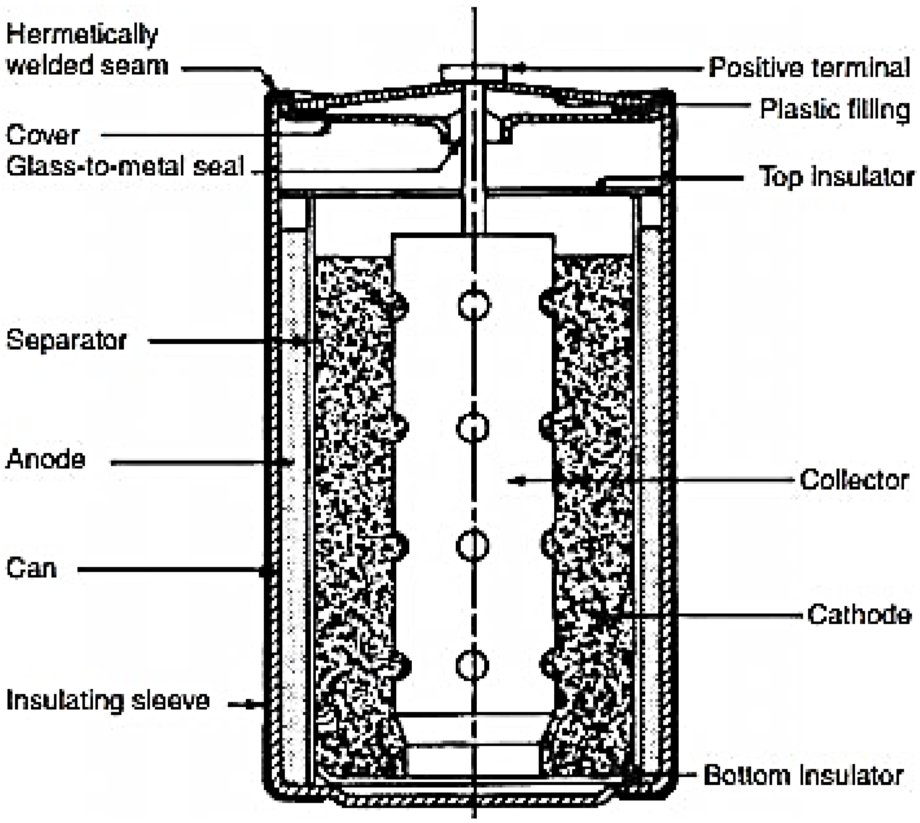

Figure 3-1: A cross-section of bobbin type Li/SOCL$_2$ battery (Linden Reddy, 2002).

3.1 Chemistry of Li/$SOCL_2$

Lithium thionyl chloride (Li/$SOCL_2$) batteries consist of cells made up of lithium foil anode, highly porous cathode Teflon-bonded carbon black as a cathode, and $SOCL_2$: $LiALCL_4$ (Lithium Aluminium Tetrachloride) salt is used for the electrolytic solution. Thionyl chloride ($SOCL_2$), here served as the dual nature; it acts as an electrolyte solvent as well as active cathodic material. For different lithium batteries, the proportions of anode, cathode, and thionyl chloride will differ, depending upon the required performance characteristics and the manufacturer. General description of the material used in the cell for SL-2780 batteries are presented in the below table 3-2:

Table 3-2: Description of different materials used in the cell of SL-2780 batteries. (Batteries, 2018)

S.no	Material	Description
1	Anode	Lithium foil is used as the anode for the battery to provide a mechanically sound and reliable electrical connection,
2	Cathode	Highly porous Teflon-bonded carbon black is used to transfer charge due to its high conductivity property
3	Electrolyte	A solution of lithium aluminum tetrachloride in thionyl chloride is used as an electrolyte, which helps to retain its ionic conductivity even at low temperature.
4	Separator	The non-oven glass is used to prevent immediate discharge and short-circuit to take place.

3.1.1 Performance characteristics of LTC batteries

Battery performance characteristics generally are described by plotting voltage, current, or power vs. discharge time and temperature, starting from a fully charged condition. Typical discharge performance data for LTC batteries are illustrated in Figures 3-2 and 3-3. Figure 3-2 shows the effect of temperature on the capacity when discharged. At the same temperature, the internal resistance (IR) shows variation with voltage. With high IR, the voltage of the batteries seems to be remained constant throughout their lifetime, but by the end, a slight decline in the voltage may occur at medium current discharge as shown in figure 3-2. The open circuit voltage of LTC cell is 3.6 V; typically, operating voltage ranges between 3.3 and 3.6 V with an end voltage of 3.0V (Batteries, 2018; Linden Reddy, 2002).

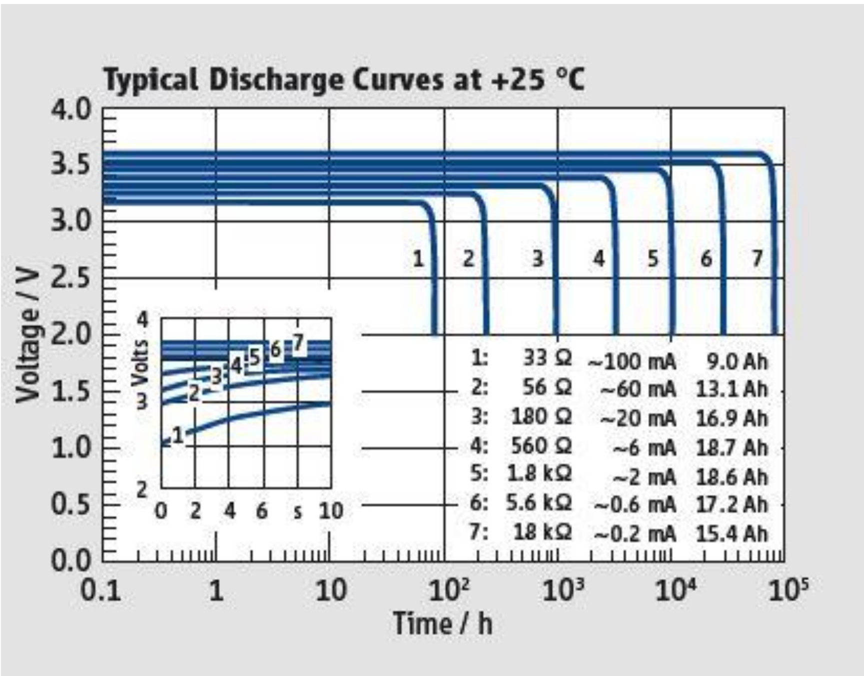

Figure 3-2: discharge characteristics of LTC battery at 25˚C at different voltage response (Batteries, 2018)

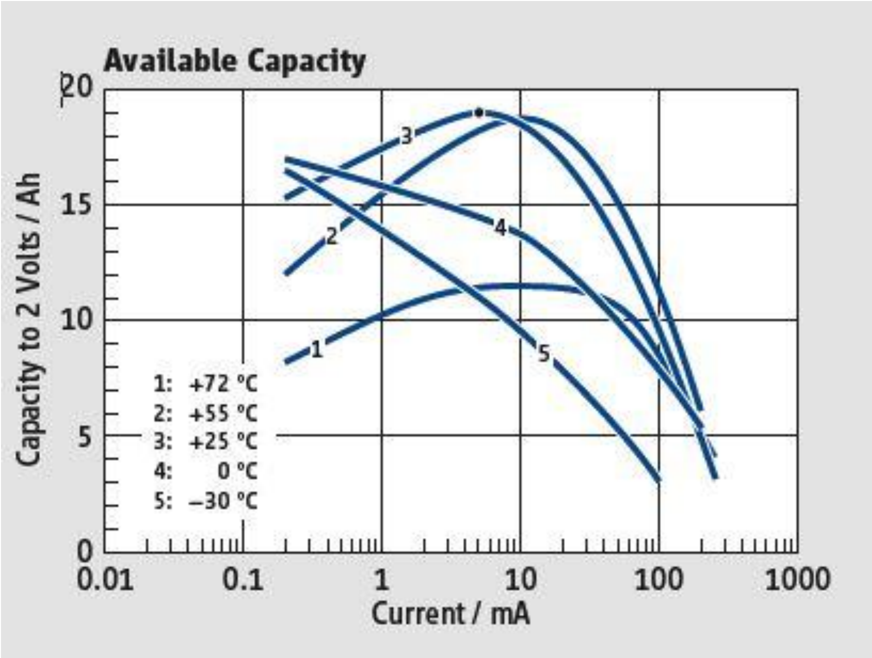

Figure 3-3: performance characteristic as a function of drain rate at various temperature (Batteries, 2018).

Figure 3-3 show the voltage capacity with the current at different temperature ranges from -30 to 72° C. The performance of a battery shows high capacity at a higher temperature. Low-temperature results in more top voltage drops and increase the depth of discharge (DOD) by increasing discharge current or drain current. In nominal discharge current range, the capacity of battery achieves the maximum value. At the lower current range, the self-discharge of the battery become significant due to long discharge time and capacity is reduced accordingly. Similarly, at the higher current range, capacity is increasing due to a rise in the internal resistance of the battery.

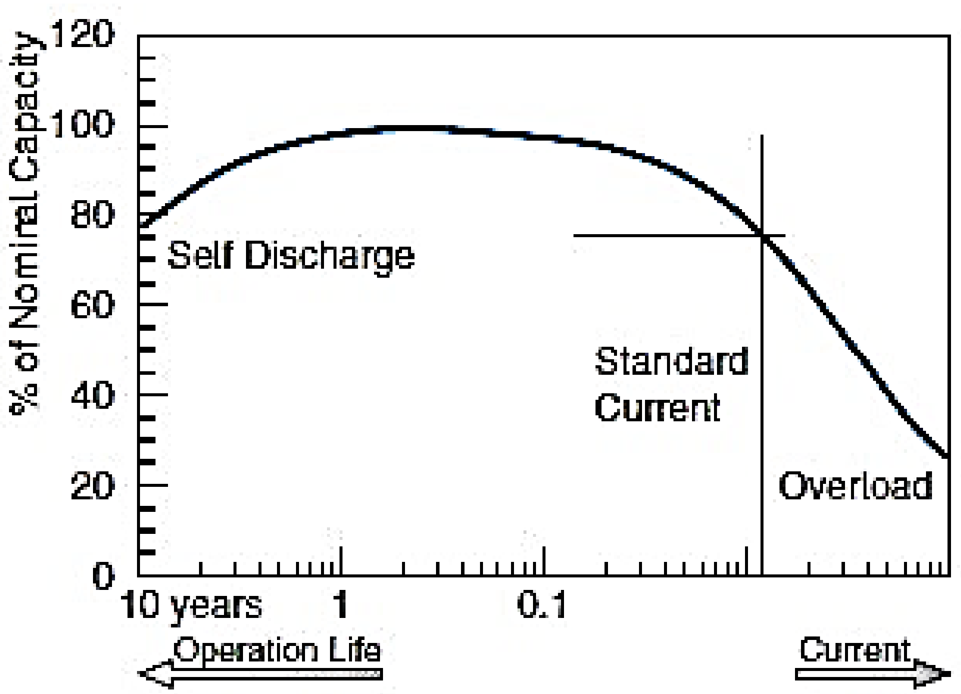

Figure 3-4: self-discharge phenomena and dependence of capacity on current.(Batteries, 2018)

Self-discharge increases with operation life, shown in figure 3-4. The current flow at which a battery brings 76% of its capacity (figure 3-4) is referred to as standard current and a further increase in current results in an overload situation.

Capacity loss can be measured from the shelf life of the battery. Shelf life is the time which battery spent in the storage under a specified condition. From figure 3-5, the shelf life of battery at 20˚C results in the capacity loss of about 1 to 2 % per year while storage at a higher temperature at 70˚C, the loss accounts more which is around 5% per year (Linden Reddy, 2002).

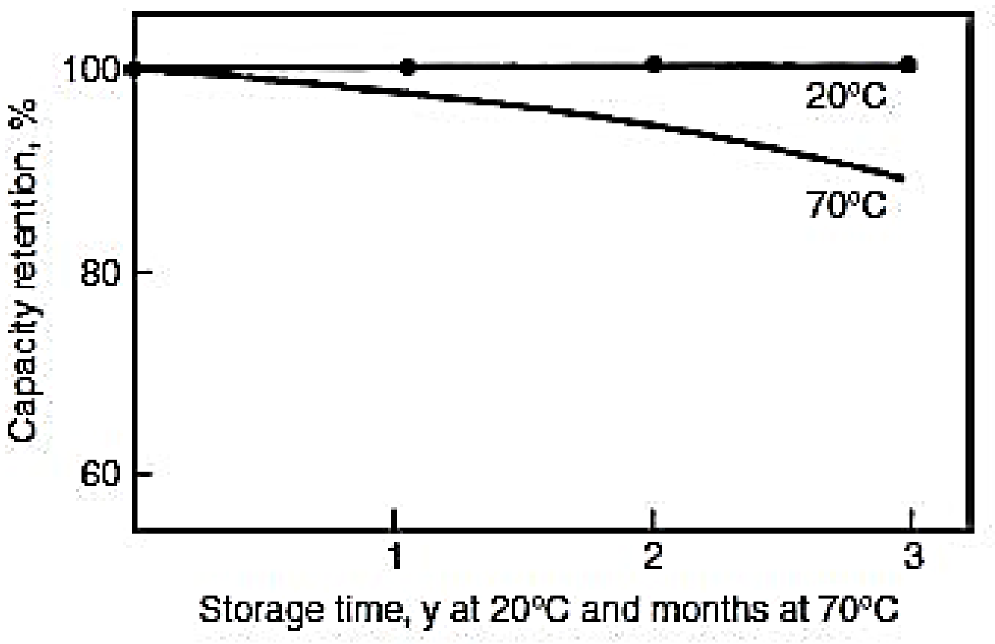

Figure 3-5: Capacity loss due to storage time of LTC battery in years at a different temperature
(Linden Reddy, 2002)

3.2 Lifetime consideration of LTC batteries

To estimate the lifetime of the battery, it is crucial to understand the fundamental and root cause of failure of the battery. The main factor, which influences the failure rate, is:

- Effect of temperature
- Shelf life or storage of the battery
- The depth of discharge or discharge rate

At higher temperatures, there is intensely more chemical movement inside a battery than at lower temperatures. Therefore, the capacity of a battery is lesser when the ambient temperature is too small. The hotter the battery, the faster chemical activity will occur. High temperatures can thus provide improved performance, but at the same time, the rate of an unwanted chemical reaction will rise, ensuing in a corresponding reduction of battery life. The shelf life and charge retention rely on the self-discharge rate, and self-discharge is the consequence of an undesirable chemical reaction in the cell. Correspondingly, adverse chemical reactions such as passivation of the electrodes, corrosion, and gassing are common causes of reduced life. Temperature, hence, disturbs both the shelf life and the cycle life in addition to charge retention since they are all result of chemical reactions. The Arrhenius temperature model can well describe this phenomenon. It states," the relationship between temperature and the reaction rate at which a chemical action proceeds"(Laidler, 1984). It shows that the rate of chemical reaction exponentially increases with rising in temperature. It is given by:

$$K = A * e^{(-Ea/RT)} \qquad \text{Eq. 3-1}$$

Where,

K is the coefficient which describes the reaction rate.

A is a frequency factor related to the frequency of collisions among molecules, generally taken as a constant over a small temperature range.

e is the mathematical constant = 2.71828

Ea. is the activation energy. A constant showing the least energy required for the reaction to occur.

R is the Universal Gas Constant = 8.314 $J.mol.K^{-1}$.

T is the temperature in degrees Kelvin.

RT is the average kinetic energy (K.E.) of the reaction.

The nominal operating temperature of most elementary series of Tadiran Lithium Batteries ranges from -40 °C to +85 °C. The temperature has an impact on the ion mobility in the electrolyte and on the morphology of the protective layer. Thus, current capability increases with temperature, but the effect is compensated to a certain extent by the increase of passivation[1] during storage and self-discharge during

[1] The phenomenon by which a metal, although in conditions of thermodynamic instability, is protected due to the formation of a surface layer(Linden Reddy, 2002).

operation. So, the temperature above and below room temperature has a profound effect on the discharge rate of the battery. It leads to the capacity loss and increases the depth of discharge and lifetime of the battery. Moreover, the usage of battery has a vital role in increasing the intensity of discharge, the longer the battery works, the sooner it fails.

According to (Cheng et al., 2016), the lifetime prediction of Battery is critical for lithium/thionyl chloride cells with a long storage life. This study was to develop models for quickly estimating the storage life of Li/$SOCl_2$ cells using the semi-empirical approach. An accelerated degradation (process shown in figure 3-6) test was conducted for LTC cells stored at a different range of temperatures (room temperature or RT, 40, 50, 60, and 70 °C) to examine the effect of the storage time and temperature on discharge or degradation of the battery. The degradation law can be summarized based on the test data for constructing the semi-empirical equation; this law reveals that the residual capacity of aging cells exponentially changes with the storage time and temperature.

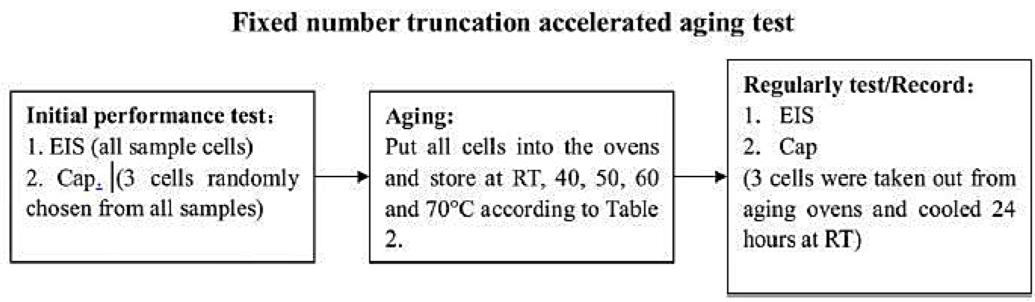

Figure 3-6: flow diagram for acceleration aging test (Cheng et al., 2016).

The experiment lasts for several weeks, and the test interval was different for different temperatures. The process adopted for the aging test is clearly explained in the above figure 3-6. The relationship between the actual capacity rate and storage time was examined by using curve-fitting data achieved during the experiment, as shown in figure 3-7:

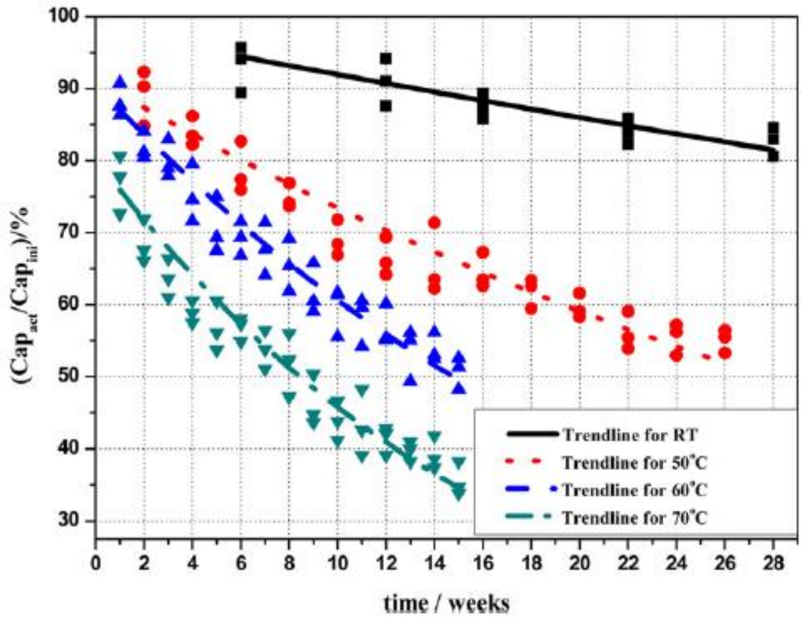

Figure 3-7: curve fitting data showing different trend line at various temperature(Cheng et al., 2016).

The ratio $\frac{CAP_{act}}{CAP_{ini}}$ is the ratio of actual capacity to the initial capacity of the battery. The initial capacity is always considered as 100 % (i.e., without any degradation or discharge).

Figure 3-7 clearly shows that degradation is linear, dependent on the increasing temperature. The less discharged is observed at room temperature, and further increment in temperature shows higher degradation and thus contributes to the capacity loss of battery. In this study, the reaction rate K from the Arrhenius equation, which is explained above, shows the actual capacity-fading rate. Consequently, the reaction rate can be real capacity fading rate, defined in the formula below:

$$\frac{CAP_{act}}{CAP_{ini}}\% = A * e^{(-Ea/RT)} \qquad \text{Eq. 3-2}$$

Now by introducing logarithm on both sides,

$$\ln\frac{CAP_{act}}{CAP_{ini}}\% = \left(-\frac{Ea}{R}\right)\left(-\frac{1}{T}\right) + \ln A \qquad \text{Eq.3-3}$$

From this formula, it can be assumed that, the $\frac{CAP_{act}}{CAP_{ini}}\%$ and $\frac{1}{T}$ are linearly dependent upon each other. Hence, it can be concluded that the effect of temperature on the degradation process follows the Arrhenius equation and indicate the exponential function. Also, the actual capacity fading rates varies exponentially with storage time.

$$\bar{r} = \left[\frac{CAP_{ini} - CAP_{act}(t,T)}{CAP_{ini}} * 100\%\right] \div t \qquad \text{Eq.3-4}$$

Where,

$\bar{r}$ = the average self-discharge rate

T = Temperature

t = storage time

During the initial aging process, the capacity of Li/SOC1$_2$ batteries degraded significantly with increasing time in storage. The capacity loss gradually decreased compared with that during the primary stage. It is due to the reaction between Li and $SOCl_2$ rapidly forming a passivation layer (LiCl), which can effectively slow down the reaction between the anode and $SOCl_2$ and plays a crucial role in protecting the anode. Furthermore, the passivation film gradually thickened with time. The average self-discharge rate for each year is defined in below equation Eq. (4). Based on Eq. (4), the average self-discharge rate in 10 years for the cells stored at 27 °C is $\bar{r}$ =7.181(% per Year)

Chapter 4

Wireless Sensor Networks.

In this chapter, the detailed study of relevant literature was carried out, and three different research articles were compared and summarised.

4.1 Introduction

As described in section 1.2, the main problems of this project are:

1. The battery's inventory management in the storage room and
2. The battery replacement strategies that are being used to supply power in wireless sensor nodes at different facilities.

Out of these two problems mentioned above, the first problem is associated with the inventory management policy. The number of batteries to be stored in the storage room is based on the types and discharge phenomenon of battery at various storage temperature. The lifetime consideration concerning the change in temperature is described in section 3.2. For inventory management, some basic maintenance policy includes:

- Regular monitoring of the storage room through visual inspection and check for any leakage of electrolytic material, since it degrades the battery. Leaking battery must be replaced or repaired,
- The battery should be stored in a pleasant place and dry condition. The storage temperature should be at room temperature for extended shelf life. It should be monitored using a thermometer.
- Storage life should not be more than six months since after that it started to lose its capacity and increase degradation.

- Use of battery in sensors should be monitored after every hour. Continuous use increases the temperature and degrades the battery faster.

The second problem relates to the wireless sensor nodes since it uses the battery for power. Therefore, this chapter is mainly focused on the battery or sensor replacement strategies to meet the constraints, which require less staff and reduced maintenance costs. A maintenance operation is performed once the battery is discharged or sensor nodes fail to function. Thus, the primary purpose of this literature study is to find the research related to the battery replacement strategies and policy as a part of maintenance operation for wireless sensor nodes.

4.2 Related work and further approach

During the search process, the literature found is related to sensor node replacement rather than battery replacement since sensor node is cheaper (Misra et al., 2010) and takes less time during maintenance, which reduces the mean downtime and ultimately minimizes the maintenance cost(Barroso et al., 2004; Taboun et al., 2015). Also, various researches have been conducted in the past to optimize and improve the sensor network lifetime, but these are limited to optimizing network properties such as threshold coverage area and energy efficient methods. Many types of research contain maintenance operation where they use a rechargeable battery for sensor nodes. The maintenance operation for wireless sensor networks, powered by rechargeable batteries, can be achieved by recharging the batteries and is a time-consuming process. So, this is out of scope for this project, since in our case, it uses non-rechargeable batteries (Barroso et al., 2004; Dutta et al.; Misra et al., 2010; Taboun et al., 2015).

Therefore, the above-stated research studies help to limits the search process and provide motivation to stick to the problem. Hence, further exploration was conducted by considering the maintenance operation where these parameters are essential. They are:

- Replacement of sensor nodes, not the battery.
- Minimum mean downtime for maintenance operation.
- Maintenance Cost optimization and less workforce requirement.

Further, very less research has been done on this topic, and the availability of only three research articles shows that this area has not been explored so much. The central theme and approach of these researches are further summarised in below sections 4.3, 4.4, and 4.5, respectively.

4.3 Maintenance cost optimization for WSN

Here the summarised article is "Probabilistic approach to minimize the conjunctive costs of node replacement and performance loss in the management of wireless sensor networks"(Misra et al., 2010).

4.3.1 Introduction

Wireless sensor networks are highly used for data processing and sensing. Each WSN consists of several nodes, requires a certain amount of energy to function, which is powered by the battery. Although these nodes are cheaper, it is restricted by their onboard energy and wireless sensor network (WSN) becomes unable to serve the purpose once the energy level of these sensor nodes gets drained. So, the maintenance operation is required to encounter this problem. Feasible maintenance operation for these failed nodes could be either recharging the battery or replacement of the battery and maintenance operation is always associated with maintenance cost(Barroso et al., 2004). So appropriate cost optimization policies are required to sustain the network.

Based on the nature of maintenance operation, WSNs can be two types:

1. WSNs in which nodes are randomly distributed in the field like battlefield surveillance, and replacement or recharging is not feasible once it gets exhausting. A new set of nodes is required once it gets exhausted.
2. WSNs in which nodes are located by manual planting used as fire alarm sensors and traffic monitoring are examples of this kind of WSN. These kinds of nodes are possible to maintain, and maintenance operation could be done by either recharging or replacing of the battery. Recharging process is very slow, which increases the mean downtime during maintenance. The second-best option is to replace the battery. However, since the sensor nodes are cheaper, it is possible to replace the nodes in the network. So, the node replacement and battery replacement are identically the same(Misra et al., 2010).

4.3.2 Maintenance operation cost and performance loss cost.

The main objective to conduct maintenance operation for the wireless sensor network is to replace the degraded nodes and to find the optimal policy for nodes replacement, as a part of maintenance strategies. The maintenance cost in case of node replacement consists of hardware cost and non-hardware cost. Total maintenance operation cost increases while the cost of maintenance per node decreases with increasing nodes.

Also, the tasking ability of WSN decreases with the increase in the number of exhausted nodes, which is termed as performance loss. Performance loss also accounts for monetary loss. It is the probability that WSN fails to perceive the signal.

4.3.3 Maintenance strategies and modeling approach

The policy that must be able to explain the desired trade-off between the node replacement cost and the performance of the network. So, to determine the optimal strategy, the Markov Decision Process (MDP) is used. Markov decision process is further proceeding to estimate this policy which is optimal only in the long run, since, it utilizes the statistical data obtained from past behavior of the network and provide ultimate solutions through Markov chains (Misra et al., 2010).

4.4 Replacement strategies for WSN

The summarised research article is "Replacement Strategies for Minimizing Maintenance Cost and Maximizing Sensor Network Lifetime" (Dutta et al.) .

4.4.1 Introduction

The main significant issue nowadays for any wireless sensor networks is its repairing and replacement for its better functioning. Wireless sensor networks are composed of several distributed sensor nodes, and the performance of the sensor nodes highly determines the quality of Sensor Networks. Each sensor nodes comprises of four functional units: - sensing unit, processing unit, transceiver unit, and the power unit. The entire node fails if any of this above-mentioned functional unit fails, and the network becomes unstable. Therefore, the better solution to deal with these problems is either to replace the faulty nodes or to repair these nodes through proper maintenance action. Service agents (SA) for all stationary nodes (SN) can do this maintenance task. Repairing of nodes may take a significant time that increases the mean downtime and the maintenance cost. On the other hand, replacing these nodes reduce downtime, but the price is higher. For that reason, the cost optimization technique, which provides the optimal solution for node replacement cost at lower maintenance cost, is required.

There are three factors, which determine the requirements of sensor node replacement:

- First, the failed node may have become very old and weak in performance so that it may not be economical to repair and retain it in the network.
- Secondly, the failed node may have been destroyed in an accident or otherwise or permanently damaged.
- Thirdly, the degraded node might have become outdated because of the availability of better and improved design of equipment and discoveries.

4.4.2 Maintenance strategies

Generally, the efficiency of sensor nodes decreases with age, but it is also observed, that sometimes these nodes do not degrade with time and become fails suddenly. So, it is a problem for service agents (SA) since it results in the formation of a queue of stationary nodes (SNs) waiting for service. Therefore, for each node, the frequency of failure is very much necessary to study, and data can be obtained from experience through a probability distribution of failure. So, some replacement strategies are proposed in this study to minimize the total cost of the system. They are:

1. Replacement Strategy for sensor node or components whose maintenance cost increase with time and money value is not considered:

Here the only maintenance cost increases with time but not the value of money and interest rate is regarded as zero. The calculations are based on an annual basis. (Yearly average price). The replacement strategy is based on viewing the time as a continuous variable.

<u>When time 't' is a continuous variable:</u>

Let,

C = Capital cost of the sensor node or components of the sensor node.

S = Scrap value of the sensor node or components of the sensor node.

n = number of years the sensor node or parts of the sensor node to be in use.

A(n) = Average annual total cost of the sensor node or components of the sensor node.

f (n) = Operating & maintenance cost of the sensor node or elements of the sensor node at time t.

We are to determine the value of n, which minimizes A (n), the cost incurred during 'n' years.

The annual cost of the sensor node or components of the sensor node at any time t = Capital Cost – Scrap Value + Maintenance cost at time t.

$$TC = C - S + \int_0^n f(t)dt \qquad \text{Eq.4-1}$$

Now, the Average annual cost is

$$A(n) = (C - S)/n + \frac{1}{n}\int_0^n f(t)dt \qquad \text{Eq.4-2}$$

We shall find out the minimum value of n for which average is minimum. For minimum cost at *f (n) =A (n)*, which gives $\frac{d^2[A_n]}{dn^2} \geq 0$. it suggests that the replacement should be done when the average annual cost equals the current maintenance cost.

2. Replacement Strategy for sensor node or components whose maintenance cost increase with time and value of money also changes with time:

Here the value of money is also considered, and it continually decreases with time known as depreciation ratio.

3. Individual replacement strategy

Here, immediately replace the sensor node or components of sensor node after its failure. Here we are considering the failure of any sensor node as death and the replacement of any sensor node on failure at birth.

4. Group replacement strategy

Under this scheme, the replacement of all the sensor node or component is replaced as a group. It includes the replacement of failing items simultaneously as a group. This strategy is implemented irrespective of whether all nodes have failed or not.

4.5 Homogenous and heterogeneous WSN management

The name of the article is “Distributed Agent-based Approach to Manage Industrial Wireless Sensor Networks” (Taboun et al., 2015).

4.5.1 Introduction

Wireless sensor networks (WSNs) are being used in several applications, which includes manufacturing industries and factory automation. WSNs are relatively cheaper and easy to handle compared to traditional wireless sensor networks. But, due to some industrial environmental condition, implementation of WSNs poses some challenges. Notably, in any factory, different industrial equipment, and people, are inclined to produce noise and disturbances, which can hinder the transmission of the signal. For example; hindrance is in the form of reflection, interference, scattering, and diffraction. Two types of wireless sensor networks are discussed here. The first is the Homogeneous wireless sensor networks, and the second category falls under heterogeneous wireless sensor network. Homogenous sensor networks are considered good that can provide a system of minimal setup, but also creates constraints concerning the consumption of energy and raise an overall reliability issue of the network due limited node lifetime caused by depletion of power in the battery. To circumvent this problem, heterogeneous wireless sensor networks can be used, which consists of two or more than two different types of sensors in the same network. As a result, a method to maintain and replace the failed node can be done through a distributed, a multi-agent cluster management system that helps to regulate node death and encourages a maintenance approach. The significant advantage of this system is to eliminate the mean downtime and allows the system network to operate continuously. Further, in this work, the Java Agent Development Environment (JADE) simulator was developed to test the design, which provided the platform for the management approach for wireless sensor networks, adaptable for heterogeneous systems.

4.5.2 Clustering classification of wireless sensor networks

Due to the different size and complexity of an automation system, wireless sensor networks consist of thousands of nodes. One way to manage these nodes is to arrange them or group them into clusters. Two types of most common algorithms that are used to classify the clustering of wireless sensor networks are:

1. Algorithms for homogenous and heterogeneous networks

It is constructed on the functionality and characteristics of sensors in the cluster. In a homogenous system, each sensor can act as a cluster head (can be rotated periodically to balance the load and to achieve uniform energy consumption throughout the network), since all the nodes in the network have similar characteristics in terms of their physical hardware and processing functionalities. While in a heterogeneous network, generally two or more different kinds of sensors are used; conventional sensors (with lower functionality) as well as sensors with higher processing capabilities and sophisticated hardware.

2. Static or dynamic cluster formation

It is based on the technique used to form or define the cluster. The formation process is said to be progressive when re-election procedure of cluster head role is preferred in such a way that it can be rotated regularly to achieve energy efficiency throughout the network and can be able to adapt with changes in the network topology.

4.5.3 Clustering algorithms

Two statistical approaches are used to classify the clustering algorithm for wireless sensor networks. This classification is based upon the cluster formation criteria and parameters used for cluster head election. These two approaches are:

1. Probabilistic clustering algorithms

It is a probability allocated to each sensor node, which is used to determine the initial cluster heads. The allocation of possibilities to each node serve as the primary norm, helps the nodes to agree on their election as cluster heads. However other secondary standards may also be considered either during cluster head election process (i.e., the residual energy) or during the cluster formation method (i.e., the proximity of the communication cost) to obtain better energy consumption and network lifetime. Some example of this category includes; LEACH (Low Energy Adaptive Clustering Hierarchy) and EEHC (Energy Efficient Hierarchical Clustering). LEACH is a probabilistic, distributed, one-hop, hierarchical algorithm while EECH is a distributed, multi-hop hierarchical clustering algorithm. The primary goal of these both algorithms is to maximize the lifetime of network and reduction in the energy consumption of the nodes in wireless sensor networks.

- Single hop and multi-hop

In a single hop, all the sensor nodes send the gathered information directly to the base station while in the multi-hop system, data is sent to neighboring nodes also known as aggregation node, represented by the blue circle in the figure 4-1.

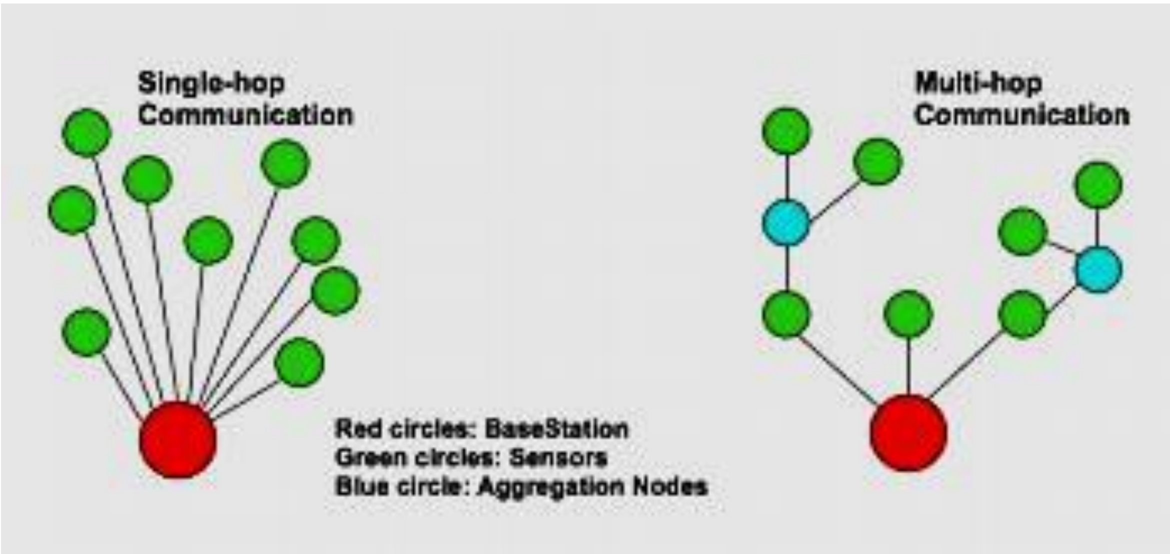

Figure 4-1: single hop and multi-hop system for communication of nodes(Tsitsigkos et al., 2012)

2. Non-probabilistic clustering algorithms

More specific (deterministic) principles for cluster head election and cluster development are primarily considered, which are mainly created on the nodes' proximity (connectivity, degree, etc.) and the information received from other closely located nodes. The cluster formation technique here is largely based on the communication of nodes with their neighbors (one-either hop or multi-hop neighbors). Examples of this category include; Energy Efficient Data Collection (EEDC) and Clustering Algorithm via Waiting Timer (CAWT).

4.5.4 Multi-agent cluster management

This section is split into three parts; the first part starts with the description of WSN architecture; the second part includes the development of battery depletion model, and the last part is about node maintenance.

1. Wireless sensor node Architecture

For a heterogeneous model, the wireless sensor networks consist of three different kinds of nodes:

- Sink nodes
- Anchor nodes and
- Mobile Nodes

Sink nodes are used in WSNs to receive a physical property of interest that is monitored or measured by other nodes. The tangible property of interest is the point-to-point distance between mobile and anchor nodes. Sink nodes are logically selected as the cluster heads given their advanced processing abilities. Anchor nodes and mobile nodes are considered as cluster members. However, mobile nodes are a temporary associate of the clusters, by definition, as they can move from one cluster to another.

In the heterogeneous model, two different agents are used to assign the role for sink nodes and anchor nodes. However, further, the cluster management for the extension of the homogenous network, a single software agent is proposed for performing both tasks, but the problem is that it is unable to assign the role to both nodes simultaneously. So, Role Assignment and Mediation (RoAM) agent is used to decide the role selection criteria and is responsible for assigning tasks to the sink nodes as a cluster head, as shown in figure 3-2.

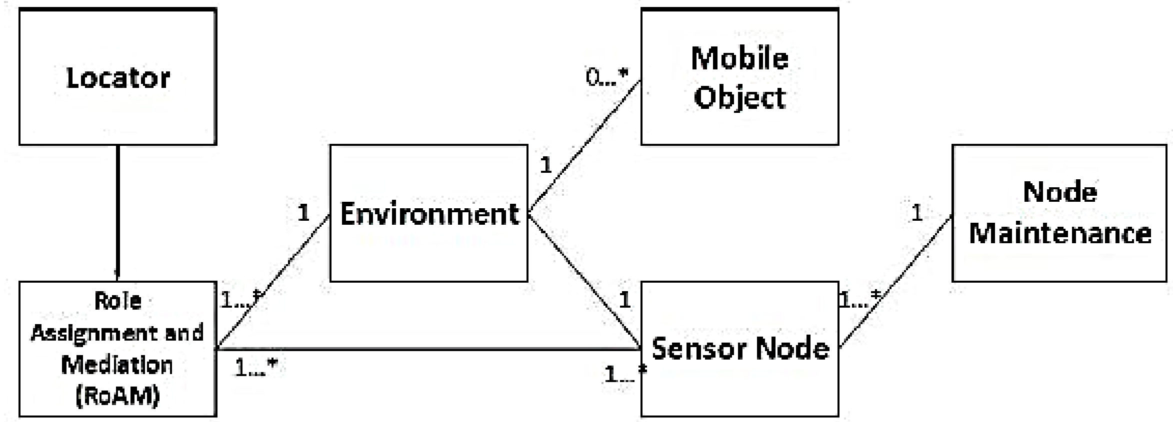

Figure 4-2: Agent classification diagram showing the extension approach for homogenous WSNs (Taboun et al., 2015).

All the boxes in the above figure 3-2 represent the software agents, and the number indicates the ratio of agent numbers (i.e., one to one, one to many). Finally, node maintenance is introduced, as shown in figure 3-2. When the energy level of the sensor nodes reaches below the threshold energy ET, this node maintenance agent receives a signal from sensor node agents and replace the nodes.

2. Wireless node depletion model

To develop this model, the wireless sensors used, which is based on the Cricket Sensor described in(Taboun et al., 2015), and the assumption is made where the sensor utilizes 2 AA Energizer E91 batteries. There are two types of transducers used in sensors; RF and ultrasonic. If voltage output remains 100%, then the remaining power of the sensor node (Pr) can be modeled as follows:

$$P_r = P_{max} - L_{OP} \sum_0^{n_{OP}} t_{n_{OP}} - L_{IDLE} \sum_0^{n_{IDLE}} t_{n_{IDLE}} - \sum_0^{n_{SRF}} LS_{RF} t_{n^{SRF}} - \sum_0^{n_{SUS}} LS_{US} t_{n^{SUS}} - \sum_0^{n_{RRF}} LR_{RF} t_{n^{RRF}} - \sum_0^{n_{RUS}} LS_{US} t_{n^{RUS}} - L_{RANDOM} \quad \text{Eq.4-3}$$

Where,

P_{max} is the total battery capacity

L_{OP} is operating current consumption

L_{IDLE} is idle current consumption

LS_{RF} and LR_{RF} is the RF transducer show send and receive current consumption

L_{RANDOM} is power loss due to random factors

LS_{US} is power consumption of ultrasonic transceivers

nSRF is the number of sending RF signals

nSUS is the number of sent US signals

nRRF is the number of received RF signals

nRUS is the number of received US signals

tnSRF is the duration of the sent RF signal

tnSUS is the duration of the sent US signal

tnRRF is the duration of the reception of the RF signal

tnRUS is the duration of the reception of the US signal

tnOP is operating time

tnIDLE is idle time

3. Simulating wireless node depletion and maintenance

Here the effect of battery life is only considered on the anchor nodes since wireless sensor network that we are looking upon is heterogeneous. The multi-agent system has a maintenance agent, which is in direct contact with anchor agents.

Once the level of a battery of an anchor node drops below a critical threshold, the anchor node sends the signal to its corresponding mediator agent and maintenance agent informing them of its low level of the battery. The anchoring agent then proceeds to store its status and data and suspend its operations until its battery is changed. At this stage, the anchor agent enters a so-called hibernation state. The maintenance agent, on the other hand, gathers the maintenance crew to change the battery of the dead anchor node. This period is stated as the maintenance time that is modeled by a uniformly distributed random variable. Upon the completion of the maintenance, the maintenance agent reactivates the anchor agent.

Chapter 5

Modelling of WSN.

This chapter includes a description of the modelling approach for managing wireless sensor network (WSN). We intend to develop a model that gives the optimal policy for a maintenance operation as a part of maintenance strategies. Following assumptions were made to determine the optimal policy:

- Maintenance operation includes replacement of failed sensor nodes.
- For each maintenance operation, some cost is associated with the replacement of failed nodes. Replacement cost increases with an increase in the number of failed nodes.
- Failed nodes also account for some performance loss in WSN, which is the loss of tasking capability of the network due to a smaller number of functioning nodes in WSN. This performance loss occurs due to the situation that the working sensors in WSN do not detect the event. This performance loss also needs to be quantified in terms of monetary loss. In general, it increases with the increase in the number of failed sensors in WSN.

5.1 Maintenance operation cost

Maintenance operation includes replacement of failed sensor nodes. For each maintenance operation, some cost is associated with the replacement of failed nodes. Replacement cost increases with the number of failed nodes.

The maintenance operation cost for the failed node replacement consists of hardware cost and non-hardware cost. Non-hardware cost is the cost associated with personnel effort. If there is i number of failed nodes in the WSN, then the total maintenance operation cost of i failed nodes is $C_m(i)$, is the sum of the total hardware cost $C_h(i)$ and total personnel cost $C_P(i)$. Mathematically, these costs can be expressed as:

$$C_m(i) = C_h(i) + C_P(i) \qquad \text{Eq.5-1}$$

Further, the personnel cost $C_P(i)$ includes the cost associated for the personnel who assess the sensor field $C_l(i)$ and the personnel involved in the planting the nodes $C_a(i)$. So

$$C_P(i) = C_l(i) + C_a(i) \qquad \text{Eq.5-2}$$

So equation 1 can be written as:

$$C_m(i) = C_h(i) + C_l(i) + C_a(i) \qquad \text{Eq. 5-3}$$

The cost shown in equation 5-3 is for total maintenance operation cost. For i number of failed nodes and if all the nodes are identical, $C_h(i)$ and $C_a(i)$ in equation 5-3 is reduced to $i^*\ S_h(i)$ and $i^*\ S_a(i)$ respectively. While $C_l(i) = C_l$ since this cost is independent of the number of failed nodes. Therefore, the total maintenance operation cost per node is node is

$$\frac{C_m}{i} = S_h + \frac{C_l}{i} + S_a \qquad \text{Eq.5-4}$$

Equation 5-3 and 5-4 shows that the maintenance operation cost $C_m(i)$ and maintenance operation cost per node $\frac{C_m}{i}$ are increasing and decreasing variables. C_m

(i) increases with the increasing number of failed nodes with a decreasing rate. In other word, the cost for replacement of one node is higher, but it decreases with the number of failed nodes.

Therefore, from the above equations, it is concluded that the total maintenance operation cost of i failed nodes is C_m (i) is a concave-shaped increasing function. Since the maintenance operation is the replacement of failed nodes in the wireless sensor network (WSN), we assume, the total maintenance operation cost of i failed nodes C_m (i) is equivalent to the replacement cost of i failed nodes C (i)

$$C_m(i) = C(i) \qquad \text{Eq.5-5}$$

Therefore, due to the concavity nature of the maintenance cost, for n number of sensor nodes, out of which if there is i failed nodes, the cost function can be assumed and defined as:

$$C(i) = C_n\left(\frac{i}{n}\right)^p \qquad \text{Eq.5-6}$$

Where,

n = total number of nodes

i = number of failed nodes

C (i) = replacement cost of i failed nodes

The replacement cost of i failed nodes satisfies the boundary condition if:

- The cost of replacing zero nodes is zero, represented as C (0) = 0 and
- The cost of replacing all the n nodes is Cn, expressed as C (n) = Cn.

The Cn and p are the scale parameter of C (i) curve, and both shows concavity. So p<1. This $C_n(\frac{i}{n})^p$ is the function of proportional of failed nodes. It indicated the percentage of failed nodes. The parameter p depends upon the input from practice and can be modified in several ways.

Verification of cost function: Let us assume the WSN consists of 20 sensor nodes and assume a random value of Cn = 100 Nok and plot of Cn at various values of p (p=0.9, p=0.7, 5 and p=0.5) shown in figure 5-1. The value of p is chosen to satisfy its concavity nature. Figure 5-1 clearly shows that for different values of p<1, the cost for replacement of nodes is increasing with decreasing rate.

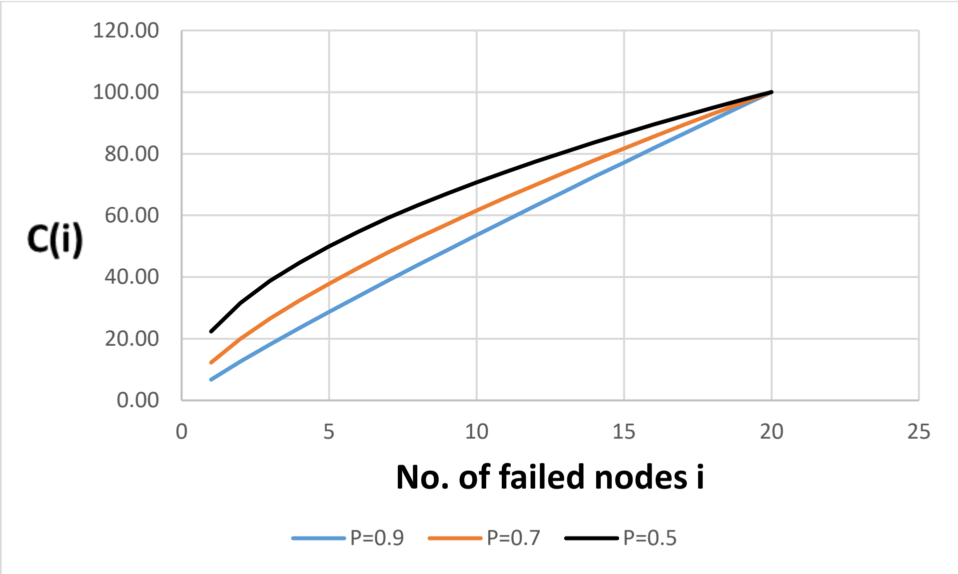

Figure 5-1: Replacement cost at different p.

In the second case, figure 5-2, the scale parameter p (p=0.9) is kept the same, and the different curve is obtained at various Cn (= 100, 200, 300) Nok. Similarly, figure 5-3 and figure 5-4 are a plot for p (=0.7 and 0.5) at different Cn. For all three values of p, the replacement cost is higher for the high value of Cn (=300 Nok) followed by Cn (=200 Nok) and Cn (=100 Nok) respectively.

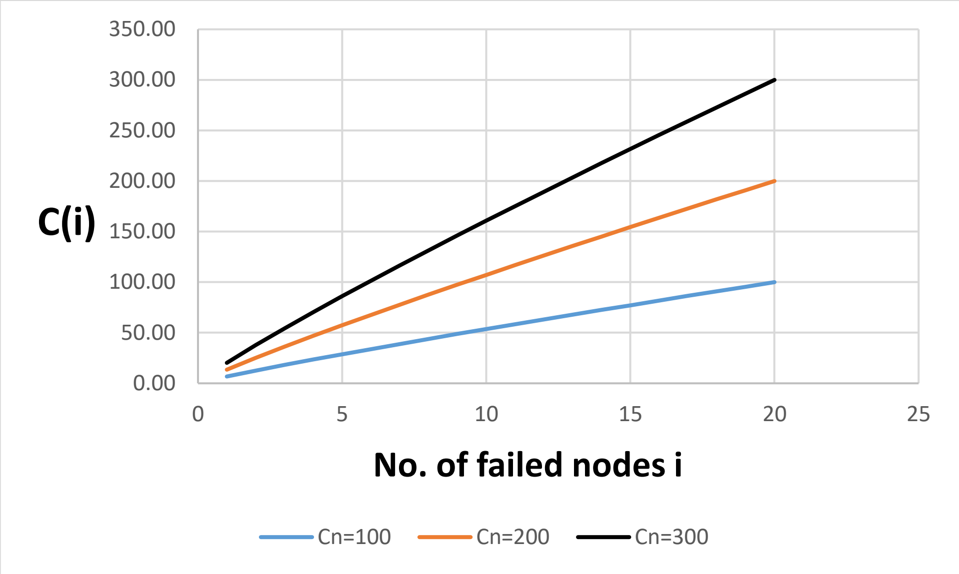

Figure 5-2: Replacement cost at p = 0.9.

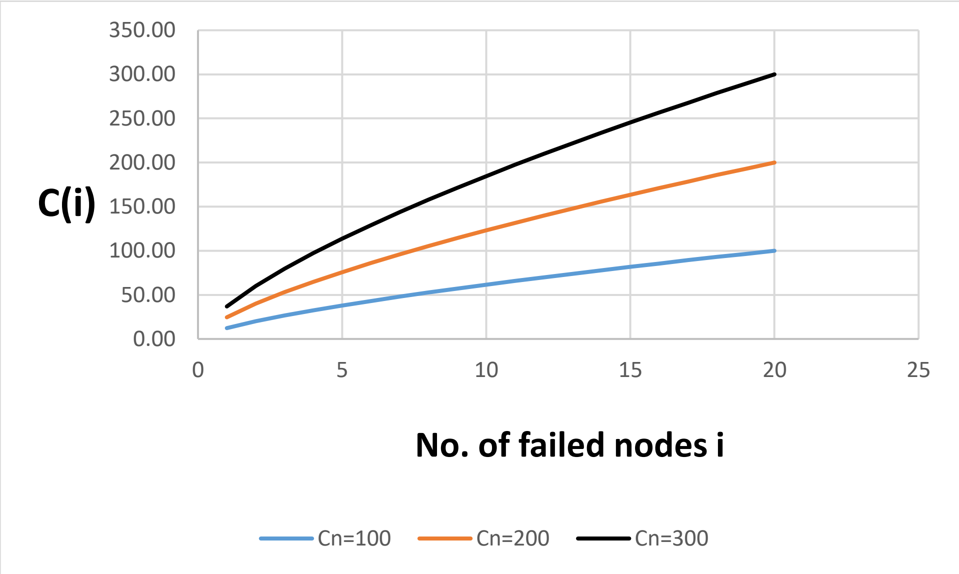

Figure 5-3: Replacement cost at p=0.7.

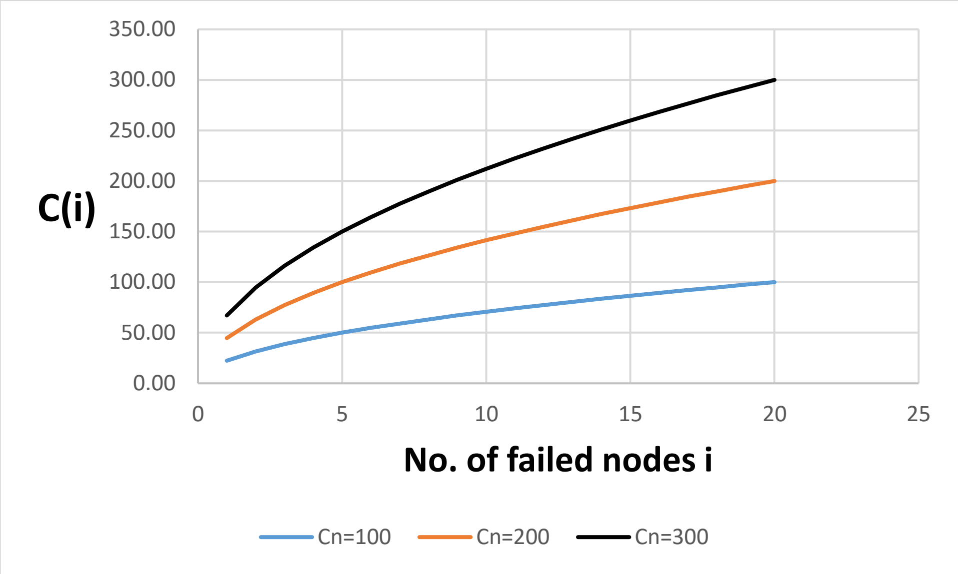

Figure 5-4: Replacement cost at p=0.5.

5.2 Performance loss cost

The tasking capability of WSN decreases with the increase in the number of failed nodes, which is termed as performance loss L (i). It is a rate which increases with time. Performance loss rate also accounts for monetary loss. It is the probability that the working sensor nodes in WSN fail to detect the signal. So, the performance loss due to i number of failed nodes is $L(i) = (1 - \mathrm{p})^{n-1}$. Therefore, the performance loss rate increases with the increase in the number of failed nodes. Also, the marginal loss of performance increases with the rise in the number of failed nodes. Thus, the performance loss rate is a convex-shaped increasing function and, the performance loss function $M(i)$ can be assumed and defines as:

$$M(i) = M_n(\frac{i}{n})^q \qquad \text{Eq.5-7}$$

Where the performance loss function $M(i)$ satisfies the boundary condition if

- The performance loss rate in terms of cost is zero when no nodes are in a failed state, represented as $M(i) = 0$ and

- The performance loss rate in terms of cost is Mn when all the n nodes are in the failed state, expressed as M(n) = Mn.

The performance loss cost function Mn and q are the convex shapes increasing function and scale parameters of $M(i)$ so q > 1. $M_n(\frac{i}{n})^q$ is the function of proportional of failed nodes. It indicated the percentage of failed nodes in terms of monetary loss. The parameter q depends upon the input from practice and can be modified in several ways.

<u>**Verification of Performance loss cost function**</u>

Here also we assume a WSN consists of 20 nodes and consider a random value of *Mn* =100 Nok and *Mn* is plotted for various values of q (q=3, q=5, q=7). Figure 5-5 clearly shows that for all values of q, the graph shows an increasing trend. The performance loss rate in terms of cost increases with failed nodes.

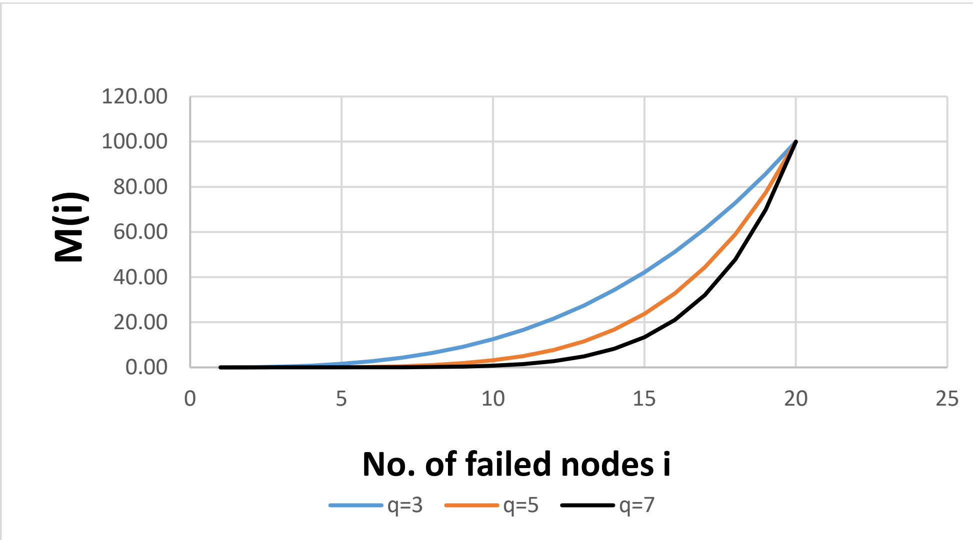

Figure 5-5: Performance loss at same Mn and three different value of q.

Here in figure 5-6, the scale parameter q (=1) is kept the same, and a different curve is obtained at various *Mn* (= 100, 200, 300) Nok. Similarly, figure 5-7 and figure 5-8 is a plot for q (=5 and 7) at different *Mn*. The Performance Loss cost is higher with higher *Mn* value, as shown in figure 5-8.

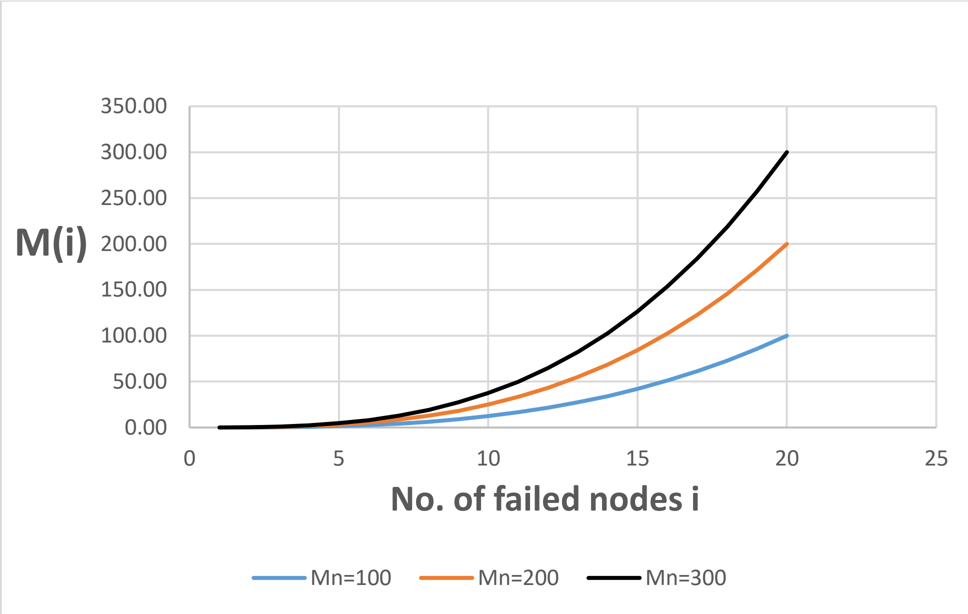

Figure 5-6: Performance Loss for different value of Mn at q=3.

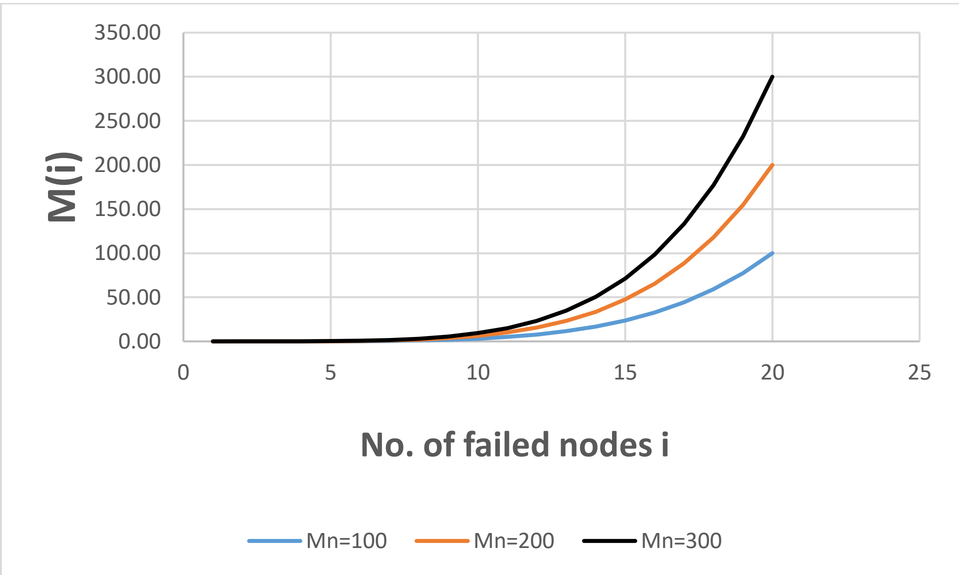

Figure 5-7: Performance Loss for different value of Mn at q=5.

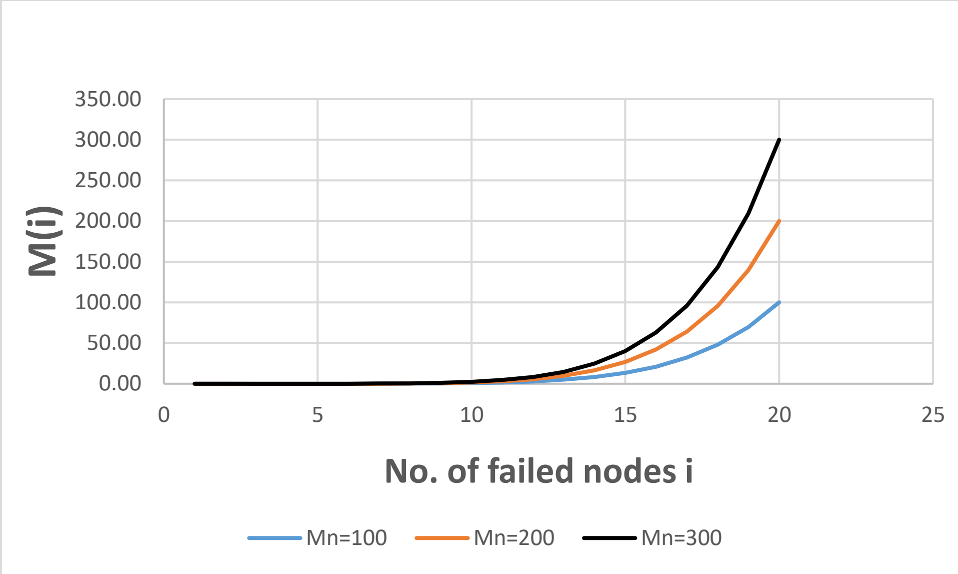

Figure 5-8: Performance Loss for different value of Mn at q=7.

5.3 Optimal policy and long-run cost.

Optimization of cost is the necessary entity for any maintenance strategies. The maintenance activities are associated with the maintenance operation cost and performance loss cost due to *i* number of failed nodes. Both operation cost and performance loss cost are defined above, but the main problem is to determine the policy, which must be able to explain the desired trade-off between the node replacement cost and the performance of the network. So, to determine the optimal strategy and long-run cost, the Markov Decision Process (MDP) is used. It utilizes the statistical data obtained from the past behavior of the network and provides future solutions through Markov chains.

Some assumptions were made, and state parameters were defined during the Markov process analysis. The premises are;

- During maintenance operation, all failed nodes are identical and
- All the failed nodes should equally affect the performance of the sensor network.

- For i number of failed nodes, maintenance operation for node replacement is termed as Oi and replacement policy as Pi. it means replacement should be done right after at least i number of failed nodes detected. Mathematically it can be represented as:

  $Pi = (\Phi i, s)$

  Where, $\Phi i = (Oi, \ldots, On)$ is the number of maintenance operation from i to n and s: O_{j+1} is executed only if it is not possible to do O_j, O_j belongs to O_i.

  For example, let's say a wireless sensor network consists of three sensor nodes. Since three nodes are used, three different policies exist, that is P (1), P (2) and P (3) respectively. For i number of failed nodes, different policies are defined as:

  Policy P (1): When one or more nodes fail i >= 1. A failed node could be either Node A, Node B or Node C.

  Second policy P (2): When two or more nodes fails i>=2.

  Third policy P (3): When all three node fails. i>=3.

- Interval of inspection Δt, the fixed time between two successive inspections since it is not possible to inspect the network constantly. Let the time of the inspection is denoted by t_m at m^{th} inspection.
- For i number of failed nodes, we define the state of the wireless sensor network to be Si. Let's assume state space S containing all possible states and n+1 are the possible states for sensor network having n nodes. Mathematically, we can represent as:

  $S = (S_0, \ldots., S_n)$

Now the optimal policy, which sustains minimum cost, is determined by the two parameters. The parameters are node replacement cost C and performance loss rate in terms of cost, M. so mathematically, the optimal policy can be expressed as:

$$Op = F(C,M)$$

Now to find the expected optimal policy, the probability transition matrix should be used to estimate the future state of the network. As a result, we have three parameters now, which is to be determined to evaluate the expected optimal policy.

$$Op = F(C, M, T)$$

Where C is the cost of a node replacement operation, M is the performance loss cost since the performance loss is also associated with monetary loss, and T is the state transition matrix. The parameters C and M are expressed in vector form, and Transition matrix T is determined by using the Markov process and the expected long-run cost is estimated to determine the optimal policy. The minimum value for the expected long-run cost gives the optimal policy.

5.3.1 Explanation of Transition Matrix

It defines the probabilities of failure of "j" node given that "i" node of the sensor network has already failed. Let us assumes this transition matrix as fixed (independent of time). It can be obtained from the past behavior of the sensor network. This matrix is required to define the failure dynamics of sensor nodes. T denotes it.

$$T = \begin{bmatrix} t00 & t01 & t02 & \dots & t0n \\ 0 & t11 & t12 & \dots & t1n \\ 0 & 0 & t22 & \dots & t2n \\ 0 & 0 & \vdots & \dots & \vdots \\ 0 & 0 & tn2 & \dots & tnn \end{bmatrix}$$

5.3.2 Properties of Transition Matrix

The transition matrix must satisfy the following two conditions:

- The number of failed nodes either increases or remains the same in the subsequent inspection; T is the upper triangular Matrix.
- In any row, the sum of all the elements must be equal to 1.

Mathematically, these two properties can be expressed as:

$$\sum_{j=0}^{n} t_{ij} = \sum_{j=1}^{n} t_{ij} = 1 \qquad \text{Eq.5-8}$$

Now, from the definition of Markov chains, the state probability vector at the *m*th inspection (π^{m}) can be expressed by two identities:

- The state probability vector at (*m-1*)th inspection multiplied with state transition matrix T.

$$\pi^{m} = \pi^{(m-1)} * T$$

- m^{th} power of transition matrix multiplied with initial state probability vector ($\pi^{(0)}$)

$$\pi^{m} = \pi^{(0)} * T^{(m)}$$

To define this transition matrix most simply, let's assume WSNs consists of four sensor nodes, and the Markov process is used to illustrate the transition of the state. Four states are presented with all the possible transition in figure 9. The 4x4 matrix explains the transition matrix of these states.

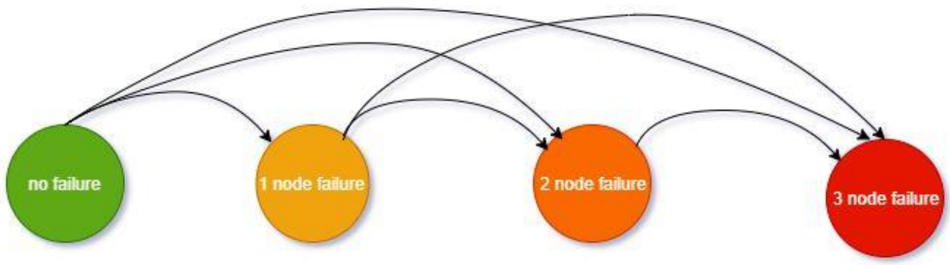

Figure 5-9: Markov chains for four state.

$$T=\begin{bmatrix} t_{00} & t_{01} & t_{02} & t_{03} \\ 0 & t_{11} & t_{12} & t_{13} \\ 0 & 0 & t_{22} & t_{23} \\ 0 & 0 & 0 & t_{33} \end{bmatrix}$$

Explanation of transition matrix:

- The above matrix shows the transition probability matrix for four states, denoted by T.
- For any inspection, it is possible that there is no node fails. It means the WSNs is either in state 0 (no failure reported) or maybe 1 or more node fails (in state 2 or 3). The probability that no nodes fail denoted by t_{00}, one node fails t_{01} and so on.

5.3.3 Estimation of long-run cost

The total expected long-run cost (e_p^m) during m^{th} inspection for policy P is determined by summing up the total replacement cost (r_p^m)and the total performance loss rate in terms of cost (l_p^m). Mathematically, it can be expressed as:

$$e_p^m = l_p^m + r_p^m \quad \text{Eq.5-9}$$

Further, the total expected long-run cost for one inspection at the *m*th inspection for policy P is:

$$e_p^m = \Delta t * \pi^m * M + \pi^m * C_P \quad \text{Eq.5-10}$$

Now, if there is N number of inspections, the optimization problem for many inspections N is to minimize the total expected long-run cost. Let us assume the new notation *Ep* for the total expected long-run cost for N. Mathematically *Ep* is expressed as:

$$E_p = \lim_{N \to \infty} \sum_{m=0}^{N} [\Delta t * \pi^m * M + \pi^m * C_P] \quad \text{Eq.5-11}$$

This equation 5-11 can be further reduced to

$$E_p = \pi^{(0)} * [I - T_P]^{-1} * [\Delta t * M + C_P] \quad \text{Eq.5-12}$$

Where,

$\pi^{(0)}$ is the initial state probability. For n x n Matrix, it is equal to

$$\pi^{(0)} = [1 \quad 0 \quad \dots \quad 0]$$

$[I - T_P]^{-1}$ is the square matrix and T_P is the transition matrix for policy P.

M is the performance loss rate in terms of cost and,

C_P is the replacement cost for policy P.

Both C_P and M are a column vector and represent cost. C_P Varies with policy and different for each policy while *M* be the same for all policies since it is independent of policy. The minimum value for *Ep* gives the optimal policy for the maintenance of a wireless sensor network (WSN).

5.3.4 Method to obtain a square matrix

$[I - T_P]^{-1}$ is the square matrix. I is an identity matrix, and Tp is the transition matrix for policy P. It is different for different policies.

The method to obtain Tp form T is:

If there is 'n' number of total nodes and 'i' number of failed nodes in WSN. The Tp consists of three matrices:

1. Rectangle matrix of order *i x (n+1*)
2. A column vector of *n+1-i* elements, all of which are 1's and
3. Null matrix of order (*n+1-i) x n.*

Taking the inverse of $I\text{-}T_P$ gives the square matrix.

5.3.5 Estimation of C_p *and M.*

We described earlier that C_P depends upon the chosen policy to estimate the long run cost. For *i* number of failed nodes and *n* number of total nodes, C_P for policy P_i, should be chosen such that, in the column matrix C_P, the top values are kept zeros, and rest values should be left intact because replacement is done only if we have *i* number of failed nodes is observed. Accordingly,

$$Cp = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ C(i) \\ C(i+1) \\ \vdots \\ C(n) \end{bmatrix}$$

As well as, the column vector *M* for the range of *M (i)* is represented as:

$$M = \begin{bmatrix} M(0) \\ M(1) \\ M(2) \\ M(3) \\ \vdots \\ M(n) \end{bmatrix}$$

5.3.6 Determination of optimal policy

We begin from defining the fix inspection interval, the time between two successive inspections and assigning values of other variables like a total number of sensor nodes, replacement cost *C*, and performance loss rate *M*. The second step is to generate the transition probability and determine all possible policies, which exists in the network. After defining policies, expected long-run cost is estimated by computing replacement cost for that policy *Cp*, initial state probability (a row vector) and square matrix.

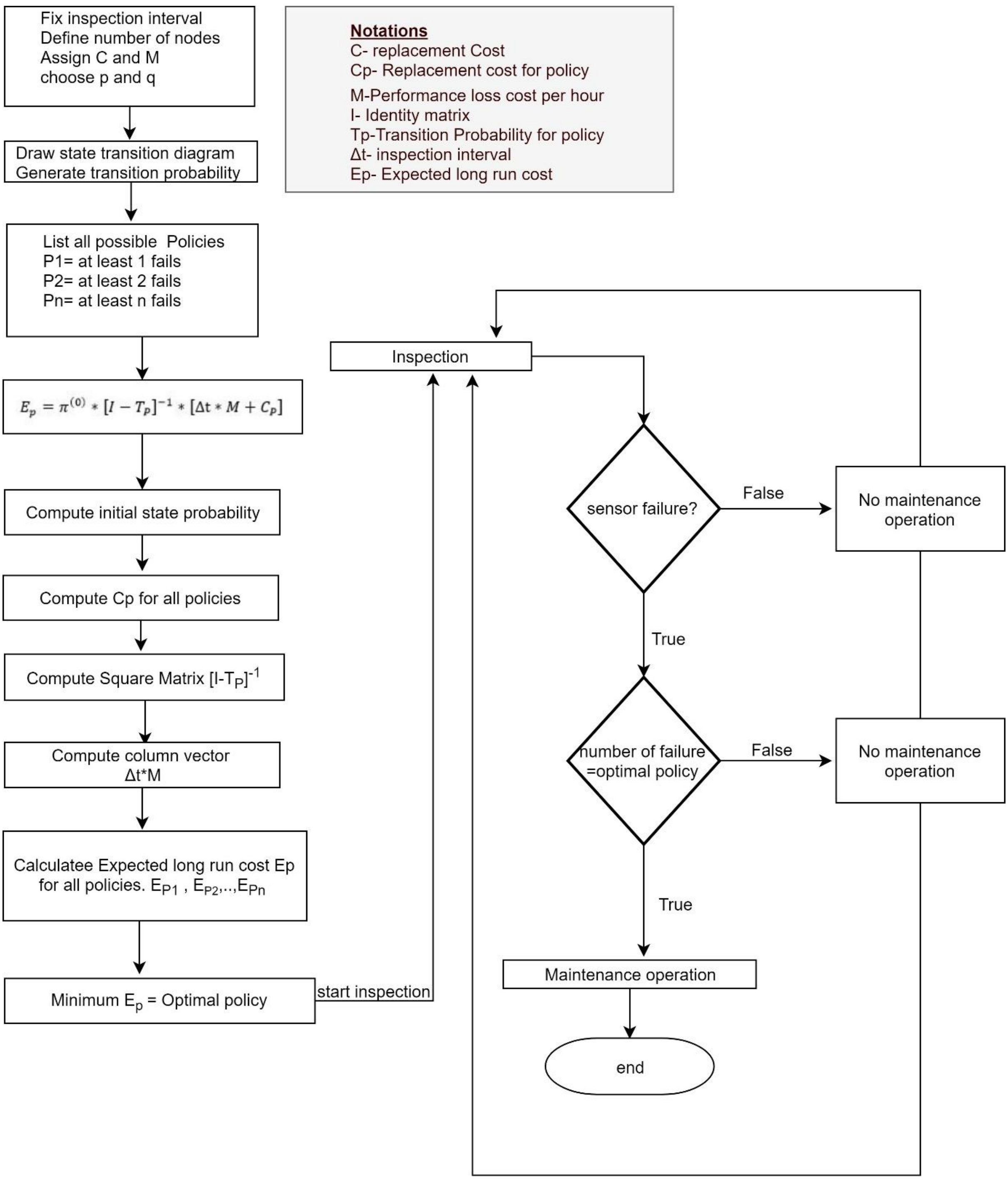

Figure 5-10: Flowchart of the expected long-run cost and optimal policy.

The final step is to check which policy gives the minimum cost, is the optimal policy, which can be used as a key decision indicator for conducting maintenance operation. Figure 5-10 shows the flowchart that demonstrates the systematic procedure to estimate the expected long-run cost and thus determine optimal policy.

Chapter 6 Numerical analysis

In this chapter, a qualitative study is performed to estimate the expected long-run cost for various types of policies. A setup of hypothetical Wireless sensor network (WSN) of ten sensor nodes is considered for the analysis and assumes all the sensor nodes are identical. We divide this study into two parts:

- The first part is the estimation of expected long-run cost to determine the optimal policy and
- The second part is the sensitivity analysis, a comparative study by changing parameters.

6.1 Qualitative study

The considered value for different parameters for this study are:

- Replacement cost (*Cn*)= 500 Nok
- Performance loss cost per hour (*Mn*) = 1000 Nok
- p=0.8 and q=2
- Δt = c (10, 20, 100) hours (time of interval between two successive inspections).

The selected sensor nodes have a failure probability of 0.7. The working probability of each sensor is 1-0.7= 0.3. The above case is solved systematically via steps:

- Step1: Define and evaluate transition matrix.

Since there are ten sensor nodes in WSN, therefore, at each inspection, either one node fails, two node fails, or possibly all ten can fails. There exists a transition matrix and consider this transition matrix is fixed (independent of time), which explains the failure pattern and gives the failure probabilities at a different state. In this case, the transition matrix for ten sensor nodes consists of 11x11 matrix, represented by *T.*

$$T = \begin{bmatrix} t00 & t01 & t02 & t03 & t04 & t05 & t06 & t07 & t08 & t09 & t010 \\ 0 & t11 & t12 & t13 & t14 & t15 & t16 & t17 & t18 & t19 & t110 \\ 0 & 0 & t22 & t23 & t24 & t25 & t26 & t27 & t28 & t29 & t210 \\ 0 & 0 & 0 & t33 & t34 & t35 & t36 & t37 & t38 & t39 & t310 \\ 0 & 0 & 0 & 0 & t44 & t45 & t46 & t47 & t48 & t49 & t410 \\ 0 & 0 & 0 & 0 & 0 & t55 & t56 & t57 & t58 & t59 & t510 \\ 0 & 0 & 0 & 0 & 0 & 0 & t66 & t67 & t68 & t69 & t610 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & t77 & t78 & t79 & t710 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t88 & t89 & t810 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t99 & t910 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & t1010 \end{bmatrix}$$

The state transition probability $t00$ means all nodes are working, and the state transition probability $t01$ means one sensor node in the WSN fails, it reaches from state 0 to state 1 during the inspection. Similarly, $t02$, $t03$, until $t010$ is the transition probability from state 0 to higher states.

$t11$ is the transition probability to remain in state 1. Here at state 1, one of the sensors is already failed. $t22$ is the probability to remain in state 2; two out of ten sensor nodes are already failed here. A similar explanation holds for $t22$, $t33$, ..., $t1010$.

Since each node is independent of each other, so for ten independent nodes:

$t00 =$ Working probabilities of all ten nodes= 0.3^10 = 5.90E-06

$t01$ means one sensor node in the WSN fails. It reaches from state 0 to state 1 during the inspection. However, we do not know exactly which one among three nodes fail. It can be calculated by

$$t01 = one\ failure\ probability * rest\ nine\ working\ probability \\ * the\ combintion\ C(10{,}1)$$

$$C(10{,}1) = \frac{10!}{1! * (10-1)!} = 10$$

Now,

$t01 = 0.3 * (0.7)^9 * 10$ =1,38E-04

A similar method is used to calculate other probabilities. The generalized form to calculate the transition probabilities is expressed as follows:

$$t_{ij} = \begin{cases} t_{nn} = 1, \\ t_{ij} = 0 \; if \; j < 1, \\ {}^{n-i}_{j-i}C * p^{n-i} * (1-p)^{j-1} \qquad j > i \end{cases}$$

Here and now, the transition probability matrix for our case is:

$$T = \begin{bmatrix} 5{,}90E-06 & 1{,}38E-04 & 1{,}45E-03 & 9{,}00E-03 & 3{,}68E-02 & 1{,}03E-01 & 2{,}00E-01 & 2{,}67E-01 & 2{,}33E-01 & 1{,}21E-01 & 2{,}82E-02 \\ 0 & 1{,}97E-05 & 4{,}13E-04 & 3{,}86E-03 & 2{,}10E-02 & 7{,}35E-02 & 1{,}72E-01 & 2{,}67E-01 & 2{,}67E-01 & 1{,}56E-01 & 4{,}04E-02 \\ 0 & 0 & 6{,}56E-05 & 1{,}22E-03 & 1{,}00E-02 & 4{,}67E-02 & 1{,}36E-01 & 2{,}54E-01 & 2{,}96E-01 & 1{,}98E-01 & 5{,}76E-02 \\ 0 & 0 & 0 & 2{,}19E-04 & 3{,}57E-03 & 2{,}50E-02 & 9{,}72E-02 & 2{,}27E-01 & 3{,}18E-01 & 2{,}47E-01 & 8{,}24E-02 \\ 0 & 0 & 0 & 0 & 7{,}29E-04 & 1{,}02E-02 & 5{,}95E-02 & 1{,}85E-01 & 3{,}24E-01 & 3{,}03E-01 & 1{,}18E-01 \\ 0 & 0 & 0 & 0 & 0 & 2{,}43E-03 & 2{,}84E-02 & 1{,}32E-01 & 3{,}09E-01 & 3{,}60E-01 & 1{,}68E-01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8{,}10E-03 & 7{,}56E-02 & 2{,}65E-01 & 4{,}12E-01 & 2{,}40E-01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2{,}70E-02 & 1{,}89E-01 & 4{,}41E-01 & 3{,}43E-01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9{,}00E-02 & 4{,}20E-01 & 4{,}90E-01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3{,}00E-01 & 7{,}00E-01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1{,}00E+00 \end{bmatrix}$$

- Step 2: Define the policy.

Since there are ten nodes in WSN, ten different policies exist, that is P (1), P (2),…,P (10), respectively. For *i* number of failed nodes, different policies are presented as in table:

Table 6-1: Different policy for wireless sensor network.

P1	If at least one node fails.	i >=1
P2	If at least two node fails.	i >=2
P3	If at least three node fails.	i >=3
P4	If at least four node fails.	i >=4
P5	If at least five node fails.	i >=5
P6	If at least six node fails.	i >=6
P7	If at least seven node fails.	i >=7
P8	If at least eight node fails.	i >=8
P9	If at least nine node fails.	i >=9
P10	All ten node fails.	i =10

For different policy, the expected long-run cost can be calculated by:

$$E_p = \pi^{(0)} * [I - T_P]^{-1} * [\Delta t * M + C_P]$$

Where,

$\pi^{(0)}$ is the initial state probability. For 11x11 Matrix, it is equal to

$$\pi^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$[I - T_P]^{-1}$ is the square matrix and T_P is the transition matrix for policy P.

M is the performance loss rate in terms of cost and,

C_P is the replacement cost for policy P.

- Step 3: Define a square matrix for policy all Policies.

$[I - T_P]^{-1}$ is the square matrix. I is an identity matrix and T_P is the transition matrix for policy P. It is different for different policies. The method to obtain T_P form T is explained in section 5.3.4. Matlab is used for the calculation of a square matrix. The Matlab code is presented in Appendix A.

- Step 4: Calculate Replacement cost for individual policies C_P and performance loss rate in terms of cost M.

First, we compute replacement cost $C(i)$ and performance loss rate in terms of cost $M(i)$ for all failed nodes. Both C_P and M are a column vector and computed for policies after computing $C(i)$ and$M(i)$. C_P is different for different policies since it is dependent upon the number of failed nodes, while M is similar for all policies; it is independent of policy.

For i number of failed nodes and n number of total nodes, replacement cost $C(i)$ can be determined by using the below cost function

$$C(i) = C_n(\frac{i}{n})^p$$

Also, M can be determined by using the performance loss function.

$$M(i) = M_n\left(\frac{i}{n}\right)^q$$

In our case, we have 10 sensor nodes. So for different *i*, 10 different cost exist. These costs are illustrated in table 6-2.

Table 6-2: values for C(i) and M(i) for all failed nodes.

Number of failed nodes, *i*	Replacement cost *C(i)* in	Performance loss rate *M(i)*
For *i* =1	79,24 NOK	10,00 NOK
For *i* =2	137,97 NOK	40,00 NOK
For *i* =3	190,84 NOK	90,00 NOK
For *i* =4	240,22 NOK	160,00 NOK
For *i* =5	287,17 NOK	250,00 NOK
For *i* =6	332,27 NOK	360,00 NOK
For *i* =7	375,88 NOK	490,00 NOK

For i =8	418,26 NOK	640,00 NOK
For i =9	459,58 NOK	810,00 NOK

Now to estimate the expected long-run cost for different policies, we need replacement cost C_P for policy. Therefore, for different policies, the replacement cost will be different. The method to obtain replacement cost C_P for policy P is explained in chapter 5. The C_P for all policies is calculated in Matlab. The matlab code is presented in Appendix 1.

- Step 5: Expected long-run cost for all policies

In this step, expected long run cost is calculated for individual policy. The replacement cost and performance loss cost is used to calculate the Expected long run cost for all policies at three different inspection intervals.

For P1 at $\Delta t = 10$ hours

$$E_{p1} = \pi^{(0)} * [I - T_{P1}]^{-1} * [\Delta t * M + C_{P1}] = 1{,}35\text{E+07 Nok}$$

For P1 at $\Delta t = 20$ hours

$$E_{p1} = \pi^{(0)} * [I - T_{P1}]^{-1} * [\Delta t * M + C_{P1}] = 2{,}61\text{E+07}$$

For P1 at $\Delta t = 100$ hours

$$E_{p1} = \pi^{(0)} * [I - T_{P1}]^{-1} * [\Delta t * M + C_{P1}] = 1{,}53\text{E+08}$$

Similarly, expected long-run cost for other policiesis calculated and for timesaving purpose, the calculations are performed in Matlab. The code to estimate is given in Appendix A. The result of all policies is presented in the table 6-3.

Table 6-3: Expected long-run cost at three-different inspection interval for all policies.

	Δt=10 hr	Δt=20 hr	Δt=100 hr
E_{P1}	1,35E+07	2,61E+07	1,53E+08
EP2	1,35E+07	2,61E+07	1,53E+08
EP3	1,35E+07	2,61E+07	1,53E+08
EP4	1,38E+07	2,66E+07	1,56E+08
EP5	1,50E+07	2,90E+07	1,41E+08
EP6	1,63E+07	3,16E+07	1,54E+08
EP7	3,18E+07	6,19E+07	3,03E+08
EP8	4,20E+07	8,22E+07	4,04E+08
EP9	5,69E+07	1,12E+08	5,53E+08
EP10	8,71E+07	1,72E+08	8,54E+08

- Step 6: Determine the optimal policy

For 10 policies, in figure 6-1, at three different inspections, the cost is lower and remains constant until P6.

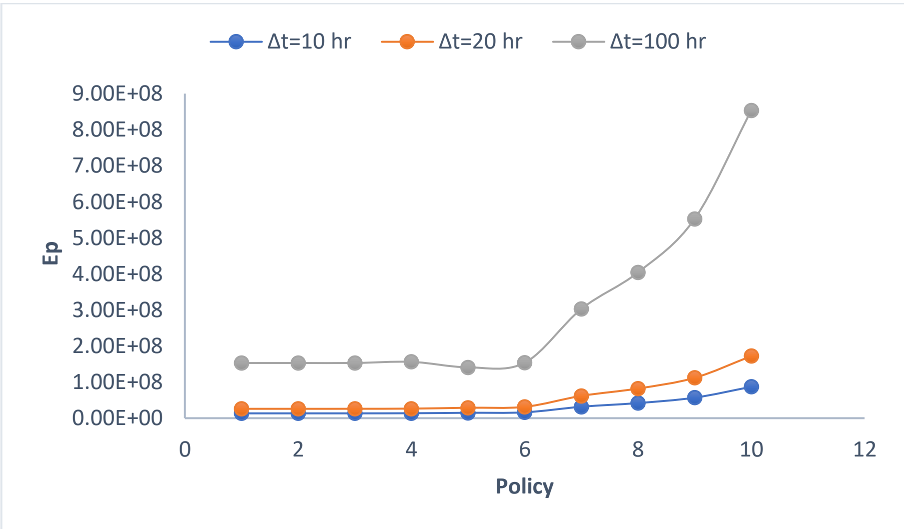

Figure 6-1: Expected long-run cost for all policies

The network seems to be working although having six nodes fails out of 10. The cost starts to increase after P6 and reaches a maximum at P10 (figure 6-1). The reason for the higher cost after policy 6 is due to a decrease in performance of the network, and the performance loss cost increases after P6. Consequently, the best strategy for maintenance operation is after six nodes fail. Therefore, with these supporting reasons, we can choose P6 as the optimal policy.

6.2 Sensitivity Analysis

Sensitivity Analysis is a method used to study and examine the impact of input variables on output, where the output is a function of several inputs. The aim of sensitivity analysis in our model is to determine how the long run cost is changed when we change the other scale parameters. Here the results are compared by varying each of the parameter – *Cn*, *Mn*, *p*, *q*, and the probability of failure- while other parameters are kept the same as we considered in a qualitative study. The chosen time of interval between two successive inspections is 100 hours.

6.2.1 Changing of parameter *Cn*.

Three different values of *Cn* is chosen to estimate the expected long run cost. The obtained results are illustrated in figure 6-2.

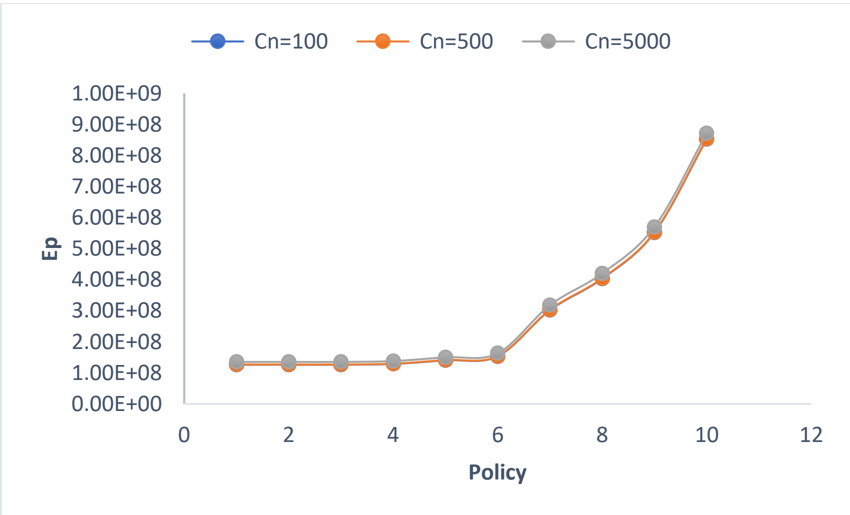

Figure 6-2: The expected long-run cost with Δt = 100 is plotted for different values of *Cn*= (100, 500, and 5000).

Decreasing and increasing the scale parameter *Cn* does not show any significant changes in the calculation of the expected long-run cost.

6.2.2 Changing of parameter *Mn*.

Here the scale parameter *Mn* is changed, keeping another parameter constant and obtained the result is shown in figure 6-3. As the *Mn* decreases, the performance loss cost is less significant in the calculation of expected long-run cost, and it becomes more significant at a higher value of *Mn*. Hence, the plot of Ep for *Mn*= 500 and *Mn*= 1000 lies below the plot of Ep at *Mn*=5000. The variation is more significant with higher policies.

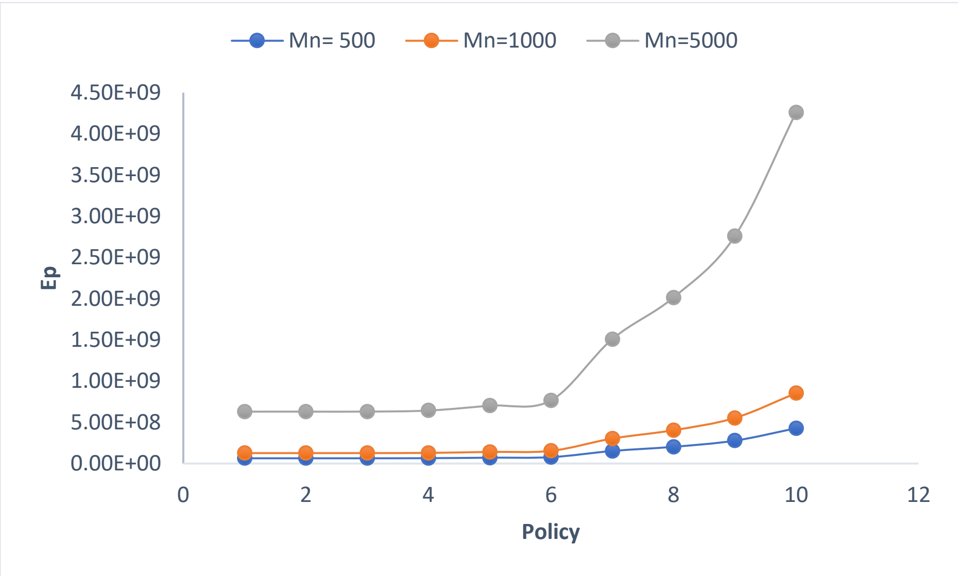

Figure 6-3: The expected long-run cost with Δt = 100 is plotted for different values of *Mn*= (500, 1000 and 5000).

6.2.3 Changing of parameter p.

Here the concavity parameter p is changed, keeping other parameters constant. Figure 6-4 shows that increasing and decreasing in p does not have any major changes in the long run cost.

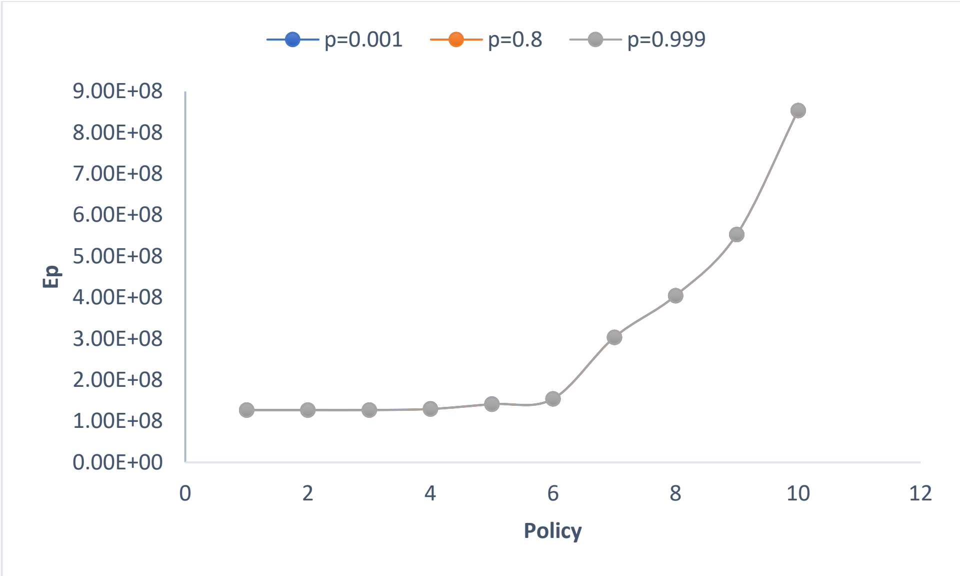

Figure 6-4: The expected long-run cost with Δt = 100 is plotted for different values of p= (0.001, 0.8 and 0.009).

6.2.4 Changing of parameter q.

Increasing and decreasing of convexity parameter q, has clear effect in the long run cost as presented in figure 6-5. Long run cost increases with a decrease in q, and the opposite result is observed with a higher value of q. The variation is more at higher policies due to decline in performance loss cost.

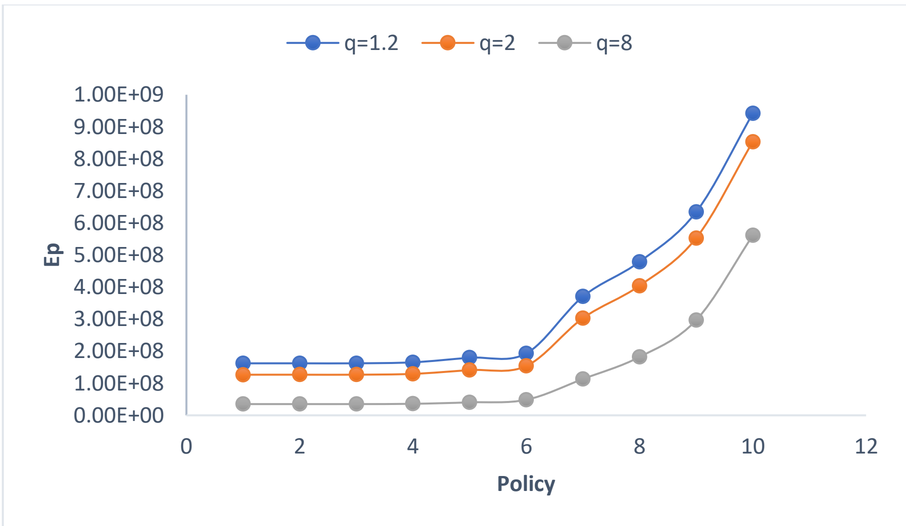

Figure 6-5: The expected long-run cost with Δt = 100 is plotted for different values of q= (1.2, 2, and 8).

6.2.5 Changing probability of failure.

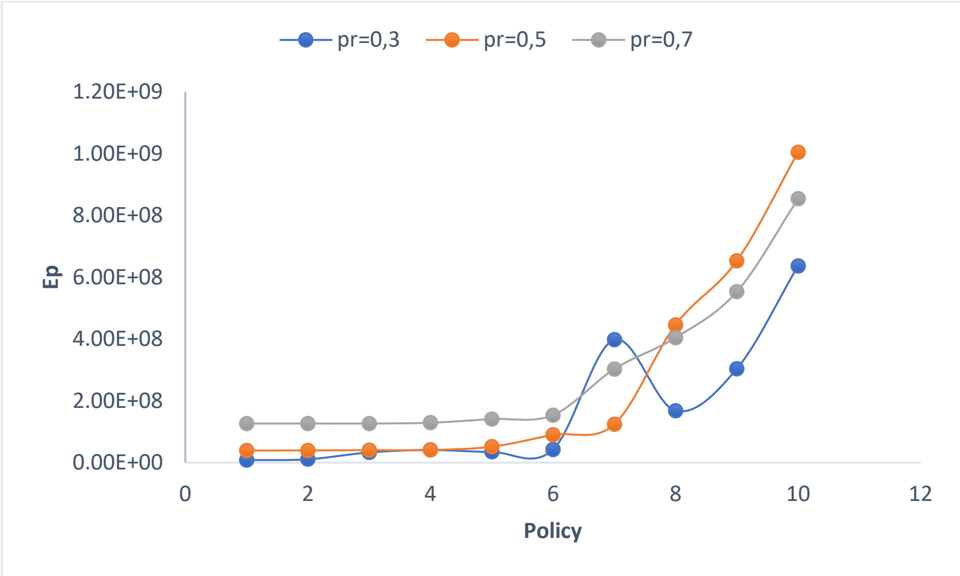

Figure 6-6: The expected long-run cost with Δt = 100 is plotted for different failure probability (0.3, 0.5, and 0.7).

Finally, we change the failure probability to observe the changes in the long run cost. Figure 6-6 shows that the long-run cost is less with good quality of sensor nodes (at low failure probability), and the cost increases with higher failure probability. At higher failure probability, the cost is higher due to more failure of sensor nodes occurring within less interval of time. The variation of Ep with failure probability is less significant until P6; the effect is more meaningful after P6.

Sensitivity analysis concludes that the expected long-run cost increases with the increasing failed nodes. At the lower policies, the long run cost is minimum and follow the same trend until the policy P6, and the cost is more significant with higher policies.

Chapter 7

Conclusion and future works

This chapter presents the essential discussions and conclusions about all the task done in this thesis. Finally, future recommendations are presented.

7.1 Discussion and conclusion

This thesis has performed an in-depth literature survey to check the degradation phenomena of LTC batteries. First, the relevant study of battery technology is conducted to know much about the performance characteristics of the battery, including discharge phenomena and lifetime consideration of the battery. Through literature, it can be concluded that the operating temperature and storage temperature profoundly influence the lifetime of the battery. The temperature effect follows the Arrhenius law, which describes the behavior of the battery at a different temperature. At higher temperature, the ratio $\frac{CAP_{act}}{CAP_{ini}}$ (the ratio of actual capacity to the initial capacity of the battery) shows a higher discharge rate than at lower temperature. Similarly, the shelf life of a battery at room temperature shows a loss of 2 % capacity per year while at a higher temperature at 70°C, it accounts for 5 % capacity loss per year.

The second part of a literature study is conducted for maintenance of the failed battery in a WSN. Most of the found literature is limited for rechargeable batteries and not for the LTC battery which we are searching. One study shows that battery replacement and sensor replacement is identically the same since sensor nodes are cheaper. Therefore, a literature survey for this thesis is shifted from the idea of battery replacement to sensor replacement as a part of the maintenance operation. Many types of research have been conducted on past to optimize and improve wireless sensor networks lifetime, but almost all the research is limited only to optimize threshold coverage area and energy efficient methods. Hence, after further exploration, three relevant studies regarding the replacement of sensor nodes is found and is summarised in this project.

The first article provides insight into the optimal replacement policy of degraded nodes as a part of a maintenance strategy. The researcher adopts a probabilistic approach and establish the mathematical model by using the Markov decision process to solve the maintenance problem. The second articles proposed four different replacement strategies with a mathematical model. The last article describes the homogenous and heterogeneous nature of wireless sensor networks and performs a maintenance action by introducing a multi-cluster management technique to replace the failed node. Thus, the availability of only three research articles shows that this area is not explored well and need further research.

The third part of this thesis begins with an intention to develop a probabilistic model that provides the optimal policy for a maintenance operation, as a part of maintenance strategies. The reliable data source is rare because the nature of the problem is new, and due to the limited accessibility of enough data and information, the establishment of the physical model seems to be tough. That is the reason we decided to move towards developing a probabilistic model. A general probabilistic model is established to find the expected long-run cost for different policies. Markov decision process is used for this purpose. Two costs are considered to develop a model. The first cost includes the failed node replacement cost (Hardware and personnel cost) and second cost accounts for performance loss in WSN due to the failed nodes. Replacement cost function and performance loss cost function are defined, and the concavity and convexity nature of these functions are verified with clear assumptions. Further transition probability matrix is generated and defined with the help of Markov chains. In addition, with the help of the cost functions and transition probability matrix, a model is developed to estimate the expected long-run cost. The systematic procedure is established to determine the optimal policy.

For analysis of this model, some inputs and experimental data is required. Since we do not have any specific information from the company, it is decided to verify this model by setting up a hypothetical wireless sensor network comprised of 10 identical sensor nodes. A qualitative study is performed to calculate the expected long-run cost for all policies that exist in WSN. The results obtained from the study concludes that it is appropriate to do the maintenance operation only after six failure nodes in a wireless sensor network. Therefore, P6 is the optimal policy that gives the minimum cost without comprising much with the performance of the network.

Finally, the sensitivity analysis is performed to examine how the long-run cost is changed when other parameters; replacement cost (*Cn*), performance loss cost (*Mn*), failure probability, and concavity and convexity scale parameters (p and q) are changed. There is no significant change observed with the variation of the parameters except performance loss cost (*Mn*) and convexity parameter q. *Ep* is increasing with the decreasing value of q, whereas *Ep* is rising with the increasing value of *Mn.*

7.2 Recommendations for future work

This thesis focused on finding the replacement strategies for failed battery in wireless sensor nodes. The developed empirical model is based on many general assumptions that are not enough, and further research is necessary. Hence, this project will be continued, and the opportunity for additional work lies within the following activities:

- It would be more interesting to choose the network for either one specific purposes for example used for noise measurement or gas detection, Where the network should be considered as working state, although 50 % of its sensor nodes shows failure.

- Where wireless sensor networks (WSNs) depends upon the critical nodes, the increased cost due to Performance loss of network depends upon critical nodes. Failure of only this critical node shows the complete network failure.
- The present model is based on the fixed (time independent) transition probability. The replacement cost and strategy could be different with a time-dependent probability matrix.
- The present strategy is based on only two states either working or fail, but a replacement could be performed not only when a node fails, but also when remaining useful life (RUL) is small, provided there is an opportunity for the third state. The three states will be now working or degraded or failed, respectively. The replacement strategy and expected long-run cost will be more realistic in this case.

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Appendix

Appendix A

Matlab code for long-run cost.

For policy 1:

```
%define Cp
Cp = [0;50.12;100.16;150.18;200.18;250.17;300.15;350.12;400.09;450.05;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=  [5.9e-06  0.121  0.233  0.267  0.2  0.103  0.0368  0.009  0.00145  0.000138
0.0282;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1
0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0
0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0
0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 2:

```
%define Cp
Cp = Cp =
[0;0;100.16;150.18;200.18;250.17;300.15;350.12;400.09;450.05;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp =[5.9e-06  0.121  0.233  0.267  0.20  0.103  0.0368  0.009  0.00145  0.000138
0.0282;0  1.97e-05  0.156  0.267  0.267  0.172  0.0735  0.021  0.00386  0.000413
0.0404;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1
0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0
0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
```

```
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 3:

```
%define Cp
Cp=[0;0;0;150.18;200.18;250.17;300.15;350.12;400.09;450.05;500
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0
0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0
0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 4:

```
%define Cp
Cp =
[0;0;0;0;200.180000000000;250.170000000000;300.150000000000;350.12000000000
0;400.090000000000;450.050000000000;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
```

```
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;1 0 0 0 0 0 0 0 0 0

0;1  0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0
0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 5:

```
%define Cp
Cp =
[0;0;0;0;0;250.170000000000;300.150000000000;350.120000000000;400.090000000
000;450.050000000000;500];
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;1 0
0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0
0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 6:

```
%define Cp
```

```
Cp =
[0;0;0;0;0;0;300.150000000000;350.120000000000;400.090000000000;450.0500000
00000;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];

%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;0 0
0 0 0 0.00243000000000000 0.0284000000000000 0.132000000000000
0.309000000000000 0.360000000000000 0.168000000000000;1 0 0 0 0 0 0 0 0 0
0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0
0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 7:

```
define Cp
Cp =
[0;0;0;0;0;0;0;350.120000000000;400.090000000000;450.050000000000;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
```

```
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;0 0
0 0 0 0.00243000000000000 0.0284000000000000 0.132000000000000

0.309000000000000 0.360000000000000 0.168000000000000;0 0 0 0 0 0
0.00810000000000000 0.0756000000000000 0.265000000000000 0.412000000000000
0.240000000000000;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0
0 0 0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 8:

```
%define Cp
Cp = [0;0;0;0;0;0;0;0;400.090000000000;450.050000000000;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;0 0
0 0 0 0.00243000000000000 0.0284000000000000 0.132000000000000
0.309000000000000 0.360000000000000 0.168000000000000;0 0 0 0 0 0
0.00810000000000000 0.0756000000000000 0.265000000000000 0.412000000000000
0.240000000000000;0 0 0 0 0 0 0 0.0270000000000000 0.189000000000000
0.441000000000000 0.343000000000000;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0
0 0;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
```

```
Ep = Firstpart*Secondpart
```

For policy 9:

```
%define Cp

Cp = [0;0;0;0;0;0;0;0;0;450.050000000000;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000
0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;0 0
0 0 0 0.00243000000000000 0.0284000000000000 0.132000000000000
0.309000000000000 0.360000000000000 0.168000000000000;0 0 0 0 0 0
0.00810000000000000 0.0756000000000000 0.265000000000000 0.412000000000000
0.240000000000000;0 0 0 0 0 0 0 0.0270000000000000 0.189000000000000
0.441000000000000 0.343000000000000;0 0 0 0 0 0 0 0 0.0900000000000000
0.420000000000000 0.490000000000000;1 0 0 0 0 0 0 0 0 0 0;1 0 0 0 0 0 0 0 0
0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 10:

```
%define Cp
Cp = [0;0;0;0;0;0;0;0;0;0;500];
%define M
M = [0;10;40;90;160;250;360;490;640;810;1000];
%define delta_t
delta_t = 100;
% define pi
pi=[1 0 0 0 0 0 0 0 0 0 0];
%define Tp
Tp=[5.90000000000000e-06 0.000138000000000000 0.00145000000000000
0.00900000000000000 0.0368000000000000 0.103000000000000 0.200000000000000
0.267000000000000 0.233000000000000 0.121000000000000 0.0282000000000000;0
```

```
1.97000000000000e-05 0.000413000000000000 0.00386000000000000
0.0210000000000000 0.0735000000000000 0.172000000000000 0.267000000000000
0.267000000000000 0.156000000000000 0.0404000000000000;0 0
6.56000000000000e-05 0.00122000000000000 0.0100000000000000
0.0467000000000000 0.136000000000000 0.254000000000000 0.296000000000000
0.198000000000000 0.0576000000000000;0 0 0 0.000219000000000000

0.00357000000000000 0.0250000000000000 0.0972000000000000 0.227000000000000
0.318000000000000 0.247000000000000 0.0824000000000000;0 0 0 0
0.000729000000000000 0.0102000000000000 0.0595000000000000
0.185000000000000 0.324000000000000 0.303000000000000 0.118000000000000;0 0
0 0 0 0.00243000000000000 0.0284000000000000 0.132000000000000
0.309000000000000 0.360000000000000 0.168000000000000;0 0 0 0 0 0
0.00810000000000000 0.0756000000000000 0.265000000000000 0.412000000000000
0.240000000000000;0 0 0 0 0 0 0 0.0270000000000000 0.189000000000000
0.441000000000000 0.343000000000000;0 0 0 0 0 0 0 0 0.0900000000000000
0.420000000000000 0.490000000000000;0 0 0 0 0 0 0 0 0 0.300000000000000
0.700000000000000;1 0 0 0 0 0 0 0 0 0 0];
I=eye(11);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

Appendix B

Long run calculation for two sensor nodes

Question 2:

Let's assume the WSN with two sensor nodes A and B. The given Values are same as question one. Calculate the Long run Cost.

- Δ t= 5 hours (time of interval between two successive inspections)
- Probability of failure Pr (F_A) =0.3 = Pr (F_B)
- Mission Time (T) = 10000 hours
- Replacement Cost (C)= 500 Nok
- Performance loss cost per hour (M)= 1000 Nok/hour
- p = 0.8 and q= 2

Solution:

There are two sensor nodes in WSN. So at each inspection, either one node fails or both.

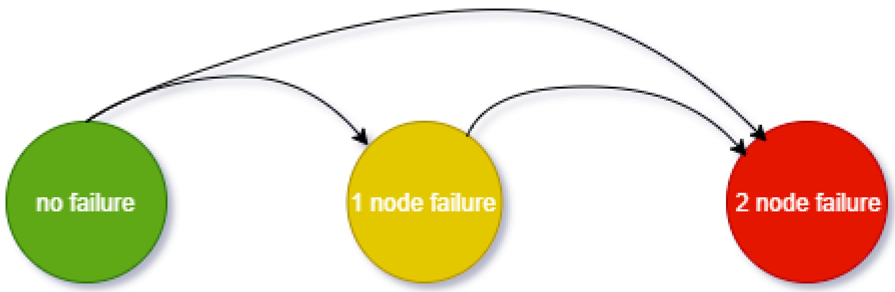

Figure 7: Markov Chains for 2 sensor nodes.

The transition matrix is:

$$T = \begin{matrix} t00 & t01 & t02 \\ 0 & t11 & t12 \\ 0 & 0 & t22 \end{matrix}$$

Now we have to find the states probability,

The probability of failure for both sensor is:

Pr (F_A) =0.3 = Pr (F_B)

And the working probability for both sensor is:

Pr (W_A) =1- Pr (F_A) = 1-0.3 = 0.7 = Pr (W_B)

Now, the state transition probability t00 means both nodes are working. For independent nodes,

$t00$ = Pr (W_A)* Pr (W_B) = 0.7*0.7= 0.49

Similarly, the state transition probability $t01$ means one sensor nodes in the WSN failed. It reaches from state 0 to state 1 during inspection.

$t01$ = Pr (W_A) * Pr (F_B) + Pr (W_B)* Pr (F_A) = 0.7*0.3+0.7*0.3=0.42

$t02$ is the transition probability from state 0 to state 2, which means both sensor nodes in the WSN failed.

$t02$ =0.3*0.3= 0.09

$t11$ is the transition probability to remain in state 1. Here at state 1, one of the sensor is already failed. So only one out of two is working and the probability to remain in same state is the working probability of that sensor which is:

$t11 = 0.7$

$t12$ is the probability of moving from state 1 to state 2.

$t12 = 0.3$

$$t22 = 1$$

Now,

$$T = \begin{matrix} 0.49 & 0.42 & 0.09 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{matrix}$$

To calculate the expected long run cost under policy P, let's define the policy first. Since there are two nodes are used, two policy exists.

Policy P (1): When one or more nodes fails i >= 1. Failed node could be either Node A or Node B.

Second policy P (2): When both nodes fails i>=2.

The long run cost for policy one is:

$$E_p = \pi^{(0)} * [I - T_P]^{-1} * [\,\Delta\,\mathrm{t} * M + C_P]$$

Where,

- $\pi^{(0)}$ is the initial state probability. For 3x3 Matrix, it is equal to

$$\pi^{(0)} = [1 \qquad 0 \qquad 0]$$

- $[I - T_P]^{-1}$ is square Matrix.

Now let's define Tp.

Tp is a square matrix under certain policy. If we have 'n' number of total nodes and 'i' number of failed nodes in WSN. The method to obtain Tp from transition matrix T under policy P is consists of three matrices:

1. Rectangle matrix of order i x (n+1)
2. Column vector of n+1-i elements, all of which are 1's and
3. Null matrix of order (n+1-i) x n.

Now, let's assign the value of Tp for Policy 1.

Step 1: under policy 1, the number of failed nodes is 1. So i =1 and n=2. So rectangular matrix i x (n+1) =1 x (2+1) = 1 x 3 matrix.

$$Tp = [0.49 \quad 0.42 \quad 0.09]$$

Step 2: column vector of n+1-i elements. So it is 2+1-1= 2 elements. So we change the first two elements of second and third row with 1.

$$Tp = \begin{matrix} 0.49 & 0.42 & 0.09 \\ 1 & 0.7 & 0.3 \\ 1 & 0 & 1 \end{matrix}$$

Step 3: Null matrix of order (n+1-i) x n. which is (2+1-1) x 2= 2 x 2. Now we replace the last two row of second column and third column with 0.

$$Tp = \begin{matrix} 0.49 & 0.42 & 0.09 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$$

Now, $[I - T_P]^{-1}$ can be obtained by calculating the inverse of the square matrix by using Gauss Jordan Elimination method.

$$[I - T_P] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.49 & 0.42 & 0.09 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{matrix} 0.51 & -0.42 & -0.09 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{matrix}$$

Now,

$$[I - T_P]^{-1} = \begin{bmatrix} 3{,}60287970189640\text{e+}16 & 1{,}51320947479649\text{e+}16 & 3{,}24259173170676\text{e+}15 \\ 3{,}60287970189640\text{e+}16 & 1{,}51320947479649\text{e+}16 & 3{,}24259173170676\text{e+}15 \\ 3{,}60287970189640\text{e+}16 & 1{,}51320947479649\text{e+}16 & 3{,}24259173170676\text{e+}15 \end{bmatrix}$$

Similarly for policy 2, we have

Step 1: under policy 2, the number of failed nodes is 2. So i =2 and n=2. So rectangular matrix i x (n+1) =2 x (2+1) = 2 x 3 matrix.

$$Tp = [\begin{matrix} 0.49 & 0.42 & 0.09 \\ 0 & 0.7 & 0.3 \end{matrix}]$$

Step 2: Column vector of n+1-i elements. So it is 2+1-2= 1 elements. So we change the first elements of third row with 1.

$$Tp = \begin{matrix} 0.49 & 0.42 & 0.09 \\ 0 & 0.7 & 0.3 \\ 1 & 0 & 1 \end{matrix}$$

Step 3: Null matrix of order (n+1-i) x n. which is (2+1-2) x 2= 1 x 2. Now we replace the two elements of last row of second column and third column with 0.

$$Tp = \begin{matrix} 0.49 & 0.42 & 0.09 \\ 0 & 0.7 & 0.3 \\ 1 & 0 & 0 \end{matrix}$$

Now,

$$[I - T_P]^{-1} = \begin{bmatrix} 7{,}68532359619539e{+}15 & 1{,}07594530346735e{+}16 & 3{,}91951503405965e{+}15 \\ 7{,}68532359619539e{+}15 & 1{,}07594530346735e{+}16 & 3{,}91951503405965e{+}15 \\ 7{,}68532359619539e{+}15 & 1{,}07594530346735e{+}16 & 3{,}91951503405965e{+}15 \end{bmatrix}$$

- $\Delta \mathrm{t} * M + C_P$, it's a column vector

C_P is the cost under policy P. C_P can be determined by using the below cost function

$$C(i) = C_n (\frac{i}{n})^p$$

For i=1,

C(1)= 500*(1/2) ^0.8 = 287,17 Nok

For i=2,

C (2) = 500 Nok

So for policy 1, we have 1 sensor node fails. In this case

$$C_{P1} = \begin{bmatrix} C(0) \\ C(1) \\ C(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 287{,}17 \\ 500 \end{bmatrix}$$

Similarly, for policy 2, two nodes fails. In this case

So

$$C_{P2} = \begin{bmatrix} 0 \\ 0 \\ 500 \end{bmatrix}$$

$\Delta \mathrm{t} * M$, here M can be determined by using the function

$$M(i) = M_n \left(\frac{i}{n}\right)^q$$

For i=1,

$M(1) = 1000(1/2)$^2 = 250 Nok and

For i=2,

$M(2) = 1000\ Nok$

$$M = \begin{bmatrix} 0 \\ 250 \\ 1000 \end{bmatrix}$$

$$\Delta\,\mathrm{t} * M = \begin{bmatrix} 0 \\ 250 * 5 = 1250 \\ 1000 * 5 = 5000 \end{bmatrix}$$

Now,

Long run cost for policy 1,

$$E_p = \pi^{(0)} * [I - T_P]^{-1} * [\,\Delta\,\mathrm{t} * M + C_{P1}]$$

$$= [1 \quad 0 \quad 0] * \begin{bmatrix} 7{,}68532359619539\text{e+}15 & 1{,}07594530346735\text{e+}16 & 3{,}91951503405965\text{e+}15 \\ 7{,}68532359619539\text{e+}15 & 1{,}07594530346735\text{e+}16 & 3{,}91951503405965\text{e+}15 \\ 7{,}68532359619539\text{e+}15 & 1{,}07594530346735\text{e+}16 & 3{,}91951503405965\text{e+}15 \end{bmatrix} * \begin{bmatrix} 0 \\ 1250 + 287{,}17 = 1537{,}17 \\ 5000 + 500 = 5500 \end{bmatrix}$$

= 4.1095e+19 Nok

Long run cost for policy 2,

$$E_p = \pi^{(0)} * [I - T_P]^{-1} * [\,\Delta\,\mathrm{t} * M + C_{P2}]$$

= 3.5007e+19 Nok

Policy 2 is adopted for the optimal policy, since the long-run cost for the second policy is minimum. Both the calculation is performed in Matlab.

Matlab Code:

For policy 1,

```
%define Cp
Cp = [0;287.17;500];
%define M
M = [0;250;1000];
%define delta_t
delta_t = 5;
% define pi
pi= [1 0 0];
%define Tp
Tp=[0.49 0.42 0.09; 1 0 0; 1 0 0];
I=eye(3);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```

For policy 2,

```
%define Cp
```

```matlab
Cp = [0;0;500];
%define M
M = [0;250;1000];
%define delta_t
delta_t = 5;
% define pi
pi=[1 0 0];
%define Tp
Tp=[0.49 0.42 0.09; 0 0.7 0.3; 1 0 0];
I=eye(3);
%calculation
A = I - Tp;
Inv_A = inv(A);
% pi(INV of A)
Firstpart = pi*Inv_A;
% second part
Secondpart = delta_t*M + Cp;
% Ep calaculation
Ep = Firstpart*Secondpart
```