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Pricing weather derivatives, and constructing portfolios that reduce the weather risk for ski resorts

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Preface

This master's thesis is an end to a two-year Master's in Business and Administration, majoring in Finance and investments.

Many, if not all, industrial sectors are influenced by weather. After witnessing poor economic performances, or in some cases bankruptcy, due to adverse weather conditions, we decided to explore new ways for reducing risk. Further reading about weather derivatives, made us realize its growing importance, as the climate continuously changes. We firmly believe that companies, of all sizes, will face great challenges regarding changes in weather in the coming years, and that weather derivatives will be useful facing them.

We want to thank Denis Becker for introducing us to such an exciting topic as weather derivatives, and his help and insight during the last few months as well. We would also like to thank Voss fjellheisar AS for being helpful and providing us with financial data.

The master thesis is written as a collaboration between Alf Martin Haugan and Jon Berge Rasmussen. We take full responsibility for the content of this thesis.

Abstract

The weather changes continuously, and climate change affects businesses to an increasing degree. Many businesses' economic performance highly depends on the weather. Tourism and leisure are one of several industries exposed to weather risk. High exposure to weather risk may lead to financial stress, or in worst case scenario bankruptcy. Many consider weather risk as an uncontrollable factor. However, there is an increasing conviction in the market, that weather derivatives can turn weather risk into a more controllable factor. One of the many reasons why weather derivatives are not widely seen as a risk management tool is that weather derivatives are not well-known among possible end-users. These are the reasons why we want to showcase weather derivatives as a risk management tool, and also present the challenges associated.

We have narrowed our problem statement to only include ski resorts and their ability to hedge weather risk. This paper is going to focus on a real-life specific situation, Voss ski resort. However, the general purpose of the paper is to use a specific case to show that it is possible to apply weather derivates to reduce weather risk, and to show how it can be done.

The approach we have chosen is to price snow level put options and temperature call options by using different pricing methods and constructing portfolios combining the two different derivatives. We have used four different pricing methods, Historical burn analysis, ADS, Historical densities, and Edgeworth densities, and after comparing the results of them, decided to use the Edgeworth densities method when constructing portfolios. To create optimal portfolios for our cause, we have chosen the minimum variance principle.

Our study suggests that investing a small portion of the firm's capital in a risk management tool such as weather derivatives leads to a decrease in the firm's weather risk as a consequence of reduced variability in revenue. In some cases, when short-sales are possible, the expected revenue is higher than without an investment in weather derivatives. When short-sales are restricted, the expected revenue is less or equal to revenue without derivatives, which makes sense since weather derivatives are a zero-sum game. We hope our analysis will inspire firms exposed to weather risk to consider investing in weather derivatives or at least acquire more knowledge.

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Abbreviations

ADS	Alaton, Djehiche, and Stillberger
ARA	Absoulute risk aversion
B&S	Black & Scholes
CAL	Capital allocation line
CAPM	Capital asset pricing model
CAT	Cumulative average temperature
CAWS	Cumulative average wind speed
CDD	Cooling degree days
CE	Certainty equivalent
CME	Chicago mercantile exchange
CR	Cumulative rainfall
CSL	Cumulative snow level
E	Expectation
EUMETSAT	European organization for meteorological satellites
HBA	Historical Burn analysis
HDD	Heating degree days
ISO	Independent system operators
MRP	Market risk premium
NOAA	National Oceanic and Atmospheric Administration
OTC	Over the counter
OLS	Ordinary least squares
RRA	Relative risk aversion
SD	Standard deviation
SL	Snow level
SMBRP	Small medium business risk premium
VIF	Variance inflation factor

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1. Introduction

"*People probably think it is funny that we are talking about the weather, but* that is *the only thing we are talking about in the office. We live on the Yr application*" (Hopland, 2018).
These are the words of a former CEO of a Norwegian sports retailer. According to him, disadvantageous weather conditions were among reasons responsible for disappointing financial results in the first two quarters of 2018 (Nilsen, 2018). EUMETSAT, a global operational satellite agency that gathers data on weather, claims that over a third of the European economy is weather sensitive, and that several sectors such as energy, construction, and tourism require accurate forecasts (EUMETSAT, 2019). Similar estimations are found for the U.S. economy by NOAA (NOAA , 2019).

Weather derivatives are financial derivatives that can be used as a risk management tool by companies and industries against adverse weather conditions, or by other stakeholders hoping to yield a profit. This is opposed to *traditional* derivatives, which values depend on underlying assets such as stocks, commodities, and (market) indexes. A weather derivative's value depends on weather measurements such as precipitation, temperature, humidity, and wind. The difference between them is that these (underlying measurements) hold no value and cannot be stored or traded. They are quantified in indices and introduced as underlying assets (Alexandridis & Zapranis, 2013, p. 1).

The weather derivatives market emerged in the '90s as a result of the energy and utility industry deregulation. Competition grew, and demand was uncertain as monopolies faded. Stakeholders identified weather conditions as the primary source of revenues uncertainty due to the change in short-term demand and long-term supply of energy (Cao & Wei, 2003). To transfer the risk of adverse weather, Koch Energy and Enron made the first public transaction of weather derivatives in 1997. A simplified version is that Enron would pay Koch a specific amount for each degree the temperature fell below normal or vice versa (Perin, 1999). Rapid OTC marked expansions followed in Europe, Australia and Asia, and in September 1999 the first exchange-listed weather derivatives were presented by the CME - most of them temperature-based futures and options.

This master thesis aims to analyze whether and how weather derivatives can improve economic performance in terms of revenue and risk. This aim can be divided into two main parts. The first part looks at the differences and similarities of the existing pricing methods when applying them to the Norwegian ski resort market. The second part aims to show that even though the market for weather derivatives is often characterized by imperfections, uncertainties, and lack of knowledge, that it is possible to create a portfolio of weather derivatives that reduces the amount of risk for a company. To illustrate this, we will use Voss fjellheisar AS as a case example, hereby known as Voss ski resort. The way we conduct the case is to gather financial data from Voss ski resort and meteorological data in order to analyze the statistical relationship/correlation between weather and operating income, and hedge volumetric risk by using (constructed) weather derivatives as a tool of risk management

The thesis will have the following structure. Chapter 2 will contain existing literature including derivative usage motives, hedging pros and cons, characteristics of the weather derivative market (risk exposure) and recent studies of snow derivative usage for ski resorts. Chapter 3 presents data for the cumulative snow level, data for average monthly temperature and financial data for Voss ski resort. In the third chapter are also data description, descriptive statistics, and regression analysis emphasized. In chapter 4 are we discussing the methodology, and we will have a closer look at different pricing methods and difficulties in pricing weather derivatives. Additionally, we will explain portfolio theory. Finally, in chapter 5, two weather derivatives are constructed for Voss ski resort, and we will apply existing pricing methods on the constructed derivates to obtain their values and compare revenue with and without a risk management strategy. We will also assemble the derivatives into a portfolio by using the minimum variance principle, then illustrate the optimal portfolio with and without a shorting opportunity.

2. Existing Literature

2.1 Motives for using weather derivatives

(McDonald, 2013) lists four motives for using derivatives. The first motive is speculation; using a derivative to construct a bet that is highly levered and tailored to a specific view. Initial costs of placing bets can be relatively small to the potential gains or losses from the bets. The second motive states that in some cases, derivatives provide a less costly financial outcome compared to combining underlying assets and will lead to reduced transaction costs. As a third motive, he mentions the possibility to circumvent regulatory restrictions, taxes and accounting rules by trading derivatives, defined as regulatory arbitrage. The fourth and last motive he lists is risk management. For this thesis' purpose, we are more interested in weather derivatives as a risk management tool, hedging weather risk/revenue affected by the weather. Therefore, we consider this last motive most vital.

In addition to McDonald's motives, (Smith & Stulz, 1985) debate that in a progressive tax system, volatile cash flows can lead to a higher fraction of its income getting taxed compared to a steady cash flow. A firm can also redistribute income according to the tax rate, generating low income at a high tax rate, and more income at a low tax rate. Low volatility can also lead to a cost-of-debt reduction, and the possibility to reach a higher debt level, thus, greater tax deductions. (Tang & Jang, 2012) claims weather derivatives can help reduce the probability of default and the expected cost of default. According to (Froot, et al., 1993), cash flow stability leads to a steadier flow of profits, which are useful when investment opportunities arise. An additional argument, made by (DeMarzo & Duffie, 1995), is that required return on equity from shareholders lowers, as their ability to evaluate the administration's performance increases. According to (Leggio, 2007) smoothing revenues, covering excess costs, reimbursing lost opportunity costs, stimulating sales and diversifying investment portfolios are reasons for hedging weather risk.

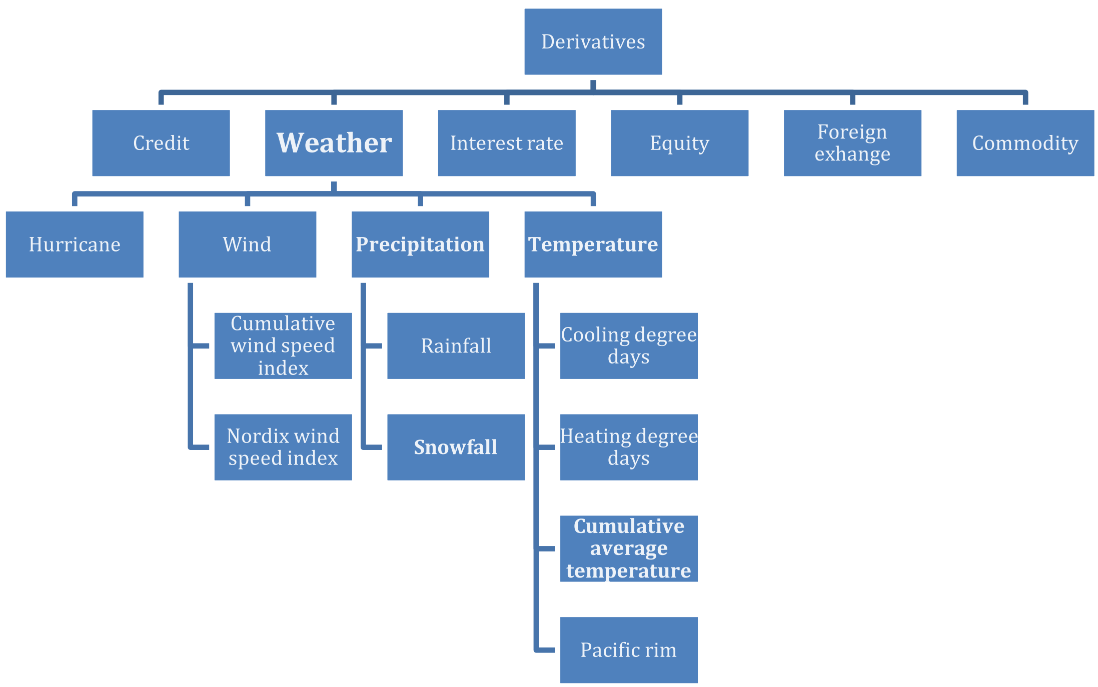
A frequent objection towards weather derivate is that insurance already serves the purpose of a weather derivative, and according to McDonald every form of insurance is a derivative. In travel insurance, for example, the buyer must pay a premium to the insurance company. If a buyer is a victim of theft or robbery, then insurance is valuable; if not, it holds no value. However, weather derivatives have some qualities that separate them from insurances. (Leggio, 2007) argues that weather derivatives protect the buyer from frequent small losses that occur at a high probability, as opposed to insurance that reduces the risk of an unexpected, low probability event, for example, a hurricane. (Tang & Jang, 2012) also, make a similar argument, but adds that an insurance claimant needs to submit a proof of loss caused by weather, which can be a demanding process. Two problems often occur in the insurance business. The first one is moral hazard, which is opportunistic behavior after signing a contract (Kenton & Abbott, 2019). As an example: signing car insurance and driving recklessly. The other one, adverse selection, emerges due to premium pricing mechanisms. Average risk level determines insurance premium, resulting in high-risk firms or persons more likely to buy insurance compared to a low-risk firm/person. Neither moral hazard nor adverse selection is present when dealing with weather derivatives, and contract costs are consequently lower, since the payout of a weather derivative depends on an underlying objectively measured index.

Unfortunately, multiple challenges may occur when using weather derivatives. In a survey conducted by (Tang & Jang, 2012), firms were asked to rank a list of eleven possible concerns. "Lack of awareness and expertise," "not beneficial," "problems in quantifying risk exposure," and "exposure already managed" was among the most dominant. One of the other major concerns, given a weather derivative is applied, is counterparty risk, which is a risk that the counterparty does not or cannot honor the contract. This risk usually is present in over-the-counter markets. Another primary concern is basis risk, which is the risk that there is a difference between potential loss and derivative payout, i.e., the financial loss is not correlated with the adverse weather condition. The lack of a standardized pricing method and pricing difficulties is a common worry as well. We will, in section 4.1, discuss the latter thoroughly.

2.2 Characteristics of weather derivative market

In this section, we will clarify different types of weather derivative products, weather derivative indexes, and market participants. Figure 1, below, offers a categorization of financial derivatives that Chicago mercantile exchange is, or has been, offering as futures and options (CME Group, 2007), (CME Group, 2015) and (CME Group, 2019). Weather derivatives are often categorized into four main classes, namely hurricanes, wind, precipitation, and temperature.

Figure 1: Categorization of derivatives



In the following section, 2.2.1, are the different weather derivative products explained, while in section 2.2.2, we will derive different indexes for measuring weather phenomena. Afterward, in section 2.2.3, we will look at some of the market's major stakeholders.

2.2.1 Weather derivative products

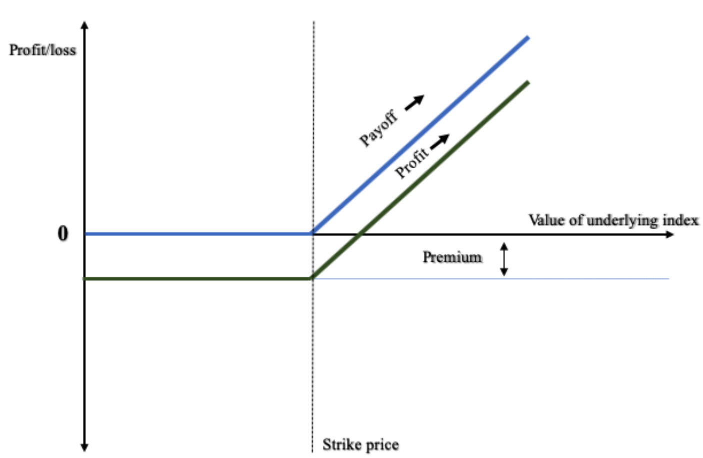
There exist many different products of weather derivatives products that are both exchange-traded and traded over the counter. Swaps, futures, and forwards are explained in more detail in chapter 4. In this section, however, we want to briefly present existing derivative theory describing put and call options, as we are going to apply these options in the empirical analysis in chapter 5.

Figure 2 illustrate a call option, which is a right, but not an obligation, to buy an underlying asset at a predetermined price. For example, imagine Google is traded at 1000 USD. There is a common belief among experts; Google's stock price will increase. An investor wants to secure his rights to buy Google for 1005 USD and pays a premium for this right. This investor will have a positive payoff if Google's stock price exceeds 1005 USD, and he will choose to execute his right to buy. However, he will have a profit if, and only if, the payoff exceeds the option price.

$$payoff\ call = \ \theta * \max[S - K, 0]$$
$$profit, or\ loss\ = payoff - premium$$

Where the payoff depends on the tick size, θ, the predetermined execution price, K, and the value of the underlying, S. The investor will choose to not execute his contract right if $S - K \leq 0$. The investor's potential profit is, in theory, unlimited. Call options' premiums tend to be more expensive than put option premiums for this reason.

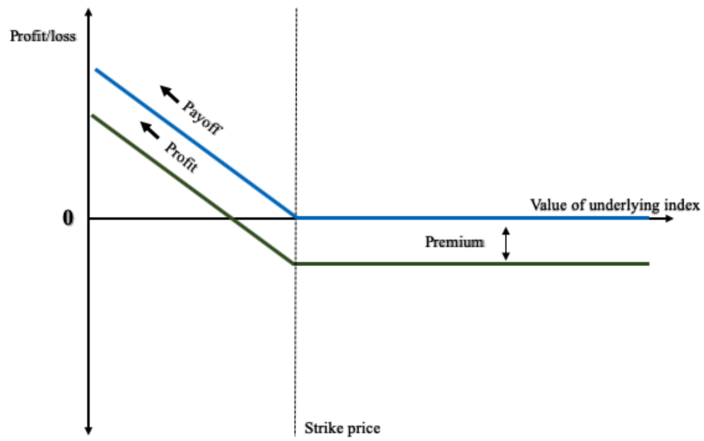
Figure 2: Call option



In figure 3 a put option is illustrated. A put option gives a buyer the right to sell an underlying asset at a predetermined price. The main difference between a put option and a call option is that the payoff-structure is inverted:

$$payoff\ put = \theta * \max[K - S, 0]$$

Figure 3: Put option



2.2.2 Weather indexes

In this section, we will present the most commonly used underlying weather indexes featured in exchange trade and over-the-counter markets. Note that in the latter, more specialized indexes can be constructed to please the buyer of risk protection. Although this thesis do not cover all weather variables, it important to explain them to describe the market's characteristics.

The cumulative wind speed index measures the sum of daily average wind speeds during a period, where duration is defined as $[t_1, t_2]$, and $W(i)$ is the daily average wind speed on day i (Alexandridis & Zapranis, 2013, p. 233).

$$CAWS = \int_{t_1}^{t_2} W(i) ds$$

The Nordix wind speed index aggregates the daily deviations from a 20 year mean over a specified period. $w_{20}(i)$ is the 20-year average wind speed on day i (Benth, 2018).

$$NWSI = 100 + \int_{i=t_1}^{t_2} W(i) - w_{20}(i)$$

A cumulative rainfall index measures the sum of daily rainfall, R, on a specific date, i, over period, $[t_1, t_2]$.

$$CR = \sum_{i=t_1}^{t_2} R(i)$$

Similarly, a cumulative snow level (CSL) index measures the sum of daily snow level, SL, on a specific date, i, over the period, $[t_1, t_2]$.

$$CSL = \sum_{i=t_1}^{t_2} SL(i)$$

The cumulative average temperature (CAT), measures the sum of average temperature, T.

$$CAT = \int_{t_1}^{t_2} T(i)di$$

Pacific rim divides CAT index over the same duration.

$$Pacific\ rim = \frac{1}{t_2 - t_1} CAT$$

Heating degree days (HDD), and cooling degree days (CDD) measure the sum of deviations from a base temperature, usually 65º Fahrenheit, or 18º Celsius. (Alexandridis & Zapranis, 2013, p. 8) We are dismissing negative values.

$$HDD = \int_{t_1}^{t_2} max(18 - T(i), 0)di$$

$$CDD = \int_{t_1}^{t_2} max(T(i) - 18, 0)di$$

This thesis highlights precipitation and temperature-based derivatives with cumulative snow level and average temperature respectively as the derivatives' underlying indexes.

Other potential indexes, although not available on CME, are displayed in the following table.

Table 1: Over the counter indexes

Energy	
Critical hot days	Weather protection based of the number on days the protection period where the daily maximum temperature is equal or above a specific level.
Consecutive Hot Days	Number of consecutive days the maximum temperature is above or equal to a specific limit.
Consecutive cold days	Number of consecutive days the minimum temperature is below or equal to a specific limit.
Renewables wind	
Critical low wind average	
Critical high wind	
Renewables solar	
Solar radiation global critical days	
Cumulative solar radiation global	
Agriculture	
Frost	Based on the number of days when the minimum daily temperature drops below a set limit
Construction	
Too many rainy days	

(weatherxchange, Industries)

2.2.3 Market participants

There are many different participants in the weather derivatives market. Table 2, by (Brockett, et al., 2005), shows us how several end-users are affected by the weather. The list could easily have been extended but was only meant to illustrate. Among other providers and end-users are insurance and reinsurance companies, hedge funds, pension funds, state governments, sellers of meteorological data, and brokerages of structured products.

Table 2: Risk exposure for end users

Hedger	Weather/Index	Risk
Agriculture industry	Temperature/precipitation	Crop losses on days with extreme temperature or rain.
Airports	Frost, snow, wind	Higher operational costs
Beverage producers	Temperature	Cold summer equal lower sales
Building material companies	Temperature/snowfall	Construction sites may shut down due to extreme cold, which disrupts business.
Construction companies	Temperature/snowfall/rainfall	Unusually weather, either hot, cold, or rainy, can lead to deadlines not being met, declining labor efficiency, and stoppage
Energy consumers	Temperature	Cold winters, warm summers
Energy/power industry	Temperature	Warm winters, and cool summers resulting in decreased demand.
Wind farms	Wind speed	
Hydroelectric power generation	Precipitation	Drought periods.
Municipal governments	Snowfall	High costs of snow removal
Ski resorts	**Snowfall/temperature**	Low snow level may impact the number of visitors and the cost of making artificial snow.
Theme parks	Temperature/precipitation	Low attendance in cold and/or rainy days
Transportation	Wind/snowfall	Cancellations

In addition to end-users, there is also a range of providers in the weather derivatives market. Below we have illustrated an example of weather protection sellers, with the information provided by (weatherxchange, 2019).

Table 3: Weather derivative providers

Company	Information
Sompo Global Weather	Global Specialty Insurance Company
Swiss Re Corporate Solutions	Global Weather Protection Seller
Liberty Speciality Markets	Global Specialty Insurance and Reinsurance Company
MSI GuaranteedWeather	Global Weather Risk Manager
AXA Global Parametrics	-
Coriolis Capital Ltd	Investment Fund
	Weather Risk Management Specialist
Nephila Capital Ltd Bermuda	Investment Fund
	Weather Risk Management Specialist
Allianz Risk Transfer	Global Insurance and Reinsurance Company
Munich Re	Global Insurance and Reinsurance Company

2.3 Former studies snow derivatives

Constructing a snow level put option, and a temperature call option are essential tasks in our assignment. Studies and theory on temperature-based derivatives are widespread. A few among many contributors are (Benth & Šaltytė-Benth, 2005), (Benth & Šaltytė-Benth, 2007) (Mraoua & Bari, 2007), (Alexandridis & Zapranis, 2008) and (Alexandridis & Zapranis, 2013). The following sections focus on results from the limited amount of studies on snow-based derivatives.

(Tang & Jang, 2011), perform an empirical analysis that examines geographical diversification and financial hedging as two strategies against snowfall risk. They split risk management into two terms, namely operational hedging and financial hedging. Operational hedging deals with risk by changing the operations, while financial hedging transfer risk by insurances or acquiring derivatives. The article highlights that geographical diversification, as a mean by multinational companies, has various outcomes when it comes to the exchange rate risk. This ambiguity can arguably be explained by a price risk from the exchange rate, and volume risk from uncertainty in demand. Cash flow estimations for small and large companies are simulated by using the Monte Carlo-method and are regressed on snowfall and a dummy for property acquisition. Subsequently, by adding a property to the cash flow, they tested its effect on the cash flow exposure of snowfall risk.

Furthermore, the strategy (of reducing snowfall risk) is more effective for a one-property company compared to a large company with several properties. The latter may benefit more from financial hedging, while hedging effectiveness depends on capital levels for small companies. If a one-property company has vast capital reserves, geographical expansion is better for risk reduction if the cash flow from the newly acquired property is negatively correlated with the original property, while financially stressed companies that hope to reduce short-term risk should opt for a hedge. Another key argument in their article is the difficulty of finding the optimal hedge ratio for ski conglomerates. Geographical basis risk, or just spatial risk, which is the difference between snowfall value of a weather station and the actual snowfall value at an arbitrary station. Spatial risk occurs at every single property, and the correlation between and within the bases and snowfall indices of different properties must be considered.

(Tang & Jang, 2012), constructed a snowfall forward to hedge the snowfall risk for Winter Sports Inc., a public traded, single-property resort. Demonstrating hedging effectiveness is optimal in this case, as the firm is not diversified geographically nor business-wise. The forward's strike price is set to the historical mean, which equals the probability of each outcome, that is a positive or negative payoff. The objective is to minimize cash flow volatility. To find optimal hedge they regress unhedged, operating cash flow against quarterly and annual snowfall to find an effective hedge. Monte-Carlo simulation is used as well, generating estimates of cash flow alternatives in order to test the statistical significance of the regression. Furthermore, Tang & Yang present the results of this hedging strategy in the period 1991-2003 by displaying cashflow with and without a derivative present. Cash flow volatility was reduced by 25.8% at most, when actual snowfall was high. They argue that it is more efficient to use snowfall forwards as a risk management tool in months with high snowfall levels.

(Beyazit & Koc, 2010), construct a snow level put option within the framework of Black & Scholes option pricing for Palandoken, a ski resort in the east of Turkey. The rationale of the article/study is that tourism has a significant impact on Turkey's economic development; hence it is crucial to reduce risks associated. They have a strong belief that the market potential for skiing resorts is significant, as 55% of Turkey lies between 1500 and 3000 meters above sea level. Daily average snow level data is gathered from November to March in the period 1975-2006.

They were not able to discover a particular distribution nor correlation between profits and snow level, as opposed to for example temperature. "Generalized Edgeworth Expansion", developed by (Jarrow & Rudd, 1982), (Rubenstein, 1994) and (Rubenstein, 2000), is applied to deal with non-normality present in cumulative snow level data, such as skewness and kurtosis. Two regressions are performed between profit as its dependent variable, and cumulative snow level as an independent variable to find the models' theta, which is a multiplier for the put option's payoff. The period for cumulative snow level differs between 90 (December-February) and 151 (November-March) days. All this gave put option prices from three different pricing methods, later discussed in our methodology chapter, and their implied volatilities of profits.

(Bank & Wiesner, 2011), conduct a qualitative survey, interviewing 61 Austrian ski lift operators regarding weather derivatives. Vulnerability to climatological risk, the importance of climatological risk, adaptability of ski lift operators, the reasoning for weather derivative usage or non-usage are the five topics in the survey. Determining factors of usage of weather derivatives was the purpose of this study. A large fraction of the responders answered they thought climate change is important, or very important, and approximately 80 percent rely on artificial snow, whereas only one responder considered weather derivatives as a risk management tool. Regarding vulnerability, 72,1 percent said their risk exposure from the weather was very high. A quarter replied that over 50 percent of their operating income was at risk due to sub-optimal weather. In 55 percent of the cases, the board was responsible for risk management, while 21 percent got help from external consultants. More surprisingly is the fact that over 50 percent told the questionnaires they did not estimate risk. Those who did were mainly large companies with a low degree of weather exposure. The results display the lack of basic framework and capacity to use derivatives in question. Most commonly, the main reason for using weather derivatives, is that insurance products do not cover weather risk. Arguments against weather derivatives are counterparty risk, basis risk, and high transaction costs, which aligns with theory. Other arguments are lack of expertise, knowledge, and even awareness. The survey finally revealed the market potential for weather derivatives, as 40 percent responded they might be willing to try this instrument

3. Data

When collecting data, we have chosen to use daily data transformed into monthly data. The main reason is that if we were to use seasonal data, we would lose the variations between the different months of the year, while using daily data would give a lot of spurious correlations. It would most likely be tough to find any correlation between the weather and revenue.

3.1 Meteorological data

The data used was collected from The Norwegian Meteorological Institute and was retrieved from their websites www.senorge.no and http://eklima.met.no

When collecting data on the weather, it is crucial to find the right quantity and quality of data. "Without an appropriate quantity of relevant, high-quality data, pricing and management of weather risk would be unfeasible" (Dunis & Karalis, 2003). As we can see "previous studies use datasets containing historical data from 5 to 230 years to fit various models. However, considering a very long period, the datasets will be affected by trends like urban effects. On the other hand, when studying tiny datasets, there is a possibility that important dynamics of the temperature process will not be revealed which will result to an incorrect model and mispricing of the corresponding weather contracts" (Alexandridis & Zapranis, 2013, p. 38).

One of the problems that we encountered were as Alexandridis and Zapranis described it, that "some stations had to be moved during the years or to be replaced by more modern equipment; as a result, jumps will occur on the data" (Alexandridis & Zapranis, 2013, p. 38). Relocation of stations led to some difficulties with collecting the data for both snow level and temperature. We will address this problem later.

In the regression, we are only using data from December 2007 to December 2018, as this is the period where we have financial data. For the period from December 2012 to December 2014, we do not have snow level data hence this period has been excluded. When pricing weather derivatives using historical Burn analysis, ADS, historical densities and Edgeworth densities we will use the full period of collected data from 1978 to 2018 with the exceptions that we will mention later.

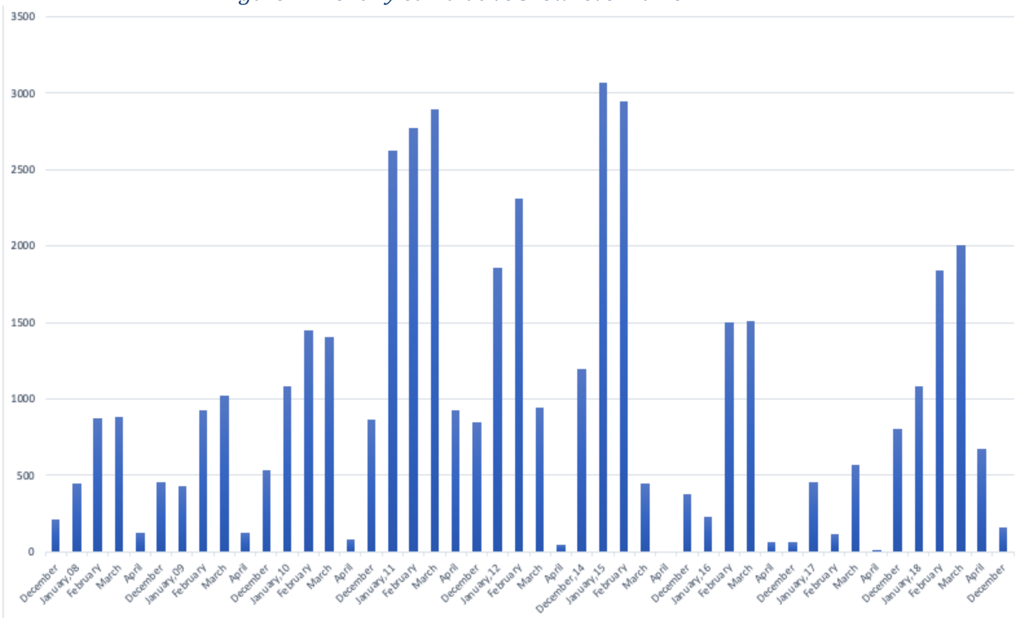
3.1.1 Snow level

One of the challenges with collecting data for snow level is that there are no weather stations located in the area of Voss ski resort. Therefore, we chose to use data from the weather station Bulken. This weather station is an official weather station operated by The Norwegian Meteorological Institute and can be categorized as nonpartisan and trustworthy for both an issuer and a buyer of a weather derivative. Bulken is located an “as the crow flies distance” of 8,25 km from Voss ski resort and lays at an altitude of 328 meters above sea level, while the altitude of Voss ski resort varies between 285 and 964 meters above the mean sea level. To verify that we can use the data from Bulken, we have used satellite data from senorge.no and checked the correlation between the places.

When it comes to deciding the right amount of quantity of samples, we have chosen to collect data from December 1978 up until April 2018. As discussed earlier moving or upgrading weather stations can cause missing data, as is the case for Bulken. From December 2012 to December 2014 the weather station was closed for maintenance. So, the total quantity of samples became therefore 38 seasons (December to April) or 180 months based on 8170 daily observations. Previous studies have as mentioned earlier contained between 5 and 230 years of historical data. We therefore think that 38 seasons is a satisfying number of samples and have therefore chosen not to compute variables for the period missing and instead exclude the period from the collected data. The data we have collected consists of daily snow levels converted into cumulative monthly and seasonally data.

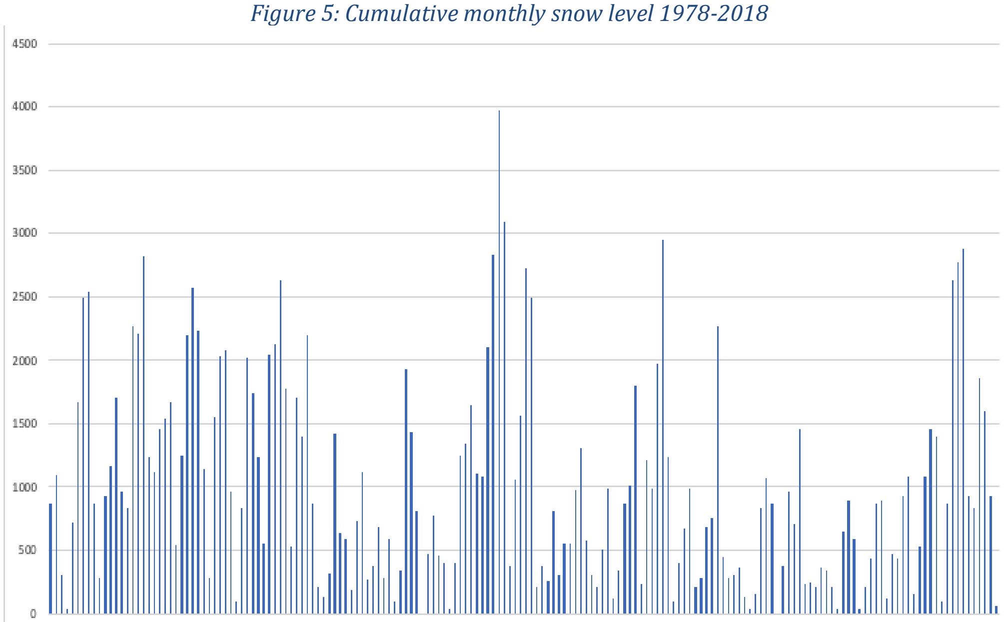
In figure 4, we can see the snow level data from 2007-2018 that is used in the regression. The data we have found shows monthly cumulative snow levels. We can observe that we have months with almost no snow at all and months with high levels of snow. The range between the minimum and maximum snow level is respectively 143 cm and 3065 cm, while the average for months is 983,28.

Figure 4: Monthly cumulative snow level Bulken 2007-2018



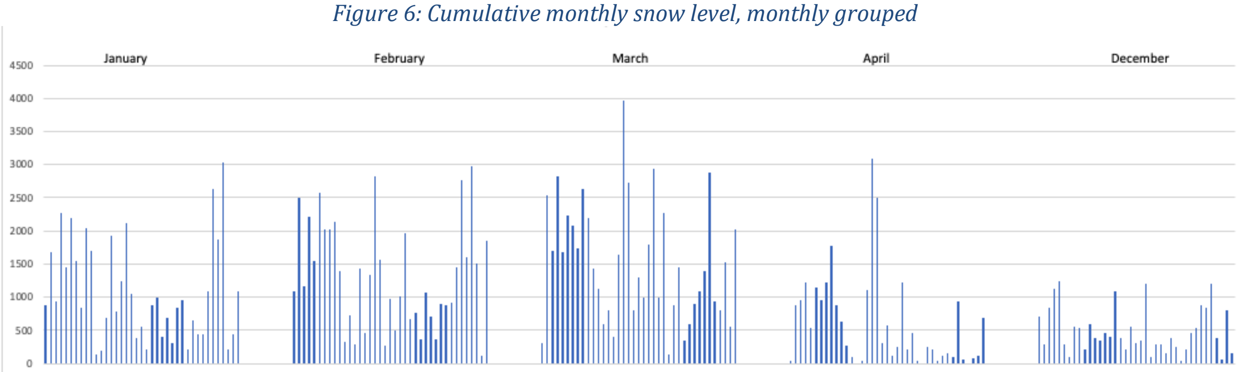
In figure 5, which shows snow level from 1978, we observe that the range has increased in comparison to the data set from 2007, which is not unexpected as we here have more samples. We also see that the average has also increased which indicates that there was more snow in the years before 2007 than after.

Figure 5: Cumulative monthly snow level 1978-2018



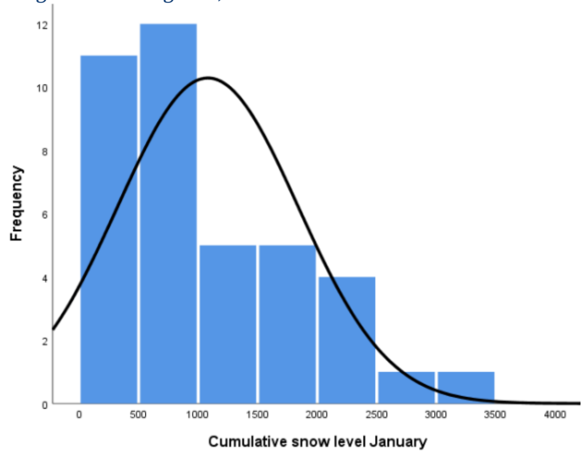
In figure 6 we have organized the monthly cumulative snow level according to the different months, and we can see that the three first months of the year has higher cumulative snow levels than April and December.

Figure 6: Cumulative monthly snow level, monthly grouped



In figure 7, we present cumulative snow level for January, in the period 1978-2018 as an illustration of non-normality. Normally distributed data is an assumption in several pricing methods for weather derivatives. It is therefore necessary to address issues regarding non-normally distributed data when pricing weather derivatives.

Figure 7: Histogram, cumulative snow level 1978-2018

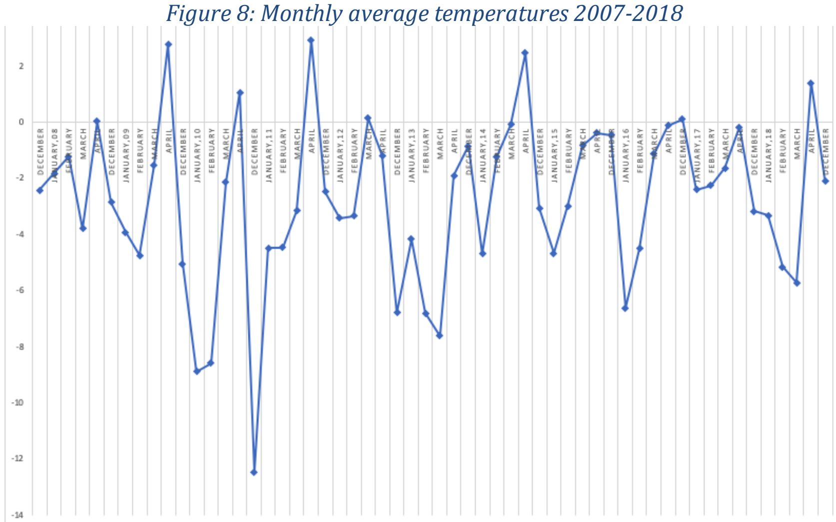


3.1.2 Temperature

The data for temperature is collected from two locations, Bø up to 2003 and Hangur from December 2007 up to December 2018. The reason for this is what we have discussed earlier that stations can be closed down or moved. When it comes to the temperatures, we have chosen to collect data from December 1978 up until December 2018. Unfortunately, we do not have data for the period from January 2004 to December 2007, as a consequence we have chosen to exclude this period. We have collected the daily average temperature and used it to compute the monthly and seasonal average temperature. The data gives us a total of 36 seasons and 180 months based on 7740 daily observations.

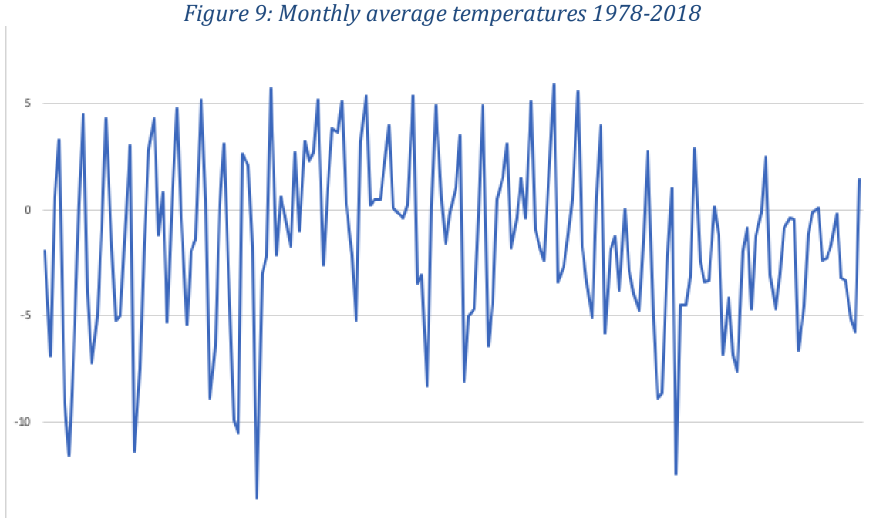
Figure 8 illustrates the monthly average temperatures for the period 2007-2018. As the data illustrates, there is considerable variation in the monthly average temperature for different months.

Figure 8: Monthly average temperatures 2007-2018



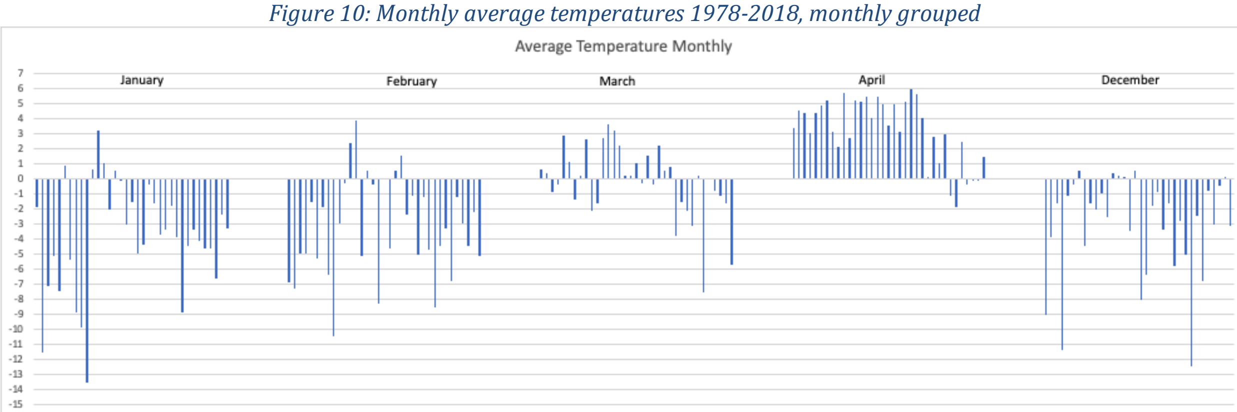
Moreover, in Figure 9 we have the temperatures from 1978 to 2018. What is interesting to see is that if we calculate the average temperature in figure 8, it is more than one degree higher than in figure 9, which indicates that it has been a lower average temperature has been measured in recent years.

Figure 9: Monthly average temperatures 1978-2018



Since we are going to look at different weather options for the different months, it is also necessary to take a look at the evolution of the average temperatures for the different months, as illustrated in figure 10.

Figure 10: Monthly average temperatures 1978-2018, monthly grouped



3.2 Financial data

With help from Voss ski resort we have collected data for their daily income from December 2007 to December 2018. Voss ski resort regards December-April to be their main season, and we have therefore collected data for this period. Unfortunately, we have not been able to obtain an outline of their operating cost. We have therefore taken a closer look at what is the most significant costs for a ski resort. Payroll costs and costs connected to having employees are two of the significant costs for a ski resort, and even though ski resorts often hire temporary staff, they cannot just send the staff home from day to day without pay, due to tariffs and the employment protection act. We can therefore not conclude that the cost to the staff is much different on a day with little snow from a day with more snow. One of the other significant costs for a ski resort is snow machines. These machines are often costly and have high fixed cost. Even though electricity and water are substantial costs, the expenses of buying and maintaining the snow machines are so high that we can say that the variations in cost are not substantial. We have also checked the yearly labor costs and operating costs for Voss resort on (Proff.no, 2019), and observe that the variations in costs are minimal. We are therefore confident that we can use the total income.

Due to a confidentiality agreement between ourselves and Voss ski resort, we are not going to project any of the exact numbers of financial data. All calculations are done with the correct data, and all final products are found by using the correct data. However, we will not show any calculations containing this data, nor will we show an overview of the financial data. We will show all the correct payoffs. The revenue without derivative will be illustrated as "100%" and we will show revenue with derivative as the exact percentage of the revenue without derivative.

3.3 Regression and descriptive statistics

3.3.1 Regression in general and variables used

The first thing that we have to do before we can determine if we can generate any weather derivatives is that we have to find out if there is any causation between snow level/temperature and annual sales. "Causal inference is the identification of the cause or causes of a phenomenon, by establishing covariation of cause and effect, a time-order relationship with the cause preceding the effect, and the elimination of plausible alternative causes" (Shaughnessy, et al., 2000, p. 25). To be able to find out if there are any causal inference, we are going to use regression analysis." Regression analysis is a statistical technique that attempts to "explain" movements in one variable, the dependent variable, as a function of movements in a set of other variables, called the independent (or explanatory) variables, through the quantification of a single equation" (Studenmund, 2013, p. 5). To conduct the regression analysis we are going to use the software SPSS by IBM.

To find out how much of the variation in sales is explained by the weather, we are going to use the coefficient of determination (R^2). R^2 is the ratio of the explained sum of squares to the total sum of squares. "The higher R^2 is, the closer the estimated regression equation fits the sample data" (Studenmund, 2013, p. 51). R^2 can not decrease if another variable is added. We are therefore going to use the adjusted coefficient of determination (adjusted R^2).

We use the Ordinary Linear Squares method to estimate as correctly and in an as straightforward manner as possible. (Studenmund, 2013) lists three main reasons why to use OLS. Firstly, OLS is relatively easy to use. Secondly, the goal of minimizing the sum of the squared residuals is entirely appropriate from a theoretical point of view. Lastly, OLS estimates have several useful characteristics. (Studenmund, 2013, p. 37) It is crucial when we are using OLS as an estimation technique for the regression that we check if conclusions are reliable and consistent with underlying assumptions. Hence, we have to check for autocorrelation, multicollinearity, and heteroskedasticity.

According to (Studenmund, 2013), autocorrelation or serial correlation implies that the value of the error term from one period depends in some systematic way on the value of the error term in other periods, and it occurs most frequently in time series data sets. Since we are using a time-series data set, autocorrelation may be a problem. The most common consequence of serial correlation is that it causes OLS to no longer be the minimum variance estimator (of all the linear unbiased estimators) and causes the OLS estimates of the Standard Error to be biased, leading to unreliable hypothesis testing (Studenmund, 2013, p. 331). To uncover any autocorrelation, we are going to use the Durbin-Watson d test. In the Durbin-Watson d test, we have a null hypothesis that the correlation coefficient rho is equal to zero and an alternative hypothesis of rho being greater than zero. If we end up rejecting the null hypothesis, we can conclude that we have significant autocorrelation.

Multicollinearity can cause troubles for the regression equation. If we have multicollinearity, it can lead to rejecting a relevant variable (type 2 error). In type 2 errors, according to (Studenmund, 2013), will estimates remain unbiased and the variances and standard errors of estimates will increase if we have multicollinearity. To detect if we have any severe multicollinearity, we are using the variance inflation factor (VIF). If the VIF index shows that we have multicollinearity it does not necessarily mean that it is a problem. If all the variables are significant and we have a high R^2, then multicollinearity will not be a problem.
One of the traditional assumptions that must be met in order to use OLS is as explained by (Studenmund, 2013, p. 102) that the error term has a constant variance (no heteroskedasticity). If there is heteroskedasticity the variance of the distribution of the error term would change for each observation or range of observations. (Studenmund, 2013) recommend using a white test to detect heteroskedasticity. Detecting heteroskedasticity can be done by finding the residuals and then see how much of the residual is explained by the dependent variables and the squared of the dependent variables. If then $n * R^2$ is higher than the chi-squared value then we have heteroskedasticity, according to (MacKinnon & White, 1985).

We have used two main weather variables to try to explain the changes in the revenue of Voss ski resort. The first variable is snow level, and the second variable is temperature, both explained earlier. In addition to the two weather variables, we have also used a variable for when most people have a day off. This variable has the value equal to the total number of days off during the month.

In our search to find out if the weather can explain the variations in revenue, we are using both linear and non-linear regression, and we are going to look at it at a monthly perspective. In the non-linear regression, we will create two new variables "$snow\ level * snow\ level$" and "$temperature\ *\ temperature$" which will give us quadratic equations. Weather and revenue do not necessarily have a linear correlation, and by using quadratic equations, it might give us a higher coefficient of determination.

3.3.2 Monthly regression

In an effort to explain the variations in monthly sales, we have conducted a regression with monthly sales as the dependent variable, with the following variables:

$$S = Sales\ per\ month$$
$$SL = Cumulative\ Snow\ Level\ per\ month$$
$$T = Average\ Temperature\ in\ the\ respective\ months$$
$$D = The\ number\ of\ days\ off\ during\ the\ month$$

We have checked for autocorrelation, multicollinearity, and heteroskedasticity for all regressions, and we have found none with significant impact on any of the monthly regressions. The regression that gave the highest adjusted coefficient of determination is a non-linear regression with the following independent variables: Snow level, snow level squared, temperature, temperature squared and days off. The adjusted coefficient of determination is 0,738, which shows a high degree of the revenue's variation is explained by these variables. We get the following regression equation;

$$S = 284\,489{,}467 + 2656{,}650SL - 0{,}725SL^2 - 380\,422{,}23T - 27\,862{,}790T^2 + 98\,563{,}433D$$

One thing we need to take in to consideration, when we are going to use this regression to create weather derivatives, is that we cannot use days off as a variable since it is not a weather phenomenon. Additionally, we can only use simple linear regression, as opposed to multivariate regression. We have not been able to find pricing methods that take multivariate or non-linear regression into consideration. We are therefore going to do two univariate linear regressions with one independent variable in each of them — one with cumulative snow level as the independent variable, and one with temperature as the independent variable.

We then get the following regressions for snow level and temperature, respectively.

$$Sales = 2\ 328\ 206{,}44\ +\ 1093.037SL$$
$$Sales = 2\ 567\ 528{,}89 - 308{,}154.76T$$

The regressions have adjusted coefficients of determination of respectively 0,370 and 0,319. The last two values in the regression equations above, 1093,037 and -308,154.76, for snow level and temperature respectively, are the thetas in the pricing of the weather derivatives. The concept of the theta will be explained in chapter 4.

The correlation for monthly cumulative snow level and sales is illustrated in figure 11, while the correlation for monthly average temperature and sales in figure 12.

Figure 11:Regression line for the equation with actual observations plotted for snow level

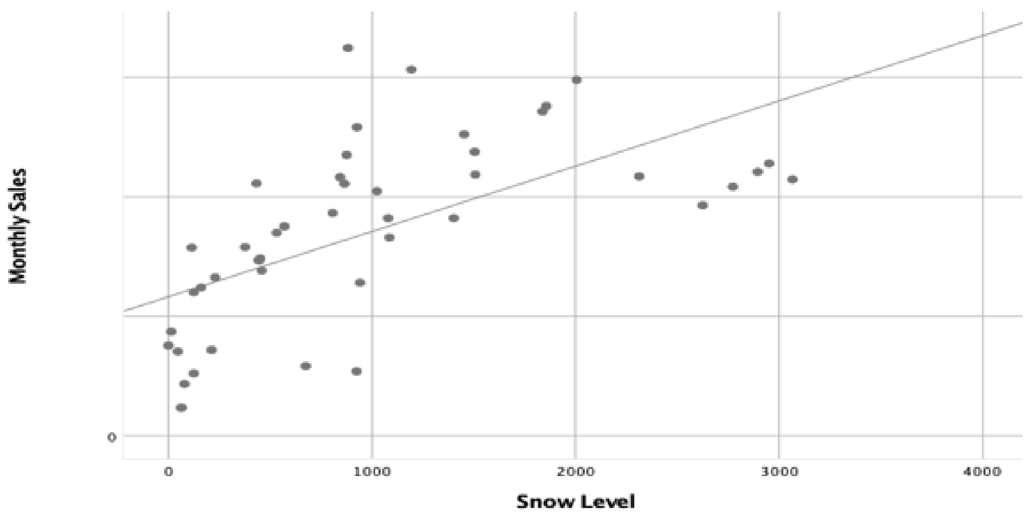
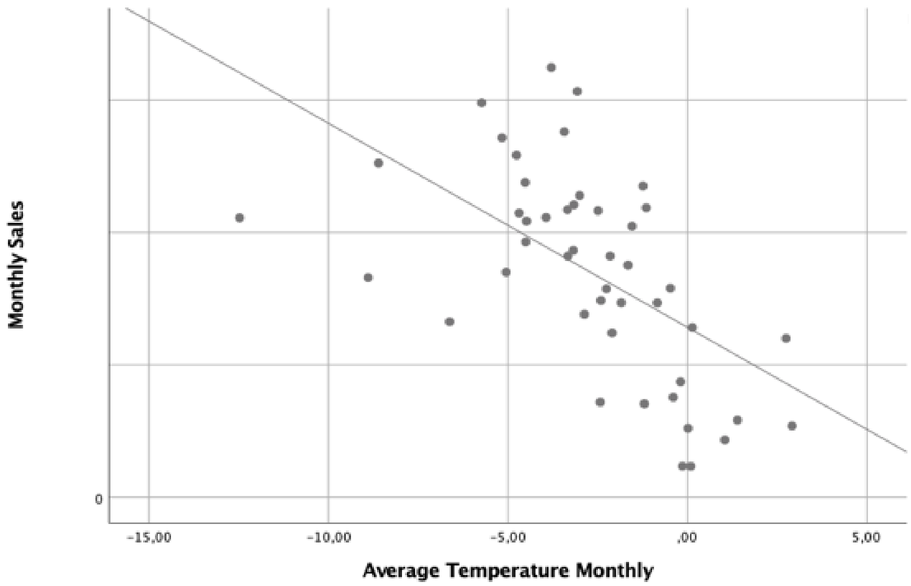


Figure 12:Regression line for the equation with actual observations plotted for temperature



3.4 Simulation and estimations of values

The benefit of simulating and estimating values is that we get more comprehensive data. We have therefore decided to do simulations of earlier values and estimations of missing values. However, there is a risk involved when estimating values. The estimated values are only an approximation of the true value, not the exact value.

For temperature, we are missing values for the period 2002 up to and including 2007, and for snow level, we are missing the values from December 2012 up to and including April 2014. To estimate these missing values, we are using an estimation technique recommended by (Alexandridis & Zapranis, 2013, p. 90). In this technique, you first the average of the seven years before and after the missing value, and the average of all years, then the mean of these two will be the estimated value. This estimation is illustrated by Alexandridis and Zapranis (2013) with the following three equations.

$$T_{t,miss} = \frac{(T_{Avy,t} + T_{Avd,t})}{2} \qquad (1)$$

$$T_{Avy,t} = \frac{1}{N} \sum_{yr=1}^{N} T_{t,yr} \qquad (2)$$

$$T_{Avd,t} = \frac{\sum_{i=1}^{7} T_{t-1} + \sum_{i=1}^{7} T_{t+1}}{2} \qquad (3)$$

Since we have weather data from 1978, we wanted to simulate the sales revenue for every year from 2007 back to 1978. We believe that this simulation will give us a better foundation for making an optimal portfolio. To simulate data, we are going to use the Monte Carlo method, with the following regression as the backbone of the simulation.

$$S = Sales\ per\ month$$
$$SL = Cumulative\ Snow\ Level\ per\ month$$
$$T = Average\ Temperature\ in\ the\ respective\ months$$

$$S = 1\,204\,814{,}37 + 3\,038{,}142SL - 0{,}812SL^2 - 391\,722{,}32T - 28\,872{,}184T^2$$

The simulated value will be created based on the regression above added a term with the inverse of the normal cumulative distribution of the standard deviation (error), where we have put the mean equal to zero, the standard deviation equal to the standard error of the regression and probability as random.

It is important to emphasize that we have not used any simulated or estimated values in the construction of the regression or the creation of the derivative prices. The reason is that the regression is used to create the simulations and because we wanted to only include actual values in the estimation of the derivative prices. We have only used the simulated and estimated values to find the optimal strike prices and to find the optimal portfolio.

4. Methodology

We will in the following sections look at challenges occurring when pricing weather derivatives, as well as different pricing methods, but first we present a framework, heavily inspired by (Alaton, et al., 2002) and CME Group. A general weather option/derivative can be described using:

- The contract type
- The length of the contract period
- The underlying (weather) index.
- The strike price, K.
- The tick size
- Location.
- Premium paid to seller from buyer (negotiable)
- Upper payout limit (contract dependent)

The contract type states which type of derivative we are dealing with. Most common types are options, forwards, futures, and swaps. Recall that weather cannot be stored or traded, so it is essential to highlight the fact that only cash will change hands when a contract expires. An option is a choice in the future whether to buy or sell the underlying index at a predetermined strike price, K. A call option will only be exercised if the value of the underlying index exceeds the strike price at contract end, while the same is true for a put option if the strike price exceeds the underlying index. A forward contract is a customized contract between two parties to buy or sell an asset, in this case, the underlying weather index, at a specified price at a future date (Dhir, 2019). This contract type is particularly well suited for hedging, considering the non-standardization. Swaps are over the counter agreements to exchange risk during a specified period between two parties. Usually, one side is paying a fixed price, while the other paying a variable price. Futures are standardized forward contracts.

The contract length can theoretically be any chosen duration, but most commonly as monthly or seasonally lengths. Due to meteorologist's ability to forecast weather, are daily and even weekly contracts to an extent redundant. Taking CME as an example, they offer futures and options on a monthly and a seasonal basis. It is possible to choose to buy a single winter month, or the winter period, November to February. We will opt for a similar approach as well, considering how rare snowfall or low temperatures are in spring and summer months.

The underlying index states what kind of weather variable we are dealing with. There are different kinds of weather variables such as rainfall, snowfall, and temperature. Existing literature mainly uses temperature indexes such as HDD or CDD. They are the sums of deviations from a reference level of temperature, often 18 degree Celsius. As an example, if the temperature is one degree Celsius below 18, it is referred to as 1 HDD. We will use both a temperature and a snowfall index, separately, in our case study, as these are most relevant. A contract written on the cumulative amount of snow during a period $t = 0$ to T is:

$$CSL = \sum_{t}^{T} SL_t$$

Where CSL and SL_t denotes cumulative snow level and snow level respectively. The payoff, X, of a snow level put option would then be:

$$X = \theta \max\{K - CSL, 0\}$$

Where K is the strike price, and θ is tick size. A contract written on mean temperature during a period of n days is written:

$$\bar{T}_n = \frac{1}{n} \sum_{i=1}^{n} T_i$$

Where $\bar{T}_n$ denotes mean temperature, and T_i temperature of day i. and n the number of days in a certain period. The payoff, γ, of a temperature call option would then be:

$$\gamma = \theta\max\{\bar{T}_n - K, 0\}$$

The tick size represents the amount of money that the option holder receives for each index point the underlying index exceeds (call option) or fall below (put option) the strike price. One may look at it as a sort of payoff multiplier, and the payoff's magnitude is decided of the strike level additionally.

Location means the weather station that is used to obtain meteorological data for the underlying index. The station should be an official station, to avoid inaccurate data. Most transactions are based on using one weather station, although weighted combinations of multiple stations can be used (Alexandridis & Zapranis, 2013, p. 9).

4.1 Derivative pricing difficulties

There are many risks when trying to price a weather derivative correctly. Two major types of risks arising in weather derivative trading, or pricing, are geographical basis risk, or spatial risk, and basis risk. Geographical basis risk is the risk of writing a derivative contract from a (non-local) weather station where the weather deviates from local conditions. (Brockett, et al., 2005), defines this risk as present when a contract is written on a different place the hedger wants to cover. Weather risk is highly localized, and weather is tough to predict accurately and consistently, even though meteoritical science has advanced. According to (Tang & Jang, 2011) this is due to varying micro-climates of different locations. It would not be reasonable for a Stockholm based firm to buy heating degree contracts based on a weather station located in Dallas. (Woodard & Garcia, 2008) display an increase in hedging effectiveness if a non-local derivative for a weather variable is highly geographically correlated. Basis risk is represented by the risk of low correlation between hedged volume and the underlying weather index (Alexandridis & Zapranis, 2013). This risk can be illustrated by a CPU producer like for example Intel hedging wind-speed. There is probably low or no correlation present. (Ederington, 1979) has shown that the degree of basis risk can have a significant effect on a hedging instrument's utility. Research on the basis risk is relatively limited. In a study of four U.S ISOs, (Brockett, et al., 2005) concluded that undiversified companies should not use a linear hedging strategy. One possible way to reduce basis risk is to write a weather derivative on the difference between two weather indexes at two different places. There is an apparent tradeoff between hedging effectiveness, spatial risk, and basis risk.

Comparing the two, (Brockett, et al., 2005) underline that established financial derivatives are based on share price, bonds, exchange rates, commodities, or currency, while the underlying of a weather derivative defines the measure of weather conditions. The inability to store an underlying weather variable, for example, snow, from one year to another is a fact that (Tang & Jang, 2011) highlight to mark property differences between traditional derivatives and weather derivatives. A reminder given by them is that, unlike underlying assets of classic derivatives, a man holds no control over the weather. For example, in real life, a stock price can be influenced by information such as press releases on the stock exchange.

(Xu, et al., 2008), concludes that an application of standard pricing model for financial derivatives to price weather derivatives is impossible, because weather is untradeable, i.e., the weather risk market is incomplete. This belief is supported by (Cao & Wei, 2004), who stress the fact that weather derivatives cannot provide a riskless hedge. Risk-neutral valuation by no-arbitrage does not hold. These opinions are also shared by (Brockett, et al., 2005) who states the Black-Scholes method, which is the most successful derivative pricing method, works under the assumption of complete markets. It does not work in incomplete markets, such as the weather derivate market, i.e., where a self-financing strategy cannot replicate all claims. The lack of liquidity in weather derivatives does not coincide with the B&S-based no-arbitrage model. Weather variables do not share similar properties, and there is no similarity in the distributional assumption in weather events, which results in the absence of market pricing. The weather derivative market's illiquidity is the consequence of valuation problems (Masala, 2014).

OTC market has historically been relatively big compared to exchange-traded weather derivative market, as it eliminates basis risk, but it does not provide price transparency, and counterparty risk is present when trading. Credit risk is another obstacle when pricing weather derivatives, at least in the OTC market. Credit risk is the risk that a counterparty fails a contractual obligation.

A central topic, after Enron's demise, is hedging effectiveness when the buyer faces credit risk (Brockett, et al., 2005).

Not all weather derivative pricing methods takes the financial market into consideration. An actuarial valuation is formulated within a framework that ignores the financial markets. It neglects the fact that weather affects liquid asset prices in some way, and that weather derivatives partially could be hedged by these (Brockett, et al., 2005). Local weather indices have a low correlation with prices of other financial assets, such as exchange and interest rate risk. Thus, it is difficult to substitute the underlying with a linked exchanged security to solve the problem with an incomplete market (Hamisultane, 2008).

4.2 Derivative valuation and pricing methods

4.2.1 Traditional methods

A method based on statistical analysis is the actuarial method or insurance method. Users of this methodology may be companies in property, automobile, health, and life insurance among others (Cao & Wei, 2003). To derive the distribution of all possible outcomes for the settlement index, are we using meteorological data and forecasts (Jewson, et al., 2005), as quoted in (Alexandridis & Zapranis, 2013). The risk-free rate is used to discount the expected payoff calculated by historical data. In other words, we are linking a probabilistic analysis to the insured event. Actuarial pricing relies on the law of large numbers, which states that repeating an experience, independently, several times, the estimated expected value continuously improves the approximation of the true value (Hamisultane, 2008). The actuarial method is not the most common method, considering most weather derivatives contracts have underlying variables following predictable patterns that have a high probability of reoccurring (Cao & Wei, 2003), in contrast to a low probability event like a typhoon.

Another method is the Historical burn analysis (HBA). HBA is a simulation based on historical data. The average payoff of the weather derivatives in previous n periods is calculated. Ten to thirty years of time series are commonly used (Cao, et al., 2004). In actuarial methods, expected payoffs are being discounted at the risk-free rate, which is not the case when using HBA.

Assume that finding a snow level put option for February is the objective. Start by gathering historical snow level values in February, $y_1, y_2, \dots y_n$ for *n* years/periods.
With these we get the historical payoffs, x_i, as well as the put option's price by averaging the historical payoffs,

$$Price = \frac{1}{n}\sum_{i=}^{n} x_i.$$

$$x = max(K - y, 0)$$

Where $K = Strike\ Level$ and $y =\ cumulative\ snow\ level$

The most important assumption is that historical weather derivative payoff reflects the potential payoff distribution, a notion that history will repeat itself with equal probability (Hamisultane, 2008), as quoted in (Alexandridis & Zapranis, 2013). A notable benefit of this model is that the computations are simple, since temperature follows a normal distribution, and there is no need to solve stochastic differential equations. Another advantage is that the model is based on a few assumptions, such as constant (temperature) mean, variance, and autocorrelation over time, hence stationarity. Independent and identically data distribution for different years is also assumed. Even though many consider HBA as a benchmark approach, it has major flaws, seeing that none of the assumptions hold. (Temperature) time series does contain seasonality, jumps, and trends, while there is evidence that volatility and temperature average are heterogeneous for different historical periods (Dischel, 1999), which can be confirmed by our meteorological data on snow level and temperature. Additionally, HBA does not include forecasting and is therefore deemed partial and faulty. (Benth & Šaltytė-Benth, 2012, pp. 110-111), point out that in a time series model for precipitation on monthly aggregated values, the number of data points for cumulative precipitation in a given month can get very small, which in turn results in very few non-zero payoff data, and yields a highly uncertain value of expectation, hence price estimate.

According to them using a stochastic model with vast amounts of data instead of HBA will lead to higher statistically reliability of the description of the underlying variable. Their last argument is HBA being unsuited for finding changes in price over time, as burn analysis produces a constant forward price that does not change with time. (Cao & Wei, 2003) apply burn analysis to a call option written on the three-month (January, February, and March) cumulative HDDs for Atlanta, Chicago, and New York. They found massive variability in the pricing estimate when using different period lengths. In light of all this, HBA remains an acceptable proxy of pricing, as its implementation is straightforward (Jewson, et al., 2005).

4.2.2 ADS, Historical Densities and Edgeworth densities

Pricing by Alaton, Djehiche, and Stillberger (ADS) is a methodology based on the studies of (Alaton, et al., 2002), which used (Black & Scholes, 1973) as a framework. Pricing by ADS can be used on any weather variable. Mostly temperature modeling has been conducted in prior studies, but we will use this method to develop a pricing tool for snow level options, as in (Beyazit & Koc, 2010), and a temperature-based call option. In their study from 2010, they start by uncovering a stochastic process describing the weather variable, S_t, is used to form monthly or yearly cumulative snow levels, M_n, for year/month *i*.

The following notation is used in calculations shown in this section. Historical meteorological data is used to compute mean and variance.

Φ: cumulative distribution function for the standard normal distribution.

σ_n: standard deviation of data.

μ_n: mean of average cumulative monthly/annual/seasonal temperature or snow level.

K: strike level.

r: risk-free rate.

M_n: cumulative snow level or average temperature on a yearly/monthly or seasonal basis.

S_i: daily snow level or daily average temperature.
M_n is $N \sim (\mu_n, \sigma_n^2)$.
θ: option payout multiplier/tick size. A beta-coefficient in a (simple) regression model is a proxy for this value since the parameter shows how much the dependent variable changes when the independent variable changes with one unit.

$$E(M_n) = \mu_n$$

$$Var(M_n) = \sigma_n^2$$

$$\alpha_n = \frac{K - \mu_n}{\sigma_n}$$

The price of a put option is now calculated from the following formula:

$$p(t) = \theta e^{-r(t_n - t)} E[\max\{K - M_n, 0\}]$$
$$= \theta e^{-r(t_n - t)} \int_0^K (K - x) f_{M_n}(x) dx$$
$$= \theta e^{-r(t_n - t)} \left[(K - \mu_n) \left(\Phi(\alpha_n) - \Phi\left(-\frac{\mu_n}{\sigma_n}\right) \right) + \frac{\sigma_n}{\sqrt{2\pi}} \left(e^{-\frac{\alpha_n^2}{2}} - e^{-\frac{1}{2}\left(\frac{\mu_n}{\sigma_n}\right)^2} \right) \right] \quad (1)$$

Equation 1 determines the price of a put option in terms of a normal distribution.
$t_n - t$ is the contract holding period. For example will February in a non-leap year be $t_n - t \approx 7{,}67\%$. $e^{-r(t_n - t)}$ represents a discount factor. This discount factor will be bigger if the contract period is bigger. When using the mean as a measure, it will give each years' observations equal weights.
Inversely to a put option, a pricing formula for a call option is displayed by equation 2:

$$c(t) = \theta e^{-r(t_n - t)} E[max\{M_n - K, 0\}]$$
$$= \theta e^{-r(t_n - t)} \int_K^\infty (x - K) f_{M_n}(x) dx$$

$$= \theta e^{-r(t_n - t)}((\mu_n - K)\Phi(-\alpha_n) + \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\alpha_n^2}{2}}) \qquad (2)$$

$$M_n = \sum_{i=1}^{n} S_i$$

Historical and Edgeworth densities are frequently used methods when pricing weather derivatives. In a table of cumulative snow level statistics, (Beyazit & Koc, 2010) showed us that normality was not present, as measures for skewness and kurtosis did not coincide with this. Skewness and kurtosis should be close to 0 and 3, respectively. Due to this (Beyazit & Koc, 2010) applies densities of the normal distribution function, $a(x)$, drawn out from historical data of cumulative snow levels using the first two moments. This pricing method is known as "Historical Densities." In addition to the notation mention earlier in this section, the following notation will be used when calculating historical and Edgeworth densities:

D: number of days in the period
ξ: skewness.
κ: kurtosis.
$a(x)$: density of normal distribution function of cumulative snow levels or average temperature.
x: cumulative snow level or average temperature on a yearly/monthly or seasonal basis.

Below we have a put option price formula for historical densities:

$$p = e^{-r(t_n - t)} \frac{1}{\sum_{j=1}^{N} a_j(x)} \sum\nolimits_{j=1}^{N} a_j(x) \max(K - \sum_{i=1}^{D} S_i(t_n), 0) \qquad (3)$$

$$a(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{(x-\mu_x)^2}{2\sigma_x^2})}$$

A difference between historical densities and "ADS" is that each observation (of weather phenomena) is weighted differently. Lower cumulative values of snow levels are weighted more heavily; hence a higher payoff is more influential in the historical densities approach.

Recall both "ADS" and historical densities assume normality, which does not necessarily hold for weather data. To modify a non-normal distribution, we use "Generalized Edgeworth Series Expansion," as shown by (Stuart & Ord, 1987), as a method to adjust densities. We then get the following put-formula when pricing weather derivatives using Edgeworth densities.

$$p = e^{-r(t_n - t)} \frac{1}{\sum_{j=1}^{N} f_j(x)} \sum\nolimits_{j=1}^{N} f_j(x) \max(K - \sum_{i=1}^{D} S_i(t_n), 0) \qquad (4)$$

$$f(x) = \left[1 + \left(\frac{1}{6}\right)\xi(x^3 - 3x) + \left(\frac{1}{24}\right)(\kappa - 3)(x^4 - 6x^2 + 3) + \left(\frac{1}{72}\right)\xi^2(x^6 - 15x^4 + 45x^2 - 15)\right] a(x)$$

To transform the put-formula into a call-formula for historical and Edgeworth densities, we need to reverse the last term of the formula in both equations 3 and 4 from: $\max(K - \sum_{i=1}^{D} S_i(t_n), 0)$ into $\max(\sum_{i=1}^{D} S_i(t_n) - K, 0)$.

4.2.3 Indifference pricing method

The following section is heavily inspired by the work of (Brockett, et al., 2005), (Xu, et al., 2008), and (Alexandridis & Zapranis, 2013).

The foundation of the *indifference pricing* approach uses the principle of equivalent utility. The idea is that we define the indifferent buy price, F_i, as the price at which a buyer or seller is indifferent between

i) buying the insurance/derivative now and receiving payout at expiration and

ii) not paying the price, and not receiving a payoff.

For complete derivation of price formulas, please go to appendix A1.

This two-state model has following assumptions

- Two market players, namely investor/hedger and seller/issuer. Both players want to construct their portfolio to optimize final wealth at time *T*.

- At current time ($t = 0$) both hedger and issuer try to optimize their wealth at time T. In between these states no exchanges are permitted.
- Two assets are present in the financial market, a risky asset with a return r, and a risk-free rate of return r_f. Both are gross rates of return. A weather derivative's arbitrary underlying weather variable/index is also present.
- Wealth maximization of a buyer is subject to a utility function, more specifically a negative exponential one

The calculations in this section will be conducted using the following notation:

X: Revenue

λ: Absolute risk aversion

X_i^{wo}: Portfolio value without a weather derivative/risky asset, i denotes b or s (buyer or seller).

X_i^{w}: Portfolio value with derivative/risky asset

$(x_b - a_b)$: Amount spent on a risk-free asset

x_b: Investor's initial wealth

x_s: Seller's initial wealth

a_b: Amount invested in risky production

a_s: Issuer's invested amount in capital market

r_f: Risk-free rate of return

r_b: Return of risky production

r_s: Return from capital market

u: utility

$\hat{X}$: Certainty equivalent (CE) of stochastic wealth

CE: the required certain amount that makes the investor indifferent whether he invests in a weather derivative or not.

$\tilde{X}$: Stochastic wealth

q_i: 1+r_i , where $i =$ s, b, or f.

k: Units/shares of the derivative.

W_T: Payoff at time T. This is a function of an underlying index, (I).

μ_x: Expected wealth

σ_x^2: variance wealth at time T

$E(W)$: Expected payoff at time T.

$\rho_{q_{s,W}}$: Correlation between payoff and market return.

$\sigma_{q_{b,W}}$: Co-variance between return on risky activity and payoff.

Recall that a_s is the amount of the initial wealth the seller invests in the capital market. The asterisk denotes the optimal amount invested, in this case with (w) and without (wo) a derivative present.

$$a_s^{wo*} = \frac{E(q_s) - q_f}{\lambda_s \sigma_{q_s}^2} \qquad (5)$$

$$a_s^{w*} = \frac{E(q_s) - q_f + \lambda_s k \sigma_{q_{s,W}}}{\lambda_s \sigma_{q_s}^2} \qquad (6)$$

Intuitively, with a derivative present, a_s^{w*}:

i) Increases if risky production expected rate of return is high.

ii) If covariance between market's rate of return and the derivative payoff is negative, a_s^{w*} decreases. This statement makes sense since W leads to negative cash flow.

iii) If risk-free return is relatively "good" compared to the return of the market, the amount invested will decrease.

Below, we have the price formula for the issuer/seller. For a complete derivation of the price formula, please go to appendix A.1. In the formulas, F_s equals the present value of expected derivative payoff, $E(W)$, in an additional to a risk premium, π_s.

$$F_s = \frac{1}{q_f}\left(E(W) - \frac{1}{2}\lambda_s k\sigma_W^2\left(\rho_{q_s,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_s}}((E(q_s) - q_f)\rho_{q_{s,W}}\right) \qquad (7)$$

$$\pi_s = -\frac{1}{2}\lambda_s k\sigma_W^2\left(\rho_{q_s,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_s}}((E(q_s) - q_f)\rho_{q_{s,W}}$$

$$F_s = \frac{1}{q_f}E(W) + \pi_s$$

i) Under the condition that $E(q_s) - q_f > 0$, will $\pi_s > 0$, but only if $\rho_{q_{s,W}} \leq 0$.

ii) The risk premium might become negative if the correlation between q_s and W has a high positive value.

iii) If $\lambda_s = \rho_{q_{s,W}} = 0$, then $\pi_s = 0$.

We can derive similar steps as in the previous section for the hedger, to find optimal amount of wealth put in to risky production with the following formulas:

$$a_b^{wo*} = \frac{E(q_b) - q_f}{\lambda_b \sigma_{q_b}^2} \qquad (8)$$

$$a_b^{w*} = \frac{E(q_b) - q_f - \lambda_b k \sigma_{q_{b,W}}}{\lambda_b \sigma_{q_b}^2} \qquad (9)$$

i) A negative covariance between return on production and derivative payoff makes the hedger increase a_b^{w*}.

ii) Amount invested in production decreases absolute risk aversion increases, all other parameters constant.

Below we have the indifferent price formula of the buyer:

$$F_b = \frac{1}{q_f}\left(E(W) + \frac{1}{2}\lambda_b k \sigma_W^2\left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W}\right.\right. \qquad (10)$$

$$\pi_b = \frac{1}{2}\lambda_b k \sigma_W^2\left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W}\right.$$

$$F_b = \frac{1}{q_f}\left(E(W) + \pi_b\right)$$

The first expression on equation 10's right-hand side is always negative. If one assumes $E(q_b) > q_f$, the second expression on the equation's right-hand side is positive and the correlation negative. Thus, the value of π_b is parameter dependent.

A trading requirement is that the hedger must be willing to pay a price that is equal or higher than the seller price, i.e., $F_b > F_s$. This will only happen if:

$$\tfrac{1}{2}\lambda_b k \sigma_W^2\left(\rho_{q_b,W}^2 - 1\right) - \tfrac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W} > -\tfrac{1}{2}\lambda_s k \sigma_W^2\left(\rho_{q_s,W}^2 - 1\right) - \tfrac{q_W}{\sigma_{q_s}}\left(\left(E(q_s) - q_f\right)\rho_{q_{s,W}}\right.\right.$$

equivalently $\pi_b > \pi_s$.

A critic on the indifference pricing method from (Xu, et al., 2008), is that it implies the buyers' possibility to adjust the optimal production level, which in many sectors is farfetched. Although the method is simple, it depends too much on its parameters, especially the absolute risk aversion parameter. (Carr, et al., 2001) argues that the application of this method in real life is not widely considered acceptable due to problems specifying input correctly, as mentioned above, which potentially makes this method very inaccurate. It is not even sure the negative exponential utility function is the best fit, as others as an alternative may be applied, for example, the mean-variance suggested by (Brockett, et al., 2005).

4.3 Portfolio Theory

Optimal portfolio choice is inspired by the mean-variance concept which was developed by Markowitz in the 1950s. It is a dominant framework for coping with security markets and investment management. Mean-variance efficiency means there is a trade-off between risk and return. Optimal portfolio choice of individuals builds the foundation of CAPM, and the following framework stems from (Copeland, et al., 2004), with supplementing notation from (Paraschiv, 2018).

Portfolio theory has the following assumptions (Copeland, et al., 2004, pp. 147-148):

- Frictionless markets
  - No transaction costs
  - No taxes
  - Perfectly divisible securities
  - Perfect competition
  - No short-sale restrictions
- Investors have mean-variance preferences
- Investor maximizes expected utility
- Investors maximize expected utility of wealth
- Under mean-variance preferences, investors only care about expected wealth and the variance of wealth. This is the case if:

  i) Investors have quadratic utility functions
  ii) Returns are normally distributed

An advantage of portfolio diversification is that investing in more assets yields the same expected return at a smaller variance than each asset individually.

Assume that we have a portfolio with N risky assets and denote the uncertain return of asset i by $\tilde{r_i}$. Let $\tilde{x_i}$ denote the fraction of wealth invested in asset i. The portfolio return is:

$$\tilde{r_p} = x_1\tilde{r_i} + x_2\tilde{r_2} + \cdots + x_N\tilde{r}_N = \sum_{i=1}^{N} x_i\tilde{r_i}$$

The portfolio return is uncertain, considering this is a futuristic measure. However, we can calculate the expected return of a portfolio by using this formula:

$$E[\tilde{r}_p] = x_1E[\tilde{r}_1] + \cdots + x_NE[\tilde{r}_N] = \sum_{i=1}^{N} x_iE[\tilde{r}_i] = \mu' x$$

where μ is a vector of expected returns.

The portfolio's variance is:

$$Var(\tilde{r}_p) = Var\left(\sum_{i=1}^{N} x_i\tilde{r_i}\right) = \sum_{i=1}^{N} x_i^2 Var(\tilde{r_i}) + \sum_{i\neq j} x_i x_j Cov(\tilde{r_i}, \tilde{r_j})$$

(Copeland, et al., 2004, p. 128)

Let V be the covariance matrix of asset returns, hence:

$$Var(\tilde{r}_p) = x'Vx$$

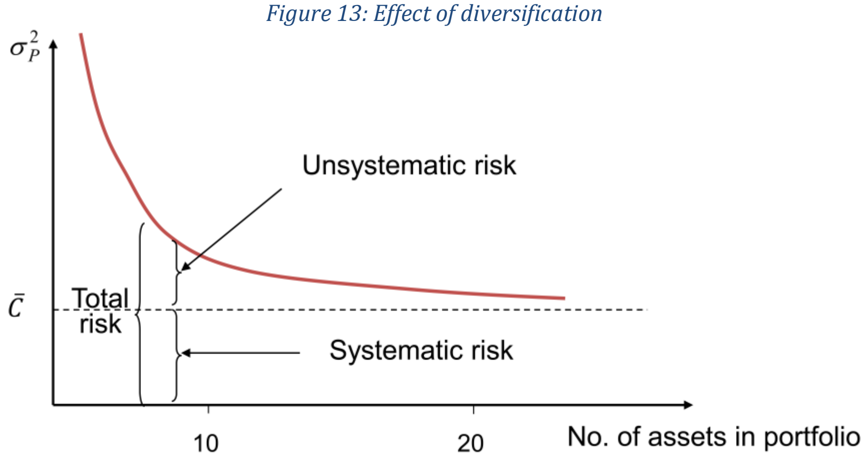
Including several assets, which are not perfectly correlated, generally reduces the portfolio volatility. This is called diversification.

If the number of assets included in the portfolio increases, the variance will approach the average covariance.

$$\lim_{N\to\infty} Var(\tilde{r_p}) = \bar{C}$$

This component, $\bar{C}$, which is called the market risk or systematic risk, this risk is undiversifiable. The other factor of the portfolio return variance is known as diversifiable risk or unsystematic risk. As the numbers of securities increases, unsystematic risk converges towards zero. A relatively small number of assets is satisfactory to achieve a substantial diversification effect.

Figure 13: Effect of diversification



(Paraschiv, 2018, p. 20)

4.3.1 Mean-Variance Portfolio Selection

Under mean-variance preferences, there is a trade-off between risk and return regarding portfolio selection. We can either

i) Set a certain level of expected return $\bar{\mu}$ and minimize risk or
ii) Set a certain level of risk and maximize expected return

Given a certain level of expected return, a portfolio is mean-variance efficient if its variance is lower than all other portfolios with the same return. The collection of these mean-variance efficient portfolios is called the efficient frontier.

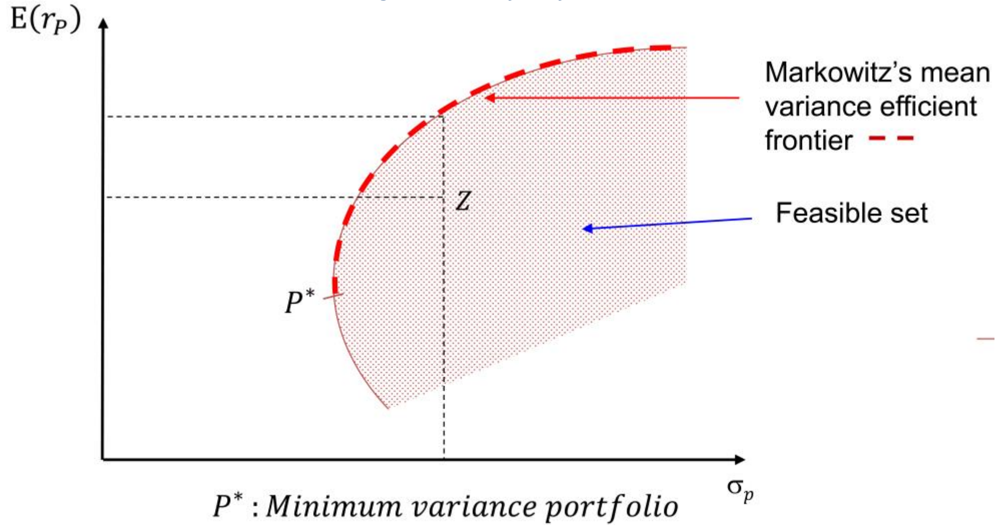
4.3.2 Portfolio Optimization

The derivation of variance minimizing portfolio weights is found in appendix A.2, in equation 1.

4.3.3 The Portfolio Frontier

By varying the level of expected return and compute the implied return standard deviations, we get the possibility to plot the efficient frontier in mean-variance space, and the efficient frontier can be illustrated as a trajectory in the mean-variance space. The portfolio that has the lowest variance overall is then called the Global Minimum Variance Portfolio.

Figure 14: Portfolio frontier



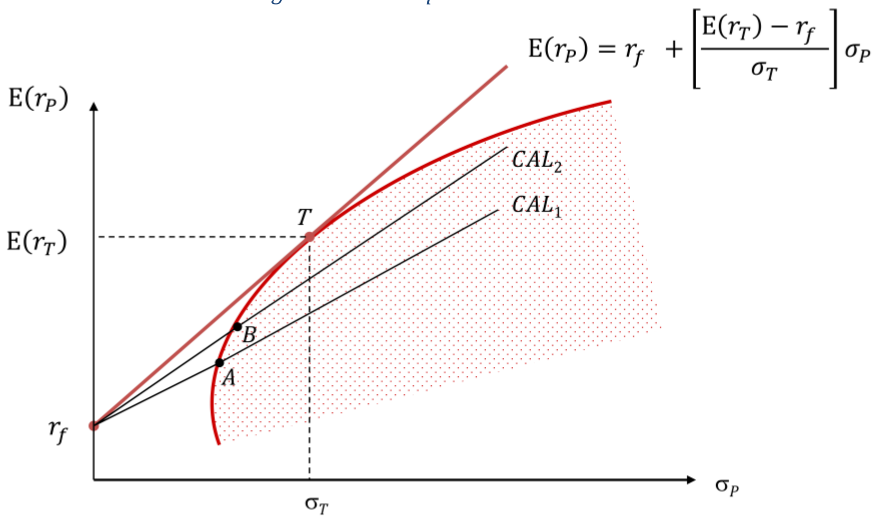
(Paraschiv, 2018, p. 27)

Most of the individual assets will lie to the right of the frontier. Risk reduction is a result of combining securities into portfolios. An investor with mean-variance preferences holds a portfolio that is located on the frontier. Adaptation below P^* will never happen, as it is inefficient, while the subset above P^* is called the efficient frontier.

4.3.4 The Capital Allocation Line

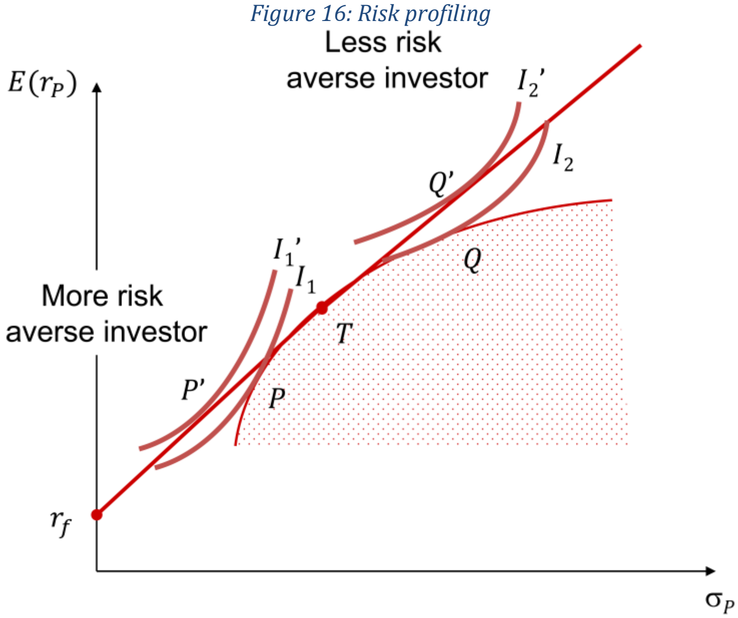
The risk-free asset in combination with any risky portfolio is called the Capital Allocation Line (CAL). The CAL with maximal slope is the efficient frontier. Its slope is often referred to as reward to variability ratio. All portfolios that are located on the efficient frontier have the same reward to variability ratio. When we later are creating optimal portfolios of weather derivatives, there will be more than one optimal portfolio, and all will be located on the efficient frontier. Which portfolios that are chosen depends on personal preferences. The efficient frontier is the half line $r_f + \sqrt{H\sigma_p}$

Figure 15: The capital allocation line



(Copeland, et al., 2004, p. 150)

Figure 16: Risk profiling



(Paraschiv, 2018, p. 38)

5. Case study: *Voss Fjellheisar AS* – Reducing risk for skiing resorts, an empirical analysis

In this case study we will assess how a ski resort can hedge the risk of unfavorable weather in their main season. The business we are using to illustrate these possibilities is Voss Fjellheisar AS. The challenges with a shortage of snow and problematically high temperatures are not challenges that will automatically solve themselves; on the contrary, they will only get worse. A research paper by (Marty, et al., 2017) as (Willsher, 2017) cited in The Guardian, explains that if global warming is limited to 2 degrees Celsius, a goal set in the Paris agreement of 2014, the snow cover in the alps will be reduced by 30 percent. If global warming exceeds 2 degrees Celsius, the loss of snow cover could be as high as 70 percent. Such a dramatic decrease in snow cover would have a dramatic impact on European ski resorts. According to Nils Magne Nedreberg, chairman of Harpefossen ski resort in Sogn og Fjordane, Norway, they need at least 100 days with snow to break even, but by 2080 may only as few as 40 to 60 percent of Norwegian ski resorts have a skiing season that lasts longer than 100 days (Kleiven, 2017). We are therefore going to create a portfolio consisting of two weather derivatives, snow level and temperature, to see if we can reduce the fluctuated revenues for the ski resort, and their risk of bankruptcy

In this case study, we will apply the derivative pricing techniques described theoretically earlier in this paper. We will start with creating the derivatives based on snow level and temperature by using Historical Burns Analysis (HBA), pricing method of Alaton, Djehiche, and Stillberger (ADS), historical densities and Edgeworth densities. Then we compare the different pricing methods and discuss which method we will use when creating a portfolio. Before we can construct a portfolio, we have to find the optimal strike levels. Here we are going to use some theory about snow production and snow levels. We will in addition use the derivative pricing technique indifference pricing. Typically, it is usual to use indifferent pricing to find the derivative price. However, we are going to stick to the four most common used methods mentioned earlier and use indifferent pricing as a tool to find out at which snow level buyer and seller are both indifferent. The indifference pricing method will help us decide the optimal strike levels. When we have found the strike levels, we will find the optimal weight of snow level derivative and temperature derivative in the portfolio.

5.1 Applying pricing methods

5.1.1 Traditional methods

5.1.1.1 Historical Burns Analysis (HBA)

When finding the derivative price by using Historical Burns Analysis, we first have to find $x = \theta max(K - y, 0)$, which is the cumulative snow level below or the average temperature above the set strike level. Then we can find the price for the different strike levels

$$Price = \frac{1}{n}\sum_{i=}^{n} x_i.$$

As we can see in table 4, do we have $n = 35$ in January, February, March and April, while we have only 34 in December. An overview of the prices can be found in table 11 and 12.

Table 4: Common parameters HBA

Common Parameters	
n Jan-Apr	35
n Des	34

5.1.2 ADS, Historical Densities and Edgeworth densities

5.1.2.1 ADS

The first pricing method we are going to use is the pricing method of Alaton, Djehiche, and Stillberger, also known as ADS. Since we have the snow level derivative as a put option and the temperature derivative as a call option, we need to use the formulas for both put and call options, as mentioned in section 4.2.2.

$$p(t) = \theta e^{-r(t_n - t)} \left[(K - \mu_n) \left(\Phi(\alpha_n) - \Phi\left(-\frac{\mu_n}{\sigma_n} \right) \right) + \frac{\sigma_n}{\sqrt{2\pi}} \left(e^{-\frac{\alpha_n^2}{2}} - e^{-\frac{1}{2}\left(\frac{\mu_n}{\sigma_n}\right)^2} \right) \right]$$

$$c(t) = \theta e^{-r(t_n - t)} \left((\mu_n - K)\Phi(-\alpha_n) + \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\alpha_n^2}{2}} \right)$$

Since we are creating different weather derivatives for the different months, we have categorized the different parameters into those who are common for all months, and those who are different.

Table 5: Common parameters, ADS

Common Parameters	
θ_{SL}	1093,037
θ_T	308154,76
r_f	0,0188
π	3,14159

Table 6: Parameters, ADS

		Parameters				
		January	*February*	*March*	*April*	*December*
Snow level	$t_n - t$	0,08493151	0,07671233	0,08493151	0,08219178	0,08493151
	μ_n	1074,66667	1313,15385	1517,41026	594,430769	489,230769
	σ_n	737,591344	779,605145	876,75476	689,613162	336,257949
	α_n	-1,3214183	-1,5561132	-1,6166553	-0,7169683	-1,1575363
Temperature	$t_n - t$	0,08493151	0,07671233	0,08493151	0,08219178	0,08219178
	μ_n	-3,9798011	-3,354475	-0,2633532	3,19586452	-3,1397201
	σ_n	3,64406287	3,22501052	2,36700096	2,13409489	3,28232933

The Thetas (θ) indicate the payout in NOK per unit (centimeter or Celsius) over or under the strike level is the respective betas of the independent variables from the regression. To find the risk-free rate, we have used the yearly average of a ten-year Norwegian government bond for 2018, which is 1,88 percent (Norges Bank, 2019). $t_n - t$ is equal to $\frac{n}{365}$, where n is equal to the number of days in each of the respective months, this term of the equation will give us the correct risk-free rate for each of the months. μ_n is the mean of the cumulative snow level or average monthly temperature, and σ_n is the standard deviation, both illustrated in table 6. Also in this table, we have α_n which equals to $\frac{(Strike\ level\ -\mu_n)}{\sigma_n}$ for all the particular strike levels.

We have chosen to calculate the derivative prices for snow level in the range 100 to 2000, and for temperature from -5 to 3,5 degrees Celsius. An overview of the prices can be found in table 11 and 12.

5.1.2.2 Edgeworth and Historical

To calculate the derivative price, we are using these formulas for respectively historical densities and Edgeworth densities, as explained earlier.

$$p = e^{-r(t_n - t)} \frac{1}{\sum_{j=1}^{N} a_j(x)} \sum_{j=1}^{N} a_j(x) \max(K - \sum_{i=1}^{D} S_i(t_n), 0)$$

$$p = e^{-r(t_n - t)} \frac{1}{\sum_{j=1}^{N} f_j(x)} \sum_{j=1}^{N} f_j(x) \max(K - \sum_{i=1}^{D} S_i(t_n), 0)$$

To transform the put-formula into a call, we need to reverse the last term of the formula in both the equations from: $\max(K - \sum_{i=1}^{D} S_i(t_n), 0)$ into $\max(\sum_{i=1}^{D} S_i(t_n) - K, 0)$.

Table 7: Common parameters, historical and Edgeworth densities

Common Parameters	
r_f	0,0188
θ_{SL}	1093,037
θ_T	308154,76

Table 8: Parameters, historical and Edgeworth densities

		January	February	March	April	December
		Parameters				
Snow Level	1/∑a(x)	68,15209371	73,28539442	81,2175638	58,4813	30,4758978
	1/∑f(x)	66,94221408	76,33332606	82,5628912	49,5830238	30,5428749
Temperature	1/∑a(x)	0,362634173	0,332550737	0,22888271	0,22003854	0,32272769
	1/∑f(x)	0,406192162	0,358643019	0,25047287	0,22050821	0,31072125

The risk-free rate (r_f) and the thetas for snow level and temperature are found the same way as in the ADS calculations, including using $t_n - t$ equal to $\frac{n}{365}$, to find the monthly rate.

We start by calculating $a(x)$ and $f(x)$, and then we calculate the $1/\sum a(x)$ and $1/\sum f(x)$ as seen in table 8. When we have computed the put price from the price formula, we multiply the answer with the respective thetas, and we get the derivative prices for the different strike levels. Moreover, an overview of the prices can be found in table 11 and 12.

5.1.4 Indifference pricing method

In this paper, we are going to use indifference pricing as a method in addition to theory of weather to find the optimal strike levels, not the derivative price, opposed to HBA, ADS, Historical, and Edgeworth densities. The reason for this is that we already have many useful pricing techniques, but we think that conducting an estimation using the indifference pricing technique will give us further insight that will be helpful. Indifference pricing method gives us an insight of the negotiation process between buyer and seller. We conduct the indifferent pricing method by solving the following to equations.

$$F_s = \frac{1}{q_f}\left(E(W) - \frac{1}{2}\lambda_s k \sigma_W^2 \left(\rho_{q_s,W}^2 - 1\right) - \frac{\sigma_W}{\sigma_{q_s}}\left(\left(E(q_s) - q_f\right)\rho_{q_{s,W}}\right)\right)$$

$$F_b = \frac{1}{q_f}(E(W) + \frac{1}{2}\lambda_b k \sigma_W^2 \left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W}\right.$$

F_s calculates for a seller's perspective, and F_b for a buyer's perspective.

Table 9: Common parameters, indifference method

Common Parameters	
r_f	0,16 %
$E(q_s)$	100,57 %
$E(q_b)$	100,62 %
K	1,00
λs	0,0000010
RRA	1,5

Table 10: Parameters, indifference method

Parameters

	January	*February*	*March*	*April*	*December*
ARA	0,0000004274273	0,0000003293439	0,0000003571142	0,0000012761168	0,0000004606376

r_f is found using the same risk-free rate as in the previous pricing methods, divided by 12 months, giving 0,16%.

$E(q_s)$ and $E(q_b)$ are based on respectively $E(r_s)$, and $E(r_b)$ added 100 percent.

To calculate $E(r_s)$, we use the equation:

$$E(r_s) = MRP + r_f$$

So, to find $E(r_b)$ we are going to use the CAPM with an added Small Business Risk Premium. This gives us the equation:

$$E(r_b) = MRP * \beta + r_f + SBRP$$

The Markets Risk Premium (MRP) is based on a publication from PwC stating that the MRP of the Norwegian market is 5 percent (Pwc and The Norwegian Society of Financial Analysts, 2016). Since we are looking at it with a monthly perspective, we have to divide the MRP on 12 months.

To find the beta (β) of the market, we have gone across the border to Sweden and used the β of Skistar AB, which is traded on the Nasdaq Stockholm index. We acknowledge that this β may not be 100% representative to Voss ski resort, due to geographical basis risk and the fact that it is two different companies. However, in the calculation of indifference prices, we will make adjustments below to obtain a more valid measure. To find the β we had to perform regression using monthly data from January 1994, when Skistar AB went public an up until today's date. The regression is conducted by using two variables:

$$Excess\ return\ Skistar = Return\ Skistar - r_f$$

And

$$Risk\ Premium\ Market = Return\ Nasdaq\ Stockholm - r_f$$

It is important to emphasize that the risk-free rate used above is not the same as the risk-free rate used earlier. This risk-free rate is based on the Swedish 10-year government bond for each of the months from January 1994. The regression gives then a β of 0,505. The reason we are using Skistar is that they are controlling large portions of the ski resort business in both Norway and Sweden. We are adding a Small Business Risk Premium (SBRP) because Voss ski resort is a relatively small company, especially compared to the multi-billion conglomerate Skistar. We have based on information from an article in Magma magazine (Kaldestad, 2017) and a publication from (Pwc and The Norwegian Society of Financial Analysts, 2016) chosen to set the SBRP to 3 percent. The SBRP is a yearly risk premium, so we are dividing it on 12 months.

We are only going to look at indifference pricing with a contract size equal to 1 for simplicity. Therefore, $K = 1$.

The Relative Risk Aversion (RRA) is not straightforward to estimate. However, Gandelman and Hernandez-Murillo estimate that the RRA for Norway is around 1,16 (Gandelman & Hernández-

Murillo, 2015). Because of the uncertainty have we decided to use a precautionary approach, and set the RRA equal to 1,5. The Absolute Risk Aversion (ARA) is initially found by dividing the RRA with the operating income. Since we only operate with revenue, we will divide the RRA with the revenue.

$$ARA = \frac{RRA}{Revenue}$$

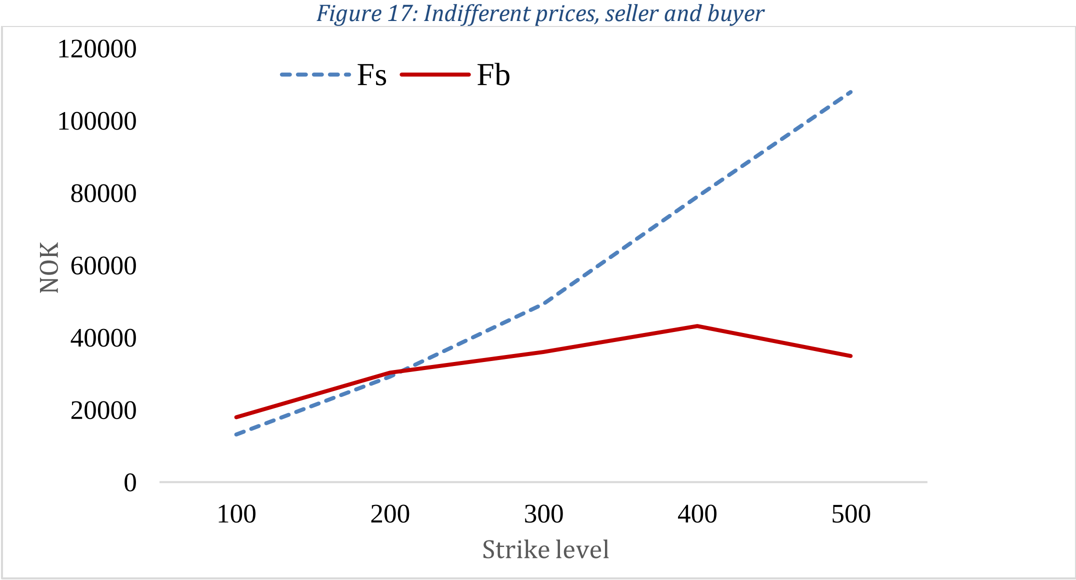
We will, therefore, get a different ARA for the different months as illustrated in table 10.

λ_s is the Absolute Risk Aversion for the seller. “The typical risk aversion parameters for market participants are around 10^{-6}” (Monoyios, 2004, p. 251). We have therefore chosen to set:

$$\lambda_s = 10^{-6}$$

In figure 17, the sellers’ and buyers’ indifferent prices, F_s, and F_b respectively, for December are illustrated.

Figure 17: Indifferent prices, seller and buyer



These illustrations, like this for December, will be helpful when we later discuss which strike prices to use in the portfolio.

5.2 Comparing pricing methods

In the following section, we would like to showcase the results of using different pricing methods. Illustrations in this section only cover the month of January and will be illustrated with tables and figures.

5.2.1 Derivative price

In the table below, we can see the relation of the weather derivative's price and strike level for different methods. When comparing the results of the different pricing methods, we can see that ADS has the lowest pricing range. ADS has the most expensive price at strike level 200, but at strike level 1100 it has the lowest price.

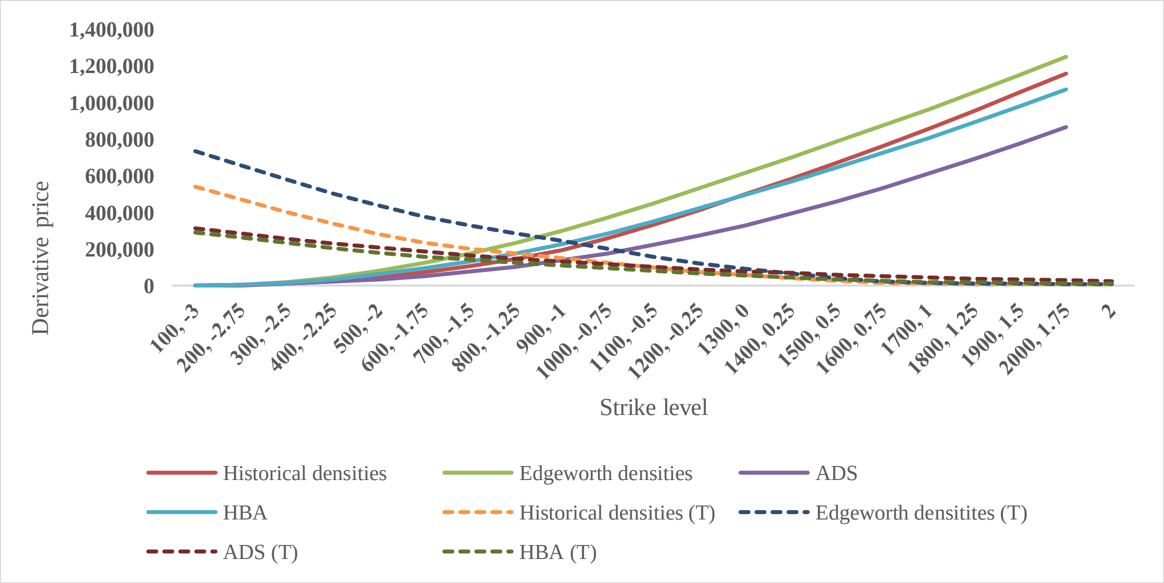
Table 11: Put option price (NOK), January

Strike	Historical densities	Edgeworth densities	ADS	HBA
100	-	-	1,092	-
200	1,605	3,127	4,655	2,589
300	10,399	19,674	11,146	15,504
400	23,068	42,893	21,043	33,280
500	45,851	81,403	34,830	61,900
600	72,850	125,778	52,977	94,749
700	104,602	175,323	75,926	131,711
800	144,745	232,983	104,065	175,576
900	194,954	297,957	137,711	226,144
1000	257,882	371,100	177,097	284,161
1100	331,153	449,652	222,355	349,628

Table 12: Call option price (NOK), January

Strike	Historical densities	Edgeworth densities	ADS	HBA
-3	540,307	734,171	313,075	290,105
-2.75	471,248	656,162	283,735	261,491
-2.5	402,189	578,152	256,386	232,876
-2,25	337,986	504,407	230,977	205,654
-2	279,558	435,973	207,452	180,121
-1.75	232,466	374,476	185,745	158,039
-1.5	200,993	326,555	165,786	140,870
-1.25	174,993	284,624	147,499	125,463
-1	148,993	242,693	130,804	110,055
-0.75	122,994	200,762	115,618	94,647
-0.5	96,994	158,832	101,855	79,240
-0.25	74,115	121,466	89,428	65,153
0	57,256	93,122	78,251	53,707
0.25	41,381	66,272	68,234	42,701
0.5	25,506	39,423	59,293	31,696
0.75	15,885	23,150	51,341	24,212
1	10,649	14,435	44,299	19,370
1.25	9,439	12,795	38,085	17,169
1.5	8,229	11,154	32,624	14,967
1.75	7,019	9,514	27,844	12,766
2	5,808	7,874	23,677	10,565

Figure 18: Temperature call option, snow level put option, and strike price



Visually, these graphs are in the intersection of linearity and convexity. Recall the methodology chapter. We highlighted the fact that lower cumulative snow level, thus high payoff, are more weighted in the historical densities and Edgeworth densities approach. Hence when a relatively high strike level is chosen for snow level derivatives, derivative price increase more in the methods as mentioned earlier. Stated differently – when the probability of payout grows, the price increase is more significant for densities approaches. Put-call-parity implies the opposite is true for the temperature-based call option. HBA weights all payoffs equally, with constant probability, which was one of the most significant drawbacks, considering it is not in line with the real distribution of cumulative snow level nor temperature.

5.2.2 Derivative payoff

In the tables below, 13 and 14 respectively, yearly payoff is calculated by deducting the sum of revenue with a derivative from the sum revenue without derivative at a set strike level and dividing it by the number of years in the period we are looking at, which is 41 years in this case. That way, we get an average yearly payoff in a particular month considered. HBA yields payoff close to zero, with the least variability, while ADS' performance has historically given highest payoff (ranking wise), as well as deviation from the mean. Averagely, density approaches give the most negative results due to the pricing mechanisms. Weather derivative are known as a zero-sum game, and as we can see in table 13, are Edgeworth densities and HBA the only one that gives a negative payoff at all strike levels. This means that historical densities and ADS gives the buyer a positive payoff, and a possible seller would therefore be reluctant to sell such a derivative. This increases Edgeworth densities and HBAs standings as the best pricing methods for this situation. For the temperature derivatives are we most interested in the temperatures below zero degrees Celsius. In table 14, we can see that all of the pricing methods has a negative yearly payoff when the strike level is below zero.

Table 13: Yearly average payoff 1978-2018, snow level put (JAN)

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
100	0	0	-1,090	0
200	795	-728	-2,249	-189
300	3,970	-5,305	3,240	-1,134
400	7,777	-12,048	9,833	-2,435
500	11,520	-24,032	22,592	-4,529
600	14,967	-37,962	34,916	-6,933
700	17,472	-53,249	46,258	-9,637
800	17,984	-70,253	58,817	-12,847
900	15,495	-87,507	72,940	-15,694
1000	9,005	-104,212	90,049	-17,273
1100	727	-117,773	109,849	-17,749

Table 14: Yearly average payoff 1978-2018, temperature call (JAN)

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
-3	-291,589	-485,453	-63,899	-41,386
-2.75	-248,024	-432,937	-60,096	-38,266
-2.5	-203,392	-379,355	-57,214	-34,079
-2.25	-162,427	-328,849	-55,081	-30,095
-2	-125,796	-282,211	-53,387	-26,359
-1.75	-97,554	-239,564	-50,562	-23,127
-1.5	-80,738	-206,299	-45,288	-20,615
-1.25	-67,891	-177,522	-40,181	-18,360
-1	-55,044	-148,744	-36,663	-16,105
-0.75	-42,197	-119,966	-34,652	-13,851
-0.5	-29,350	-91,188	-34,063	-11,596
-0.25	-18,497	-65,848	-33,680	-9,534
0	-11,408	-47,275	-32,289	-7,859
0.25	-4,928	-29,820	-31,682	-6,249
0.5	1,552	-12,365	-32,149	-4,638
0.75	4,783	-2,481	-30,598	-3,543
1	5,886	2,100	-27,699	-2,835
1.25	5,217	1,862	-23,373	-2,512
1.5	4,548	1,623	-19,799	-2,190
1.75	3,880	1,384	-16,905	-1,868
2	3,211	1,146	-14,623	-1,546

Figure 19: Yearly average payoff 1978-2018, snow level put

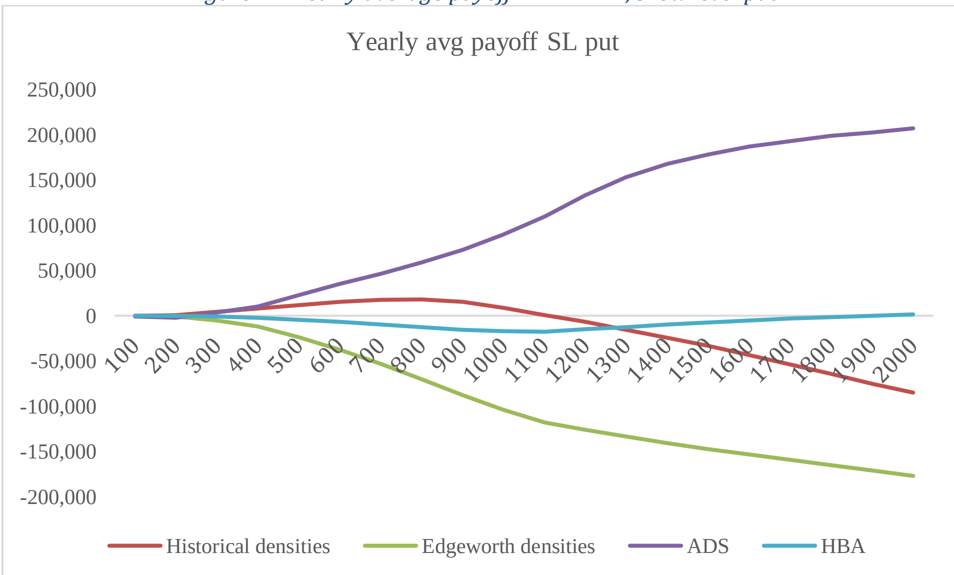
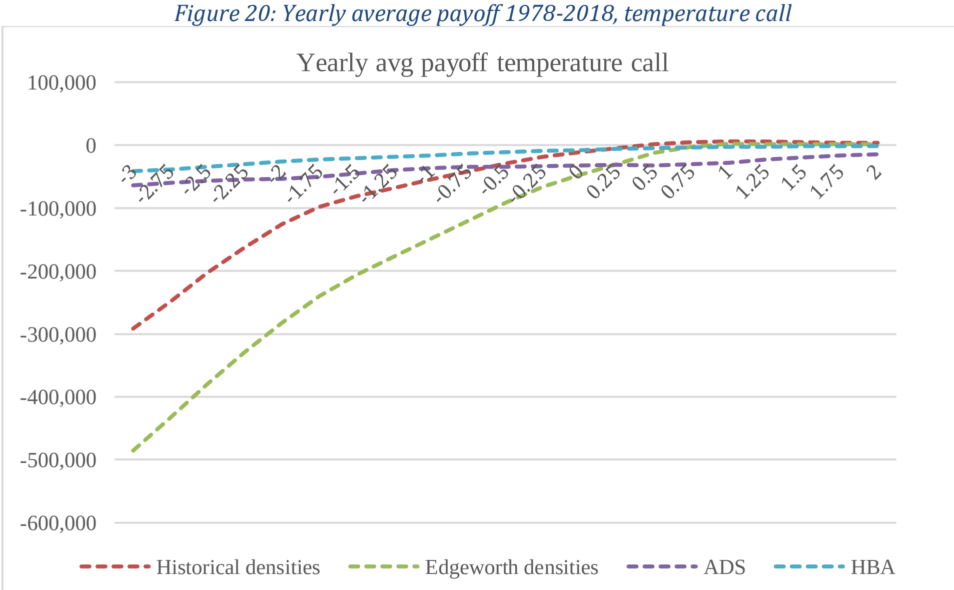


Figure 20: Yearly average payoff 1978-2018, temperature call



In table 15 & 16 total payoff for the period 1978-2018 is displayed. We are computing total payoff by subtracting the sum revenue with a derivative from the sum revenue without derivative at a given strike level.

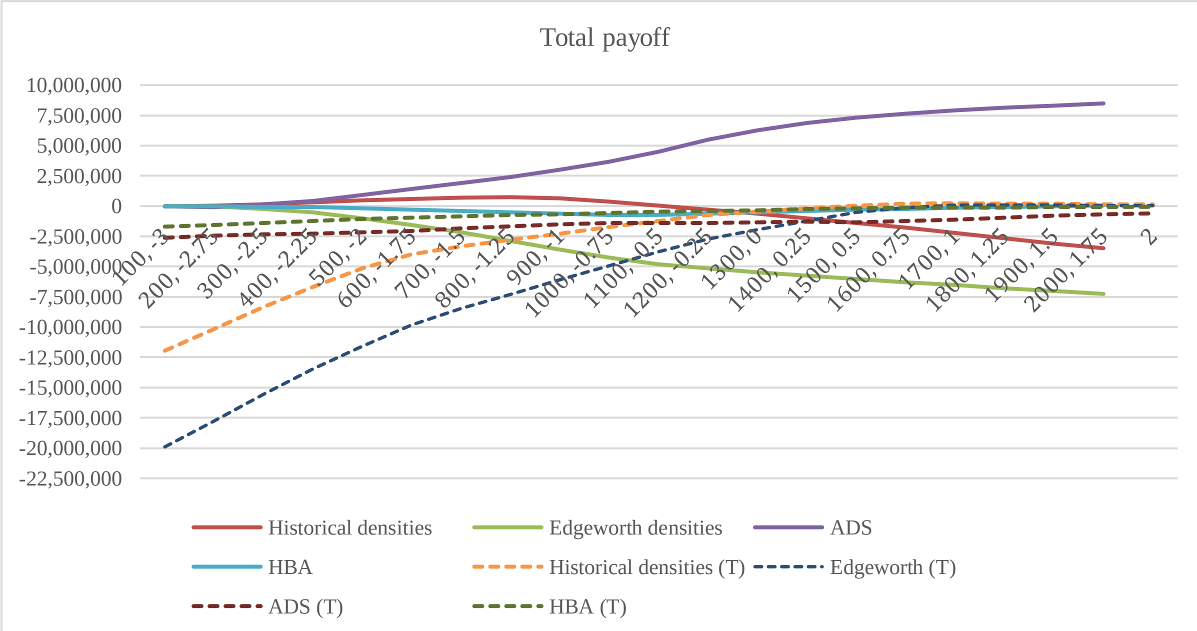
Table 15: Total payoff 1978-2018, snow level put

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
100	0	0	-29,780	0
200	51,122	2,331	-31,738	-7,594
300	133,430	-10,029	-38,833	-21,314
400	322,258	-113,912	176,915	-60,405
500	561,219	-290,905	477,312	-115,287
600	821,708	-546,355	819,664	-183,975
700	1,079,751	-817,401	1,046,329	-255,425
800	1,294,897	-1,221,142	1,419,806	-351,728
900	1,482,120	-1,670,952	1,738,017	-458,213
1000	1,572,925	-2,185,657	2,143,811	-587,306
1100	1,559,016	-2,726,200	2,591,100	-726,898

Table 16: Total payoff 1978-2018, temperature call

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
-3	-17,470,372	-23,165,450	-1,048,876	-1,746,165
-2.75	-15,040,093	-20,719,839	-967,860	-1,702,146
-2.5	-12,550,773	-18,215,187	-923,598	-1,599,087
-2.25	-10,229,842	-15,796,540	-907,047	-1,486,330
-2	-8,310,821	-13,608,996	-876,787	-1,391,686
-1.75	-6,495,349	-11,469,262	-827,682	-1,243,074
-1.5	-4,878,935	-9,465,233	-787,951	-1,089,880
-1.25	-3,418,388	-7,577,786	-784,647	-944,610
-1	-2,591,260	-6,245,266	-654,199	-834,645
-0.75	-1,841,287	-4,980,652	-573,294	-728,994
-0.5	-1,091,314	-3,716,038	-562,776	-623,343
-0.25	-483,604	-2,629,713	-556,547	-528,256
0	-148,704	-1,899,327	-488,425	-454,301
0.25	91,594	-1,301,729	-430,870	-388,268
0.5	331,892	-704,130	-425,552	-322,236
0.75	343,878	-516,250	-313,920	-282,617
1	355,864	-328,369	-245,699	-242,998
1.25	367,850	-140,489	-216,695	-203,379
1.5	379,836	47,392	-222,909	-163,759
1.75	343,070	92,354	-183,522	-137,347
2	306,304	137,317	-172,029	-110,934

Figure 21: Total payoff 1978-2018 all methods



5.2.3 Ratio of revenue & variability/risk

The first two tables in this section display revenue with derivatives (mean values in the period 1978-2018) for a given strike price divided by revenue without derivative. These are based on data from January. For snow level put options, a recurring feature is that HBA stays relatively close to 100% in the given strike range. ADS' ratio varies, while historical and Edgeworth densities decrease when strike price increases. Inversely, the different methods have comparable properties for temperature call options.

Table 17: Mean revenue with derivative divided by revenue without derivative. Snow level put.

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
100	100.00%	100.00%	99.98%	100.00%
200	100.01%	99.99%	99.96%	100.00%
300	100.07%	99.90%	100.06%	99.98%
400	100.14%	99.78%	100.18%	99.96%
500	100.21%	99.56%	100.42%	99.92%
600	100.28%	99.30%	100.64%	99.87%
700	100.32%	99.02%	100.85%	99.82%
800	100.33%	98.71%	101.08%	99.76%
900	100.29%	98.39%	101.34%	99.71%
1000	100.17%	98.08%	101.66%	99.68%
1100	100.01%	97.83%	102.02%	99.67%

Table 18: Mean revenue with derivative divided by revenue without derivative. Temperature call.

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
-3	94.64%	91.07%	98.82%	99.24%
-2.75	95.44%	92.04%	98.89%	99.30%
-2.5	96.26%	93.02%	98.95%	99.37%
-2.25	97.01%	93.95%	98.99%	99.45%
-2	97.69%	94.81%	99.02%	99.52%
-1.75	98.21%	95.59%	99.07%	99.57%
-1.5	98.51%	96.20%	99.17%	99.62%
-1.25	98.75%	96.73%	99.26%	99.66%
-1	98.99%	97.26%	99.33%	99.70%
-0.75	99.22%	97.79%	99.36%	99.75%
-0.5	99.46%	98.32%	99.37%	99.79%
-0.25	99.66%	98.79%	99.38%	99.82%
0	99.79%	99.13%	99.41%	99.86%
0.25	99.91%	99.45%	99.42%	99.89%
0.5	100.03%	99.77%	99.41%	99.91%
0.75	100.09%	99.95%	99.44%	99.93%
1	100.11%	100.04%	99.49%	99.95%
1.25	100.10%	100.03%	99.57%	99.95%
1.5	100.08%	100.03%	99.64%	99.96%
1.75	100.07%	100.03%	99.69%	99.97%
2	100.06%	100.02%	99.73%	99.97%

Below, we have two tables that illustrate the standard deviation of revenue with derivative in the period 1978-2018 divided by the standard deviation of revenue without derivative in the same period.

Table 19: Standard deviation of revenue with derivative divided by standard deviation of revenue without derivative. Snow level put.

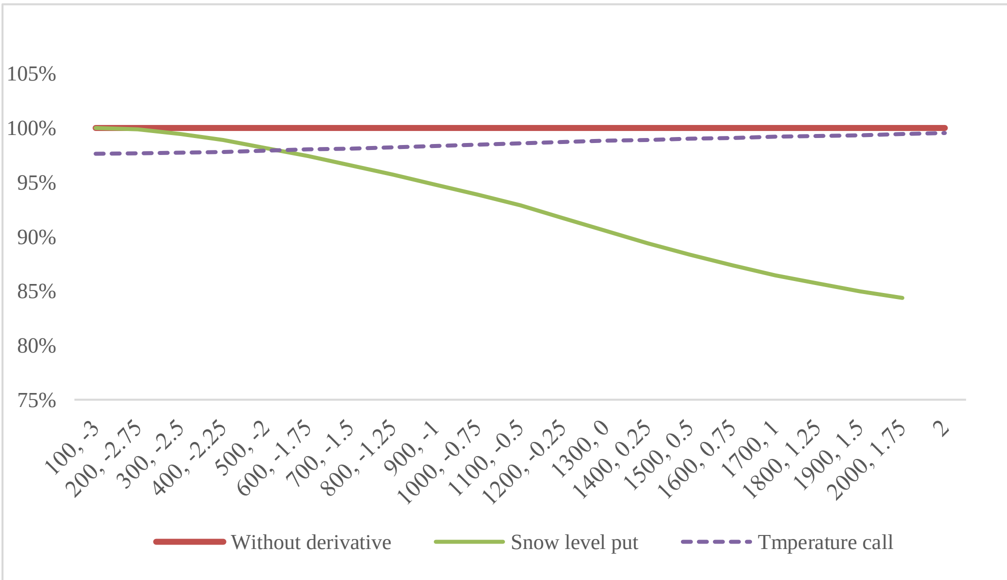
STRIKE	Historical densities	Edgeworth densities	ADS	HBA
100	100.00%	100.00%	100.00%	100.00%
200	99.90%	99.90%	99.90%	99.90%
300	99.45%	99.45%	99.45%	99.45%
400	98.91%	98.91%	98.91%	98.91%
500	98.18%	98.18%	98.18%	98.18%
600	97.39%	97.39%	97.39%	97.39%
700	96.58%	96.58%	96.58%	96.58%
800	95.68%	95.68%	95.68%	95.68%
900	94.76%	94.76%	94.76%	94.76%
1000	93.84%	93.84%	93.84%	93.84%
1100	92.89%	92.89%	92.89%	92.89%

Table 20: Standard deviation of revenue with derivative divided by standard deviation of revenue without derivative. Temperature call.

STRIKE	Historical densities	Edgeworth densities	ADS	HBA
-3	97.63%	97.63%	97.63%	97.63%
-2.75	97.68%	97.68%	97.68%	97.68%
-2.5	97.75%	97.75%	97.75%	97.75%
-2.25	97.80%	97.80%	97.80%	97.80%
-2	97.89%	97.89%	97.89%	97.89%
-1.75	98.02%	98.02%	98.02%	98.02%
-1.5	98.12%	98.12%	98.12%	98.12%
-1.25	98.23%	98.23%	98.23%	98.23%
-1	98.34%	98.34%	98.34%	98.34%
-0.75	98.47%	98.47%	98.47%	98.47%
-0.5	98.61%	98.61%	98.61%	98.61%
-0.25	98.71%	98.71%	98.71%	98.71%
0	98.81%	98.81%	98.81%	98.81%
0.25	98.91%	98.91%	98.91%	98.91%
0.5	99.03%	99.03%	99.03%	99.03%
0.75	99.07%	99.07%	99.07%	99.07%
1	99.17%	99.17%	99.17%	99.17%
1.25	99.26%	99.26%	99.26%	99.26%
1.5	99.35%	99.35%	99.35%	99.35%
1.75	99.44%	99.44%	99.44%	99.44%
2	99.54%	99.54%	99.54%	99.54%

One of this case study's primary purposes was using weather derivatives to hedge weather risk. These tables clearly show an equal reduction in standard deviation for all pricing methods, albeit a small one.

Figure 22: Illustration of standard deviation, with and without derivatives



The line chart suggests snow level derivative as a more efficient tool for reducing revenue variability compared to the temperature derivative.

5. 3 Constructing portfolios to reduce variance in revenue

5.3.1 Finding the optimal strike level and pricing method

When deciding the strike price, many variables need to be considered. The temperature is essential for it to snow and for the snow not to melt, but it is also essential for the production of snow through snow cannons. According to a publication from the Norwegian Department of Culture, the temperature should be below minus two degrees Celsius for a snow cannon to work optimal (Gjerland & Ødegaard Olsen, 2014). It is important to emphasize that even though the optimal condition is that the temperature is below minus two degrees, Celsius does it not mean that it has to be, nor does it mean that the average monthly temperature must be as low as minus two degrees Celsius. The mean of the average temperatures for the different months are ranging from -3,9 to 3 degrees Celsius, so the cost of a weather derivative at a strike level equal to -2 or lower will be substantial. We have therefore chosen to use strike levels in the range -2 to -1.

To find a strike level for the snow level derivative we will try to find a bare minimum snow level, that will still give the ski resort the possibility to run efficiently. The reason why we are looking at the bare minimum is that the weather derivatives purpose is, in this case, is to give the business a safeguard against the worst periods of snowfall. If we first look at it at a daily basis, then a 10 cm snow level would be a bare minimum, if we also assume an average of 30 days each month, will we have a monthly cumulative snow level of 300 cm. Since this is a bare minimum, we are also going to use a strike level of 400, 500 and 600. The reason for this increase in strike level is because 300 is the bare minimum and most months have significant higher snowfall and therefore are also the expectations of the costumers higher.

We have chosen to use Edgeworth densities as our preferred pricing method when creating our portfolios both for snow level and temperature. One of the reasons is that it takes into account that snow level is not normally distributed. Another reason is that Edgeworth densities gives the seller an extra risk premium, which could be necessary considering that the weather derivatives market still is uncharted territory for many.

5.3.2 Key summary of optimal strike levels

In the following tables, we present key summaries, such as derivative price, payoff, and revenue ratios, and standard deviation ratios, when using Edgeworth densities for the optimal strike levels.

Table 21: Derivative prices, Edgeworth densities

Derivative Price Edgeworth					
	Strike level	*January*	*February*	*March*	*December*
Snow Level	300	19,674	6,829	3,894	69,908
	400	42,893	21,440	10,639	136,875
	500	81,403	42,712	22,189	218,340
	600	125,778	70,164	36,239	306,123
Temperature	-2	435,973	532,459	2,975,418	1,024,098
	-1	242,693	271,076	1,936,982	438,125

Table 22: Yearly derivative payoff, Edgeworth densities

Derivative Payoff Edgeworth					
	Strike level	*January*	*February*	*March*	*December*
Snow Level	300	-5,305	-245	798	-27,359
	400	-12,048	-2,778	691	-46,147
	500	-24,032	-7,095	-275	-64,122
	600	-37,962	-13,326	-1,955	-76,166
Temperature	-2	-282,211	-331,927	-2,374,633	-858,156
	-1	-148,744	-152,324	-1,571,995	-369,984

Table 23: Mean revenue with derivatives as a fraction of mean revenue without derivatives

Mean revenue with derivative divided by mean revenue without					
	Strike level	*January*	*February*	*March*	*December*
Snow Level	300	99.90%	100.00%	100.01%	99.21%
	400	99.78%	99.96%	100.01%	98.66%
	500	99.56%	99.89%	100.00%	98.14%
	600	99.30%	99.79%	99.97%	97.79%
Temperature	-2	94.81%	94.86%	67.13%	75.15%
	-1	97.26%	97.64%	78.24%	89.29%

Table 24: Standard deviation of revenue with derivatives as a fraction of the standard deviation without

Standard deviation of revenue with derivative (Edgeworth)

	Strike level	*January*	*February*	*March*	*December*
Snow Level	300	99.45%	99.86%	99.91%	97.27%
	400	98.91%	99.56%	99.79%	95.31%
	500	98.18%	99.17%	99.61%	92.84%
	600	97.39%	98.69%	99.39%	90.17%
Temperature	-2	97.89%	96.54%	102.13%	93.01%
	-1	98.34%	97.36%	101.64%	95.75%

As seen in this key summary, an investment in any derivative, except March temperature, will decrease the standard deviation of the revenue. The mean revenue will also decrease or stay approximately the same.

5.3.3 Creating portfolios with minimum variances in revenue

The matrix below shows the weight combinations of snow level derivative and temperature derivative, which yield the lowest possible portfolio variance at a given strike level combination.

Table 25: Minimum variance weights

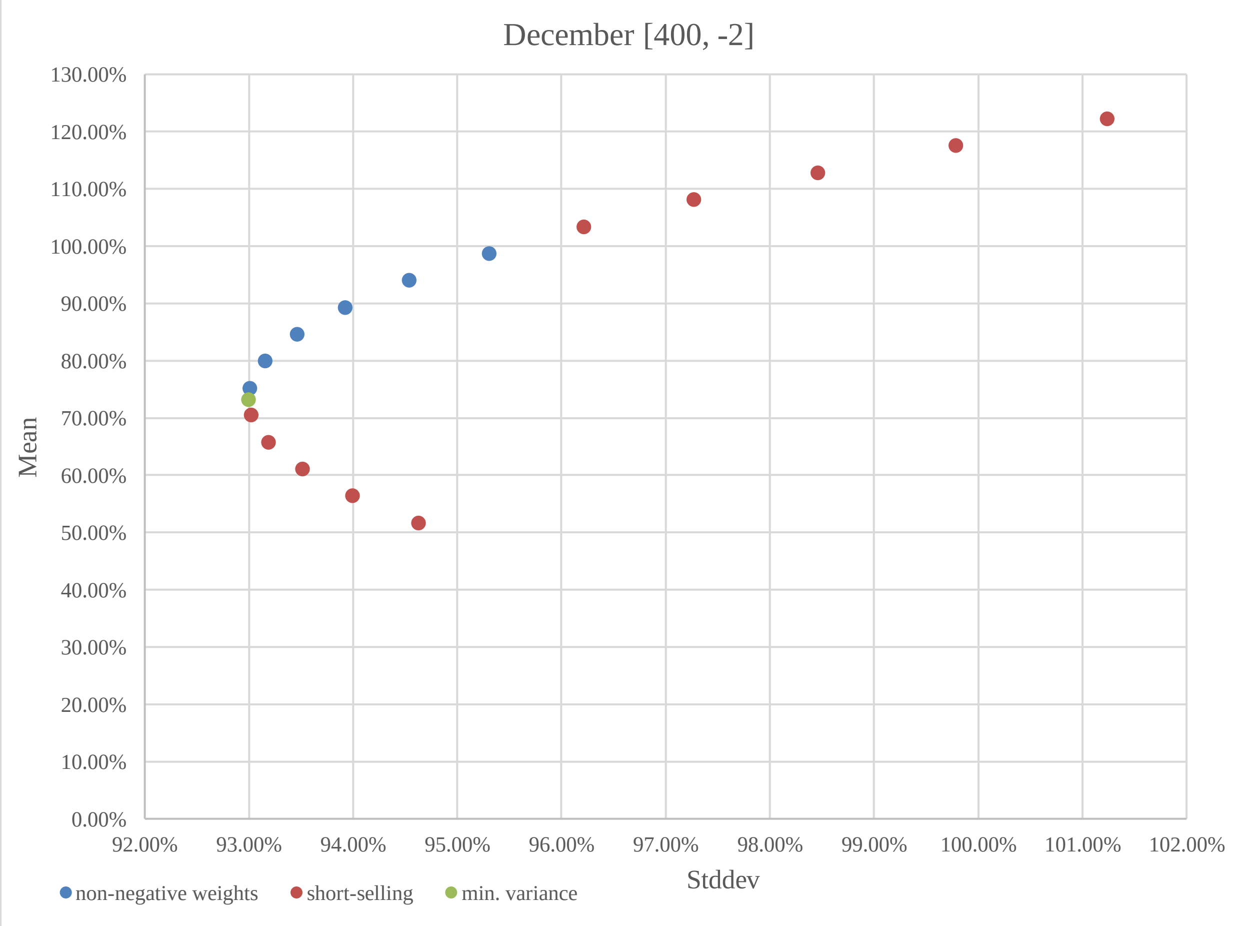
Minimum Variance Weights of Portfolios

	January		*February*		*March*		*December*	
Strike levels	SL	T	SL	T	SL	T	SL	T
300, -1	-151.98%	251.98%	-368.71%	468.71%	124.52%	-24.52%	-84.25%	184.25%
300, -2	-89.45%	189.45%	-266.79%	366.79%	159.28%	-59.28%	-72.02%	172.02%
400, -1	-60.38%	160.38%	-333.50%	433.50%	140.78%	-40.78%	79.89%	20.11%
400, -2	-46.81%	146.81%	-246.56%	346.56%	170.73%	-70.73%	-8.73%	108.73%
500, -1	80.21%	19.79%	-268.43%	368.43%	167.41%	-67.41%	209.61%	-109.61%
500, -2	22.09%	77.91%	-211.30%	311.30%	189.94%	-89.94%	54.11%	45.89%
600, -1	206.81%	-106.81%	-187.88%	287.88%	194.92%	-94.92%	296.95%	-196.95%
600, -2	93.89%	6.11%	-169.70%	269.70%	209.69%	-109.69%	113.72%	-13.72%

From portfolio theory, recall that our model has no restrictions on non-negative weights, which means short-selling is not disallowed. For example, in January at strike price 300, to minimize portfolio variance, short-sell the snow level derivative and go long in the temperature derivative. In stock trading, a relevant argument against this strategy would be the possibility for unlimited losses (if the stock price increases). The same risk does not apply in this option-trade since cumulative snow levels cannot have negative values, and the maximal potential loss is $300 * \theta$. If we were to (short)-sell a temperature call, there are natural risk limitations as well, considering a temperature's volatility cannot be compared to stocks.

The distribution of weights is very uneven, heavily favoring one derivative. Only in December, at strike combination 500, -2, we get approximately one half in each derivative. We want to showcase one portfolio in an example below.

Figure 23: Portfolio frontier - a combination of snow level derivative and temperature derivative



In figure 23, we have drawn the portfolio frontier. The horizontal axis displays the portfolio standard deviation as a fraction of the standard deviation of revenue without derivative in December, while the vertical axis returns the portfolio mean as a fraction of mean revenue without derivative in December. Blue dots indicate that the portfolio weights are non-negative, i.e. $0 \leq w_i \leq 1, i = 1{,}2$, in this area. Red dots highlight negative weights, while the green dot marks the combination that yields the lowest possible portfolio variance. Investors will never adapt to a point below this on the frontier, due to inefficiency. Ski resorts can set a specific expected return, and minimize risk, or set a certain level of risk and maximize expected return. Regardless, there are several ways to spend initial wealth. It will always be a trade-off between risk and return, as well as the buyer's risk aversion. A risk-neutral investor would try to maximize mean-variance-ratio, $\frac{R_p}{\sigma_p}$. In this example, one can argue that it is not worth losing approximately 25% of the company's income to reduce variance by 7%

Table 26: Expected values and standard deviations of portfolios with derivatives as ratios of revenue without derivatives

	Without derivative	**400, -2**	**400, -1**	**300, -2**	**300, -1**	**600, -2**	**600, -1**	**500, -2**	**500, -1**
December									
E	100%	73.10%	96.78%	57.82%	80.93%	100.90%	114.55%	87.59%	107.85%
SD	100%	93.00%	95.28%	92.08%	95.34%	90.12%	85.53%	92.40%	91.73%
January									
E	100%	92.48%	95.75%	90.25%	93.25%	99.03%	101.48%	95.86%	99.10%
SD	100%	97.77%	98.25%	97.43%	97.70%	97.39%	97.05%	97.86%	98.17%
February									
E	100%	82.28%	89.91%	81.15%	88.95%	86.48%	93.59%	84.22%	91.60%
SD	100%	93.33%	94.09%	87.68%	93.17%	95.10%	96.37%	94.23%	95.28%
March									
E	100%	123.26%	108.89%	119.50%	105.35%	136.00%	120.60%	129.56%	114.66%
SD	100%	99.57%	99.74%	99.75%	99.89%	98.83%	99.08%	99.23%	99.45%

Short-selling is not necessarily a possibility. Few if any financial institutions would be willing to facilitate a shorting where they would most likely lose money. The only companies who would possibly be interested are local businesses in Voss who would benefit from hedging the risk of too high temperatures or too little snow. So, the only possibilities are to find such business or pay an extra premium to a financial institution. Since it could be difficult to short weather derivatives, we have chosen to also look at it under the assumption that shorting is not possible.

Table 27: Minimum variance weights - short-sales not permitted

Minimum Variance Weights of Portfolios

	January		*February*		*March*		*December*	
Strike levels	SL	T	SL	T	SL	T	SL	T
300, -1	0%	100%	0%	100%	100%	0%	0%	100%
300, -2	0%	100%	0%	100%	100%	0%	0%	100%
400, -1	0%	100%	0%	100%	100%	0%	79.89%	20.11%
400, -2	0%	100%	0%	100%	100%	0%	0%	100%
500, -1	80.21%	19.79%	0%	100%	100%	0%	100%	0%
500, -2	22.09%	77.91%	0%	100%	100%	0%	54.11%	45.89%
600, -1	100%	0%	0%	100%	100%	0%	100%	0%
600, -2	93.89%	6.11%	0%	100%	100%	0%	100%	0%

Table 28: Expected values and standard deviations of portfolios with derivatives as ratios of revenue without derivatives. Short-sales not permitted

	Without derivative	**600, -2**	**600, -1**	**500, -2**	**500, -1**	**400, -2**	**400, -1**	**300, -2**	**300, -1**
December									
E	100 %	97,79 %	97,79 %	87,59 %	98,14 %	75,15 %	96,78 %	75,15 %	89,29 %
SD	100 %	90,17 %	90,17 %	92,40 %	92,84 %	93,01 %	95,28 %	93,01 %	95,75 %
January									
E	100 %	99,03 %	99,30 %	95,86 %	99,10 %	94,81 %	97,26 %	94,81 %	97,26 %
SD	100 %	97,39 %	97,39 %	97,86 %	98,17 %	97,89 %	98,34 %	97,89 %	98,34 %
February									
E	100 %	94,86 %	97,64 %	94,86 %	97,64 %	94,86 %	97,64 %	94,86 %	97,64 %
SD	100 %	96,54 %	97,36 %	96,54 %	97,36 %	96,54 %	97,36 %	96,54 %	97,36 %
March									
E	100 %	99,97 %	99,97 %	100,00 %	100,00 %	100,01 %	100,01 %	100,01 %	100,01 %
SD	100 %	99,39 %	99,39 %	99,61 %	99,61 %	99,79 %	99,79 %	99,91 %	99,91 %

When excluding the possibility of shorting, the expected value decreases, and the standard deviation increases. So, it is no doubt that that shorting would be beneficially, but it could be challenging to do.

Table 29: Expected value and standard deviation of revenue of final portfolio

		Shorting allowed				*Shorting Not allowed*			
		December	**January**	**February**	**March**	**December**	**January**	**February**	**March**
	Without derivative	**(600, -1)**	**(600, -1)**	**(300, -2)**	**(600, -2)**	**(600, -2/-1)**	**(600, -1)**	**(any, -2)**	**(600, -2/-1)**
E	100 %	114,55 %	101,48 %	81,51 %	136,00%	97,79 %	99,30 %	94,86 %	99,97 %
SD	100 %	85,53 %	97,05 %	87,68 %	98,83 %	90,17 %	97,39 %	96,54 %	99,39 %

When deciding which strike levels to use, we have chosen the levels based on the minimum variance principle. It is possible to use various principles, and none is better than the other. For example, could we have used the highest expected value or best Sharpe ratio, because peoples' adaptations are based on personal preferences. It is important to emphasize that all calculations of expected value and standard deviations are made using 40 years of historical data and are therefore not an accurate prediction of the future, but an illustration of historical values. In table 29, we can see the end-portfolio for each of the months, with and without the possibility of shorting.

5.4 Conclusion

In conclusion, we can see that different pricing methods gives widely different prices. However, all the pricing methods we have chosen to use returns a lower variation in revenue. We can also conclude that by using the Edgeworth densities pricing method and combining the two weather derivatives, snow level, and temperature, to a portfolio, will further reduce the variance in revenue. The strike levels and the weighting of snow levels and temperatures in each portfolio depend on personal preferences. We have chosen to use the principle of minimum variance and have ended up with the portfolio illustrated in table 29.

Our main objective in this paper has not been to create a perfect hedging tool for Voss ski resort. It has been to show that even though the market for weather derivatives often is characterized by imperfections, uncertainties and lack of knowledge, that it is possible to use weather derivatives to reduce the risk of unfavorable weather, not just on paper but also in real life.

Although we have been able to use weather derivatives as a tool of risk management, there still exists imperfections and weaknesses that are important to address. Basis risk, as explained in depth previously, is an imperfection that should always be taken into consideration. In chapter 3, the adjusted coefficient of determination confirms that revenue variation is not explained in full by the independent variable(s), which is more or less the definition of basis risk. In the same chapter, we demonstrated the challenges of collecting data, more specifically gathered data from multiple weather stations, none of them 100% representative for Voss Ski Resort. Gathering data from different stations no located at the ski resort contributes to an increase in the geographical spatial/basis risk.

For further research and improvement, we will recommend looking at the possibility of using multivariate, and/or non-linear regression when calculating the tick size used in derivative pricing. This could improve the adjusted coefficient of determination, which in turn should decrease basis risk. Using not only simple linear regression, would most likely further improve weather derivatives as a risk management tool.

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Appendix

A.1 Indifferent pricing method.

For both the seller and buyer assume utility as a function of risk aversion and revenue.

$$u(X) = -e^{-\lambda X} \qquad (1)$$

Revenue without derivative, X_s^{wo}, depends on initial wealth, x_s, and amount invested in capital markets, a_s.

$$X_s^{wo} = (x_s - a_s)q_f + a_s q_s \qquad (2)$$

$$q_f = 1 + r_f, \qquad q_s = 1 + r_s, \qquad q_b = 1 + r_b \quad (3)$$

r_b is buyer's return, while r_s is seller's return, and r_f is risk free rate of return.

X_s^w is the seller's revenue with a derivative. K is the number of units, and F_s is the derivative price a seller needs. W_T is the derivative's payoff, which depend on the underlying weather index I.

$$X_s^w = (x_s - a_s + kF_s)q_f + a_s q_s - kW_T(I) \qquad (4)$$

Equalizing equation (2) and equation (4):

$$\sup_{a_s} E[u(X_s^{wo})] = \sup_{a_s} E[u(X_s^w)] \qquad (5)$$

Approximating the certainty equivalent, $\hat{X}$, of the utility function using Taylor expansion or Pratt's theorem in equations in six to nine. μ_x represents expected wealth, while $\tilde{X}$ is stochastic wealth.

$$U(\hat{X}) = E[U(\tilde{X})] \qquad (6)$$

$$\rightarrow \hat{X} = \mu_x - \frac{1}{2}\lambda\sigma_x^2 \qquad (7)$$

- Put (7) into (6) and get (8).

$$\sup_{a_s}\left[E(X_s^{wo}) - \frac{1}{2}\lambda_s\sigma^2(X_s^{wo})\right] = \sup_{a_s}\left[E(X_s^w) - \frac{1}{2}\lambda_s\sigma^2(X_s^w)\right] \qquad (8)$$

- Setting (3), then (4), into (7), we get expressions for certainty equivalents with and without derivative respectively. E denotes expectation, λ the risk parameter (absolute), and σ is either variance or co-variance.

$$CE_s^{wo} = x_s q_f + a_s\big(E(q_s) - q_f\big) - \frac{1}{2}\lambda_s a_s^2 \sigma_{q_s}^2 \qquad (9)$$

$$CE_s^{w} = x_s q_f + k\, F_s q_f + a_s\big(E(q_s) - q_f\big) - kE(W) - \frac{1}{2}\lambda_s a_s^2 \sigma_{q_s}^2 - \frac{1}{2}\lambda_s k^2 \sigma_W^2 + \lambda_s a_s k \sigma_{q_{s,W}} \qquad (10)$$

- $F.O.C$: $\frac{\partial CE_s^{wo}}{\partial a_s} = 0$

$\rightarrow \frac{\partial CE_s^{wo}}{\partial a_s} = E(q_b) - q_f - \lambda_s a_s \sigma_{q\ s}^2 = 0$

$$a_s^{wo*} = \frac{E(q_s) - q_f}{\lambda_s \sigma_{q\ s}^2} \qquad (11)$$

- $F.O.C$: $\frac{\partial CE_s^{w}}{\partial a_s} = 0$

$\rightarrow \frac{\partial CE_s^{w}}{\partial a_s} = E(q_s) - q_f - \lambda_s a_s \sigma_{q\ s}^2 + \lambda_s k \sigma_{q_{s,W}} = 0$

$$a_s^{w*} = \frac{E(q_s) - q_f + \lambda_s k \sigma_{q_{s,W}}}{\lambda_s \sigma_{q\ s}^2} \qquad (12)$$

- We set (11) into (9) and (12) into (10) and solve for F_s to find the indifference price for the seller. ρ is the correlation coefficient.

$$F_s = \frac{1}{q_f}\left(E(W) - \frac{1}{2}\lambda_s k \sigma_W^2\big(\rho_{q_{s,W}}^2 - 1\big) - \frac{\sigma_W}{\sigma_{q_s}}((E(q_s) - q_f)\rho_{q_{s,W}}\right),$$

$$\pi_s = -\frac{1}{2}\lambda_s k \sigma_W^2\big(\rho_{q_{s,W}}^2 - 1\big) - \frac{\sigma_W}{\sigma_{q_s}}((E(q_s) - q_f)\rho_{q_{s,W}} \qquad (13)$$

$$F_s = \frac{1}{q_f}E(W) + \pi_s \qquad (14)$$

- Buyer's solution

$$a_b^{wo*} = \frac{E(q_b) - q_f}{\lambda_b \sigma_{q_b}^2} \tag{15}$$

$$a_b^{w*} = \frac{E(q_b) - q_f - \lambda_b k \sigma_{q_b,W}}{\lambda_b \sigma_{q_b}^2} \tag{16}$$

$$F_b = \frac{1}{q_f}\left(E(W) + \frac{1}{2}\lambda_b k \sigma_W^2 \left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W}\right.\right.,$$

$$\pi_b = \frac{1}{2}\lambda_b k \sigma_W^2 \left(\rho_{q_b,W}^2 - 1\right) - \frac{\sigma_w}{\sigma_{q_b}}\left(\left(E(q_b) - q_f\right)\rho_{q_b,W}\right. \tag{17}$$

$$F_b = \frac{1}{q_f}(E(W) + \pi_b) \tag{18}$$

A.2 Portfolio theory

Activum j. $(j = 1,2, \dots, N)$

Return: R_j

Expectation: $\mu_j = E(\tilde{R}_j)$

Variance: $\sigma_j^2 = \sigma_{jj} = Var(\tilde{R}_j)$

Co-variance: $\sigma_{jk} = Cov(\tilde{R}_j, \tilde{R}_k)$

Vector/matrix notation:

Return vector $(Nx1)$: $\tilde{R} = (\tilde{R}_j)$

Expectation vector $(Nx1)$: $\mu = (\mu_j)$

Variance co-variance matrix (NxN): $V = (\sigma_{jk})$

Weight vector $(Nx1)$: $W = (W_j)$

Inverted variance co-variance matrix (NxN): V^{-1}

Transposition operator: T

Assumptions:

- Linearly independent activas. The variance-covariance matrix **V** is symmetric and positive definite, such that the symmetric, inverse variance-covariance matrix V^{-1} exists.

- At least two activas have different expectation

Portfolio relations and matrix rules:

- Stochastic return: $\tilde{R}_p = W'\tilde{R} = \tilde{R}'W$
- Mean return: $\mu_p = E(\tilde{R}_p) = W'\mu = \mu'W$
- Variance: $\sigma_p^2 = Var(\tilde{R}_p) = W'VW$
- N-summation vector: $e_N = \mathbf{1}$ $(a\ Nx1\ vector\ of\ ones)$
- $0_N = \mathbf{0}$ $(a\ Nx1\ vector\ of\ zeros)$
- Note that negative weights ("short sales") are not precluded.

The optimization problem is to choose portfolio weights that:

- Minimize the portfolio variance σ_p^2
- Subject to a given portfolio mean $\mu_p = \bar{\mu}$
- And subject to the budget restriction $\sum_{j=1}^{N} x_j = 1$

$$\min_{W} \frac{1}{2} W^T V W$$

$$s.t. \quad W^T = \bar{\mu}$$

$$s.t. \quad W^T e = 1$$

- Lagrange: $L = \frac{1}{2} W^T \boldsymbol{V} W - \lambda_1 [W^T - \bar{\mu}] - \lambda_2 [W^T e - 1]$
- Lagrange multipliers: $\lambda_1(\bar{\mu})$ and $\lambda_2(e)$
- F.O.C

$$\frac{\partial \mathrm{L}}{\partial \mathrm{W}} = \mathrm{VW} - \lambda_1 \mu - \lambda_2 e = 0_N$$

- Reformulated: $VW = \lambda_1 \mu + \lambda_2 e$
- Premultiplying by the inverse inverse covariance matrix V^{-1} to find the weights of the frontier portfolio $W = \lambda_1 V^{-1} \mu + \lambda_2 V^{-1} e$ (**)

- The optimal weight for asset $j = 1,2,..,N$ in frontier portfolio having mean $\bar{\mu}$:

$$x_j = \lambda_1(\bar{\mu}) \sum_{k=1}^{N} V_{jk}^{-1} \mu_k + \lambda_2(\bar{\mu}) \sum_{k=1}^{N} V_{jk}^{-1} \qquad (1)$$

- Where V_{jk}^{-1} is the element in j-th row and k-th column of V^{-1}.
- Premultiply (**) by mean vector: $\mu^T W = \lambda_1 \mu^T V^{-1} \mu + \lambda_2 \mu^T V^{-1} e$
- And then by summation vector: $e^T W = \lambda_1 e^T V^{-1} \mu + \lambda_2 e^T V^{-1} e$
- Definition of "information constants":

$$a \equiv \mu^T V^{-1} \mu > 0$$
$$b \equiv \mu^T V^{-1} e = e^T V^{-1} \mu$$
$$c \equiv e^T V^{-1} e > 0$$
$$d \equiv ac - b^2 > 0$$

- The left-hand side are, respectively, the desired mean $\bar{\mu}$ and the number one, giving two equations in the unknown Lagrange multipliers:

$$a\lambda_1 + b\lambda_2 = \bar{\mu}$$
$$b\lambda_1 + c\lambda_2 = 1$$
$$\lambda_1 = \lambda_1(\bar{\mu}) = \frac{c\bar{\mu} - b}{d}$$
$$\lambda_2 = \lambda_2(\bar{\mu}) = \frac{a - b\bar{\mu}}{d}$$

- *All* frontier portfolios (without any additional restrictions on the weights) satisfy:

$$\sigma_p^2 = \frac{\left(a - 2b\mu_p + c\mu_p^2\right)}{d}$$