Visual Abstract

The Strain energy density to estimate lifetime of notched components subjected to variable amplitude fatigue loading

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ABSTRACT. In the present paper, the approach based on the strain energy density (SED) averaged over a structural volume is reformulated to estimate the lifetime of notched components subjected to variable amplitude (VA) uniaxial fatigue loading. The accuracy and reliability of the proposed reformulation of the SED approach was checked against a large number of data taken from the literature and generated, under two different load spectra, by testing specimens of carbon steel C40 containing notches of different sharpness. Such a validation exercise allowed us to demonstrate that the extension of the SED approach as proposed in the present paper is capable of accurately estimating fatigue damage in notched components subjected to in-service VA fatigue loading.

KEYWORDS. Strain energy density; Notch; Variable amplitude loading.

OPEN ACCESS

Citation: Hu Z, Berto F, Susmel L, The Strain energy density to estimate lifetime of notched components subjected to variable amplitude fatigue loading, Frattura ed Integrità Strutturale, 47 (2019) 383-393.

Received: 15.11.2018
Accepted: 04.12.2018
Published: 01.01.2019

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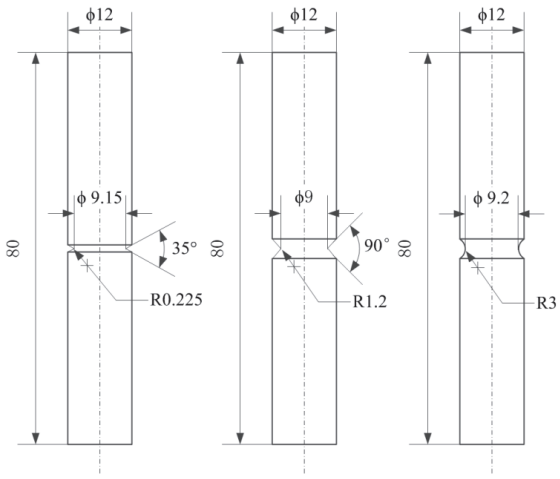
INTRODUCTION

Most components contain material defects or geometric discontinuities. Examination of the state of the art demonstrate that the effect of stress concentrators on the fatigue strength of engineering materials has been investigated by considering mainly constant amplitude (CA) fatigue situations. However, most components experience variable amplitude (VA) fatigue load histories which make the characteristics of the critical local stress/strain states more complex, with this affecting markedly fatigue strength.

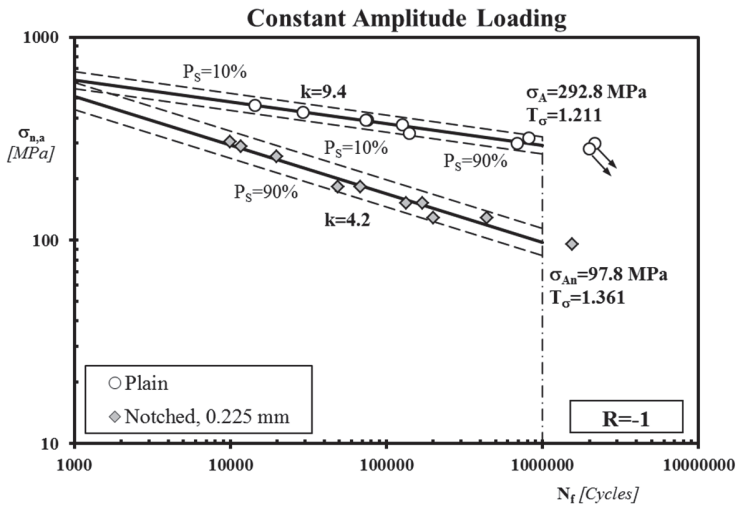
In order to predict in-service lifetime of notched components under VA loading, it is important to make use of specific methods suitable for taking into account the way fatigue damage accumulates itself. Palmgren [1] and Miner [2] first formulated this concept in a mathematical form as:

$$D = \Sigma(n_i \,/\, N_{\mathrm{f},i}) \tag{1}$$

where D denotes the cumulative damage sum, and n_i and $N_{\mathrm{f},i}$ are the applied cycles and the total number of cycles to failure under the ith constant amplitude stress level, respectively. When engineering metallic materials are damaged by VA load histories, failure is supposed to occur as soon as the critical value of cumulative damage sum D_{cr} is equal to unity. However, experimental evidence under VA loading condition indicates that in situations of practical interest D_{cr} varies in the range 0.02-5 [3, 4]. Clearly, this fact can produce a remarkable discrepancy between estimated and actual lifetime, with this calculation resulting in non-conservative predictions.



(a)



(b)

Figure 1: (a) Geometries of the tested notched samples (dimensions in millimetres). (b) Fatigue results generated under CA uniaxial fatigue loading [3].

Fatigue life estimation techniques based on local approaches require some detailed information about the stress-strain state around the notch tip [3, 5-9]. Different SED-based approaches to deal with notch fatigue problems have been formulated since the beginning of the last century. Recently, Lazzarin et al. [8, 10, 11] developed a volume-based approach, in which the SED calculations are carried out in a material-related control volume. The averaged SED approach

is based both on a precise definition of the control volume and on the fact that the critical energy does not depend on the notch sharpness. The control radius R_0 of the volume is a material property, which involves the plain specimen's fatigue limit and the threshold stress intensity factor range [10]. The approach was successfully used under both static and CA fatigue loading conditions to assess the strength of notched components subjected to uniaxial [12-14] as well as to multiaxial loading [15-19].
Even if a lot of theoretical and experimental work has been carried out to check the accuracy and reliability of the SED approach, surprisingly, so far no systematic attempt has been made to extend the use of this powerful method to those situations involving VA fatigue loading. Accordingly, the ultimate goal of the research work summarized in the present paper was to reformulate the SED approach to make it suitable for assessing fatigue lifetime of notched components subjected to VA uniaxial fatigue load histories, with this being done by using a suitable cycle counting method (i.e., the Rain-Flow method [20]) as well as a suitable cumulative damage model (i.e., Palmgren and Miner's rule [1, 2]).

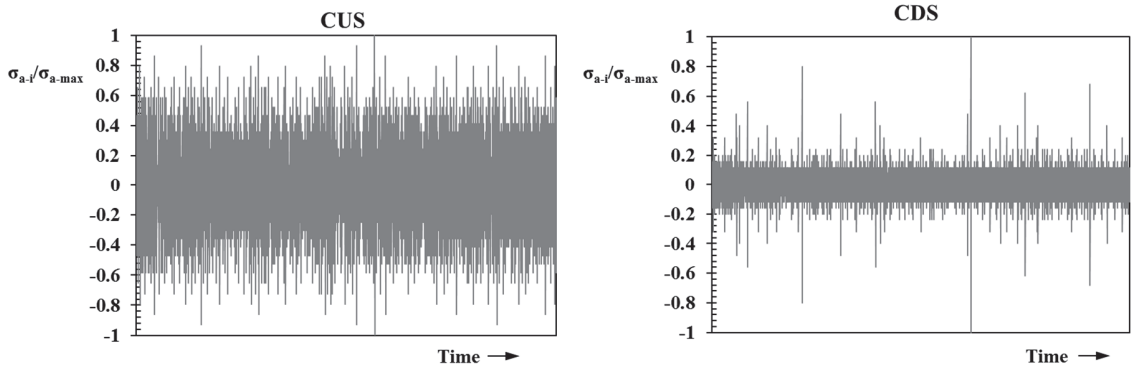
EXPERIMENT DETAILS

In a recent work, Susmel and Taylor [3] tested under uniaxial VA fatigue loading circumferentially notched specimens made of a medium-carbon steel C40 and containing three different geometrical features. The geometry of the tested specimens is reported in Fig. 1a along with details of the notches that were characterized by three different values of the notch tip radius and opening angle. Static tensile tests were performed in order to evaluate the mechanical properties according to the ASTM standard procedure. The yield stress, tensile stress and Young's modulus of the investigated materials were 672 MPa, 852 MPa and 209000 MPa, respectively. The net stress concentration factor, K_t, was calculated by solving standard linear-elastic Finite Element (FE) models where the mesh density in the notch region was gradually increased until convergence occurred. This standard numerical procedure returned a K_t value under tension equal to 4.42, 2.2 and 1.66 for the specimens with a root radius of 0.225, 1.2 and 3 mm, respectively.
Both CA and VA fatigue tests were carried out in axial load control under a nominal load ratio, $R=\sigma_{min}/\sigma_{max}$, equal to -1 at a frequency of 4 Hz. All the CA fatigue results are summarised in the Wöhler diagrams of Fig. 1b in terms of nominal stress amplitude evaluated onto the net cross-sectional area of the specimens. Fig. 1b shows the fatigue data generated by testing both the un-notched specimens and the V-notched samples with root radius equal to 0.225 mm (K_t = 4.42). The statistical analyses have been performed assuming a log-normal distribution of the number of cycles to failure for each stress amplitude. All data obtained from specimens characterized by a fatigue life between 10^4 and 2×10^6 have been taken into account in the statistical analyses. It is worth observing here that the scatter bands plotted in the chart of Fig. 1b were calculated for each stress level by assuming a confidence level equal to 95%. The results from the statistical re-analysis are summarised in Tab. 1 in terms of: nominal stress amplitudes (σ_A for the plain specimens and σ_{An} for the notched samples) for a probability of survival P_S =50% at a reference number of cycles to failure, N_A, equal to 10^6; inverse slope, k, of the Wöhler curves and the scatter index T_σ that provides the width of the scatter band between the curves with a probability of survival of 10% and 90% respectively (with a confidence level equal to 95%).

Specimen Type	r_n(*mm*)	d_g(*mm*)	d_n(*mm*)	2α(°)	k	σ_A, σ_{An}* (*MPa*)	T_σ	K_t
Plain	-	12	6	-	9.4	292.8	1.211	1.0
Sharp	0.225	12	9.15	35	4.2	97.8	1.361	4.42

Table 1: Summary of the experimental results generated under fully-reversed CA loading [3].

As to the performed VA tests, the specimens were tested under the two different random spectra (Fig. 2) having sequence length, $n_{tot} = \Sigma n_i$, equal to 1000 cycles. All the random spectra were repeated until failure of the specimens occurred. In particular, the Concave Upwards Spectrum (CUS) was derived from a conventional Rayleigh distribution, whereas the Concave Downwards Spectrum (CDS) was defined so that we could have few cycles with high stress ranges combined with a relatively large number of cycles with low stress ranges [21]. The aim of two different spectra of tests was to determine the effect of spectrum shape on the fatigue lifetime of the notched material being investigated and to determinate the fatigue lifetime under random block loading.
All the results generated by Susmel and Taylor [3] are summarized in Tabs. 2-4, where the performed tests are described in terms of adopted spectrum, maximum force, $F_{a,max}$, maximum amplitude in the spectrum of the axial, $\sigma_{a,max}$, load ratio, R, and experimental value of the number of cycles to failure, N_f. For comparison, Tab. 5 shows the fatigue results generated by testing plain samples under the two load spectra. Tab. 5 also reports the corresponding experimental values of the critical damage sum, $D_{cr,exp}$, calculated by using experimental CA plain Wöhler curve.



(a) (b)

Figure 2: Adopted load spectra (a) CUS and (b) CDS [3].

Code	Spectrum	$F_{a,max}$(kN)	$\sigma_{a,max}$(MPa)	R	N_f(Cycles)
S2_VA	CUS	34.2	520.1	-1	20975
S3_VA	CUS	34.2	520.1	-1	21568
S4_VA	CUS	18.4	279.8	-1	170155
S6_VA	CUS	18.4	279.8	-1	182155
S7_VA	CUS	25.6	389.3	-1	66703
S9_VA	CUS	25.6	389.3	-1	61594
S10_VA	CDS	65.8	1000.7	-1	8620
S12_VA	CDS	49.3	749.7	-1	45620
S13_VA	CUS	14.4	219.0	-1	391420
S14_VA	CDS	65.8	1000.7	-1	7620
S15_VA	CDS	49.3	749.7	-1	42620
S16_VA	CDS	35.4	538.4	-1	179620
S17_VA	CUS	14.4	219.0	-1	339763
S19_VA	CDS	28.4	431.9	-1	367620
S20_VA	CDS	35.4	538.4	-1	172620
S21_VA	CDS	28.4	431.9	-1	376620

Table 2: Summary of the experimental results generated by testing the notched specimens with root radius equal to 0.225 mm under VA fatigue loading [3].

Code	Spectrum	$F_{a,max}$(kN)	$\sigma_{a,max}$(MPa)	R	N_f(Cycles)
I1_VA	CUS	35.5	520.4	-1	37568
I2_VA	CUS	29.5	432.4	-1	84463
I3_VA	CUS	23.5	344.5	-1	170581
I4_VA	CUS	35.5	520.4	-1	34044
I5_VA	CUS	29.5	432.4	-1	83663
I6_VA	CUS	23.5	344.5	-1	253927
I7_VA	CDS	65.4	958.6	-1	10620
I8_VA	CDS	48.5	710.9	-1	78620
I9_VA	CDS	65.4	958.6	-1	11905
I10_VA	CUS	20.5	300.5	-1	715859
I11_VA	CDS	31.0	454.4	-1	732620
I12_VA	CDS	31.0	454.4	-1	531620
I13_VA	CDS	37.0	542.4	-1	285277
I14_VA	CUS	20.5	300.5	-1	588009
I15_VA	CDS	48.5	710.9	-1	76620
I16_VA	CDS	37.0	542.4	-1	330620

Table 3: Summary of the experimental results generated by testing the notched specimens with root radius equal to 1.2 mm under VA fatigue loading [3].

Code	Spectrum	$F_{a,max}$(kN)	$\sigma_{a,max}$(MPa)	R	N_f(Cycles)
B1_VA	CUS	42.0	631.8	-1	18736
B2_VA	CUS	42.0	631.8	-1	15715
B3_VA	CUS	33.0	496.4	-1	65733
B4_VA	CUS	24.0	361.0	-1	274715
B5_VA	CUS	27.5	413.7	-1	118044
B6_VA	CUS	24.0	361.0	-1	300763
B7_VA	CUS	33.0	496.4	-1	63450
B8_VA	CUS	27.5	413.7	-1	138975
B9_VA	CDS	65.0	977.8	-1	13620
B10_VA	CDS	49.0	737.1	-1	85397
B11_VA	CDS	39.0	586.7	-1	352620
B12_VA	CDS	39.0	586.7	-1	300216
B13_VA	CDS	65.0	977.8	-1	12620
B14_VA	CDS	33.5	503.9	-1	665620
B15_VA	CDS	33.5	503.9	-1	749620
B16_VA	CDS	49.0	737.1	-1	78392

Table 4: Summary of the experimental results generated by testing the notched specimens with root radius equal to 3 mm under VA fatigue loading [3].

Spectrum	$\sigma_{a\text{-}max}$(MPa)	R	N_f(Cycles)	D_{exp}
CUS	728.6	-1	14763	0.59
CUS	728.6	-1	13525	0.54
CUS	594.2	-1	49450	0.29
CUS	512.8	-1	264594	0.38
CUS	512.8	-1	219086	0.32
CDS	891.3	-1	32620	1.31
CDS	735.6	-1	136620	0.91
CDS	618.9	-1	356620	0.47
CDS	891.3	-1	23620	0.95
CDS	618.9	-1	380085	0.50
			Average Value=	0.63

Table 5: Summary of the experimental results generated by testing the plain samples under fully-reversed VA loading and experimental values of the critical damage sum, D_{cr} [3].

THE SED TO ESTIMATE LIFETIME OF NOTCHED COMPONENTS UNDER CA FATIGUE LOADING

In recent years, the strain energy density (SED) has been widely used to estimate the fatigue lifetime of notched components subjected to CA fatigue loading [10, 12, 13, 22, 23].

The averaged SED approach states that failure occurs when the mean value of the strain energy density averaged over a control volume surrounding the notch tip is equal to a critical energy value. For rounded V-notches, the control volume is as depicted in Fig. 3, where its radius R_0 does not depend on the notch geometry. In more detail, R_0 is the depth of the crescent shape as measured along the notch bisector line. The outer radius of the control volume equals R_0+r_0, where r_0 is defined as a function of both the notch opening angle 2α and the notch root radius ρ, according to the following relationship:

$$r_0 = \rho \cdot \frac{\pi - 2\alpha}{2\pi - 2\alpha} \tag{2}$$

Dealing with the fatigue assessment of notched components, the control radius, R_0, can be defined by imposing the equality, with reference to the fatigue limit condition [24], between the averaged SED relevant to the plain specimens and that obtained from V-notched specimens with known geometry, i.e.:

$$\Delta W_0^{\text{plain}} = \frac{\sigma_{\text{A,p}}^2}{2E} = \Delta W^{\text{notch}}(R_0) \tag{3}$$

where $\sigma_{\text{A,p}}$ is the fatigue strength of the plain sample at N_A cycles and E is the elastic modulus. By varying the control radius R_0 around the notch tip in an FE model under the nominal stress $\sigma_{\text{A,n}}$, the corresponding values of strain energy density ΔW^{notch} averaged over the control volume can be determined. Then the numerical value $\Delta W^{\text{notch}}(R_0)$ as a function of the control radius R_0 is generated by a fitting equation. Finally, the control radius R_0 of the critical volume can be calculated since it is equal to the strain density energy of the plain sample, $\Delta W_0^{\text{plain}}$. Due to the absence of experiment data related to stress intensity factor, the control radius can be easily evaluated by the procedure above.

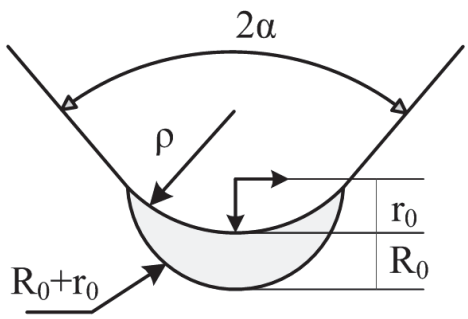


Figure 3: Definition of the control volume for specimens weakened by rounded V-notch.

The averaged SED can also easily and quickly be calculated from FE analyses by summing the energy contributions $W_{\text{element,i}}$ for all the finite elements within the control volume V, i.e.:

$$\Delta W = c_w \frac{\sum_V W_{\text{element},i}}{V} \tag{4}$$

Eq. (4) defines the so-called direct approach to evaluate the SED parameter. In order to consider the influence of the nominal load ratio R, the weighting parameter c_w has to be adopted by using the following expressions [25, 26]:

$$c_w(R) = \begin{cases} \dfrac{1+R^2}{(1-R)^2} & \text{for} \quad -\infty \le R < 0 \\ 1 & \text{for} \quad R = 0 \\ \dfrac{1-R^2}{(1-R)^2} & \text{for} \quad 0 \le R < 1 \end{cases} \tag{5}$$

Additionally, it is important to know that refined mesh in the control volume around the notch tip is not necessary to determine the value of the SED, because this parameter can be determined via the nodal displacements, without involving their derivatives. This means that the adopted FE meshes can be very coarse inside the control volume having radius R_0 when accurately evaluating the value of the local SED [27].

The assumption of the SED as a damage parameter allows to summarize a large number of fatigue data obtained for notched specimens in a reasonable scatter band [10, 23, 28]. Generally, it is important to understand the fatigue behaviour of notched components and to assess the fatigue strength without performing a large number of experiments. Therefore, in order to predict the fatigue life of components having different geometrical features, the SED, ΔW, versus the fatigue life, N_f, relationship is calculated by considering the notched samples as sharp as possible. The averaged SED, ΔW, can

be obtained from the post-processing results by applying the value of nominal stress in the FE models. The relationship of the SED, ΔW , and the fatigue life, N_f, can be written as:

$$\Delta W = A \cdot N_f^B \tag{6}$$

where A and B are material constants. Keeping constant the material and the nominal load ratio, the fatigue life can be assessed by using Eq. (6) for any geometrical configuration of the notch.

THE SED TO ESTIMATE LIFETIME OF NOTCHED COMPONENTS UNDER VA FATIGUE LOADING

In the presence of VA load histories, the value of the SED will change as the amplitudes of the applied load spectrum vary. This clearly implies that, to correctly extend the use of the SED to those situations involving VA loadings, the procedure of the SED has to be redefined coherently according to the specific features of the assessed load spectrum.

When the notched sample is subjected to the VA fatigue loading, the VA load spectrum can be counted by the Rain-Flow method [20]. Such a spectrum is formed by j different stress levels that are characterized by a nominal stress amplitude equal to $\sigma_{a\text{-}i}$ as shown in Fig. 4. The corresponding number of cycles is equal to $n_i(i=1,2,\ldots,j)$, where:

$$n_{tot} = \sum_{i=1}^{j} n_i \,.$$

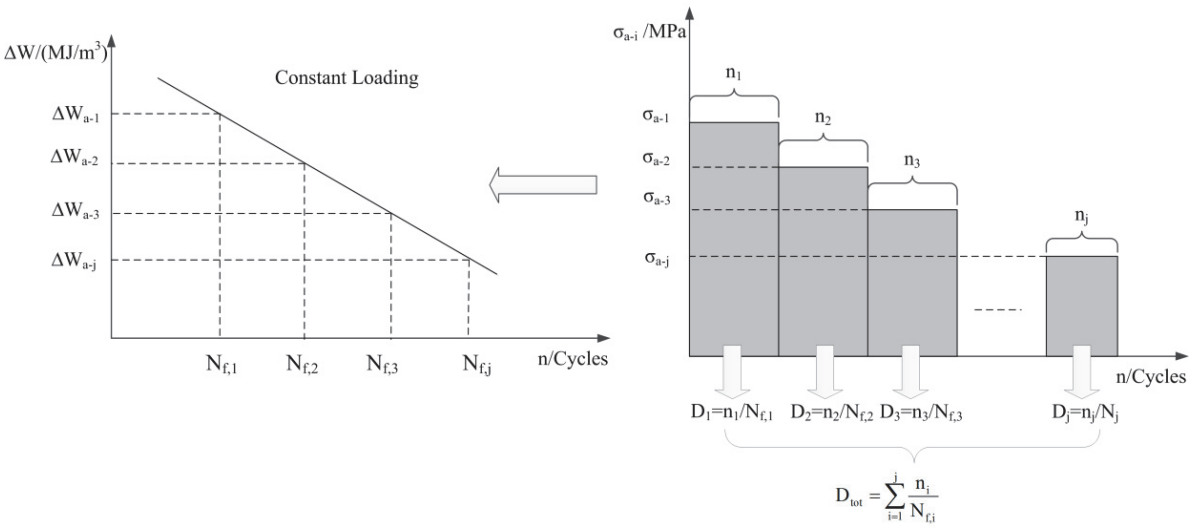


Figure 4: Estimation of the fatigue lifetime under VA loading by SED.

The different stress levels forming the above load spectrum can be treated as an independent CA loading, which can be applied in the FE model to obtain the value of the SED. Then, the corresponding number of cycles to failure $N_{f,i}$ can directly be determined according to the relation of SED ΔW versus the fatigue life N_f obtained by Eq. (6). As formalized by Palmgren and Miner [1,2], the fatigue damage content associated with any stress level can be calculated as follows:

$$D_i = \frac{n_i}{N_{f,i}} \tag{7}$$

The resulting total damage is equal to:

$$D_{\text{tot}} = \sum_{i=1}^{j} \frac{n_i}{N_{\text{f},i}} \tag{8}$$

Finally, the number of cycles to failure, $N_{\text{f,e}}$, can be estimated from the predicted number of blocks to failure, $N_{\text{b,e}}$, i.e.:

$$N_{\text{b,e}} = \frac{D_{\text{cr}}}{D_{\text{tot}}} \Rightarrow N_{\text{f,e}} = n_{\text{tot}} \cdot N_{\text{b,e}} \tag{9}$$

where D_{cr} is the critical value of the damage sum.

VALIDATION BY EXPERIMENTAL DATA

The data used to validate the proposed extension of the SED approach are those generated by Susmel and Taylor by testing the notched samples of C40 under both the Concave upwards Spectrum (CUS) and Concave Downwards Spectrum (CDS) [3]. As to the strain energy density analysis, the cylindrical samples were modelled by using axisymmetric bi-dimensional model with coarse meshes inside the volume.

By applying the net nominal stress σ_{An} at 10^6 cycles to the FE model, the numerical values of ΔW^{notch} as a function of the critical radius R_0 were obtained as shown in Fig. 5. It is observed that the value of the critical radius R_0 is about 0.11 mm when the value of the SED for the V-notched specimen equals to that of plain specimen, $\Delta W_0^{\text{plain}}$.

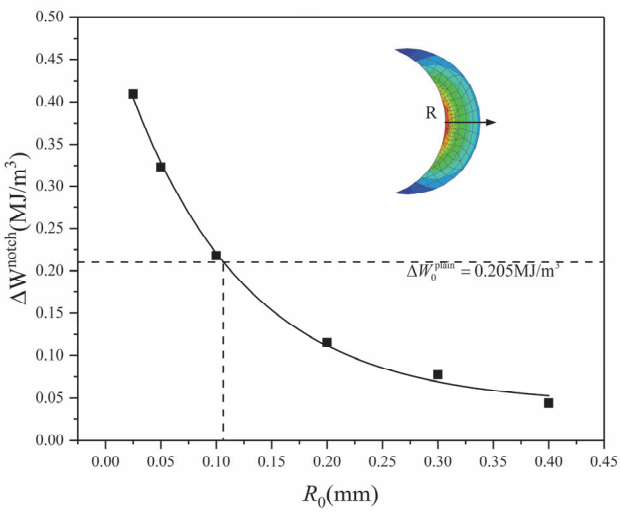


Figure 5: Numerical values of ΔW^{notch} as a function of the critical radius R_0.

The value of the SED averaged over the control volume was also calculated numerically by using the FE method. As shown in Fig. 4, a set of stress levels are acquired after calculating the load spectrum by Rain-Flow method. Then, the values of the SED of notched specimens with the sharpest radius are evaluated by applied above stress levels in the FE model. According to Eq. (6), the relationship of the SED $\Delta\overline{W}$ versus fatigue life N_{f} under CA fatigue loading is

$$W = 242.4 \times N_{\text{f}}^{-0.5266} \tag{10}$$

where the constants in Eq. (10) are determined by a best fit to a series of calculated data.

As to the critical value of the damage sum, D_{cr}, initially the hypothesis was formed that it could be taken equal to its average value experimentally determined from the plain samples, $D_{\text{cr,exp}}$=0.63. The obtained results with different notch

root radius are summarized in the experimental, N_f, vs. estimated, $N_{f,e}$, fatigue lifetime diagrams reported in Fig. 6a. The predictions with small notch radius made using the SED are seen to fall always within the parent material scatter band. For the medium notch radius, the predicted results mostly fall within the patent material scatter band. The predictions with large notch radius nearly fall outside of the parent material scatter band, and on the non-conservative side. In this case, the accuracy of the predicted results is higher as the notch becomes sharper. As shown in Fig. 6a, the novel formalization of the SED is seen to be capable of accurately assessing the fatigue lifetime of notched specimens under VA loading by adopting simple Palmgren-Miner rule. Using this approach can reduce the time and costs associated with the design process.

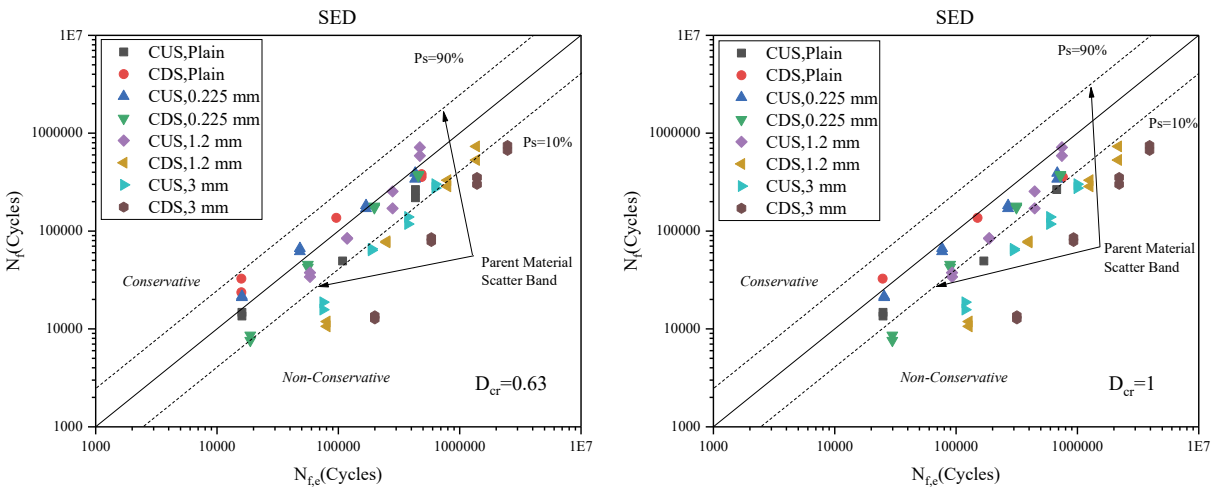


Figure 6: The accuracy of SED in estimating the fatigue lifetime of the notched samples under two critical value of the damage sum, D_{cr}. (a) D_{cr}=0.63. (b) D_{cr}=1.

The same analysis as above was performed also by taking, as suggested by the classical theory by Palmgren and Miner, a critical value for the damage sum equal to unity. The diagram reported in Fig. 6b shows that for the damage sum equal to unity, the level of non-conservatism of increases, resulting in poor accurate estimates. The predictions with small notch radius have the highest accurate estimates, but on the non-conservative side. Other predictions with medium and large notch radius mostly fall outside of the parent material scatter band and on the non-conservative side. It is also interesting to note that the estimated results under CUS are higher accuracy than the results under CDS, which shows that the loading shape has obvious effect on the fatigue behaviour. In the CDS, the cycles of low stress levels take up the majority of the sequence length, which would make the prediction a little inaccurate.

According to the experimental evidence (Tab. 5), the critical value D_{cr} of plain specimens varied as the shape of the load spectrum change. D_{cr} under the CUS spectrum is lower than the value of critical damage sum under the CDS spectrum. Such a high variability shows that the critical value of critical damage sum strongly affects the accuracy of predicted fatigue lifetime. So far, the only way to correctly obtained D_{cr} is by running enough experiments. Though many damage sum theories have also been developed [29-35], unfortunately, none of them enjoys universal acceptance due to the complexity of the problem, such as nonlinear damage, load sequence and small amplitude stress below fatigue limit. Consequently, the Palmgren-Miner rule is still dominantly used in design, in spite of its major shortcomings. More efforts on the cumulative damage are needed in order to provide design engineers with a general and reliable fatigue damage analysis and life prediction approach.

CONCLUSIONS

In the present work, the fatigue criterion based on the strain energy density (SED) has been applied to re-analyse some experimental results reported in the recent literature. The specimens being considered were characterized by three notch tip radius and tested under uniaxial loading. The relationship of the averaged SED and the number of cycles to failure was calculated by considering the sharper specimens. According to above the relationship and the Rain-Flow method, the novel formalization of the SED proposed was seen to be accurate in estimating fatigue lifetime of

notched components damaged by VA uniaxial fatigue loading. It has also been observed that the critical value of damage sum has obvious effects on the accuracy of estimated fatigue life.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support by the Innovation Program (237099000000170004) and the State Key Laboratory Program (614221305020617) and also by the China Scholarship Council (Grant No. 201705290009).

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