HydroCen
NORWEGIAN RESEARCH CENTRE
FOR HYDROPOWER TECHNOLOGY

CENTRE FOR
ENVIRONMENT-
FRIENDLY ENERGY
RESEARCH

HydroCen Report nr.12

Environmental Constraints in Seasonal Hydropower Scheduling

Survey and Feasibility

NTNU SINTEF NINA

HydroCen

The main objective of HydroCen (Norwegian Research Centre for Hydropower Technology) is to enable the Norwegian hydropower sector to meet complex challenges and exploit new opportunities through innovative technological solutions.

The research areas include:

- Hydropower structures
- Turbine and generators
- Market and services
- Environmental design

The Norwegian University of Science and Technology (NTNU) is the host institution and is the main research partner together with SINTEF Energy Research and the Norwegian Institute for Nature Research (NINA).

HydroCen has about 50 national and international partners from industry, R&D institutes and universities.

HydroCen is a Centre for Environment-friendly Energy Research (FME). The FME scheme is established by the Norwegian Research Council.

The objective of the Research Council of Norway FME-scheme is to establish time-limited research centres, which conduct concentrated, focused and long-term research of high international calibre in order to solve specific challenges in the field.

The FME-centres can be established for a maximum period of eight years HydroCen was established in 2016.

Environmental Constraints in Seasonal Hydropower Scheduling

Survey and Feasibility

Arild Helseth, SINTEF Energi AS

12

HydroCen

Environmental Constraints in Seasonal Hydropower Scheduling– HydroCen Report nr.12

Trondheim, Mai 2019]

ISSN:
ISBN: 978-82-93602-13-2

AUTHOR/S
Arild Helseth, SINTEF Energi AS

COVER PHOTO
Pixabay.com

KEY WORDS
Hydro Power Scheduling
Modelling
SDDP

CONTACT DETAILS

HydroCen www.hydrocen.no
Vannkraftlaboratoriet,
NTNU
Alfred Getz vei 4
Gløshaugen,
Trondheim

Abstract

Environmental Constraints in Seasonal Hydropower Scheduling– HydroCen Report nr.12

The work documented in this report has been carried out in project WP3.4 in the HydroCen research center. We have surveyed the need for improved modelling of environmental and technical constraints related to operational hydropower scheduling software. The results have been processed and summarized, and we have selected the two major types of constraints that seems the most relevant for this project. The two constraint types are described and their possible inclusions in the stochastic dual dynamic programming (SDDP) algorithm are assessed

Arild Helseth, Sem Sælands vei 11, 7034 Trondheim, arild.helseth@sintef.no

Innhold

1 Introduction

1.1 Project Background

In the future, the Nordic power system will have tighter connections with Europe and an increasing proportion of intermittent renewable generation from, for example, wind, solar, and unregulated hydroelectric systems. Rapid and unpredictable fluctuations in intermittent generation will offer new possibilities for controllable generation to be able to respond to these fluctuations. Flexible and fast-responding power plants able to produce at demand peaks will therefore see a higher profit potential. To capture the impact on rapid fluctuations in generation and power prices when computing the expected marginal value of water (water values), long- and medium-term hydropower scheduling tools are run using finer time resolution and with a more detailed representation of the physical system.

In symphony with the ongoing power market changes, the physical and environmental requirements associated with hydropower operation are changing, e.g. through proposed revisions of hydropower concessions conditions and the implementation of EU Water Framework Directive. Consequently, hydropower producers need to both adjust their operational schedules according to the new price patterns seen in the market and at the same time relate to new operational constraints. In this context, the producers need operational planning (or scheduling) tools (or models) that relate to physical and environmental constraints in a precise and consistent manner.

Operational scheduling models have been widely used by Nordic hydropower producers for several decades. Although these have been developed along different methodological tracks, the stochastic dual dynamic programming (SDDP) method is currently the state-of-the-art method for medium-term hydropower scheduling. The SDDP method allows for optimization of hydropower schedules provided a detailed and complex system description and uncertainties in e.g. inflow and power price. SINTEF Energy Research maintains and develop the software tool ProdRisk which is based on the SDDP method, and have lately conducted multiple research activities involving SDDP and ProdRisk prototypes, see e.g. [1], [2], [3] and [4].

Methodological research within WP3.4 primarily relates to the SDDP method, and ProdRisk prototypes will be developed to test methodological advances in realistic case studies.

Many of the environmental constraints that are relevant to study involve state dependencies which are not easily treated in the SDDP method. The main complicating factor is the nonconvexities associated with such constraints, since the SDDP method requires a convex model formulation. Through this project we aim at
approximating nonconvexities within the SDDP method for more accurate modelling of environmental constraints.

1.2 Survey and Feasibility Study

The work documented in this report has been carried out in project WP3.4 in the HydroCen research center. We have surveyed the need for improved modelling of environmental and technical constraints related to operational hydropower scheduling software. The results have been processed and summarized, and what we find as the most relevant types of constraints for this project are presented in Section 3. In Section 4 we evaluate how these constraints can be included in the SDDP algorithm.

1 Background

1.1 Environmental Constraints

In this context we use the term 'environmental constraints' to broadly cover limits or boundaries that we put on the reservoir volumes and release plans. Obviously, such boundaries are necessary in order to the describe the physics of the watercourse. Environmental constraints can be seen as additional boundaries, either internally defined or enforced as a part of the concession, to further bound reservoir volumes and releases. Environmental constraints typically serve to improve conditions for the nature, fishing and recreational purposes. Note that this report concerns the treatment of environmental constraints in hydropower scheduling models, and that we therefore will not go into detail of the origins of such constraints.

1.2 Basic Modelling Features

In the following we describe the basic modelling capabilities represented in SINTEF's medium-term (or "seasonal") scheduling models. These models typically takes the viewpoint of a hydropower producer trying to maximize the expected profit from operation of a watercourse over a certain time period, subject to all relevant physical and legislative constraints.

A hydropower watercourse is built by connecting a set of *hydropower modules*. The basic variables and parameters being modelled in a hydropower module are illustrated in Figure 1. The presentation is valid for the model ProdRisk, but many of the modelling capabilities are available in SINTEF's other long- and medium-term scheduling models.

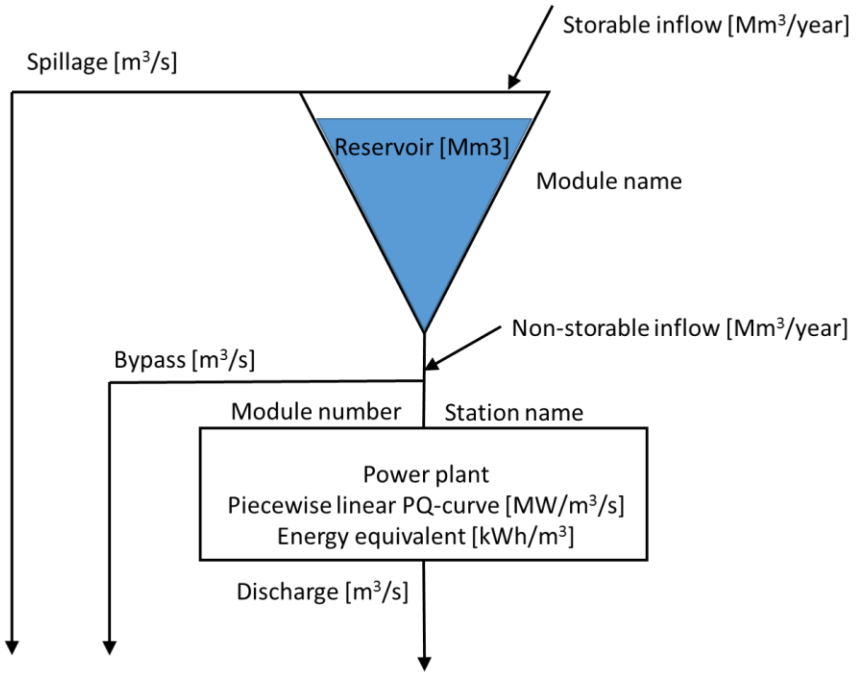

Figure 1 The hydropower module with associated variables and parameters.

As illustrated in Figure 1, there are three waterways leading water out of the reservoir; spillage, bypass and discharge. We will sometime use the term release when referring to the sum of discharge and bypass. Spilling water is normally presented to the model as the last resort: It should not be used unless there are no other options. Both the bypass and discharge waterways often lead to a downstream river and have bounds on minimum, maximum and even the ramping[1].

SINTEFs medium-term models allow all or some of the following constraints and variable bounds:

- A power balance enabling the sales of produced energy to the market
- A reservoir balance for each hydropower module
- Head-dependent power generation is represented through head coefficients in an approximate and iterative scheme, described in [5]
- Time-dependent variable bounds on all variables
- All constraints have slack provided by auxiliary variables to ensure that a solution is always found
- A segmented discharge variable allowing generation at successively lower efficiencies (a concave PQ-curve)
- Flow between hydraulically coupled reservoirs, but without consideration of pressure conditions

The reservoir volume (v) is defined as a variable constrained by time-dependent upper and lower boundaries:

$$\underline{V} \leq v \leq \overline{V} \tag{0.1}$$

One can further constrain the reservoir operation by adding ramping constraints in (1.2). The change in reservoir volume from one time step to the next is controlled by the ramping rates for downward (Δ^-) and upward ramping (Δ^+).

$$\Delta^- \leq v_t - v_{t-1} \leq \Delta^+ \tag{0.2}$$

The variable boundaries on discharge (q^D) and bypass (q^B) can be formulated as:

$$\begin{aligned} \underline{Q}^D &\leq q^D \leq \overline{Q}^D \\ \underline{Q}^B &\leq q^B \leq \overline{Q}^B \end{aligned} \tag{0.3}$$

In the current version of the model ProdRisk it is not possible to define a ramping limit on discharge and bypass, but the rate of change for these variables can to some extent be controlled by newly added functionality:

- Ramping constraints on reservoir level as described in (0.2)
- Start-up costs on hydropower stations (limiting the frequency of starting/stopping hydropower generation)

1.3 SDDP, State Variables and Concavity

[1] By ramping we mean the change in a specific variable's value from one time period to the next.

1.3.1 Problem Solving by Decomposition and Sampling

The objective of the seasonal hydropower scheduling problem is to find the release policies that maximize the expected profit over the period of analyses. It is normally solved for a period of 2-5 years starting at a known initial state. It is important to include uncertainty both in future inflow to the reservoirs and electricity prices.

We let the model take decisions in weekly stages. That is, for each week the value of the uncertain variables for that week are known to the model, and the optimal release policy can be made for that week.

The structure of decision sequences can be represented in a scenario tree. For each decision node the tree further branches into a set of new decision nodes in each decision stage. Let's say we plan for a period of 104 weeks and have 12 branches each week, the full scenario tree would have 1.72x10112 nodes, which is far beyond what can be stored in memory and be solved in reasonable computation times. For this reason we rely on algorithms that:

a) Decompose the optimization problem into weekly decision problems that can be solved independently;
b) Apply sampling algorithms to avoid searching the entire tree, and;
c) Apply cut sharing to efficiently "collapse" the scenario tree.

A main iteration of the SDDP algorithm is illustrated in Figure 2. It consists of a forward and backward iteration, as briefly outlined below. Note that this illustration and the explanation below refer to the SDDP-part of the combined SDP/SDDP model used e.g. in ProdRisk. Stochastic price is treated by an outer SDP-loop, as described in detail in [6].

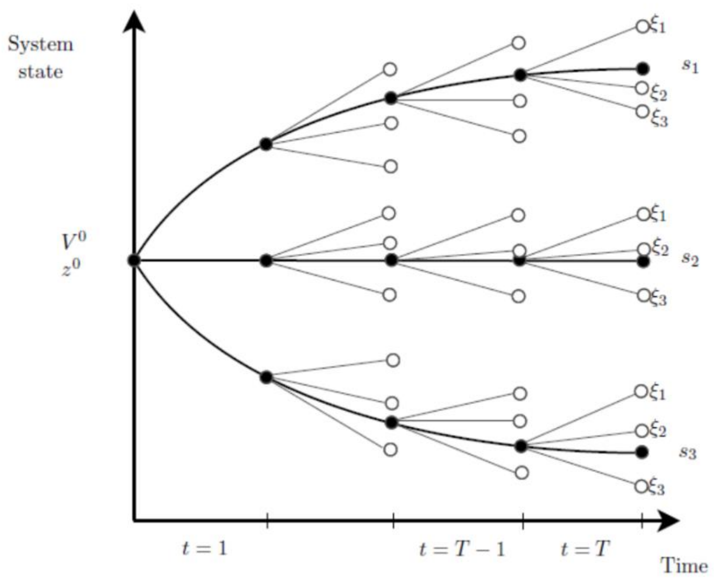

Figure 2 Illustration of a main iteration in the SDDP algorithm.

In the forward iteration of the SDDP algorithm, we sample a set of inflow scenarios {s1-s3} and simulate week-by-week along these scenarios by solving an LP problem for each week. All scenarios share the initial state, which is defined by a vector of initial reservoirs and inflows in the first week. The simulated state trajectory along the sampled scenarios is shown as the thick black line in Figure 2. We keep track of the simulated reservoir levels and find the expected simulated profit over these scenarios, and that value serves as a lower bound.

In the backward iteration cuts are built for each time stage, and added to a list of cuts representing that stage. Consider the state obtained in scenario 1 in stage T-1 in Figure 2. The simulated state trajectory (reservoir level and inflow) up to this point is known from the forward iteration. We now sample 3 vectors of errors or "white noise", so that inflow samples for the coming week can be calculated. For each sample, we solve an LP problem. The average dual values on state variables are used to create a cut constraining the expected future profit seen from previous decision stage (T-2). An important feature here is cut sharing; Cuts computed for a specific initial state at a given stage can be shared among all states for that stage.

We use an autoregressive inflow model with lag-1 both in the forward and backward iterations. The inflow for the current week can be found partly by the inflow from the previous weeks, and partly by the sampled white noise. In the literature, the SDDP algorithm is normally described using the same statistical inflow model both in the forward and backward iteration. The inflow representation is different in ProdRisk, as historical inflows are used directly in the forward iteration, and no resampling is carried out.

1.3.2 State Variables and Problem Concavity

The details of the decomposed LP problem is discussed in detail in e.g. [7] and [8]. The basic set-up is as follows:

$$\begin{aligned}&\max \lambda_t p_t + \alpha_{t+1}\\&\text{S.t.:}\\&\text{- Reservoir balances}\\&\text{- Power balances}\\&\text{- Cuts constraining } \alpha_{t+1}\end{aligned} \tag{0.4}$$

For this maximization problem, the nonconvexity requirement of the SDDP algorithm translates into the following:

The expected future profit (α_{t+1}) should be concave in all state variables.

In the SDDP problem formulated in ProdRisk, there are three types of **state variables**: reservoir volumes, average weekly inflow and average weekly power price. The expected future profit is concave in reservoir volume and inflow, but not in price, as illustrated in Figure 3. Without going into mathematical proofs of this property, we discuss the concavity below.

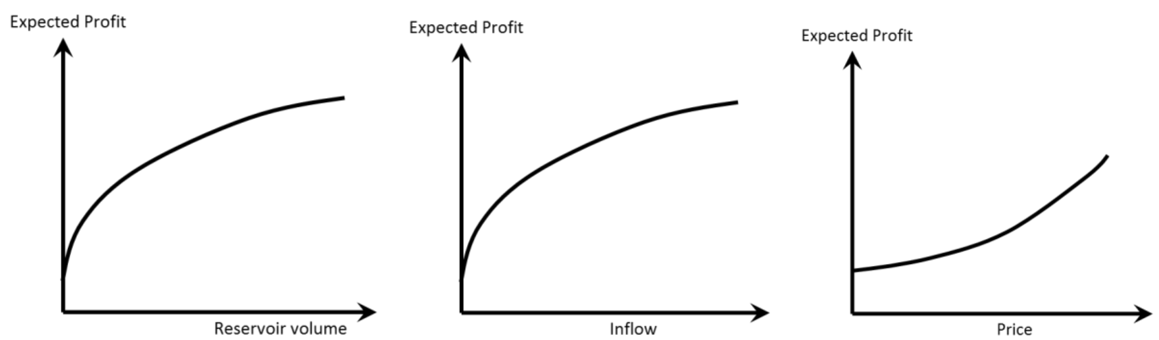

Figure 3 Illustration of the expected future profit as a function of reservoir volume, inflow and price.

Volume:
Reservoir volume is a decision variable in the optimization problem. What we decide to leave in the reservoir at the end of the current decision period would impact the expected future profit. Gradually increasing the reservoir volume at the end of the current decision period for a specific reservoir would increase our expectation for future profit. A marginal increase in reservoir volume will give higher increases in expected revenue for lower volumes than for higher, because of the possibilities of selling power in high-price periods. As one approaches full reservoir, the increasing risk of spillage gradually reduces the marginal increase in expected revenue. This leads to the functional relationship illustrated in the left part of Figure 3.

Inflow:
Unlike volume, inflow is not a decision variable, but rather a parameter in the optimization problem. Inflow in one week is normally strongly correlated to inflow in the previous week, and this correlation in time needs to be presented in the model. The correlation in time makes inflow a state variable; the future expected profit depends on the inflow in the past. A standard way of representing the correlation in SDDP is through a lag-1 autoregressive process:

$$z_t = \phi_t z_{t-1} + \varepsilon_t \tag{0.5}$$

Where z_t and z_{t-1} are the inflows in the current and previous decision stage, respectively, ϕ_t is the correlation matrix, and ε_t is the random "noise". z_t is typically a transformation of the physical inflows, e.g. by normalizing:

$$z_t = \frac{I_t - \mu_t}{\sigma_t} \tag{0.6}$$

Where I_t is the physical inflow, μ_t is the expected inflow, and σ_t is the standard deviation in inflow. The normalization serves to remove seasonal patterns, and the model parameters in (0.5) are fitted using the normalized data.

Equation (0.5) shows that the normalized inflow is linearly dependent on the normalized inflow in the previous period. This equation can be formulated differently, e.g. as a lag-2 model, but care should be taken so that it does not violate the concavity requirement of the SDDP algorithm. Moreover, the data transformation in (0.6) could follow different scheme, but one should ensure that concavity is not sacrificed. A logarithmic transformation of data would e.g. violate the concavity requirement.

By gradually increasing the expectation for future inflow (through the inflow state), the risk of spillage increases, giving the concave curve illustrated in the middle part of Figure 3.

Price:
Price is a parameter in the optimization problem. As seen from Figure 3, the relationship between expected profit and price is convex. The price (λ_t) enters the objective function in (0.4). The price in the current week λ_t generally depends on the price in the previous week λ_{t-1}, and thus the price is a state variable. The obvious consequence of increasing the price state is that the term $\lambda_t p_t$ increases. Assuming that the production (p_t) is independent of price would lead to a linear relationship between expected profit and price. However, the production will often increase when increasing the prices state, and therefore the expected profit is convex in the price state. To deal with this non-concavity, ProdRisk discretizes the price into discrete price nodes and apply an outer SDP loop to deal with the price state, as described in [6]. A recent research article has proposed an algorithm to deal with uncertain variables in the objective function without discretization [9].

As long as the problem is formulated so that the expected profit function is concave in all state variables, cuts can be shared within a given decision stage independently of the initial state of the system. As an example, a cut that was created for a high inflow scenario can be applied in a low inflow scenario. This ability to share cuts between states in a given stage is crucial for SDDP convergence properties. Thus, introducing new state-dependent constraints needs careful treatment to ensure that the concavity-requirement of the expected future profit functions holds.

2 Survey

2.1 Introduction

A user survey was conducted among all known model users contributing to the HydroCen centre. The survey was submitted in September 2018 and the actual wording is shown in Appendix A.

The survey asked for responses to the following question:

Which constraints (covered by concession/license) do you consider the most relevant addressing within the HydroCen project? Provide a prioritized list of 3 types of constraints and, if possible, a practical example for each of those.

The criteria of relevance should relate to that the constraints are:

a) Difficult to represent in existing models
b) Assumed to significantly impact the results

Although the emphasis in the survey is on environmental constraints, we did not limit the survey to only address environmental constraints. Afterall, the model itself does not need to distinguish between environmental and physical constraints.

We received 7 answers to the survey. A complete list of the suggested constraints is presented in Appendix B. In Section 2.2 below we summarize the results and point to the most pronounced constraints that should be further assessed in in this project.

2.2 Summary of Results

The primary motivation behind the survey was to identify constraints that are difficult to handle in the existing models and that are believed to be important for the model users to represent accurately. Within the scope of HydroCen WP3.4, we will primarily focus on environmental constraints and on the SDDP methodology.

We find that a majority of the suggested constraints can be dealt with if state-dependent maximum discharge and virtual reservoirs[2] can be modelled. The modelling of these two features are discussed in the following.

2.2.1 State-dependent Maximum Discharge

Most of the responses pointed to the importance of state-dependent maximum discharge constraints, either directly or indirectly. We will first discuss the direct interpretation of this constraint, and then show how it can be used to control time-dependent local minimum reservoir requirements.

The model keeps track of discharge through the power station by a variable q^D, as illustrated in Figure 4.

[2] The term "virtual reservoir" is often used in river systems with multiple competing owners, where each owner operates it's share of the system seeing reservoir characteristics that are different from the physical characteristics. In this report we use the term with a slightly different meaning. More on that in Section 3.3.

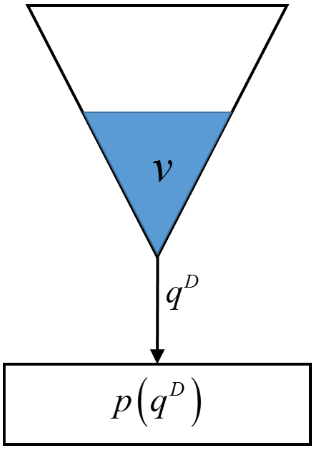

Figure 4 Discharge through power station.

The discharge has limits:

$$\underline{Q}^D \le q^D \le \overline{Q}^D \tag{0.7}$$

In practice, the upper boundary ($\overline{Q}^D$) often depends on the reservoir volume, such that the maximum discharge decreases at low reservoir volumes. This functional relationship can be expressed as:

$$\underline{Q}^D \le q^D \le \overline{Q}^D(v) \tag{0.8}$$

Recall that the reservoir volume is classified as a state variable. This type of constraint is state-dependent because it depends on the current reservoir volume. We generally categorize it as a ***state-dependent maximum discharge*** constraint, and discuss further how it can be treated within the SDDP method in Section 3.1.

Many of the answers pointed to the need for controlling the discharge from a station if the upstream reservoir volume is below a threshold. An illustration is provided in Figure 5.

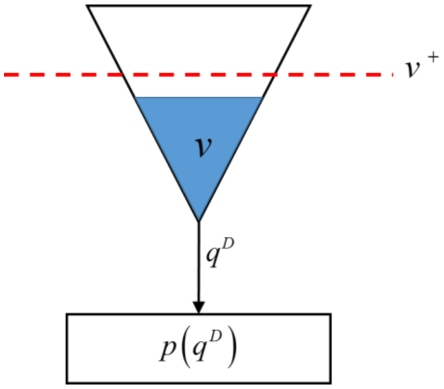

Figure 5 Discharge depending on reservoir volume.

In mathematical terms, this type of constraint can be expressed as follows:

$$\begin{aligned} v \geq v^{+} &\rightarrow \underline{Q}^{D} \leq q^{D} \leq \overline{Q}^{D} \\ v < v^{+} &\rightarrow q^{D} = \underline{Q}^{D} \end{aligned} \tag{0.9}$$

This type of constraint is state-dependent because it depends on the current reservoir volume. It can be seen as a "local" minimum reservoir constraint, telling the model to reduce the local discharge to a minimum $\underline{Q}^{D}$ if the reservoir is below a critical limit (v^{+}). There are no explicit requirements for upstream reservoirs to release water to meet this constraint. Minimum discharge or bypass requirements downstream the reservoir can be met by using the bypass waterway.

The survey reveals that this type of constraint comes in many variants, and the major variation seems to be in defining the time interval in which the constraint should be active. The constraint periods defined in the survey can be summarized as follows:

- A fixed time period, e.g. from week 20-35.
- A time period initiated when the inflow in a previous short-term period exceeds a threshold, e.g. when the weekly inflow in the previous week exceeds the average weekly inflow.
- A time period initiated when the accumulated inflow to the system exceeds a threshold.

We categorized this type of constraint as ***state-dependent maximum discharge*** constraint, and reformulate Equation (0.9) to:

$$\underline{Q}^{D} \leq q^{D} \leq \overline{Q}^{D}(v, z) \tag{0.10}$$

We have expressed the maximum discharge boundary $\overline{Q}^{D}(v, z)$ as a function of the reservoir volume and the inflow (z). The treatment of this constraint within the SDDP method will be discussed in Section 3.1.

2.2.2 Virtual Reservoirs

In some cases, the use of water available in a single physical reservoir is constrained differently depending on the origin of the water and at what time it arrived the reservoir. Two cases are described below:

CaseA:

Consider a reservoir that has state-dependent maximum discharge constraint for a predefined time interval, as discussed in Section 2.2.1, with the following additional rules:

- The water present in the reservoir *at the beginning* of the constraint interval can be freely controlled within the constraint interval. We refer to this water as the "initial water".
- All water arriving the reservoir *during* the constraint interval needs to be stored until a certain reservoir level is reached. We refer to this water as the "constraint water".

In this case we need to keep track of the initial water since it can be operated more freely than the constraint water. Although the water is physically stored in the same reservoir, the initial water and the constraint water should be explicitly represented in the model to reflect the differences in water values.

CaseB:
Consider a reservoir with a downstream power plant that has a maximum discharge constraint for a defined summer period. This reservoir has multiple upstream river strings connected to it, and the accumulated water discharged through one of these strings in a predefined time interval can be used to exceed the maximum discharge rate. In this case we need to keep track of the water from that string, since it has an additional value.

The two cases are conceptually similar. Water arriving to a reservoir may have different uses or rules associated with it. In CaseA there is a *time-dependency* of water, and in CaseB there is both a *path- and time-dependency*. In both cases one could represent the water differentiation by establishing a virtual reservoir in addition to the physical reservoir. The virtual reservoir should be a part of the water value computation, i.e. it should have a separate water value.

3 Constraint Evaluation

In this section we evaluate how the most relevant constraints detected from the survey can be modelled in the SDDP algorithm. These constraints will be a part of the decomposed optimization problems described in Section 1.3.

3.1 Volume-dependent Discharge Boundaries

The volume-dependent boundaries on discharge (q^D) can be expressed as follows:

$$\begin{aligned} v \geq v^+ &\rightarrow \underline{Q}^D \leq q^D \leq \overline{Q}^D \\ v < v^+ &\rightarrow q^D = \underline{Q}^D \end{aligned} \tag{0.11}$$

The constraint set is governed by a logical condition; If the volume is above a threshold v^+, discharge is a free variable, if it is below discharge is a fixed variable. The relationship is illustrated by the red line in Figure 6.

The constraint set in (0.11) can be characterized as a logical constraint. Logical constraints are generally not possible to represent exactly in LP formulations, but in some cases linear approximations will provide acceptable results. One example of a successfully approximated constraint of similar type is the linearized start-up costs used in the EMPS model [10]. The logical constraint in (0.11) has similarities to the irrigation constraints presented in [11]. The authors in that article experimented with a sub-optimal approach, compromising the SDDP convergence properties.

We now proceed with the aim of linearizing (0.11). From the discussion on problem concavity above, we need to ensure that the logical constraint reformulation leaves the Expected Future Profit (EFP) function a concave function in the state variables. If maximum discharge is a concave function in reservoir volume, there is an increasing production (and revenue) potential with increasing volume, but the marginal increase is diminishing. Since the PQ-curve is concave, we can therefore claim that a concave relationship between maximum discharge and reservoir volume preserves the concavity requirement of the SDDP algorithm.

A straightforward relationship is indicated by the two dotted green lines in Figure 6. The green lines are inequalities constraining q^D from above according to (1.12).

$$\begin{aligned} q^D &\leq \gamma v \\ q^D &\leq \overline{Q}^D \end{aligned} \tag{0.12}$$

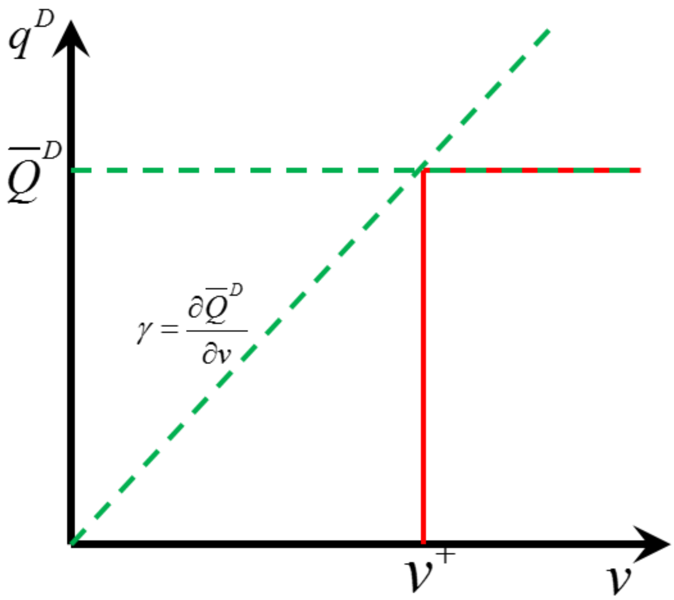

Figure 6 Maximum discharge limit as a function of reservoir volume.

The linear approximation illustrated in Figure 6 will to some extent limit generation at low reservoir levels. However, the approximate modelling in (0.12) will not necessarily limit the operation at best efficiency points at lower discharge rates.

The discharge through a power station is modelled by a set of variables, one per efficiency-curve segment for the station's PQ-curve. Thus, the maximum discharge through the station equals the sum of maximum segment discharges:

$$\overline{Q}^D = \overline{Q}_1^D + \overline{Q}_2^D + \ldots + \overline{Q}_S^D \tag{0.13}$$

It is possible to improve the modelling by moving the constraints in (0.12) from the sum discharge (q^D) to the segmented discharge variables in (0.13). One could constrain each discharge segment $s = 1 \ldots S$ as follows:

$$\begin{aligned} q_s^D &\le \gamma v \\ q_s^D &\le \overline{Q_s}^D \end{aligned} \tag{0.14}$$

The relationship illustrated in Figure 6 would still hold, but the detailed constraints in (0.14) ensures that operation at the best efficiency point is limited once the volume dips below v^+.

3.2 Inflow-dependent Discharge Boundaries

3.2.1 Scenario Dependency

The release boundaries in (0.3) can be given to the model as time series at a weekly time resolution. They can also be associated with inflow series, meaning that the time-dependency in release is given by the inflow scenarios.

$$\begin{aligned} &\underline{Q}^D\left(z_t\right) \le q^D \le \bar{Q}^D\left(z_t\right) \\ &\underline{Q}^B\left(z_t\right) \le q^B \le \bar{Q}^B \end{aligned} \tag{0.15}$$

As will be discussed in the previous section, the inflow is a state variable in ProdRisk, and thus needs to be treated carefully. In the current version of ProdRisk the observed inflow scenario profiles are used directly in (0.15) in the forward iteration, and an average profile in the backward iteration. The averaging in the backward iteration is to ensure concavity of the future expected profit function. This separation between the forward and backward iteration introduces an inconsistency that most likely impacts the results. We have not quantified the impact of this inconsistency.

This inflow-dependency is also a state-dependency due to the inflow's correlation in time. This is clearly seen when re-writing (0.15) to:

$$\begin{aligned} &\gamma_1\left(\phi_t z_{t-1}+\varepsilon_t\right) \le q^D \le \gamma_2\left(\phi_t z_{t-1}+\varepsilon_t\right) \\ &\gamma_3\left(\phi_t z_{t-1}+\varepsilon_t\right) \le q^B \le \bar{Q}^B \end{aligned} \tag{0.16}$$

Here γ_1- γ_3 are constants scaling the inflow profile. We could allow the use of observed inflow scenario profiles also in backward iteration, but only if the state-dependencies are accounted for when creating and later using the Cuts.

Consider the lower discharge boundary:

$$\gamma_1\left(\phi_t z_{t-1}+\varepsilon_t\right) \le q^D \tag{0.17}$$

Increasing the lower bound will decrease the expected profit, so that

$$\mu_-^D = \frac{\partial \alpha_t}{\partial\left[\gamma_1\left(\phi_t z_{t-1}+\varepsilon_t\right)\right]} \le 0 \tag{0.18}$$

Where μ_-^D is the dual value of (0.17). This value is not readily available in standard LP solvers, but one can use information provided by the reduced cost of variable q^D. If q^D is at its lower boundary, μ_-^D equals the reduced cost of q^D.

As detailed in [12], the dual value of the state variable z_{t-1} is computed a posteriori since it is treated as a parameter in the model. Consequently, the entrance of z_{t-1} in (0.17) provides an additional term to the expression for finding the dual of z_{t-1}:

$$\pi_z = \ldots + \gamma_1 \phi_t \mu_-^D \tag{0.19}$$

Similarly, the contributions from the upper discharge and the lower bypass boundaries from (0.15) can be accounted for in (0.19):

$$\pi_z = \ldots + \gamma_1 \phi_t \mu_-^D + \gamma_2 \phi_t \mu_+^D + \gamma_3 \phi_t \mu_-^B \tag{0.20}$$

Where μ_+^D and μ_-^B are the dual values for discharge at its upper bound and bypass at its lower bound, respectively. Note that $\mu_+^D \geq 0$, $\mu_-^B \leq 0$ and that $\mu_-^D \cdot \mu_+^D = 0$.

In conclusion, we believe that scenario dependency which is approximated in ProdRisk can be improved by using scenario values in the backward iteration and accounting for the impact of the state-dependency in the inflow cut coefficients.

3.2.2 Threshold Dependency

Several of the reported inflow-dependent discharge boundaries are formulated so that discharge should be stopped once a certain inflow threshold (typically last 5 days) exceeds a defined threshold (Z^+). This type of constraint could be formulated as follows:

$$\begin{aligned}
&\bar{z}_t = \max\left(z_i, i = n...t-1\right) \\
&\bar{z}_t < Z^+ \rightarrow \underline{Q}^D \leq q^D \leq \overline{Q}^D \\
&\bar{z}_t \geq Z^+ \wedge v \geq v^+ \rightarrow \underline{Q}^D \leq q^D \leq \overline{Q}^D \\
&\bar{z}_t \geq Z^+ \wedge v < v^+ \rightarrow q^D = \underline{Q}^D
\end{aligned} \qquad (0.21)$$

The set of equations in (0.21) comprises the volume-dependent discharge boundaries described in (0.11).
Since the maximum operator is over a set of known parameters, this type of constraint is easier dealt with in a scenario-based solution method. It is much harder to implement this constraint in a stage-wise decomposition model such as SDDP. Thus, as simplification we assume that once the threshold has been reached, all subsequent inflows are above that threshold.

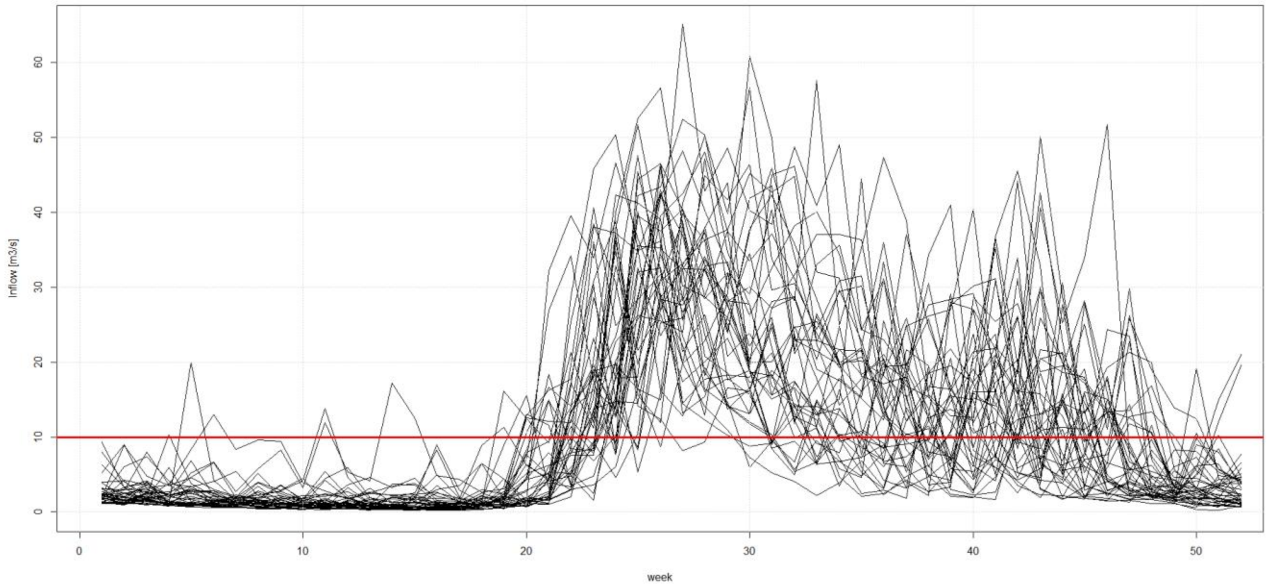

Figure 7 Example of inflow threshold. The "low water period" end whenever inflow exceeds the threshold (red line) in a set of consecutive days.

Maximum discharge is also in some cases described as a function of both reservoir volume and inflow. An example is shown in Figure 8 where the maximum allowed discharge is at the physical maximum only when the reservoir volume is above 80% or the inflow is below 30%. The red lines indicate the linear constraints that can be introduced to approximate the discrete nature of the maximum discharge function.

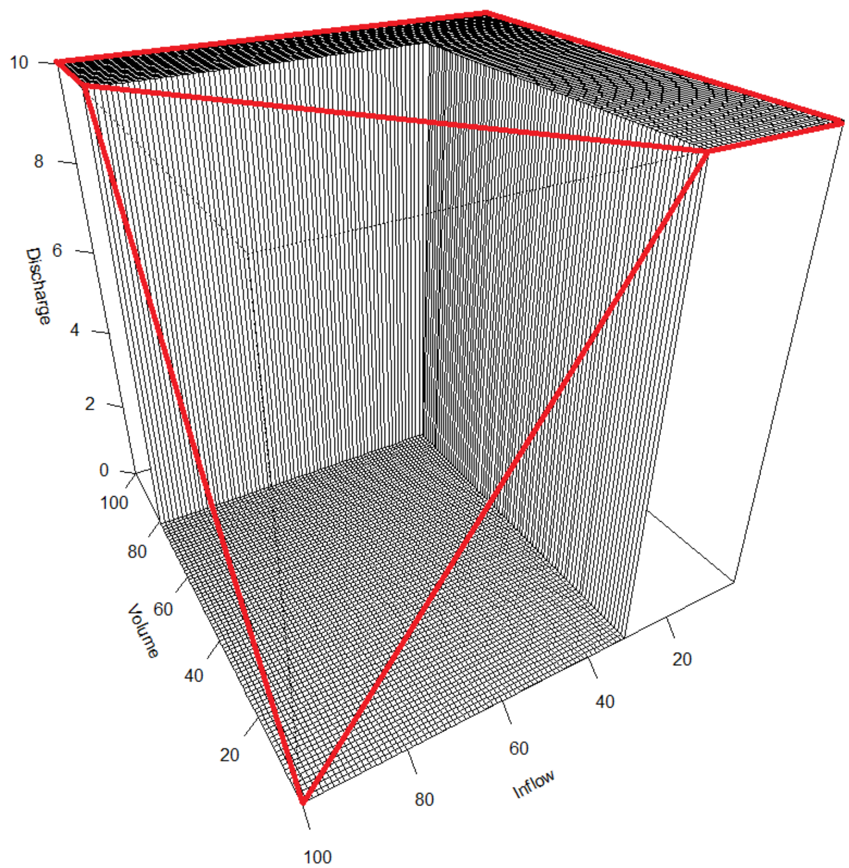

Figure 8 Maximum discharge depending on volume and inflow.

$$
\begin{aligned}
&q_s^D \le \gamma v - \mu I + \beta \;\text{ if }\; \tilde{I} \le I \le \bar{I} \;\text{ and }\; v \le v^+ \\
&q_s^D \le \overline{Q_s}^D \;\text{ otherwise}
\end{aligned}
\tag{0.22}
$$

3.3 Virtual Reservoirs

In this section we elaborate on the modelling of virtual reservoirs. Our interpretation of virtual reservoirs in this context is reservoirs where the water is valued differently depending on the time it arrives and/or the path it has taken to arrive in the reservoir.

3.3.1 Case A – Time-dependent Use of Water

We start by illustrating case A described in Section 2.2.2 in Figure 9. Recall that we defined a constraint interval, and classified the water present in the reservoir before the interval as "initial water" and the arriving the reservoir during the constraint interval as "constraint water". The initial water can be used freely, but the constraint water should be stored until a certain reservoir level is reached.

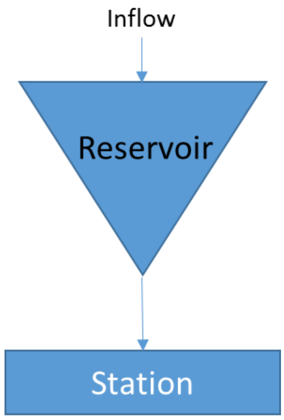

Figure 9 Illustration of system in case A from Section 2.2.2.

We suggest solving this type of problem by introducing a virtual reservoir and introducing the state-dependent discharge boundaries described in Section 3.1, see illustration in Figure 10.

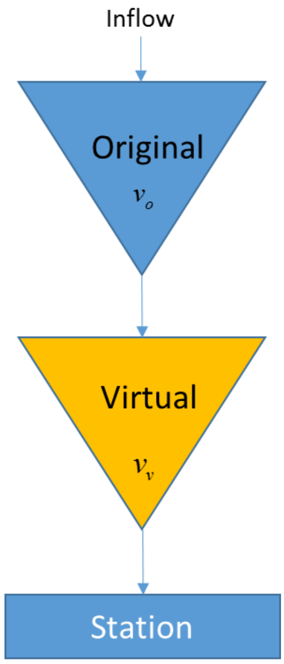

Figure 10 Illustration of possible solution for case A.

The virtual reservoir in Figure 10 should have the same physical properties as the original reservoir and it should be included as a state variable in the SDDP algorithm. One should give the model an incentive to move all water to the virtual reservoir prior to the constraint period. This can be done by adding a small income from discharging from the original to the virtual reservoir the previous time period.

In the constraint period we limit the waterways (discharge q^D, bypass q^F and spillage q^S) out of the original reservoir as long as the sum of water in the two reservoirs are below v^+:

$$\begin{aligned} & v_0 + v_v \geq v^+ \rightarrow \underline{Q}^D \leq q^D \leq \overline{Q}^D \\ & v_0 + v_v < v^+ \rightarrow q^D = \underline{Q}^D \\ & q^B = 0 \\ & q^S = 0 \end{aligned} \tag{0.23}$$

The set-up of a virtual reservoir is done by the model user. Note that the maximum discharge limit refers to the sum of the volumes in the two reservoirs in (0.23). The volume sum should see the physical and tactical limits of the original reservoir on the form:

$$\underline{V}_o \leq v_0 + v_v \leq \overline{V}_o \tag{0.24}$$

Such sum constraints are currently not available in ProdRisk, but are straightforward to introduce.

The accuracy of the presented modelling approach will particularly depend on two details:

- The accuracy of modelling state-dependent discharge boundaries according to 3.1
- The success of leading all water into the virtual reservoir at the beginning of the constraint period while keeping the properties of the two reservoirs as similar as possible.

3.3.2 Case B – Time- and Path-dependent Use of Water

Case B was described in Section 2.2.2 and the system is illustrated in Figure 11. We assume a maximum discharge rate $\overline{Q}^D$ that is constraining system operation in a defined *constraint period*. Water arriving the reservoir from "upstream discharge" within a specified *accumulation period* is allowed to be discharged through the station in addition to the maximum discharge.

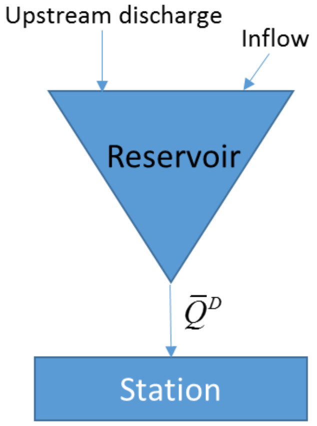

Figure 11 Illustration of system in case B from Section 2.2.2.

A possible solution is presented in Figure 12. A virtual and a dummy reservoir are introduced. The virtual reservoir should have the following volume boundaries:

$$\begin{aligned} & v_v \leq \text{Big} \quad \text{Within accumulation period} \\ & v_v = 0 \quad \text{Outside accumulation period} \end{aligned} \tag{0.25}$$

The dummy reservoir has zero reservoir volume. The maximum bypass limit from the virtual to the dummy reservoir should be non-zero in the constraint period. Similar to case A, the reservoir volume boundaries will now apply to the sum of the virtual and the original volume:

$$\underline{V}_o \le v_0 + v_v \le \overline{V}_o \tag{0.26}$$

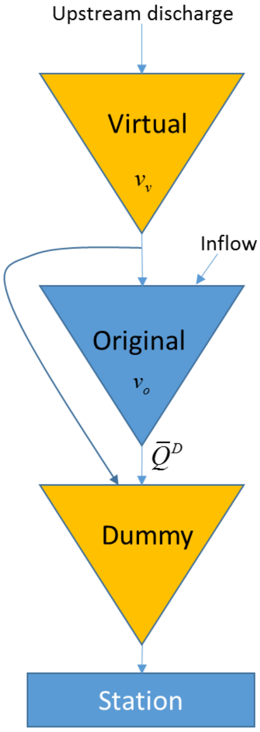

Figure 12 Illustration of possible solution for case B.

4 Summary and Further Work

We have surveyed the need for improved modelling of environmental and technical constraints related to operational hydropower scheduling software. The results have been processed and summarized, and we have selected the two major types of constraints in the context of environmental constraints, namely the state-dependent discharge boundaries and the use of virtual reservoirs. Both constraint types are nonconvex due to their logical nature, and thus need to be linearized to fit well into the SDDP algorithm. Mathematical methods for including both types of constraints in the SDDP algorithm have been elaborated.

The next steps in this work is to implement the constraint modelling and test the implementation on relevant cases provided by the model users. Such tests will reveal if the proposed approximations (through linearization) are accurate enough for practical use. If not, alternative approaches such as the SDDiP algorithm presented in [4] or the scenario fan simulator in [13].

5 References

[1] A. Helseth, M. Fodstad og B. Mo, «Optimal Medium-Term Hydropower Scheduling Considering Energy and Reserve Capacity Markets,» *IEEE Transactions on Sustainable Energy,* vol. 7, pp. 934-942, 2016.

[2] O. Wolfgang, A. L. Henden, M. M. Belsnes, C. Baumann, A. Maaz, A. Schäfer, A. Moser, M. Harasta og T. Døble, «Scheduling when Reservoirs are Batteries for Wind- and Solar-power,» *Energy Procedia,* vol. 87, pp. 173-180, 2016.

[3] A. Helseth, M. Fodstad og B. Mo, «Optimal Hydropower Maintenance Scheduling in Liberalized Markets,» *IEEE Transactions on Power Systems,* vol. 33, pp. 6989-6998, 2018.

[4] M. N. Hjelmeland, J. Zou, A. Helseth og S. Ahmed, «Nonconvex Medium-Term Hydropower Scheduling by Stochastic Dual Dynamic Integer Programming,» *IEEE Transactions on Sustainable Energy,* vol. 10, pp. 481-490, 2018.

[5] A. Gjelsvik og A. Haugstad, «Considering head variations in a linear model for hydro scheduling,» i *The backbone of Sustainable Energy Supply*, Trondheim, 2005.

[6] A. Gjelsvik, M. M. Belsnes og A. Haugstad, «An algorithm for stochastic medium-term hydrothermal scheduling under spot price uncertainty,» i *Proc. 13th Power System Computation Conference*, Trondheim, 1999.

[7] A. Gjelsvik, B. Mo og A. Haugstad, «Handbook of Power Systems I,» Springer, 2010, pp. 33-55.

[8] A. Helseth, A. Gjelsvik, B. Mo og U. Linnet, «A model for optimal scheduling of hydro thermal systems including pumped-storage and wind power,» *IET Generation, Transmission \& Distribution,* vol. 7, pp. 1426-1434, 2013.

[9] A. Downward, O. Dowson og R. Baucke, «Stochastic dual dynamic programming with stagewise dependent objective uncertainty,» *http://www.optimization-online.org/DB_HTML/2018/02/6454.html,* 2018.

[10] G. Warland, A. Haugstad og E. S. Huse, «Including thermal unit start-up costs in a long-term hydro-thermal scheduling model,» i *Proc. 16th Power System Computation Conference*, Glasgow, 2008.

[11] E. Pereira-Bonvallet, S. Pushel-Lovengreen, M. Matus og R. Moreno, «Optimizing hydrothermal scheduling with non-convex irrigation constraints,» *Energy Procedia,* vol. 87, pp. 132-150, 2016.

[12] A. Gjelsvik, «Løysingsmetodikk i SDDP-algoritmen,» 2001.

[13] A. Helseth, B. Mo, A. L. Henden og G. Warland, «Detailed Long-Term Hydro-Thermal Scheduling for Expansion Planning in the Nordic Power System,» *IET Generation, Transmission and Distribution,* vol. 12, pp. 441-447, 2018.

[14] A. Gjelsvik, «Hydropower scheduling with variable water head,» 2016.

[15] A. Helseth og A. L. Henden, «Start-up costs in ProdRisk - Mathematical formulation and user's guide,» 2017.

[16] A. Helseth, «Treating nonconvex PQ-curves in ProdRisk final simulation,» 2017.

A Survey Letter

An email with this core text (in Norwegian, formatted in italics) was sent to the model users in August 2018.

I Forskningssenteret HydroCen, arbeidspakke 3, prosjekt 3.4 jobber vi med å forbedre representasjonen av fysiske- og miljørelaterte restriksjoner i metodikk og modeller som anvendes i produksjonsplanleggingen. Fokus er særlig på sesongplanleggingen og verktøy (ProdRisk, Vansimtap) og metodikk (SDDP, SDP) som anvendes til dette formålet.

Vi ønsker innspill fra brukergruppen på hvilke konsesjonsbelagte og fysiske restriksjoner (krav/regler/begrensninger) i vassdrag og produksjonssystem som er mest relevant å jobbe med i denne sammenheng. Relevans kan knyttes til at restriksjoner er både:

a. *vanskelige å representere i eksisterende modeller, og*

b. *antas å påvirke modellresultater i vesentlig grad.*

Noen eksempler på restriksjoner er listet under. Vi gjør oppmerksom på at listen under ikke er utfyllende; det finnes helt sikkert andre typer restriksjoner som er relevante for brukergruppen.

- *Begrensninger på endringer i magasintilstand og endringer i vannføring ("ramping")*
- *Magasinoppfyllingskrav til en definert vannstand som er avhengig av hvor mye tilsig som kommer fra et gitt tidspunkt til man når vannstandskravet*
- *Minstevannførings- og magasinoppfyllingskrav som inntrer først når det er et visst tilsig i systemet*
- *Tappekapasiteter som er avhengig av vannstand i et magasin, og som begrenser kjøringer i kraftverk eller overføring mellom magasiner*
- *Tidsforsinkelser i vassdrag*
- *Flyt mellom hydraulisk koblede magasiner i henhold til fysiske lover*

Vi oppfordrer derfor den enkelte bruker til å bidra med følgende:

1. ***En liste med maks. 3 typer restriksjoner som du mener er de mest relevante (prioritert rekkefølge)***

2. ***Konkrete eksempler på hvor slike krav og regler er gjeldende.***

Alle svar behandles konfidensielt, med mindre annet avtales. Resultatene fra undersøkelsen vil (på generalisert form) bli presentert i en teknisk rapport.
Tilbakemelding kan gis til arild.helseth@sintef.no
Frist for tilbakemelding: 28. September

B Survey Responses

Constraint	Comment
No discharge from the reservoir before a reservoir threshold level is reached. This type of constraint should hold for either: • A fixed time interval • A time interval initiated when the inflow in a previous short-term period exceeds a threshold • A time interval initiated when the accumulated inflow to the system exceeds a threshold In some cases, the available water at the beginning of the constraint interval can be freely controlled within the constraint interval	Variants of this constraint were reported in many responses. The treatment of this type of constraint is described in detail in the report.
Maximum discharge dependent on reservoir level	This constraint can be legislative or physical. The legislative version corresponds to the constraint described above. The physical can be treated similarly.
Flow between reservoirs according to physical laws	
Constraints on the sum of two or more reservoirs	The need for sum constraints arises when introducing virtual reservoirs, as discussed in the report.
Power plants with forbidden zones, e.g. to avoid cavitation	This constraint is captured in the short-term models and may to some extent be captured in the final simulation in ProdRisk. It is difficult to represent in the SDDP methodology since it is essentially nonconvex.
Improved head-dependent production function	The head-dependency applied in ProdRisk follows an approximate scheme, where head coefficients are computed in a first iteration and included in the optimization in a second iteration. Experience shows that this scheme works well in many watercourses, but is unstable in others.
Limit the sum of discharge and "utjevning"	
Time-delay in river systems	Modelling of time-delay becomes increasingly relevant as users are using finer time-resolution in the model. The modelling of time-delay is not particularly complex.
Maximum ramping rates (up/down) on discharge	Two related functionalities are included in V10 of ProdRisk: a) Ramping rates on reservoir level b) Start-up costs (linearized) Both these functionalities can be used to limit ramping on discharge.

Disallow increased discharge to iced rivers
Virtual reservoirs to account for shared ownership in reservoir
Hydraulic coupling between reservoirs with different head to a downstream power station
Spillage topology dependent on reservoir level

www.hydrocen.no

ISSN:1504-3312
ISBN: 978-82-426-[xxxx-x]

HydroCen

HydroCen
v/ Vannkraftlaboratoriet, NTNU
Alfred Getz vei 4,
Gløshaugen, Trondheim

www.hydrocen.no
HydroCen
@FMEHydroCen