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Emission Constrained Unit Commitment: Impacts of the Shale Gas Revolution

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Master project

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Background

The shale-gas revolution in the US has caused a significant increase in the use of natural gas in power production. This increase is both due to the abundance of natural gas and the low sales prices due to the shale-gas boom, but also driven by an incentive to replace coal plants to meet domestic targets in reduction of carbon emissions. Natural gas power plants (NGPP), however, still rely on burning fossil fuels. In power production and planning, often formulated and solved as a (hydro-thermal) unit-commitment (UC) problem, there may often be an incentive to minimize emissions or to keep the emissions below a certain limit, while at the same time meeting the short and long-term power demands through scheduling of different power generating units. Keeping emissions low may be particularly important for power production close to populated areas.

Task description:

1. Do a thorough review on the unit-commitment (UC) problem. Topics that should be reviewed include, but are not limited to, different time-horizons, min cost/max profit formulations, necessary (model) constraints for each unit, various scheduling constraints such as min up/down times and sources of uncertainty.
2. Review the integration of natural-gas power plants in context of the UC problem, to what extent it has been common to include natural gas units in the UC problem, and which advantages/disadvantages it creates both for the computations and in practice.
3. Consider different versions of emission constraints on the UC problem.
4. Formulate a mixed integer nonlinear program (MINLP) for the UC problem with different types of emission constraints, such as hard constraints, cost functions for emissions and pure minimizations of emissions while meeting power demands. Implement the MINLP in suitable software (GAMS, AMPL etc.). Consider multi-objective formulations for including emissions constraints.
5. Do a short review on different methods for solving the deterministic UC problem, and choose a suitable method for solving the problem implemented in item 4. above.
6. Do sensitivity testing of the emission constrained UC problem with different formulations, and consider how the increased use of gas impacts costs and emissions in different power production scenario. Consider different mixtures of coal and natural-gas power production for meeting peek demands, related to varying fuel prices, demands and availability of the fuels.

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Trondheim, 14.01.2014

Bjarne Foss
Professor/supervisor

Preface

This Master Thesis is written at the Department of Engineering Cybernetics at the Norwegian University of Science and Technology (NTNU), spring 2014. The report is written as a sequel to the author's project thesis written in fall 2013, *The Role of Shale Gas as a Regulator in Energy Supply Systems*, but may be read independently as well.

The motivation has been to gain knowledge of emission constrained unit commitment problems, and the environmental aspects of replacing coal with natural gas in the power industry. This knowledge is important in terms of serving the world's growing power demand in a sustainable way.

I would like to thank my supervisors, professor Bjarne Foss and PhD candidate Brage Rugstad Knudsen, for their commitment and guidance throughout the whole process. It has been exciting cooperating with you. I would like to give a special thank to Brage Rugstad Knudsen for always taking his time helping me, and answering my questions. I would also like to thank Shaurya Sharma for introducing me to his Objective Feasibility Pump heuristic, and helping me out tuning it for my problem.

Trondheim, 2014-06-13

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Einar Wold

Abstract

In view of global warming, the world is facing a common responsibility in terms of reducing its CO_2 emissions. The extensive use of fossil fuels in the power industry is a significant source of carbon emissions, and in such matter a reasonable arena to take action. On the other hand, there is a sustainable increase in the global power demand, where European energy intensive industry is struggling against high power prices.

This thesis presents a diverse view on unit commitment (UC) problems in electric power production with emission constraints, and presents different approaches for solving them. In terms of trying to find a reasonable trade-off between costs and CO_2 emissions, and taking into account a CO_2 cap-and-trade system like the European Union Emissions Trading System (EU ETS), a problem formulation including a CO_2 allowance system is presented. The problem is formulated as a mixed integer nonlinear program (MINLP). Several approaches for solving the problem is implemented and assessed, including direct MINLP based branch-and-bound, piecewise linearization, and using an Objective Feasibility Pump (OFP) heuristic. The latter OFP heuristic is demonstrated to give promising results on the problem instances in this report, in terms of finding good feasible solutions to MINLPs within short computation times. The results are compared to solutions retrieved using the branch-and-bound method in BONMIN. Computation time is of significant importance to UC problems, though they occur in the intraday market, where power is traded close to real time.

Based on the comprehensive success of replacing coal with shale gas in the US' power industry, the role of natural gas within EU is payed significant attention in this thesis. Different applications of natural gas in the power industry is presented, and its potential of contributing to lower CO_2 emissions is evaluated. A reflection upon benefiting from shale gas within the EU is also included.

Sammendrag

I lys av utfordringene med global oppvarming, står verden i dag ovenfor et felles ansvar for å redusere CO_2-utslippene. Den omfattende bruken av fossile brensler i kraftindustrien er et betydelig bidrag til utslippene. Det er derfor rimelig at dette er en av industriene som tar grep, og verner om våre felles interesser. Samtidig er den globale etterspørselen etter elektrisk kraft økende, mens energikrevende europeisk industri lider grunnet høye strømpriser.

Denne oppgaven presenterer en flersidig beskrivelse av enhetstilkoblingsproblemer for kraftindustri med utslippsbegrensninger, og ulike måter å løse dem på. Med tanke på å finne en rimelig balansegang mellom kostnader og CO_2-utslipp, og samtidig inkludere et handelssystem for utslipp, slik som Europa Unionens Handelsprogram for Utslipp (EU ETS), presenteres det et enhetstilkoblingsproblem som inkluderer et CO_2-kvote system. Problemet er formulert som et ulineært blandet heltallsproblem (MINLP). Ulike tilnærminger for å løse problemet er implementert, inkludert direkte MINLP basert branch and bound, stykkevis linearisering og bruk av en Objective Feasibility Pump-heuristikk. Sistnevnte har vist seg å gi lovende resultater for probleminstansene som inngår i denne rapporten, med tanke på å finne gode, gyldige løsninger til MINLP-problemer på liten kjøretid. Resultatene er sammenlignet opp mot branch and bound-metoden i BONMIN. Kjøretid er av stor viktighet for enhetstilkoblingsproblemer, da de bl.a. må løses i justeringsmarkedet, hvor krafthandel skjer tett opptil sanntid.

Med utgangspunkt i den omfattende suksessen med å erstatte kull med skifergass i USAs kraftproduksjon, er naturgass innenfor EU viet stort fokus i denne rapporten. Ulik bruk av gass i kraftindustrien er presentert, og gass' muligheter til å bidra til lavere CO_2-utslipp er evaluert. Det følger også en refleksjon over mulighetene for å dra nytte av skifergass innenfor EU.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

In the US, the shale gas revolution has caused a significant increase in the use of natural gas in power production, and a drastic loss of market shares for coal fired power plants. The increase is both due to the abundance of natural gas, and the low sale prices due to the shale gas boom, but also driven by an incentive to replace coal as an power source to meet domestic targets in reduction of carbon emissions. In power production and planning, often solved as an unit commitment (UC) problem, there may often be an incentive to minimize emissions or to keep them below a certain limit, while at the same time meeting the short and long-term power demands through scheduling of different power generating units. Besides carbon emissions, among other nitrogen oxides and sulfur emissions has to be constrained. Keeping emissions low may be particularly important for power facilities near populated areas.

Pollution caused by power production is a global scale problem, and most first world countries have at this point committed to reduce, or at least set a maximum limit for their annual carbon emissions. An important corner stone in the work achieving these commitments is the Kyoto protocol. As a response to the growing number of countries committing to this protocol, there have been implemented measures such as the European Union Emissions Trading Scheme (EU ETS). This has expanded the financial complexity of the power industry drastically, due to economical measures to polluting facilities.

The change in the market has forced many generating companies (GENCOs) to look for new, less conventional power sources to remain competitive. In such matter, US' success replacing coal with natural gas in power production, is interesting for several markets outside the US.

Problem Formulation

To take advantage of all available power sources in an as sustainable way as possible, it is required to have thorough understanding of both the strategical, technical and environmental aspects of the business. As (Wold [115]) was focused on the macro aspect of the industry, as well as technical features of different power sources, this research is focused mainly on optimization and production strategies concerning cost and emissions. In specific this thesis presents:

- A thorough review on the UC problem, including among other different time-horizons, min cost/max profit formulations, necessary model constraints for different units, various scheduling constraints, sources of uncertainty and different versions of emission constraints for the problem.

- An evaluation of integration of natural gas power plants in context of the UC problem, including to what extent it has been common to include natural gas units in the UC problem, and which advantages and disadvantages this causes both for the computations and in practice.

- A CO_2 emission constrained UC problem formulated as a mixed integer nonlinear program (MINLP). The main focus is on constraining emissions through emission allowances, but approaches using hard constraints, conventional cost functions (emission weighting) and pure minimization of emissions are also included.

- A review on methods for solving deterministic UC problems.

- Practical approaches solving different setups of the presented UC problem using CPLEX[28], BONMIN B-BB[12] and Objective Feasibility Pump (OFP)[100], analyzing which one is most suitable for solving the problem both concerning accuracy and computation time.

- Sensitivity testing of the emission constrained UC problem with different formulations, and analysis of how the increased use of natural gas impacts costs and CO_2 emissions in different power production scenarios.

This thesis got several target groups. The emission constrained UC problem and issues related to finding a sustainable balance between keeping the production profitable and limiting emissions, are important for all parts of the industry, as well as governmental control organizations. The nature of the UC problem being a MINLP, makes it an interesting problem in terms of mathematical optimization as well, and the analysis presented regarding this can in large extent also be taken advantage of solving general MINLP formulations.

Literature Survey

The issues related to optimization within power production are not new. The oldest articles which the author has come across throughout this research are (Hara et al. [50]) and (Kerr et al. [61]), both published in 1966. From that point and until today, the complexity of the problems and the solutions has grown along with the industry, and the increased availability of affordable computational resources. (Padhy [87]) and (Bhardwaj et al. [9]) summarizes some of the previous work presented regarding general UC problems.

What differs this thesis from most previous work is its focus on limiting emissions along side optimizing the production costs. This is an approach which seem to have started occurring throughout the 1990s, and grown since then. In terms of limiting emissions, the work presented in among other (Gjengedal [46]), (Yamin and Shahidehpour [118]) and (Simopoulos et al. [102]) are benefiting from using hard constraints. Another approach, presented in (Mendes et al. [77]), is including the emissions as a part of a cost function in the objective. (Rebennack et al. [94]) presents an approach using emission quotas system, where it has to be paid a fee for violating the allowed amount of emissions. The latter publication has been a significant inspiration for the work presented in this thesis.

Among other publications worth mentioning are (van den Bosch and Hondered [110]),(Takriti et al. [108]),(Takriti et al. [107]), (Huang et al. [53]) and (Dudek [34]), all presenting different approaches of solving UC problems.

What Remains to be Done?

As presented in the former section, there already exist a lot of work within the field of the UC problem. As the consequences of global warming have slowly become clearer, there has throughout the last decades been an increased focus on the emissions from the power industry. From the authors point of view, it seems like a lot of the research presented so far are focused on hard constraints limiting the emissions. As will be argued in this thesis, this is not necessarily the most attractive solution for the parties in the industry. In the future it will be necessary to be aiming for models which are flexible enough for the industry, taking emission allowances and allowance trading into consideration.

When it comes to increased use of natural gas, and natural gas as a replacement for coal, it is important to have the complexity of distribution in mind. In (Wold [115]) this was discussed, as well as the difference in infrastructure from the US to Europe. In terms of filling the gap from the work presented in (Wold [115]) to this research, a natural and interesting path would be to link the limitations in the natural gas distribution networks as dynamic constraints to an emission constrained UC problem. (Liu et al. [70]), (Liu et al. [69]), (Damavandi et al. [29]) and (Wu et al. [117]) present some approaches related to this, though without any form of limitation of emissions.

In terms of the UC problem in general, there will always be a challenge making as exact models as possible, and at the same time keeping them simple enough to be solvable within reasonable time. A reasonable question to ask is whether it is necessary with smarter models, better solvers or simply more computational power. The authors guess is all three. The work presented in this thesis shows that formulating the system model in a smart way, and at the same time making the right trade-offs considering exactness, is crucial in terms of computation time.

Using standard solvers on an UC problem, often requires the model to be tailored for the solver. Maybe a better approach can be to tailor make or tailor modify a solver to the model instead? This way there might be possible finding more efficient ways of handling nonlinearities and nonconvexities which are typical for many UC problems.

In terms of hardware, it is still a fast growing industry, and the prices of computational power are still decreasing. An important change the last ten years have been an increased focus on multi core processing units and concurrent programming, instead of expanding the computational capacity of each single core. To take advantage of this, it is essential that solvers in the future are designed to run concurrent processes. Such features may provide the ability of solving larger problems with better exactness in the future.

1.2 Objectives

The main objectives of this master project are:

1. Present the complexity of the UC problem, and its importance in terms of both economical and environmental aspects.

2. Introduce an UC problem formulation including limitations of CO_2 emissions.

3. Find a suitable solver and apply it on different variations of the UC model introduced.

4. Account for how different constraints and/or penalties affects the CO_2 emissions and the fuel mixes used in power production.

1.3 Structure of the Report

Chapter 2 starts presenting the UC problem, including different approaches of limiting emissions. This part is followed by a section giving a short introduction to the Norwegian electric power trading market, and a section covering the role of natural gas in the general electric power market and UC problems. The chapter is ended with a section presenting different approaches of solving MINLP formulations of the UC problem.

Chapter 3 presents an UC problem formulation using an emission allowance system, punishing generators exceeding their CO_2 allowances.

The following chapter is divided into two sections. The first presents the setup used carrying out the practical work, while the second presents the different simulations which have been performed.

Chapter 5 presents a summary of the most important results found.

Chapter 6 is divided into five sections. The first one argues why the strategic choices were made the way they were, when choosing an appropriate solver. The second evaluates the effects the model presented in chapter 3 has to costs, emissions and energy mixes. The next section compares the allowance system to other emission constrained UC problem approaches, while the fourth evaluates the possibilities of improving computation time using an Objective Feasibility Pump (OFP) heuristic. The chapter is ended with a section reflecting over of the role of natural gas in EU's power industry, especially focusing on shale gas.

In chapter 7 the conclusion is presented.

Chapter 2

Theory

This chapter presents a thorough review of the UC problem. The theory presented is in a formal manner, and will be used to argue for the choices made in the practical part, presented in the next chapter.

The first section covers different approaches to the UC problem relative market and available resources. It is followed by a section presenting different types of emission constraints on the UC problem, an introduction to the Norwegian power market and a section putting natural gas fired power plants into context of the UC problem. In the end, the chapter is rounded off by a short review presenting different approaches solving the UC problem.

The use of variable and parameter names within this chapter is consistent, meaning that if they are explained in context of one equation, they represent the same in all other equations throughout this chapter.

2.1 The Unit Commitment problem

Trading Power

The conventional UC problem is commonly defined as finding the least cost dispatch of available generation resources to satisfy the demand of electricity[24],[37]. In a regulated market with one utility controlling the power production for a whole region, this approach is the same as maximizing the profit. The traditional picture of having a single utility controlling the market in a specific region is fading though. Instead, there have been an extensively growth of partly and fully deregulated markets[108]. The industry has developed from being vertically integrated and highly regulated, to being horizontally integrated in which the generation, transmission and distribution are unbundled[118].

A deregulated market consist of several utilities competing of being the most attractive provider of electric power. A partly deregulated market means that some of the energy production is controlled by governmental regulations. This might for example be orders requiring that each generating company (GENCO) supply the market with a certain amount of energy below a maximum price. There is also common to have rules of competition, avoiding large GENCOs dumping cheap electricity for short periods to squeeze smaller ones out of the market.

There are several different designs of deregulated electricity markets, but most of them share some equalities. They all feature some kind of auction system, where the GENCOs can provide their offers to the market. The collection of bids available in a market is often referred to as the pool. To maintain non-discriminatory pricing and fair access to transmission lines,

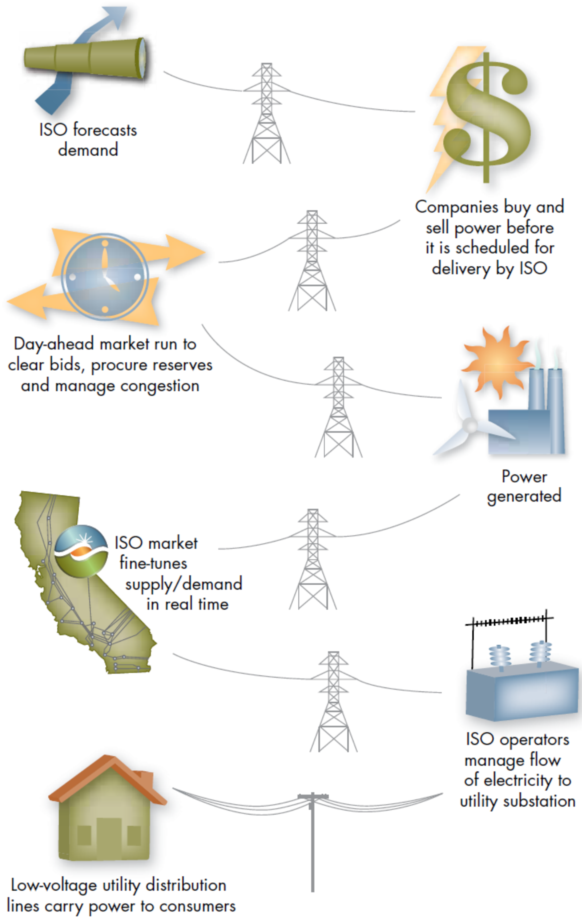


Figure 2.1: Simplified sketch of the electricity market organized by CAISO. *Source: California ISO, Company Informations and Facts*

it is common to have an independent system operator (ISO) facilitating and controlling the market[118]. An example of an ISO is California ISO (CAISO), responsible for 80% of the California power grid and a small part of the Nevada grid[22]. A rough sketch of how they organize their market is shown in figure 2.1.

In some deregulated markets, each GENCO is responsible for its own decisions of what and how to bid on load supply and reserves to maximize its profit. In this kind of market, the GENCOs submit single part bids (a price for supplying a certain load in a certain period of time) to the ISO. This is to minimize the risk related to actually not knowing the number of hours they will have to be producing, or the exact market clearing price (MCP) during these hours. After the GENCOs have made their bids, the ISO is trying to combine them in the best way possible maintaining the system security. In other words: The GENCO's are responsible of maximizing their own profit, while the ISO is safeguarding the interests of consumers. This kind of deregulated market is used in: New England Power Pool, California Market, New Zealand and Australia[118].

An other approach is that the ISO gathers bid-in costs submitted by the GENCOs. In this situation the GENCOs submit multi part bids (bids which include startup costs, minimum generation cost, running costs etc.). This information is used by the ISO to calculate which units which are going to be dispatched at which time to minimize costs, without violating the system security. It also calculates the MCP. This kind of deregulated market is used in PJM interconnection, New York Market and UK Power[118].

Independent of how the formal standard for bids is set, all markets can usually be divided into two branches. The main branch is the ahead market. In this part of the market there is made a priori (typical 24 hours ahead) bids to meet the estimated load in a certain period[23], [22]. There is also traded reserve contracts in this market, which means agreements regarding generators which are to be kept ready in case of underestimated load. These generators are commonly referred to as spinning reserves.

The majority of the trading takes place in the ahead market. However, incidents may take place between the closing of the ahead auction and the delivery. A nuclear power plant might face an emergency shutdown, or shiny weather may cause unforeseen availability of solar power. This is where the adjustment market (often referred to as intraday market) comes in handy. In this branch of the market, buyers and sellers can trade volumes close to real time to maintain market balance[105].

Besides the trading in the auctions, it is common with direct bilateral agreements between large industrial consumers or distribution companies (DISCOs) and GENCOs.

Due to a multiple number of auctions in the market, there are several instances of the UC problem occurring every day. The ISO might solve different versions of it at the closing of the ahead market, and for each update in the adjustment market. The latter ones has to be calculated fast. For example is CAISO updating its adjustment market every fifth minute[22]. This has established a need of efficient algorithms. These types of UC problems are the conventional ones, serving the demand to the lowest cost possible.

The changeover from regulated to more deregulated markets has created a sister instance of the UC problem. Rather than minimizing the sum of costs the focus is on maximizing profit, the difference between revenue and total cost. With this objective in fully deregulated markets, the GENCOs have the possibility of choosing how much energy they are going to sell in each hour[23]. This means a GENCO have no obligation to sell, unless it is sufficiently profitable. In the same way, they can choose to provide more energy to the market despite increased costs, as long as revenue rises more than total cost[87]. Due to the increased flexibility from ignoring the demand constraint, this formulation is not to be considered a complete UC problem, and it is often referred to as the self-scheduling problem.

Nevertheless, the two problems are closely related. A major modeling challenge related to this problem is the interaction between the company's bids and the rest of the system. The simplest case, is obviously when the company is small enough to be considered a price taker not influencing the market price. The situations where a GENCO's decisions alone can affect the market price, are way more complex. These situations requires modeling of market response and strategic behavior of competing GENCOs[91]. This kind of modeling is beyond the scope of this thesis, and it will not be elaborated further. Good sources of oligopoly theory and approaches to game theory are (Anderson and Bergman [3]), (Berry et al. [7]), (Borenstein and Bushnell [15]), (Hobbs et al. [52]) and (Jing-Yuan and Smeers [56]).

The MCP is determined by the intersection of the aggregate sell offer and the buy bid curves. If a bid is lower than the MCP it will be rejected, and in the same way an offer will be rejected if it is above[23]. This is illustrated in figure 2.2. In many situations it requires a probability density function (PDF) to model the future MCP while doing theoretical experiments on the self scheduling problem. In lack of this, robust optimization might be an alternative. This concept is presented in (Soroudi [103]).

In (Wold [115]) the concept of smart grid is presented, and the challenges this implies in terms of solving the UC problem. Considering estimating the MCP for the self scheduling problem, smart grid might actually come in handy. Information gained by the multi way communication can be used to identify patterns of price and demand, which can be used in

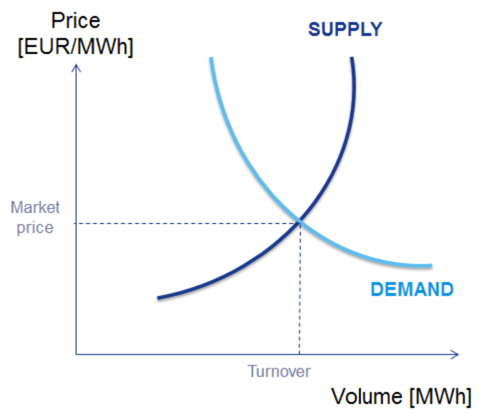


Figure 2.2: The intersection between the curves of supply and demand defines the MCP. *Source: Nord Pool Spot, Elspot, The day-ahead market*

models estimating MCP[103].

Planning Horizons

At first sight, it might seem like a good idea for a GENCO to optimize its profit hour by hour, but this is not necessarily the case. The target for a GENCO is to maximize its long term profit. In such matter, it is required to evaluate several time horizons while planning production and scheduling. It is common to distinguish between at least three planning horizons: Long-, medium- and short-term. Long-term planning extends over periods longer than two years. In this kind of planning operational decisions play a smaller role, but are not insignificant. More important are capacity expansion decisions and long-term demand forecasting[91]. These two are closely related, though the need of expanded capacity follows the growth in demand.

Considering capacity expansions, there are several options. The most obvious one is to build an all new power plant and keep maintaining the others. This will expand the total capacity significantly, but the investment cost is high, and the GENCO's total maintenance cost will increase. In such matter, choosing this solution requires a significant increase in revenue.

Another option is to replace an old plant with a new one. With this approach there is still a large investment cost, but maintenance cost relative amount of energy produced will most likely decrease. There might also be an environmental gain, by replacing old technology. The downside is that the total capacity is not increasing nearly as much as with the first approach.

The third possible solution is to renew an already existing plant. This may have the same benefits and downsides as the previous suggestion, but for a possibly lower price. In addition the replacement might cause a temporary shutdown of the plant, and in such matter a reduced total capacity in the respective period.

Which option is the best depends on the situation. Both long-term demand forecasts, financial resources and potential moves of competitors have to be carefully considered.

To take advantage of the long-term demand forecast the market has to be analyzed. It has to be verified what other GENCOs can provide, and if there are any reasons for expecting changes in the market. Further, patterns of the last years' demand must be evaluated.

For time horizons ranging from a month to two years, it is common to talk about medium-term planning. In these plans, there is natural to include unit availability schedules and expected monthly energy production[91]. Further, analyzes of availability of fuel and resources, as well as maintenance schedules, should be included.

There are significant costs related to having generators running idle during periods of low demand. In the same way, there are significant costs related to run cold startups of generators[46]. In such matter, it might be natural to shut down some plants or generators for periods of low demand. In a Norwegian climate, the summer months are examples of such periods. This planning is closely related to the maintenance scheduling. There will be natural to serve maintenance to the generators, in the same periods as they are scheduled to be shut down.

The expected energy production can be estimated from among other long-term weather forecasts and previous years' demand patterns. From this estimate, the need of fuel and other resources can be calculated. Depending on price and availability, this estimate will affect the choices made in the availability schedule.

Short-term planning extends over periods shorter than one week, and hourly stages are used[91]. In this planning the detailed production schedules are set, and the UC problem and the self-scheduling problem gets into context.

In terms of maximizing profit, it is necessary to calculate revenue, as well as total cost. The revenue is found by summing all incomes. The total cost consists of two main components: Fixed and variable costs. Though fixed costs are constant, they do not really matter for the optimization problems, and can be left out of the objective functions. Calculating the variable costs might be a complex task, depending on several variables.

The fuel costs depend on the price of each fuel type. These prices may follow local or global markets depending on type of fuel. Natural gas and oil are fuels which follow local and global market prices, respectively[115]. Further, the total fuel cost depends on consumption and storage costs.

Different operations such as boiler startups, turbine startups, load following and unit shutdowns entail costs. These costs are dependent of the characteristics of the hardware, and factors such as the amount of time which an unit has been shut down or operating[87]. It is a common mistake to only link these costs to increased fuel usage. Just as important are wear and tear to the equipment, caused by mechanical accelerations and thermal variations[68]. Examples of such damages are shown in figure 2.3 and 2.4.

Figure 2.3 shows a heat recovery steam generator's[1] (HRSG) superheater tube, which is cracking at the header. This type of damage is very costly, and might occur from thermal quenching[68].

Figure 2.4 shows an economizer[2] tube failure caused by assisted corrosion[68], which is a chemical corrosion process that causes wall loss in piping. The pipe wall thickness is progressively reduced to the point of rupture. Frequent ramping, startups and shutdowns in thermal power units, may affect the amount of corrosion in a negative manner[64].

Maintenance costs may both be fixed and variable. Most plants require maintenance periodically. Besides this, some units also require extra maintenance depending on the amount of energy produced, and the number of startups and ramping actions performed. After an unit has been maintained, it might also require a slow startup which price may differ from a regular startup.

A thorough review on operation and maintenance (O&M) costs for thermal units is presented in (Kumar et al. [64]). It also includes analyzes on the extra costs related to increased

[1]An energy recovery heat exchanger that recovers heat from a hot gas stream. The steam can be used to run a steam turbine.

[2]Heat exchange devices that heat fluids, usually water, up to, but normally not beyond, their boiling point.

Figure 2.3: Superheater tube cracking at header. *Source: Intertek AIM, The Increased Cost of Cycling Operations at Combined Cycle Power Plants*

Figure 2.4: Flow assisted corrosion causing economizer tube failure. *Source: Intertek AIM, The Increased Cost of Cycling Operations at Combined Cycle Power Plants*

failure rates due to cycling abrasion. Shorter component life expectancies will result in higher plant equivalent forced outage rates (EFOR), and increased costs to replace components at, or near, the end of their service lives[64].

Modeling Costs

In terms of implementing UC problems, it is hard to model all different aspects of costs exactly, and reasonable simplifications have to be made. In such matter the total cost of operating a generator may be divided into four sub-costs, and can be formulated as

$$c_{it} = c_{oper_{it}} + c_{ramp_{it}} + c_{start_{it}} + c_{stop_{it}} \tag{2.1}$$

where c_{it}, $c_{oper_{it}}$, $c_{ramp_{it}}$, $c_{start_{it}}$ and $c_{stop_{it}}$ are total cost, operational cost, ramping cost, startup cost and shutdown cost for unit i during period t, respectively.

The operational cost can be modeled on the form

$$\begin{aligned} c_{oper_{it}} &= (\epsilon_i \cdot f_{it} + \eta_i \cdot w_{it}) \cdot u_{on_{it}} \\ f_{it} &= \frac{w_{it}}{\theta_{it} \cdot \iota_i} \end{aligned} \tag{2.2}$$

where ϵ_i is the fuel price for unit i, f_{it} is the fuel/water consumption for unit i during period t, η_i is an unit specific maintenance parameter, w_{it} is the output from unit i during period t, $u_{on_{it}}$ is a binary variable which is 1 if unit i is on in period t and 0 else, θ_{it} is the efficiency of unit i during period t and ι_i is the energy content for the fuel consumed by unit i.

By using this approach, it is assumed that all fuel prices are kept constant during the whole simulation period, though ϵ_i is not time varying. This is a reasonable simplification for short-term simulations. If the formulation is to be extended with time varying fuel prices, there has to be added a fuel price estimated for each period t.

Considering each period t to last for one hour, the unit specific maintenance parameter η_i can be set to the average maintenance cost for each MWh produced. This way of implementing maintenance costs does not take extra maintenance due to much or fast cycling into account though, unless it is estimated a priori to the simulation and calculated into η_i. Another alternative could be to add this type of maintenance as a term in the calculation of $c_{ramp_{it}}$.

The efficiency variable, which is updated for each period t, can be calculated from the efficiency characteristic of the specific unit i. Examples of such characteristics are presented in (Wold [115]). A challenge including these types of characteristics in the objective function is that they often are highly nonlinear. This may increase the computational load significantly.

Equation 2.3 and 2.4 presents two different ways of modeling ramping costs, where the first in large extent is inspired by (Wang and Shahidehpour [111]).

$$c_{ramp_{it}} = (\zeta_{1_i} + \zeta_{2_i} \cdot |\Delta w_{it}| + \zeta_{3_i} \cdot l_{it} + \zeta_{4_i} \cdot l_{it}^2) \cdot u_{on_{it}} \tag{2.3}$$

$$c_{ramp_{it}} = \gamma_i \cdot |\Delta w_{it}| \cdot u_{on_{it}} \tag{2.4}$$

$\zeta_{1_i}, \zeta_{2_i}, \zeta_{3_i}$ are ζ_{4_i} are unit specific parameters which have to be estimated for each unit. Δw_{it} and l_{it} are change in output and load change gradient of unit i during time t, respectively. A good reason for having a time dependent load change gradient, is that it might be desirable not to ramp with maximum gain at all times. This is due to increased fuel consumption, emissions and tear on hardware.

Besides being nonlinear, the fact that equation 2.3 allows use of time varying load change gradient, increases the complexity of the problem drastically. Allowing load change gradients being time varying is a realistic approach, but it makes all load change gradients optimization variables, which has to be set for each time step.

In (Wold [115]) and (Scowcroft and Nies [98]) the load change gradient is described as the ability of changing output from one minute to another. In most power auctions, the bids are given as amount of power provided through one hour. This combined with the reduced computational load, have caused most approaches to the UC problem to be solved on hourly basis. In such matter, there is natural to define the load change gradient as the ability of changing output from one hour to another.

In comparison to equation 2.3, equation 2.4 adds much less complexity to the objective, by being linear for constant values of $u_{on_{it}}$, and using a constant load change gradient. The ramping cost parameter γ_i, can easily be estimated, though the equation is linear when the unit is operating. The significant downside of this approach is that it excludes a lot of the actual dynamics in the generators.

It is common to formulate startup costs as a function of the time which an unit has been turned of. This approach is most realistic for thermal units, where the startup costs depends on the units' temperature. An unit which has stayed off for a long time, will be cold, and in such matter be more expensive to start up than a warm one. An approach to model this can be formulated as

$$c_{start_{it}} = (\beta_{1_i} + \beta_{2_i} \cdot t_{off_{it}}) \cdot x_{it} \tag{2.5}$$

where $t_{off_{it}}$ is the time which unit i has been turned off at time t and β_{1_i} and β_{2_i} are unit specific parameters. x_{it} is a binary startup variable, which is 1 if unit i is started in period t and 0 else.

Equation 2.5 presents a simple approach, and keeps the computational load down. For simulations running for long periods of time, there might be beneficial to extend the formulation with a maximum price.

In terms of thermal units, startup procedures can roughly be divided into three main groups: Hot, warm and cold startups[64]. Using this approach, there is possible to model the startup cost after which startup procedure which is used. This can be formulated on the form

$$c_{start_{it}} = \begin{cases} \beta_{hot_i} \cdot w_{max_i} \cdot x_{it} & t_{off_{it}} < t_{warm_i} \\ \beta_{warm_i} \cdot w_{max_i} \cdot x_{it} & t_{warm_i} \leq t_{off_{it}} < t_{cold_i} \\ \beta_{cold_i} \cdot w_{max_i} \cdot x_{it} & t_{off_{it}} \geq t_{cold_i} \end{cases} \tag{2.6}$$

where β_{hot_i}, β_{warm_i} and β_{cold_i} are startup cost parameters for unit i while in state hot, warm and cold, respectively. t_{warm_i} and t_{cold_i} are the time before unit i turns from hot to warm and hot to cold, respectively. $t_{off_{it}}$ is a variable stating how long the respective unit has been off.

This approach is easy to implement, and realistic values for β_{hot_i}, β_{warm_i} and β_{cold_i} can be found in (Kumar et al. [64]). Significant disadvantages of modeling the cost this way, are that the function is not continuously differentiable, and that the steps increases the number of integer variables in the problem. For many solvers these drawbacks may cause the problem to become unsolvable, or to generate more computational load than using a smooth nonlinear model. In terms of being realistic, the steps also causes the model to be much more sensitive to small changes in unit down time compared, the actual situation.

In terms of achieving smoother transitions, and at the same time maintaining some of the flexibility offered in formulation 2.6, it is necessary with a nonlinear approach. In such matter both exponential and polynomial formulations may be considered. An example of an exponential formulation is

$$c_{start_{it}} = \left(\beta_{1_i} + \beta_{2_i} \cdot (1 - e^{\frac{-t_{off_{it}}}{\beta_{3_i}}})\right) \cdot x_{it} \tag{2.7}$$

where β_{1_i}, β_{2_i} and β_{3_i} are startup cost parameters for unit i. This formulation is in large extent inspired by (Gjengedal [46]).

In terms of having a realistic approach, a nonlinear formulation is most likely the best. Nevertheless it has to be taken into consideration that too many nonlinearities in the objective function, may cause the computation time to become unacceptable.

The shutdown cost is often modeled as an unit specific constant. This can be formulated as

$$c_{stop_{it}} = \alpha_i \cdot w_{max_i} \cdot (x_{it} + u_{on_{i(t-1)}} - u_{on_{it}}) \tag{2.8}$$

where α_i and w_{max_i} are an unit specific shutdown cost parameter and the maximum capacity of unit i, respectively. An alternative, but less common approach, is to model the shutdown cost as a function of the period of time which the unit has been operating. An example of such approach is presented in (Gjengedal [46]).

Modeling Constraints

In the same way as the costs, also the constraints of the UC problem has to be formulated. In the following paragraphs some approaches are presented and discussed.

Considering the ISO's responsibility of meeting the market demand, the UC problem is constrained to produce at least as much energy as the sum of demand and distribution losses in total. The losses depend on distribution path, and transmission line quality.

To maintain sufficient robustness and flexibility, and avoid interruptions, an electric power supply system depends on having a spinning reserve. This constraint may be defined in many ways. Some common approaches are to define it as a percentage of the estimated demand, a percentage of total capacity or a constant value.

In many grids there exist so called "must run units". These units include prescheduled units which must be online, due to operating reliability, economic considerations or governmental regulations. In the same way, in all grids there will from time to time be "must out units". This is usually due to mandatory maintenance or closure [87]. A good example of must run units are American nuclear power plants, which by regulations are prohibited from providing automatic load following or fast changes in output[115].

From a mathematical point of view, this can be implemented as

$$u_{on_{it}} = 1 \qquad \forall i \in A \tag{2.9}$$

$$u_{on_{it}} = 0 \qquad \forall i \in B \tag{2.10}$$

where A and B are the domains which includes all must run units and all must out units, respectively.

In terms of the ahead auction typically being arranged 24 hours ahead, and the fact that the UC problem rarely span over more than a few days, an easier approach is to exclude the must run and must out units from the simulation. This can be done by not including the units in domain B in the simulation at all, and expect the units in domain A to always operate on optimal efficiency. Then the output from units in A can be subtracted from the demand constraint, as shown in equation 2.11.

$$\sum_{i=1}^{n} w_{it} \cdot u_{on_{it}} \geq d_t + \xi_t - w_{At} \tag{2.11}$$

n, d_t, ξ_t and w_{At} are the number of units in the simulation, demand at time t, total expected transmission loss at time t and the total output provided by the must run units, respectively.

In terms of the technical limitations for each generator, there are several constraints to the UC problem. Every generator got a maximum capacity and a minimum output. The minimum output is defined as the lowest output which the generator can provide, without getting into a state which will shut it down[115]. Every generator also got limitations on its load change gradient. This constrains how fast a generator can change its output[98].

This chapter presents two suggestions for ramping constraints. The first, and most complex one, is on the form

$$\begin{gathered} w_{i(t-1)} \cdot (1 - l_{i(t-1)}) \cdot u_{on_{it}} \leq w_{it} \cdot u_{on_{it}} \leq w_{i(t-1)} \cdot (1 + l_{i(t-1)}) \cdot u_{on_{it}} \\ l_{it} \leq l_{max_i} \end{gathered} \tag{2.12}$$

where l_{max_i} is the maximum load change gradient per hour for unit i.

The constraint suggested in equation 2.12 is in large extent inspired by (Wold [115]), but instead of defining the gradient as a percentage gain per minute it is defined per hour.

The second suggestion for ramping constraint is

$$(w_{i(t-1)} - \Delta w_{max_i}) \cdot u_{on_{it}} \leq w_{it} \cdot u_{on_{it}} \leq (w_{i(t-1)} + \Delta w_{max_i}) \cdot u_{on_{it}} \tag{2.13}$$

where Δw_{max_i} is maximum load change per hour. This is a much simpler and less realistic way of constraining ramping. Using this formulation causes significant loss of dynamic for each power unit in the model. On the other hand, in terms of problem complexity and computation time, equation 2.13 got a great advantage compared to 2.12, though it limits the numbers of variables.

When an unit has finished its start up procedure, it will usually have to stay operative for a certain amount of time. The length of the period depends on generator characteristics and fuel type. In the same way, when a generator is shut down, it has to remain turned off for a certain period of time [107]. There have been made several different approaches to implement these constraints into the UC problem. Some good examples which in large extent has been used as templates in this thesis, are presented in (Hedman et al. [51]), (Rajan and Takriti [93]) and (Takriti et al. [108]). The second is partly a continuation of the third, while the first can be seen as a extension to the second. The results presented in the articles can be summarized as

$$x_{it} - y_{it} = u_{on_{it}} - u_{oni(t-1)} \tag{2.14}$$

$$\sum_{j=t-t_{min-on_i}+1}^{t} x_{ij} \leq u_{on_{it}} \qquad \forall i,j \in \{t_{min-on_i}+1...m\} \tag{2.15}$$

$$\sum_{j=t-t_{min-off_i}+1}^{t} y_{ij} \leq 1 - u_{on_{it}} \qquad \forall i,j \in \{t_{min-off_i}+1...m\} \tag{2.16}$$

$$x_{it} \in [0,1] \qquad \forall i,t$$

$$y_{it} \in [0,1] \qquad \forall i,t$$

$$u_{on_{it}} \in \{0,1\} \qquad \forall i,t$$

where y_{it} is a shutdown variable, which is 1 if and only if unit i is turned off at time t. Under any other circumstances it is 0. t_{min-on_i} and $t_{min-off_i}$ are the minimum time unit i has to stay on while started and off while turned off, respectively.

It is worth noting that both x_{it} and y_{it} can be treated as continuous variables since, by the constraints set within the model, their final values will always be binary once the value of $u_{on_{it}}$ is determined[51].

The inequalities 2.15 and 2.16 are not defined for all $t \in \{2...m\}$. This might cause challenges while implementing the constraints for problems where $t \leq t_{min-on_i}$ or $t \leq t_{min-off_i}$ occur. This can be handled by defining values for $t \leq t_{start} - max(t_{min-on_i}, t_{min-off_i}) + 1$, and let the simulation starts at time $t = t_{start}$.

Considering factors which are indirectly constraining the UC problem, it is natural to mention fuel supply, reservoir levels and network capacity. To optimize the fuel supply to a power plant might be a advanced optimization problem itself. It is desirable to always have sufficient fuel for providing the power production, and buy it for the lowest price possible. On the other hand, there are significant costs related to storing fuel, and especially natural gas. The complexity of natural gas transportation and storage is covered in detail in (Wold [115]).

While thermal units are constrained by fuel supply, conventional hydropower and pumped storage units are constrained in the same manner by reservoir levels. The conventional hydropower units are strictly depending on weather and melting water, and therefor there is nothing which can be done to supply the reservoirs.

In terms of pumped storage facilities, the situation is quite opposite, and there are several strategic decisions to be made. To make the plants profitable, there is necessary to buy cheap power to refill the reservoirs, and only sell while prices are high[60]. To succeed with this, significant analyzes of power price variations and informed bids in the power auctions are required.

In general modeling water reservoir levels might be challenging, and it is considered beyond the scope of this thesis. Two different approaches are presented in (Soroudi [103]) and (Rebennack et al. [94]).

Another important factor when it comes to hydro power, is whether there is desirable to use all available hydro power, even though it is the most profitable short-term solution. As discussed in (Wold [115]), hydropower provides an unique flexibility compared to other power sources, and there might be beneficial to limit the consumption to maintain it as an emergency resource. The decision depends strictly on the market, and the amount of hydropower available[115].

A simple way of modeling fuel and water consumption constraints for a short-term UC problem is

$$\sum_{t=1}^{m} f_{it} \leq f_{max_i} \tag{2.17}$$

where f_{max_i} and m are the maximum fuel/water consumption for unit i during the simulation and the time when the simulation ends, respectively.

To avoid having units produce while the MCP is too low to make the production profitable, there can be introduced a constraint on the form

$$u_{on_{it}} = 0 \ \text{ if } \mu_t < \mu_{min_i} \tag{2.18}$$

where μ_t and μ_{min_i} are the MCP at time t and minimum acceptable MCP for unit i, respectively. To model the MCP is a complex task, but if the capacity available in the market is approximately constant, it is reasonable to assume that MCP can be calculated based on market demand alone. An alternative to constraint 2.18 can then be formulated on the form

$$u_{on_{it}} = 0 \ \text{ if } d_t < d_{min_i} \tag{2.19}$$

where d_t and d_{min_i} are demand at time t and minimum required demand to make it profitable to run unit i, respectively.

When an ISO is solving the UC problem, and schedules the production, it also has to take into account capacity limitations in transmission lines. This information is usually provided from the transmission companies (TRANSCOs), through a real time information system[118]. In terms of the UC problem, these limitations can be included by a constraint on the form

$$w_{it} \leq w_{net_{it}} \tag{2.20}$$

where $w_{net_{it}}$ is the maximum available network capacity for unit i at time t. $w_{net_{it}}$ has to be updated every time step t, by solving a flow problem for the transmission network.

2.2 Emission Constraints

The Intergovernmental Panel on Climate Change was established in 1988, jointly by the World Meteorological Organization and the United Nations Environment Programme. This panel led the way for the United Nations Framework Convention on Climate Change in 1992, which became the framework of the Kyoto protocol in 1997[81]. The protocol states to which extent the parties are to reduce their future green house gas (GHG) emissions, relative to the emissions in 1990. The average reductions are 5.2%, while the EU has committed itself to reduce its emissions with 8%[102]. The protocol specifically remarks the power industry as a sector which can contribute achieving the stated targets, and defines the following gases as GHGs: Carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons ($HFCs$), perfluorocarbons ($PFCs$) and sulfur hexafluoride (SF_6).

On annual basis, the world emits approximately 27 gigatonnes of CO_2 from multiple sources. Electric power production accounts for approximately 37% (10 gigatonnes) of these emissions[75]. At the same time, the global demand of electricity is increasing fast[115].

Nitrogen oxides (NO_x) and sulfur dioxide (SO_2) emissions are not accounted for by the Kyoto protocol, but most Western-European countries got regulations which limit these emissions. NO_x forms quickly from combustion power plants, and may further react to form smog and acid rain. It is also being central to the formation of tropospheric ozone.

SO_2 is formed when fuel containing sulfur is combusted. The gas itself is toxic, and got a rotten smell. If the gas oxidates further, which often happens in the presence of a catalyst such as NO_2, sulfuric acid (H_2SO_4) might form. When the H_2SO_4 reaches the atmosphere, it causes acid rain.

There have been made several approaches around the world to limit emissions. The US got a cap and trade system for SO_2. The idea is to have a maximum cap for the emissions. Each part in the market gets its emission allowances for a certain period. If an operator wants to exceed its allowances, it has to buy allowances from other operators. If emissions exceed the respective allowances, the responsible actor will face a huge financial penalty[36]. Within EU SO_2 and NO_x emissions got specific constraints imposed by the EU directives[102].

The American cap and trade market for SO_2 emissions has been an important source of inspiration for the EU ETS, which is the world's first large-scale CO_2 emissions trading program, started January 1st, 2005. Examples of previous smaller scale emission trading programs are: UK Emissions Trading Scheme (UK ETS), the Danish CO_2 trading program, the Dutch offset programs, and BP's internal experiment with emissions trading. The EU ETS was in large extent motivated by the Kyoto protocol, but it is expected to continue further, regardless of what happens to the protocol in the future. The scheme includes approximately 11500 emission sources located in the participating countries, including all combustion sources exceeding 20 MW thermal rating[36].

In short terms, the EU ETS regulates the CO_2 emission allowances for each participating part, and states the rules of international trading of allowances. How allowances are distributed within a specific country, and how internal trading is organized is handled by the respective country[36]. How the GENCOs are to distribute their allowances during different periods, are typically decided during medium-term planning.

While considering GHG emissions and construction of power plants in the future, there is a natural approach to evaluate the total amount of emissions during the lifecycle of the plant. Such evaluation should include emissions related to construction, operation (including fuel supply chain) and decommissioning. (McIntyre et al. [75]) have made a research on this, and their results are presented in table 2.1. What they have not taken into account, is Carbon Capture Sequestration (CCS), that is a technology which often is cited to reduce carbon emissions from coal fired power plants drastically. This is due to the fact that the technology is new, expensive and not have a widespread commercial application. Nevertheless, in the future CCS might make fossil fueled plants more competitive in terms of CO_2 emissions[75].

So far, this paper has covered scheduling and unit specific constraints, as well as economical factors related to the conventional UC problem. Minimizing fuel costs and emissions are conflicting objects[77], though, in view of the last decades' climate changes, this conventional approach is inadequate. To run the power industry in a sustainable way, it is necessary to take emissions into consideration.

In terms of the UC problem, the emissions related to construction have already occurred. In such matter, the focus should be on constraining emissions directly related to operations. Table 2.2 presents typical average values for CO_2 emissions during operation, with respect to fuel type.

Certain emissions, such as SO_2 and CO_2, are directly related to the amount of fuel consumed. Such emissions can be calculated directly based on the fuel consume, which is directly related to the generator load. The emissions are found by multiplying emission factors

Technology	Mean	Low	High
Lignite	1054	790	1372
Coal	888	756	1310
Oil	733	547	935
Natural Gas	499	362	891
Solar PV	85	13	731
Biomass	45	10	101
Nuclear	29	2	130
Hydroelectric	26	2	237
Wind	26	6	124

Table 2.1: Average CO_2 emissions [tonnes CO_2e/GWh] during the lifecycles of power plants. *Source: World Nuclear Association (WNA) Report, Comparison of Lifecycle Greenhouse Gas Emissions of Various Electricity Generation Sources*

Fuel	kg CO_2/kWh
Bituminous (black coal)	0.94
Sub-bituminous	0.98
Lignite (brown coal)	0.99
Natural Gas	0.55
Distillate Oil (No. 2)	0.76
Residual Oil (No. 6)	0.82

Table 2.2: Average CO_2 emissions from power plants during combustion of fuel. *Source: U.S. Energy Information Administration (EIA), Independent Statistics and Analysis*

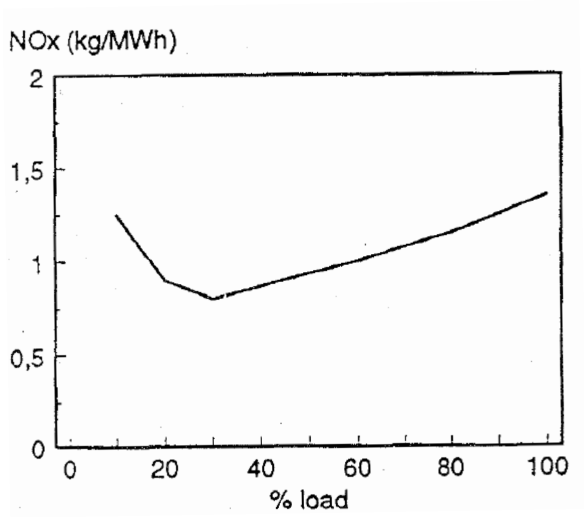


Figure 2.5: Specific NO_x emission characteristic for a 500 MW coal unit. *Source: Gjengedal, Emission Constrained Unit-Commitment (ECUC)*

with the fuel consumption[46]. In modern plants there might be installed purifiers limiting these types of emissions. These can be taken into account by reducing the emission factors.

Other types of emissions, such as NO_x, are primarily combustion process dependent. These emissions cannot be described accurately from input or output information, and require specific combustion characteristics to be calculated. An example of such characteristic is shown in figure 2.5.

It is important notifying that emission rates during startup, shutdown and ramping processes deviates considerably from while operating in a steady state mode[46].

There have been made different approaches making emission constraints for the UC problem. The simplest and most obvious approach is to add an inequality constraint on the form

$$\sum_{i=1}^{n}\sum_{t=1}^{m} e_{start_{it}} + e_{oper_{it}} + e_{stop_{it}} \leq e_{max} \tag{2.21}$$

to the problem, where $e_{start_{it}}$, $e_{oper_{it}}$, $e_{stop_{it}}$ and e_{max} are startup emissions from unit i during period t, operational emissions from unit i during period t, shutdown emissions from unit i during period t and maximum permitted total emissions, respectively. Approaches similar to this are presented in (Yamin and Shahidehpour [118]), (Gjengedal [46]) and (Simopoulos et al. [102]).

In general, this is a straightforward way of including emissions to the UC problem. The emission cap can easily be adjusted, and if there is desirable to have separate constraints for different types of emissions, the model can easily expanded. The adverse feature of this approach is that there is no constraint on each individual unit. From a GENCO's point of view, in terms of the self-scheduling, this approach might be sufficient. If the optimal solution requires one unit to release emissions above its allowances, this can be covered for by using allowances from other units belonging to the same GENCO (unless there are geographical regulations associated with the allowances).

From an ISO's point of view, and the UC problem, this approach might be found deficient while trying to maximize the total profit for a group of GENCOs. While solving the problem, it will lack information regarding the different GENCO's allowances, and in such matter the solution will be useless. A solution to this problem is to keep the GENCOs responsible of not offering more energy in the auctions than their allowances can cover for.

Another option using hard constraints, would be constraining the emissions from each generating unit. This approach will solve the problem from the ISO's point of view, but will limit the GENCO's possibility of moving quotas between their own units.

Nevertheless, using hard emission constraints got one major downside which is difficult to get away from. The formulations become very sensitive in terms of changes in demand. If the demand increases, and the constraints are not adjusted, the problem will become infeasible. This has to be taken into consideration if, the model is going to be used in an adjustment market or similar.

In (Mendes et al. [77]) a different approach of including emissions to the UC problem is presented. Instead of using a conventional constraint, the sum of emissions are included in the objective function. It is represented on the form

$$f = \sum_{i=1}^{n} \sum_{t=1}^{m} \omega \cdot c_{it} + \rho \cdot (1-\omega) \cdot e_{it} \tag{2.22}$$

where e_{it}, ω and ρ are emissions caused by unit i during period t, a weighting factor between 0 and 1 and a scaling factor respectively.

This approach is obviously not very suitable for either GENCOs or ISOs regulated by the EU ETS, though it does not benefit from any form of emission cap. On the other hand, it could be very useful in a market where the GENCOs are paying a price per unit emission released. Further, in a research perspective this approach comes in handy. By making simulations with different values of ρ and ω, there can be seen how the dispatch of generators varies. For example will a high value of ρ combined with a small ω, most likely make coal units less attractive compared to natural gas units, which have lower emission rates.

A third approach is combining elements from both the suggestions above. This can be done by having defined allowances for each plant. If a plant is exceeding its allowances, it will be charged a penalty fee, or be forced to buy extra allowances. There are several ways of implementing this. One example is presented in (Rebennack et al. [94]). In terms of the EU ETS, this approach is very realistic and useful both for ISOs and GENCOs. An objective function for this approach can be formulated on the form

$$f = \sum_{i=1}^{n} \left(p_i + c_{allowance_i} + \sum_{t=1}^{m} c_{start_{it}} + c_{oper_{it}} + c_{stop_{it}} \right) \tag{2.23}$$

where p_i is the sum of fees generated by unit i. For units which not exceed their allowances, this value will be set to zero. $c_{allowance_i}$ is the cost from buying extra allowances for unit i. If the model is very complex, this value may be allowed to become negative in terms modeling sales of allowances.

To include allowance trading in the UC problem may increase the problem complexity significantly, though this is a separate market with varying prices depending on demand. This might make it reasonable to simplify the model only to include penalty fees.

The penalty fees can be modeled in several ways. The easiest is to make it proportional to the exceeding emissions. This can be expressed on the form

$$p_i = \begin{cases} 0 & \text{if } e_{total_i} - e_{max_i} \leq 0 \\ (e_{total_i} - e_{max_i}) \cdot c_{emission} & \text{if } e_{total_i} - e_{max_i} > 0 \end{cases} \tag{2.24}$$

where e_{total_i} and $c_{emission}$ are sum of emissions from unit i and the cost for each emission unit exceeding e_{max_i}, respectively. Considering CO_2, a typical value for $c_{emission}$ would be 100 €/tCO_2, which was the penalty the parties agreed upon for the first period of EU ETS[102].

Other opportunities are to express p_i as a polynomial or exponential function with respect to the difference between e_{total_i} and e_{max_i}. These approaches give the opportunity of penalizing large violations of the allowances stricter than small ones. A disadvantage choosing this way of penalizing, is that the complexity of the problem increases significantly due to nonlinearities.

In terms of allowances, and real world trading schemes, there is a significant challenge for the GENCOS related to allocating the allowances. This planning has to be included in resource management during the medium-tern planning. A power plant is typically assigned a number of allowances for periods in the range from 6 months to 4 years. These have to be allocated in a way maintaining flexibility, such that the plant is allowed releasing more emissions in periods of high demand. On the other hand, the GENCO operating the plant should also make sure it got sufficient allowances to operate throughout the full period which the allowances last for. An important factor in this planning is whether an unused allowance can be brought into a new period or not.

A way of modeling the stock of allowances is purposed in (Rebennack et al. [94]). It suggests modeling allowances as a stored resource, similar to hydro power reservoirs. The unused allowances controlled by the GENCO is seen as the level of a reservoir containing emissions. The emissions released from the unit is equivalent to outflow from the reservoir. In the same way new, acquired allowances are seen as inflow.

2.3 The Norwegian Power Market

In terms of deregulation, Norway was kind of a pioneer when it in 1991, became the first country in the world to introduce a fully deregulated marked for electric power trading[49]. From this it followed that Statnett Marked, a trading facility for electric power, was founded in 1993. In 1996 Sweden joined Statnett Marked, and the organization was rebranded Nord Pool. The expansion did continue, and in 1998 Finland joined. In 1999 Nord Pool launched Elbas, the first international intraday market, followed by Denmark joining in 2000. In total, these events constituted the foundation of what is today the world's largest marked of it's kind, including among other Germany, UK and the Baltic countries. Today 370 companies from 20 countries operate in Nord Pool's markets, and in 2013 there was sold more than 493 TWh within Nord Pool Spot (suborganization of Nord Pool). For comparison, this is equivalent to the electric power consumption of Oslo for 61 years[104].

Nord Pool is to be considered an ISO for all its markets, and is regulated by the Norwegian Water Resource and Energy Directorate (NVE). The stakeholders are Statnett (Norway, 28.2%), Svenska Kraftnät (Sweden 28.2%), Fingrid (Finland, 18.8%), Energinet.dk (Denmark, 18.8%), Elering (Estonia, 2%), Litgrid (Lithuania, 2%) and AST (Latvia, 2%)[104].

In terms of time perspective, the Nord Pool markets can be divided into 4 submarkets: Financial market, day-ahead market (Elspot), intraday market (Elbas) and Balancing market. The financial market is used for managing risks. The Elspot system price is used as a reference for power supply contracts, lasting for up to six years[104].

Elspot is the main trading market, and is organized through a day-ahead auction of power delivery the next day. Nord Pool Spot calculates power prices based on supply and demand for every hour the following day[104].

According to (Spot [104]) the daily routines of trading in Elspot can be summarized as:

- 0700-1000: Power transmission capacities are given by the system operators (TRANSCOs) to each bidding area in the market.
- 0700-1200: Buyers plan how much power they will need, and the sellers (GENCOs) plan how much power they will produce.
- 0800-1200: Buyers and sellers enter their bids and offers into the trading system.
- 1200: Auction closes.
- 1200-1245: Based on orders and transmission capacity, the prices are calculated in the trading system. The price is calculated for each hour of the day.
- 1245: The prices are announced to the market.
- 1300-1500: The trades are invoiced between buyers and sellers.

Elbas provides continuous trading up to 30 minutes before delivery to adjust power production or consumption plans[104].

The balance market is operated by the respective transmission system operators. Final adjustments are made to ensure the correct frequency in the grid and security of supply[104].

It is worth noting that all the markets mentioned are open for international actors, and in the same way are Norwegian GENCOs allowed to export their products to customers beyond Norwegian borders.

The Norwegian power network can roughly be divided into three main parts: Central network, regional network and distribution network. The central network is the core of the transmission system, and takes care of all long distance distribution. It consists of approximately 11000 km of cord, where 90% is owned by Statnett. Other TRANSCOs with significant shares of the central network are BKK Nett AS, SKL Nett AS, Lyse Elnett AS and Hafslund Nett AS[83].

The regional network consists of approximately 19000 km cord, transmitting high voltage power within specific regions. The largest owners, with more than 1000 km cord each, are Hafslund Nett AS, Eidsiva Nett AS and Skagerak Nett AS[83].

The distribution network is also often referred to as the local network. It takes care of distribution to regular end users, such as residents and retail stores. The total size of this network is about 305000 km. The largest shareholders in the distribution network are Hafslund Nett AS, BKK Nett AS, Agder Energi Nett AS, Skagerrak Nett AS and Eidsiva Nett AS[83].

The deregulation of the power market has caused a increased streamlining of the industry. The investments in both production and transmission facilities have dropped, and this is despite the fact that the demand has kept growing. A direct consequence of this is that the existing facilities are utilized to maximum capacity. Especially the central network in the northern areas are undersized[83].

In general, effort is being made to maintain an as equal market price as possible all over the country, but due to lower production in the northern areas, and the bottle neck in the transmission system this is challenging. To meet the demand of this areas in a sustainable way, there is an extensive cooperation with Sweden, which has more of its production located up north. The cooperation among other includes sharing transmission lines and mutual exchange of power. For example, to keep transmission losses to a minimum, facilities in northern parts Sweden may export power to the northern parts of Norway, while facilities located south-east in Norway are exporting to southern parts of Sweden.

2.4 Natural Gas Fired Power Plants and the UC Problem

As illustrated in figure 2.6, in terms of power production, natural gas is one of the fastest growing fuels[88]. Due to the shale gas revolution in the US, they are having the fastest growth percentage both considering general natural gas consumption and production. The increased production has affected the prices in the national market significantly, and has turned the US Henry Hub from one of the most expensive regional spot markets in the early 2000s, to one of the significantly cheapest today[89]. The price development is shown in figure 2.7.

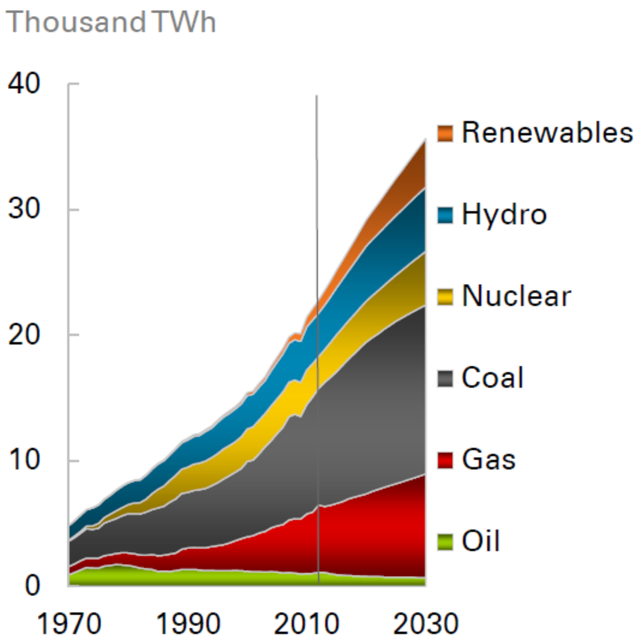


Figure 2.6: Estimated world power production by source. *Source: British Petroleum, BP Energy Outlook 2030*

As discussed in (Wold [115]), the decreased prices have made natural gas a more competitive energy source in the US' power industry. As illustrated in figure 2.8, it has nearly doubled its market share between 2002 and 2012, and it is expected to increase even further. A important observation, is that fuel types loosing market shares are primarily coal and oil. In terms of emissions, this is a positive development, though natural gas in general is considered to have a lower emission intensity per unit electricity generated than coal.

The US decreased their CO_2 emissions with 8.6% between 2005 and 2012. Not all the reduction came from the power industry, where shale gas has had its most significant impact, and neither is all the replacement of coal due to low natural gas prices. Nevertheless, research has shown that it is reasonable to assume that the reduced natural gas price is the reason for 35-50% of the reduction[17].

In terms of the increased use of natural gas fired generators, it is of interest to evaluate to which extent they affect the UC problem. The conventional hydrothermal UC problem includes hydropowered units and thermal units in general. The term thermal units covers combustion units, which also includes natural gas fired units. Hydrothermal UC has been a subject of comprehensive research the last decades[9], and in such matter also UC problems including natural gas fired units.

What directly differs the natural gas fired units from other thermal units, are faster load

Figure 2.7: Price variations of four significant natural gas markets. *Source: British Petroleum, BP Statistical Review of World Energy June 2013*

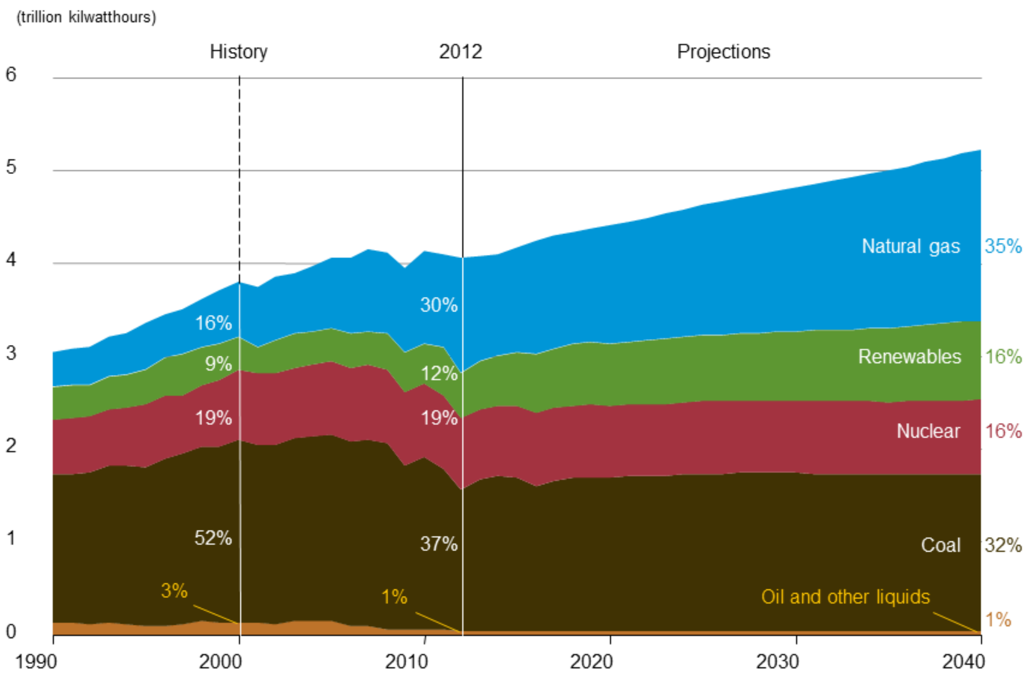


Figure 2.8: Estimated US' electricity generation by source. *Source: U.S. Energy Information Administration (EIA), Independent Statistics & Analysis, Annual Energy Outlook 2014 Early Release Overview*

change gradients[98] and startup procedures[70]. This is mainly because of the physical features of natural gas, but also because of the design of the power plants. Going back before the shale gas boom in the US, natural gas units were mainly considered peak demand units due to the high fuel prices[115]. For this reason, these units were also designed to provide fast changes in output. On the other hand, coal fired units are often designed to operate as baseload suppliers and are therefore slower. These units also provides the highest ramping costs if both fuel consumption and tear on hardware are taken into consideration[64].

Considering modeling natural gas units for a UC problem, it will in general be natural to model the units similar to other fossil fueled units, and add the unit specific features. The part about adding unit specific features is nothing special for natural gas units, but something which is necessary for any unit. The potentially faster load change gradients of the units will provide increased flexibility to the system in terms of ramping. On the other hand, units which are designed to operate only as peak demand units may have limited maximum operational time. This may increase the complexity and computational load of the problem.

Focusing on a broader aspect than just the technical features of the units, there is one practical constraint which significantly works against natural gas compared to other fossil fuels: Fuel supply. The complexity of distributing and storing natural gas is presented in (Wold [115]), and it is clear that storing natural gas causes significant costs.

In terms of the American market, the natural gas and electricity infrastructures are highly intertwined. The electric power supply system is special in the sense that real time balance between generation and load always is required. If there occurs contingencies, this has to be responded for quickly, by ramping up a spinning reserve or starting up a quick starting unit. If an imbalance not is compensated for fast enough, the frequency of the power system will change, which may cause instability due to electromechanical and electromagnetic transients. This might further escalate to voltage collapses and potential blackouts. For this reason, predictable supply of natural gas is critical for the stability of electricity supply systems[70].

Natural gas fired power plants are usually high priority customers. Nevertheless, the pipeline system may saturate, a congestion may occur or a well may have reached its maximum discharge. Further, the units require a high gas pressure to function well, and are therefor sensitive to pressure drops[70].

To model the supply and having sufficient emergency handling are important challenges for the GENCOs. Instabilities and blackouts may cause damage to the system and claims. The complexity of integrating the natural gas supply chain to the UC problem is significant. Some approaches are presented in (Liu et al. [70]), (Liu et al. [69]), (Damavandi et al. [29]) and (Wu et al. [117]).

2.5 Solving a Deterministic UC Problem

Background

During the last 50 years, there have been made several approaches solving the UC problem. In the early beginning, exhaustive enumeration and priority listing was common methodologies. In short terms the exhaustive enumeration approach consists of enumerating all possible combinations of generating units, and then choose the combination which accumulates the least total operational cost[87]. For obvious reasons, this method is inefficient and not suitable for large scale problems. (Hara et al. [50]) and (Kerr et al. [61]) presents solutions to UC problems using exhaustive enumeration.

The concept of priority listing is to arrange the units based on lowest operational cost characteristics. The units which generates least operational costs get the highest priority. Then the UC problem is solved by combining the units which got the highest priorities, until the demand constraint is satisfied[87]. This solution is not necessary optimal, and rarely efficient when there are many constraints involved. (Lee [66]) and (Burns and Gibson [20]) presents examples of UC problems solved using priority listing.

The deterministic UC problem can be described as a mixed integer nonlinear program (MINLP) problem[46]. In general this kind of problems can be formulated on the form

$$\begin{aligned} min\ f(x,y) \\ subject\ to\ g(x,y) \leq 0 \\ x \in X \subseteq \mathbb{R}^n \\ y \in Y \subseteq \mathbb{Z}^s \end{aligned} \tag{2.25}$$

where function $f : \mathbb{R}^{n+s} \to \mathbb{R}$ and $g : \mathbb{R}^{n+s} \to \mathbb{R}^m$ are a possibly nonlinear objective function and a possibly nonlinear constraint function, respectively. x and y are decision variables, and y is required to be integer valued. The sets $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{Z}^s$ are bounding-box-type restrictions on the variables[21].

One of the earliest (possibly the first), commercial software packages solving MINLP problems was SCICONIC, developed in the 1970s. This algorithm replaced low dimensional nonlinear terms with piecewise linear approximations, and was able to obtain a solution from the approximation of the original MINLP problem[42]. The work of Grossman, Kocis and Duran in the 1980s led to DICOPT (Discrete Continuous OPTimizer), which featured an interface for solving convex MINLP problems in GAMS[62]. ALPHABB, BARON and GLOP are considered some of the first solvers providing global solutions for nonconvex MINLP problems[21].

MINLPs are NP-hard problems[45]. This means, in worst case scenario, the computation time solving the problem increases exponentially with the size of the problem. As a consequence, large MINLPs might be unsolvable within reasonable time[100].

The computational tractability while solving problem 2.25, depends significantly on whether it is convex or not. The MINLP is to be considered convex if both $f(x,y)$ and $g(x,y)$ are convex over $X \times Y$[21]. A function h is convex if its domain S is convex, and if for any two points x and y in S

$$h(a \cdot x + (1-a) \cdot y) \leq \ a \cdot h(x) + (1-a) \cdot h(y) \quad \text{for all } a \in [0,1] \tag{2.26}$$

is true. The set $S \in \mathbb{R}^n$ is convex if any two points $x \in S$ and $y \in S$ satisfies $a \cdot x + (1-a) \cdot y \ \in S$ for all $a \in [0,1]$[82].

During the last 30 years, the complexity and number of methodologies for solving UC problems has developed significantly. Modern algorithms usually take advantage of combining several of them. Further in this section, some of the methodologies are presented, followed by an overview of the most common MINLP solvers.

Methodologies

The nature of a MINLP problem is such that some of the variables can take only discrete values, while others are continuous[67]. In formulation 2.25 these variables are located in Y and X, respectively. In two cases, the problem complexity is drastically reduced. At the

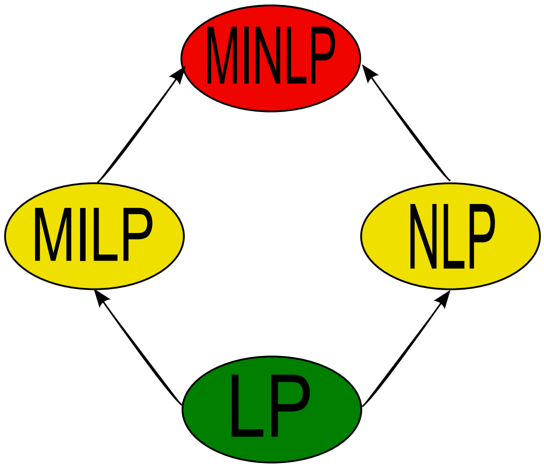


Figure 2.9: Optimization problem hierarchy

extreme where Y is empty, the problem reduces to a NLP problem, and if both f and g are linear the problem reduces to a MILP problem. If both cases occurs at the same time, problem 2.25 reduces to a standard LP problem[67]. The hierarchy of theses different problems are presented in figure 2.9

Due to the relation between MINLP, MILP, MIP and LP problems, MINLP solvers are often built by combining solvers for the three latter problem types[21]. The traditional approach to MINLP is either relaxing the integrality restrictions and using an NLP solver, or replacing the nonlinear functions with a piecewise linear one. The former suggestion, may cause difficulties finding a solution satisfying the integrality constraints. Furthermore, there is common that the NLP relaxation does not have a convex feasible region, and it follows that most NLP methods seek a local optimum. In complex models, there is virtually impossible to verify if a solution is a global one. The latter approach may increase the number of discrete variables drastically, which tends to increase the computation time dramatically. The number of discrete variables can be limited by using coarser intervals for the piecewise linear functions, but this will affect the accuracy[67].

In terms of MINLP problems of high complexity, including many UC problems, the traditional approaches alone become insufficient. It becomes necessary to combine different algorithms for MILP, NLP and other approaches to gain a sufficient solver. One of the most common methodologies used in such solvers is dynamic programming (DP)[87]. The idea of DP is solving complex problems by breaking them down into simpler subproblems, and it is applicable to problems exhibiting the properties of overlapping subproblems and optimal substructure[31]. While applied successfully, each subproblem is only solved ones during the process solving the problem. This approach is often way more efficient than more naive approaches such as dept first search[27].

In terms of the UC problem, depending og the constraints taken into account, the state of the system would typically be the states of the generating units or the number of hours which each unit has been on or off. Using one of these approaches, often makes DP a suitable strategy for solving UC problems[91]. Instead of solving the UC problem directly, there might be possible to relax some of the constraints to decouple the problem. This may result in each generator having its own optimization problem, which can be solved using DP[108]. A possible difficulty is the dimensionality if the state space, which may grow fast, is large. This may cause the approach to become too time consuming[91]. Examples of previous work which have benefited from using DP as a part of their solvers are: (Rebennack et al.

[94]),(Mendes et al. [78]) and (van den Bosch and Hondered [110]).

(Mendes et al. [78]) is combining DP with Lagrangian relaxation (LR). In terms of commercial solutions to the UC problem, the utilization of LR is much more recent than DP. Compared to DP, LR is more beneficial for problems with many generating units. This is due to the fact that the suboptimality goes to zero, as the number of generating units increases. LR methodology is also easily modified to model characteristics of specific generating units, and there is relatively easy to add unit constraints. The most significant disadvantages are that it got a inherent suboptimality[87], and that the solution not necessarily is feasible for the original problem.

In short terms, LR is a way of relaxing a constrained problem to a simpler problem. LR penalizes violations of inequality constraints by multiplying them with a Lagrangian multiplier, and relaxing the result into the objective function[41]. The solution of the relaxed problem is an approximation of the solution to the original problem. For a maximization problem solved using LR, the resulting value provides an upper bound of the solution for the original problem. If the solution is feasible for the original problem, it provides a lower bound as well[48]. Since

$$min\ h(x) = max\ -h(x) \tag{2.27}$$

LR is also well applicable for minimization problems as well.

Problem 2.25 can be reformulated to

$$\begin{aligned} &min\ f(x,y) \\ subject\ to\ &g_1(x,y) \le 0 \\ &g_2(x,y) \le 0 \\ &x \in X \subseteq \mathbb{R}^n \\ &y \in Y \subseteq \mathbb{Z}^s \end{aligned} \tag{2.28}$$

where $g_1 : \mathbb{R}^{n+s} \to \mathbb{R}^{m_1}$, $g_2 : \mathbb{R}^{n+s} \to \mathbb{R}^{m_2}$ and $m = m_1 + m_2$. If g_2 contains the constraints which are to be relaxed, the LR of problem 2.28 can be formulated as

$$\begin{aligned} &min\ f(x,y) + \lambda^T g_2(x,y) \\ subject\ to\ &g_1(x,y) \le 0 \\ &x \in X \subseteq \mathbb{R}^n \\ &y \in Y \subseteq \mathbb{Z}^s \end{aligned} \tag{2.29}$$

where $\lambda \in \mathbb{R}^{m_2}$ and contains the Lagrangian multipliers.

(Yamin and Shahidehpour [118]) and (Takriti et al. [107]) are former articles which presents results to UC problems, using solvers benefiting from LR.

(Takriti et al. [107]) refers to LR, DP and branch and bound methods as the three main types of approaches to the UC problem. Branch and bound is a basic, or general, algorithm for solving MIP problems[90]. The basic idea of the method is to divide and conquer. Since the original problem is hard to solve, it is divided into smaller and smaller subproblems, until they can be easily solved. The branching of the original problem is done by partitioning the entire feasible set into smaller subsets, and if necessary do another branching on the subsets. The conquering, often referred to as fathoming, consists of two main steps: Giving a bound for the best possible solution in the subset, and discarding the subset if the bound indicates that it cannot contain an optimal solution[19]. (Huang et al. [53]), (Cohen and Yoshimura [25]) and (Lauer et al. [65]) present different approaches of solving the UC problem, taking advantage of the branch and bound methodology.

For a long time, approaches to the UC problem using simulated annealing (SA) gave few encouraging results[108]. The reason for this can partly be explained by SA's disadvantage of requiring much CPU time[74]. Throughout the recent years, available computational capacity has grown, and so has the use of SA solving UC problems. The technique has attracted a lot of attention, because of its ability of solving difficult combinatorial optimization problems, while easily handling complex nonlinear constraints[102].

The original SA algorithm simulates the process of gradually cooling a metal, until the energy of the system acquires the global minimal value. The process starts with the metal having a high temperature, and it is slowly cooled such that the system is in thermal equilibrium at every stage. At each stage, there is performed an iterative procedure consisting of a sequence of trials. The trials are results of a Markov chain, where the outcome of each trial depends strictly on the previous outcome. In each trial, the state of a atom is randomly perturbed. This results in a change to the energy of the system. If the energy has decreased or is unchanged, the configuration is accepted and becomes the starting point for a new trial. If the energy has increased, the change is accepted with a probability calculated by a Boltzmann distribution[102].

The analogy of SA is suitable for solving UC problems, by letting the configuration of the physical system correspond to a schedule of generating units. Further, the costs can be seen as analogous to the energy of the atoms. Then, the final ground state of the physical is equivalent to the global minimum of the cost function. The temperature can be treated as equivalent to the control parameter of the optimization process[102]. (Simopoulos et al. [102]) and (Dudek [34]) provides two different approaches to the UC problem taking advantage of SA.

(Madrigal and Quintana [72]) is using an interior point method to solve an UC problem. Interior point methods have been used to solve very large linear and nonlinear programming problems, as well as combinatorial and nondifferentiable problems[87]. In general, these methods are a class of algorithms that achieves optimization by going through the middle of the solid defined by the problem, rather than around the surface[112].

(Wang and Shahidehpour [111]) presents an approach solving a scheduling problem combining LR and Benders decomposition. The latter is an algorithm which provides the opportunity of solving large LP problems that have a special block structure. As it progresses towards a solution, the algorithm adds new constraints [6], an approach which often is referred to as "row generation" as a contrast to the Dantzig-Wolfe decomposition relying on column generation[30].

(Mantawy et al. [73]) is solving an UC problem by dividing into a combinatorial optimization problem and a NLP problem. The former is solved using a tabu search (TS) algorithm, while the latter is solved using a quadratic programming routine. TS is a metaheuristic, employing neighborhood search methods for mathematical optimization. It can be considered a iterative improvement procedure that starts from a feasible initial point, and attempts to determine a better solution taking advantage of the greatest descent algorithm. TS seeks to counter the danger of entrapment at a local optimum, by incorporating a memory structure that avoids moves which may return to recently visited solutions[73].

The number methodologies used to solve UC problems is huge. In terms of what has been recovered in this research, and to the best of the author's knowledge, the most common approaches have now been introduced. Other methodologies which curios readers might find exciting are expert systems[63], fuzzy systems[58], artificial neural networks[76], genetic

Extended MIP solver	Extended NLP solver	From scratch solver
BONMIN	BNB	alphaBB
Couenne	FICO Xpress-SLP	AlphaECP
CPLEX	fminconset	AOA
FICO Epress-Optimizer	Knitro	BARON
FilMINT	MILANO	DICOPT
LindoAPI without global solver option	MINLP_BB	GloMIQO
MOSEK	MISQP with OA extension	LaGO
SCIP	OQNLP	LindoAPI
	SBB	MIDACO

Table 2.3: MINLP solvers grouped by the basis they take advantage of. *Source: Bussieck and Vigerske, MINLP Solver Software*

algorithms[99], evolutionary programming[57] and ant colony search algorithm[106].

MINLP Solvers

Commercial MINLP solvers are often implemented in modeling systems. Roughly divided, these can be split into general systems like AIMMS[95], AMPL[43] and GAMS[44], and vendor specific systems such as FICO Xpress-Mosel[38], LINGO[97] and OPL[33]. The solvers are often designed by integrating LP, MIP and NLP solvers which are specifically implemented for the respective modeling system[21]. Therefor, solvers designed for a specific modeling system, which are not fully open source, can rarely be transfered to other modeling systems.

Considering the structures of the most common solvers, they can be divided into three groups: Extended MIP solvers, extended NLP solvers and from scratch solvers. The latter group consist of solvers, which are more or less developed from scratch, but they may still solve LP, MIP and NLP subproblems[21]. An overview of some solvers which belong in the different groups are shown in table 2.3.

There is important to be aware that not all the solvers presented in table 2.3 accepts all MINLPs as input. CPLEX, FICO Epress-Optimizer, GloMIQO and MOSEK can only handle MILPs, MIQPs and MIQCPs[21], where the two latter can be considered a very specific group of MINLPs[4],[10]. Therefor, utilizing these solvers requires the problem formulation only to consist of linear and quadratic mixed integer terms. In terms of the UC problem, the mentioned requirements are rarely satisfied in its original form. An approach solving the problem is to reformulate it into a MIQP or MIQCP using piecewise linearization on all nonlinear terms which are not quadratic. By also using piecewise linearization on the quadratic terms, the problem can be turned into a MILP. As will be illustrated later, there is important to be aware of the trade-offs made in terms of reduced accuracy and increased number of variables, when using piecewise linearization.

MIDACO, MISQP and OQNLP are solvers which can take general MINLPs as input, but they do not guarantee optimal solutions for either nonconvex or convex problems, and have to be considered heuristics[79], [80], [109].

AlphaECP, AOA, BNB, BONMIN, DICOPT, FICO Xpress-SLP, FilMINT, fminconset, Knitro, LaGO, LindoAPI without global solver option, MILANO, MINLP_BB, MISQP with OA extension and SBB are all solvers which guarantee optimal solutions for general convex MINLPs. In cases of nonconvexity these solvers are to be considered heuristics[21]. It is worth notifying that AlphaECP also guarantee optimal solutions for pseudo-convex problems[113].

In terms of guaranteeing global optimal solutions for nonconvex general MINLPs, there is required to have an algebraic representation of the functions $f(x, y)$ and $g(x, y)$ in equation 2.25. This is for the computation of convex under estimators. In the cases where this is fulfilled, the solvers alphaBB, BARON, Couenne, LindoAPI and SCIP can guarantee optimal global solutions[21], but whether the solution is found within reasonable time is a whole different question.

An overview of the availability of the different solvers can be found in appendix F.

For the purpose of the practical part of this research, BONMIN, CPLEX and an Objective Feasibility Pump[100] (OFP) heuristic for MINLPs will be used.

BONMIN

BONMIN, which is short for Basic Open-source Nonlinear Mixed INteger programming, is an open source code for solving general MINLP problems. It is worth noting that BONMIN may require f and g in equation 2.25 to be twice continuously differentiable. The solver is distributed by COIN-OR (www.coin-or.org) under a Common Public License (CPL), which is approved by the Open Source Initiative (OSI)[12].

According to (Bonami and Lee [12]), BONMIN can be utilized in six different modes, using different algorithms:

- B-BB: A branch and bound algorithm, based on solving continuous nonlinear programs at each node of the search tree and branching on variables. This mode also provides the possibility of Special Ordered Set of type 1 (SOS1)[3] branching.
- B-OA: An outer-approximation based decomposition algorithm.
- B-QG: An outer-approximation based branch and cut algorithm inspired by (Quesada and Grossmann [92]).
- B-Hyb: A hybrid outer-approximation/nonlinear programming branch and cut algorithm.
- B-Ecp: An outer-approximation based branch and cut algorithm inspired by (Abhishek et al. [1]).
- B-iFP: An iterated feasibility pump (FP) algorithm.

Some of the algorithmic choices made by BONMIN require the ability of solving MILPs and NLPs. By default Cbc and Ipopt are used for this purpose, respectively. Both solvers are based on codes provided by COIN-OR. Cbc uses among other Clp and Cgl as submodules for solving LPs and generating MILP cutting planes, respectively, also these available from COIN-OR. Cbc and Ipopt can be replaced by CPLEX and FilterSQP, respectively. It is worth noting that the two latter options are not open source[12],[13].

What might be a disadvantage of the default setup of BONMIN is that it does not have multi core CPU support (meaning it only uses one core at the time). The only way having BONMIN taking advantage of such features are by replacing Cbc with CPLEX[11]. Remark that in such cases, there are only subproblems solved by CPLEX in B-OA which will benefit from using concurrent threads operating on separate cores.

In terms of nonconvex MINLPs, the B-BB is the most suitable algorithmic option[12], and is therefore the mode chosen during implementation. This is also the default mode used by BONMIN. The main challenges handling this type of problems are that the solutions found by Ipopt might be only locally optimal, rather than globally, and that the outer-approximation constraints are not necessarily valid for the problem. In terms of the challenges related to nonconvexity, BONMIN B-BB may be configured not to always trust the solutions from Ipopt, and also seek other solutions. These features are to be considered very experimental at this point[12].

While using BONMIN B-BB, there are a few parameters which the user should be specifically aware of. To allow continued branching even if the current node is worse than the best known solution, the fathoming rule has to be changed. This is necessary because the solution from Ipopt not

[3]A set of variables where at most one variable can be nonzero at the time[32]

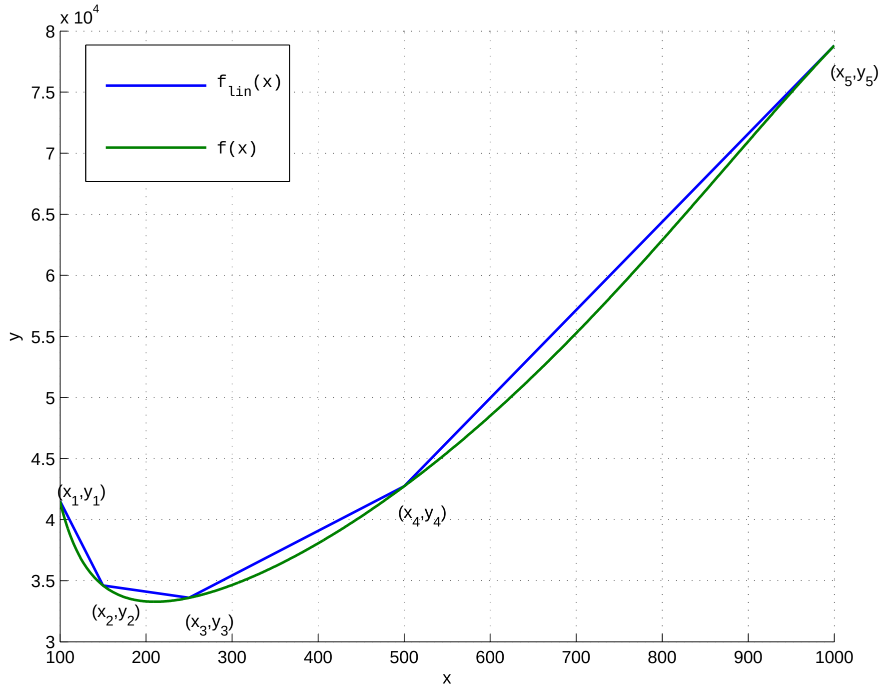


Figure 2.10: Piecewise Linearization with $n = 5$

truly provides a lower bound if the NLP relaxation is nonconvex. This can be achieved by setting options *allowable_gap*, *allowable_fraction_gap* and *cutoff_decr* to negative values[12].

Though Ipopt may return different answers for different starting points, it might be desirable to solve a node from several starting points. The parameters *num_resolve_at_root* and *num_resolve_at_node*, allow the user to decide how many randomly chosen starting points which shall be tested for the root node and each node of the tree, respectively. The algorithm automatically saves the best solution found. It is important to note that the function generating starting points is quite naive, though what it basically does is to uniformly choose starting points between the bounds of each variable[12].

CPLEX

CPLEX is the property of IBM, and is limited to solve MILPs, MIQPs and MIQCPs.

As mentioned earlier, most UC problems require some modifications in terms of piecewise linearization to be solvable using CPLEX. The concept of piecewise linearization is shown in figure 2.10. Remark that the intervals between the breakpoints do not have to be of the same length. In mathematical terms, the piecewise linearization can be achieved by adding weighting variables to each breakpoint[47]. For a piecewise linearized function having n breakpoints, this can be formulated as shown in equation 2.30, requiring all weighting variables ϕ_i, where $i = 1, 2, ..n-1, n$ to be in a Special Ordered Set of type 2 (SOS2)[5].

$$\begin{aligned} x &= x_1\phi_1 + x_2\phi_2 + ... + x_{n-1}\phi_{n-1} + x_n\phi_n \\ f_{lin}(x) &= y_1\phi_1 + y_2\phi_2 + ... + y_{n-1}\phi_{n-1} + y_n\phi_n \\ 1 &= \phi_1 + \phi_2 + ... + \phi_{n-1} + \phi_n \end{aligned} \tag{2.30}$$

Among all variables in an SOS2, only two can be nonzero at the same time. Further, if two variables in the set are nonzero, then they must be adjacent[32]. Some solvers include features which allow the user to assign variables to different SOS sets without implementing. In terms of using CPLEX implemented for AMPL, it does recognize SOS2 variables automatically, when using the specific syntax for piecewise linearization included in AMPL[28],[43]. For languages not featuring automatic detection of SOS2 variables, it can be implemented by adding the following constraints to the problem (inspired by (Williams [114])):

$$\begin{aligned}
&i = 1, 2 \ldots n-1, n \\
&1 = \psi_1 + \psi_2 + \ldots + \psi_{n-1} + \psi_n && \psi_i \in \{0, 1\} \\
&\phi_1 \leq \psi_1 \\
&\phi_i \leq \psi_{i-1} + \psi_i && 1 < i < n \\
&\phi_n \leq \psi_{n-1}
\end{aligned} \tag{2.31}$$

Assuming constraints 2.31 are satisfied, then for any $x \in [x_0, x_n]$ the linearization $f_{lin}(x)$ can be written as

$$f_{lin}(x) = y_i \phi_i + y_{i+1} \phi_{i+1} \qquad 1 \leq i < n \tag{2.32}$$

Now, let

$$\Delta y_i = y_{i+1} - y_i \qquad 1 \leq i < n \tag{2.33}$$

Then equation 2.32 can be written as

$$\begin{aligned}
f_{lin}(x) &= y_i \phi_i + y_{i+1} \phi_{i+1} \\
&= y_i \phi_i + (y_i + \Delta y_i) \phi_{i+1} \\
&= \Delta y_i \phi_{i+1} + y_i (\phi_i + \phi_{i+1}) \\
&= \Delta y_i \phi_{i+1} + y_i && 1 \leq i < n
\end{aligned} \tag{2.34}$$

which shows that $f_{lin}(x)$ consists of $n-1$ linear pieces.

By combining piecewise linearization and use of CPLEX, there may be possible to solve advanced problems faster with reasonable exactness. The main downside is the loss of accuracy. This can be verified by looking at figure 2.10. An approach to improve the accuracy is to have a very large number of breakpoints. The obvious disadvantage is that this generates a corresponding number of new binary variables and constraints. An increased number of variables may increase the computation time drastically, and in worst case cause the problem to become impossible to solve within reasonable time.

In terms of limiting the number of breakpoints with as little loss of accuracy as possible, there is reasonable to have higher density of breakpoints in areas where the function got significant curvature, and a lower one where it is closer to a linear behavior. This is illustrated in figure 2.10. The job of choosing breakpoints does easily get cumbersome, and for advanced functions there might be necessary to use specialized algorithms for this purpose.

OFP

The OFP used in the practical part is an extension of BONIMIN B-iFP and is thoroughly presented in (Sharma [100]), while a more compact presentation can be found in (Sharma et al. [101]). The algorithm is a modification of the FP heuristic for MINLPs, which aims at balancing the two goals of quickly obtaining feasible solutions and preserving the quality of the solution with respect to the original objective[101]. It is originally designed for convex MINLPs, but is by experiments found to perform well for some types of nonconvex MINLPs as well[100].

The original FP is a heuristic for solving difficult MILPs[40], where the main idea is to have two sequences of points, one which is constraint feasible and another which is integer feasible. If the two sequences converges to the same point, then a feasible solution is found[100]. The constraint feasible point is initially found by computing the LP-relaxation of the original MILP. Besides this, the objective value is not accounted for in the solution process. This often causes the FP to provide poor solution quality with respect to the objective value[8].

Instead of excluding the original MILP objective after the initialization, the OFP for MILPs includes it through the whole process, weighting it as a part of the cost function. As the number of iterations increases, the weight is reduced. At the same time, as the number of iterations increases, the weight of a cost corresponding to the integer infeasibilities increases[100].

There have been made different approaches extending the original FP to heuristics for convex MINLPs. One such approach is described in (Bonami et al. [14]). It finds a feasible solution by solving a sequence of NLPs. Initially, a constraint feasible point is found by solving a NLP relaxation of the original MINLP. This is followed by a process rounding all integer variables to the nearest integer point. Further, the distance to the integer feasible point is minimized. This approach is similar to what is implemented in BONIMIN B-iFP[14].

The results presented in (Sharma [100]), show that BONMIN B-iFP is able of finding feasible results for convex MINLPs significantly faster than BONMIN B-BB. The downside is that the solution quality is poor. The purpose of the OFP presented in the same paper, is to improve this while keeping computation time low.

The basis of the OFP is to formulate the MINLP as a multiobjective problem with two conflicting objectives: One subproblem minimizing the distance to the integer feasible point and the NLP relaxation of the MINLP. As in the MILP-case, the subproblems are weighted, mainly focusing on minimizing the NLP relaxation at first, and weighting the integer feasibility more as the number of iterations increases[100].

When applying the solver, there are four parameter values which might be of interest of varying. The weighting of the two terms in the objective function can be set manually. For the remainder of the text, the weighting of the integrality condition and the original objective will be referred to as u_1 and u_2, respectively. In terms of the variable weighting of the two terms throughout the simulation process, the geometrical reduction factor can be adjusted.

To prevent the algorithm from getting stuck in an integer infeasible point, a state often referred to as stalling, the algorithm flip a random number of integer variables when this state is detected. The maximum number of variables to be flipped can also be set manually.

For the remainder of this thesis, the term OFP refers to the OFP heuristic designed to handle MINLPs.

Chapter 3

Emission Constrained UC Problem, an Allowance System Approach

This chapter presents the problem formulation which is used for the practical work presented in this thesis. The choice of model design is based on the different approaches presented in chapter 2. For the purpose of this thesis, the model is in terms of emissions limited to include CO_2 emission constraints only, but can easily be extended for later use. The objective function is divided into two main parts: Penalty costs and regular costs. These are representing costs related to emissions exceeding allowances and operating expenses, respectively. All variables and parameters are explained in table 3.1.

3.1 Problem Formulation

Symbol	Explanation
α_i	Shutdown cost parameter for unit i [€/MW capacity]
β_{1_i}	Startup cost parameter for unit i
β_{2_i}	Startup cost parameter for unit i
γ_i	Ramping cost parameter for unit i
δ	Emission penalty parameter [€/kg CO_2]
Δe_i	Amount of emissions from unit i exceeding allowances [kg CO_2]
Δw_{it}	Absolute value of change of output from unit i at time t [MW]
Δw_{max_i}	Maximum load change per hour for unit i [MW/h]
ϵ_i	Fuel price for unit i [€/fuel unit]
η_i	Maintenance cost parameter for unit i [€/MWh]
θ_{it}	Characteristic efficiency function for unit i
ι_i	Fuel energy density for unit i [MJ/fuel unit]
κ	Spinning reserve scaling parameter
ν_i	Emission parameter for unit i [kg CO_2/per unit fuel consumed]
ξ	Transmission and distribution loss factor
c_{it}	Total cost for unit i during period t [€]
$c_{oper_{it}}$	Operational costs for unit i during period t [€].
$c_{ramp_{it}}$	Ramping costs for unit i during period t [€].
$c_{start_{it}}$	Startup cost for unit i during period t [€]]
$c_{stop_{it}}$	Shutdown cost for unit i during period t [€]
d_t	Power demand during period t [MW]
e_{max_i}	Maximum allowed emissions from unit i [kg CO_2]
e_{total_i}	Total emissions from unit i during the period of simulation [kg CO_2]

f_{it}	Fuel/water consumption for unit i during period t
f_{max_i}	Maximum fuel/water consumption for unit i during the simulation
m	Total number of periods
n	Total number of units
p_i	Penalty for exceeding emission constraint for unit i [€/$kg\ CO_2$]
t	Time [h]
$t_{min-off_i}$	Minimum period of time which unit i has to continuously stay off when stopped [h]
t_{min-on_i}	Minimum period of time which unit i has to continuously stay on when started [h]
$t_{off_{it}}$	Time which unit i has been shut down [h]
$u_{on_{it}}$	1 if unit i is on during period t, 0 else
w_{it}	Output from unit i at time t [MW]
w_{max_i}	Maximum capacity of unit i [MW]
w_{min_i}	Minimum stable output from unit i [MW]
x_{it}	Startup parameter, 1 if unit i starts up during period t, 0 else
y_{it}	Shutdown parameter, 1 if unit i is shut down during period t, 0 else
z_i	1 if unit i is exceeding its CO_2 allowance, 0 else

Table 3.1: Nomenclature.

Objective function

Objective function:

$$min \sum_{i=1}^{n} \left(p_i + \sum_{t=1}^{m} c_{it} \right) \tag{3.1}$$

Penalty costs:

$$p_i = \delta \cdot \Delta e_i \cdot z_i \tag{3.2}$$

$$\Delta e_i = e_{total_i} - e_{max_i} \tag{3.3}$$

$$e_{total_i} = \sum_{t=1}^{m} f_{it} \cdot \nu_i \tag{3.4}$$

$$f_{it} = \frac{3600 \cdot w_{it}}{\theta_{it} \cdot \iota_i} \tag{3.5}$$

$$z_i = \begin{cases} 1 & \Delta e_i > 0 \\ 0 & \Delta e_i \leq 0 \end{cases} \tag{3.6}$$

Regular costs:

$$c_{it} = c_{oper_{it}} + c_{ramp_{it}} + c_{start_{it}} + c_{stop_{it}} \tag{3.7}$$

$$c_{oper_{it}} = (\epsilon_i \cdot f_{it} + \eta_i \cdot w_{it}) \tag{3.8}$$

$$c_{ramp_{it}} = \gamma_i \cdot \Delta w_{it}^2 \tag{3.9}$$

$$\Delta w_{it} = (w_{it} - w_{i(t-1)}) \cdot (u_{on_{it}} - x_{it}) + (w_{it} - w_{min_i}) \cdot x_{it} + (w_{i(t-1)} - w_{min_i}) \cdot y_{it} \tag{3.10}$$

$$c_{start_{it}} = (\beta_{1_i} + \beta_{2_i} \cdot t_{off_{it}}) \cdot x_{it} \tag{3.11}$$

$$t_{off_{it}} = (t_{off_{i(t-1)}} + 1) \cdot (1 - u_{on_{it}}) \tag{3.12}$$

$$c_{stop_{it}} = \alpha_i \cdot w_{max_i} \cdot y_{it} \tag{3.13}$$

Constraints

Demand:

$$\sum_{i=1}^{n} w_{it} \geq d_t \cdot \xi \tag{3.14}$$

Spinning Reserve:

$$\sum_{i=1}^{n} w_{max_i} \cdot u_{on_{it}} \geq d_t \cdot \xi \cdot \kappa \tag{3.15}$$

$$\kappa \geq 1$$

Capacity:

$$w_{min_i} \cdot u_{on_{it}} \leq w_{it} \leq w_{max_i} \cdot u_{on_{it}} \tag{3.16}$$

Ramping:

$$(w_{i(t-1)} - \Delta w_{max_i}) \leq w_{it} \leq ((w_{i(t-1)} + \Delta w_{max_i}) \cdot (u_{on_{it}} - x_{it})) + (x_{it} \cdot w_{max_i}) \tag{3.17}$$

$$\Delta w_{max_i} \geq w_{min_i}$$

Fuel consumption:

$$\sum_{t=1}^{m} f_{it} \leq f_{max_i} \tag{3.18}$$

Minimum up and down time:

$$x_{it} - y_{it} = u_{on_{it}} - u_{on_{i(t-1)}} \tag{3.19}$$

$$\sum_{j=t-t_{min-on_i}+1}^{t} x_{ij} \leq u_{on_{it}} \qquad \forall i, j \in \{1...m\} \tag{3.20}$$

$$\sum_{j=t-t_{min-off_i}+1}^{t} y_{ij} \leq 1 - u_{on_{it}} \qquad \forall i, j \in \{1...m\} \tag{3.21}$$

$$x_{it} \in [0,1] \qquad \forall i, t$$

$$y_{it} \in [0,1] \qquad \forall i, t$$

$$u_{it} \in \{0,1\} \qquad \forall i, t$$

Explanation

As can be seen from the previous section, the objective function is divided into two parts: Penalty costs and regular costs. The penalty for each unit is growing linearly with the amount of CO_2 released exceeding the unit specific allowance defined by e_{max_i}. If an unit complies its allowances the penalty $p[i]$, will be set to zero.

The regular costs are divided into four sub-costs: Operational, ramping, startup and shutdown costs. The operational costs are a sum of fuel consumption and maintenance costs. The fuel costs are proportional to the nonlinear fuel consumption. The maintenance costs are assumed to vary linearly with the power generated.

Ramping costs are assumed to be quadratic functions of the change in output Δw_{it}. As can be seen in equation 3.10, the calculation of Δw_{it} varies depending on if the unit is in operational, startup or shutdown mode. This is done to avoid double charging of the ramping cost between $w_{it} = 0$ and $w_{it} = w_{min_i}$, though this is assumed to be included in the startup and shutdown cost terms.

Startup costs are calculated proportional to the period of time which an unit has been off, before the respective startup procedure takes place.

Shutdown costs are constant for each unit, and are calculated from a shutdown cost parameter α_i, multiplied with the maximum capacity of the respective unit.

In terms of the constraints, the electricity demand is described as the actual demand multiplied by a transmission and distribution loss factor ξ, which is assumed to be constant.

The requirement of having a spinning reserve is satisfied through constraint 3.15. If for instance $\kappa = 1.10$, then there are required for the operating units to in total have a free capacity of 10% relative the demand.

As discussed in section 2.1, the output from each unit is constrained to operate between and minimum and maximum value. This is included in the model by constraint 3.16.

The ramping constraint was originally formulated as

$$(w_{i(t-1)} - \Delta w_{max_i}) \cdot u_{on_{it}} \leq w_{it} \leq ((w_{i(t-1)} + \Delta w_{max_i}) \cdot (u_{on_{it}} - x_{it})) + (x_{it} \cdot w_{max_i}) \tag{3.22}$$

but through practical experiments, it was found that the binary variable $u_{on_{it}}$ on the left hand side of the constraint, caused significant growth in computation time compared to the formulation presented in constraint 3.17. The main consequence of this change is that if $\Delta w_{max_i} < w_{min_i}$, then it is impossible to shut down unit i. For this reason, there is set as a requirement that $\Delta w_{max_i} \geq w_{min_i}$. This requirement is not unreasonable though. In terms of the model presented in (Wold [115]), can a typical coal fired unit provide a load change gradient of 2%/min. For an unit operating on a minimum level of 200 MW, this is equivalent to approximately 650 MW/h, which is more than three times the minimum output.

The rightmost term in constraint 3.17 is included to allow the units to start operating at whatever outputs are preferred, straight after startup.

Fuel and water consumption, as well as minimum up and down times, are implemented as discussed in section 2.1.

As discussed in section 2.1, there is reasonable to assume that constraints for "must run" and "must out units" can be excluded from the model, for simulations ranging over short periods of time. None of the simulation periods explored in this thesis exceed 24 hours. For this reason these constraints are omitted.

To limit the complexity of the model, MCP and network capacity constraints are excluded from the model.

A weakness of the model is that it does neither take extra fuel consumption nor CO_2 emissions due to ramping into account. This causes the fuel constraint to be slightly unrealistic, and the CO_2 emissions to appear lower than what is the actual case.

Chapter 4

Methods

This chapter introduces the setup for all simulations, and presents how they have been conducted to meet the objectives.

4.1 Setup

Model Specific

For the purpose of solving some example cases of the problem formulated in section 3.1, four different sets of demand have been used. These can be seen in figure 4.1 or be found in table B.1. The demand sets were scaled for being in suitable range of problems with 3, 6, 9 and 12 generating units, and will for the remainder of this thesis be referred to as DS1, DS2, DS3 and DS4, respectively. All demand sets were fictitious, but were designed to represent typical variations during a 24 hour consumption cycle.

For each demand set there was made a generator setup which can be found in table C.1. These will for the remainder of the text be referred to as GS1, GS2, GS3 and GS4. For simplicity, the different setups only includes three different types of power sources: Hydropower, coal and natural gas. In terms of available maximum capacity per type of source, the distributions were quite different for the four setups. This can be seen in figure 4.2.

GS1 was dominated by hydro and coal, and does in such matter remind a little of the former situation in the US, described in (Wold [115]), where natural gas was a peak demand source only.

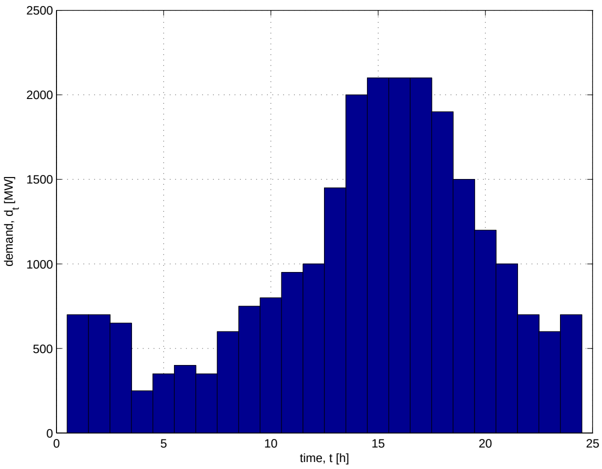
The distribution of maximum capacity by power source for GS2 was chosen with a small regional market in mind, having the main parts of its supply from coal fired plants in Sweden and hydropower in Norway, complemented by a significant, but not dominating part of power produced with natural gas.

As will be discussed later, the simulations performed using GS2 found hydropower to be the dominating choice as a baseload due to its low variable costs. To gain more data regarding energy mixes and costs related to use of natural gas, the capacity distribution for GS3 is dominated by two equal shares of coal and natural gas.

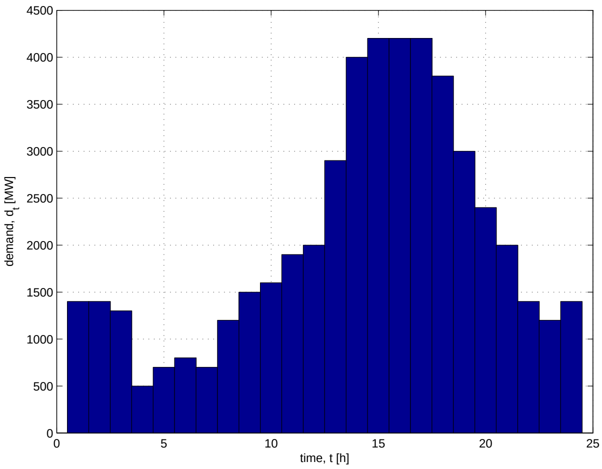
GS4 can be seen to be dominated by natural gas. This choice was not made for any specific reason, though this setup mainly was used for testing computation time with different solvers.

In general, the average maximum capacity of a natural gas unit can be seen to be somewhat lower than for the other units. This was done deliberately, because conventional baseload plants often are coal fired units (and in some countries, like Norway, hydropower units), which are likely to have larger maximum capacity than typical peak demand units. It is important to note that this not necessarily has to be the case.

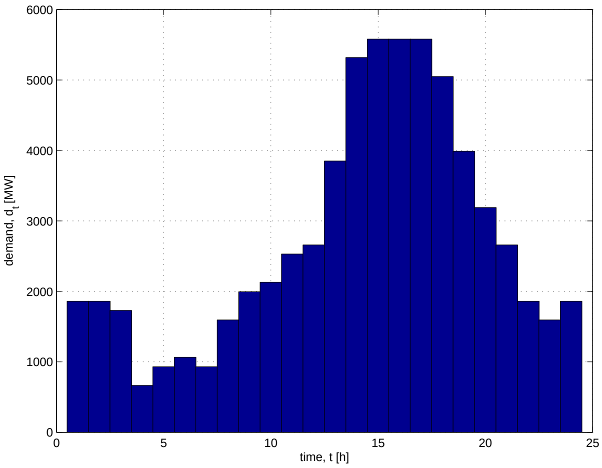
All parameter values presented in table C.1 are to be considered fictional, but still based on reasonable assumptions. In terms of coal and natural gas units, the startup parameters, β_{1_i} and β_{2_i}, were inspired by the results presented in (Kumar et al. [64]). The hydropower units were all assumed



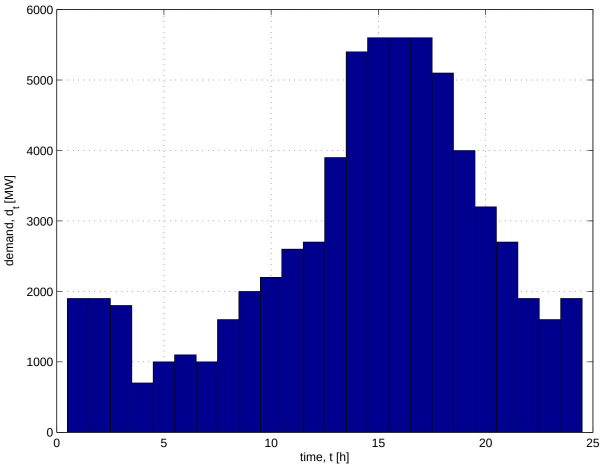
(a) Demand Set 1 (DS1)



(b) Demand Set 2 (DS2)



(c) Demand Set 3 (DS3)



(d) Demand Set 4 (DS4)

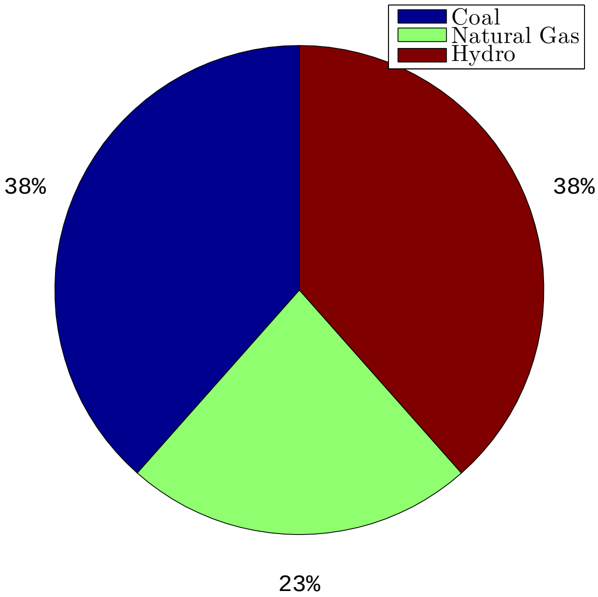
Figure 4.1: Demand sets designed for simulations with 3 (a), 6 (b), 9 (c) and 12 (d) generating units.

to have a constant startup costs, due to their nature of not having the same thermal variations as thermal units. This was implemented by setting $\beta_{2_i} = 0$. All startup parameters were set to values which were assumed to include the cost of extra fuel consume, maintenance and manpower required by the respective process. The same applies for the shutdown parameters α_i. These values are strictly fictional though, and were set to what the author considers reasonable relative the startup costs.

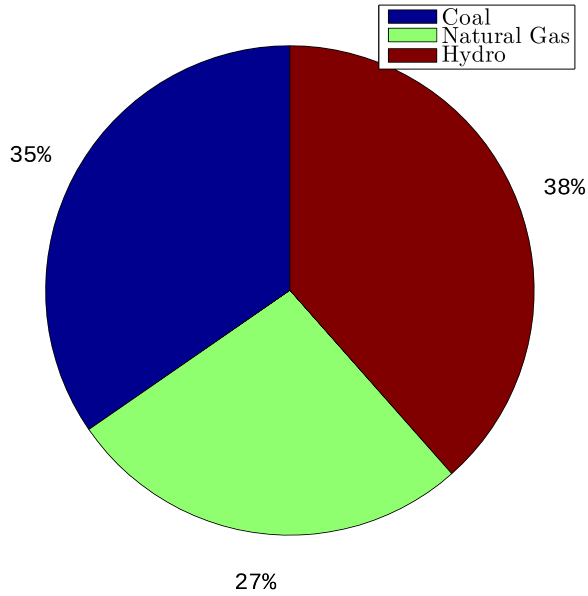
For coal and gas units, there were pursued to find values for γ_i which gave average ramping costs in the same range as the ones presented in (Kumar et al. [64]). There is important to remark that the values presented in (Kumar et al. [64]) are based on average numbers, and that all $c_{ramp_{it}}$ were defined as quadratic functions. For this reason, large changes in output within short periods of time, might have caused significant deviations between the ramping costs found in the simulations and the values found in (Kumar et al. [64]). The ramping parameters for hydro units were set to reasonable values, relative the values set for coal and natural gas units. The ramping costs were assumed to include the cost of extra fuel consumption, maintenance and manpower required for the respective process.

The load change parameters Δw_{max_i}, were set to moderate values of those presented in (Wold [115]).

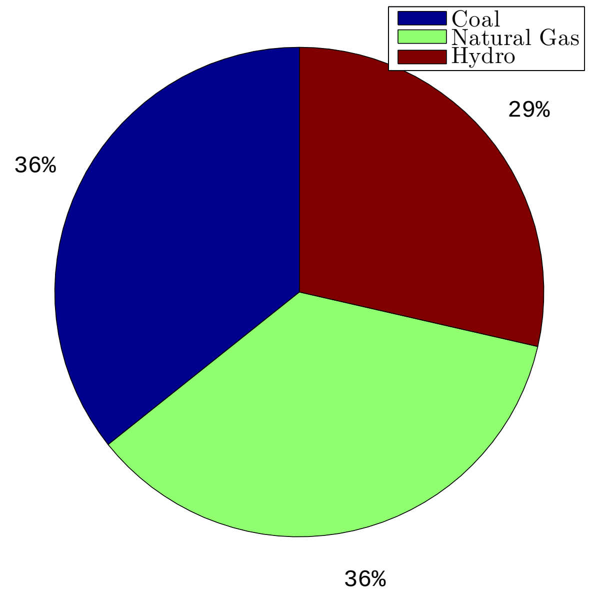
The coal price was set to 60 €/tonnes. This choice was made based on data from (InfoMine [55]). Remark that this price was based on February 2014 numbers, which was straight after a significant



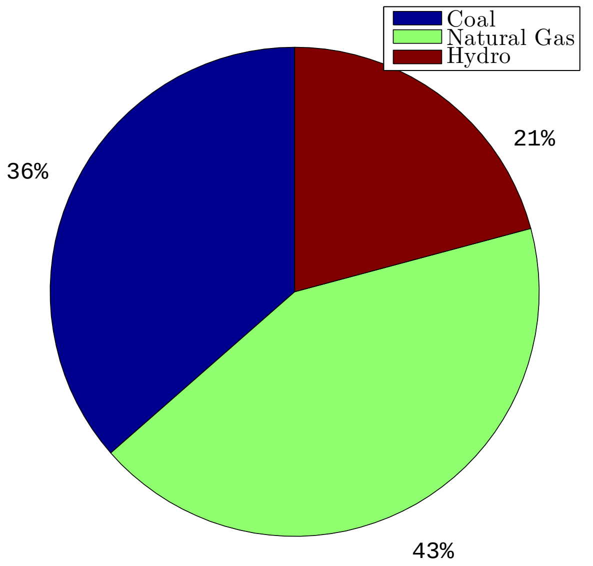
(a) Capacity distribution GS1



(b) Capacity distribution GS2



(c) Capacity distribution GS3



(d) Capacity distribution GS4

Figure 4.2: Distribution of maximum capacity for the different generator setups.

price drop, due to a warmer January than expected. The price of natural gas was set to 4.7 €/Mcf based on numbers from (InfoMine [54]). The price for water consumption was set to be within the price range presented by (Worldwatch [116]).

The values for η_i are fully collusive, and were not based on any scientific research. For this reason they were set with small variations.

The energy densities of coal and natural gas, may vary depending on purity. In this case the densities were set to 28 MJ/kg and 1.1 MJ/ft^3, respectively. The values were based on the numbers presented in (Wold [115]), and represent fuel of high quality. In terms of the hydro units, ι_i was set to 0.5MJ/tonnes. This is a realistic number while having a waterfall slightly higher than 50 meters.

The emission parameters ν_i, for coal and natural gas were set to 2.3 kg CO_2/kg and 0.05 kg/ft^3, respectively. There were not assumed to be any carbon emissions related to production done by the hydro facilities.

Each power plant got its own emission allowance, e_{max_i}. They were calculated equally for all units, by multiplying the capacity with a factor of 2000000 kg/GW capacity.

The maximum fuel consumption set for each unit was calculated by multiplying its capacity with 6200000 kg/GW capacity, 3333333333 ft^3/GW capacity or 190000000 m^3/GW capacity, depending on if the unit was a coal, natural gas or hydro unit, respectively.

To gain realistic efficiency characteristics, all θ_{it} were assumed to be on the form

$$\theta_{it} = p_{1_i}\left(\frac{w_{it}}{w_{max_i}}\right)^3 + p_{2_i}\left(\frac{w_{it}}{w_{max_i}}\right)^2 + p_{3_i}\left(\frac{w_{it}}{w_{max_i}}\right) + p_{4_i} \tag{4.1}$$

The values of p_{1_i}, p_{2_i}, p_{3_i} and p_{4_i} were set by parameter estimation based on the efficiency characteristics presented in (Wold [115]). Graphical illustrations of the estimates can be seen in appendix E.

Minimum up and down times were set based on the expected dynamics of the power sources, but were not based on any specific publication. An important remark in this matter, is that by the start of all simulations, all units were assumed to be in a state where $t_{off_{it}} = 1$ h, and where the minimum downtime constraint not applied. This means that for all units, the first startup was allowed to occur without satisfying $t_{off_{it}} \geq t_{min-off_i}$.

Minimum stable operational output for the units were set relative maximum capacity, according to what was presented in (Wold [115]).

Computational Platform

All simulations were carried out with the following system specifications:

- CPU: Intel®Core™i7-2600 (3.40 GHz x 8).
- Memory: 16 GB.
- Operative System (OS): Ubuntu 12.04 64-bit, Kernel Linux 3.2.0-57-generic.
- AMPL Version 20120328 (Linux x86_64).

The CPLEX version used was 12.4.0.0. In all simulations taking advantage of this solver scaling was turned off, because it was detected feasibility problems during the scaling process. Besides this, all CPLEX specific parameters were set to their default values.

The BONMIN B-BB version used was 1.5.2 with Cbc 2.7.5 and Ipopt 3.10.1. The OFP was implemented with BONMIN version 1.6 using Cbc 2.7.7 and Ipopt 3.10.2. The BONMIN parameters *allowable_gap*, *allowable_fraction_gap* and *cutoff_decr* were all set to -0.00001. Except for these, all BONMIN specific parameters were set to their default values.

4.2 Practical Approach

In total there is presented 36 simulations in appendix D, which can be divided into 6 groups according to formulation and solver:

- Simulation 1-5: MILP problems solved using CPLEX in multi core mode.
- Simulation 6-7: MILP problems solved using CPLEX in single core mode.
- Simulation 8-11: MIQP problems solved using CPLEX in multi core mode.
- Simulation 12-15: MIQP problems solved using CPLEX in single core mode.
- Simulation 16-33: MINLP problems solved using BONMIN B-BB.
- Simulation 34-36: MINLP problems solved using OFP.

While studying the simulations in appendix D, please remark that the numbers of variables and constraints occur with two different values, one enclosed by parentheses and one regular. The one enclosed by parenthesis, represents the number of variables or constraints handled by the solver after the problem has been through AMPL's presolve procedure. This procedure is supposed to optimize the problem formulation in terms of computational load. The other value represents the number of variables and constraints, as the problem was implemented by the author.

It can also be observed that the results are presented with two different types of computation time, referred to as CPU time and real time. The former is the sum of time consumed from each CPU core by the solver, while the latter is the time from AMPL starts presolving until the answers are presented. In such matter, problems solved using solvers with multi core CPU support, will often have much higher CPU time than real time.

The simulations mentioned above mainly served two purposes. The first was to find a suitable solver for the allowance model. Primarily, the simulations with 3 and 6 units with $\delta = 0.1$ €$/kg\ CO_2$ served this purpose. The selection process was based on the simulation results, and is discussed in the first part of the discussion chapter. The second was evaluating the effects to costs, emissions, energy mix and computation time using different setups and penalties for CO_2 emissions. In terms of computation time, there were also made several experiments which will be presented at the end of this chapter.

During the practical work, all MILP formulations were implemented as piecewise linearizations of the model presented in chapter 3. This was achieved by replacing equation 3.5 and 3.9 with piecewise linear terms as discussed in section 2.5, and by reformulating equation 3.2, 3.10, 3.11, 3.12 and 3.17 using sets of linear logical expressions providing equivalent effects.

Unlike what was shown i figure 2.10, the steps between the breakpoints were kept constant within each equation. The linearizations of equation 3.5 and 3.9 were done with respect to w_{it} and Δw_{it}, respectively. Three different combinations of step sizes between the breakpoints were used. For the remainder of this thesis these are referred to as piecewise linearization with step size 100, 200 and 100/200. By step size 100, it is meant that the breakpoints are placed with intervals of 100 MW for both equation 3.5 and 3.9, and equivalently for step size 200. By step size 100/200, it is meant that equation 3.5 is piecewise linearized with a step size of 200 MW, while equation 3.9 is piecewise linearized with a step size of 100 MW. There were made simulations with 3 and 6 units using the MILP approach, all with $\delta = 0.1$ €$/kg\ CO_2$.

The simulations based on the MIQP formulation were very similar to the ones formulated as MILPs. The only difference was that equation 3.9 was left unchanged from the original model. This left all $c_{ramp_{it}}$ as quadratic terms in the objective function, while all other terms and constraints were linear, which made the formulation an MIQP problem. During the practical work, there were carried through simulations with 3 and 6 units, using step sizes of 100 and 200 for the piecewise linearization of equation 3.5. All simulations were performed with $\delta = 0.1$ €$/kg\ CO_2$.

Both the MILP and the MIQP simulations were performed using CPLEX. A nice way of evaluating the quality of the solutions found while using this solver is studying the MIP optimality gap. This especially comes in handy when the simulation is interrupted before the solver has finished, though it gives an indication of how far the problem is from being fully solved. In the results part, as well as in appendix D, the terms absolute MIP gap and relative MIP gap are used. The former is the difference between the current best integer solution and the optimal value of the LP relaxation, while the latter is defined as:

$$relative\ MIP\ gap = \frac{|best\ bound - current\ best\ integer\ solution|}{|best\ bound|} \tag{4.2}$$

Some of the CPLEX simulations were performed twice, once in single core mode and once in multi core mode. This was done to compare runtime with other solvers, and to retrieve data regarding the importance of multi core CPU support. All CPLEX simulations were interrupted after 3600 sec.

The MINLP formulation was implemented as presented in chapter 3, and the same formulation was used for both BONMIN B-BB and OFP. In total appendix D includes 18 simulations using BONMIN B-BB. Simulation 16 is the only one of them having three units, and was performed to evaluate the performance relative other formulations and solvers. Simulation 17 to 28 were all carried through with six generating units, with δ varying from 0 $€/kg\ CO_2$ to 0.2 $€/kg\ CO_2$. These simulations were done to evaluate the CO_2 allowance model, and to see how different CO_2 fines affected the energy mixes and the emissions. There were not set any time limits on these simulations.

Simulation 29 to 32 are to be considered special, in the sense that they are not based on the CO_2 allowance model. These were performed to test other potential approaches of constraining CO_2 emissions, and compare them to the allowance model. Simulation 29 is formulated as a pure minimization of CO_2 emissions. Though it is assumed that hydro units are not releasing any emissions, minimizing the regular costs of these units and minimizing the emissions are not conflicting objectives. For this reason both were included in the objective. The setup of the simulation was implemented by removing the penalty costs and replacing the objective presented in equation 3.1 with

$$min\left(\left(\sum_{i=1}^{n}\sum_{t=1}^{m} f_{it}\cdot v_i\right) + \left(\sum_{i\ \in\ hydro}\sum_{t=1}^{m} c_{it}\right)\right) \tag{4.3}$$

while the constraints were kept unchanged. The notation $i \in hydro$ refers to all generating units i which are hydropower units. The simulation was conducted using DS2 and GS2, but remark that the parameters e_{max_i} and δ in this case were ignorable.

There is worth noting, that there is a potential weakness of the way the objective presented in equation 4.3 is formulated. If there are two different solutions which both are optimal in terms of the objective, there is no warranty that the solver will choose the one of them providing the lowest total cost.

Simulation 30 is formulated as a strict unit emission constraint approach. This was implemented by removing the penalty costs from the original objective function, and adding a constraint on the form

$$e_{total_i} \le e_{max_i} \tag{4.4}$$

to the problem. The simulation was conducted using DS2 and GS2, but with modified values for e_{max_i}. They were calculated equally for all units, by multiplying the respective maximum capacity with a factor of 2500000 kg/GW capacity. It is worth noting that also here δ was ignored.

Simulation 31 is closely related to simulation 30. It is formulated with a strict total emission constraint. This was achieved by using the exact same setup as in simulation 30, except that constraint 4.4 was replaced by

$$\sum_{i=1}^{n} e_{total_i} \le \sum_{i=1}^{n} e_{max_i} \tag{4.5}$$

Simulation 32 is a conventional cost function approach. This was formulated easily by setting $e_{max_i} = 0$ for all i. Besides this modification standard GS2 and DS2 were used, and δ was set to 0.001 $€/kg\ CO_2$.

Simulation 33 was simulated using the CO_2 allowance model, and is the only simulation in appendix D which includes 9 generating units, that was conducted using BONMIN B-BB. The simulation was manually interrupted after approximately ten days, and for that reason the lower bound is included in table D.33. This value is the lower bound provided by Ipopt, at the point when the simulation was interrupted. Remark that as discussed in section 2.5, this value is not necessarily truly the lowest bound.

Simulation 33 was the only simulation in appendix D using BONMIN B-BB which was interrupted. All others were ended by the solver itself.

The specifications of simulation 34, 35 and 36 are identical with 16, 18 and 33, respectively, except that they were performed using OFP instead of BONMIN B-BB. All of the respective simulations were performed with $u_2 = 0.01$. Throughout the conduction of simulation 36, the same issue related to computation time as in simulation 33 was experienced.

In terms of the computation time issues experienced while the number of units were increasing, there were made another 45 simulations using the allowance model, besides those recently described. These were divided into three groups of fifteen simulations, were the first was using GS2 and DS2, the second was using GS3 and DS3 and the third was using GS4 and DS4.

Each group consisted of 13 simulations solved by OFP with different weighting, u_2, on the original objective. All these simulations were set to end when the first feasible solution was found. All OFP simulations presented in this thesis were performed with $u_1 = 1$, a geometrical reduction factor of 0.9 and a maximum limit of 10 integer variables to be flipped if stalling. Further, each group included a BONMIN B-BB simulation where all support heuristics were turned off (in default mode BONMIN B-BB uses an FP heuristic in the first node to find a feasible solution). Also this simulation was set to end after finding the first feasible solution. The last simulation in each set, was conducted using the same BONMIN B-BB setup as in simulation 18 in appendix D, but it was set to end after 3600 seconds.

An overview of the results from these simulations can be found in table 5.4, 5.5 and 5.6. The three rightmost columns are comparisons of the objective relative the result of the B-BB simulation without heuristics, the objective relative the best OFP solution and CPU time relative the CPU time of the OFP solution providing the best objective. The best objective value and CPU time retrieved using OFP are market with green text.

Chapter 5

Results

This chapter presents a summary of the most important results found from the simulations described in the previous chapter. A more comprehensive overview of simulation 1 to 36 can be found in appendix D.

The first section presents series of simulations of the CO_2 allowance model, performed using different solvers. The section includes simulations for 3, 6 and 9 generating units. The results presented in this section have mainly been used to choose a suitable solver for the presented UC problem.

The results presented in the next section show how different values of δ affect costs, emissions and choice of power sources. It also presents data from simulations using more conventional emission constraints.

The last section presents data comparing different setups of BONMIN B-BB and OFP solving the presented UC model.

5.1 Problem Type and Solvers

Solver	Mode	Problem	Step Size	Objective [€]	Absolute MIP gap	Relative MIP gap	CPU time [sec]	Real Time [sec]
CPLEX	Multi Core	MILP	100	534613	53	10^{-4}	20635	2850
CPLEX	Multi Core	MILP	200	539428	0	0	17	4
CPLEX	Multi Core	MILP	100/200	535364	0	0	19	5
CPLEX	Single Core	MILP	100	Infeasible problem	-	-	3589	Interrupted after 3600 sec.
CPLEX	Single Core	MILP	100/200	535364	0	0	3	4
CPLEX	Multi Core	MIQP	100	533573	25516	0.0478	26554	Interrupted after 3600 sec.
CPLEX	Multi Core	MIQP	200	534305	0	0	252	34
CPLEX	Single Core	MIQP	100	593414	78631	0.1325	3589	Interrupted after 3600 sec.
CPLEX	Single Core	MIQP	200	534330	26	$5 \cdot 10^{-5}$	30	31
BONMIN B-BB	Single Core	MINLP	-	533211	-	-	29	35
OFP	Single Core	MINLP	-	533212	-	-	26	30

Table 5.1: Summary of simulations using the CO_2 allowance model with 3 units using: GS1, DS1, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

Solver	Mode	Problem	Step Size	Objective [€]	Absolute MIP gap	Relative MIP gap	CPU time [sec]	Real Time [sec]
CPLEX	Multi Core	MILP	100	Infeasible problem	-	-	24300	Interrupted after 3600 sec.
CPLEX	Multi Core	MILP	200	1345279	482749	0.3588	26480	Interrupted after 3600 sec.
CPLEX	Multi Core	MIQP	100	1386631	553216	0.3990	26854	Interrupted after 3600 sec.
CPLEX	Multi Core	MIQP	200	1531848	667892	0.4360	26785	Interrupted after 3600 sec.
CPLEX	Single Core	MIQP	100	1361414	520030	0.3820	3588	Interrupted after 3600 sec.
CPLEX	Single Core	MIQP	200	1495863	628113	0.4199	3588	Interrupted after 3600 sec.
BONMIN B-BB	Single Core	MINLP	-	892196	-	-	1313	1320
OFP	Single Core	MINLP	-	892196	-	-	1136	1140

Table 5.2: Summary of simulations using the CO_2 allowance model with 6 units using: GS2, DS2, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

Solver	Mode	Problem	Step Size	Objective [€]	Absolute MIP gap	Relative MIP gap	CPU time [sec]	Real Time [sec]
BONMIN B-BB	Single Core	MINLP	-	1517534	-	-	898155	Interrupted after 898200 sec.
OFP	Single Core	MINLP	-	1517534	-	-	898085	Interrupted after 898200 sec.

Table 5.3: Summary of simulations using the CO_2 allowance model with 9 units using: GS3, DS3, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

5.2 Emission Constraints and Energy Mixes

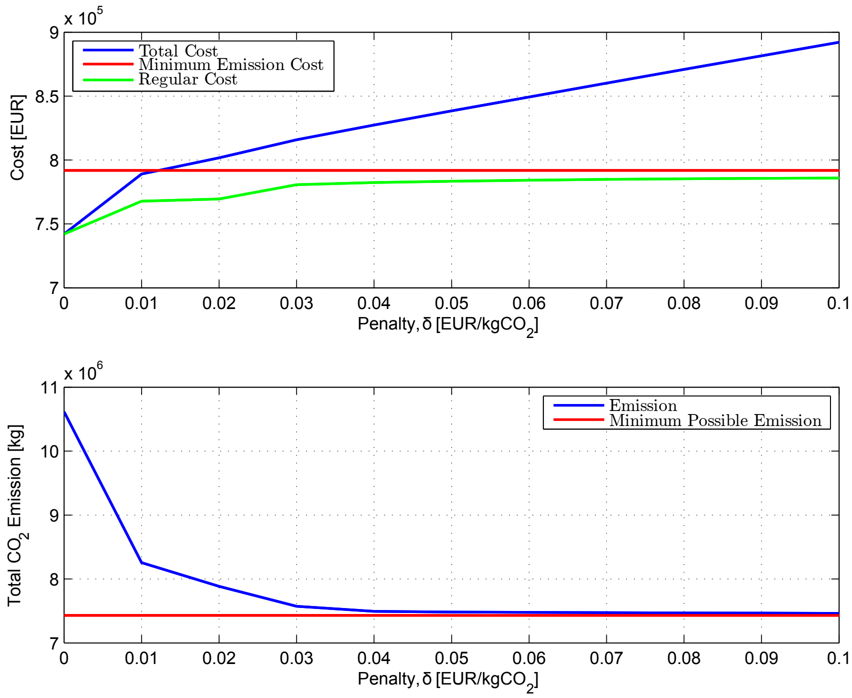


Figure 5.1: Cost and emission with respect to δ, compared to the cost and emission when minimizing emissions as much as possible. The results are based on simulations using the CO_2 allowance model with 6 units using: GS2, DS2, $\kappa = 1.10$ and $\xi = 1.07$. For a more extensive overview, see simulation 18 to 29 in appendix D.

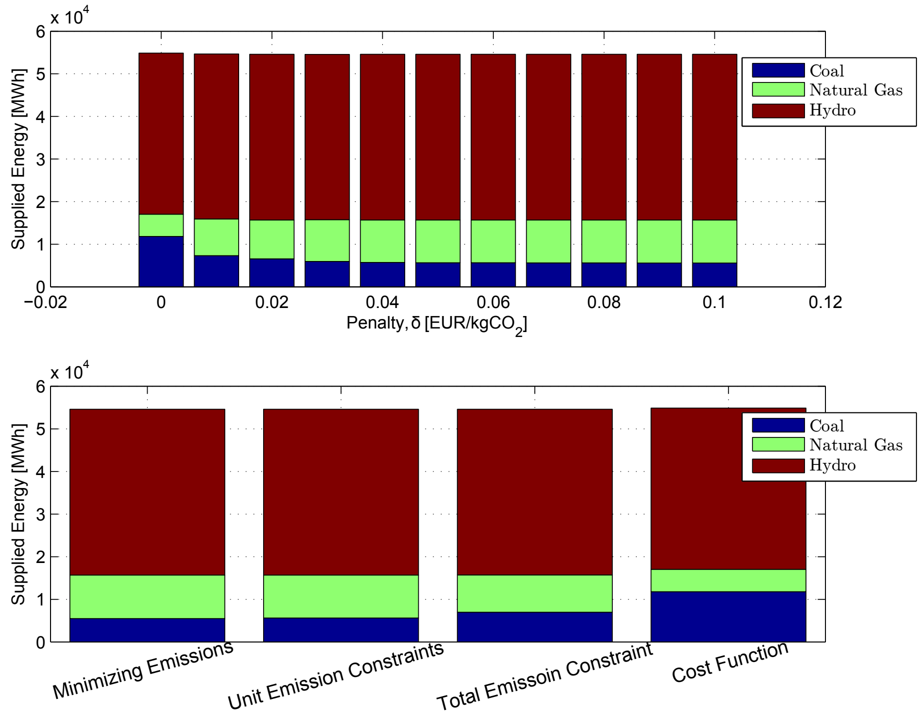


Figure 5.2: *Upper:* Energy mix with respect to δ. The results are based on simulations using the CO_2 allowance model with 6 units using: GS2, DS2, $\kappa = 1.10$ and $\xi = 1.07$. For a more extensive overview, see simulation 18 to 28 in appendix D. *Lower:* Energy mix for simulations with six units, using other types of emission constraints. From left to right the bars correspond to simulation 29 to 32 in appendix D.

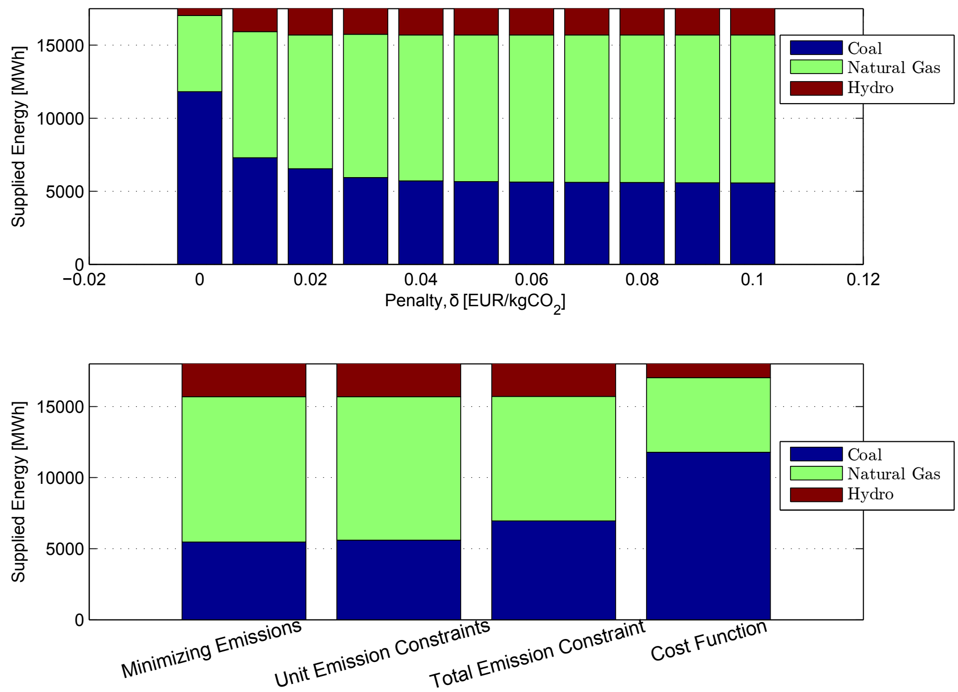


Figure 5.3: Same as figure 5.2, but zoomed in on the shares counting for coal and natural gas.

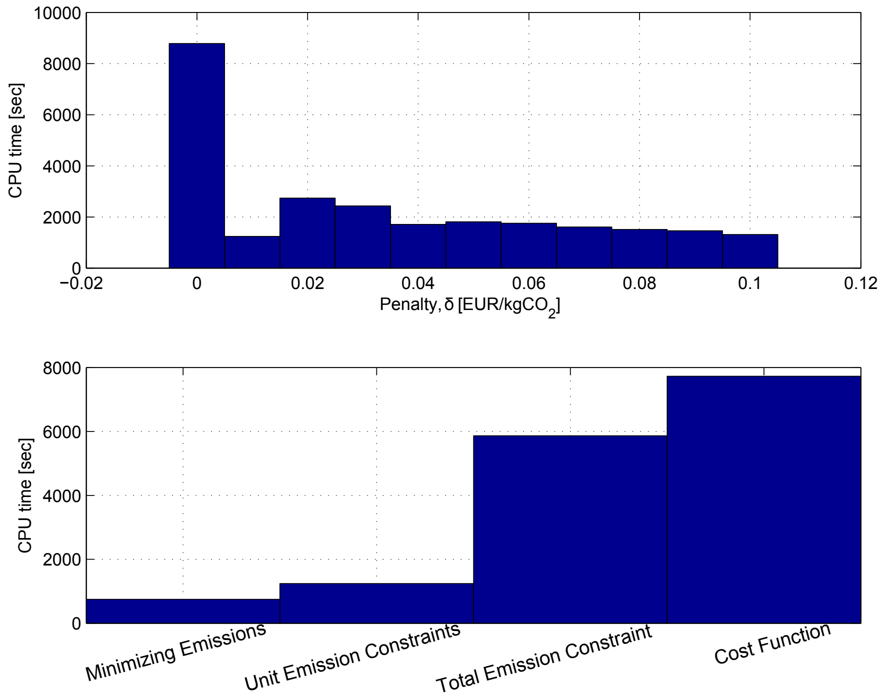


Figure 5.4: *Upper:* CPU time with respect to δ. The results are based on simulations using the CO_2 allowance model with 6 units using: GS2, DS2, $\kappa = 1.10$ and $\xi = 1.07$. For a more extensive overview, see simulation 18 to 28 in appendix D. *Lower:* CPU time for simulations with six units, using other types of emission constraints. From left to right the bars correspond to simulation 29 to 32 in appendix D.

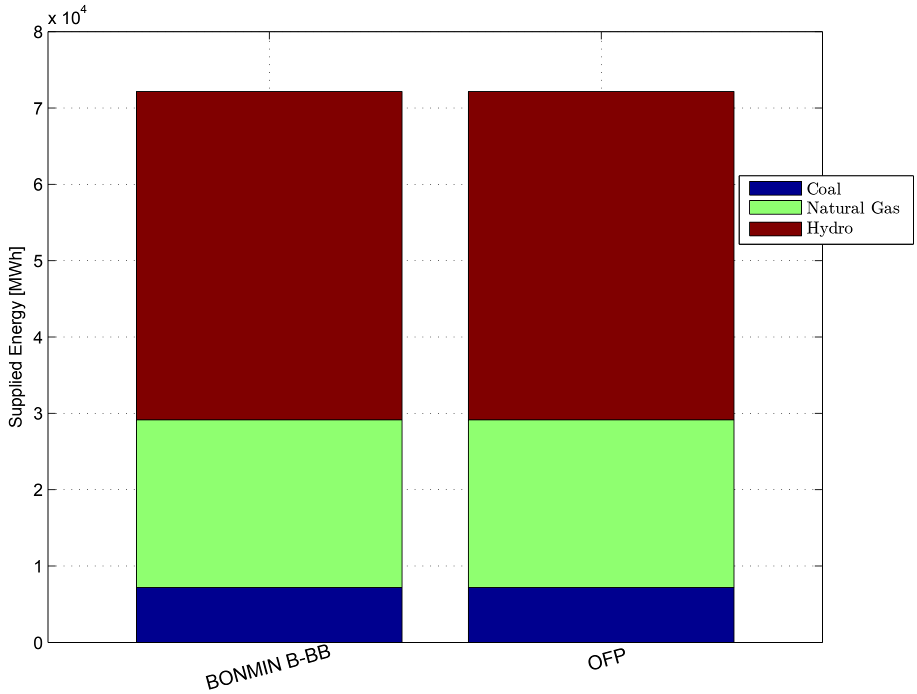


Figure 5.5: Energy mix for simulations using the CO_2 allowance model with 9 units using: GS3, DS3, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$. Both simulations had identical setup, but were using different solvers (BONMIN B-BB and OFP).

5.3 Computation Time for Larger Problems

Solver	u_2	Objective [€]	CPU time [sec]	Obj. rel. B-BB, no FP-heuristic, 1st feasible solution found [%]	Obj. rel. best solution found with OFP [%]	CPU time rel. time best obj. found with OFP [%]
OFP	0,001	909667	114	0.09	1.43	-1.72
OFP	0,01	909667	113	0.09	1.43	-2.59
OFP	0,1	909667	116	0.09	1.43	0.00
OFP	1	896881	116	-1.31	0.00	0.00
OFP	10	937394	154	3.14	4.52	32.76
OFP	100	955412	190	5.13	6.53	63.79
OFP	150	1054210	200	16.00	17.54	72.41
OFP	200	2679010	229	194.78	198.70	97.41
OFP	250	2128040	225	134.15	137.27	93.97
OFP	300	997999	224	9.81	11.27	93.10
OFP	400	2177900	206	139.64	142.83	77.59
OFP	500	1335270	219	46.92	48.88	88.79
OFP	1000	1022110	195	12.47	13.96	68.10
BONMIN B-BB, without heuristics, first solution found	-	908817	379	0.00	1.33	226.72
Regular BONMIN B-BB (max runtime: 3600 sec.)	-	892196	1313	-1.83	-0.52	1031.90

Table 5.4: Comparison of results using OFP with different weighting on the original objective. The results are based on simulations using the CO_2 allowance model with 6 units using: GS2, DS2, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

Solver	u_2	Objective [€]	CPU time [sec]	Obj. rel. B-BB, no FP-heuristic, 1st feasible solution found [%]	Obj. rel. best solution found with OFP [%]	CPU time rel. time best obj. found with OFP [%]
OFP	0,001	1611850	485	-10.06	5.19	-33.29
OFP	0,01	1611850	479	-10.06	5.19	-34.11
OFP	0,1	1611850	474	-10.06	5.19	-34.80
OFP	1	1622400	486	-9.47	5.88	-33.15
OFP	10	1583190	546	-11.66	3.32	-24.90
OFP	100	1559570	714	-12.98	1.78	-1.79
OFP	150	1687120	728	-5.86	10.10	0.14
OFP	200	1532290	727	-14.50	0.00	0.00
OFP	250	1626250	770	-9.26	6.13	5.91
OFP	300	1586560	743	-11.47	3.54	2.20
OFP	400	1802640	801	0.58	17.64	10.18
OFP	500	1756430	823	-2.00	14.63	13.20
OFP	1000	2356400	926	31.48	53.78	27.37
BONMIN B-BB, without heuristics, first solution found	-	1792210	4007	0.00	16.96	451.17
Regular BONMIN B-BB (max runtime: 3600 sec.)	-	1611850	3611	-10.06	5.19	396.70

Table 5.5: Comparison of results using OFP with different weighting on the original objective. The results are based on simulations using the CO_2 allowance model with 9 units using: GS3, DS3, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

Solver	u_2	Objective [€]	CPU time [sec]	Obj. rel. B-BB, no FP-heuristic, 1st feasible solution found [%]	Obj. rel. best solution found with OFP [%]	CPU time rel. time best obj. found with OFP [%]
OFP	0,001	1419550	2427	-3.49	11.57	-41.39
OFP	0,01	1419590	2487	-3.49	11.57	-39.94
OFP	0,1	1319400	2571	-10.30	3.70	-37.91
OFP	1	1306860	3053	-11.15	2.71	-26.27
OFP	10	1357550	3734	-7.71	6.70	-9.83
OFP	100	1272350	4141	-13.50	0.00	0.00
OFP	150	1310360	4082	-10.91	2.99	-1.42
OFP	200	1283770	4167	-12.72	0.90	0.63
OFP	250	1273990	3688	-13.39	0.13	-10.94
OFP	300	1290180	3454	-12.29	1.40	-16.59
OFP	400	1303140	3460	-11.40	2.42	-16.45
OFP	500	1288780	3385	-12.38	1.29	-18.26
OFP	1000	1343290	3560	-8.68	5.58	-14.03
BONMIN B-BB, without heuristics, first solution found	-	1470890	11858	0.00	15.60	186.36
Regular BONMIN B-BB (max runtime: 3600 sec.)	-	1419550	3580	-3.49	11.57	-13.55

Table 5.6: Comparison of results using OFP with different weighting on the original objective. The results are based on simulations using the CO_2 allowance model with 12 units using: GS4, DS4, $\delta = 0.1$, $\kappa = 1.10$ and $\xi = 1.07$.

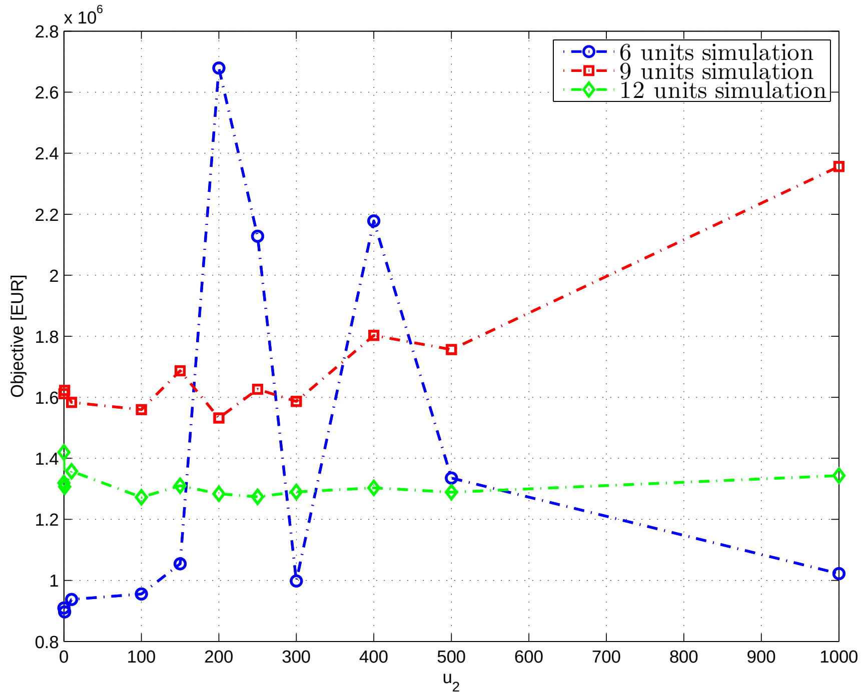


Figure 5.6: Graphical overview of the objectives from the OFP simulations in table 5.4, 5.5 and 5.6 with respect to u_2.

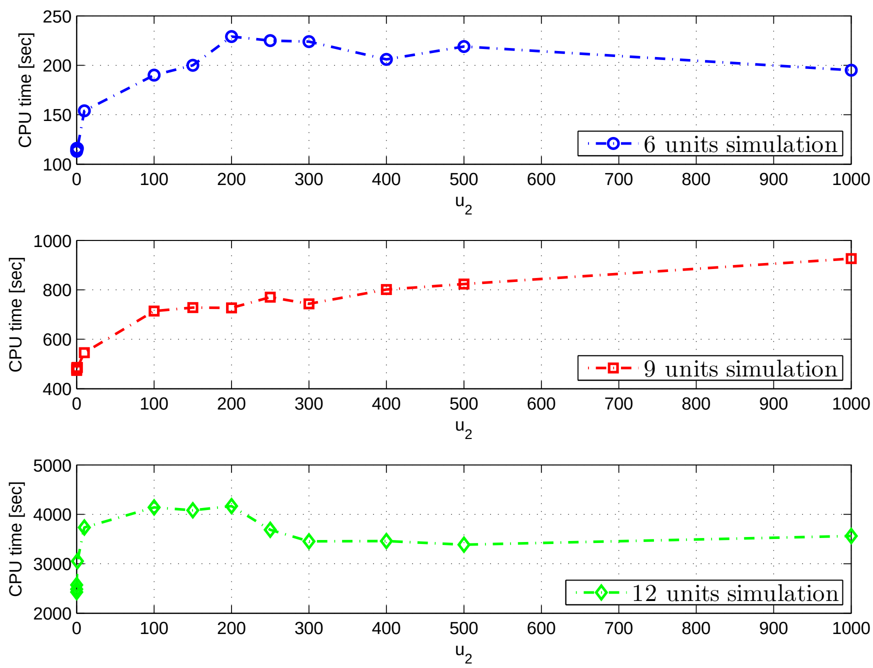


Figure 5.7: Graphical overview of the CPU times from the OFP simulations in table 5.4, 5.5 and 5.6 with respect to u_2.

Chapter 6

Discussion

This chapter is divided into 5 sections. The first one presents the evaluation of the early results, used to choose the most suitable setup for testing the allowance model. The next discusses the impacts of the model to costs, emissions and energy mixes, followed by a section comparing the model to other approaches of emission constrained UC problems. The fourth section focuses on the possibilities of improving the computation time using the OFP, while the fifth presents a reflection upon the role of natural gas in the power industry within the EU.

6.1 Choosing Solver

Objective value, computation time and accuracy were set as the main criteria, when choosing a solver for exploring different setups of the problem presented in chapter 3. In terms of finding a problem formulation and a solver which could provide these features, simulations with 3 and 6 units with $\delta = 0.1$ €$/kg\ CO_2$ were performed.

A summery of these simulations are presented in table 5.1 and 5.2. Starting with the former table, studying the objective values shows that solving with BONMIN B-BB and OFP gave the lowest objective values, but many of the CPLEX solutions were very close of being equally good. The MILP formulation with step size 100, solved using CPLEX in multi core mode, provided an objective value which was only 0.26% worse than the one from the B-BB simulation. As can be seen from the MIP gap values for this formulation, which were low, most of this deviation did most likely come from the inaccuracy introduced by the piecewise linearizations. Comparing the generator schedules in table D.1 and table D.16 gives that the schedules are close to identical, except for slightly higher output in some periods when using the MILP formulation.

The big downside of the MILP formulation with step size 100, was that the computation time was very poor relative both BONMIN B-BB and OFP. On the other hand, the simulation conducted with the same setup, except for having a step size of 200, provided significantly better real time than both the MINLP simulations. As can be seen by comparing the CPU time and the real time, may this have been much due to the solver's multi core CPU support, but also the CPU time was in fact better. The reason for the tremendous reduction in computation time relative the simulation with step size 100, can in large extent be assumed to be due to the reduction in variables and constraints. Comparing the number of variables for the two simulations, presented in table D.1 and D.2, gives that the total number of variables and constraints before presolve were reduced with 2688 and 1344 respectively. This is a drastic reduction in complexity. In terms of binary variables, was the number reduced with 1344. This can be assumed to have had significant impact on the amount of branching done by the solver.

The faster runtime was followed by a small price in terms of accuracy though, and it can been seen that by increasing the step size, the difference in objective value relative the B-BB simulation increased to almost 1.17%. As can be seen was the MIP gap zero, which indicates that the deviation was caused by the linearizations alone.

Even though the deviation was small, there is always interesting to find the formulation which gives the best trade-off between objective value and computation time. In this case, the MILP setup which seemed to be providing this, was the one using step size 100/200. The full overview of this simulation can be found in table D.3. The objective value was only 0.40% higher than the one provided by the B-BB simulation, and the computation time was insignificantly higher than for the MILP simulation using step size 200.

In terms of the MILP simulations in table 5.1 using CPLEX in single core mode, there was no surprise, in view of the multi core simulation with step size 100, that the single core one failed. What was remarkable though, was that the simulation with step size 100/200 gave the same solution as in multi core mode, but with significantly lower CPU time. From the same table, it can be seen that this behavior applied for the MIQP simulations with step size 200 as well. For this reason, these setups were simulated several times, both in single and multi core mode, always with the same result.

A potential explanation to this behavior could be that the use of several cores increased the memory usage, causing the data feed to slow down, but in this case this was unlikely the reason. The memory usage was monitored during the simulations, and there was not found any significant difference in memory usage during the simulations, and the total system memory usage was constantly below 10% of maximum capacity.

Another more probable reason, might be that the synchronization process and time coordinating among the threads undertaking independent branches in the solution tree, affected the computation performance in a negative way. An example of such behavior could be a thread which is standing locked up waiting for another thread to pass on information. The waiting thread will then be adding CPU time, without performing any useful work. Also the process of handing over information between threads may be time consuming.

In terms of the MIQP simulations, there can be observed for step size 100, that the objective value was significantly closer to the B-BB solution than the MILP simulation with equivalent step size. Despite this, the MIP gap was much larger, and the simulation was not able of finishing within 3600 seconds. Comparing the number of variables and constraints in table D.1 and D.8, it could be natural to assume significantly lower computation time for the latter, but this is obviously not the case. The reason for this is that CPLEX is using different strategies solving MILPs and MIQPs. The former is solved using branch and cut, while the latter utilizes barrier methods. The improved objective value was most likely due to improved accuracy, caused by not applying piecewise linearization to equation 3.9.

For the equivalent MIQP simulation with step size 200, it can, as expected, be observed that the objective value was a little higher, even despite a significantly better MIP gap. On the other hand, was the computation time much more acceptable. The objective value was slightly better than the MILP simulation with step size 100/200, which also can be considered reasonable.

In terms of comparing computation times of the simulations, there are reasonable to compare real time values due to BONMIN B-BB's and OFP's lack of multi core support. Based on table 5.1, there can be argued that MILP with step size 100/200 solved with CPLEX is the best choice for the $C0_2$ allowance model with 3 units, if the user is willing to slightly compromise on objective value in favor of computation time. If objective value is more important than time, a MINLP solved by BONMIN B-BB or OFP should be preferred. Based on the data in table 5.1 alone, the difference in computation time for the OFP simulation relative the B-BB simulation is to small to argue which solver is the best choice.

Focusing on a problem only including 3 units is very synthetic in terms of the UC problem. As the number of generating units increases, the number of variables and constraints grows drastically. For example did the total number of variables and constraints before presolve in the MILP with step size 100, increase with 9783 and 9243, respectively, as GS1 and DS1 were replaced by GS2 and DS2. The increased number of variables did also affect the CPLEX simulations drastically. This became very clear from the results presented in table 5.2. Neither the objective values nor the computation times, were even close to competing with the B-BB or the OFP result. There is worth noting that also with six units, the MIQP simulation using step size 200 performed significantly better in single core than multi core mode.

Based on the results in table 5.2, there was decided to exclude CPLEX from further testing. In terms of choosing between BONMIN B-BB and OFP, the significantly better runtime counted to OFP's favor. On the other hand, former work using OFP has detected varying objective quality compared to B-BB. This, together with the fact that the computation time of B-BB was to be considered acceptable, made B-BB the preferred solver for exploring the allowance model. Another factor influencing the choice was that the OFP did not seem to provide extreme improvements, in terms of fully solving larger problems. This impression was mainly based on the results for 9 units simulations, presented in table 5.3. Based on the obvious computation time challenges related to the simulations with 9 units, GS2 and DS2 were chosen as a suitable platform for further analyzing the presented UC model.

So far, this section has presented a deliberate argumentation regarding choosing a suitable solver, and based on the results presented so far, there seem natural that CPLEX was turned down. From another point of view, what seemed to be limiting the performance of CPLEX was the large number of variables and constraints. In such matter, there is natural to ask critical questions regarding the implementation of the problem, and see if there is any room for improvement.

As mentioned earlier, do large numbers of binary and integer variables have crucial effect on the computation time. The main sources causing the MILP and MIQP formulations to have more binary variables than the equivalent MINLP, are the piecewise linearizations and the replacements of nonlinear constraints using logical linear expressions. In terms of piecewise linearization, a reasonable approach would be to implement variable step size between the break points. This could have improved the accuracy of the approximation, and possibly reduced the number of binary variables. Smarter use of logical expressions could most likely reduced the number of binary and integer variables significantly. As an example, all non-binary integer variables in the MILP and MIQP simulations are caused by the replacement of equation 2.18. It is considered beyond the scope of this thesis to include an analysis aiming to optimize the number of variables in the MILP and MIQP formulation, but there is still important to have in mind that CPLEX might had performed better with more sophisticated implementations.

6.2 Costs, Emissions and Energy Mixes

Figure 5.1 does in large extent summarize simulation 18 to 29. The upper plot focuses on how the costs varies along with varying values of δ from 0 to 0.1 $€/kg\ CO_2$. The blue graph indicates the total cost, including both what were referred to as penalty costs and regular costs in chapter 3. The green graph shows the variations in regular costs. The difference between the blue and the green graph is the fee paid, due to CO_2 emissions which exceed the allowances. The red line indicates the regular costs when minimizing with respect to total emission instead of total cost. The blue graph in the lower plot indicates how the CO_2 emissions vary with the value of δ, and the red line marks the minimum possible emission without causing periods of brownouts or blackouts.

Studying the plots gives that even small values of δ compared to $\delta = 0$ $€/kg\ CO_2$, have drastic effects on both regular costs and emissions, and that the effect slows down for $\delta \geq 0.05$ $€/kg\ CO_2$. There is important to note though, that the magnitudes of the y-axis are high, which might give an impression of smaller effects than what is the reality. As an example: There is a reduction in CO_2 emission of about 52 tonnes between $\delta = 0.05$ $€/kg\ CO_2$ and $\delta = 0.1$ $€/kg\ CO_2$.

There would be natural to expect for the regular costs to become equal to the cost of optimizing with respect to emissions, as the value of δ grows. This is not the case for any of the allowance system simulations included in figure 5.1. Neither is it the case for simulation 17, where $\delta = 0.2$ $€/kg\ CO_2$. Actually does the total cost raise with about 105000 € when the value of δ is increased from 0.1 $€/kg\ CO_2$ to 0.2 $€/kg\ CO_2$. At the same time, the emissions are reduced with approximately 24.3 tonnes of CO_2, and the regular costs raise with a little less than 3600 €.

To find the reason why not even simulation 17 is operating on the same emission level nor regular cost level as while minimizing emission, table D.29 has to be studied. There it can be seen, that even when optimizing with respect to emissions, some units are exceeding their CO_2 allowances. With $\delta = 0.2$ €$/kg\ CO_2$, these emissions qualify for a penalty of 213602 €, and the total cost becomes 1005490 €. For comparison is the total cost of the allowance model with $\delta = 0.2$ €$/kg\ CO_2$ 997207 €, and for that reason a preferred financial choice.

Based on these observations, and the fact that the EU ETS operates with a penalty equivalent to $\delta = 0.1$ €$/kg\ CO_2$, the setup would most likely be better in terms of limiting emissions in a realistic manner, if the emission allowances were raised. In terms of forcing the system to perform on the same level as during minimization with respect to CO_2 emissions, a good approach would be to set the allowance for each unit close to the value of released emission presented in table D.29, and at the same time operate with high values of δ. Despite that the parameters could need some slight adjustments, the results show that the allowance system is well suited for limiting CO_2 emissions efficiently and at the same time serving the demand.

In the upper diagrams in figure 5.2 and 5.3, the energy mixes of the simulations recently discussed can be seen. From figure 5.2 it can be seen that hydropower in general is dominating all simulations, and it is only when $\delta \leq 0.01$ €$/kg\ CO_2$ that the thermal sources are able of taking significant shares from it. This is not surprising, though the regular costs of the hydro units are set very competitive. Further, as these units are assumed to have zero CO_2 emissions, the costs of operating them are not affected by changes in δ. These reasons make hydropower units a natural choice for being baseload sources in this case. It also helps that hydropower has the dominating share of total available power in GS2.

An important observation from figure 5.3 is the exchange of coal for natural gas as the value of δ increases. Comparing this to the blue, total emission graph in figure 5.1, the patterns are pretty much the same, and it is clear that one of the key measures reducing emissions is replacing coal with natural gas. Comparing the generator schedules for simulation 17 to 28 shows a significant change in how natural gas is used. For low values of δ, coal is serving the intermediate demand, while natural gas only is used as a strict peak demand source. For higher values, the roles are totally switched. In such matter, it is interesting to draw lines to the work presented in (Wold [115]), which concluded that coal fired units can serve as peak demand sources, despite their slower dynamic compared to natural gas units.

In the upper diagram in figure 5.4, the corresponding computation time of the presented simulations are summarized. For δ between 0.01 and 0.1 €$/kg\ CO_2$ the variations are relatively small, and there cannot be seen any obvious pattern. Though there might be several factors affecting the computation time, and the small variations taken into consideration, comparing these data upon each other are not left much attention in this thesis. What does stand out in the diagram though, is the runtime of the simulation without any penalty fee. A reasonable assumption of what is causing this, is that the lack of penalty fee makes different generator combinations more similar and competitive, and in such matter forces the solver to explore more solutions.

6.3 Allowance System vs. Conventional Approaches

As discussed in section 2.2 there are many ways of constraining emissions in the UC problem. The allowance system presented is to be considered an experimental approach, trying to tailor make an UC model to the modern industry taking part in emission quotas trading schemes. As already mentioned, the model got its limitations and inaccuracies, which limits its usability in for real trading platform.

On the other hand, the presented approach solves the feasibility problems of using hard emission constraints. At the same time it handles emission allowances, which are a actual part of the real market, better than regular cost function approaches.

In terms of minimizing with respect to emissions, this would be the ideal solution considering the environmental aspects. The question is if this approach is realistic in a large scale, without doing a huge step backwards in terms of free competition and deregulated markets.

In the lower part of figure 5.2 and figure 5.3, the energy mixes of the simulations minimizing with respect to CO_2 emissions, using hard emission constraints on each unit, using hard constraint on total emission and using a conventional cost function are presented. Except from the first one, it is not reasonable to compare these mixes with the ones using the allowance system, though they obviously depends on the setup. Considering the minimizing emissions bar, the energy sources are distributed quite similar to the one for the allowance system using $\delta = 0.1$ €$/kg\ CO_2$, which confirms the earlier observation, stating that replacing coal with natural gas is an efficient measure to decrease the CO_2 emissions.

In terms of the simulations using hard constraints, these can be compared to each other though they are based on the same parameter values. From figure 5.3, there can easily be seen that the simulation using one emission constraint per unit is using a much larger share of natural gas. Studying simulation 30 and 31 in appendix D, also shows that the respective setup releases 6.25% less CO_2 and has 2.78% higher regular costs. The reason for this is that the simulations using a total emission constraint got a better flexibility in terms of choosing generators within its constraints, while the other simulation maximizes its allowed amount of CO_2 from coal units, and then starts using the more expensive natural gas sources. In such matter, a high value of the total emission constraint favors coal in the same way as low values of δ in the allowance system model.

Looking at the energy mix of the conventional cost function simulation, there can be seen that the value of δ is set so low that natural gas becomes a critical peak demand source only. For increasing values of δ, this model will most likely develop after a similar pattern as the one presented for the allowance model.

When comparing the allowance model to other emission constrained UC approaches, also computation time is important. The lower part of figure 5.4 shows the CPU time of the four alternative approaches. The data presented here is too narrow to draw a conclusion, but comparing the upper and lower part of the figure, at least indicates that the allowance system model performs fairly compared to the other approaches.

6.4 Computation Time

As can be seen from studying table 5.3 and figure 5.5 in the results part, or alternatively simulation 33 and 36 in appendix D, both solvers (BONMIN B-BB and OFP) perform equally on the allowance system with 9 generating units and $\delta = 0.1$ €$/kg\ CO_2$. There was no success of fully solving the problem for any of them. As explained in chapter 2, the nature of the UC problem being used in auctions requires it to be solved within reasonable time, and even stricter time requirements occur for computations within intraday and balancing markets.

For sure there might be room for improving the implementation of the problem, and measures which can simplify the solution process without affecting the results may be performed. For instance, there was observed by coincidence that changing v_i for hydro units to 0.001 decreased the CPU time of simulations with six units and $\delta = 0.1$ €$/kg\ CO_2$ with about 30%. This reduction was observed for both BONMIN B-BB and the OFP.

In terms of more methodological approaches to improve computation time, one option is to implement the penalty fee in objective 3.1 slightly different. From equation 3.2 and 3.6 it can be seen that the penalty is formulated using a switch constraint, requiring one binary variable per unit. This could have been avoided by replacing equation 3.2, 3.3 and 3.6 with

$$p_i = \delta \cdot \|\Delta\hat{e}_i\|_1 \tag{6.1}$$

$$e_{total_i} \leq e_{max_i} + \Delta\hat{e}_i \tag{6.2}$$

$$\Delta\hat{e}_i \geq 0 \tag{6.3}$$

where $\Delta\hat{e}_i$ is used instead of Δe_i and operates as a slack variable. This formulation is inspired by the work on soft constraints presented in (Maciejowski [71]), and could potentially decrease the computational load of the problem to some extent. On the other hand, this formulation will only slightly reduce the number of binary variables in the problem. The way the model is implemented in chapter 3 also has its advantage, in terms of being more general. In particular, this formulation can easily incorporate penalties of the form

$$p_i = (p_0 + \delta \cdot \Delta e_i) \cdot z_i \tag{6.4}$$

where p_0 is a penalty for actually *violating* the allowance, while the rest of the fee is growing proportionally with the exceeding emissions. Nonlinear increasing penalties for exceeding the emission allowances can also be implemented.

In a larger perspective, as the problem size increases, in many situations there will be impossible to fully solve the problem within reasonable time. In such cases, the goal would be to find an as good as possible solution within the given time constraints. These are requirements which are very similar to the features the OFP wants to provide. Based on the results presented in table 5.4, 5.5 and 5.6, this is exactly the goal in terms of choosing suitable values of u_2.

Starting with the simulations with six units, it can be seen that among the OFP simulations the best objective value is provided by the simulation using $u_2 = 1$. This is also the only OFP simulation which performs better in terms of objective value than the B-BB without heuristics simulation. Also the computation time is very competitive, both relative the other OFP variants and the B-BB simulations. It is worth noting that fully solving the problem only improves the objective value with 1.83% relative the best OFP solution, and at the same time requires more than 10 times as much computation time.

As can be seen from the blue graph in figure 5.6, the quality of the objective value depends significantly on the value of u_2. Besides the simulations using $u_2 = 1$, the setups with $u_2 = 0.001$, $u_2 = 0.01$ and $u_2 = 0.1$ are all providing good results with short computation times. Also the simulation with $u_2 = 10$ performs fairly good.

Considering the simulations with 9 generating units, it can be seen that $u_2 = 200$ gives the best objective value among all the simulations, performing way better than both the B-BB simulations. Relative the result presented in simulation 36, the objective is less than 1% higher, which is remarkable in terms of the large difference in computation time. Compared to the other OFP simulations, the computation time is no more than mediocre. For comparison is the simulation using $u_2 = 0.1$ almost 35% faster. On the other hand, this simulation offers an almost 5.2% weaker objective value.

In terms of making a reasonable trade-off between runtime and objective value, there can be seen by studying the red graphs in figure 5.6 and 5.7, that $u_2 = 10$ might be a good choice. This setup improves the runtime with close to 25%, and does only sacrifice about a 3.3% increase in the objective value.

When studying the simulations with 12 units in table 5.6, there can be observed that the objective values in general are lower than the objective values presented in table 5.5. This might occur confusing at first, because it is reasonable to assume higher costs due to the higher values in DS4 relative to DS3, and worse objective quality due to the increased complexity from the larger number of units. The reason for this, is that the relative gap between the total maximum capacity in GS4 and the demand in DS4 is much higher then for GS3 and DS3. From this, it follows that the simulations with 12 units got a wider aspect of available units, which is helpful in terms of reducing regular costs and avoiding CO_2 penalty fees.

Studying table 5.6 gives that BONMIN B-BB without heuristics performs very poorly. The regular B-BB simulation presents an objective value in the range of the poorest OFP simulations, but with a significantly worse computation time. Among the OFP simulations, does the one with $u_2 = 100$ give the best objective value. The downside of this setup is that it provides the second worst computation time among the OFP simulations. The very best computation time is for the simulation with $u_2 = 0.001$, but this also retrieves the second worst objective among the OFP simulations.

In terms of trying to make a fair trade-off between cost and computation time the use of $u_2 = 0.1$

and $u_2 = 500$ can be good choices. The former is the best choice if runtime is the most important criterion, improving runtime with almost 38% and only sacrificing 3.7% of the objective value, relative the OFP simulation providing the best objective value. The latter is only raising the objective value with about 1.3%, and improves the computation time with almost 18.3%.

Considering figure 5.6, gives that it is hard to find any specific pattern of how the solutions develops relative u_2, for the different number of units. As already mentioned, there are significant differences in terms of maximum capacity relative demand for the different setups. This obviously impacts the complexity of the simulations, but there would be natural to expect a common pattern where the objective values are improved in line with increased weighting of the original objective[101]. This is obviously not the case for the presented results.

In terms of very low values of u_2, the OFP almost becomes identical to the FP-heuristic implemented in BONMIN. In such matter, should the first solutions found by these setups retrieve similar objective values as the first solution found by BONMIN B-BB using the FP heuristic in the first node. This makes very much sense, though for both the simulations with 9 and 12 units, the objective values of the simulations with $u_2 = 0.001$ are identical to the results returned by the regular B-BB setup.

For larger values of u_2, there can be seen that the objective values of the simulations with 6 generating units oscillates drastically. There can also be seen such tendencies for the simulations with 9 units, but with much lower amplitude. A reasonable explanation for this behavior is the stochastic mechanism in the OFP stall handling. When detecting stalling, the algorithm flips a random number of integer variables. As the maximum limit is set to 10 variables in the respective simulations, the number of variables flipped will be between 1 and 10 for each simulation. In terms of evaluating this further, a reasonable approach would be to repeat the respective simulations several times and compare the changes in the observed oscillations in figure 5.6.

Another factor which may have a significant influence is the nonconvexities in the problem formulation. The OFP is originally designed for convex MINLPs, and throughout testing on nonconvex problems in this thesis, it has provided varying results. Nevertheless, for the right values of u_2, the OFP has performed significantly better than BONMIN B-BB for the CO_2 allowance system, considering finding acceptable solutions within reasonable time. This is an important observation, which opens for new approaches for solving the UC problem. Further testing can reveal if this is a viable approach in terms of solving problems to optimality within reasonable computation time. An alternative use of the OFP can be as a support heuristic, finding better start points for other algorithms.

Based on the graphs of objective values and computation times presented in figure 5.6 and figure 5.7, respectively, it would be reasonable to assume that a good compromise for a common value of u_2 for all simulations, would be near 10 or 100. On the other hand, in terms of the discussed oscillations and stalling, this is a hard decision to make without a broader aspect of data.

In terms computation time for MINLPs in general, an interesting approach in the future will be to evaluate the possibility of taking advantage of multiple CPU cores. Such support can possibly, but as presented in this thesis not necessarily, improve the runtime significantly, by utilizing more of the computational power in modern hardware.

In terms of really large problems, there will also be natural to benefit from using decomposition schemes. This is not considered within the scope of this thesis, but different approaches can be found in (Borghetti et al. [16]), (Finardi et al. [39]) and (Sagastizabal [96]).

6.5 European Shale Gas Revolution?

Based on this research and other approaches to the UC problem including emission constraints, it is reasonable to state that natural gas is a highly relevant power source, in terms of reducing CO_2 emissions and at the same time serving the demand in a sustainable way. The practical part of this research was limited to include only three types of power sources. In terms of a more holistic view of the contamination aspect, all available sources must be taken into consideration.

From the data presented in table 2.1, it can be seen that hydropower and wind provide the best mean values for lifecycle CO_2 emissions. This research has shown hydropower to be a very attractive option, both when it comes to costs and emissions. On the other hand, there are few countries which got sufficient hydro resources to use it as a dominating baseload source.

The main downside of using wind, and solar power as well, is the strict dependence on weather conditions. As discussed in (Wold [115]), a power supply system with large shares of these sources, will require very robust units compensating for weather changes to avoid brown- and blackouts. There is also important to have in mind that installing these types of facilities, including hydropower, may cause significant ecological footprints in their respective areas.

Energy from biomass can also be seen to be an attractive option, but it got some major drawbacks. It usually requires large areas of land to be efficient, and there might also be challenges with materials not being available the whole year around. The extraction of biomass is also often expensive.

As presented in (Wold [115]), nuclear power does provide great features in terms of GHG emissions and of being a stable baseload source. Despite this, it is a very controversial power source, due to its nature of producing nuclear waste.

Drawing the full picture, all power sources got their pros and cons. What is for sure, is that measures have to be made if Europe is going reach their goals of reducing CO_2 emissions, and whatever strategy which is chosen there will have to be made trade-offs.

As earlier discussed, as an effect of the shale gas revolution, the US has experienced great results in terms of reducing its CO_2 emissions. In (Wold [115]) the difference in infrastructure in the US compared to Europe was discussed, and it came clear that the latter is modest compared to the former. Regardless of this, natural gas is an important power source in the European market. Norway and Russia are supplying extensively throughout Europe using pipelines, and there is a significant import of LNG. On top of this, there is estimated to exist significant instances of shale gas on the continent.

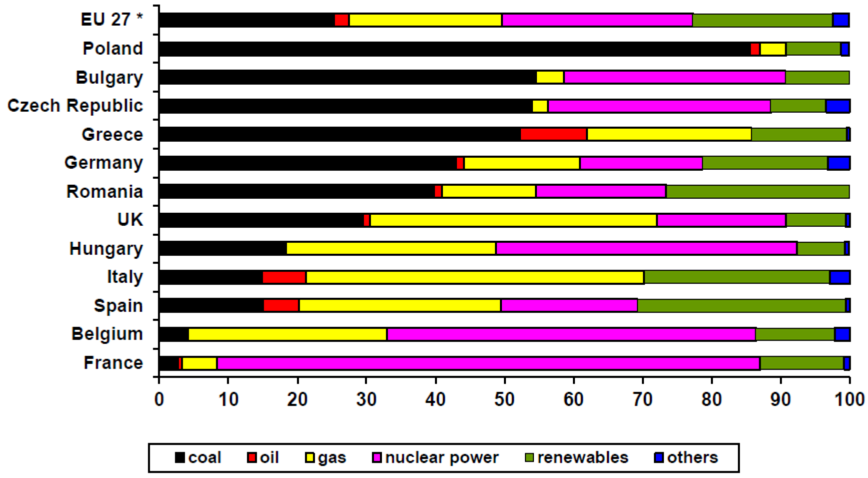
The mentioned factors make it reasonable to ask if Europe can achieve similar results as the US, by replacing shares of coal with natural gas in the power industry. This is a question of complex nature, and if it is possible, is it the best solution? It is beyond the scope of this thesis to evaluate all potential strategies for a continental CO_2 reduction plan, and state which one is the best. The focus is therefor left on whether it is possible, and which trade-offs it may require.

Between 2005 and 2012, average natural gas prices for industry consumption decreased with 66% in the US. In the same period of time, EU faced an increase of 35%. Further has the electricity price index for industry in US and EU decreased 4% and increased 38%, respectively[18]. At the same time, European coal prices have dropped significantly due to an oversupplied market. This can partly be seen in context of high import levels from Colombia, South Africa and the US. The increased import from the US can naturally be seen as a consequence of them replacing coal with natural gas.

In the same period, Europe has been facing a significant recession, and especially are Central and Southern Europe affected. The increased power prices have contributed to worsen the situation, in terms of making industry depending on electric power less competitive to the US' industry. CO_2 put aside, this whole situation in large scale seems to favor coal before natural gas, which by studying figure 6.1 also comes clear is the case for many countries. From a financial point of view, there are two factors which can change this, an increase in the total cost of using coal and a reduction in the natural gas prices.

The former can indirectly be achieved by financial measures to CO_2 emissions, such as the allowance system presented. This kind of measures have indirectly been implemented in the EU in terms of the EU ETS. On the other hand, may it be difficult to gather support for measures which may cause further increase in electricity prices. Such measures may cause increased relocation of power consuming industry outside the respective countries, and worsen the situation for an already struggling population.

As with all consumables, the price of natural gas follows demand and availability. Considering the idea of replacing coal with natural gas, this will cause a raise in demand. Assuming that the availability stays on today's level, this will raise the prices, and from this there will follow a raise in electric power



	TWh	Share of coal %
EU 27 *	3,280	25
Poland	164	86
Bulgary	51	54
Czech Republic	88	54
Greece	59	52
Germany	609	43
Romania	62	40
UK	368	30
Hungary	36	18
Italy	303	15
Spain	292	15
Belgium	90	4
France	562	3

Figure 6.1: Diversity of power generation in selected EU countries (2011) *Source: Eurostat with graphics from Euracoal*

prices as well. Therefore, it is natural to focus on availability, in terms of making natural gas a more attractive option.

As discussed in (Wold [115]), can today's natural gas market be divided into two main branches: Liquefied natural gas (LNG) and pipeline gas. A rough overview of EU's trade patterns in these markets are showed in figure 6.2. In terms of LNG, the market is tight due to large demand both in Europe and Asia. Considering the pipeline market, Russia and Norway are the dominating actors providing for 29% and 24% of EU's total imports, respectively[89]. This market is also very tight, and there is common with oil price indexed supply contracts, keeping the prices high.

At this point, there are few realistic options of increasing the availability of natural gas in the pipeline market, besides exploring the opportunities of retrieving shale gas. Conventional domestic production is scarce for the EU countries, and possibilities of increased supply from the North African actors are limited due to the unrest in the region.

The estimated volume of recoverable shale gas within the EU countries varies a lot, though little test drilling has been carried out so far. In 2012 the EU Joint Research Centre (JRC) presented numbers stating best, high and low estimates of technical recoverable shale gas within the EU. These values were set to 17.6, 15.9 and 2.3 trillion cubic meters (tcm), respectively. For comparison were the best and low numbers for the US 47.2 and 13 tcm[18].

Even though the estimates are modest relative the numbers for the US, these amounts of natural gas got the potential of supplying the whole region for decades[18]. Despite this, the enthusiasm has been rather varying so far. Among the countries which are assumed to hold shale gas resources are Poland, France, Great Britain, Bulgaria, Germany and Romania, where the former two are assumed to hold the largest amounts[35].

The most negative parties have been France and Bulgaria, which both have banned fracking, due to the risks of polluting ground water and potential seismic disturbances. Another significant factor when it comes to France, is that fossil fuels only counts for a very small share of their power generation, as can be seen in figure 6.1. On the other hand, Bulgaria got a power production dominated by coal, similar to the US before the shale gas revolution. In such matter, this is a country with significant opportunities of improving their carbon emissions by retrieving their own shale gas, if it shows off to be recoverable.

Within Germany the opinions regarding fracking are divided, but some drilling has taken place. Germany can also be seen to have a significant share of electric power coming from coal fired plants, but they have already made large investments in their renewable sector, which are assumed to change

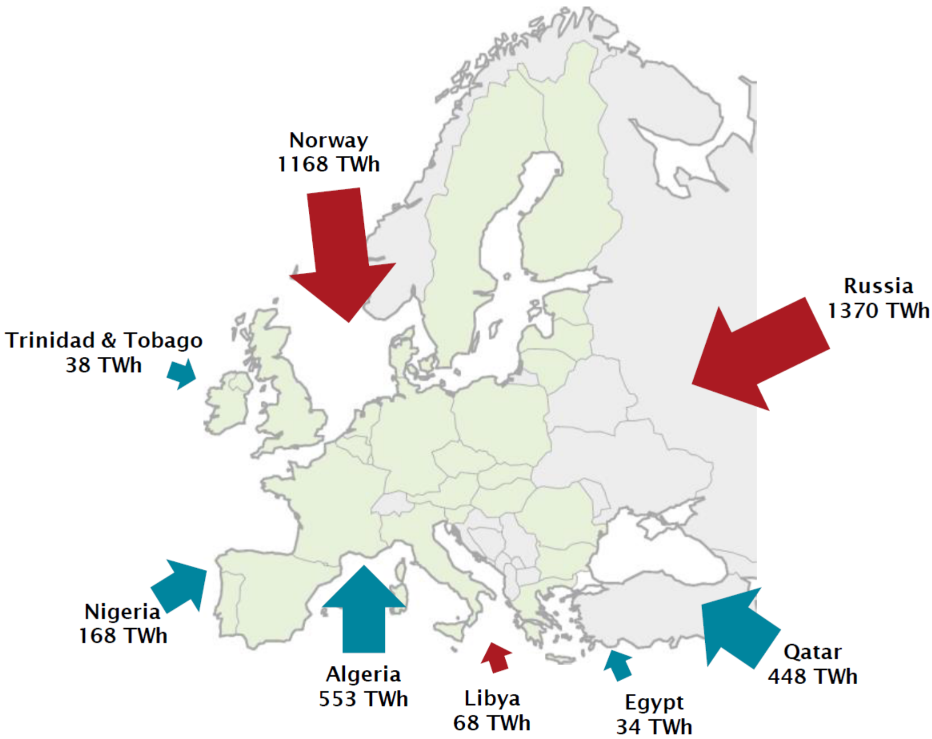


Figure 6.2: Imports of natural gas to the EU. Red arrows indicate imports by pipeline, and blue arrows indicate LNG imports. *Source: Gas in Focus (Observatoire du Gaz), Imoports of natural gas into the European Union*

their energy mix drastically. The official goal is having a 35% share of the total gross electricity consumption supplied by renewable sources within 2020[2]. A natural consequence of this, is that recovering shale gas got a minor priority at this point.

Romania has taken a similar stand as Germany. They have allowed drilling of a few test wells, but have not given any approval for recovering gas or fracking[18].

In Britain, shale gas has been considered a cheap option to alternative renewable energy sources, and there is an ongoing argue where the main initiative should be. A significant number of test wells are being drilled these days, but under strict regulations and requirements of geological assessment before drilling[18]. In large extent, Britain is one of the most exploring countries within the field.

Poland is the country which in largest extent has started exploring its potential shale gas resources. As discussed in (Wold [115]), Poland have traditionally been using small amounts of natural gas. As shown in figure 6.1 is the electric power industry dominated by coal fired plants. This has led to comprehensive pressure from the EU, regarding reduction in their carbon emissions. This can be seen as an impetus to their eager of assessing the possibilities of recovering shale gas.

There have been done comprehensive drilling of test wells, and the government has stated that they are aiming for commercial operation within the end of 2015. So far, the project has not been a road without bumps though. During 2012 Exxon pulled out of Poland, stating that their technology was not sufficient for the type of shale in the basin. In spring 2013 Marathon and Talisman followed the same path, due to challenges with the geology and uncertainty regarding tax policy[18].

Nevertheless, in January 2014 San Leon Energy (SLE) informed the press (articles among other published at www.bloomberg.com and www.reuters.com) that they had drilled a successful vertical well in Lewino, in northern Poland. The well was said to produce as much as 60000 cubic feet daily during tests, and the company stated they were aiming to drill a commercial horizontal well no later than July 2014.

To draw a line, there are so far made small steps within the EU to start commercial recovering of shale gas. At where it stands now, some countries might face an economical growth, and achieve reduced carbon emissions as an effect of their ongoing projects. On the other hand, at this point, the initiatives are to small to achieve nearly the same impacts within the union, as have been seen in the US.

Only time will show how the width of the upcoming production in Poland and the resources in Britain. Nevertheless, as long as potentially large resource holders, such as France, are holding back, there will be hard to gain sufficient cooperation regarding infrastructure, which was pointed out as one of the main challenges in (Wold [115]).

Besides the environmental gain of replacing coal with natural gas, it is also clear that natural gas is an important energy source within Central, Southern and Western Europe. In view of the unstable relationship between Russia and the EU, and Russia's role as a dominating supplier of natural gas, there is natural to consider the union's ability of becoming more self provided. This concern is strengthened by the recent escalation of the conflict between Russia and Ukraine, which affects a significant transfer path for EU's natural gas supply. There is a slim possibility that this situation may trigger the parties now holding back in their shale gas projects.

The pressure on many of the EU member's economies, can also be seen as an argument not making drastic investments in the shale gas industry. On the other hand, the potential gain is huge, and such initiatives may be a significant part in terms of turning the negative trends. A major initiative in the shale gas industry will require significant investments both when it comes to equipment, environmental protection measures and infrastructure, but this will also create an increased demand of work labor, especially considering craftsmen and engineers. In such matter, an investment may besides increasing future financial income and securing the supply of natural gas, also provide a near immediate effect in terms of reducing unemployment. As the unemployment rates are decreasing, also social service costs will decrease and tax incomes raise.

To conclude this section, it can be stated that it is unrealistic to expect the same success of shale gas within the EU, as have been experienced in the US in terms of decreasing CO_2 emissions. Some of the ongoing projects are showing positive results, which may strengthen the role of natural gas in the power industry in the future. As a consequence of both geological and structural differences, there will be hard to copy the American success, but a wise approach will be to learn from their experiences, both in terms of making the business profitable and protecting the environment.

Chapter 7

Conclusion

In this project there has been created a model combining the a conventional UC problem with a CO_2 allowance system. Reasonable simplifications have been made, but the action patterns of the generating units are close to realistic. The presented formulation is relevant, because it got potential of adapting the changes which will follow from the EU ETS, in larger extent than more conventional approaches. A challenge which has to be taken into consideration though, is to find optimal daily distributions for the long therm quotas, which often range from 6 months to 4 years.

Based on experimental simulations with 3 and 6 generating units, BONMIN B-BB and OFP have been found to be suitable solvers for the problem. BONMIN B-BB was chosen for further exploration of the features of the model, due to a better historical foundation of retrieving good solutions to similar problems. The results of these simulations proved the model efficient and flexible, in terms of limiting CO_2 emissions while sufficiently serving the demand. As expected, there was verified that natural gas is favored for coal while using the respective allowance system, and that natural gas is a good replacement to coal, considering CO_2 emissions.

For simulations with 6 generating units, BONMIN B-BB was able of fully solving the problem within reasonable time. For simulations with 9 and 12 units, this was not the case. By comparing the objective values and the computation times for the first feasible solutions found by BONMIN B-BB and OFP, for these problems, there was found that OFP in general was providing better trade-offs between computation time and objective values. For some weightings of the original objective, the results retrieved by OFP were remarkably good. Taking simulations with 6, 9 and 12 units into consideration, a reasonable common value for the weighting was found to be 10.

For the EU to gain a growth in popularity of natural gas relative to coal, similar to what has been experienced in the US, the relative price gap has to decrease. One potential approach is to add penalties for using coal, which indirectly is the case with the emission penalty in the allowance system. Another option is to reduce the natural gas price, by increasing the availability in the market. Within the EU, shale gas got a potential of contributing to this, but at this point there are divided opinions if this is the right card to play. If the region is going to see an effect anything near what has been experienced in the US, it will require large investments in infrastructure and geological research.

Appendix A

Acronyms

BP British Petroleum

CCS Carbon Capture Sequestration

CAISO California Independent System Operator

COIN-OR Computational Infrastructure for Operations Research

CPL Common Public License

CPU Central Processing Unit

DS1 Demand set 1

DS2 Demand set 2

DS3 Demand set 3

DS4 Demand set 4

DISCO Distributing Company

DICOPT Discrete and Continuous Optimizer

DP Dynamic Programming

EFOR Equivalent Forced Outage Rates

ECUC Emission Constrained Unit Commitment

EIA Energy Information Administration

EU European Union

EU ETS European Union Emissions Trading Scheme

FP Feasibility Pump

GENCO Generating Company

GHG Greenhouse Gas

GS1 Generator Setup 1

GS2 Generator Setup 2

GS3 Generator Setup 3

GS4 Generator Setup 4

ISO Independent System Operator

LP Linear Program

LR Lagrangian Relaxation

MCP Market Clearing Price

MIP Mixed Integer Program

MILP Mixed Integer Linear Program

MINLP Mixed Integer Nonlinear Program

MIQCP Mixed Integer Quadratically Constrained Program

MIQP Mixed Integer Quadratic Program

NLP Nonlinear Program

NTNU Norwegian University of Science and Technology (Norges Tekniske Naturvitenskapelige Universitet)

NVE Norwegian Water Resources and Energy Directorate (Norges Vassdrags- og Energidirektorat)

OFP Objective Feasibility Pump

OSI Open Source Initiative

O&M Operation and Maintenance

PDF Probability Density Function

QCP Quadratically Constrained Program

SA Simulated Annealing

SOS1 Special Ordered Set of Type 1

SOS2 Special Ordered Set of Type 2

TRANSCO Transmission Company

TS Tabu Search

UC Unit Commitment

UK ETS United Kingdom Emissions Trading Scheme

US United States

WNA World Nuclear Association

Appendix B

Demand Setup

DS1

Time	Demand [MW]
1	700
2	700
3	650
4	250
5	350
6	400
7	350
8	600
9	750
10	800
11	950
12	1000
13	1450
14	2000
15	2100
16	2100
17	2100
18	1900
19	1500
20	1200
21	1000
22	700
23	600
24	700

(a) Demand Set 1 (DS1)

DS2

Time	Demand [MW]
1	1400
2	1400
3	1300
4	500
5	700
6	800
7	700
8	1200
9	1500
10	1600
11	1900
12	2000
13	2900
14	4000
15	4200
16	4200
17	4200
18	3800
19	3000
20	2400
21	2000
22	1400
23	1200
24	1400

(b) Demand Set 2 (DS2)

DS3

Time	Demand [MW]
1	1860
2	1860
3	1730
4	665
5	930
6	1065
7	930
8	1595
9	1995
10	2130
11	2530
12	2660
13	3850
14	5320
15	5580
16	5580
17	5580
18	5050
19	3990
20	3190
21	2660
22	1860
23	1595
24	1860

(c) Demand Set 3 (DS3)

DS4

Time	Demand [MW]
1	1900
2	1900
3	1800
4	700
5	1000
6	1100
7	1000
8	1600
9	2000
10	2200
11	2600
12	2700
13	3900
14	5400
15	5600
16	5600
17	5600
18	5100
19	4000
20	3200
21	2700
22	1900
23	1600
24	1900;

(d) Demand Set 4 (DS4)

Table B.1: Demand sets

Appendix C

Generator Setup

GS1

Unit	α	β_1	β_2	γ	Δw_{max}	ϵ	η	ι	ν	e_{max}	f_{max}	p_1	p_2	p_3	p_4	$t_{min_{off}}$	$t_{min_{on}}$	w_{max}	w_{min}
Coal 1	25	10000	5000	0.1	400	0.06	1	28	2.3	2000000	6200000	0	-0.3205	0.5688	0.1592	5	5	1000	400
Natural Gas 1	10	1000	2000	0.03	300	0.0047	1.5	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	1	2	600	300
Hydro 1	4	6000	0	0.01	800	0.006	1.5	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	200

(a) Generator Setup 1 (GS1)

GS2

Unit	α	β_1	β_2	γ	Δw_{max}	ϵ	η	ι	ν	e_{max}	f_{max}	p_1	p_2	p_3	p_4	$t_{min_{off}}$	$t_{min_{on}}$	w_{max}	w_{min}
Coal 1	25	10000	5000	0.1	400	0.060	1	28	2.3	2000000	6200000	0	-0.3205	0.5688	0.1592	5	5	1000	400
Natural Gas 1	10	1000	2000	0.03	300	0.0047	1.5	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	2	3	600	300
Hydro 1	4	6000	0	0.01	800	0.006	1.5	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	200
Coal 2	20	8000	4500	0.08	300	0.060	2	28	2.3	1600000	4960000	0	-0.3205	0.5688	0.1592	3	5	800	300
Natural Gas 2	15	500	2500	0.03	400	0.0047	0.75	1.1	0.05	1600000	2700000000	0	-0.4463	0.7922	0.2217	1	2	800	350
Hydro 2	3	6500	0	0.015	700	0.006	1	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	300

(b) Generator Setup 2 (GS2)

GS3

Unit	α	β_1	β_2	γ	Δw_{max}	ϵ	η	ι	ν	e_{max}	f_{max}	p_1	p_2	p_3	p_4	$t_{min_{off}}$	$t_{min_{on}}$	w_{max}	w_{min}
Coal 1	25	10000	5000	0.1	400	0.06	1	28	2.3	2000000	6200000	0	-0.3205	0.5688	0.1592	5	5	1000	400
Natural Gas 1	10	1000	2000	0.03	300	0.0047	1.5	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	1	2	600	300
Hydro 1	4	6000	0	0.01	800	0.006	1.5	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	200
Coal 2	20	8000	4500	0.08	300	0.060	2	28	2.3	1600000	4960000	0	-0.3205	0.5688	0.1592	3	5	800	300
Natural Gas 2	15	500	2500	0.03	400	0.0047	0.75	1.1	0.05	1600000	2700000000	0	-0.4463	0.7922	0.2217	1	2	800	350
Hydro 2	3	6500	0	0.015	700	0.006	1	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	300
Coal 3	18	8000	4000	0.08	300	0.060	1.8	28	2.3	1400000	4340000	0	-0.3205	0.5688	0.1592	4	4	700	300
Natural Gas 3	10	500	2000	0.03	300	0.0047	1	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	1	2	600	300
Natural Gas 4	5	500	1500	0.03	250	0.0047	1	1.1	0.05	1000000	1666666667	0	-0.4463	0.7922	0.2217	1	2	500	250

(c) Generator Setup 3 (GS3)

GS4

Unit	α	β_1	β_2	γ	Δw_{max}	ϵ	η	ι	ν	e_{max}	f_{max}	p_1	p_2	p_3	p_4	$t_{min_{off}}$	$t_{min_{on}}$	w_{max}	w_{min}
Coal 1	25	10000	5000	0.1	400	0.06	1	28	2.3	2000000	6200000	0	-0.3205	0.5688	0.1592	5	5	1000	400
Natural Gas 1	10	1000	2000	0.03	300	0.0047	1.5	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	1	2	600	300
Hydro 1	4	6000	0	0.01	800	0.006	1.5	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	200
Coal 2	20	8000	4500	0.08	300	0.060	2	28	2.3	1600000	4960000	0	-0.3205	0.5688	0.1592	3	5	800	300
Natural Gas 2	15	500	2500	0.03	400	0.0047	0.75	1.1	0.05	1600000	2700000000	0	-0.4463	0.7922	0.2217	1	2	800	350
Hydro 2	3	6500	0	0.015	700	0.006	1	0.5	0	2000000	190000000	169.3	-440.9	379.3	-16.34	1	1	1000	300
Coal 3	18	8000	4000	0.08	300	0.060	1.8	28	2.3	1400000	4340000	0	-0.3205	0.5688	0.1592	4	4	700	300
Natural Gas 3	10	500	2000	0.03	300	0.0047	1	1.1	0.05	1200000	2000000000	0	-0.4463	0.7922	0.2217	1	2	600	300
Coal 4	25	10000	5000	0.1	400	0.060	1	28	2.3	2000000	6200000	0	-0.3205	0.5688	0.1592	5	5	1000	400
Natural Gas 4	5	500	1500	0.03	250	0.0047	1	1.1	0.05	1000000	1666666667	0	-0.4463	0.7922	0.2217	1	2	500	250
Natural Gas 5	15	500	2500	0.03	400	0.0047	0.75	1.1	0.05	1400000	2362500000	0	-0.4463	0.7922	0.2217	1	2	700	300
Natural Gas 6	15	500	2500	0.03	450	0.0047	0.75	1.1	0.05	1800000	3037500000	0	-0.4463	0.7922	0.2217	1	2	900	400

(d) Generator Setup 4 (GS4)

Table C.1: Generator Setups

Appendix D

Simulations

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MILP	GS1	DS1	100	9495 (7331)	3621 (2510)	291 (250)	5583 (4571)	9147 (5788)	2148 (1790)	6999 (3998)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	528
5	0	0	428
6	0	0	428
7	0	0	528
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	770
12	0	300	770
13	400	300	870
14	547	593	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	533	500	1000
19	400	300	984
20	0	300	984
21	0	300	849
22	0	0	749
23	0	0	749
24	0	0	749
Total	**3821**	**4693**	**18880**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	102366	3023660
Natural Gas 1	21472	1414720
Hydro 1	0	0
Total	**123838**	**4438380**

Regular Costs

Type	Cost [€]
Fuel	220891
Maintenance	39181
Ramping	15705
Startup	104000
Shutdown	31000
Total	**410777**

Results

Objective	534613
Best Bound	534560
Absolute MIP Gap	53.332
Relative MIP Gap	10^{-4}
CPU Time [sec]	20635
Real Time [sec]	2850

Simulation Specific Parameters

Parameter	Value
δ [€/kg CO_2]	0.1
κ	1.10
ξ	1.07

Table D.1: Simulation 1

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MILP	GS1	DS1	200	6807 (4956)	2277 (1398)	291 (250)	4239 (3308)	7803 (4600)	2148 (1742)	5655 (2858)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	496
5	0	0	442
6	0	0	442
7	0	0	442
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	852
12	0	300	852
13	400	300	852
14	540	600	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	447	586	1000
19	400	300	984
20	0	300	984
21	0	300	784
22	0	0	749
23	0	0	749
24	0	0	749
Total	**3728**	**4786**	**18869**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	97663.9	2976640
Natural Gas 1	24224.9	1442250
Hydro 1	0	0
Total	**121889**	**4418890**

Regular Costs

Type	Cost [€]
Fuel	222276
Maintenance	39211
Ramping	21052
Startup	104000
Shutdown	31000
Total	**417539**

Results

Objective	539428
Best Bound	539428
Absolute MIP Gap	0
Relative MIP Gap	0
CPU Time [sec]	17
Real Time [sec]	4

Simulation Specific Parameters

Parameter	Value
δ [€$/kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.2: Simulation 2

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MILP	GS1	DS1	100/200[a]	8247 (6236)	2997 (1958)	291 (250)	4959 (4028)	8523 (5160)	2148 (1742)	6375 (3418)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	528
5	0	0	428
6	0	0	428
7	0	0	528
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	770
12	0	300	770
13	400	300	870
14	540	600	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	500	533	1000
19	400	300	984
20	0	300	984
21	0	300	849
22	0	0	749
23	0	0	749
24	0	0	749
Total	**3781**	**4733**	**18880**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	100529	3005290
Natural Gas 1	22786	1427860
Hydro 1	0	0
Total	**123315**	**4433150**

Regular Costs

Type	Cost [€]
Fuel	221664
Maintenance	39201
Ramping	16185
Startup	104000
Shutdown	31000
Total	**412050**

Results

Objective	535364
Best Bound	535364
Absolute MIP Gap	0
Relative MIP Gap	0
CPU Time [sec]	19
Real Time [sec]	5

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.3: Simulation 3

[a] Step size is 100 for $\hat{c}_{ramp_{it}}$, and 200 for all other piecewise linear approximations.

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MILP	GS2	DS2	100	19278 (15369)	7386 (5447)	582 (510)	11310 (9412)	18390 (12315)	4296 (3674)	14094 (8641)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	900	0	0	598
2	0	300	900	300	0	0
3	0	300	200	300	350	300
4	0	300	0	300	350	0
5	0	0	0	300	0	449
6	0	0	200	300	0	500
7	0	0	200	0	0	700
8	0	0	400	0	0	900
9	0	0	605	0	0	1000
10	0	0	712	0	0	1000
11	0	0	800	300	0	1000
12	0	0	900	300	0	1000
13	0	423	1000	300	380	1000
14	500	600	1000	400	780	1000
15	600	600	1000	494	800	1000
16	600	600	1000	500	794	1000
17	700	300	1000	694	800	1000
18	666	0	1000	600	800	1000
19	400	0	1000	300	510	1000
20	400	0	900	300	350	940
21	0	0	900	300	0	940
22	0	0	800	0	0	900
23	0	0	800	0	0	900
24	0	0	900	0	0	900
Total	**3866**	**3423**	**17117**	**5988**	**5914**	**19027**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	104854	3048540
Natural Gas 1	0	0
Hydro 1	0	0
Coal 2	339836	4998360
Natural Gas 2	17869	1778690
Hydro 2	0	0
Total	**462559,3**	**9825590**

Regular Costs

Type	Cost [€]
Fuel	491618
Maintenance	70115
Ramping	51775
Startup	200500
Shutdown	103000
Total	917008

Results

Objective	infeasible problem
Best Bound	∞
Absolute MIP Gap	551115
Relative MIP Gap	0.3982
CPU Time [sec]	24299
Real Time [sec]	interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.4: Simulation 4

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MILP	GS2	DS2	200	13614 (10645)	4554 (3152)	582 (510)	8478 (6983)	15558 (10019)	4296 (3682)	11262 (6337)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	598	0	0	900
2	0	0	200	300	0	998
3	0	0	200	300	0	891
4	0	0	0	300	0	691
5	0	0	0	300	0	491
6	0	0	200	300	0	356
7	0	0	200	300	0	300
8	0	300	684	300	0	0
9	0	300	684	0	0	621
10	0	300	400	0	350	800
11	0	0	683	0	350	1000
12	0	0	840	300	0	1000
13	0	403	1000	300	400	1000
14	580	600	1000	300	800	1000
15	694	600	1000	400	800	1000
16	694	600	1000	400	800	1000
17	694	600	1000	400	800	1000
18	400	566	1000	300	800	1000
19	0	300	1000	300	610	1000
20	0	0	1000	300	350	918
21	0	0	1000	0	350	790
22	0	0	1000	0	350	500
23	0	0	1000	0	350	0
24	0	0	1000	0	0	498
Total	**3062**	**4569**	**16689**	**5100**	**7110**	**17754**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	35044	2350440
Natural Gas 1	16260	1362600
Hydro 1	0	0
Coal 2	285787	4457870
Natural Gas 2	56628	2166280
Hydro 2	0	0
Total	**393719**	**10337190**

Regular Costs

Type	Cost [€]
Fuel	526292
Maintenance	68236
Ramping	53532
Startup	200500
Shutdown	103000
Total	951559,5

Results

Objective	1345279
Best Bound	862530
Absolute MIP Gap	482749
Relative MIP Gap	0.3588
CPU Time [sec]	26480
Real Time [sec]	interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.5: Simulation 5

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MILP	GS1	DS1	100	9495 (7331)	3621 (2510)	291 (250)	5583 (4571)	9147 (5788)	2148 (1790)	6999 (3998)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	528
5	0	0	428
6	0	0	428
7	0	0	528
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	770
12	0	300	770
13	400	300	870
14	547	593	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	533	500	1000
19	400	300	984
20	0	300	984
21	0	300	849
22	0	0	749
23	0	0	749
24	0	0	749
Total	**3821**	**4693**	**18880**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	102366	3023660
Natural Gas 1	21472	1414720
Hydro 1	0	0
Total	**123838**	**4438380**

Regular Costs

Type	Cost [€]
Fuel	220891
Maintenance	39181
Ramping	15705
Startup	104000
Shutdown	31000
Total	**410776,5**

Results

Objective	infeasible problem
Best Bound	∞
Absolute MIP Gap	11128
Relative MIP Gap	0.0208
CPU Time [sec]	3589
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€$/kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.6: Simulation 6

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MILP	GS1	DS1	100/200[a]	8247 (6236)	2997 (1958)	291 (250)	4959 (4028)	8523 (5160)	2148 (1742)	6375 (3418)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	528
5	0	0	428
6	0	0	428
7	0	0	528
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	770
12	0	300	770
13	400	300	870
14	540	600	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	500	533	1000
19	400	300	984
20	0	300	984
21	0	300	849
22	0	0	749
23	0	0	749
24	0	0	749
Total	**3781**	**4733**	**18880**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	100529	3005290
Natural Gas 1	22786	1427860
Hydro 1	0	0
Total	**123315**	**4433150**

Regular Costs

Type	Cost [€]
Fuel	221664
Maintenance	39201
Ramping	16185
Startup	104000
Shutdown	31000
Total	**412050**

Results

Objective	535364
Best Bound	535364
Absolute MIP Gap	0
Relative MIP Gap	0
CPU Time [sec]	3
Real Time [sec]	4

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.7: Simulation 7

[a] Step size is 100 for $c_{ramp_{it}}$, and 200 for all other piecewise linear approximations.

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MIQP	GS1	DS1	100	6327 (4563)	2037 (1278)	291 (250)	72 (72)	3927 (2963)	7347 (4388)	2004 (1678)	5343 (2710)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	515
5	0	0	427
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	764
12	0	300	770
13	400	300	852
14	544	596	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	519	514	1000
19	400	300	943
20	0	300	984
21	0	300	817
22	0	0	749
23	0	0	700
24	0	0	749
Total	**3804**	**4710**	**18682**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	101434	3014340
Natural Gas 1	21988	1419880
Hydro 1	0	0
Total	**123422**	**4434220**

Regular Costs

Type	Cost [€]
Fuel	221047
Maintenance	38892
Ramping	15211
Startup	104000
Shutdown	31000
Total	**410151**

Results

Objective	533573
Best Bound	508057
Absolute MIP Gap	25516
Relative MIP Gap	0.0478
CPU Time [sec]	26554
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.8: Simulation 8

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MIQP	GS1	DS1	200	5079 (3468)	1413 (726)	291 (250)	72 (72)	3303 (2420)	6723 (3760)	2004 (1630)	4719 (2130)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	516
5	0	0	427
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	765
12	0	300	770
13	400	300	852
14	540	600	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	508	525	1000
19	400	300	943
20	0	300	984
21	0	300	817
22	0	0	749
23	0	0	701
24	0	0	749
Total	**3789**	**4725**	**18682**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	100982	3009820
Natural Gas 1	22558	1425580
Hydro 1	0	0
Total	**123540**	**4435400**

Regular Costs

Type	Cost [€]
Fuel	221485
Maintenance	38900
Ramping	15379
Startup	104000
Shutdown	31000
Total	**410764**

Results

Objective	534305
Best Bound	534305
Absolute MIP Gap	0
Relative MIP Gap	0
CPU Time [sec]	252
Real Time [sec]	34

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.9: Simulation 9

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MIQP	GS2	DS2	100	12942 (9608)	4218 (2785)	582 (510)	144 (144)	7998 (6169)	14790 (9290)	4008 (3432)	10782 (5858)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	598	0	0	900
2	0	0	515	0	0	983
3	0	300	200	0	0	900
4	0	300	200	0	0	655
5	0	300	200	0	0	456
6	0	300	200	0	356	0
7	0	300	0	0	350	500
8	0	300	200	0	0	784
9	0	0	200	0	405	1000
10	0	0	500	0	350	925
11	0	300	900	0	0	900
12	400	300	500	0	350	600
13	400	303	1000	0	400	1000
14	500	600	1000	390	790	1000
15	694	600	1000	400	800	1000
16	774	520	1000	400	800	1000
17	905	300	1000	489	800	1000
18	966	0	1000	300	800	1000
19	600	0	1000	0	610	1000
20	400	0	901	0	350	925
21	0	0	900	0	350	900
22	0	0	900	0	0	598
23	0	0	684	300	0	300
24	0	0	400	300	0	798
Total	**5640**	**4723**	**14997**	**2578**	**7511**	**19125**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	235147	4351470
Natural Gas 1	26532	1465320
Hydro 1	0	0
Coal 2	55682	2156820
Natural Gas 2	69851	2298510
Hydro 2	0	0
Total	**387212**	**10272120**

Regular Costs

Type	Cost [€]
Fuel	540587
Maintenance	65134
Ramping	72198
Startup	225500
Shutdown	96000
Total	**999420**

Results

Objective	1386631
Best Bound	833415
Absolute MIP Gap	553216
Relative MIP Gap	0.3990
CPU Time [sec]	26854
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.10: Simulation 10

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Multi Core)	MIQP	GS2	DS2	200	10158 (7534)	2826 (1700)	582 (510)	144 (144)	6606 (5180)	13398 (8204)	4008 (3440)	9390 (4764)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	400	0	580	0	0	518
2	400	0	611	0	0	487
3	400	0	591	0	0	400
4	400	0	200	0	0	331
5	400	0	200	300	0	0
6	0	0	200	300	0	356
7	0	300	200	300	0	400
8	0	300	200	300	0	578
9	0	300	455	300	0	800
10	0	0	800	0	0	912
11	0	0	849	0	350	834
12	0	0	990	0	350	800
13	400	303	1000	0	400	1000
14	563	600	1000	353	764	1000
15	694	600	1000	400	800	1000
16	694	600	1000	400		1000
17	664	600	1000	430	800	1000
18	518	584	1000	300	664	1000
19	400	410	1000	0	400	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	200	0	350	951
23	0	0	200	0	350	951
24	0	0	0	498	0	1000
Total	**5933**	**5197**	**15035**	**3881**	**6378**	**18317**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	285587	4855870
Natural Gas 1	36373	1563730
Hydro 1	0	0
Coal 2	167862	3278620
Natural Gas 2	35225	1952250
Hydro 2	0	0
Total	**525046**	**11650470**

Regular Costs

Type	Cost [€]
Fuel	559513
Maintenance	67142
Ramping	36146
Startup	219000
Shutdown	125000
Total	**1006801**

Results

Objective	1531848
Best Bound	863956
Absolute MIP Gap	667892
Relative MIP Gap	0.4360
CPU Time [sec]	26785
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.11: Simulation 11

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MIQP	GS1	DS1	100	6327 (4563)	2037 (1278)	291 (250)	72 (72)	3927 (2963)	7347 (4388)	2004 (1678)	5343 (2710)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	515
5	0	0	427
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	764
12	0	300	770
13	400	300	852
14	544	596	1000
15	647	600	1000
16	647	600	1000
17	691	556	1000
18	733	300	1000
19	605	0	1000
20	400	0	884
21	0	300	770
22	0	300	668
23	0	0	661
24	0	0	749
Total	**4666**	**4153**	**18472**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	162271	3622710
Natural Gas 1	5959	1259590
Hydro 1	0	0
Total	**168230**	**4882300**

Regular Costs

Type	Cost [€]
Fuel	221756
Maintenance	38603
Ramping	18826
Startup	109000
Shutdown	37000
Total	**425184**

Results

Objective	593414
Best Bound	514783
Absolute MIP Gap	78631
Relative MIP Gap	0.1325
CPU Time [sec]	3589
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.12: Simulation 12

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MIQP	GS1	DS1	200	5079 (3468)	1413 (726)	291 (250)	72 (72)	3303 (2420)	6723 (3760)	2004 (1630)	4719 (2130)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	516
5	0	0	427
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	780
12	0	300	800
13	400	300	852
14	540	600	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	508	525	1000
19	400	300	943
20	0	300	984
21	0	300	817
22	0	0	749
23	0	0	701
24	0	0	749
Total	**3789**	**4425**	**18727**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	100982	3009820
Natural Gas 1	22558	1425580
Hydro 1	0	0
Total	**123540**	**4435400**

Regular Costs

Type	Cost [€]
Fuel	221504
Maintenance	38967
Ramping	15318
Startup	104000
Shutdown	31000
Total	**410789**

Results

Objective	534330
Best Bound	534305
Absolute MIP Gap	26
Relative MIP Gap	$5 \cdot 10^{-5}$
CPU Time [sec]	30
Real Time [sec]	31

Simulation Specific Parameters

Parameter	Value
δ [$€/kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.13: Simulation 13

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MIQP	GS2	DS2	100	12942 (9608)	4218 (2785)	582 (510)	144 (144)	7998 (6169)	14790 (9290)	4008 (3432)	10782 (5858)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	700	0	0	798
2	0	0	717	0	0	781
3	0	0	500	300	0	591
4	0	0	200	300	350	0
5	0	0	200	300	350	308
6	0	0	200	300	0	356
7	0	0	449	300	0	0
8	0	0	200	300	0	784
9	0	0	600	300	0	900
10	0	412	1000	0	0	300
11	0	300	1000	0	0	733
12	0	300	951	0	0	900
13	0	403	1000	300	400	1000
14	500	600	1000	400	780	1000
15	500	600	1000	594	800	1000
16	500	600	1000	594	800	1000
17	500	600	1000	594	800	1000
18	400	559	1000	400	707	1000
19	0	400	1000	300	510	1000
20	0	300	721	300	350	975
21	0	300	540	300	0	1000
22	0	300	200	0	0	998
23	0	300	200	0	0	973
24	0	300	200	0	0	998
Total	**2400**	**6274**	**15578**	**5282**	**5847**	**18396**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	0	0
Natural Gas 1	69129	1891290
Hydro 1	0	0
Coal 2	333339	4933390
Natural Gas 2	15611	1756110
Hydro 2	0	0
Total	**418078**	**8580790**

Regular Costs

Type	Cost [€]
Fuel	540260
Maintenance	69723
Ramping	48352
Startup	198000
Shutdown	87000
Total	**943336**

Results

Objective	1361414
Best Bound	841384
Absolute MIP Gap	520030
Relative MIP Gap	0.3820
CPU Time [sec]	3588
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.14: Simulation 14

Simulation Setup

Solver	Problem	Generator Set	Demand	Step Size	Variables	Binary Variables	Integer Variables	Nonlinear Variables	Linear Variables	Constraints	Equality Constraints	Inequality Constraints
CPLEX (Single Core)	MIQP	GS2	DS2	200	10158 (7534)	2826 (1700)	582 (510)	144 (144)	6606 (5180)	13398 (8204)	4008 (3440)	9390 (4764)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	300	800	0	0	398
2	0	300	800	0	0	423
3	400	300	200	0	0	491
4	400	0	200	0	0	376
5	400	0	349	0	0	0
6	400	0	233	0	0	331
7	400	300	0	0	350	400
8	0	300	200	0	350	578
9	0	300	800	0	0	800
10	0	0	757	0	350	812
11	0	0	810	0	350	873
12	0	300	899	0	0	941
13	400	303	1000	0	400	1000
14	563	600	1000	353	764	1000
15	694	600	1000	400	800	1000
16	694	600	1000	400	800	1000
17	694	600	1000	400	800	1000
18	400	600	1000	339	727	1000
19	0	510	1000	300	400	1000
20	0	300	918	0	350	1000
21	0	0	790	0	350	1000
22	0	0	450	0	350	917
23	0	0	200	0	350	883
24	400	0	200	0	0	898
Total	**5845**	**6213**	**15606**	**2192**	**7491**	**18120**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	280967	4809670
Natural Gas 1	68593	1885930
Hydro 1	0	0
Coal 2	23262	1832620
Natural Gas 2	72361	2323610
Hydro 2	0	0
Total	**445182**	**10851830**

Regular Costs

Type	Cost [€]
Fuel	585879
Maintenance	66696
Ramping	41106
Startup	230000
Shutdown	127000
Total	**1050681**

Results

Objective	1495863
Best Bound	867750
Absolute MIP Gap	628113
Relative MIP Gap	0.4199
CPU Time [sec]	3588
Real Time [sec]	Interrupted after 3600 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.15: Simulation 15

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS1	DS1	483 (375)	78 (56)	288 (223)	117 (96)	750 (573)	216 (175)	534 (398)	267 (202)	483 (371)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	516
5	0	0	428
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	764
12	0	300	770
13	400	300	852
14	554	586	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	512	521	1000
19	400	300	943
20	0	300	984
21	0	300	817
22	0	0	749
23	0	0	701
24	0	0	749
Total	**3807**	**4407**	**18683**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	101459	3014590
Natural Gas 1	21785	1417850
Hydro 1	0	0
Total	**123244**	**4432440**

Regular Costs

Type	Cost [€]
Fuel	220858
Maintenance	38893
Ramping	15218
Startup	104000
Shutdown	31000
Total	**409968**

Results

Objective	533211
CPU Time [sec]	29
Real Time [sec]	35

Simulation Specific Parameters

Parameter	Value
δ [€/kg CO_2]	0.1
κ	1.10
ξ	1.07

Table D.16: Simulation 16

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	715
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	491	600	1000	389	800	1000
15	594	600	1000	500	800	1000
16	594	600	1000	500	800	1000
17	586	600	1000	508	800	1000
18	400	547	1000	319	800	1000
19	0	300	1000	300	610	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2665**	**4747**	**17452**	**2816**	**5463**	**21455**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	24451	2122250
Natural Gas 1	45989	1429940
Hydro 1	0	0
Coal 2	137324	2286620
Natural Gas 2	0	1599550
Hydro 2	0	0
Total	**207764**	**7438360**

Regular Costs

Type	Cost [€]
Fuel	418476
Maintenance	67149
Ramping	29820
Startup	215000
Shutdown	59000
Total	**789445**

Results

Objective	997207
CPU Time [sec]	541
Real Time [sec]	550

Simulation Specific Parameters

Parameter	Value
δ [€/kg CO_2]	0.2
κ	1.10
ξ	1.07

Table D.17: Simulation 17

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	505	559	1000	417	800	1000
15	584	600	1000	510	800	1000
16	580	600	1000	514	800	1000
17	567	600	1000	527	800	1000
18	400	492	1000	374	800	1000
19	0	300	1000	300	610	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2636**	**4650**	**17453**	**2942**	**5463**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	10663	2106630
Natural Gas 1	20130	1401300
Hydro 1	0	0
Coal 2	75517	2355170
Natural Gas 2	0	1599550
Hydro 2	0	0
Total	**106310**	**7462650**

Regular Costs

Type	Cost [€]
Fuel	417165
Maintenance	67225
Ramping	27495
Startup	215000
Shutdown	59000
Total	**785886**

Results

Objective	892196
CPU Time [sec]	1313
Real Time [sec]	1320

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.18: Simulation 18

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	(1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	506	553	1000	420	800	1000
15	583	600	1000	511	800	1000
16	578	600	1000	516	800	1000
17	564	600	1000	530	800	1000
18	400	486	1000	380	800	1000
19	0	300	1000	300	610	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2632**	**4639**	**17453**	**2957**	**5463**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	9404	2104490
Natural Gas 1	17832	1398130
Hydro 1	0	0
Coal 2	68717	2363520
Natural Gas 2	0	1599550
Hydro 2	0	0
Total	**95952**	**7465690**

Regular Costs

Type	Cost [€]
Fuel	417029
Maintenance	67235
Ramping	27334
Startup	215000
Shutdown	59000
Total	**785598**

Results

Objective	881550
CPU Time [sec]	1461
Real Time [sec]	1470

Simulation Specific Parameters

Parameter	Value
δ [€/ $kg\ CO_2$]	0.09
κ	1.10
ξ	1.07

Table D.19: Simulation 19

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	508	548	1000	424	800	1000
15	582	600	1000	512	800	1000
16	576	599	1000	518	800	1000
17	562	600	1000	532	800	1000
18	400	480	1000	386	800	1000
19	0	301	1000	300	609	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2628**	**4628**	**17453**	**2973**	**5462**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	8192	2102400
Natural Gas 1	15595	1394930
Hydro 1	0	0
Coal 2	61783	2372290
Natural Gas 2	0	1599290
Hydro 2	0	0
Total	**85570**	**7468910**

Regular Costs

Type	Cost [€]
Fuel	416879
Maintenance	67246
Ramping	27178
Startup	215000
Shutdown	59000
Total	**785303**

Results

Objective	870873
CPU Time [sec]	1514
Real Time [sec]	1520

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.08
κ	1.10
ξ	1.07

Table D.20: Simulation 20

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	256 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	510	542	1000	428	800	1000
15	580	600	1000	514	800	1000
16	577	595	1000	523	800	1000
17	560	600	1000	534	800	1000
18	400	474	1000	428	800	1000
19	0	308	1000	300	602	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2626**	**4619**	**17453**	**2991**	**5455**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	7107	2101520
Natural Gas 1	13423	1391760
Hydro 1	0	0
Coal 2	54726	2381800
Natural Gas 2	0	1597660
Hydro 2	0	0
Total	**75256**	**7472740**

Regular Costs

Type	Cost [€]
Fuel	416652
Maintenance	67260
Ramping	26980
Startup	215000
Shutdown	59000
Total	**784892**

Results

Objective	860149
CPU Time [sec]	1611
Real Time [sec]	1650

Simulation Specific Parameters

Parameter	Value
δ [€/ kg CO_2]	0.07
κ	1.10
ξ	1.07

Table D.21: Simulation 21

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	512	536	1000	432	800	1000
15	578	600	1000	516	800	1000
16	576	590	1000	528	800	1000
17	556	600	1000	538	800	1000
18	400	477	1000	402	787	1000
19	0	319	1000	300	591	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2621**	**4622**	**17453**	**3017**	**5431**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	5927	2098780
Natural Gas 1	11494	1391570
Hydro 1	0	0
Coal 2	47773	2396220
Natural Gas 2	0	1590490
Hydro 2	0	0
Total	**65194**	**7477060**

Regular Costs

Type	Cost [€]
Fuel	416265
Maintenance	67294
Ramping	26593
Startup	215000
Shutdown	59000
Total	**784153**

Results

Objective	849347
CPU Time [sec]	1757
Real Time [sec]	1800

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.06
κ	1.10
ξ	1.07

Table D.22: Simulation 22

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	514	529	1000	437	800	1000
15	577	599	1000	519	800	1000
16	575	585	1000	535	800	1000
17	551	600	1000	543	800	1000
18	400	482	1000	414	770	1000
19	0	332	1000	300	578	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2616**	**4627**	**17453**	**3047**	**5401**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	4795	2095910
Natural Gas 1	9576	1391520
Hydro 1	0	0
Coal 2	40642	2412840
Natural Gas 2	0	1582030
Hydro 2	0	0
Total	**55014**	**7482300**

Regular Costs

Type	Cost [€]
Fuel	415823
Maintenance	67334
Ramping	26244
Startup	215000
Shutdown	59000
Total	**783401**

Results

Objective	838415
CPU Time [sec]	1812
Real Time [sec]	1815

Simulation Specific Parameters

Parameter	Value
δ [€/ $kg\ CO_2$]	0.05
κ	1.10
ξ	1.07

Table D.23: Simulation 23

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	520	523	1000	448	789	1000
15	580	586	1000	528	800	1000
16	576	575	1000	543	800	1000
17	547	600	1000	547	800	1000
18	400	488	1000	426	753	1000
19	0	344	1000	300	566	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2623**	**4616**	**17453**	**3091**	**5361**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	3990	2099750
Natural Gas 1	7459	1386480
Hydro 1	0	0
Coal 2	33490	2437240
Natural Gas 2	0	1570100
Hydro 2	0	0
Total	**44939**	**7493570**

Regular Costs

Type	Cost [€]
Fuel	414965
Maintenance	67383
Ramping	26017
Startup	215000
Shutdown	59000
Total	**782365**

Results

Objective	827304
CPU Time [sec]	1711
Real Time [sec]	1720

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.04
κ	1.10
ξ	1.07

Table D.24: Simulation 24

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	317	1000	350	436	1000
14	535	536	1000	503	706	1000
15	587	575	1000	564	769	1000
16	579	571	1000	572	772	1000
17	537	600	1000	559	798	1000
18	400	522	1000	451	693	1000
19	0	394	1000	300	516	1000
20	0	300	918	0	350	1000
21	0	0	807	0	350	983
22	0	0	639	0	0	909
23	0	0	566	0	0	886
24	0	0	586	0	0	912
Total	**2638**	**4415**	**17385**	**3297**	**5391**	**21423**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	3238	2107950
Natural Gas 1	3362	1312050
Hydro 1	0	0
Coal 2	28570	2552350
Natural Gas 2	0	1600000
Hydro 2	0	0
Total	**35170**	**7572350**

Regular Costs

Type	Cost [€]
Fuel	413952
Maintenance	67398
Ramping	25284
Startup	215000
Shutdown	59000
Total	**780634**

Results

Objective	815804
CPU Time [sec]	2434
Real Time [sec]	2440

Simulation Specific Parameters

Parameter	Value
δ [€/ kg CO_2]	0.03
κ	1.10
ξ	1.07

Table D.25: Simulation 25

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	0	769	300	0	964
12	0	0	864	300	0	976
13	0	0	1000	447	656	1000
14	494	500	1000	527	759	1000
15	547	577	1000	570	800	1000
16	550	581	1000	573	790	1000
17	528	600	1000	565	800	1000
18	400	526	1000	447	694	1000
19	0	396	1000	300	514	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2519**	**3779**	**17453**	**4029**	**5363**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	875	2043760
Natural Gas 1	0	1109810
Hydro 1	0	0
Coal 2	31331	3166560
Natural Gas 2	0	1564520
Hydro 2	0	0
Total	**32206**	**7884650**

Regular Costs

Type	Cost [€]
Fuel	405999
Maintenance	67902
Ramping	24625
Startup	212000
Shutdown	59000
Total	**769527**

Results

Objective	801733
CPU Time [sec]	2741
Approx. Absolute Time [sec]	2800

Simulation Specific Parameters

Parameter	Value
δ [€/ $kg\ CO_2$]	0.02
κ	1.10
ξ	1.07

Table D.26: Simulation 26

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	966 (778)	(156 (122)	576 (479)	234 (185)	1452 (1184)	432 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	696
5	0	0	300	0	0	655
6	0	0	274	0	0	661
7	0	0	330	0	0	714
8	0	0	469	0	0	815
9	0	0	663	0	0	942
10	0	0	738	0	0	974
11	400	0	727	0	0	933
12	427	0	814	0	0	942
13	557	0	1000	0	546	1000
14	664	484	1000	434	698	1000
15	713	543	1000	493	745	1000
16	712	549	1000	488	745	1000
17	685	586	1000	448	774	1000
18	574	516	1000	300	676	1000
19	400	349	1000	0	461	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**5132**	**3628**	**17367**	**2162**	**4995**	**21384**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	20105	4010540
Natural Gas 1	0	1068100
Hydro 1	0	0
Coal 2	1142	1714170
Natural Gas 2	0	1462580
Hydro 2	0	0
Total	**21247**	**8255390**

Regular Costs

Type	Cost [€]
Fuel	405840
Maintenance	66079
Ramping	26378
Startup	210500
Shutdown	59000
Total	**767798**

Results

Objective	789045
CPU Time [sec]	1242
Real Time [sec]	1250

Simulation Specific Parameters

Parameter	Value
δ [€/ $kg\ CO_2$]	0.01
κ	1.10
ξ	1.07

Table D.27: Simulation 27

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2	DS2	954 (770)	150 (115)	582 (480)	222 (175)	1446 (1180)	432 (368)	1014 (812)	528 (409)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	655
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	469	0	0	815
9	0	0	663	0	0	942
10	0	0	737	0	0	975
11	0	0	770	300	0	963
12	0	0	835	359	0	957
13	585	0	1000	518	0	1000
14	717	423	1000	619	521	1000
15	790	463	1000	659	582	1000
16	813	461	1000	647	572	1000
17	815	497	1000	614	568	1000
18	745	430	1000	491	400	1000
19	621	300	989	300	0	1000
20	568	0	1000	0	0	1000
21	451	0	783	0	0	905
22	400	0	561	0	0	790
23	400	0	432	0	0	725
24	400	0	391	0	0	707
Total	**7306**	**2574**	**16995**	**4508**	**2642**	**20841**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	0	5624600
Natural Gas 1	0	755553
Hydro 1	0	0
Coal 2	0	3447450
Natural Gas 2	0	787523
Hydro 2	0	0
Total	**0**	**10615126**

Regular Costs

Type	Cost [€]
Fuel	399958
Maintenance	68499
Ramping	30116
Startup	209500
Shutdown	34000
Total	**742073**

Results

Objective	742073
CPU Time [sec]	8786
Real Time [sec]	8800

Simulation Specific Parameters

Parameter	Value
δ [€$/kg\ CO_2$]	0
κ	1.10
ξ	1.07

Table D.28: Simulation 28

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2[a]	DS2	954 (770)	(150 (115)	486 (406)	318 (249)	1446 (1184)	432 (368)	1014 (812)	528 (409)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	344	1000	300	459	1000
14	580	600	1000	300	800	1000
15	608	600	1000	486	800	1000
16	608	600	1000	486	800	1000
17	608	600	1000	486	800	1000
18	400	585	1000	300	781	1000
19	0	390	1000	300	520	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2803**	**4920**	**17453**	**2659**	**5309**	**21454**

Emission Penalty

Unit	Emission [kg CO_2]
Coal 1	2197460
Natural Gas 1	1470780
Hydro 1	0
Coal 2	2199770
Natural Gas 2	1562460
Hydro 2	0
Total	**7430470**

Regular Costs

Type	Cost [€]
Fuel	418525
Maintenance	67115
Ramping	32247
Startup	215000
Shutdown	59000
Total	**791888**

Results

Objective	7518170
CPU Time [sec]	749
Real Time [sec]	760

Simulation Specific Parameters

Parameter	Value
κ	1.10
ξ	1.07

Table D.29: Simulation 29 - Minimizing emissions

[a] Modified as described in section 4.2

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2[a]	DS2	954 (774)	150 (115)	582 (480)	222 (175)	1452 (1184)	432 (368)	1020 816	528 (409)	924 (775)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	336	1000	300	467	1000
14	489	600	1000	391	800	1000
15	592	600	1000	502	800	1000
16	596	600	1000	498	800	1000
17	597	600	1000	497	800	1000
18	437	578	1000	300	752	1000
19	400	346	1000	0	464	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**3110**	**4860**	**17453**	**2489**	**5232**	**21454**

Emission Penalty

Unit	Emission [kg CO_2]
Coal 1	2500000
Natural Gas 1	1456930
Hydro 1	0
Coal 2	2000000
Natural Gas 2	1542660
Hydro 2	0
Total	**7499590**

Regular Costs

Type	Cost [€]
Fuel	418044
Maintenance	66935
Ramping	28687
Startup	215000
Shutdown	59000
Total	**787666**

Results

Objective	787665
CPU Time [sec]	1241
Real Time [sec]	1245

Simulation Specific Parameters

Parameter	Value
κ	1.10
ξ	1.07

Table D.30: Simulation 30 - Strict unit emission constraints

[a] Modified as described in section 4.2

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2[a]	DS2	954 (770)	150 (115)	582 (480)	222 (175)	1447 1181	432 (368)	1015 (8131)	528 (409)	919 (772)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	663	0	0	942
10	0	0	737	0	0	975
11	0	0	769	300	0	964
12	0	0	853	317	0	969
13	0	0	1000	490	613	1000
14	520	469	1000	584	706	1000
15	583	537	1000	630	745	1000
16	581	545	1000	626	742	1000
17	540	582	1000	601	771	1000
18	400	513	1000	479	674	1000
19	0	397	1000	300	513	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2624**	**3643**	**17443**	**4328**	**5114**	**21447**

Emission Penalty

Unit	Emission [kg CO_2]
Coal 1	2100290
Natural Gas 1	1069540
Hydro 1	0
Coal 2	3340590
Natural Gas 2	1489590
Hydro 2	0
Total	**8000010**

Regular Costs

Type	Cost [€]
Fuel	401176
Maintenance	68190
Ramping	26085
Startup	212000
Shutdown	59000
Total	**766451**

Results

Objective	766451
CPU Time [sec]	5865
Real Time [sec]	5900

Simulation Specific Parameters

Parameter	Value
κ	1.10
ξ	1.07

Table D.31: Simulation 31 - Strict total emission constraint

[a] Modified as described in section 4.2

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS2[a]	DS2	954 (770)	150 (115)	582 (480)	222 (175)	1446 (1180)	432 (368)	1014 (812)	528 (409)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	655
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	469	0	0	815
9	0	0	663	0	0	942
10	0	0	737	0	0	975
11	0	0	770	300	0	963
12	0	0	835	357	0	957
13	586	0	1000	517	0	1000
14	715	425	1000	617	523	1000
15	786	466	1000	655	586	1000
16	809	465	1000	644	576	1000
17	811	501	1000	611	571	1000
18	742	434	1000	490	400	1000
19	619	300	991	300	0	1000
20	568	0	1000	0	0	1000
21	450	0	784	0	0	906
22	400	0	562	0	0	791
23	400	0	432	0	0	725
24	400	0	391	0	0	707
Total	**7286**	**2590**	**16999**	**4491**	**2656**	**20842**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	5612	5612380
Natural Gas 1	760	759595
Hydro 1	0	0
Coal 2	3437	3437180
Natural Gas 2	791	790809
Hydro 2	0	0
Total	**10600**	**10599964**

Regular Costs

Type	Cost [€]
Fuel	400062
Maintenance	68488
Ramping	30030
Startup	209500
Shutdown	34000
Total	**742080**

Results

Objective	752680
CPU Time [sec]	7726
Real Time [sec]	7800

Simulation Specific Parameters

Parameter	Value
δ [$€/kg\ CO_2$]	0.001
κ	1.10
ξ	1.07

Table D.32: Simulation 32 - Cost function

[a]Modified as described in section 4.2

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
BONMIN B-BB	MINLP	GS3	DS3	1449 (1204)	225 (184)	864 (756)	351 (264)	2154 (1823)	648 (575)	1506 (1248)	801 (632)	1353 (1191)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]	Coal 3 [MW]	Natural Gas 3 [MW]	Natural Gas 4 [MW]
1	0	0	894	0	0	847	0	0	250
2	0	0	853	0	0	887	0	0	250
3	0	0	739	0	0	862	0	0	250
4	0	0	563	0	0	779	0	0	0
5	0	0	479	0	0	745	0	0	0
6	0	0	485	0	0	760	0	0	0
7	0	0	580	0	0	824	0	0	0
8	0	0	770	0	0	937	0	0	0
9	0	0	864	0	0	970	0	300	0
10	0	0	979	0	0	1000	0	300	**0**
11	0	351	1000	0	0	1000	0	357	0
12	0	420	1000	0	0	1000	0	426	0
13	0	459	1000	0	545	1000	300	464	351
14	492	588	1000	413	761	1000	357	591	490
15	565	600	1000	510	800	1000	395	600	500
16	573	600	1000	544	800	1000	354	600	500
17	571	600	1000	600	800	1000	300	600	500
18	400	600	1000	504	800	1000	0	600	500
19	0	496	1000	300	615	1000	0	472	386
20	0	430	1000	0	400	1000	0	333	250
21	0	546	1000	0	0	1000	0	300	0
22	0	300	838	0	0	941	0	0	0
23	0	0	775	0	0	932	0	0	0
24	0	0	756	0	0	935	0	300	0
Total	**2601**	**5990**	**20575**	**2871**	**5521**	**22419**	**1706**	**6243**	**4227**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	8770	2087700
Natural Gas 1	54524	1745240
Hydro 1	0	0
Coal 2	61222	2212220
Natural Gas 2	831	1608310
Hydro 2	0	0
Coal 3	0	1400000
Natural Gas 3	65713	1857130
Natural Gas 4	25489	1254890
Total	**216549**	**12165490**

Regular Costs

Type	Cost [€]
Fuel	776847
Maintenance	88289
Ramping	34749
Startup	318500
Shutdown	82600
Total	**1300985**

Results

Objective	1517534
Lower Bound	1466413
CPU Time [sec]	898155
Real Time [sec]	Interrupted after 898200 sec.

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.33: Simulation 33

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
OFP	MINLP	GS1	DS1	(483) 375	78 (56)	288 (223)	117 (96)	750 (573)	216 (175)	534 (398)	267 (202)	483 (371)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]
1	0	0	749
2	0	0	749
3	0	0	696
4	0	0	516
5	0	0	428
6	0	0	428
7	0	0	490
8	0	0	642
9	0	0	803
10	0	0	856
11	0	300	764
12	0	300	770
13	400	300	852
14	554	586	1000
15	647	600	1000
16	647	600	1000
17	647	600	1000
18	512	521	1000
19	400	300	943
20	0	300	984
21	0	300	817
22	0	0	749
23	0	0	701
24	0	0	749
Total	**3807**	**4707**	**18683**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	101459	3014590
Natural Gas 1	21785	1417850
Hydro 1	0	0
Total	**123244**	**4432440**

Regular Costs

Type	Cost [€]
Fuel	220858
Maintenance	38893
Ramping	15218
Startup	104000
Shutdown	31000
Total	**409968**

Results

Objective	533212
CPU Time [sec]	26
Real Time [sec]	30

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.34: Simulation 34

Simulation Setup

Solver	Problem	Generator Set	Demand	Variables	Binary Variables	Nonlinear Variables	Linear Variables	Constraints	Nonlinear Constraints	Linear Constraints	Equality Constraints	Inequality Constraints
OFP	MINLP	GS2	DS2	966 (778)	156 (122)	576 (479)	234 (185)	1452 (1184)	423 (368)	1020 (816)	534 (413)	918 (771)

Generator Schedule

Time	Coal 1 [MW]	Natural Gas 1 [MW]	Hydro 1 [MW]	Coal 2 [MW]	Natural Gas 2 [MW]	Hydro 2 [MW]
1	0	0	746	0	0	752
2	0	0	703	0	0	795
3	0	0	606	0	0	785
4	0	0	409	0	0	697
5	0	0	300	0	0	656
6	0	0	274	0	0	661
7	0	0	329	0	0	714
8	0	0	468	0	0	816
9	0	0	662	0	0	943
10	0	0	737	0	0	975
11	0	300	769	0	0	964
12	0	300	864	0	0	976
13	0	300	1000	300	503	1000
14	505	559	1000	417	800	1000
15	584	600	1000	510	800	1000
16	580	600	1000	514	800	1000
17	567		1000	527	800	1000
18	400	492	1000	374	800	1000
19	0	300	1000	300	610	1000
20	0	300	918	0	350	1000
21	0	300	840	0	0	1000
22	0	0	661	0	0	920
23	0	0	578	0	0	890
24	0	0	588	0	0	910
Total	**2635,849**	**4650**	**17453**	**2942**	**5463**	**21454**

Emission Penalty

Unit	Penalty [€]	Emission [kg CO_2]
Coal 1	10663	2106630
Natural Gas 1	20130	1401300
Hydro 1	0	0
Coal 2	75517	2355170
Natural Gas 2	0	1599550
Hydro 2	0	0
Total	**106310**	**7462650**

Regular Costs

Type	Cost [€]
Fuel	417165
Maintenance	67225
Ramping	27495
Startup	215000
Shutdown	59000
Total	**785886**

Results

Objective	892196
CPU Time [sec]	1136
Real Time [sec]	1140

Simulation Specific Parameters

Parameter	Value
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.35: Simulation 35

Simulation Setup

Solver	**Problem**	**Generator Set**	**Demand**	**Variables**	**Binary Variables**	**Nonlinear Variables**	**Linear Variables**	**Constraints**	**Nonlinear Constraints**	**Linear Constraints**	**Equality Constraints**	**Inequality Constraints**
OFP	MINLP	GS3	DS3	1449 (1204)	225 (184)	864 (756)	351 (246)	2154 (1823)	648 (575)	1506 (1248)	801 (632)	1353 (1191)

Generator Schedule

Time	**Coal 1 [MW]**	**Natural Gas 1 [MW]**	**Hydro 1 [MW]**	**Coal 2 [MW]**	**Natural Gas 2 [MW]**	**Hydro 2 [MW]**	**Coal 3 [MW]**	**Natural Gas 3 [MW]**	**Natural Gas 4 [MW]**
1	0	0	894	0	0	847	0	0	250
2	0	0	853	0	0	887	0	0	250
3	0	0	739	0	0	862	0	0	250
4	0	0	563	0	0	779	0	0	0
5	0	0	479	0	0	745	0	0	0
6	0	0	484	0	0	760	0	0	0
7	0	0	580	0	0	824	0	0	0
8	0	0	770	0	0	937	0	0	0
9	0	0	864	0	0	970	0	300	0
10	0	0	979	0	0	1000	0	300	**0**
11	0	351	1000	0	0	1000	0	357	0
12	0	420	1000	0	0	1000	0	426	0
13	0	459	1000	0	545	1000	300	464	351
14	492	588	1000	413	761	1000	357	591	490
15	565	600	1000	510	800	1000	395	600	500
16	573	600	1000	544	800	1000	354	600	500
17	571	600	1000	599	800	1000	300	600	500
18	400	600	1000	504	800	1000	0	600	500
19	0	496	1000	300	615	1000	0	472	386
20	0	430	1000	0	400	1000	0	333	250
21	0	546	1000	0	0	1000	0	300	0
22	0	300	838	0	0	941	0	0	0
23	0	0	775	0	0	932	0	0	0
24	0	0	756	0	0	935	0	300	0
Total	**2601**	**5990**	**20574**	**2870**	**5521**	**22419**	**1706**	**6243**	**4227**

Emission Penalty

Unit	**Penalty [€]**	**Emission [kg CO_2]**
Coal 1	8770	2087700
Natural Gas 1	54524	1745240
Hydro 1	0	0
Coal 2	61222	2212220
Natural Gas 2	831	1608310
Hydro 2	0	0
Coal 3	0	1400000
Natural Gas 3	65713	1857130
Natural Gas 4	25489	1254890
Total	**216549**	**12165490**

Regular Costs

Type	**Cost [€]**
Fuel	776847
Maintenance	88289
Ramping	34749
Startup	318500
Shutdown	82600
Total	**1300985**

Results

Objective	1517534
Lower Bound	1466413
CPU Time [sec]	898085
Real Time [sec]	Interrupted after 898200 sec.

Simulation Specific Parameters

Parameter	**Value**
δ [€/$kg\ CO_2$]	0.1
κ	1.10
ξ	1.07

Table D.36: Simulation 36

Appendix E

Efficiency Characteristics

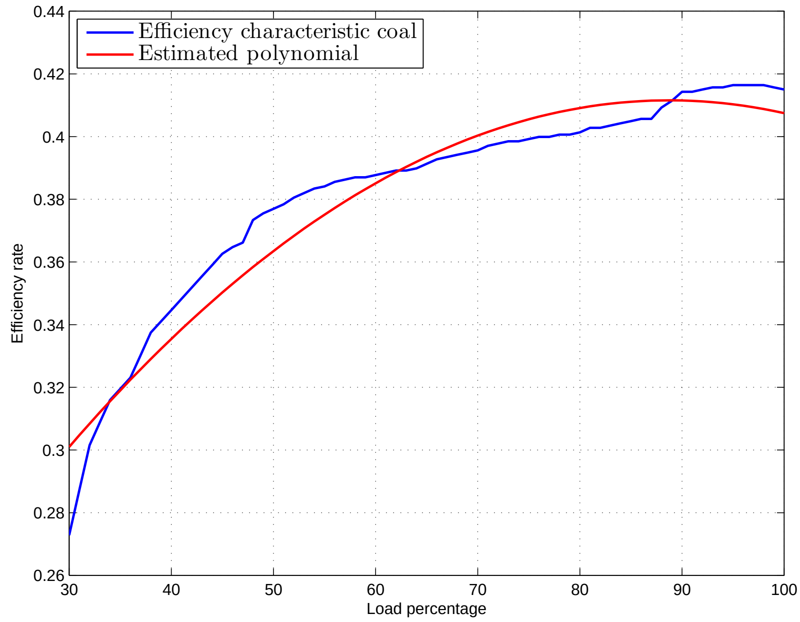


Figure E.1: Estimated efficiency characteristic for coal units.

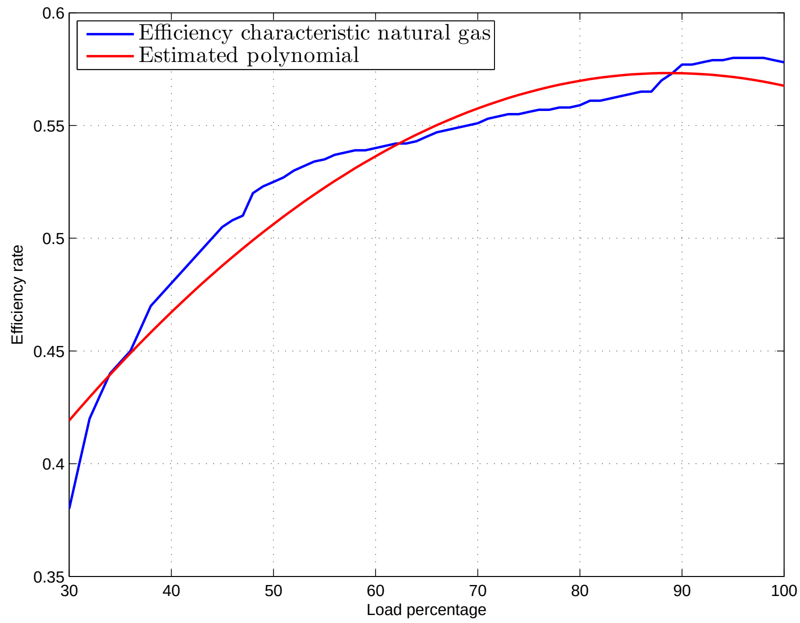


Figure E.2: Estimated efficiency characteristic for natural gas units.

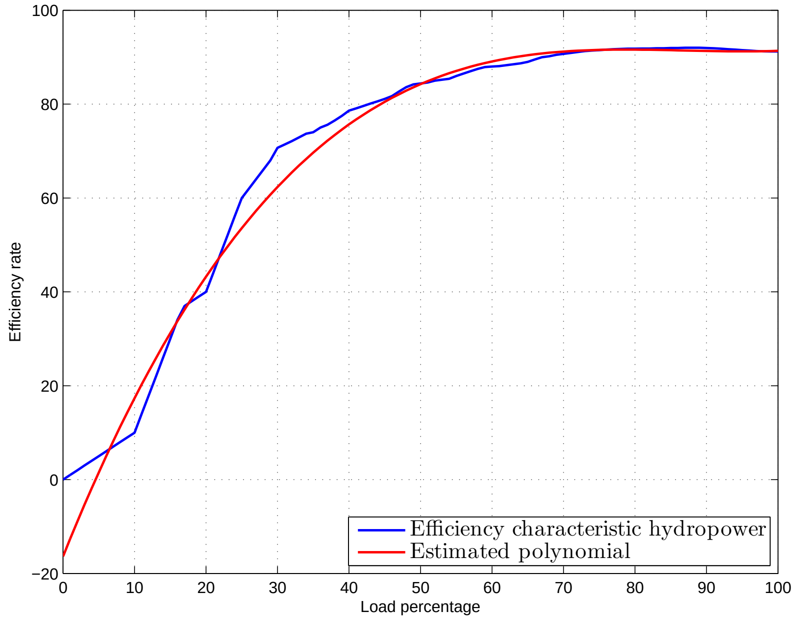


Figure E.3: Estimated efficiency characteristic for hydropower units.

Appendix F

MINLP Solvers

Please note that the data presented in this chapter is to the best of the author's knowledge. Empty fields do not necessarily mean that no such software exist, but rather that the author was not able to find reliable sources stating that it does exist.

Solver	AIMMS	AMPL	GAMS	NEOS	MATLAB	License
ALPHABB (α-Branch-and Bound)						not public
ALPHAECP (α-Extended Cutting Plane)			V	V		commercial GAMS
AOA (AIMMS Outer Approximation)	V					open AIMMS
BARON (Branch-And-Reduce Optimization Navigator)	V		V	V		commercial AIMMS & GAMS
BNB (Branch 'n Bound)					V	MATLAB source
BONMIN (Basic Open-source Nonlinear Mixed Integer Programming)		V	V	V	V[a]	open source
COUENNE (Convex Over and Under ENvelopes for Nonlinear Estimation)		V	V	V		open source
CPLEX	V	V	V		V[b]	proprietary IBM
DICOPT (Discrete and Continuous Optimizer)			V	V		commercial GAMS
FICO EPRESS-OPTIMIZER	V	V	V			proprietary FICO
FICO XPRESS-SLP[c]						proprietary FICO
FILMINT (Filter-Mixed Integer Optimizer)		V		V		
fminconset					V	MATLAB source
GLOMIQO			V			commercial GAMS
KNITRO	V	V	V	V	V	proprietary Ziena opt. LLC
LAGO (Lagrangian Global Optimizer)		V	V			open source
LINDOAPI[d]			V	V		commercial GAMS
MIDACO (Mixed Integer Distributed Ant Colony Optimization)[e]					V	available on request
MILANO (Mixed-Integer Linear and Nonlinear Optimizer)					V	
MINLP_BB (Mixed Integer Nonlinear Programming Branch-and-Bound)		V			V[f]	
MISQP (Mixed Integer Sequential Quadratic Programming)[g]						
MOSEK	V	V	V		V	commercial AIMMS & GAMS
OQNLP (OptQuest Nonlinear Programming)			V	V	V[h]	commercial GAMS
SBB (Simple Branch-and-Bound)			V	V		commercial GAMS
SCIP (Solving Constraint Integer Programs)			V	V		availavle for academic inst.

Table F.1: Overview of availability of MINLP solvers. Empty field means: No information found. *Sources: [21], [26], [59], [84], [85],[86]*

[a] Under development[26].
[b] Require TOMLAB extension package[84].
[c] Available as a FICO Xpress-Mosel module.
[d] Also available within LINDO, LINGO and What's Best!.
[e] Also available for C/C++ and Fortran.
[f] Require TOMLAB extension package.
[g] Available as standalone library with Fortran interface.
[h] Require TOMLAB extension package.

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