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To cite this article: Bo-Sture K Skagerstam 2018 *J. Phys. Commun.* **2** 125014

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Journal of Physics Communications

PAPER

On the three-slit experiment and quantum mechanics

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Abstract

It has been suggested by Sorkin that a three-slit Young experiment could reveal the validity a fundamental ingredient in the foundations of one of the cornerstones in modern physics namely quantum mechanics. In terms of a certain parameter κ_S, it was argued that a non-zero value could imply a breakdown of the fundamental Born's rule as well as the superposition principle. Here we argue that a physical realization of such arguments could lead to an erroneous conclusion and contradict the basic rules of quantum mechanics. In fact, we argue that a straightforward interpretation of the procedures involved in a physical determination of κ_S does not necessarily lead to $\kappa_S = 0$. In order to show this we consider a mono-chromatic source of photons prepared in an *arbitrary* quantum state and a simple version of the well-established photon detection theory of Glauber which, by construction, obeys all the rules of quantum mechanics. It is, however, also argued that after a proper identification of the relevant quantum-mechanical probability amplitudes one can be reach $\kappa_S = 0$. As long as one only consider a single photon detector, it is verified that, in this context, there is no fundamental difference between quantum-mechanical interference and interference as expressed in terms of classical electro-magnetic waves.
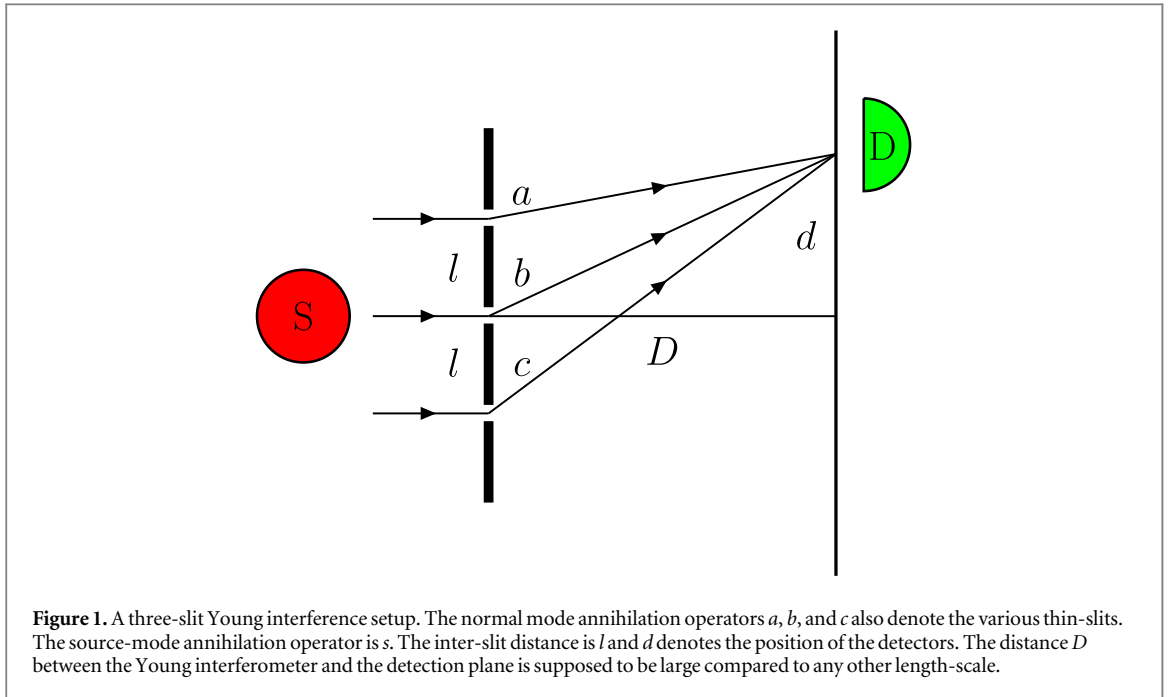
1. Introduction

Some time ago Sorkin [1] introduced a parameter k_S defined for arbitrary complex numbers α, β, and γ,

$$\kappa_S \equiv \frac{1}{P}(P_{\alpha\beta\gamma} - P_{\alpha\beta} - P_{\alpha\gamma} - P_{\beta\gamma} + P_\alpha + P_\beta + P_\gamma), \tag{1}$$

with $P_{\alpha\beta\gamma} = |\alpha + \beta + \gamma|^2$, $P_{\alpha\beta} = |\alpha + \beta|^2$, and $P_\alpha = |\alpha|^2$ and similarly for other combinations. P is a suitably chosen normalization in order to factor out possible unimportant constants. Here we put $P = 1$ but include it when convenient. Sorkin observed that $\kappa_s = 0$ as a *mathematical identity* for the arbitrary complex numbers α, β, and γ. Since the complex numbers in the definition of κ_S can be interpreted as quantum-mechanical probability amplitudes for physical events, it was, nevertheless, argued that a non-zero value of κ_S could be used as test of some of the fundamental ingredients of quantum mechanics, namely the superposition principle and Born's rule for obtaining measurable probabilities from quantum mechanical probability amplitudes. A physical realization corresponding to the symbol $P_{\alpha\beta\gamma}$ could, e.g., correspond to the detection probability in a three-slit Young interferometer as illustrated in figure 1. With one of the slits (γ) closed, $P_{\alpha\beta}$ should then be identified with the corresponding detection probability, and P_α should correspond to two slits (β and γ) closed and similarly for other combinations of the probability amplitudes α, β, and γ. Various theoretical and experimental oriented considerations of these ideas of Sorkin have recently been under investigation [2–12].

It is now of crucial importance to specify the identification above in a clear physical manner when making use of one and the same experimental setup with a given source and detector system. It is, e.g., then not obvious that closing one slit in a three-slit Young interferometer is physically equivalent to a two-slit situation to be used in the experimental determination of κ_S. By imposing proper boundary conditions for the various Young interferometer configurations, it has actually been argued that a non-zero value of κ_S quite naturally emerges [4, 7, 8, 12]. In very elementary terms, and focusing on a purely quantum field theoretical treatment, we will confirm that this is the case. Furthermore, one may raise questions on the quantum-mechanical nature of the

Figure 1. A three-slit Young interference setup. The normal mode annihilation operators a, b, and c also denote the various thin-slits. The source-mode annihilation operator is s. The inter-slit distance is l and d denotes the position of the detectors. The distance D between the Young interferometer and the detection plane is supposed to be large compared to any other length-scale.

prepared source state. We will verify, what has been known for a long period of time, that the interference pattern in all the cases we consider does not depend on the quantum nature of state of the source, at least if we consider mono-chromatic sources and a single photon detector. Apart from an overall factor, the interference pattern will therefore be the same for a source prepared in, e.g., a quantum-mechanical Fock-state of photons or for a conventional coherent state. As is well-known, the use of coherent states naturally leads to the interference of classical electro-magnetic fields (see, e.g., [13] and references cited therein). The observation of a non-zero value of κ_S is therefore not exclusively related to quantum-mechanical interference effects.

2. The photon detector

We first recall a simple quantum field theoretical treatment of quantum interference effects in a two-slit Young interferometer (see, e.g., [14, 15]), illustrated in figure 1, for a mono-chromatic source with wave-number k and angular frequency $\omega = ck$. The Glauber theory of quantum coherence [16] is then used in order to find the corresponding probability for single photon-detection. Below this analysis will be extended to a three-slit Young interferometer configuration. For a prepared quantum state $|\psi\rangle$ of the source S, and for a properly designed detector, the detection probability of one photon, with the port c closed, is related to the absorption of a photon in the detector described by the process

$$|\psi\rangle \rightarrow E^{(+)}(\mathbf{r}, t)|\psi\rangle. \tag{2}$$

Here

$$E^{(+)}(\mathbf{r}, t) = \mathcal{E}\left(a\, \frac{e^{i\phi_a}}{r_a} + b\, \frac{e^{i\phi_a}}{r_b} \right), \tag{3}$$

is the positive frequency part of one of the components of the second-quantized electric field observable $E^{(+)}(\mathbf{r}, t)$ at the position $\mathbf{r}$ and time t at the detector far from the interferometer. The field $E^{(+)}(\mathbf{r}, t)$ is expressed in terms of outgoing normal-mode annihilation operators a and b. In a more rigorous setting one should make use of appropriate Greens functions for system which, however, would make the points we are addressing less transparent. Furthermore, $\phi_a = \omega(t - \tau_a)$ and $\phi_b = \omega(t - \tau_b)$ are suitable phases expressed in terms of time-delays τ_a and τ_b. $\mathcal{E}$ is a common amplitude for the a and b modes and r_a, r_b are the in-plane distances from the various openings of the interferometer to the detector D. According to the fundamental Born's rule, the probability for single photon detection at the detector D is then, apart from unimportant constants, proportional to P_{ab} given by

$$P_{ab} = \sum_f |\langle f|E^{(+)}(\mathbf{r}, t)|\psi\rangle|^2 = \langle\psi|E^{(-)}(\mathbf{r}, t)E^{(+)}(\mathbf{r}, t)|\psi\rangle, \tag{4}$$

where we sum over all possible photon states $|f\rangle$. The probability P_{ab} may therefore be written in a well-known general form using equation (3), i.e.,

$$P_{ab} = |\mathcal{E}|^2 \langle\psi| (a^*a + a^*be^{-i(\phi_a-\phi_b)} + ab^*e^{i(\phi_a-\phi_b)} + b^*b)|\psi\rangle. \tag{5}$$

We now consider a prepared Fock state $|\psi\rangle$ for the source, i.e.,

$$|\psi\rangle = \frac{s^{*n}}{\sqrt{n!}}|0\rangle = \frac{(a^*+b^*)^n}{\sqrt{2^n n!}}|0\rangle = \frac{1}{\sqrt{2^n}} \sum_{k=0}^{n} \left(\frac{n!}{k!\,(n-k)!}\right)^{1/2} |k\rangle_a \otimes |n-k\rangle_b, \tag{6}$$

where $|k\rangle_a$ and $|n-k\rangle_b$ represent the Fock states of photons emerging from the slits a and b. Here we expressed the initial state $|\psi\rangle$ in the a and b modes using a boundary condition at the two identical thin-slits, i.e.,

$$s = \frac{1}{\sqrt{2}}(a+b). \tag{7}$$

This relation does not represent the result of a unitary transformation. However, by including an additional local source, with a mode operator $s_V = (a - b)/\sqrt{2}$, we have a unitary $U(2)$ transformation connecting the pair of independent mode operators (s, s_V) and the independent mode operators (a, b) [17]. The number operator of photons will then be conserved. Expressed in a somewhat different manner, fundamental commutation relations for mode operators applied to a completely symmetric Young interferometer naturally leads to the condition equation (7). In the discussion below on the three-slit Young interferometer two easily constructed local source operators have to be included in a similar manner and a corresponding unitary $U(3)$ transformation can easily be found preserving the number of photons. If quantum states of such local modes are present with, e.g., random phases, the visibility of interference patterns will in general be diminished. In all of the considerations below we, however, assume that the quantum states of such local modes are the vacuum state. We can therefore suppress their presence in the considerations below.

The state vectors in equation (6) describe the superposition of all possible combinations that can occur with appropriate weights, for photons passing through the various slits at the same time. We stress again that we only need the asymptotic form of the field $E^{(+)}(\mathbf{r}, t)$ at the detector and a relation like equation (7) for the mode operators in order to complete the analysis for all relevant detection probabilities. It now follows that

$$\langle\psi|a^*a|\psi\rangle = \langle\psi|b^*b|\psi\rangle = \langle\psi|b^*a|\psi\rangle = \langle\psi|a^*b|\psi\rangle = \frac{n}{2}, \tag{8}$$

and therefore

$$P_{ab} = |\mathcal{E}|^2 \frac{n}{2} \left(\frac{1}{r_a^2} + \frac{1}{r_b^2} + \frac{2\cos(\phi_a - \phi_b)}{r_a r_b} \right). \tag{9}$$

Similar expressions can be obtained for the probilities P_{ac} and P_{bc}. For the one-slit case the approximations used above lead to $P_a = |\mathcal{E}|^2 n/r_a^2$ and similarly for P_b and P_c.

With all slits open in figure 1, we extend the discussion above with the asymptotic field $E^{(+)}(\mathbf{r}, t)$ in equation (3) replaced by

$$E^{(+)}(\mathbf{r}, t) = \mathcal{E}\left(a\, \frac{e^{i\phi_a}}{r_a} + b\, \frac{e^{i\phi_a}}{r_b} + c\, \frac{e^{i\phi_c}}{r_c} \right). \tag{10}$$

Correspondingly, the initial state $|\psi\rangle$ is expressed in terms of the a, b, and c mode operators, i.e.,

$$|\psi\rangle = \frac{(a^*+b^*+c^*)^n}{\sqrt{3^n n!}}|0\rangle = \frac{1}{\sqrt{3^n}} \sum_{k=0}^{n}\sum_{l=0}^{n} \left(\frac{n!}{k!\,l!\,(n-k-l)!}\right)^{1/2} |k\rangle_a \otimes |l\rangle_b \otimes |n-k-l\rangle_c, \tag{11}$$

where we have made use of the multi-nomial theorem. As in the two-slit case, equation (11) describes the superposition of the possible combinations that can occur with appropriate weights, for photons passing through different slits at the same time. It is straightforward to verify that the extension of equation (8) is given by

$$\langle\psi|a^*a|\psi\rangle = \langle\psi|b^*b|\psi\rangle = \langle\psi|c^*c|\psi\rangle = \langle\psi|a^*b|\psi\rangle = \langle\psi|b^*a|\psi\rangle$$
$$= \langle\psi|a^*c|\psi\rangle = \langle\psi|c^*a|\psi\rangle = \langle\psi|b^*c|\psi\rangle = \langle\psi|c^*b|\psi\rangle = \frac{n}{3}. \tag{12}$$

The three-slit probability P_{abc} is therefore given by

$$P_{abc} = |\mathcal{E}|^2 \frac{n}{3} \left(\frac{1}{r_a^2} + \frac{1}{r_b^2} + \frac{1}{r_c^2} + \frac{2\cos(\phi_a - \phi_b)}{r_a r_b} + \frac{2\cos(\phi_b - \phi_c)}{r_b r_c} + \frac{2\cos(\phi_a - \phi_c)}{r_a r_c} \right). \tag{13}$$

The various detection probabilities discussed above are all proportional to the number of photons n of the initial state $|\psi\rangle$. In general all the results above will actually remain the same for any mono-chromatic initial quantum state, pure or mixed, by replacing n with the corresponding mean value $\langle n\rangle$. In order to verify this fact we make use of Glauber-Sudarshan representation [16, 18] for a general single-mode quantum state in terms of conventional coherent states $|\alpha\rangle$ [19] namely
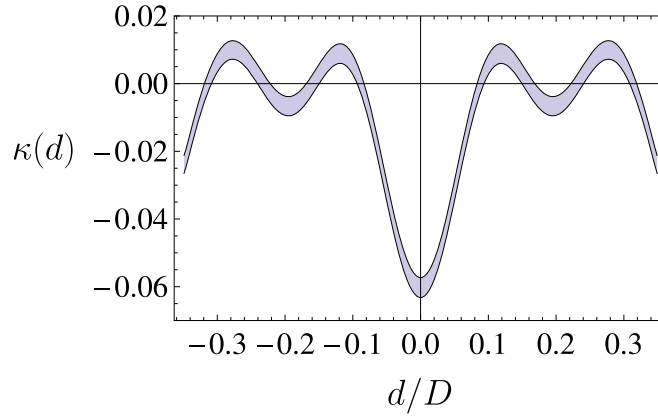
Figure 2. The Sorkin parameter $\kappa_s \rightarrow \kappa(d)$, as defined in equation (18) normalized by $P = P_{abc}(d = 0)$, as a function of d/D. The parameter used are as in [12] with $\lambda = 0.05$ m, $l = 0.13$ m and $D = 1.25$ m. For reasons of simplicity we neglect the width of the slots. The figures illustrates the sensitivity in reaching the degeneracy value $\kappa(d) = 0$ for a small range of the normalization constants n_1 and n_2 in equation (18). Here we make use of $n_2 = 2/3 + 1.3\%$. The upper curve corresponds to $n_1 = 1/3 + 1.3\%$ and the lower curve to $n_1 = 1/3 + 1.2\%$.

$$\rho = \int d^2\alpha \, \mathcal{P}(\alpha, \alpha^*) \, |\alpha\rangle \langle\alpha|. \tag{14}$$

A detection probability P_D is then evaluated according to

$$P_D \equiv \mathrm{Tr}[\, \rho \, E^{(-)}(\mathbf{r}, t) E^{(+)}(\mathbf{r}, t)\,] = \int d^2\alpha \, \mathcal{P}(\alpha, \alpha^*) \, \langle\alpha|E^{(-)}(\mathbf{r}, t) E^{(+)}(\mathbf{r}, t)|\alpha\rangle. \tag{15}$$

We illustrate the procedure in terms of the two-slit slit configuration with $P_D = P_{ab}$. Expressing the coherent state $|\alpha\rangle = D(\alpha)|0\rangle$ in terms of the displacement operator $D(\alpha) = \exp(\alpha s^* - \alpha^* s)$, and by making use of the mode operator relationship equation (7), it is clear that

$$|\alpha\rangle = \left| \frac{\alpha}{\sqrt{2}} \right\rangle_a \otimes \left| \frac{\alpha}{\sqrt{2}} \right\rangle_b. \tag{16}$$

Since $|\alpha\rangle$ is an eigenstate of the observable $E^{(+)}(\mathbf{r}, t)$, one easily finds the same expression for P_{ab} as in equation (9) with n replaced by $\langle n\rangle \equiv \langle s^*s\rangle$ using

$$\frac{1}{2}\langle n\rangle \equiv \frac{1}{2}\mathrm{Tr}[s^*s] = \frac{1}{2}\int d^2\alpha \, \mathcal{P}(\alpha, \alpha^*)|\alpha|^2 = \langle a^*a\rangle = \langle b^*b\rangle = \langle a^*b\rangle = \langle ab^*\rangle. \tag{17}$$

Apart from the replacement of n with $\langle n\rangle$, the interference pattern exhibited by P_{ab} then stays the same and does not depend on the details of the initially prepared quantum states of the source. The same reasoning applies to all detection probabilities considered above.

We now introduce the combination $\kappa(d)$, motivated by equation (1), as defined by

$$\kappa(d) \equiv P_{abc} - n_2(P_{ab} + P_{ac} + P_{bc}) + n_1(P_a + P_b + P_c), \tag{18}$$

where we have introduced two normalization parameters n_1 and n_2. With $n_1 = n_2 = 1$, the Sorkin parameter κ_S and $\kappa(d)$ are, at least symbolically, identical. But then

$$\kappa(d) = |\mathcal{E}|^2 \, \frac{\langle n\rangle}{3} \left(\frac{1}{r_a^2} + \frac{1}{r_b^2} + \frac{1}{r_c^2} - \frac{\cos(\phi_a - \phi_b)}{r_a r_b} - \frac{\cos(\phi_b - \phi_c)}{r_b r_c} - \frac{\cos(\phi_a - \phi_c)}{r_a r_c} \right), \tag{19}$$

is in general non-zero as a function of the position d of the photon detector. Due to the two-slit conditions equation (8) and three-slit conditions equation (12) it is, however, clear that the parameters n_1 and n_2 have to be adjusted in order to have the same average number of photons passing through the various slit combinations under consideration. Without loss of generality, we should therefore use $n_1 = 1/3$ and $n_2 = 2/3$. The physical conditions are then the same for the various slit configurations and one then finds that $\kappa(d) = 0$. As an example, we illustrate in figure 2 the sensitivity in the approach to the degeneracy point, defined by $\kappa(d) = 0$, for various choices of the normalization parameters n_1 and n_2. Other parameters used correspond to a recent experiment by Rengaraj *et al* [12]. We find it remarkable that we can reproduce some features of [12] in view of the simplicity of the arguments put forward in these comments.

3. Final remarks

In accordance with other considerations, as in [4, 7, 8, 12], we have argued that one cannot make a straightforward physical identification of the quantum-mechanical probability amplitudes to be used in the definition of the Sorkin parameter k_S. As we have seen in the case of the one-, two-, and three-slit Young interferometer configurations, the intensity of the source considered has to be adjusted in order to describe the same physical conditions. It then follows that the corresponding identification of the Sorkin parameter κ_S is naturally zero. If not properly adjusted a non-zero value emerges without any contradiction with the basic rules of quantum mechanics. This is our main conclusion.

For a mono-chromatic source, the interference patterns as discussed above do not depend on the nature of the quantum state of the source and only the mean number of photons is of importance. This has the consequence that there is no fundamental difference between classical and quantum-mechanical interference when making use of a single photon detector, a fact that is well established [20]. Even though the concept of a photon has been disputed [21], the interference pattern as builded up by single-photon events can, with current technology, rather easily be demonstrated (see, e.g., [22]) and agrees with the interference pattern as obtained in terms of classical optics.

The Feynman-path integral approach [23] to quantum interference and the notion of non-classical paths for photons appears to play an important role in many of the current discussions on the Sorkin parameter κ_S (see, e.g., the Supplementary Material in [7] and references cited therein). In the Feynman-path integral approach to the quantum mechanics of a non-relativistic particle the notion of a, not necessarily classical, path expressed in terms of co-ordinates makes much sense even though this has to be used with care (see, e.g., [19, 24]). For highly relativistic particles the notion of a co-ordinate and a corresponding path needs clarification. This is so since one can argue that the components of a position observable for a massless particle with non-zero helicity, like a photon, are, due to topological reasons, non-commuting [25]. It therefore appears that, in general, the physical meaning of non-classical paths for photons is not clear. In the quantum field theoretical approach to interference phenomena as discussed in this work, such considerations do, however, not play any role.

Acknowledgments

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