Real Time Phasor Estimation based on Recursive Prony with Several Channels of One PMU

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***Abstract*—Phasor estimation is crucial for monitoring and control of smart power systems. The classical signal processing method named Prony has been used for estimating the parameters of measured signals such as frequency, damping factor and phasor. To reduce the impact of noise on the parameters estimated by Prony, multi-channel Prony has been previously explored and presented in the literature. The basic approach for multi-channel Prony is a generalized solution, in which new rows are added to matrices for every channel. Since the generalized multi-channel Prony is time-consuming, a new method based on recursive solution is proposed in this paper to make it suitable for real-time application. Here, several channels of one Phasor Measurement Unit (PMU) are used to estimate the phasor of current/voltage in a recursive pattern, in which the phasor is computed recursively over time based on previously calculated estimates and new measurements. The proposed method is compared with three other solutions for multi-channel Prony: a) data fusion which is based on the Kalman filter concept, b) an alternating direction method of multipliers (ADMM), and c) a consensus update approach which is based on an iterative procedure. Simulation results demonstrate the ability of the proposed method for real-time phasor estimation, both in terms of maintaining the accuracy and reducing computation time.**

Keywords-Multi-channel Prony; Phasor estimation; Prony algorithm; Recursive least square; Weighted least square.

I. Introduction

The power systems get increasing complexity due to new components and intermittent resources at the same time as there is a need for enhanced utilization of the existing infrastructure. Reduced margins require more precise knowledge about the system state. Considerable penetration of renewable energy resources and inverter-fed generations are changing the dynamics of the system. Use of Synchronized Measurement Technology represented by Phasor Measurement Units (PMUs) appears to be an approach that will contribute to cover the requirements of monitoring and smart control [1], [2].

Different identification algorithms can be applied to signals measured by Phasor Measurement Unit (PMU) to estimate states of systems. Kalman filters [3], [4], Least square estimation [5], [6], Wavelet transform [7], Hilbert-Huang Transform [8] and the Prony algorithm [9] are some of these algorithms, which are presented in the literature. The Prony algorithm is a promising method for extracting the oscillatory modes. The method models the main signal into a sum of damped sinusoidal signals.

There are a number of research initiatives that investigate algorithms for phasor estimation [10], [11]. An adaptive filter is suggested in [12] to estimate phasors. An algorithm for digital filter design named Shanks' method, can also be used for estimating phasors and some results are presented in [13]. Additionally, a new algorithm based on Fast Recursive Gauss-Newton (FRGN) is introduced in [14] to extract frequency and phasor simultaneously. To remove the DC component, an enhanced Fourier Transform is applied in [15]. Moreover, an energy operator based on a shift angle is proposed in [16] to estimate the amplitude during dynamic conditions. Recursive Wavelet Transform (RWT) is another method used for extracting amplitude and phase of signals [17]. The Prony algorithm is also a promising method proposed in [18] for phasor estimation where estimates are calculated adaptively, based on estimated frequency.

Despite the promising performance of the Prony algorithm for phasor estimation, the accuracy of phasor estimates is challenging under noisy conditions. To address this problem, a multi-channel Prony is proposed in the literature [19]–[21]. In the generalized multi-channel Prony, the basic principle is to estimate the phasor based on measurements from several channels of one or more PMUs. However, due to the size of the matrices, different strategies are used for solving it. A new strategy based on a classical data fusion is proposed in [19] for modal information estimation. In [20], an alternating direction method of multipliers (ADMM) is proposed to estimate the slow frequency eigenvalues. In [21], a consensus and subgradient update is used to solve the multi-channel Prony. Both aforementioned methods are iterative algorithms, which increase the computational time. One of the general solutions for reducing computation time is a recursive pattern as proposed in [22]–[24]. In [22], a recursive approach for Prony is proposed for model information extraction. Modified recursive Prony is also used in [23] to estimate the monotonous trend of an oscillating system. In addition, an oscillation detection method based on the recursive Prony is proposed in [24] to automatically detect the ringdown data. However, in these references, only one signal is used in the estimation process.

In this paper, a recursive solution is proposed for the multi-channel Prony algorithm to estimate the phasor of cur-

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rent/voltage. Since the proposed method is used in a phasor estimation concept, different channels of one PMU are used. Firstly, a recursive multi-channel Prony is examined with different test signals to show that it is possible to reach good accuracy with reduced computation time. Next, the proposed method is compared with three different implementations of the multi-channel Prony presented in the literature (ADMM, Consensus updating, data fusion).

II. One channel Prony Algorithm

The basics of Prony analysis are included in this section to make the paper self-sufficient. The Prony algorithm models an uniformly sampled signal by a linear sum of damped complex exponentials as:

$$y(t) = \sum_{k=1}^{L} 0.5 M_k e^{(\alpha_k + j\omega_k)nT + j\phi_k} \tag{1}$$

where n is sample number, L represents the system order, T is sampling period, M_k is amplitude, α_k is damping coefficient, ϕ_k is phase angle and ω_k angular frequency of k^{th} component. To simplify the formulations, consider $L = 2$ and then (1) is reduced to:

$$y(t) = 0.5(M_1 e^{j\phi_1} e^{(\alpha_1 + j\omega_1)nT} + M_2 e^{j\phi_2} e^{(\alpha_2 + j\omega_2)nT}) \tag{2}$$

By considering $Z_1 = e^{(\alpha_1 + j\omega_1)T}$ and $Z_2 = e^{(\alpha_2 + j\omega_2)T}$ as conjugated poles ($Z_2 = {Z_1}^*$), we have:

$$y(nT) = 0.5hZ_1^n + 0.5h^* Z_1^{*n} \tag{3}$$

where, $h = M_1 e^{j\phi_1} = (M_2 e^{j\phi_2})^*$ is the phasor of $y(t)$. By considering N samples per fundamental cycle of the main signal $y(t)$, it is possible to form (4) as detailed form and (5) as abbreviated form.

$$\begin{pmatrix} y[0] \\ \vdots \\ y[n] \\ \vdots \\ y[N-1] \end{pmatrix} = 0.5 \begin{pmatrix} 1 & 1 \\ \vdots & \vdots \\ Z_1^n & {Z_1}^{-n} \\ \vdots & \vdots \\ Z_1^{(N-1)} & {Z_1}^{-(N-1)} \end{pmatrix} \begin{pmatrix} h \\ h^* \end{pmatrix} \tag{4}$$

$$\boldsymbol{Y} = \boldsymbol{J}\ \boldsymbol{H} \tag{5}$$

The estimate of matrix $\boldsymbol{H}$ (phasor data) is obtained by a least square method as:

$$\boldsymbol{H} = (\boldsymbol{J}^T \boldsymbol{J})^{-1} \boldsymbol{J}^T \boldsymbol{Y} \tag{6}$$

In order to estimate the phasor data ($\boldsymbol{H}$), Z_1 should be determined first. By assuming poles ($Z_2 = Z_1^*$) as the characteristic equation roots, (7) is obtained.

$$F(z) = (z - Z_1)(z - Z_1^*) = a_0 z^2 + a_1 z + a_2 \tag{7}$$

To estimate the roots of the characteristic equation, the coefficients ($a_0 = 1$, a_1 and a_2) should be determined first. It is demonstrated that the coefficients of the characteristic equation are calculated based on shifted samples of the input signal as:

$$\begin{pmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[N-1] \end{pmatrix} = \begin{pmatrix} y[-1] & y[-2] \\ y[0] & y[-1] \\ y[1] & y[0] \\ \vdots & \vdots \\ y[N-2] & y[N-3] \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{8}$$

$$\boldsymbol{Y} \ = \ \boldsymbol{Q}\ \boldsymbol{a}$$

Finally, the matrix ($\boldsymbol{a}$) is obtained based on least square as:

$$\boldsymbol{a} = (\boldsymbol{Q}^T \boldsymbol{Q})^{-1} \boldsymbol{Q}^T \boldsymbol{Y} \tag{9}$$

III. Generalized Multi-channel Prony

The accuracy of the phasor estimates is reduced under noisy conditions. To address this problem, multi-channel Prony is proposed in the literature. In the generalized multi-channel Prony, the basic principle is to estimate the phasor based on measurements from several channels of one or more PMUs. Assume that there are m channels of PMU data for a voltage or current that should be analyzed together as multi-channel Prony. In the first step of Prony (to estimate of coefficients of $\boldsymbol{a}$ vector based on (9)), it is possible to formulate the $\boldsymbol{Q}$ matrix and $\boldsymbol{Y}$ vector for each channel (denoted as $\boldsymbol{Q}_i$ and $\boldsymbol{Y}_i$ for the i^{th} channel) separately and then combine them as:

$$\begin{pmatrix} \boldsymbol{Y}_1 \\ \boldsymbol{Y}_2 \\ \boldsymbol{Y}_3 \\ \vdots \\ \boldsymbol{Y}_m \end{pmatrix} = \begin{pmatrix} \boldsymbol{Q}_1 \\ \boldsymbol{Q}_2 \\ \boldsymbol{Q}_3 \\ \vdots \\ \boldsymbol{Q}_m \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{10}$$

According to (10), it is possible to calculate the coefficients of vector $\boldsymbol{a}$ of the multi-channel formulation by a least square algorithm. It is important to note that, the dimension of the vector $\boldsymbol{a}$ in one-channel is the same as for multi-channel and only depends on the system model order. Therefore, the roots of the polynomial (second step in the Prony algorithm) can be extracted similarly to one-channel Prony based on (7). As a last step of the Prony algorithm, the estimate of the phasor (amplitude and phase) is calculated. Similarly to the first step of Prony, it is possible to formulate the $\boldsymbol{J}$ matrix and $\boldsymbol{Y}$ vector for each channel (notation: $\boldsymbol{J}_i$ and $\boldsymbol{Y}_i$ for the i^{th} channel) separately and then combined as:

$$\begin{pmatrix} \boldsymbol{Y}_1 \\ \boldsymbol{Y}_2 \\ \boldsymbol{Y}_3 \\ \vdots \\ \boldsymbol{Y}_m \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \boldsymbol{J}_3 \\ \vdots \\ \boldsymbol{J}_m \end{pmatrix} \begin{pmatrix} h \\ h^* \end{pmatrix} \tag{11}$$

Finally, we can extract the phasor of a signal measured from different channels of one PMU. According to (11), we calculate the phasor matrix $\boldsymbol{H}$ of multi channels by a least square algorithm, as well. According to (10) and (11), the middle matrices have the size of $(m \times N) - by - 2$ where m is number of channels and N is number of sample per cycle.

In Generalized Prony that is based on least square, matrices $(m \times N) - by - 2$ must be inverted (pseudo) for every new time window of data. By increasing the number of channels m, the number of rows will be m times the single channel case. Therefore, it will become time consuming to conduct the multi-channel Prony. Since there are two steps of least square in Prony, recursive least square can be applied to reduce computation time.

IV. Recursive One-Channel Least Square

In the recursive version of the least square algorithm, the initial conditions are used first and followed by updates when new samples are available. Accordingly, the cost function is:

$$\zeta(n) = \sum_{i=1}^{n} \beta(i,n)|x(i)|^2 \tag{12}$$

where β is weighting factor, and $x(i)$ is obtained from subtracting the intended response $d(i)$ from the output $y(i)$:

$$x(i) = d(i) - y(i) = d(i) - \sum_{k=0}^{M-1} \boldsymbol{w}_k(n)\boldsymbol{u}(i-k) \tag{13}$$

where $\boldsymbol{u}(i)$ and $\boldsymbol{w}(n)$ are input and weight respectively. By considering an exponential form for weighting (λ^{n-i}), where $0 < \lambda < 1$, the standard least square can be written as:

$$\boldsymbol{w}(n) = (\sum_{i=1}^{n} \lambda^{n-i}\boldsymbol{u}(i)\boldsymbol{u}(i)^T)^{-1}(\sum_{i=1}^{n} \lambda^{n-i}\boldsymbol{u}(i)d(i))$$
$$\boldsymbol{w}(n) = (\boldsymbol{\psi}(n))^{-1}\boldsymbol{\varphi}(n) \tag{14}$$

According to previous sample information, we have:

$$\boldsymbol{w}(n-1) = (\boldsymbol{\psi}(n-1))^{-1}\boldsymbol{\varphi}(n-1) \tag{15}$$

It is possible to relate variables of sample n with its previous sample as:

$$\boldsymbol{\psi}(n) = \lambda\boldsymbol{\psi}(n-1) + \boldsymbol{u}(i)\boldsymbol{u}(i)^T \tag{16}$$

To estimate the parameter by least square based on (15), the inverse matrix $\boldsymbol{\psi}$ is needed. To avoid the full matrix inversion, the inverse matrix modification lemma is used as:

$$\boldsymbol{\psi}^{-1}(n) = \boldsymbol{P}(n) = \lambda^{-1}\boldsymbol{P}(n-1)(1 - \boldsymbol{k}(n)\boldsymbol{u}(n)^T) \tag{17}$$

$$\boldsymbol{k}(n) = \frac{\lambda^{-1}\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{1 + \lambda^{-1}\boldsymbol{u}(n)^T\boldsymbol{P}(n-1)\boldsymbol{u}(n)} \tag{18}$$

where $k(n)$ is a gain vector and $P(n)$ is inverse of the correlation matrix. Finally, the main update equation is:

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \boldsymbol{k}(n)(d(n) - \boldsymbol{u}(n)^T\boldsymbol{w}(n-1)) \tag{19}$$

We can summarize all equations of Recursive Least Square (RLS) as:

$$\boldsymbol{k}(n) = \frac{\lambda^{-1}\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{1 + \lambda^{-1}\boldsymbol{u}(n)^T\boldsymbol{P}(n-1)\boldsymbol{u}(n)} \tag{20}$$

$$\alpha(n) = d(n) - \boldsymbol{u}(n)^T\boldsymbol{w}(n-1) \tag{21}$$

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \boldsymbol{k}(n)\alpha(n) \tag{22}$$

$$\boldsymbol{P}(n) = \lambda^{-1}\boldsymbol{P}(n-1) - \lambda^{-1}\boldsymbol{k}(n)\boldsymbol{u}(n)^T\boldsymbol{P}(n-1) \tag{23}$$

Detailed explanations of recursive least square can be found in [25].

V. Recursive Multi-channel Prony

Generally, different measurement channels have different noise levels. For instance, some of channels are with low noise, while others are with high noise. Even though the second set of measurements is less reliable, we should never discard measurements, no matter how unreliable they are. Therefore, presume that each channel is taken under different conditions so the variance of the measurement noise is different. Therefore, the covariance matrix is given by:

$$\boldsymbol{R} = \begin{pmatrix} \delta_1^2 & 0 & ... & 0 \\ 0 & \delta_2^2 & ... & 0 \\ \vdots & \vdots & ... & 0 \\ 0 & 0 & ... & \delta_M^2 \end{pmatrix} \tag{24}$$

where δ_1^2 , δ_2^2 ,$\cdots$, δ_M^2 are variances of different channels. In this circumstance, the cost function (ξ) is expanded as:

$$\xi = (\boldsymbol{d} - \boldsymbol{w}\boldsymbol{u})^T\boldsymbol{R}^{-1}(\boldsymbol{d} - \boldsymbol{w}\boldsymbol{u}) \tag{25}$$

According to the partial derivative of the cost function, the best estimate of $\boldsymbol{u}$ is calculated as:

$$\boldsymbol{u} = (\boldsymbol{w}^T\boldsymbol{R}^{-1}\boldsymbol{w})^{-1}\boldsymbol{w}^T\boldsymbol{R}^{-1}\boldsymbol{d} \tag{26}$$

According to the equations of recursive least square and weighted least square, we can replace the gain (20) with (27) in the recursive weighted least square as:

$$\boldsymbol{k}(n) = \frac{\lambda^{-1}\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{\boldsymbol{R} + \lambda^{-1}\boldsymbol{u}(n)^T\boldsymbol{P}(n-1)\boldsymbol{u}(n)} \tag{27}$$

As discussed in the section III, the solution of the multi-channel Prony method involves three estimation stages where stage 1 and stage 3 involve least square algorithms, while stage 2 involves finding the roots of the polynomial. The equations of the recursive least square are presented in (20-23). Additionally, with multiple channels, the channels may represent different quantities and have to be treated with different weights as shown in (24). This property leads to more accurate estimates and improved alignment of reconstructed signals. Therefore, the objective functions of the two stages of weighted least square in multi-channel Prony analysis are:

$$\underset{\boldsymbol{a}}{minimize} \sum_{i=1}^{i=M} (\boldsymbol{Q}_i\boldsymbol{a} - \boldsymbol{Y}_i)^T\boldsymbol{R}(\boldsymbol{Q}_i\boldsymbol{a} - \boldsymbol{Y}_i) \tag{28}$$
$$\underset{\mathbf{H}}{minimize} \sum_{i=1}^{i=M} (\boldsymbol{J}_i\boldsymbol{H} - \boldsymbol{Y}_i)^T\boldsymbol{R}(\boldsymbol{J}_i\boldsymbol{H} - \boldsymbol{Y}_i)$$

Finally, these two objective functions are solved with recursive least square.

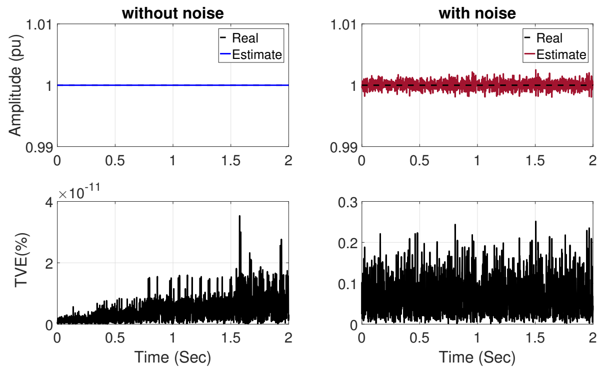

Fig. 1. Amplitude estimation (analysis of noise impact)

VI. Simulation Results

To analyze the performance of the recursive multi-channel Prony (proposed method), the following subsections are included: Impact of noise on Prony, Generalized multi-channel Prony, Recursive multi-channel Prony, Performance under step change of amplitude and phase, Performance under ramp variation of frequency, Comparison with three other methods and finally Phasor estimate based on real data.

A. Impact of Noise on traditional Prony

As the first step, the influence of noise on the phasor estimation using the Prony algorithm is given and thereby the motivation for the multi-channel Prony approach. The two main signals in (29) are: the first one (y_1) is without noise condition and the second one (y_2) is under noisy condition.

$$y_1(t) = M_1 e^{-\alpha t} cos(2\pi f_1 t + \theta_1) \tag{29}$$
$$y_2(t) = M_1 e^{-\alpha t} cos(2\pi f_1 t + \theta_1) + \epsilon(t)$$
$$M_1 = 1; \alpha = 0.05; f_1 = 50; \theta_1 = 0; \delta_1^2 = 10^{-4}$$

The signals presented in (29) are synthesized in MATLAB. The fundamental frequency is $50Hz$ and there are 20 samples per cycle. Size of the data window equals one fundamental cycle (20 samples) and the phasors are obtained by the sliding window. The noise (ϵ) is white noise with zero mean and distinct variance value (δ_1^2). Amplitude estimation without-noise and under noisy conditions are shown in Fig.1. Total Vector Error (TVE) as a criterion for phasor estimation error (defined in the IEEE standard for synchro-phasors) is used in this section and the results are shown in Fig.1 for both conditions (with and without noise). The TVE is presented in (30) where X_r is the real phasor and X_e is the estimated phasor. By comparing the results, it is clear that the noise has negative impact on performance of the Prony.

$$TVE = \frac{|X_r - X_e|}{|X_r|} \tag{30}$$

B. Generalized multi-channel Prony

According to the previous section, noise reduces the accuracy of the estimates. To solve this problem, more channels are applied in the estimation process. In this subsection, results using two-, three- and four-channels are presented as well as the impact of adding more channels are examined. Presume that there are four channels with different noise levels as:

$$y_m(t) = M_1 e^{-\alpha t} cos(2\pi f_1 t + \theta_1) + \epsilon_m(t)$$
$$m = 1, 2, 3, 4; M_1 = 1; \alpha = 0.05; f_1 = 50; \theta_1 = 0;$$
$$\delta_1^2 = 10^{-4}; \delta_2^2 = 10^{-5}; \delta_3^2 = 10^{-6}; \delta_4^2 = 10^{-7}; \tag{31}$$

TABLE I
Simulation results of subsection VI.B

Number of channels	**(TVE%)**	**FLOPs**
One-channel	0.2151	71516
Two-channel	0.0658	539780
Three-channel	0.0205	1788844
Four-channel	0.0063	4202708

The fundamental frequency is $50Hz$ and there are 20 samples per cycle. Since different channels have different noise level, weighted generalized multi-channel Prony is used in this section. The simulation results of this section are summarized in Table I. Floating Point Operations (FlOPs) is used as an index of computational burden. FLOP is a count of operations carried out by a given algorithm or a computer program.

The superiority of multi-channel Prony can be observed from Table I. According to this table, the accuracy of the estimates increases when more channels are added to the estimation process. Four-channel is better than three-channel, three-channel is better than two-channel and finally two-channel is better than one-channel. However, multi-channel Prony improves the accuracy, while the computation burden (number of FLOPs for estimating one sample of phasor with 20 samples per data window) increases when employing more channels. This is the motivation for proposing a recursive solution.

C. Recursive multi-channel Prony

According to subsection VI.B, the accuracy of the estimates are increased by adding new channels to the estimation process. However, by increasing the number of channels, the dimensions of the matrices in the Generalized multi-channel Prony increases significantly and the computation burden will increase accordingly. To reduce the processing time and make multi-channel Prony applicable for real time applications, a recursive algorithm is used in this subsection. Again, consider the four channels data presented in (31). The fundamental frequency is $50Hz$ and the sampling frequency is $F_s = 1KHz$. These channels are processed by recursive multi-channel Prony and the results are tabulated in Table II. According to Table II, the same conclusion as in subsection VI.B can be made. The accuracy of estimates is improved with more channels while computational burden (number of FLOPs for estimating one sample of the phasor) is a bit increased. However, by comparing Table II with Table I, it is clear that the computational burden of the recursive solution is significantly lower than the generalized solution of multi-channel Prony.

TABLE II
SIMULATION RESULTS OF SUBSECTION VI.C

Number of channels	(TVE%)	FLOPs
One-channel	3.5850	171
Two-channel	0.3620	336
Three-channel	0.2126	589
Four-channel	0.0388	887

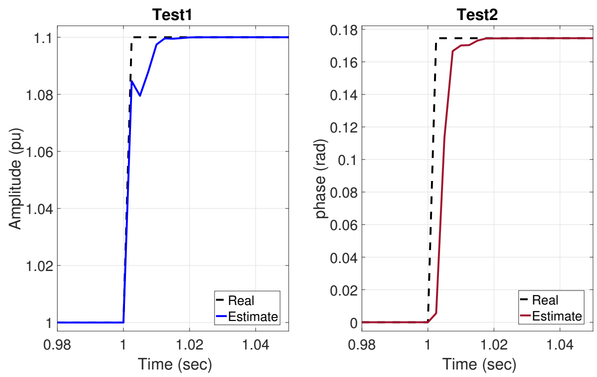

Fig. 2. Step changes of amplitude and phase

It is important to note that error estimation of the recursive solution is higher due to the inherent property of recursive algorithms that error of the previous estimate will go to the next. However, the accuracy is lower than the defined threshold (TVE=1%) in the IEEE synchrophasor standard when there are more than one channel in the estimation procedure. Therefore, it can be concluded that the recursive multi-channel Prony is a better choice than the generalized multi-channel Prony.

D. Performance under step changes in phase and magnitude

According to the IEEE standard for synchrophasor measurements for the power systems, a step change is considered for the amplitude and phase to examine the dynamic performance of the recursive Prony. The test case has the form of:

$$y(t) = A[1 + k_x f(t)]cos(\omega_0 t + k_a f(t)) \quad (32)$$
$$A = 1; \omega_0 = 2\pi 50;$$
$$k_x = 0.1; k_a = pi/18; f(t) = u(t - t_0); t_0 = 10$$

where A is the amplitude of the input signal, ω_0 is the nominal power system frequency, $f(t)$ is a unit step function, k_x is the magnitude step size and k_a is the phase step size. This case is a transition test between two steady state conditions (amplitude and phase are increased at $t = 1sec$ by ten percent). Two tests are specified: *Test1*: amplitude step ($k_x = 0.1$, $k_a = 0$), *Test2*: phase step ($k_x = 0$, $k_a = pi/18$) and the results are shown in Fig.2. According to this figure, the estimated amplitude and phase track their real values accurately after a transient period.

E. Performance under ramp variation of system frequency

Performance under ramping of the system frequency is another defined test in the IEEE standard for synchrophasor.

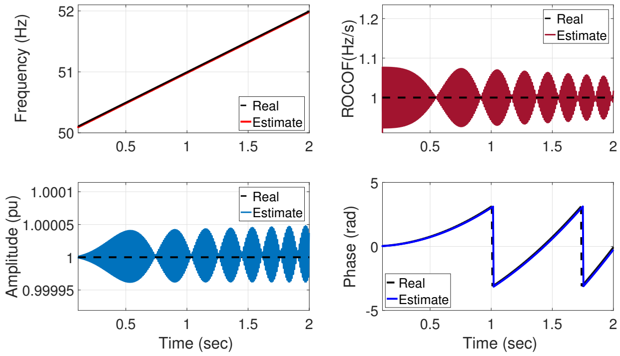

Fig. 3. Performance during ramp of system frequency

Mathematically the input signal may be represented as:

$$y(t) = Acos(\omega_0 t + \pi R_f t^2)$$
$$A = 1; \omega_0 = 2\pi 50; R_f = 1; \quad (33)$$

where R_f is the frequency ramp rate in Hz/s. The signal test is applied and the estimated frequency, Rate Of Change Of Frequency ($ROCOF$), amplitude and phase are shown in Fig.3. According to the figure, the recursive Prony can track all these quantities accurately so this satisfies the requirement of the IEEE standard for PMU (TVE=1%, frequency error (FE)= 0.005 Hz as well as Rate of change of frequency error (RFE) =0.1 Hz/s).

F. Comparison with three other methods

In this subsection, the proposed method is compared with three other methods for multi-channel Prony, which have already been presented in literature. These methods are:

Data fusion: Data fusion-based multi-channel Prony analysis [19]: This method implements the Kalman filter based data fusion approach in the Prony analysis with multi-channel.

Consensus: multi-channel Prony by consensus and sub-gradient update [21]: This method is an iterative method that deal with multi-channel Prony by utilization of consensus and sub-gradient update.

ADMM: This method is an iterative method based on an optimization algorithm, namely, alternating direction method of multipliers ($ADMM$) [20]. To compare these four methods, four channels of one PMU is considered as presented in (31). The fundamental frequency is $50Hz$ and the sampling frequency is $F_s = 1kHz$. Amplitude estimates using these four methods are shown in Fig.4. In addition, the index of TVE and the computational burden (number of FLOPs for estimating one sample of phasor) of all these methods are tabulated in Table III (ADMM and Consensus methods are optimized by 40 iterations). According to these results, although all these methods managed to estimate the phasor accurately, the recursive multi-channel Prony provides the best results since it offers the lowest computational burden with acceptable accuracy ($TVE < 1\%$).

TABLE III
SIMULATION RESULTS OF SUBSECTION VI.F

Methods	**(TVE%)**	**FLOPs**
Data fusion	0.0064	4159
Consensus	0.1074	77600
ADMM	0.1458	92320
Proposed method	0.0375	887

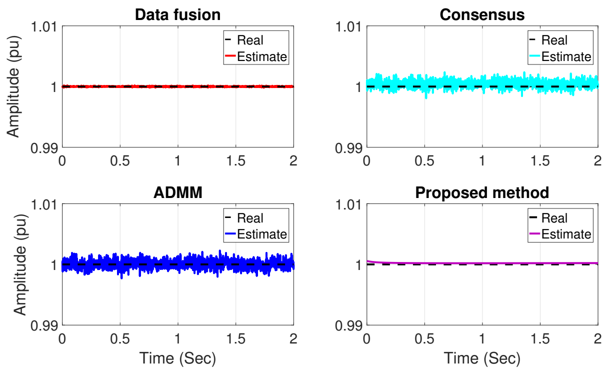

Fig. 4. Amplitude estimation by four methods

VII. CONCLUSION

PMU measurements provide multiple channel signals and these may be optimally utilized by an adequate method for multiple channel data. With more channels, the Prony algorithm is more accurate. Additionally, by considering multiple signals the influence of noise in the phasor estimation process is reduced. Therefore, it is possible to reach significant improvements in phasor estimation by using multiple signals under noisy condition. However, multi-channel Prony significantly increases the problem size when the number of channels is increased. Therefore, real time implementation of the generalized multi-channel Prony method is limited mainly due to the requirement of matrix inversion. For multiple-channel Prony, this paper proposed a recursive solution. The approach shows to be much more efficient compared to other established multi-channel Prony algorithms. Different test signals are employed in different test cases to show the capabilities of the proposed method. By using the recursive multi-channel Prony, the computation effort is reduced for generalized multi-channel Prony and makes it feasible for real time applications. As a future research direction, nonlinear signal processing algorithms will be studied and used to improve the performance of the phasor estimation algorithms under different conditions.

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