EMD Mode Mixing Separation of Signals with Close Spectral Proximity in Smart Grids

Olav Bjarte Fosso
Dep. of Electric Power Engineering
NTNU, Trondheim, Norway
olav.fosso@ntnu.no

Marta Molinas
Dep. of Engineering Cybernetics
NTNU, Trondheim, Norway
marta.molinas@ntnu.no

***Abstract*—The Empirical Mode Decomposition (EMD) is a signal analysis method that separates multi-component signals into single oscillatory modes called intrinsic mode functions (IMFs). These IMFs can generally be associated to a physical meaning of the process from which the signal is obtained. When the phenomena of mode mixing occurs, as a result of the EMD sifting process, the IMFs may lose their physical meaning hindering the interpretation of the results of the analysis. Previous research presents a rigorous mathematical analysis that shows how EMD behaves in the case of a composite two-components signal, explaining the roots of the mode mixing problem. Also, the frequency-amplitude region within which a good separation is achieved with EMD is well identified and discussed. However, a solution that offers good IMF separation when components reside within the same octave is not yet available. In this paper, a method to separate spectral components that reside within the same octave, is presented. This method is based on reversing the conditions by which mode mixing occurs presented in the paper *One or Two frequencies? The Empirical Mode Decomposition Answers*, in [3]. Numerical experiments with signals containing spectral components within the same octave shows the effective separation of modes that EMD can perform after this principle is applied. This separation technique has potential application for identifying the cause of different oscillatory modes with spectral proximity present in the smart grid.**

***Keywords*—EMD, Mode Mixing, Sifting process, Intermittency, Masking signal, Close spectral proximity.**

I. Introduction

Empirical Mode Decomposition (EMD) has since it was first proposed in [1], demonstrated its capabilities within many application areas. Within electric power engineering, the technique is used to study power system oscillations in a nonlinear and non-stationary perspective [5] as well as to analyze time-varying waveforms related to power quality issues [6]. Both [5] and [6], highlight the challenge of separating closely spaced spectral components (frequency components that exist within the same octave) which is the main focus of this paper. The EMD technique, being a data driven approach, does not require any a-priory assumption about the signal model and is able to handle both non-linear and non-stationary signals. However, the algorithm has shown some limitations in separating closely spaced spectral components and components appearing intermittently in the signal. In such cases, the EMD will not be able to separate well modes with spectral proximity contained in the signal. This paper addresses the problem of separation of these modes and proposes a new method for separating modes located within the same octave. The paper is structured in the following sequence: the concept and the roots of mode mixing are first discussed, then some of the existing solutions are highlighted before the new method is presented. A case study based on a synthetic signal demonstrates the method's capabilities.

A. What is Mode Mixing?

Although Mode Mixing has not been strictly defined in the literature, it is known to happen as a result of the Empirical Mode Decomposition mechanism to extract mono-components from a multi-component signal. As a result of this mechanism, only modes that clearly contribute with their own maxima and minima can be identified by the sifting process of the EMD. When a mode cannot clearly contribute with extremas, the EMD will not be able to separate the mode in a single IMF and the mode will remain mixed in another IMF. The paper in [3], provides restrictions on when it is possible to extract a component from a composite two-components signals. The ratio of the amplitudes and of the frequencies of the individual components of the signal, will determine whether the EMD will be able to separate them in two different IMFs or whether they will be interpreted as one single IMF.

In this paper, Mode Mixing is broadly categorized in two groups. Depending on the source at the origin, they can be originated by a) presence of closely spaced spectral components (one frequency is twice or less than the other), b) presence of intermittence. In general, the reported literature offers solutions to the mode mixing caused by the presence of intermittence, while separation of closely spaced spectral components has remained unsolved [11]. This paper proposes a principle that when implemented improves the ability of the EMD to separate closely spaced spectral components. An introduction of the principle is available in the preprint uploaded to the open repository arXiv [4]. In the following section, the main contributions to the solution of the mode mixing problem are discussed in light of their evolution in time. The new method is introduced within this perspective.

B. Existing Methods for Mode Mixing Separation

Mode mixing, observed in the context of the Empirical Mode Decomposition, and caused either by intermittency or close spectral proximity of the signal components, is a well recognized limitation of the EMD method [2] [3] [7].

This work was supported by NTNU Signal Analysis Research Lab.

The mechanism behind EMD works in a way that always extract the highest frequency component present is the residue of the signal. In the presence of intermittence, the next lower frequency components may appear in the same IMF where the intermittent signal is identified, even though they are in different octaves and should appear as individual components. In the case of closely spaced spectral components, the signals will appear in the same IMF unless successfully separated. In 2005, in the paper: *The Use of Masking Signal to Improve Empirical Mode Decomposition*, Ryan Deering and James F. Kaiser [2] discuss the phenomena of mode mixing and presented for the first time the idea of using a masking signal to separate mixed components caused by the presence of intermittency in the signal. A formula for choosing the masking signal was suggested based on the observations of empirical trials with several signals with mode mixing. The demonstration was essentially made for mode mixing caused by the presence of intermittency, since the frequency ratio between the signals was 0.57 which from the perspective of closely spaced spectral components, was only moderately mixed. Rilling and Flandrin presented in 2008 a theoretical analysis that explains the behavior of the EMD in the presence of two closely spaced spectral components and the roots of the mode mixing problem. Their work demonstrates which spectral components could be expected to be separated by the EMD based on their frequency and amplitude ratios. A *Boundary Map* prepared by the authors, provides a visual indication of the efficiency of separation of two components depending on their amplitude and frequency ratios. A similar map is shown in Figure 1 in this paper to illustrate how the separation boundary works depending on frequency and amplitude ratios. The authors of this work did not present a solution for the mode mixing problem caused by closely spaced spectral components. They rather established restrictions on when it is possible, using EMD, to extract a component from a composite two-components signals. In 2009, Wu and Huang presented the *Ensemble Empirical Mode Decomposition (EEMD)* as a solution to cope with the mode mixing phenomena [7] [8]. Again, this approach was intended to solve the mode mixing caused by the presence of intermittent components in the signal. The principle behind EEMD is to average the modes obtained by EMD after several realizations of Gaussian white noise that are added to the original signal. After this work by Huang, several versions of the Ensemble EMD were proposed in the literature and the method is widely used [9] but it is considered to be computationally expensive. Most recently, in April 2017, a patent application, *System and Method of Conjugate Adaptive Conjugate Masking Empirical Mode Decomposition* filed by Norden Huang et. al [10] discloses a method for directly processing an original signal into a plurality of mode functions without mode mixing. The invention claims to exclude the problem of mode mixing caused by an intermittent disturbance but does not apparently address the mode mixing caused by closely spaced spectral components.

Although Deering and Kaiser introduced the idea of a masking signal and proposed empirical formulations for frequency and amplitude selection, the principle behind the choice of frequency and amplitude of the masking signal, to achieve a good separation of closely spaced spectral components, remains an open question. The following section presents and discusses the principle that can enable EMD to separate this class of mode mixing. Based on the idea of the masking signal presented by Deering and Kaiser and the knowledge of the restrictions of the boundary map presented by Rilling and Flandrin, a masking signal can be designed in order to reverse an existing mode mixing condition.

II. The Proposed Masking Signal: principle for amplitude and frequency choice

The method is based on the original idea of Deering and Kaiser, of injecting a masking signal to the original signal. The contribution to this original idea is in the general principle used to determine the amplitude and frequency of the masking signal. In contrast to the empirical way in which these parameters are defined in [2], in this work, there is a clear mathematical principle that decides the frequency and amplitude of the masking signal. The same mathematical formulations and boundary conditions used to build up the map presented in [3] and portrayed in Figure 1, are the basis that form the principle for choosing the amplitude and frequency of the proposed masking signal in this paper. This principle is explained in the following. Figure 1 illustrates the frequency and amplitude ratios of a signal components for which mode mixing occurs (attraction region for mode mixing: red region) and for which a good separation of components is obtained (non-attraction region for mode mixing: blue region). Based on the conditions of attraction of these regions, it will be possible to identify a masking signal with the appropriate frequency and amplitude so that the existing mode mixing conditions of the original signal can be reversed. This will be achieved by adding a masking signal that can create a controlled artificial mode mixing with one of the original signal's components. By ensuring that the masking signal and the higher frequency component of the original signal become mode mixed in the first IMF of the original signal + masking signal, the first IMF of the original signal can be separated. The controlled artificial mode mixing can be easily removed, since the masking signal is known. In the process of choosing the masking signal frequency and amplitude, care must be taken to keep the masking signal outside the attraction region to the second component of the original signal. If these two conditions are observed, the two components that were mode mixed after the EMD of the original signal will be free of mode mixing.

The step by step procedure for extracting IMFs when modes are mixed, based on the proposed method in this paper is the following:

1) Construct masking signal x_m based on the new principle defined in this paper,
2) Perform EMD on $x_+ = x + x_m$ and obtain the IMF y_+. Similarly obtain y_- from $x_- = x - x_m$
3) Define IMF as $y = (y_+ + y_-)/2$

Step 1 will require to obtain the frequency information from the original data. This information is obtained as explained in section II.B.

A. Mode Mixing of Closely Spaced Spectral Components

In [3], it is explained that the amplitude and frequency ratios of the signal components are crucial for the understanding of the roots of mode mixing. In the same work, a

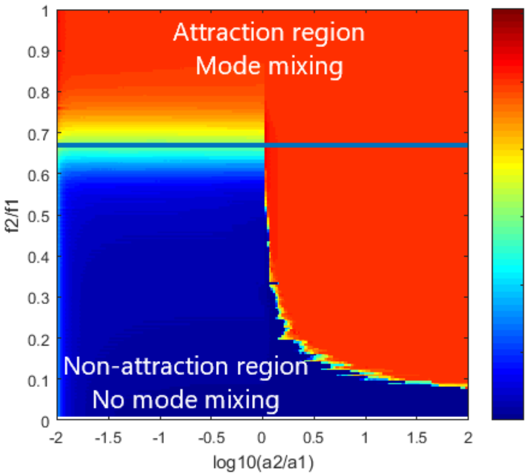

Fig. 1. Mode mixing boundary conditions map reproduced from [3] used for defining masking signal

Boundary Condition Map portrays the different regions where mode mixing is likely to occur as a function of the relative frequencies and amplitudes involved in the two signals. It can be observed from the map that in the region where the ratio between the frequencies involved are between 0 and 0.5, mode-mixing will not be observed for a range of amplitude ratios. Mode mixing is observed as the frequency ratio is higher than 0.67 and approaches 1.0, where mode mixing will always occur for all amplitude ratios. For the sake of clarity and to aid the discussions, the same *Boundary Map* is reconstructed in this paper and shown in Figure 1. It is important to note that the boundary of this map is a property of the signals involved and parameters such as sampling time, frequency resolution and EMD parameters will affect the accuracy of the boundaries between attraction and non-attraction regions. The map illustrates well how closely spaced spectral components attract each other in a mode mixing (red attraction region). This same property is exploited in this paper for constructing effective masking signals to separate closely spaced spectral components.

B. Principle for Separation of Closely Spaced Spectral Components

The principle proposed in this paper is based on the combination of the idea presented by Kaiser and the *Boundary Map conditions* presented by Flandrin [2] [3]. The boundary conditions guides in the choice of the masking signal's frequency and amplitude. To be able to extract a frequency component by applying this principle, the ratio between the frequency of that component and the frequency of the masking signal, must be located in the red region of the *Boundary Map* (mode mixing region), while the ratio between the next component frequency and the masking signal frequency should be located in the blue region of the map (non-attraction region). The amplitude ratios need to be adopted to ensure these same conditions. It is therefore necessary to operate with a frequency sufficiently close to the first mode and sufficiently distant from the next mode, to be successful. This criteria will be discussed in more detail in the case study.

Assume a signal with the two frequencies f_1 and f_2 ($f_1 > f_2$), where the ratio between them will create mode mixing due to their close spectral proximity ($f_1/f_2 \leq 2$). A masking signal of frequency f_m larger than f_1 will attract f_1 if the ratio f_1/f_m falls into the attraction region of the map (red color). If the ratio between f_2/f_m falls into the non-attraction region (blue color), adding a positive masking signal of frequency f_m will separate the two signal f_1 and f_2 and the first IMF will have a controlled mode mixing of the signals f_1 and f_m. To separate f_1 and f_m, a negative masking signal is added, and by averaging the two first IMFs, the new IMF will be a signal of frequency f_1. However, depending on how close the two frequencies f_1 and f_2 are, some amplitude modulation will be observed between the separated signals f_1 and f_2. To identify the frequencies involved in the original signal, a Fast Fourier Transform (FFT) can be used as first screening tool. In this paper, a technique has been developed to identify the involved instantaneous frequencies and amplitudes, to assist in choosing the right masking signal. Assume a signal x defined by:

$$x = A\sin(2\pi f_1 t) + B\sin(2\pi f_2 t) \tag{1}$$

After a standard EMD, these two signals will be mixed into one IMF. After a Hilbert-transform of the mode mixed IMF ($s = x + jy$) followed by an amplitude and an instantaneous frequency calculation, the required information for identifying the amplitudes and frequencies of the two signals involved, will be available. The instantaneous frequency used here is defined by:

$$f = \frac{1}{2\pi}\frac{\partial\phi}{\partial t} \tag{2}$$

where:

$$\tan\phi = \frac{y}{x} \tag{3}$$

$$\phi = \arctan\frac{y}{x} \tag{4}$$

From the amplitudes of the Hilbert-transformed signal, the following expressions can be derived:

$$K_{min} = \sqrt{A^2 + B^2 - 2AB} = (A - B) \tag{5}$$

$$K_{max} = \sqrt{A^2 + B^2 + 2AB} = (A + B) \tag{6}$$

Similarly, the expressions for the extreme values of the instantaneous frequency plots are:

$$Fmin = \frac{A\Delta f}{(A+B)} + f_2 \tag{7}$$

$$Fmax = \frac{A\Delta f}{(A-B)} + f_2 \tag{8}$$

K_{min}	Minimum value of the amplitude plot
K_{max}	Maximum value of the amplitude plot
F_{min}	Minimum value of the instantaneous frequency plot
F_{max}	Maximum value of the instantaneous frequency plot
Δf	Difference between the two frequencies ($f_1 - f_2$)

Through these derivations, it is also demonstrated that Δf is equivalent to the number of peaks/second in the instantaneous frequency and the amplitude plots. The frequencies f_1 and f_2 can now be calculated and may be further validated by using a Fast Fourier Transformation. In Equation 8, it should be noted that $A = B$ represents a discontinuity of the equation system where the first term of the equation will change from positive to negative when $B > A$ and originate a significant positive or negative spike when the signals have almost the same amplitude. The case where $A > B$ is shown with the blue curve in figure 5. Switching A and B would turn the spikes downwards.

In the case of synthetic signals, these calculations are accurate and in principle the signal components could have been calculated directly by applying the above presented technique. However, for real signals the instantaneous amplitude and frequency functions are less smooth. Still they can reveal information about the amplitudes and frequencies involved in the different periods of a mode mixed signal and from this, an optimal mask signal based on the map can be defined.

III. Case Study

To demonstrate the principle for mode mixing separation presented in this paper, a synthetic signal consisting of the following components is chosen:

$$x = 0.7\sin(2\pi 8t) + 0.7\sin(2\pi 24t) + 1.4\sin(2\pi 30t) \quad (9)$$

The individual components and the final signal are shown in Figure 2. From simple inspection of the frequency components we can expect that there will be mode mixing between the 24 and 30 Hz components (30/24=1.25 $\leq$ 2)

After separation with a standard EMD, the IMFs are shown in Figure 3. The first plot corresponds to the signal. The first IMF is the mix of the signals 24Hz and 30 Hz while the second IMF is the 8Hz signal well separated. The other IMFs reported are not relevant and only a result of end-effect conditions, sifting process and applied tolerances. To identify the frequencies and amplitudes involved in the first IMF, a Hilbert transform is performed and the instantaneous amplitudes and frequencies are calculated. The instantaneous amplitudes are shown in Figure 4 and the instantaneous frequencies in Figure 5. Both figures portray the instantaneous values from the two IMFs.

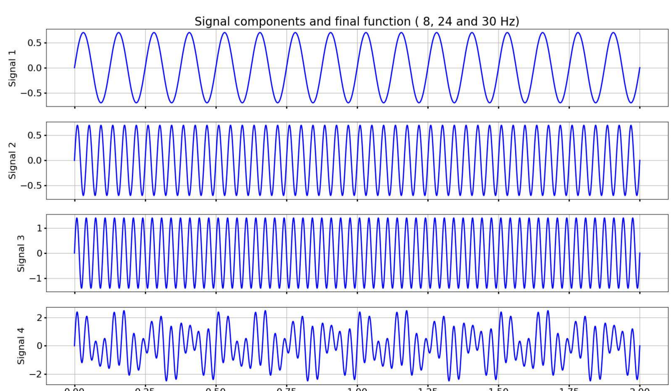

Fig. 2. Synthetic signal and its components for case study

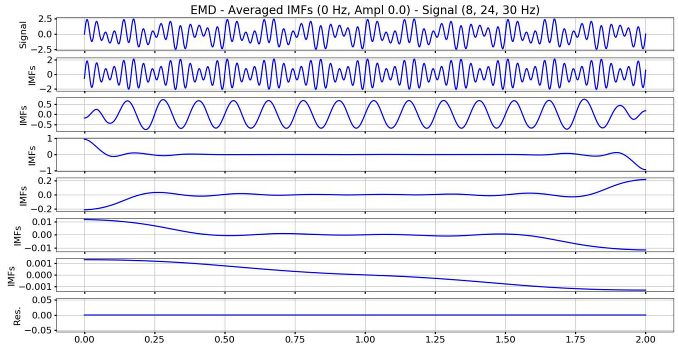

Fig. 3. Original signal, two IMFs and residuals after separation with a standard EMD

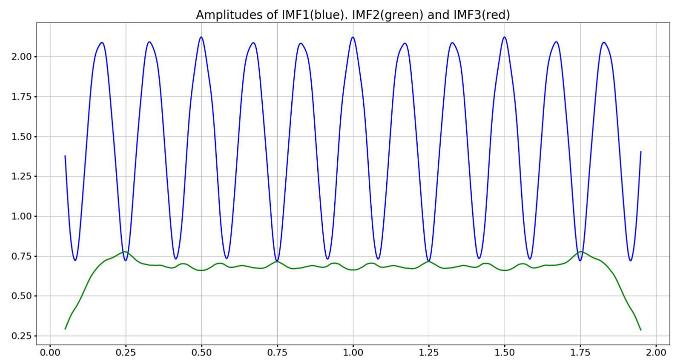

Fig. 4. Instantaneous amplitudes of IMF_1 and IMF_2

TABLE I. Maximum values obtained from Inst. Amplitude and Inst. Frequency plots

Frequency (Peaks/s)	Δf	6
Extreme values	**Minimum**	**Maximum**
Amplitudes	0.74	2.08
Instantaneous Frequencies	27.9	35.5

TABLE II. Estimated (Amplitudes / Frequencies) from Model

Estimated parameters	Mode 1 (A and f_1)	Mode2 (B and f_2)
Amplitudes	1.41	0.67
Frequencies	30.06	24.06

To estimate the amplitudes and frequencies of the components involved, we need to find the maximum and minimum of the amplitude function and of the instantaneous frequency function. Additionally we have to find the Δf which is the number of *peaks/second* in either the instantaneous amplitude or the instantaneous frequency plots. The maximum values extracted from the plots are indicated in Table 1. Substituting the obtained values in Table I into Equations 5-8, the estimated amplitudes and frequencies involved in the IMF with mode mixing are shown in Table II. These values are in good agreement with the original signal components. Using the estimated values of A and B in Equation 8, the estimated frequency is: f_2 = 24.06. The process of extracting the IMFs depends on the number of iterations in the sifting process and may introduce some inaccuracies. This is verified by applying the technique directly on the signal, which gives exact values. For a synthetic signal, the separation could have been done directly. When applied in conjunction with EMD, the purpose

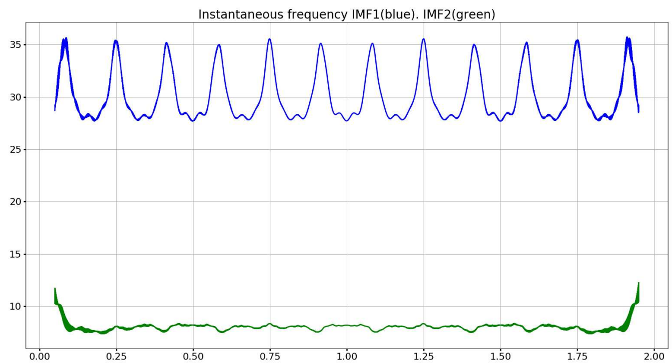

Fig. 5. Instantaneous frequencies of the IMF_1 and IMF_2

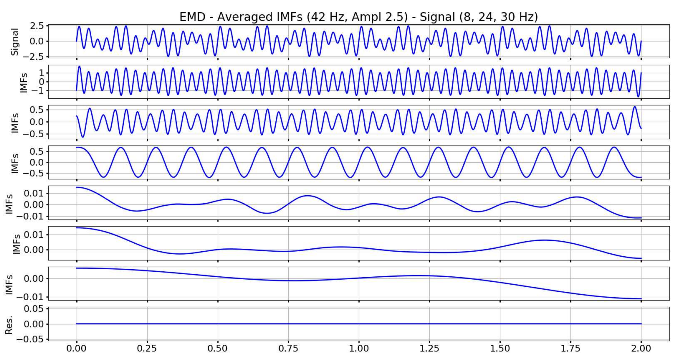

Fig. 6. IMFs after applying the masking signal defined by the proposed method

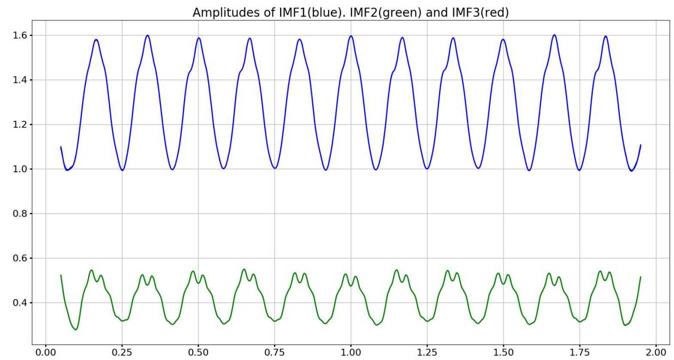

Fig. 7. Instantaneous amplitudes of IMF_1 and IMF_2 after proposed masking signal

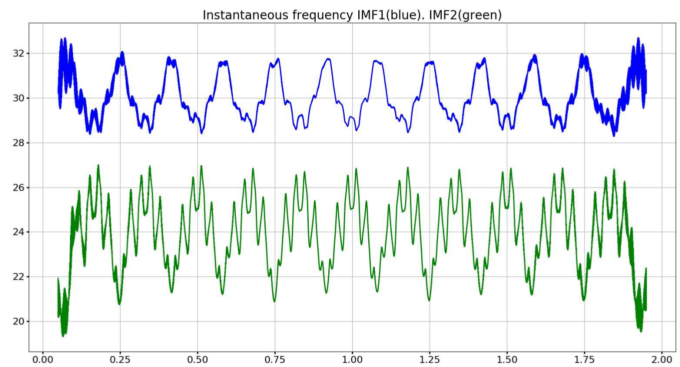

Fig. 8. Instantaneous frequencies of IMF_1 and IMF_2 after proposed masking signal

is to extract amplitudes and frequencies for different parts of the signal to assist in constructing an optimal masking signal. Based on the values obtained in Table II and following the recommendation of amplitude and frequency ratios observed in the *Boundary Map* for mode mixing $f_1/f_m > 0.67$ (red attraction region) and separation $f_2/f_m << 0.67$ (blue non-attraction region), an appropriate masking signal is defined. Generally, a good compromise will be to keep $f_1/f_m >$ 0.7 and $f_2/f_m < 0.6$ if possible. According to the map, a value of $a_m = 2.5$ for the amplitude will be appropriate as $log10(a_1/a_m) = -0.25$ and $log10(a_2/a_m) = -0.4$. With these values, IMF_1 after masking, will be located in the attraction region (red color) while IMF_2 in the non-attraction region (blue color).

Now, using the defined masking signal, a new separation with EMD is conducted on the original signal + the masking signal:

$$x_m = 2.5cos(2\pi f_m t) \tag{10}$$

Following the step by step procedure described in section II, the IMFs obtained with this procedure are shown in Figure 6. The three spectral components are now well separated. However, some amplitude modulation is observed on the 24 Hz and 30 Hz signals. When increasing the masking signal frequency, the amplitude modulation moves from the 30 Hz signal to the 24 Hz signal, while reducing the frequency will move the amplitude modulation to the 30Hz signal as the attraction is stronger according to the *Boundary Map*. A Hilbert-transform is now performed on the two first IMFs followed by instantaneous amplitude and instantaneous frequency calculations. The results are shown in Figures 7 and 8. The instantaneous amplitude plots clearly shows the amplitude modulation and the presence of the other frequency component in both, IMF_1 and IMF_2. As the instantaneous frequency plots are less smooth, it is more difficult to identify the real extreme values needed in Equations 7 and 8. However, in this case we extract values from both IMF_1 and IMF_2. The extracted values are shown in Table III. Using the extracted values from Table III into Equations 5-8, the estimated amplitudes and frequencies involved in the IMF_1 and IMF_2 are shown in Table IV.

TABLE III. EXTREME VALUES FROM INST. AMPLITUDE AND INST. FREQUENCY PLOTS

Frequency (Peaks/s)	Δf	6
Extreme values	**Minimum**	**Maximum**
IMF_1: Amplitudes	1.0	1.6
IMF_1: Instantaneous Frequencies	28.8	31.79
IMF_2: Amplitudes	0.32	0.55
IMF_2: Instantaneous Frequencies	21.4	26.9

TABLE IV. ESTIMATE (AMPLITUDES / FREQUENCIES) FROM MODEL

Estimated parameters	**Mode 1 (A and f_1)**	**Mode2 (B and f_2)**
IMF_1: Amplitudes	1.3	0.3
IMF_1: Frequencies	29.96	23.96
IMF_2: Amplitudes	0.12	0.43
IMF_2: Frequencies	29.7	23.7

With reference to Equation 1, and the estimated values

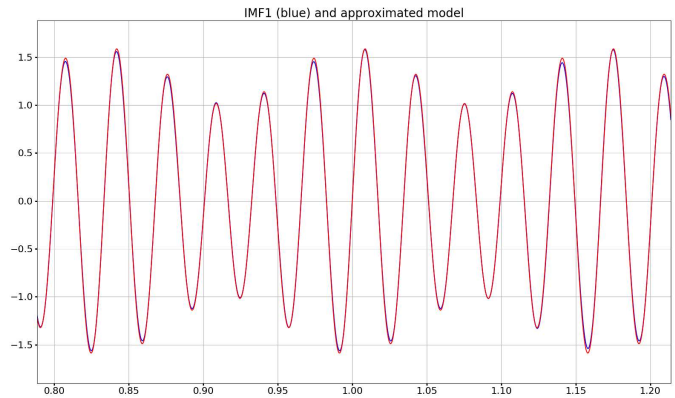

Fig. 9. First IMF and estimated model - plot for 0.5 s

from Table IV, the IMF_1 and IMF_2 are estimated by:

$$x_1 = 1.3\sin(2\pi 29.96t) + 0.3\sin(2\pi 23.96t) \quad (11)$$

$$x_2 = 0.12\sin(2\pi 29.96t) + 0.43\sin(2\pi 23.96t) \quad (12)$$

The frequency components should be exactly the same (in Table IV) but since the extreme values of the instantaneous frequency were identified with low reliability, this can explain the difference. The plot of the estimates of the IMF_1 (red curve) together with the IMF_1 from the EMD (blue curve) are shown in Figure 9 for a period of 0.5 second. A very good agreement is also observed for the period of 2 seconds. For IMF_2, the agreement between the model and the IMF was also very good. In principle, the two corresponding components could have been added to get two pure sinusoidal functions of frequency 30 Hz and 24 Hz without any amplitude modulation. However, the phase shift of the sinusoidal functions are not accurately accounted for and therefore the conclusion could be misleading. A similar accuracy is found for the IMF_2 [4].

A. *Summary of Findings*

This research aimed at enhancing the ability of the Empirical Mode Decomposition to separate closely spaced spectral components in a signal. When two components are in the same octave ($f_1/f_2 \leq 2$), the standard EMD will generate IMFs that will exhibit mode mixing. The main findings prove that by reversing the mode mixing conditions observed in the *Boundary Map* presented in [3], it is possible to define a masking signal which separates closely spaced spectral components. Even cases where the ratio f_2/f_1 exceeds 0.8, are separated with limited amplitude modulation. It is also found that by using the instantaneous amplitudes and instantaneous frequencies obtained from the Hilbert-transform of the signal, it is possible to identify the frequencies and the amplitudes of the components present in the IMF with mode mixing. This information can be used to propose an optimal masking signal for separation of the closely spaced spectral components.

IV. Discussion and Conclusions

This work has essentially proposed a new method for separating closely spaced spectral components when using EMD. The principle for separating mode mixed components was demonstrated and validated using synthetic signals. A relatively simple but challenging signal from a separation point of view, was used in this paper for demonstrating the concept. The model estimation process uses essentially local information obtained as a result of the EMD and Hilbert Transform process (number of peaks/s, extreme values of instantaneous amplitudes and instantaneous frequencies). The information is used to estimate the properties of the signal after a standard EMD has produced IMFs that may present mode mixed IMFs. This mode mixing separation method is presented in the context of smart grid, as spectral components with the proximity discussed in this paper have been reported in smart grids. The presence of power electronics units that introduces switching events and are dominated by controllers, have introduced oscillatory modes in the system often lying within the same octave.

References

[1] N. E. Huang, Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N.-C. Yen, C. C. Tung, and H. H. Liu, *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis*, Proc. of the Royal Society of London A: Math., Physical and Engineering Sciences, vol. 454, no. 1971, pp. 903995, 1998.

[2] R. Deering and J. F. Kaiser, *The use of a masking signal to improve empirical mode decomposition*, in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP 05), 2005, vol. 4, pp. 1823.

[3] G. Rilling and P. Flandrin, *One or two frequencies? The empirical mode decomposition answers*, IEEE Trans. Signal Process., vol. 56, no. 1, pp. 8595, Jan. 2008.

[4] Olav B. Fosso and Marta Molinas, *Method for Mode Mixing Separation in Empirical Mode Decomposition*, eprint arXiv:1709.05547, pp. 1-7, September 2017

[5] Arturo Roman Messina, *Inter-area Oscillations in Power Systems*, Springer, ISBN 978-0-387-89529-1, 2009.

[6] N. Senroy, S. Siddharth Suryanarayanan and P. Ribeiro, *An Improved Hilbert-Huang Method for Analysis of Time-Varying Waveforms in Power Quality*, IEEE Trans. on Power Systems., vol. 22, No. 4, pp. 18431850, November 2007

[7] Z. Wu, N. E. Huang, and X. Chen, *The multi-dimensional ensemble empirical mode decomposition method*, Adv. Adapt. Data Anal., vol. 1, no. 3, pp. 339372, 2009.

[8] Z.Wu and N. E. Huang, *Ensemble empirical mode decomposition: A noise-assisted data analysis method*, Adv. Adapt. Data Anal., vol. 1, no. 1, pp. 141, 2009.

[9] M.A. Colominas, G. Schlotthauer, M.E. Torres, P. Flandrin, 2012 : *Noise-assisted EMD Methods in Action*, Adv. Adapt. Data Anal., Vol. 4, No. 4, pp. 1250025.1-1250025.11

[10] Norden E. Huang, Zhao-Hua Wu and Jia-Rong Yeh, *System and Method of Conjugate Adaptive Conjugate Masking Empirical Mode Decomposition*, U.S. Patent 2017/0116155 A1, April 27, 2017

[11] Maans Klingspor: *Hilbert transform: Mathematical theory and applications to signal processing*, Thesis, University of Linkoping, November 2015, Electronic version: http://liu.diva-portal.org/smash/record.jsf?pid=diva2