

### **Early mathematics - Teacher communication supporting the pupil's agency**

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## Early mathematics - Teacher communication supporting the pupil's agency

### Introduction

I pretend that I ponder over the same things as they do (...) Maybe, try to be one of them instead of being the teacher, playing ball with them (...) in that way, they get used to be the ones who are supposed to find out things. Even if it is difficult to understand the task, they don't give up (Pamela).

As a researcher on a Norwegian intervention project in the early learning of mathematics, I have, over a three-year period, regularly been a part of 'Pamela's' mathematics classroom. When we started our project, I knew Pamela as a skilled teacher in the early grades, who held a master's degree in teaching mathematics. Aiming to enhance the pupils' ability in mathematical reasoning, we developed teaching designs informed by Brousseau's *Theory of Didactical Situations in Mathematics* (TDS) (Brousseau 1997). In TDS, the roles of the pupils and the teacher are rigorously regulated in different phases. The teacher's mathematical knowledge is crucial when designing tasks and creating the learning environment needed to develop the target knowledge. After handing the planned task over to the pupils, the teacher holds back until the pupils have completed the required activity and have formulated and shared their strategies. Without being too active herself, the teacher's role in this phase is to ensure that the pupils engage in mathematical thinking. Being amateurs in mathematics classrooms with pupils aged 7-10 years, I and the other researchers experienced how difficult it was to follow this plan, in other words, avoiding teaching actions that deprive the pupils of their ownership of the problem. There were always multiple invitations from the pupils to give them hints about the answer, for guidance on how to proceed, or for assurance that they were moving in the right direction. As strongly motivated

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3 researchers wanting to succeed with the teaching design, we were easily tempted to accept  
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5 their invitations. Lacking the communicative competence needed, we often ended up not  
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7 knowing whose problem we solved, the student's or ours.  
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10 In contrast, observing Pamela, I was impressed by how she managed to scaffold the learning,  
11  
12 while the problems and the strategies were still the pupils' own, when she left the scene. I  
13  
14 was struck by an immediate impression of professional teacher communication that I wanted  
15  
16 to explore more systematically. As Pamela states in the initial excerpt in this article, it is her  
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18 intention to communicate in a way that supports the pupils' ownership to the mathematical  
19  
20 problem and help them to endure the uncertainty of the situation. With this as a background, I  
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22 ask: *What characterizes the communication of a skilled teacher when scaffolding young*  
23  
24 *pupils working with mathematics?*  
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### 30 **Recent research and theory**

31  
32 A shift from traditional teaching to teaching informed by the social constructivist paradigm  
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34 changes the role of the teacher to what in Wood, Bruner and Ross's (1976) terminology can  
35  
36 be described as a scaffolder of pupils' learning. However, how to perform the role of a  
37  
38 responsive guide in developing pupils' own thinking varies. Anghileri (2006) describes three  
39  
40 different levels of scaffolding: *environmental provisions* (Level 1), *explaining, reviewing and*  
41  
42 *restructuring* (Level 2) and *developing conceptual thinking* (Level 3). Level 1 scaffolding  
43  
44 typically includes structured tasks, artefacts and classroom organisations, sequencing, pacing  
45  
46 events, and grouping. Peer collaboration is one such environmental provision. The tasks can  
47  
48 include self-correcting elements. Level 2 scaffolding is direct involvement between the  
49  
50 teacher and the pupils specifically related to the mathematics being considered. The teacher  
51  
52 responds to refocus the pupils' attention in such scaffolding. In a reviewing process, the  
53  
54 teacher is getting the students to look, touch and verbalize, to explain and justify. She also  
55  
56 interprets the pupils' actions and speech, uses prompting and probing questions and parallel  
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3 modelling. Through restructuring actions like providing meaningful contexts, simplifications,  
4  
5 rephrasing and negotiating meanings, the teacher makes ideas more accessible and brings the  
6  
7 pupils' understanding forward. Level 3 scaffolding takes the learning yet another step  
8  
9 forward aiming to develop concepts through specialized processes such as generalisation,  
10  
11 extrapolation and abstraction.  
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14  
15 The importance of language and social interaction for learning mathematics has been  
16  
17 emphasized both theoretically (Vygotsky [1934] 1987) and in research (Mercer and Sams  
18  
19 2006). However, research on mathematics in Norwegian classrooms indicates that there is  
20  
21 little time for dialogue between teachers and pupils, or among pupils (Skorpen 2006). Despite  
22  
23 knowledge on how important mathematical reasoning is for children's later achievements  
24  
25 (Nunes et al. 2009), pupil interventions are often not taken up by the teacher (Bjørkås and  
26  
27 Bulien 2010).  
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32 Even though numerous studies have pointed to valuable teacher strategies in dialogic  
33  
34 teaching, Howe et al. (2019) assert that many studies are either too small in scale or unclear  
35  
36 regarding learning outcomes to serve as compelling evidence. In their review, they present  
37  
38 five recurring themes in the proposals about characteristics of dialogic classroom discourses:  
39  
40 (1) open questions are used; (2) participants make extended contributions elaborating and  
41  
42 building on previous contributions made by themselves or others; (3) differences of opinions  
43  
44 are acknowledged, probed and critiqued, bringing in the reasons on which they are based; (4)  
45  
46 integrated lines of enquiry are used for linking the different contributions and resolving  
47  
48 differences; and (5) pupils are made aware on a metacognitive level of the value of verbal  
49  
50 interaction. In their review of small-group research in which pupils worked independently of  
51  
52 teachers, Howe et al. only found sufficient support for characteristics (3) and (5); reasoned  
53  
54 discussion of differences of opinion support both academic attainment and general reasoning,  
55  
56 as do 'awareness of good practice achieved through negotiation and display of such "ground  
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3 rules” as “We give reasons to explain our ideas” and “If we disagree, we ask why” (6). The  
4  
5 results in Howe et al.’s large-scale study were consistent with the three first characteristics;  
6  
7 the teacher-pupil dialogues manifested high levels of pupil participation, elaboration and  
8  
9 querying. For teacher communication during small group work, the findings imply that open  
10  
11 questions should be used, that the teacher needs to withhold her demonstrations, that  
12  
13 explanations permit pupil reflection and appraisal and, as proposed by Littleton and Mercer  
14  
15 (2010), that scaffolding of cumulative and explorative talk is needed.  
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20  
21 Deciding what to attend to and what not in teacher-pupils dialogues is a major challenge for a  
22  
23 teacher. Identifying important mathematical moments and using them productively is difficult  
24  
25 even for experienced teachers (Chamberlin 2005). In a qualitative Finnish study of dialogic  
26  
27 teaching patterns in mathematics and two other subjects in early school years, Muhonen et al.  
28  
29 (2016) found two teacher-initiated and two child-initiated patterns. The teacher-initiated  
30  
31 dialogues were characterized by the utilisation of diverse strategies and focused on  
32  
33 maintaining the interactional flow, while the teacher’s scaffolding in the child-initiated  
34  
35 dialogues served more as facilitation through listening and inquiry. Studying single turns and  
36  
37 their role in the teacher-pupil-dialogue in Norwegian mathematics classrooms in a larger  
38  
39 study with both quantitative and qualitative data, Drageset (2015) found that most of the  
40  
41 pupil-utterances were part of sequences where the teacher controlled the process and the  
42  
43 pupils respond to basic tasks that look like mere control questions. As the pupils typically  
44  
45 answer the questions correctly, the dialogue often ends without any further exploration of the  
46  
47 theme. By asking for one detail at the time, the teacher also, in many cases, reduces the  
48  
49 complexity of the task to the student. This affects the relationship between the teacher and the  
50  
51 pupils, changes the task and, consequently, also changes the target knowledge. Such  
52  
53 simplification processes are described by Brousseau (Strømskag 2017) as the Topaze effect.  
54  
55 The second most frequent pupil intervention in Drageset’s study, however, the ‘unexplained  
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3 answer' received about half as many follow-ups from the teacher, asking the pupil to recap or  
4  
5 for a justification. To make use of the opportunities the pupil interventions provide to explore  
6  
7 the hidden thinking involved, Drageset sees a need for studies on how to balance between  
8  
9 focusing actions and progressing actions. Before proceeding to the methodology of the  
10  
11 project, I present the TDS to explain Pamela's teaching context.  
12  
13

### 14 15 **The theory of didactical situations in mathematics**

16  
17 The TDS (Brousseau 1997) is a scientific approach to the problems related to the teaching  
18  
19 and learning of mathematics. The rigour in the description of different phases in the model  
20  
21 ensures both space for pupils to work on problem-solving and to formulate their own  
22  
23 thoughts, and space for focused use of dialogues between pupils and between the pupils and  
24  
25 the teachers. In the following, I explain the dynamics of the TDS and some concepts based on  
26  
27 Strømskag (2017, 2011) with emphasis on the teacher's role.  
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31  
32 In the TDS, the teacher is seen to have two main roles: one is to create and devolve a 'milieu'  
33  
34 that enables the students to develop particular knowledge by solving a problem (devolution);  
35  
36 the other is to decontextualize or generalize the situated knowledge developed by the pupils  
37  
38 and transform it into cultural knowledge that the pupils can reuse in other situations  
39  
40 (institutionalisation). In between these two didactical phases, the situation should be  
41  
42 adidactical according to Brousseau's teaching model. An adidactical situation is  
43  
44

45  
46 'a situation where the [pupil] takes a mathematical problem as their own and solves it  
47  
48 based on the problem's inner logic without the teacher's guidance and without trying  
49  
50 to find out the teacher's intention with the problem' (Strømskag 2017, 75, my  
51  
52 translation).  
53  
54

55  
56 The milieu that the pupils are going to act on might comprise: the problem to be solved;  
57  
58 material or symbolic tools (informative texts, centicubes, numberlines, or other artefacts  
59  
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3 relevant to the situation); the pupil's prior knowledge; other pupils; and classroom  
4  
5 arrangements and rules for operating in the situation (e.g., who is supposed to interact with  
6  
7 whom).

8  
9  
10 There are three such didactical phases: action, formulation and validation (fig. 1). In the  
11  
12 action phase, the pupils act to solve the problem without teacher interventions. In the  
13  
14 formulation phase, the pupils formulate their strategy enabling somebody else to solve the  
15  
16 problem, while the situation of validation is where the pupils attempt to explain some  
17  
18 phenomenon or verify a conjecture.  
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22  
23 (Figure 1 here)

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25  
26 The interaction between the teacher and the pupils is regulated in the 'didactical contract' in  
27  
28 TDS-informed teaching (Strømskag 2017, 75). This contract is negotiated in the devolution  
29  
30 phase, establishing a set of mutual commitment and expectations. The contract and the  
31  
32 quality of the designed milieu enables a temporary transfer of responsibility to the pupils in  
33  
34 the didactical phases. An appropriate didactical 'milieu' provides feedback to the students,  
35  
36 making substantial teacher interventions redundant. The feedback might be incorporated in  
37  
38 the task itself or e.g., in the organisation of the work.  
39

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41  
42 According to Strømskag's (2017, 76) description, the teacher's role in the three didactical  
43  
44 phases in TDS-informed teaching are: no interventions (action); to support the exchange of  
45  
46 observations and to make different formulations visible in the classroom (formulation); and to  
47  
48 be the facilitator of a scientific debate, to structure the debate and to motivate the pupils to  
49  
50 use more precise mathematical concepts (validation). With reference to Anghileri (2006), I  
51  
52 claim that the scaffolding in the didactical phases lies between Level 1 and Level 2  
53  
54 scaffolding. The teacher is supposed to be a facilitator, taking some steps reviewing the  
55  
56 pupils' thinking, but leaving most of the restructuring actions to the institutionalisation phase.  
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## Methodological questions

### *Context and participants*

This study is part of a four-year, intervention-based classroom project *PROJECT* carried out in two primary schools in CITY, Norway. The main objective of the project was to study pupils' development and use of mathematical language in order to gain knowledge that will help teachers to develop their teaching. By planning lessons and designing activities informed by the TDS in collaboration with skilled primary teachers, the aim was to develop a greater understanding of how pupils in Years 1-4 develop mathematical proficiency, and how this connects to teaching. As researchers, we entered the process together with the teachers without any ready-made solutions, engaging in a co-learning agreement, where both parties were engaged in actions and reflections (Wagner 1997). Together, we changed and developed practices as well as co-constructed knowledge and understanding.

In this article, I present a study of one of the two *PROJECT* teachers, Pamela, and her communication with pupils, while they were in the middle of very complex processes, solving mathematical problems and formulating their strategies. Case studies are suitable for describing such complex school practices (Gudmundsdottir 2001) and for understanding practices in depth (Stake 1995). Merriam (1998) finds case studies to be a particularly suitable design if you are interested in processes, an attribute of high relevance for this study.

### *Analysis of audiotapes from lessons*

To understand the content and function of Pamela's communication during what was intended to be the didactical phase (action, formulation and validation), I transcribed and analysed audiotapes from this phase during three different lessons. The analysis rests fully on these transcripts. As researcher, I was present during the lessons, but my focus was on the pupils' work. One lesson was from the very start of the intervention project in Spring 2015, when the pupils were in Year 2 (L1), the two others are from two different interventions in



1  
2  
3 Spring 2016 (L5 and L6). The chosen lessons are representative of how all seven PROJECT-  
4 lessons in Pamela's school were organized. Variation in mathematical focus and data  
5 collected over time were the reasons for choosing these exact three. In all three cases, the  
6 didactical phase lasted for approximately 30 minutes, and the pupils were working with the  
7 concept 'a half' (halving the numbers 7, 11 and 13), subtraction understood as difference and  
8 multiplication (a two-digit number multiplied with a single-digit number). In all, I have  
9 identified 40 dialogues of varying length between Pamela and pairs of pupils in the dataset.  
10  
11 To analyse the teacher utterances, I used a framework developed by Drageset (2015) for  
12 detailed studies of mathematical discourses on a turn-to-turn basis. Such sociocultural  
13 discourse analysis focuses on the functions of language for the pursuit of joint intellectual  
14 activity. Drageset's framework provides three superordinate categories of teacher  
15 interventions and five superordinate categories of pupil interventions. However, to answer  
16 my research question, what characterizes the communication of a skilled teacher in early  
17 mathematics, I have only used the part of the framework concerning the teacher's utterances,  
18 as shown in fig. 2.  
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39 (Figure 2)

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42 In *redirecting actions*, the teacher redirects the pupils' attention by either asking a correcting  
43 question, advising a new strategy or putting aside a pupil's comment. *Progressing actions*  
44 include the teacher's different ways of moving the lesson forward. The actions simplification  
45 and closed progress detail, asking a specific question with typically only one correct answer,  
46 are both used to move the pupils one-step ahead by simplifying the problem. Open progress  
47 initiatives are also used to move the work forward, but such questions do not limit the  
48 possible responses from the pupil. The fourth option is to provide a demonstration, that is  
49 when the teacher takes over and solves the problem by himself. *Focusing actions* are actions  
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3 the teacher uses to put emphasis on certain aspects. The teacher can request pupil input by  
4 asking them to enlighten details, for justification or to apply the learning to a similar problem.  
5  
6 He can also focus by requesting assessment from other pupils, by recapping at the end of the  
7 dialogue or by making the pupils notice something.  
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13 By using existing concepts and codes, there is a danger of reporting within the established  
14 expectations and missing important details (Klette 2010). Coding Pamela's utterances  
15 sentence by sentence, I found that some of her utterances did not fit Drageset's framework.  
16  
17 Looking closer at the nature of the utterances that did not fit, I found them to be more general  
18 class management comments. Utterances like "You must go to your working partner now"  
19 and "It might be smart to tidy up and make some space [for the figures] here" are different  
20 from the other utterances that had a weaker connection to the task and the mathematics. The  
21 fact that the pupils in my study were younger (year 2-4) than the pupils in Drageset's study  
22 (year 5-7) can explain the high number of such utterances. As I registered a substantial  
23 decrease in these teacher interventions, from being nearly one-third of the teacher talk in the  
24 first lesson to being only one-sixth of the teacher talk in the last lesson (see Table 1), I have  
25 chosen not to go deeper into this part of Pamela's communication.  
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41 Brousseau (1997) developed the TDS based on observations and experiments in schools over  
42 many years. The didactical situations (tasks, instructions, artefacts) used in this study were  
43 tested only once or twice. Thus, we had fewer possibilities to assure the feedback potentials  
44 of our didactical 'milieus'. This might explain some of the need for teacher interventions.  
45  
46 Even if the length of the didactical situations in the chosen cases were similar, differences in  
47 task construction will influence the results of the analysis. Thus, tendencies in the numbers of  
48 utterances within each category must be read with a critical stance. Bearing this in mind, I  
49 regard reoccurring patterns of communication on the superordinate level as a more valid  
50 indicator of Pamela's communication in the didactical phase.  
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### *Interview with Pamela*

To get an even deeper understanding of Pamela's communication skills, I conducted a semi-structured interview with her. The topics in focus were: her role as a teacher in the didactical phase of the teaching, examples of necessary communication, the need for adapting her communication to different pupils and different tasks and her motivation to let TDS inform her teaching in the future. A part of the interview was based on transcripts of three dialogues Pamela had with three different pairs of pupils. By picking out three shorter written dialogues, I hoped to set a focus on what I have found to be typical for her communication. The first dialogue was a situation in L1 where she helped two second graders compare their answers, a situation I find many examples of in the data material. I also find the second dialogue from L6 typical, showing a situation where Pamela kept the pupils focused on something essential in the task by posing simple questions or giving short supporting comments without revealing too much of the answer. [The third example was part of an unusually long dialogue in L5 where Pamela took several steps to scaffold the learning of a pupil, who seemed to have multifaceted problems.](#) I experienced that the three transcripts focused our talk in the interview. At the same time, without framing the situations in my own language, I could ensure that my analysis of her role as a teacher was reasonable, using member checking (Lincoln and Guba 1985).

### *Ethics*

The project plan had been approved by the Norwegian Centre for Research Data (NSD) and I followed the guidelines of the National Committee for Research Ethics in the Social Sciences and the Humanities in Norway (NESH 2018). Pseudonyms are used in all examples. Ethical care and credibility have also been addressed through the processes of informed consent (Bogdan and Biklen 2003). As PROJECT is an intervention project where we have come close to Pamela and her pupils over four years, I come close to 'studying my own success' -

1  
2  
3 even though the focus is on Pamela. Being aware of possible pitfalls, it has been important to  
4  
5 monitor my subjective I's (Peshkin 1988) in conducting the study. Because I initially was so  
6  
7 impressed by Pamela's actions in the classroom, I found it necessary to go through the  
8  
9 categorizing an extra time and review it on some points with a colleague in PROJECT.  
10  
11

## 12 13 **Findings**

14  
15 *I ask: What characterizes the communication of a skilled teacher when scaffolding young*  
16  
17 *pupils working with mathematics?* As shown in Table 1, Pamela's communication with the  
18  
19 pupils in the didactical phases consisted mostly of a high number of progressing (B1-4) and  
20  
21 focusing (C1-6) actions while the number of redirecting actions (A1-3) was generally low in  
22  
23 all lessons. I start by presenting the open progress initiatives (B4) and the high number of  
24  
25 closed progress details (B3) that characterised Pamela's progressing actions, before I proceed  
26  
27 to her focusing actions. Finally, I comment on Pamela's use of redirecting actions (A1-3),  
28  
29 demonstrations (B1) and simplifications (B2) in L5.  
30  
31  
32

33  
34 The presentation is based on my analysis, building on Drageset's (2015) framework and  
35  
36 mirrored by Pamela's utterances from the interview. Excerpts from the interview are used to  
37  
38 make the situations more transparent to the readers.  
39  
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41  
42 (Table 1 here)  
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## 48 ***Attends to the pupils***

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50 In all three lessons, Pamela used both a high number of open progress initiatives (B4) and a  
51  
52 high number of closed progress details (B3) to stimulate the pupils to move forward in their  
53  
54 work. Both teacher interventions were often formed as questions, either as open questions to  
55  
56 their strategy like 'How will you share these to get half each?', or as questions with typically  
57  
58 one correct answer, like 'How much older is he then?'  
59  
60

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3 The number of open progress initiatives (B4) were kept high through all three lessons.  
4  
5 Pamela walked between the desks and checked the work of each group of pupils, normally  
6  
7 opening the dialogue by asking ‘How are you doing?’ and ‘What did you find out?’ When the  
8  
9 pupils started explaining, Pamela keeps them going by adding new open questions or short  
10  
11 responses like yes!/yes?’, ‘okay./okay!/okay?’, ‘hm?’, ‘mm?’, ‘aha?’ and ‘super!’. The  
12  
13 following illustrative excerpt is from the very end of a longer dialogue in L6 where Pamela  
14  
15 was talking with George and Nora and comparing two ways of outlining the multiplication  
16  
17 task, as  $12 \cdot 2 =$  and  $10 \cdot 2 + 2 \cdot 2 =$ :  
18  
19  
20  
21

22 Pamela: Why do you think they have written it like this? Divided it like this? (points  
23  
24 to the left and then to the right side of the sheet.)  
25  
26

27 George: Because we are supposed to learn to add?  
28  
29

30 Pamela: Do you think that’s why?  
31  
32

33 Nora: That we have learned a long time ago.  
34  
35

36 Pamela: Yes, that’s right? Hm?  
37  
38

39 George: Or maybe we are going to learn to add multiplication problems?  
40  
41

42 Pamela: Mm?  
43  
44

45 George: For example, add four times ten and four times four? Add them?  
46  
47

48 Pamela: Mm? Yes?  
49  
50

51 What we see here is that Pamela introduced a new theme, the outlining of the task, by posing  
52  
53 an open question (B4) and by making the pupils notice that twelve in the left task, to the  
54  
55 right, is divided into ten and two (focusing action, C6). After another open question, she  
56  
57 follows up with several short responses, which I have categorised as open progress initiatives  
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3 (B4). This short excerpt shows a typical pattern in Pamela's dialogues with pupils, while they  
4 are working. She is what can be characterised as a very active listener, confirming that she  
5 listens and finds their thoughts important and that she wants them to go on speculating. At the  
6 same time, she avoids evaluating their thinking. Occasionally, when she seems satisfied with  
7 the pupils' work, as in this situation, such an open utterance ends the dialogue. But mostly  
8 these utterances are open progress initiatives in the middle of the dialogues.  
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17 Pamela confirmed in the interview that her communication was about reviewing and  
18 encouraging the pupils to continue working based on their initial ideas. She described her role  
19 in TDS-informed teaching like this:  
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23  
24

25 I believe that the teacher's role is to observe what the pupils are doing. And then talk  
26 with them while they are working, trying to get them to explain what they have done,  
27 put into words what they have thought, what they struggle with or what they want to  
28 try. And not giving them any input, but maybe ask some questions that might, well,  
29 not exactly push them forward, but maybe get them to think something new or get  
30 them to pursue a thought they already had and clarify it.  
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39 Pamela unveiled that this is a role that she likes and often misses in teaching, she says,  
40 'because I find it so very interesting when [the pupils] start working with something, think for  
41 themselves. And what they end up with something before anyone has told them anything'.  
42 Being part of PROJECT, she has grown even more fond of teaching through scaffolding the  
43 pupils' own thinking. Pamela also confirmed that she uses all the short open-ended responses  
44 as motivation to pursue a thought and explains:  
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54 For example, when George says that maybe you want us to add the two multiplication  
55 problems, he obviously is unsure. So, then I try not to confirm what he says, but  
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3 instead, to motivate him, to elaborate on his explanation, or to think about it a little  
4  
5 longer.  
6  
7

8 In this situation, she said she thought the pupils were able to continue alone. In other  
9  
10 situations, the pupils needed something ('a straw' to grasp at or something to 'build with').  
11  
12

### 13 ***Scaffolds comparison processes***

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16 Based on Pamela's description of her preferred role, one could expect high numbers of open  
17  
18 progress initiatives (B4). However, the number of closed progress details (B3) were also  
19  
20 high, even higher than the open progress initiatives in the first lesson. Questions categorised  
21  
22 as close progress details looked like mere control questions with little potential for the pupils  
23  
24 to elaborate on. In Pamela's teaching however, my analysis shows that such questions served  
25  
26 an important role in her scaffolding for processes of comparison. The following is an excerpt  
27  
28 from a situation in L1 where this was the case. The task was to compare how many circles,  
29  
30 stars and squares the pupils had after sharing them equally, each getting half of each sort:  
31  
32  
33

34  
35 Pamela: Let's see. Now I want to see, now you must check that you have got the  
36  
37 same.  
38

39  
40 Lucas: Okay.  
41

42  
43 Pamela: Let's see the circles first. (...) How many do you have, Lucas?  
44

45  
46 Lucas: Seven.  
47

48  
49 Pamela: How many do you have, Nadia?  
50

51  
52 Nadia: Two.  
53

54  
55 Pamela: Do you have half each?  
56

57  
58 Lucas: No.  
59  
60

1  
2  
3 Pamela: No. Then you have to share equally so you each get half of them.  
4  
5

6 Lucas: Like this. Now we have half each.  
7  
8

9 As described, the TDS designs normally include tasks where the pupils are supposed to argue  
10 for their own strategies, and communication skills regarding processes of comparison are  
11 needed to succeed. Thus, I claim that Pamela's questions, which appear as closed progress  
12 details, were model questions that the pupils needed to ask each other to compare their  
13 thoughts. In the interview, Pamela claimed that the formulation, and especially in the  
14 beginning, the validation phase was quite hard for the pupils:  
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23 Because they were so busy solving the task. To prioritise to talk together - that was  
24 not easy. That is something they have become much better at during the project. (...) I  
25 remember that I somehow had to put some pressure on [them] to get them to compare  
26 and talk together.  
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33 According to Pamela, learning this kind of collaborative talk was one of the most important  
34 outcomes of PROJECT for the pupils. By playing the role of an active partner in the  
35 comparison processes, she felt she had helped them to develop their communication skills.  
36  
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39  
40 She said:  
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42

43 In second grade, they weren't that good at listening to each other. Even less to answer  
44 each other. But that's what a teacher does. In addition, I could give a little support  
45 when they were explaining.  
46  
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### 51 *Asks questions to clarify the pupils' own thoughts*

52

53 Pamela related that she is preoccupied with how she can adapt her questions to each pupil's  
54 thinking. Ahead of the lessons, she always thinks through what she believes the different  
55 pupils will manage to find out in the constructed 'milieu', she explained why:  
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3 Because, when I start talking with them I already have some thoughts about what they  
4 most likely have started to think (...) What will they initially think? What will they  
5  
6  
7 discover next? What will the most advanced find out, and what will the others find out?  
8  
9

10 Pamela knows her pupils well, and through such preparations, she believes she will be a  
11 better scaffolder during the didactical phase. In addition to the open progress initiatives  
12 (B4), this is where the high number of focusing actions (C1-6) occurred. The increasing  
13 tendency of such focusing actions, both regarding the numbers and especially as the  
14 percentage of Pamela's total talk shows in Table 1, was probably related to the task design.  
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22 However, what I find interesting discussing my research question was the selection of such  
23 focusing actions in Pamela's communication as shown in both Table 1 and fig. 3.  
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25

26  
27 (Figure 3 here)  
28

29  
30 Requests for justifications (C2) and asking the pupils to notice something (C6) seemed to  
31 dominate her communication in the didactical phase in all three lessons. There was also an  
32 increasing number of utterances where Pamela requests for assessment from the pupil's  
33 working partner (C4) and utterances where she recaps the pupils' solutions (C5). **In this  
34 excerpt, Pamela started by asking for a justification (C2) from Naomi before she asked  
35 Bianca to assess (C4) Naomi's answer:**  
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44  
45 **Pamela: How did you find the difference between dog, one year and daughter, seven  
46 years?'**  
47

48  
49  
50 **Naomi: We counted on our fingers.**  
51

52  
53 **Pamela: Is it six years difference? Please, check it out, Bianca?**  
54  
55

56 When asked in the interview about what she believed to be important teacher support in the  
57 didactical phase, Pamela answered:  
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3 I think they need questions that clarify their own thoughts. That [the teacher] must get  
4 the pupils to justify what they have found out and believe. And maybe, ask them to  
5 show with an example or to explain why they believe what they believe.  
6  
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10 I find it reasonable to interpret Pamela's many requests for justifications as a result of the  
11 TDS design and the earlier mentioned concern Pamela had regarding the pupils' skills in  
12 collaborative communication. For example, Pamela's question to Lucas in L1 ('How do you  
13 know that you have got half [of the stars] now?') served like an open progress initiative and  
14 helped Lucas and Nadia to prepare for the next phases, in arguing for their strategy in the  
15 validation phase and then possibly sharing their thoughts with the whole class in the  
16 institutionalisation phase. To be able to formulate and validate their thinking, Pamela finds it  
17 important that they 'clarify their own thoughts'.  
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29 Many of Pamela's notices (C6) and recaps (C5) had a similar function. For example, by  
30 directing Naomi's and Knuth's attention to similarities and differences regarding their  
31 number of circles, stars and squares in L1, they were able to continue with the halving. At the  
32 same time, the pupils now had a basis for comparing and arguing. Pamela commented on her  
33 actions in the interview:  
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42 Well, what I do, I ask them to look at what they have done and check that it is ...they  
43 obviously think that they have solved the task and are done. And then I get them to  
44 look at it again, and then they see. In that way, they confirm themselves that, yes, we  
45 have got half each.  
46  
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51 In this utterance, Pamela confirmed my earlier claim, the pupils' agency was important for  
52 her, they should confirm the answers themselves.  
53  
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55

56 The high number of notice actions (C6) also seemed to relate closely to the task design, e.g.,  
57 in the mentioned situation with George and Nora in L6, where Pamela spent a lot of the time  
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1  
2  
3 directing the pupils' attention towards similarities and differences between the problems in  
4  
5 the left and the right column in fig. 4.  
6  
7

8 (Figure 4 here)  
9

10 In the interview, she explained:  
11  
12

13  
14 The point of this task was that they should find out that the sum of the answers to the  
15  
16 two problems to the right was the same as the answer in the left column. Then [when  
17  
18 the pupils hadn't seen that] I tried to focus on why. Why have they done it like this?  
19  
20 Without giving any input, just trying to get them to look at something that might take  
21  
22 them further.  
23  
24  
25

26 In such comparison processes, Pamela regularly asked for each pupil's opinion and if they  
27  
28 agreed with each other. Again, this was an initiative preparing the pupils and herself for the  
29  
30 next step forward, the institutionalisation phase, where Pamela as a teacher was supposed to  
31  
32 decontextualize the situated knowledge developed by the pupils and transform it into cultural  
33  
34 knowledge. At the same time, the ownership of the problem and the solution was still with  
35  
36 the pupils.  
37  
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39

#### 40 41 *Adapted teaching* 42

43 As shown in Table 1, Pamela's communication in L5 stood out as somewhat different from  
44  
45 the two other lessons regarding the use of redirecting actions, especially advising new  
46  
47 strategy (A2), use of demonstrations (B1) and simplifications (B2). The analysis showed that  
48  
49 most of these teacher interventions ending with Pamela taking over and solving the task  
50  
51 herself by telling the pupil what to do, took place in two shorter dialogues and one long  
52  
53 dialogue (10 minutes and 16 seconds) with Sarah and Knuth. While Knuth had solved the  
54  
55 task, Sarah seemed to have multifaceted problems related to motivation and language  
56  
57 problems as a user of Norwegian as a second language. However, Pamela's communication is  
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1  
2  
3 the focus here, and the analysis showed that before turning to simplifications, demonstration  
4 and redirecting actions, Pamela tried to adapt her communication to Sarah's situation. She  
5 took several open detail initiatives and used different focusing actions, building either on  
6 Sarah's own or Knuth's thinking. For example, on one occasion, she asked Sarah to check out  
7 Knuth's answer (C4) and in another, she asked Sarah to observe Knuth demonstrating how he  
8 used his fingers to find the difference between the age of two people (C1). Neither of these  
9 focusing actions nor an application of the problem to what Pamela expects to be a familiar  
10 situation (C3), the age difference between Sarah and her two years younger sister, worked  
11 out.  
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24 In the interview, Pamela revealed that her reason for eventually giving Sarah more direct  
25 instruction was that she found it necessary to avoid demotivation and collapse. At the same  
26 time, she underlined that this is a practice that goes against her actual belief, 'I actually don't  
27 think this is appropriate. I think there is more learning in getting it right with the support I can  
28 give than getting it right through instructions.' For Pamela, this was about the pupils  
29 developing strategies that would be available for them in their future work, 'In the  
30 institutionalisation phase, they can hear about many methods. (...) But starting with their  
31 own, I think is smart.' On this point, she failed with Sarah in this situation.  
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### 43 **Discussion**

44 My research question was: *What characterizes the communication of a skilled teacher when*  
45 *scaffolding young pupils working with mathematics?*  
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50 To answer the question, I have analysed Pamela's communication in what was intended to be  
51 the didactical phase of three TDS-informed lessons. To sum up, the analysis showed that  
52 Pamela's communication formed a pattern that was different from the classroom discourse  
53 described in recent Norwegian research. While Bjørkås and Bulien (2010) found that pupil  
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3 interventions often are ignored by the teacher, Pamela showed active interest in interventions  
4  
5 with each pupil. We do not know from this study how Pamela brought the pupils' thinking  
6  
7 into the whole class discussions, but what we see is that she, to a very little extent, corrected  
8  
9 their answers or suggested new strategies through redirecting actions (A1-3) in the  
10  
11 preparatory phase. While Drageset (2015) found that most of the pupil-utterances were part  
12  
13 of sequences where the teacher controlled the process and the pupils responded to basic tasks  
14  
15 that look like mere control questions, Pamela supported the pupils' own thinking, and used  
16  
17 open progress initiatives (B4) and different focusing actions (C1-6) to scaffold reflections  
18  
19 based on the pupils' initial ideas. There was no indication of any Topaze effect (Strømskag  
20  
21 2011), i.e. that Pamela's communication was characterised by a reduction of complexity, and  
22  
23 thereby, changed the initial target knowledge of the tasks.  
24  
25  
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27  
28

29 Being part of the PROJECT project, Pamela received support through the designed 'milieus'  
30  
31 to change her role towards being a facilitator in the didactical phases of the lesson. The  
32  
33 analysis documents that Pamela was a very active reviewer of the pupils' thinking, especially  
34  
35 by modelling comparison processes with the support of closed progress details (B3), by  
36  
37 taking a lot of open progress initiatives to motivate the pupils to put their thinking into words,  
38  
39 and by continually requesting the pupils to justify their thinking. Reviewing the pupils'  
40  
41 thinking, she prepared both the pupils and herself for the institutionalisation phase, where  
42  
43 they could complete the meaning-making process. The TDS-informed lessons in my study  
44  
45 are all based on learning through social interaction and the analysis showed that Pamela in all  
46  
47 three lessons was working to structure the debate between the pupils. Lack of communication  
48  
49 skills often reduced the outcome of group work; it caused the talk to be uncooperative, off-  
50  
51 task, inequitable and ultimately unproductive (Mercer and Sams 2006; Sfard and Kieran  
52  
53 2001). The pupils in this study were young, second and third graders at the time. Thus,  
54  
55 Pamela's active role (e.g., scaffolding comparison processes) might be explained by the fact  
56  
57  
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1  
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3 that the pupils had little experience in collaborative reasoning and how to use language  
4  
5 effectively in a joint and explicit way.  
6  
7

8 As I have shown, the interview revealed that Pamela had great belief in letting the pupils 'put  
9  
10 into words' what they have thought' and having them ask questions 'that clarify their own  
11  
12 thoughts'. The many open progress initiatives and notice actions are easily understood within  
13  
14 this context. However, upon finishing the analysis, it was still unclear to me how she was  
15  
16 able to end the pupil dialogues with so few confirming answers, and still, the pupils were able  
17  
18 to work. As an inductive researcher, I searched for theories that could explain or interpret my  
19  
20 data (Goetz and LeCompte 1984). Listening to a lecture by Judith Langer about  
21  
22 'envisonment-building classrooms' (Langer 2011), it suddenly struck me that Pamela  
23  
24 seemed to achieve exactly what Langer was describing. Langer defines envisionments as:  
25  
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28

29  
30 dynamic sets of related ideas, questions, images, anticipations, agreements, arguments  
31  
32 and hunches that fill our mind during every reading, writing, discussion, technology  
33  
34 interaction, or other experience where we gain or express thoughts and understandings  
35  
36 (Langer 2011, 17).  
37  
38

39  
40 In envisionment-building classrooms, the teacher's goal is to help pupils step into, move  
41  
42 through and revise their envisionments. The teacher's mind is on what and how the pupils are  
43  
44 thinking about the material, acting to enable the pupils 'to make sense of the ideas, explore  
45  
46 connections, think more deeply, reflect on what they know, and go beyond (2011, 51).  
47

48 According to Langer, envisionment-building classrooms invite pupils to build knowledge  
49  
50 themselves through activities and group discussions to explore the discipline's different ideas  
51  
52 and enabling strategies, and this is not dependent on an active teacher in all parts of the  
53  
54 learning process. Transporting Langer's concept into the TDS, one can say that the 'milieu'  
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3 must offer sufficient possibilities for envisionment-building in order to develop the target  
4  
5 knowledge.  
6  
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8  
9 Could Pamela's focusing actions in combination with the many open progress initiatives like  
10 'yes!/yes?', 'okay./okay!/okay?', 'mm?', 'hm?', 'aha?' and 'super!' be enough motivation  
11 and input to help pupils step into, move through and revise their envisionments? The situation  
12 where she left the third grader George and his partner to continue their reflections, based on  
13 the three responses 'Do you believe so?', 'mm?' and 'yes?', can seem quite brutal. However,  
14 this communication was part of a culture where it was normal that Pamela 'is playing ball  
15 with them' on their own level. In the opening excerpt, Pamela herself used the word 'brood'  
16 to describe what she tries to model dialoguing with the pupils. During the interview, she used  
17 many different expressions like 'think further', 'pursue a thought', 'to elaborate on an  
18 explanation' and 'to think a little longer'. To dwell on mathematical problems seems to be a  
19 value in Pamela's mathematics classroom. In the communication with George and Nora, she  
20 was both showing positive interest in their opinion and trust in their ability to think and to  
21 dwell on mathematical problems. In this situation, this seemed to be enough. In other  
22 situations, Pamela drew the pupils' attention to something in the task, asking them to read it  
23 again, or in their own words, to give them what she called 'a straw' to grasp at or something  
24 to 'build with'. In that way, she got the pupils to 'clarify their own thought', to 'think deeper'  
25 or to 'go beyond' their initial explanation, as Langer (2011, 51) expressed it.  
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48 Pamela's reviewing actions can be classified as Level 2 scaffolding (Anghileri 2006).

49  
50 However, it is not obvious that Pamela practiced restructuring actions, which is the other side  
51 of Level 2 scaffolding. Restructuring actions are described as providing meaningful contexts,  
52 simplifying the problem, rephrasing the pupils' talk and negotiating meanings. Some of these  
53 actions in Drageset's framework can be expected to appear as apply to similar problems (C3)  
54 or simplifications (B2). As the analysis showed, only very little of Pamela's communication  
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3 could be described as simplifications (B2) or apply to similar problems (C3), and the  
4  
5 examples found occurred mainly in the atypical dialogues with one pair of pupils. Rephrasing  
6  
7 and negotiating meanings might appear as recaps (C5) or notice (C6) in Drageset's  
8  
9 framework, but utterances in these categories might just as well have functioned as reviewing  
10  
11 the pupils' own thinking as being restructuring initiatives. Thus, in and of itself, the  
12  
13 categorisation of Pamela's utterances, based on Drageset's framework, is not evidence of  
14  
15 restructuring as part of Pamela's role in the didactical phase. However, interpreting  
16  
17 Pamela's communication in the light of 'envisonment-building' (Langer 2011) showed that  
18  
19 with quite small means, she managed to inspire the pupils to take the next step themselves.  
20  
21  
22

23  
24 Looking back on Strømskag's (2017) description, it is reasonable to ask whether Pamela's  
25  
26 scaffolding is compatible with the intentions of the didactical phase in TDS? **I will claim**  
27  
28 **that it is; after handing the planned task over to the pupils, Pamela held back until the pupils**  
29  
30 **had done the activity needed and had formulated and shared their strategies. Except for the**  
31  
32 **dialogue with Sarah, she withheld giving instructions or any other kind of input that was not**  
33  
34 **rooted in the pupils' present work. Her focus was on helping the pupils to formulate their**  
35  
36 **own thoughts and on enabling them to communicate those thoughts as expected in the**  
37  
38 **didactical contract. Thus, Pamela's communication during small group work follows what is**  
39  
40 **recommended by Howe et al. (2019); the teacher needs to withhold her demonstrations and**  
41  
42 **explanations to permit pupil reflection and appraisal. She also scaffolded talks that over time**  
43  
44 **could develop into cumulative and explorative talk as proposed by Littleton and Mercer**  
45  
46 **(2010). The pupils were on their way before, she said, 'she had to put some pressure on them**  
47  
48 **to get them to compare and talk together', now they have become better at it.**  
49  
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54  
55 Leaving the more fundamental negotiation of meaning - where situated knowledge is  
56  
57 transformed into cultural knowledge - to the institutionalisation phase, Pamela throughout the  
58  
59 didactical phases managed to support the pupil's agency, understood their capacity to act  
60



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3 independently and to make their own free choices. How she managed to do so through her  
4  
5 teacher communication is where this case study can add to the field.  
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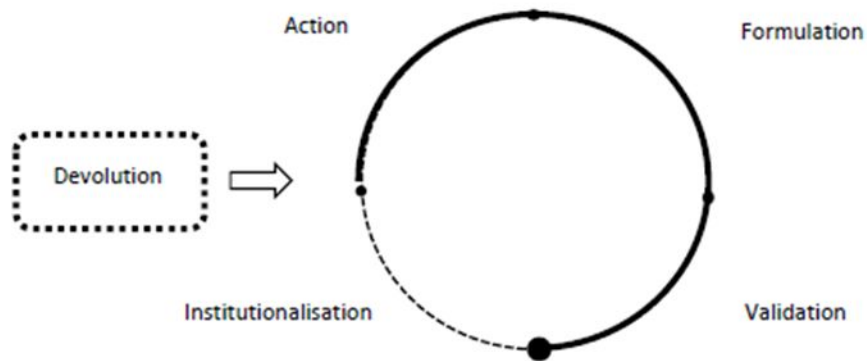


Fig. 1 The dynamics of the TDS where the didactical phases are marked with a dotted line and the adidactical phases are marked with a solid line (Strømskag 2017, 76, my translation).

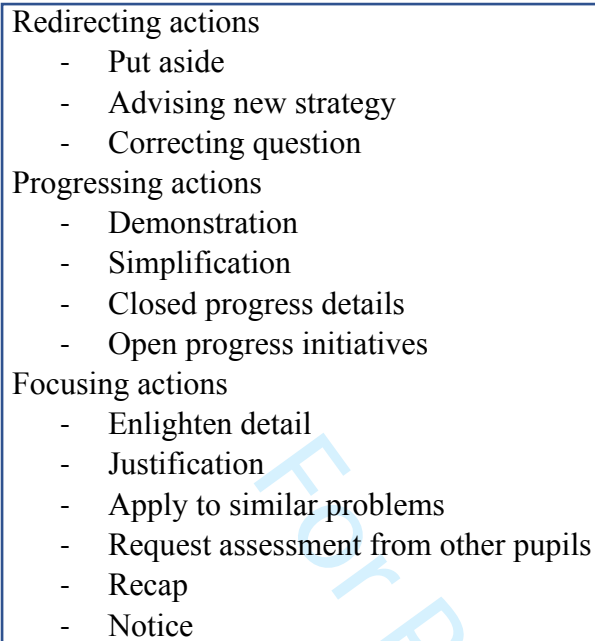


Fig. 2. Framework for detailed studies of teacher interventions in mathematical discourses (Drageset, 2015)

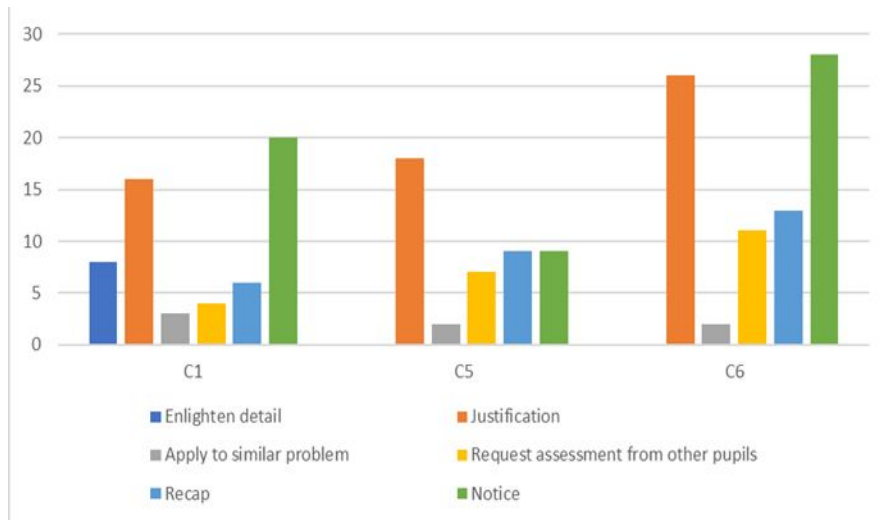


Fig. 3. Focusing actions made by the teacher in three classroom sessions.



$12 \cdot 2 = \underline{\quad}$	$10 \cdot 2 + 2 \cdot 2 = \underline{\quad}$ $\underline{\quad} + \underline{\quad} = \underline{\quad}$
$12 \cdot 4 = \underline{\quad}$	$10 \cdot 4 + 2 \cdot 4 = \underline{\quad}$ $\underline{\quad} + \underline{\quad} = \underline{\quad}$
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Fig. 4. Part of a multiplication task in L6.

Table 1: Teacher inventions during the didactical phase of three lessons; number of interventions and percentage of total teacher talk.

Lesson and text volume	L1		L5		L6	
	N	%	N	%	N	%
Teacher communication						
A1 Put aside	0	0,00	1	0,61	0	0,00
A2 Advising new strategy	1	1,33	13	11,96	0	0,00
A3 Correcting questions	4	0,97	3	0,80	0	0,00
Sum Redirecting actions	5	2,30	17	13,37	0	0,00
B1 Demonstration	0	0,00	11	12,53	0	0,00
B2 Simplification	0	0,00	13	12,12	1	1,46
B3 Closed progress details	73	18,03	18	6,89	21	13,96
B4 Open progress initiatives	50	25,44	31	12,23	58	16,85
Sum Progressing actions	123	43,47	73	43,77	80	32,27
C1 Enlighten detail	8	3,73	0	0	0	0,00
C2 Justification	16	4,46	18	11,48	26	12,10
C3 Apply to similar problems	3	1,35	2	2,39	2	4,21
C4 Request assessment from other pupils	4	2,23	7	3,17	11	4,25
C5 Recap	6	2,34	9	3,43	13	9,28
C6 Notice	20	8,03	9	7,08	29	21,83

Sum Focusing actions	57	22,16	45	27,61	81	51,66
D1 Classroom management actions	69	30,26	25	14,19	23	12,47

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