Forecasting Price Distributions in the German Electricity Market

Forthcoming Book Chapter

Sjur Westgaard

Florentina Paraschiv

Lina Lassesen Ekern

Ingrid Naustdal

Malene Roland

Department of Industrial Economics

and Technology Management,

Faculty of Economics and Management,

NTNU,

NO-7491 Trondheim

Abstract

Electricity price distributional forecasts are crucial to energy risk management. In this paper we model and forecast Value at Risk (VaR) for the German EPEX spot price using variable selection with quantile regression, exponential weighted quantile regression, exponential weighted double kernel quantile regression, GARCH models with skewed t error distributions, and various CAViaR models. Our findings are; (1) exponential weighted quantile regression tends to perform best overall quantiles and hours., and (2) different variables are selected for different quantiles and different hours. This is not surprising since the there is a non-linear relationship between fundamentals and the electricity price. This non-linear relationship is different between the different hours as the dynamics of the intra-daily prices are different. Quantile regression has the feature of capturing these effects. As the input mix has changed in Germany over the last years, exponential weighted quantile regression allowing for time-varying parameters can also capture the effect of changing quantile sensitivities over time. Exponential weighted quantile regression is also easy model to implement relative to the other models investigated in this study. Thus, we recommend this model together with carefully selecting fundamentals for given hours and quantiles when the aim is to forcast VaR for German electricity prices.

1 Introduction

Electricity is non-storable by nature, and a stable power system requires a constant demand & supply balance. This makes electricity a unique commodity with complex price dynamics and relations to fundamentals. Prices are characterised by sudden (positive and negative) spikes, high volatility and volatility clustering, and seasonality patters over the day, week, and year. Thus, forecasting in electricity markets is arguably more challenging than in traditional financial markets.

Electricity price forecasts are important inputs to energy companies decision-making. For day-to-day market operations, accurate forecasts of short-term prices are crucial. Price forecasts serve as aids for producers, retailers, and speculators who seek to determine their optimal short-term strategies for production, consumption, hedging and trading. Uncontrolled exposure to market price risk can have devastating consequences for market participants (see Deng and Oren (2006) for a discussion). This has led to an increased focus on risk management in power markets the last years.

The recent introduction of smart grids and renewable integration has created more uncertainty in future supply and demand conditions where both very low (often negative) and high prices might occur. Over the last 15 years, the bulk of research has been concerned with predicting the mean of electricity prices. As stakeholders require explicit control of the risk of both high and low extreme prices, point forecasts are inadequate in many cases (see Nowotarski and Weron (2017) for more details). Academics and practitioners have come to understand that probabilistic electricity price forecasting is now more important for energy systems planning, risk management and operations than ever before.

Value-at-Risk (VaR) is the market standard for risk measurement and is simply a given quantile that will be found directly from a price distribution forecast. Despite the importance of measure risk management in power markets, Weron (2014) and Nowotarski and Weron (2017) finds that distribution forecasting is “barely touched upon” in electricity price forecasting literature. This statement is supported by Bunn et al. (2016), who argue that for electricity markets, VaR forecasting remains a highly “under-researched” area. Maciejowska et al. (2016) claim that the lack of such research is likely due to the embedded complexity of the research problem compared to point forecasting. The sparseness in current literature, combined with the importance of density forecasting, is our motivation for investigating how state-of-the-art econometric models can be applied to forecast VaR. We have chosen to look at the German market for several reasons; (1) It is probably the most important electricity market in Europe, (2) Data quality, transparency, and access are excellent, (3) The input mix of production towards renewables has changed a lot over the years, challenging us to build models that capture this dynamics. We need to capture models that can capture the nonlinear sensitivities of electricity prices to fundamentals (because if the convex supply curve) as well as time varying sensitivities to fundamentals.

We argue for using quantile regression (QR) models when forecasting price distributions and estimate value at risk (VaR). QR models estimate each quantile with a distinct regression. Moreover, they are simple, insensitive to outliers, avoid distributional assumptions and helps us to capture non-linear sensitivities (e.g. that electricity price sensitivities to gas prices should be higher when electricity prices are higher since gas power plants are used in such a regime "at the end" of the supply or merit order curve). In addition we also apply QR models with time varying parameters taking into account that the input mix has changed over time[1]. These models are exponentially weighted QR (EWQR) and exponentially weighted double kernel QR (EWDKQR) proposed by Taylor (2008b). We compare these alternatives with common benchmarks in the VaR prediction literature using GARCH and CAViaR type of models.

By using knowledge of market conditions, we form a set of fundamental factors (supply and demand variables) and perform a variable selection procedure for each trading period and quantile with the aim of (1) proper in-sample fit, and (2) optimal out-of-sample fit for a given hour and a given quantile. Hence, in additional to the dimension of model choice, we stress the fact variable selection should be carefully monitored as different fundamentals influence hours and quantile differently.

[1] For example, the sensitivity of the electricity price to wind production is allowed to vary over the quantiles of the electricity prices in a static QR model but not over time. Since the share of wind production in Germany has increased a lot over the last years, it is reasonable that the sensitivity for a given quantile also will change over time. This feature can be captured in a dynamic QR model.

To sum up; the overall goal of our work is threefold. First, we want to identify appropriate fundamental variables for selected hours and quantiles of the price distribution. Secondly, we assess the gain of using more complex QR models compared to traditional QR models and given GARCH and CAViaR benchmarks. No such comprehensive Value at Risk prediction study for the German electricity market, according to our knowledge, has not yet been performed.

The report is structured as follows: In chapter 2 we review relevant literature on fundamental electricity price modelling and VaR forecasting. Next, we describe the German power market and price formation process in chapter 3. In chapter 4 we present and analyse the data set. In chapter 5, we give a explanation of the models and evaluation procedures for distributional forecasts. We present and discuss the empirical results in chapter 6, and conclude in chapter 7.

2 Literature review

We position ourselves between the following groups of literature; (1) VaR forecasting of asset markets and more specifically energy commodities, and (2) Fundamental analysis of electricity price formation.

VaR forecasting is complicated by the fact that high frequency asset prices (including electricity prices), exhibit challenging data features. Time varying volatility, skewness and kurtosis induce a lot of complexity to the modelling of VaR (see Hartz et al. (2006) for more discussion). Kuester et al. (2006) provide a comprehensive review of VaR prediction strategies that can solve some of this issue.

In our context, models that can be obtained for VaR forecasts are to be classified into three main categories:

- ***Fully parametric models*** *assuming a given error distribution. (e.g. a GARCH-skew-t model, EVT models etc.).*
- ***Non parametric approaches such as Historical simulation*** *where one computes empirical quantiles based on past data. Historical simulation can be filtered taking into account time-varying volatility*

- ***Semi-parametric such as Quantile regression** that directly models specific quantiles with no assumption regarding the error distribution.*

Most of the application of these models have been done without usage of fundamentals when we look at the energy market literature. Gurrola-Perez and Murphy (2015) evaluate historical and filtered historical simulation models for energy markets. The latter approach should be used as in improves the distributional forecast greatly according to the authors. At present, research on EVT for estimating VaR in energy markets is sparse. However, examples are found in Bystrom (2005), Chan and Gray (2006) and Paraschiv and Hadzi-Mishev (2016), who all report that the results are encouraging. Garcia et al. (2005) use two GARCH models to forecast spot prices in the Spanish and Californian market; one with price as the only variable, and one including demand. They benchmark these against an ARIMA model. They find that GARCH with price only out- performs ARIMA as time varying volatility and price spikes are present. Moreover, adding demand as explanatory variable further improves the forecasting performance. GARCH models when assuming t or skewed t distribution shows promising results when forecasting VaR for commodies, including energy commodities (See Giot and Laurent (2003) and Fuss et.al. (2010)). Conditional Autoregressive Value-at-Risk (CAViaR) models by Engle and Manganelli (2004) model quantiles directly as an autoregressive process. The estimation is based on a quantile regression approach. The performance of CAViaR models are promising for electricity markets (see Fuss et al. (2010) and Bunn et al. (2016)). Bunn et al. (2016) also finds that classical quantile regression models gives excellent VaR forecast for UK electricity prices. Florentina and Hadzi-Mishev (2016) use a combination of GARCH and EVT to investigate the tails of the German electricity price change distribution. They find that the model delivers relatively precise quantile estimates, but that the quality of the estimates is sensitive to the threshold selected for the tail.

The other area of research is fundamental models for electricity markets. Fundamental models try to capture price dynamics by modelling the impact of exogenous (supply & demand) factors on the electricity prices. The main motivation for using such models is that characteristic electricity price patterns are results of adaption too fundamentals. Prices are also functions of different drivers in specific trading period. Prices have also different sensitivities to fundamentals depending on the level of electricity prices. In a comprehensive review of electricity forecasting literature, Weron (2014) finds that the majority of models include fundamentals. Fundamental type of models have been applied to the Nordic electricity market where the effects of water reservoir levels, load, and gas & coal prices among other variables have been found important for the price formation. Examples of such studies are Lundby and Uppheim (2011), Huisman at al (2015a,b), and Fleten et al (2016). All these studies give empirical insights on the nonlinear influence of the fundamentals to the electricity prices and well as time varying sensitivity to these fundamentals. The methods applied are quantile regression, various non-linear models, and state space models. In the German market, there are several interesting studies. Paraschiv et al., (2014, 2016) emphasise the importance of using fundamentals, and find variables for renewable power particularly influential. Her focus is the investigation on how the sensitivities to fundamentals changes over time applying a State Space model with Kalman filtering. Time varying parameters are motivated because evolving factors, like technology, market structure and participant conduct, affect the underlying price formation dynamically over time. Follow up studies using quantile regression also highlight that sensitivities changes in relation to the level of electricity prices (Hagfors et al. (2016a)). Prediction of extreme price occurrences in the german day-ahead electricity market by using non-linear discrete choice models are found in (Hagfors et al. (2016b)). In the UK market there are several papers investigating the price formation with fundamentals using state space model (Karakatsani and Bunn (2008)), regime switch models with non-linear transition functions of fundamentals (Bunn and Chen (2008)), and quantile regression (Bunn et al. (2016), and Hagfors et al. (2016)). One of the few papers focusing on predicting prices in the UK el-market is Gonzales et al. (2012). They find improved accuracy by including fundamentals when forecasting UK spot prices. Moreover, they observe that the variable coefficients in their models evolve remarkably over time. Thus, they argue that dynamic specifications are necessary, and that forecasting models should be re-estimated day by day. They suggest constant monitoring of market conditions in order to select the appropriate model specification and fundamental drivers. The only paper we have found so far on predicting prices *distributions* in the UK el-market is Bunn et. al. (2016). They find improved forecasting accuracy by including fundamentals. Bunn et al. (2016) investigate the UK market using various forms of quantile regression and compare the out-of-sample forecast with various GARCH and CaViaR models. The general finding is that quantile regression models including fundamentals performs just as well these advanced models at a much lower cost of implementation. Quantile regression models including fundamentals are also much easier to understand and enables the risk management to perform scenario analysis and investigate the effect on VaR of changing values of risk factors directly. Market participants can use these in risk management, by planning for a range of price scenarios given different input ranges for the fundamental variables. Maciejowska and Weron (2016) also find that inclusion of fundamentals generally improves the forecasting performance of UK baseload prices. However, they also emphasise that variable selection is crucial. For example, they observe that including gas prices increases forecasting performance, whereas variables related to system-wide demand and CO_2 prices worsen price predictions. The authors conclude that there is no general answer as to which fundamentals are the best to include, as the optimal selection depends on both forecasting horizon and trading period.

We want to extend the literature and understanding of electricity price distributional forecasting using dynamic quantile regression models. This is an area that lack investigation. It is particular interest in markets such as the German electricity market where the input mix (and hence sensitivities to the drivers) clearly has changed over time. Particularly we want to follow the models proposed by Taylor (2008a). He introduces exponentially weighted quantile regression (EWQR). The extension is motivated by the trade-off between including too few observations and getting large sampling errors, and including too many and getting a model that reacts slowly to changes in the true distribution. EWQR attempts to resolve this by placing exponentially decaying weights on the observations, which gives greater emphasis to newer observations. To the best of our knowledge, EWQR has received little attention in electricity price forecasting literature. It is challenging to estimate extreme quantiles due to the sparseness of observations in the tails. This is Taylor's (2008b) motivation for extending the EWQR model further, to exponentially weighted double kernel quantile regression (EWDKQR). The EWDKQR method is based on the paper by Jones and Yu (1998), who argue that double-kernel methods are useful for calculating quantiles. In empirical studies, Taylor finds that EWDKQR performs worse than EWQR in terms of hit percentage. However, the dynamic properties of the quantiles are better explained by the EWDKQR model.

To sum up; we want to extend the analysis and understanding of electricity price formation with fundamentals using dynamic quantile regression models, and investigate whether these model improve the forecast compare to static quantile regression, GARCH, and CaViaR type of models. We also want to invest specifically which combinations of drivers / fundamentals that should be used in predicting VaR for specific hours and quantiles.

3. The German electricity market

In this chapter, we describe the German electricity market, the price drivers, and the price formation process. This will serve as a guidance and motivation for choosing the fundamental variables.

The European Power Exchange (EPEX) is the main trading platform for electricity prices in Europe. It offers trading, clearing and settlement in both the day-ahead- and intraday markets. The day-ahead, hourly prices in Germany are traded on EPEX and are referred to as ”Phelix”. The day-ahead market is the primary market for power trading. Here, buyers and sellers make hourly contracts for delivery of power the following day. This happens through a daily auction at 12.00pm, where the market clearing price is determined by matching demand and supply. The intraday market supplements the day-ahead market and helps secure necessary demand-supply balance.

Seasonal fluctuations, substantial volatility clustering, large spikes and increasing occurrences of negative prices for certain hours characterise the German electricity market (see Reisch and Micklitz (2006), Paraschiv et al. (2014), and Hagfors et al. (2016b) for more discussions).

Energy input mix

Table 3.1 shows the development of the energy mix in Germany from 2010 to 2016. It illustrates that power production in Germany mainly relies on fossil fuel power, particularly coal with 40.3% of the total production in 2016. Moreover, there is a large share of intermittent renewable energy in the form of wind and solar power. The increase in renewable energies and reduction in nuclear power are the most notable developments during the period. The latter is due to the German government’s decision to phase out nuclear energy within 2022. Regulatory changes are also the key driving force for the growth in renewables, as several subsidies and policy measures have been introduced during the recent period (see (Federal Minsitry for Economic Affairs and Energy (2017)).

Source		2010	2011	2012	2013	2014	2015	2016
Coal		41.6	42.9	44.1	45.2	43.8	42.1	40.3
Nuclear		22.2	17.6	15.8	15.3	15.5	14.2	13.1
Natural gas		14.1	14.1	12.2	10.6	9.7	9.6	12.4
Oil		1.4	1.2	1.2	1.1	0.9	1.0	0.9
Renewable energies:		16.5	20.1	22.6	23.7	25.8	29.0	29.0
Wind		6.0	8.0	8.1	8.1	9.1	12.3	11.9
Solar		1.9	3.2	4.2	4.9	5.7	6.0	5.9
	Biomass	4.6	5.2	6.1	6.3	7.7	6.9	7.0
	Hydro power	3.3	2.9	3.5	3.6	3.1	2.9	3.2
	Waste to energy	0.7	0.8	0.8	0.8	1.0	0.9	0.9
Other		4.2	4.1	4.1	4.1	4.3	4.1	4.2

Table 3.1: Electricity production in Germany by source (%). Data from AG Energibalanzen e.V. (2017) and Clean Energy Wire (2017).

Demand

Since electricity is a flow, it is produced and consumed continuously. The non-storable nature of electricity entails that a constant balance between supply and demand is necessary to ensure power system stability. Hence, hourly price variations are largely due to fluctuations in demand. Therefore, the dynamics of hourly prices will behave very different. Demand is a function of temperature, seasonality and consumer patterns, which give rise to the periodic nature of electricity prices. As few options are available to consumers in response to price changes, demand is highly price inelastic in the short term. Positive price spikes are often caused by high (unexpected) demand. Producers with market power may also offer and create market prices substantially above marginal costs in times of scarcity and high demand. Including lagged price and volatility behaviour might capture some of these effects (see Bunn et al. (2016) for more discussions of how adaptive behaviour can be specified in the model specifications).

Supply

The *merit order curve* plays a vital role in the electricity price formation process. This the sorted marginal cost curve of electricity production, starting with the least expensive technologies to the left of the curve. Generally, the plants with the lowest marginal costs are the first to use to meet demand. Thus, we can use the merit order curve to determine the price setting technology, i.e. the production technology located at the intersection between supply and demand. The German merit order curve increases areas of flat and convex regions, hence induce non-linearity in the elasticity between fundamentals and prices. During periods of low demand, base load power plants, such as nuclear and coal, usually serve as price setting technologies. These plants are inflexible, due to high ramp-up costs. Contrary, in times of high demand, prices are set by expensive peak load plants, like gas and oil power plants. These facilities have high flexibility, high marginal costs, and give rise to the convex shape of the merit order curve. With the lowest marginal cost, renewable energy sources are at the bottom of the merit order curve. Increased supply of renewable energy shifts the curve to the right, and thus lowers power prices. Coal is the largest source of electricity in Germany. Hence, coal is a generating technology in the mid-region of the supply function where demand tends to be most of the time. CO_2-producing companies are obliged to buy emission allowances. Since coal-fired power plants and to a lesser degree gas-fired power are CO_2 intensive, the price of CO_2 allowances influence the marginal cost of coal and gas power plant in a different way. During periods of high prices for emission allowances, a phenomenon called *fuel switch* may occur. This is a change in the merit order curve, where the marginal production costs of more efficient gas-fired power plants become less than those of CO_2 intense coal-fired power plants (see Erni (2012) and Paraschiv et al. (2014) for more discussion).

Among the renewable energy sources in Table 3.1, wind and solar energy have attracted the most attention in Germany over the past years. In 2016, they contributed to 18% of the total production in Germany. The supply of wind and solar energy is determined by meteorological conditions and features seasonal patterns. A notable observation from Paraschiv et al. (2014) is that wind infeed tends to be higher in the early morning and the afternoon hours. Due to intermittency, renewable energy sources pose significant challenges for modern energy markets (see EPEX Spot (2017)). Hours with increased supply of renewable energy cause difficulties for inflexible facilities that should run continuously. This is because the inflexible base load facilities have shutdown and start-up costs, forcing them to accept negative marginal returns in order to generate continuously. This has a lowering effect on electricity prices. Hagfors et al. (2016c) find that negative prices largely are caused by high wind production at times when demand is low. Thus, negative price spikes occur mainly at night.

Reserve margin is a commonly considered supply side factor in literature. It is defined as available supply minus demand. Bunn et al. (2016) argue that spot prices are sensitive to supply shocks such as plant outages, and that expectations of spot prices involve consideration of the reserve margin.

4. Data analysis

Our dependent variables are selected prices from EPEX observed between 01.01.2010 and 31.08.2016. The main reason for this choice is the Equalisation Mechanism Ordinance, which came into force January 1st 2010. This act induced a significant increase in the use of renewable energy and caused large changes in the EPEX input mix. Moreover, some of the data, like solar and power plant availability, are incomplete or not available for earlier time periods. The spot price data has hourly resolution, which means that we have 58 440 price observations. However, since each hour is a separate trading period, we treat the price data as 24 independent time series with 2435 data points each. We have selected hours 3, 8 and 19 as the periods we aim to model as representatives of hours with different dynamics.

Our selection of independent variables are based on our discussion in chapter 3. The data set applied in this analysis consists of the variables shown in table 4.1 and table 4.2;

Variable	**Daily**	**Hourly**
Phelix spot price		X
Coal price	X	
Gas price	X	
Oil price	X	
CO_2 allowance price	X	
EU Expected wind		X
Expected solar (PV) infeed		X
Expected power plant availability (PPA)	X	
Expected demand		X

Table 4.1: Data granularity for our dependent and independent variables used in the analysis. We apply hour 3,8, and 19 with the associated forecasts in our analysis. The data period is from 1Jan2010 to 31Aug2016.

Variable	Units	Description	Data Source
Spot price	EUR/MWh	Market clearing price	European Energy Exchange: http://www.eex.com
Coal price	EUR/12,000 t	Latest available price (daily auctioned) of the front-month ARA futures contract before the electricity price auction takes place	European Energy Exchange: http://www.eex.com
Gas price	EUR/MWh	Latest price of the NCG Day Ahead Natural Gas Spot Price on the day before the electricity price auction takes place	Bloomberg, ticker: EGTHDAHD Index
Oil price	EUR/bbl	Latest price of the active ICE Brent Crude futures contract on the day before the electricity price auction takes place	Bloomberg, ticker: CO1 Comdty
CO2 price	EUR 0.01/EUA 1,000 t CO2	Latest available price of the EEX Carbon Index (Carbix), daily auctioned at 10.30 am	European Energy Exchange: http://www.eex.com
Expected wind and PV infeed	MWh	Sum of expected infeed of wind electricity into the grid, published by German transmission systems operators in the late afternoon following the electricity price auction	Transmission system operators: http://www.50Hertz.com, http://www.amprion.de, http://www.transnetbw.de, http://www.tennettso.de
Expected PPA	MWh	Forecast of expected power plant availability production (voluntary publication) on the delivery day (daily granularity), published at 10:00 am	European Energy Exchange & transmission energy operators: ftp://infoproducts.eex.com
Expected demand	MWh	Sum of the total vertical system load and actual wind infeed for the same hour on the last relevant delivery day	Transmission system operators: http://www.50Hertz.com, http://www.amprion.de, http://www.transnetbw.de, http://www.tennettso.de

Table 4.2: Description of dependent and independent variables used in our analysis. We apply hour 3,8, and 19 with the associated forecasts in our analysis. The data period is from 1Jan2010 to 31Aug2016.

Figure 4.1 shows the development of spot prices for hours 3,8,19. We see occurrences of negative price spikes in hour 3, and positive spikes in hours 8 and 19. The different hours clearly display different price characteristics due to different demand conditions and usage of different technologies for electricity production. Table 4.3 gives more details of the properties of the various prices. All prices where found stationary using Dickey Fuller tests and we found significant serial correlation for several lags (not shown in the table). The mean of the prices shows in general a falling trend. For the whole period hour 8 and 19 have a significant higher mean price (Euro/MWh) than hour 3. Negative prices are found every year for hour 3 prices, with the lowest price of -221 Euro/MWh for the year 2012. There is a trend downward in the absolute values of negative prices indicating that power companies might have improved to manage negative spikes. There has also been minor changes in German energy policy over these years that might have had an effect. In hour 8 we detect negative prices some years. In hour 19 there are (not surprisingly) no negative prices as this represent the hour with highest demand. Hour 8 have few positive spikes. Hour 8 and

19 have substantial positive price spikes some years (although the trend is falling). There has in general been a falling trend of volatility for all prices. Hour 3 has negative skewness each year in prices, hour 8 have some years with negative skewed prices and some years with positive. Hour 19 have positive skewness in prices all years. The kurtosis is also high for all series. Extreme negative spikes some periods makes the kurtosis highest for hour 3. Figure 4.1 also illustrate this.

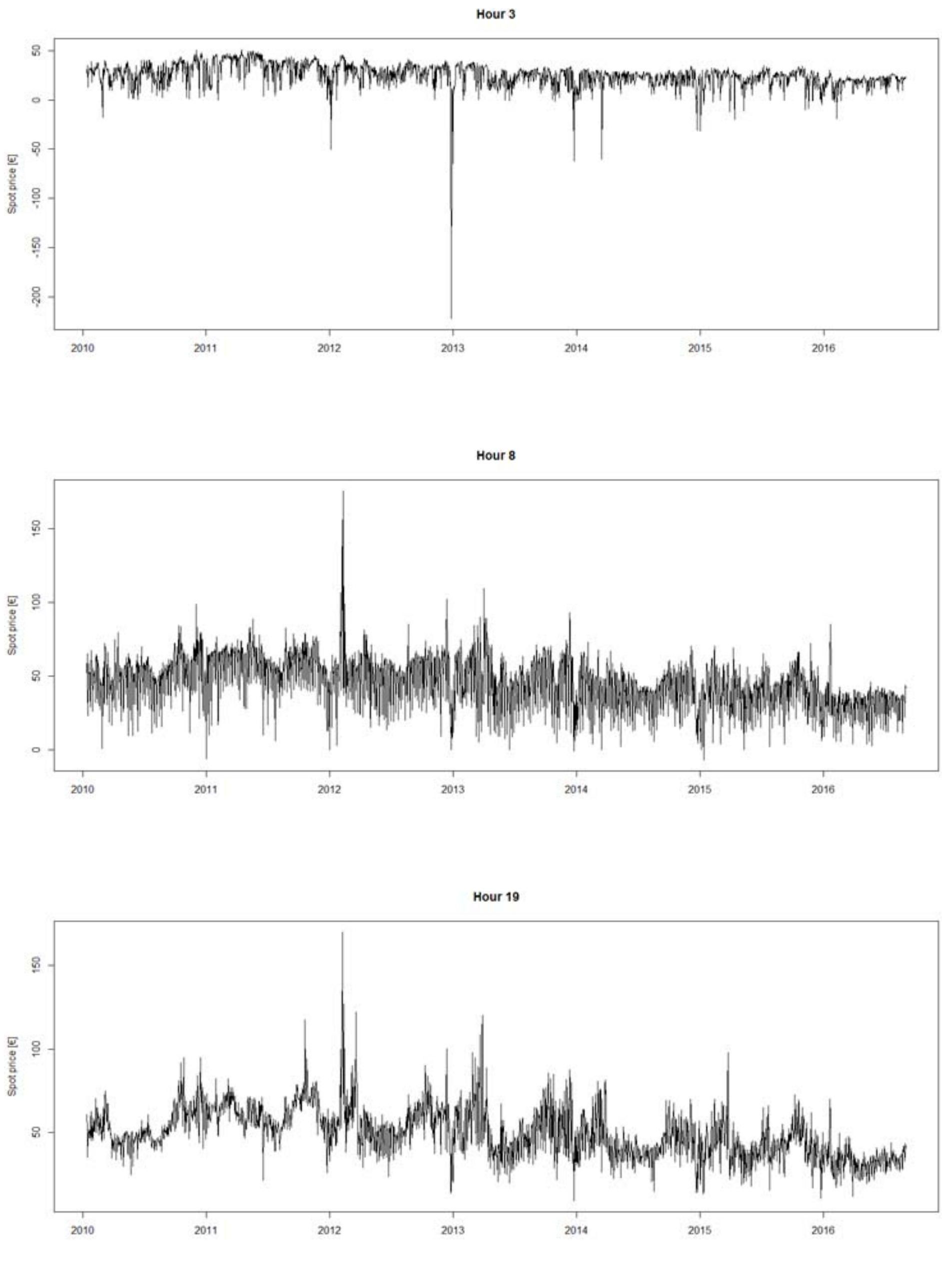

Figure 4.1: Development in selected hourly EPEX spot prices (hour 3,8,19) from 1Jan2010 to 31Aug2016

Hour 3	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
2010	29.83	27.63	-18.10	50.15	10.60	-0.77	3.47
2011	38.58	34.85	-0.10	51.08	10.71	-1.10	3.47
2012	30.08	26.21	-221.94	45.20	20.99	-8.11	86.69
2013	25.90	23.29	-62.03	39.67	10.78	-2.03	13.98
2014	23.98	21.14	-60.26	34.46	9.00	-3.09	23.16
2015	24.02	21.29	-31.41	34.92	9.21	-1.82	7.73
2016	20.10	18.48	-19.30	30.01	6.27	-1.99	8.95
Total	25.67	25.00	-221.90	51.08	13.10	-5.37	84.24

Hour 8	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
2010	51.55	50.07	1.06	98.71	14.66	-0.50	3.83
2011	60.63	57.44	-5.95	88.78	13.83	-1.18	5.10
2012	53.24	51.38	-0.09	175.55	19.35	1.15	10.20
2013	46.61	46.71	-0.98	109.36	18.14	-0.10	3.02
2014	41.03	39.65	0.05	72.94	13.81	-0.35	2.82
2015	40.46	38.85	-6.86	71.92	13.88	-0.43	3.05
2016	34.10	31.17	2.59	85.05	10.53	0.03	5.86
Total	46.37	45.76	-6.86	175.60	17.21	0.19	2.02

Hour 19	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
2010	50.85	53.52	24.76	95.00	10.78	0.92	4.15
2011	62.53	62.37	21.49	117.49	9.79	0.20	6.20
2012	55.00	56.39	13.70	169.90	15.85	1.87	12.79
2013	49.55	51.29	9.28	120.16	15.67	0.85	4.45
2014	42.44	44.20	14.34	81.51	11.56	0.67	3.73
2015	42.11	42.53	10.55	98.05	11.39	0.44	4.40
2016	33.15	33.04	11.79	70.03	6.99	0.75	6.74
Total	48.95	49.85	9.28	169.90	14.99	0.74	2.65

Table 4.3: Descriptive statistics of EPEX spot prices hour 3,8, and 19 measured in Euros. The tables show the characteristics based on daily data each year from 1Jan2010 to 31Aug2016 (that is summary statistics are only given for part of 2016).

Table 4.4 displays descriptive statistics of the fundamental variables used in the analysis. Wind, Solar, Demand have all hourly resolution, while coal, gas, oil, co2 prices and PPA are given on a daily basis in the period 1Jan2010 to 31Aug2016. Table 4.5 display the correlation between the spot price at the specific hour and the fundamentals. Wind and Solar have a negative correlation on the electricity price as expected. Demand and fuel prices have positive correlation as expected. Expected power plant availability have positive (as expected) correlation with prices at hour 8 and 19. At hour 3 the effect is neglect able. This might be due to that in the night hours, supply capacity is usually in surplus.

Hour 3	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
Wind	4491	6067	286	37322	5265	1.9	7.4
Solar	0.0	0.1	0.0	255.0	5.2	49.3	2433
Demand	31078	31219	19127	45071	3821	0.2	3.1
Coal	60.4	64.2	37.6	99.0	14.1	0.3	2.2
Gas	22.1	21.4	11.0	39.5	4.7	-0.3	2.7
Oil	45.3	40.4	15.0	56.7	10.1	-0.6	2.1
Co2	7.2	8.5	2.7	16.8	3.8	0.8	2.3
PPA	55531	55323	40016	64169	4863	-0.2	2.1

Hour 8	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
Wind	4075	5875	229	35663	5399	1.8	6.9
Solar	2087	3011	0.0	11665	2849	0.8	2.6
Demand	48673	45193	22783	62594	7800	-0.8	2.3
Coal	60.4	64.2	37.6	99.0	14.1	0.3	2.2
Gas	22.1	21.4	11.0	39.5	4.7	-0.3	2.7
Oil	45.3	40.4	15.0	56.7	10.1	-0.6	2.1
Co2	7.2	8.5	2.7	16.8	3.8	0.8	2.3
PPA	55531	55323	40016	64169	4863	-0.2	2.1

Hour 19	Median	Mean	Min	Max	Std.div.	Skewness	Kurtosis
Wind	4473	6101	270	33522	5225	1.7	6.3
Solar	74.0	736	0.0	4730	1047	1.3	3.5
Demand	45947	45496	30768	60966	5840	-0.3	2.4
Coal	60.4	64.2	37.6	99.0	14.1	0.3	2.2
Gas	22.1	21.4	11.0	39.5	4.7	-0.3	2.7
Oil	45.3	40.4	15.0	56.7	10.1	-0.6	2.1
Co2	7.2	8.5	2.7	16.8	3.8	0.8	2.3
PPA	55531	55323	40016	64169	4863	-0.2	2.1

Table 4.4: Descriptive statistics of fundamental variables used in the analysis. Note that coal, gas, oil, $CO2$ and PPA has a daily data granularity, and therefore show the same numbers for all hours. The calculations are based on daily data from EPEX from 1Jan2010 to 31Aug2016.

Hour	Wind	Solar	Demand	Coal	Gas	Oil	Co2	PPA
3	-0,571	-0,003	0,264	0,370	0,151	0,222	0,316	-0,074
8	-0,378	-0,224	0,699	0,441	0,300	0,321	0,308	0,132
19	-0,394	-0,368	0,538	0,553	0,425	0,427	0,336	0,182

Table 4.5: Correlation between spot prices and fundamental variables. The calculations are based on daily data from EPEX from 1Jan2010 to 31Aug2016.

5. Econometric Methods and Procedures

In this chapter, we describe how we implement and test quantile regression models and a set of benchmark models as well evaluation procedure for distributional forecasts. Finally, we describe our variable selection approach and our rolling window forecast approach.

We implement three different quantile regression models and three benchmark GARCH and CaViaR models:

- Traditional quantile regression (QR)
- Exponential weighted quantile regression (EWQR)
- Exponential weighted double kernel quantile regression (EWDKQR)
- GARCH(1,1) with skewed student-t distribution (GARCH-T)
- Symmetric absolute value CAViaR (SAV CAViaR)
- Asymmetric slope CAViaR. (AS CaViaR)

Linear quantile regression

We start with the original quantile regression model by Koenker and Bassett (1978). This is given by;

$$Q_\theta\left(lnP_{i,t+1}\right) = \beta_{i,0}^{\theta} + \sum_{n=1}^{N} \beta_{i,n}^{\theta} X_{n,t} \qquad (5.1)$$

Here, $\theta \in \{1\%, 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%, 99\%\}$ denotes the quantile, $i \in [0, 23]$ is the hour, and n indexes the set of explanatory variables x which has N elements. P is the price of electricity. The set of fundaments are from the ones described in chapter 4 but will vary as we perform a variable selection procedure (this will be described later). The quantile coefficients are found by minimizing;

$$\min_{\boldsymbol{\beta}_i^{\theta}} \quad \sum_{t=1}^{T} (ln\boldsymbol{P}_{i,T} - X_{i,t}\beta_i^{\theta})(\theta - I(ln\boldsymbol{P}_{i,T} \leq \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^{\theta})) \qquad (5.2)$$

$\mathbf{X}_{i,t}$ is a vector of explanatory variables at time t and $\boldsymbol{\beta}_i^{\theta}$ is a vector of regression coefficients. I() refers to an indicator function returning the value 1 or 0. We solve the minimisation problem using the ”quantreg” package in R.

Exponentially weighted quantile regression

By adding a weighting parameter λ to equation 5.2, we get the exponentially weighted quantile regression by Taylor (2008b). λ decays exponentially, amounting to simple exponential smoothing of the cumulative distribution function. Thus, the EWQR minimisation has the form;

$$\min_{\boldsymbol{\beta}_i^{\theta}} \quad \sum_{t=1}^{T} \lambda^{\mathrm{T-t}}(ln\boldsymbol{P}_{i,T} - X_{i,t}\beta_i^{\theta})(\theta - I(ln\boldsymbol{P}_{i,T} \leq \boldsymbol{X}_{i,t}\boldsymbol{\beta}_i^{\theta})) \tag{5.3}$$

Again, we solve the minimisation using R’s ”quantreg” package. The value of λ determines how fast the weights decay. If the distribution changes rapidly, a relatively low value is needed to ensure that the model adapts swiftly. However, larger values may be necessary in the extreme quantiles to give significant weight to a higher number of observations. We follow Taylor’s approach to optimise the λ-values. This is done by using a rolling window to produce one step-ahead quantile forecasts for the observations in the in-sample set, and selecting the λ that yields the minimum QR sum. This is the summation in the standard form of QR in Equation 5.2. Since λ depends on all parts of the model specification, we perform this optimisation for all combinations of hours, quantiles, explanatory variables and window sizes. We test a window of λ-values between 0.9 and 1, with a step size of 0.001.

Exponentially weighted double kernel quantile regression

We expand the exponential weighted quantile regression model further to exponentially weighted double kernel quantile regression following the approach of Taylor (2008b). In this model, we replace the observations $lnP_{i,t}$ from Equation 5.3 with a kernel function K_{h2} . Taylor argues that introducing kernels may allow faster decay of the exponential weighting parameter, and consequently, better adaption to swift distribution changes.. To perform the minimisation, we use the ”nlm” nonlinear optimisation solver in R .

Fully parametric GARCH models

In several studies investigation GARCH models for commodity VaR predictions GARCH with a skewed student-t distribution perform significantly better than Gaussian GARCH (see for example Giot and Laurent (2003) and Fuss et al. (2010)). This is the reason why we only implement skewed student-t GARCH. For details of the the model, see Giot and Laurent (2003). We first run a regression with $\ln(P_t)$ as the dependent variable against a set of fundamentals. The residuals from this regression is then modelled by a skewed student-t GARCH. This model is then used for forcasting VaR (see Giot and Laurent (2003) for details). We use here the ”fGarch” package in R.

Conditional autoregressive Value-at-Risk models

The CAViaR models by Engle and Manganelli (2004) specify the evolution of a quantile over time as an autoregressive process. They derive expressions for four different CAViaR processes; symmetric absolute value, asymmetric slope, adaptive, and indirect GARCH(1,1). We use the first two as benchmarks, as Fuss et al. (2010)) found that these generally outperformed the others in predicting VaR. These models are estimated using a developed code in R.

Out-of-sample performance analysis

To test the predictive performance of the models, we use Kupiec's unconditional coverage (UC) test (1995), Christoffersen's conditional coverage (CC) test (1998), and the dynamic conditional quantile (DQ1 and DQ2) tests by Engle and Manganelli (2004). Alexander (2008b,d) gives a nice description on how these measures are used in risk management and backtesting of VaR models in practice.

Variable selection and Forecasting Approach

Variable selection is a crucial step in building a good prediction model (e.g. Diebold (2015)). Distributional forecasting this is a complex process, and the standard goodness- of-fit tests are not sufficient. In Chapter 2, we saw evidence that fundamental variables affect specific hours and quantiles differently. As our target is forecasting, it is therefore necessary to perform variable selections for all combinations of hours and quantiles, to take full advantage of modelling each quantile separately. To achieve high predictive power, variable selection should be based on the quality of the *association* between predictors and responses, rather than causal relationships (see Shmueli (2010) and Diebold (2015) for more discussion. For each hour and quantile, we choose the combination of variables that yields the best SIC score[2] in sample while also demanding that the model pass the critical out-of-sample tests[3].

We apply a rolling window, which works as follows: If the window size is, lets say 365, observations [1, 365] are used for forecasting the VaR of observation 366. Next, we re-estimate the model with observations [2, 366] and forecast the VaR of observation 367, and so on. We test window sizes of 250, 365, 548, 730 and 913 days. The window that gives the best results for a given hour and quantile is chosen in each case.

Results

Table 6.1, 6.2, and 6.3 shows the predictive performance of the models presented in chapter 5. As evaluation criteria, we use the UC-,CC- and DQ tests and display the detailed results for hours 3, 8, and 19, respectively. Here, we only present the optimized model for each quantile and hour. That is, models that are optimized with window size and variable combinations for each hour and quantile.

It is difficult to draw general conclusions as to which model is performing best based on our results. The model with the highest predictive performance varies across both the distribution and the trading periods. Moreover, the four evaluation criteria favour different models in each case. We calculate the total number of rejected tests when we rate one model against another in Table 6.1, 6.2, and 6.3 below.

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[2] SIC measures for quantile regression are described Koenker et al. (1994) and in Vinod (2010) chapter 2. For GARCH models, it's the standard SIC measure based on the likelihood function and number of parameters.
[3] Details on which variables to include for each model, each hour, and each quantile as well as the optimal window size can be given by contacting the corresponding author.

Table 6.1: Predictive performance in hour 3

Model	Quantile	Violations	P_UC	P_CC	P_DQ1	P_DQ2
QR	0.01	1.09E-02	8.01E-01	0.00E+00*	1.86E-01	2.61E-02*
	0.05	4.92E-02	9.25E-01	3.07E-01	1.32E-01	1.65E-01
	0.10	8.48E-02	1.61E-01	2.73E-01	6.11E-01	6.07E-01
	0.25	2.42E-01	6.22E-01	1.08E-01	7.81E-03*	1.21E-01
	0.50	5.62E-01	7.51E-04*	0.00E+00*	5.54E-31*	3.38E-02*
	0.75	7.44E-01	7.17E-01	0.00E+00*	2.46E-38*	2.16E-01
	0.90	8.88E-01	2.81E-01	3.61E-02*	5.28E-04*	3.75E-01
	0.95	9.51E-01	9.25E-01	6.71E-01	1.56E-04*	9.64E-01
	0.99	9.84E-01	1.11E-01	0.00E+00*	1.90E-04*	2.83E-02*
# Rejections			1	5	6	3
EWQR	0.01	1.37E-02	3.44E-01	0.00E+00*	4.95E-01	9.23E-02
	0.05	5.06E-02	9.39E-01	3.52E-01	1.85E-01	2.70E-01
	0.10	9.44E-02	6.10E-01	7.24E-01	7.26E-01	7.97E-01
	0.25	2.46E-01	8.14E-01	8.27E-01	1.41E-02*	1.50E-01
	0.50	4.77E-01	2.22E-01	0.00E+00*	1.14E-23*	8.24E-03*
	0.75	6.87E-01	1.17E-04*	0.00E+00*	5.96E-27*	7.97E-03*
	0.90	8.82E-01	1.21E-01	4.32E-02*	2.70E-02*	1.22E-01
	0.95	9.51E-01	9.25E-01	6.71E-01	1.56E-04*	9.64E-01
	0.99	9.82E-01	5.66E-02	7.74E-02	1.60E-01	4.90E-01
# Rejections			1	4	5	2
EWDKQR	0.01	6.84E-03	3.62E-01	0.00E+00*	1.00E+00	7.49E-02
	0.05	3.97E-02	1.84E-01	3.13E-01	8.41E-01	8.72E-01
	0.10	8.62E-02	2.03E-01	4.30E-01	4.58E-01	4.73E-01
	0.25	2.37E-01	4.02E-01	0.00E+00*	3.98E-43*	2.79E-10*
	0.50	4.90E-01	5.79E-01	4.45E-02*	3.44E-02*	3.83E-08*
	0.75	7.35E-01	3.40E-01	0.00E+00*	6.60E-85*	1.48E-26*
	0.90	9.53E-01	9.73E-08*	1.53E-07*	1.84E-01	4.63E-18*
	0.95	9.53E-01	6.62E-01	4.51E-11*	2.12E-25*	2.17E-12*
	0.99	9.96E-01	6.92E-02	0.00E+00*	1.00E+00	2.29E-07*
# Rejections			1	7	4	6

Model	Quantile	Violations	P_UC	P_CC	P_DQ1	P_DQ2
GARCH	0.01	6.84E-03	3.62E-01	0.00E+00*	1.00E+00	1.80E-01
	0.05	6.29E-02	1.22E-01	3.03E-01	3.54E-01	5.66E-01
	0.10	1.07E-01	5.50E-01	5.09E-01	6.24E-01	9.45E-01
	0.25	2.38E-01	4.52E-01	7.26E-04*	1.49E-05*	4.28E-02*
	0.50	8.81E-01	0.00E+00*	0.00E+00*	7.93E-13*	1.46E-02*
	0.75	6.13E-01	3.33E-16*	0.00E+00*	3.60E-16*	2.03E-06*
	0.90	8.84E-01	1.52E-01	1.18E-01	3.11E-01	2.29E-09*
	0.95	9.49E-01	9.39E-01	7.62E-01	6.04E-02	1.37E-06*
	0.99	9.93E-01	3.62E-01	0.00E+00*	1.00E+00	2.07E-02*
# Rejections			2	5	3	6
Asym. slope CAViaR	0.01	1.37E-02	3.44E-01	1.74E-01	9.98E-01	3.13E-01
	0.05	5.75E-02	3.66E-01	3.86E-01	8.26E-01	2.71E-01
	0.10	9.44E-02	6.10E-01	1.32E-02*	1.24E-02*	2.59E-01
	0.25	2.28E-01	1.74E-01	2.20E-04*	7.02E-05*	9.86E-02
	0.50	5.27E-01	1.49E-01	7.05E-09*	6.33E-14*	1.72E-07*
	0.75	7.55E-01	7.48E-01	9.26E-04*	2.97E-11*	3.32E-06*
	0.90	9.21E-01	5.43E-02	5.29E-05*	5.49E-05*	3.99E-09*
	0.95	9.67E-01	2.34E-02*	1.79E-03*	1.30E-03*	5.79E-01
	0.99	9.86E-01	3.44E-01	0.00E+00*	1.21E-05*	2.64E-01
# Rejections			1	7	7	3
Sym. abs. value CAViaR	0.01	1.09E-02	8.01E-01	1.89E-01	2.98E-03*	2.26E-01
	0.05	5.61E-02	4.58E-01	1.77E-01	1.26E-01	1.20E-01
	0.10	1.11E-01	3.38E-01	4.76E-03*	3.11E-02*	9.41E-01
	0.25	1.44E-01	1.82E-12*	0.00E+00*	1.09E-18*	2.30E-02*
	0.50	5.40E-01	2.90E-02*	0.00E+00*	2.47E-39*	1.59E-04*
	0.75	6.80E-01	2.05E-05*	0.00E+00*	6.18E-43*	4.10E-04*
	0.90	8.96E-01	7.22E-01	4.45E-01	5.17E-01	4.05E-09*
	0.95	9.52E-01	7.91E-01	8.11E-01	9.57E-01	2.12E-10*
	0.99	9.88E-01	5.44E-01	0.00E+00*	9.97E-01	3.06E-11*
# Rejections			3	5	5	6

The table displays the violation in percent, p-values of the unconditional coverage test (UC), the conditional coverage test (CC), and the two dynamic conditional quantile tests (DQ1 and DQ2), as described in Section 5. P-values highlighted with * are significant at the 5% level, which implies poor model calibration (Under H_0 we have a correct model). Window sizes and variable selection is optimize for each model as described in chapter 5 .

Table 6.2: Predictive performance in hour 8

Model	Quantile	Violations	P UC	P CC	P DQ1	P DQ2
QR	0.01	9.58E-03	9.08E-01	0.00E+00*	9.99E-01	2.73E-01
	0.05	3.15E-02	1.39E-02*	4.60E-02*	6.06E-01	9.98E-01
	0.10	7.52E-02	2.01E-02*	6.10E-02	1.96E-02*	6.43E-01
	0.25	2.54E-01	7.82E-01	6.66E-16*	9.75E-20*	2.52E-02*
	0.50	4.90E-01	5.79E-01	1.37E-14*	6.91E-49*	1.66E-01
	0.75	7.15E-01	3.35E-02*	3.26E-13*	6.64E-23*	5.32E-02
	0.90	9.18E-01	9.66E-02	6.16E-13*	1.30E-28*	1.41E-04*
	0.95	9.49E-01	9.39E-01	2.33E-15*	1.11E-42*	1.86E-05*
	0.99	9.96E-01	6.92E-02	0.00E+00*	1.00E+00	4.56E-01
# Rejections			3	8	6	3
EWQR	0.01	1.23E-02	5.44E-01	0.00E+00*	9.97E-01	3.30E-01
	0.05	4.79E-02	7.91E-01	9.35E-01	6.77E-01	6.50E-01
	0.10	9.85E-02	8.92E-01	3.08E-01	9.48E-04*	1.23E-01
	0.25	2.60E-01	5.38E-01	8.48E-10*	4.87E-10*	2.86E-01
	0.50	4.66E-01	6.98E-02	5.07E-12*	6.05E-52*	7.33E-03*
	0.75	7.10E-01	1.40E-02*	3.72E-10*	2.90E-15*	1.28E-03*
	0.90	9.12E-01	2.53E-01	2.63E-12*	7.48E-21*	4.84E-01
	0.95	9.44E-01	4.58E-01	7.71E-11*	7.32E-24*	1.44E-01
	0.99	9.86E-01	3.44E-01	0.00E+00*	7.70E-02	1.34E-04*
# Rejections			1	7	6	3
EWDKQR	0.01	1.50E-02	2.02E-01	1.37E-02*	9.01E-08*	4.42E-01
	0.05	5.20E-02	8.07E-01	7.50E-01	2.35E-04*	9.10E-01
	0.10	9.99E-02	9.90E-01	1.65E-06*	3.50E-07*	8.85E-01
	0.25	2.50E-01	9.83E-01	1.40E-02*	1.11E-59*	1.47E-01
	0.50	5.06E-01	7.39E-01	0.00E+00*	4.43E-29*	4.71E-01
	0.75	7.47E-01	8.48E-01	6.50E-07*	5.43E-08*	5.04E-08*
	0.90	9.06E-01	6.10E-01	2.55E-15*	3.80E-28*	3.67E-01
	0.95	9.53E-01	6.62E-01	3.49E-06*	2.82E-17*	4.76E-11*
	0.99	9.88E-01	5.44E-01	2.05E-01	7.89E-04*	9.62E-01
# Rejections			0	7	9	2

Model	Quantile	Violations	P UC	P CC	P DQ1	P DQ2
GARCH	0.01	1.37E-02	3.44E-01	0.00E+00*	5.08E-01	1.01E-03
	0.05	3.15E-02	1.39E-02*	4.60E-02*	2.53E-02*	1.52E-03
	0.10	7.66E-02	2.85E-02*	8.16E-03*	1.21E-01*	1.25E-01
	0.25	1.86E-01	3.72E-05*	4.86E-07*	1.08E-02	8.81E-02
	0.50	6.05E-01	1.35E-08*	1.66E-11*	2.79E-06	2.22E-08
	0.75	7.28E-01	1.69E-01	6.20E-06*	2.20E-06	2.17E-11
	0.90	8.77E-01	4.35E-02*	1.10E-02*	1.03E-01	4.04E-12
	0.95	9.44E-01	4.58E-01	4.21E-01	1.56E-01	5.50E-10
	0.99	9.89E-01	8.01E-01	0.00E+00*	1.38E-09	4.36E-05
# Rejections			5	8	5	7
Asym. slope CAViaR	0.01	3.83E-02	4.43E-09*	2.36E-08*	4.04E-09*	5.66E-03*
	0.05	8.62E-02	4.33E-05*	1.47E-05*	2.59E-12*	1.39E-01
	0.10	1.38E-01	1.07E-03*	5.48E-04*	5.51E-25*	1.59E-02*
	0.25	2.75E-01	1.23E-01	4.23E-07*	1.25E-05*	3.79E-02*
	0.50	4.47E-01	4.36E-03*	3.50E-06*	2.12E-57*	8.85E-16*
	0.75	6.51E-01	2.72E-09*	1.23E-12*	2.60E-29*	7.47E-08*
	0.90	8.73E-01	1.80E-02*	1.56E-09*	3.33E-20*	1.67E-03*
	0.95	9.36E-01	8.88E-02	2.36E-06*	6.59E-12*	7.45E-04*
	0.99	9.90E-01	9.08E-01	0.00E+00*	6.20E-02	5.99E-01
# Rejections			6	9	8	7
Sym. abs. value CAViaR	0.01	1.78E-02	5.66E-02	0.00E+00*	7.79E-01	5.74E-02
	0.05	6.29E-02	1.22E-01	6.77E-02	3.68E-10*	2.44E-01
	0.10	9.85E-02	8.92E-01	1.64E-01	4.60E-13*	2.52E-02*
	0.25	2.60E-01	5.38E-01	2.50E-08*	1.24E-09*	1.74E-01
	0.50	4.40E-01	1.27E-03*	0.00E+00*	1.33E-65*	6.42E-04*
	0.75	6.59E-01	4.56E-08*	0.00E+00*	1.11E-33*	2.50E-04*
	0.90	9.02E-01	8.92E-01	4.12E-09*	8.98E-21*	5.26E-03*
	0.95	9.38E-01	1.66E-01	9.57E-06*	1.52E-12*	5.68E-03*
	0.99	9.90E-01	9.08E-01	0.00E+00*	6.20E-02	7.16E-01
# Rejections			2	7	7	5

The table displays the violation in percent, the p-values of the unconditional coverage test (UC), the conditional coverage test (CC), and the two dynamic conditional quantile tests (DQ1 and DQ2), as described in chapter 5. P-values highlighted with * are significant at the 5% level, which implies poor model calibration (Under H_0 we have a correct model). Window sizes and variable selection is optimize for each model as described in chapter 5.

Table 6.3: Predictive performance in hour 19

	Quantile	Violations	P_UC	P_CC	P_DQ1	P_DQ2
QR	0.01	1.23E-02	5.44E-01	0.00E+00*	3.24E-02*	9.21E-03*
	0.05	5.20E-02	8.07E-01	9.70E-01	2.80E-04*	5.39E-02
	0.10	9.58E-02	7.01E-01	9.22E-01	4.30E-04*	2.00E-01
	0.25	2.48E-01	8.81E-01	5.95E-01	1.06E-08*	5.55E-01
	0.50	4.99E-01	9.70E-01	4.95E-01	6.61E-01	1.34E-01
	0.75	7.55E-01	7.48E-01	1.31E-01	5.58E-01	3.70E-04*
	0.90	9.19E-01	7.30E-02	1.50E-03*	1.33E-02*	2.40E-01
	0.95	9.85E-01	4.18E-07*	7.84E-11*	5.81E-26*	8.27E-01
	0.99	9.96E-01	6.92E-02	0.00E+00*	1.00E+00	9.31E-01
# Rejections			1	4	6	2
EWQR	0.01	1.64E-02	1.11E-01	0.00E+00*	3.63E-01	2.58E-02*
	0.05	6.57E-02	6.32E-02	5.85E-02	5.40E-04*	5.44E-03*
	0.10	1.16E-01	1.52E-01	2.80E-01	1.91E-02*	9.33E-03*
	0.25	2.56E-01	7.17E-01	4.70E-01	4.32E-07*	4.05E-01
	0.50	5.03E-01	8.53E-01	7.07E-01	6.33E-01	7.39E-02
	0.75	7.36E-01	3.84E-01	1.18E-01	2.19E-01	2.38E-02*
	0.90	9.02E-01	8.92E-01	7.78E-02	6.55E-02	6.79E-02
	0.95	9.59E-01	2.52E-01	2.97E-10*	5.45E-24*	2.77E-01
	0.99	9.95E-01	1.78E-01	1.81E-02*	2.76E-08*	8.19E-01
# Rejections			0	3	5	4
EWDKQR	0.01	1.09E-02	8.01E-01	1.89E-01	1.70E-01	1.25E-01
	0.05	4.79E-02	7.91E-01	0.00E+00*	8.60E-02	3.97E-01
	0.10	9.71E-02	7.95E-01	9.66E-01	4.56E-03*	7.12E-02
	0.25	2.56E-01	7.17E-01	1.55E-15*	2.13E-18*	5.74E-01
	0.50	5.05E-01	7.96E-01	1.32E-01	2.24E-10*	2.71E-02*
	0.75	7.47E-01	8.48E-01	0.00E+00*	3.66E-41*	5.41E-01
	0.90	9.15E-01	1.61E-01	9.68E-02	5.92E-02	4.90E-02*
	0.95	9.70E-01	7.85E-03*	3.54E-04*	1.63E-04*	5.97E-01
	0.99	9.93E-01	3.62E-01	4.86E-02*	1.25E-04*	7.33E-01
# Rejections			1	5	6	2

	Quantile	Violations	P_UC	P_CC	P_DQ1	P_DQ2
GARCH	0.01	1.09E-02	8.01E-01	0.00E+00*	9.99E-01	2.84E-02*
	0.05	4.51E-02	5.40E-01	7.51E-01	8.82E-02	4.33E-01
	0.10	8.62E-02	2.03E-01	3.46E-01	4.85E-01	3.29E-01
	0.25	2.13E-01	2.01E-02*	8.09E-04*	3.14E-02*	4.67E-04*
	0.50	4.87E-01	4.82E-01	5.84E-02	1.10E-02*	7.33E-05*
	0.75	6.83E-01	4.19E-05*	2.65E-09*	9.73E-05*	3.65E-08*
	0.90	8.80E-01	7.41E-02	6.92E-02	2.52E-01	2.26E-05*
	0.95	9.45E-01	5.64E-01	7.28E-01	8.35E-01	2.39E-04*
	0.99	9.95E-01	1.78E-01	0.00E+00*	2.97E-08*	6.07E-01
# Rejections			2	4	4	6
Asym. slope CAViaR	0.01	2.33E-02	2.12E-03*	0.00E+00*	8.08E-01	2.41E-04*
	0.05	5.61E-02	4.58E-01	6.73E-02	1.55E-02*	4.35E-02*
	0.10	1.01E-01	9.12E-01	2.29E-01	2.75E-02*	6.87E-01
	0.25	2.68E-01	2.61E-01	4.36E-01	6.51E-08*	6.19E-01
	0.50	4.92E-01	6.84E-01	8.61E-01	8.03E-01	5.39E-02
	0.75	7.28E-01	1.69E-01	7.67E-02	1.87E-01	6.64E-05*
	0.90	9.15E-01	1.61E-01	9.68E-02	4.24E-01	1.42E-01
	0.95	9.48E-01	8.07E-01	2.89E-08*	9.87E-15*	6.05E-01
	0.99	9.85E-01	2.02E-01	5.59E-04*	6.51E-12*	9.58E-01
# Rejections			1	3	5	3
Sym. abs. value CAViaR	0.01	1.37E-02	3.44E-01	0.00E+00*	1.34E-01	6.44E-01
	0.05	5.06E-02	9.39E-01	7.22E-01	2.96E-01	6.95E-01
	0.10	1.05E-01	6.33E-01	9.03E-02	2.60E-04*	4.84E-01
	0.25	2.76E-01	1.04E-01	2.00E-01	2.00E-06*	2.06E-01
	0.50	5.06E-01	7.39E-01	8.29E-01	7.46E-01	2.21E-02*
	0.75	7.35E-01	3.40E-01	3.04E-01	4.87E-01	3.42E-05*
	0.90	9.21E-01	5.43E-02	1.25E-01	3.55E-01	5.33E-02
	0.95	9.59E-01	2.52E-01	2.97E-10*	1.31E-24*	7.20E-01
	0.99	9.92E-01	6.15E-01	1.82E-03*	8.77E-18*	2.31E-01
# Rejections			0	3	4	2

The table displays the violation in percent , the p-values of the unconditional coverage test (UC), the conditional coverage test (CC), and the two dynamic conditional quantile tests (DQ1 and DQ2), as described in chapter 5.P-values highlighted with * are significant at the 5% level, which implies poor model calibration (Under H_0 we have a correct model). Window sizes and variable selection is optimize for each model as described in chapter 5.

	UC (27)	CC (27)	DQ1 (27)	DQ2 (27)	Total (108)
EWQR	2	14	16	9	41
QR	5	17	18	8	48
SAV CAViaR	5	15	16	13	49
EWDKQR	2	19	19	10	50
GARCH-T	9	17	12	19	57
AS CAViaR	8	21	18	12	59

Table 6.4: Total number of test rejections per model over all quantiles and periods. The table displays the total number of test rejections per model at the 5% significance level. The numbers in parentheses give the maximum number of rejections. A high number of rejections indicates poor calibration. UC is the unconditional coverage test, CC is the conditional coverage test, and DQ1 and DQ2 are the two dynamic conditional quantile tests, as described in Section 5. The models are also described in section 5.

In Table 6.4 above we display the total number of test rejections per model. Based on this, we rate the EWQR as the best model overall; it outperforms both the other QR type models and the benchmarks in terms of test rejections. Another important observation is that clustering of exceedances is challenging to capture for all models. Again, EWQR is relatively the best model regarding the CC test but not with the more advanced DQ1 and DQ2 tests.

Next, we break down the analysis into performance in each hour and parts of the distribution. The results for each hour are summarised in Table 6.5. EWQR are performing best for hour 3 and 8 and third best for hour 19. For hour 19 SA CAViaR is performing best. An observation is also that hour 8 have a higher level of rejections in general than hour 3 and hour 19.

Rating	Model	Rejections
1	EWQR	12
2	QR	15
3	GARCH-T	16
4	EWDKQR	18
5	AS CAViaR	18
6	SA CAViaR	19

Hour 3 (36)

Rating	Model	Rejections
1	EWQR	17
2	EWDKQR	18
3	QR	20
4	SA CAViaR	21
5	GARCH-T	25
6	AS CAViaR	30

Hour 8 (36)

Rating	Model	Rejections
1	SA CAViaR	9
2	AS CAViaR	11
3	EWQR	12
4	QR	13
5	EWDKQR	14
6	GARCH-T	16

Hour 19 (36)

Table 6.5: Total number of test rejections per hour. The table displays the total number of test rejections per model at the 5% significance level. The numbers in parentheses give the maximum number of rejections. A high number of rejections indicates poor calibration.

In table 6.6,we assess performance across the distribution. For risk management purposes, it is particularly important to consider accuracy in the tails. Thus, we divide the distribution into three parts: i) The lower tail with quantiles 0.01%, 0.05% and 0.10%, ii) the mid-region with quantiles 0.25%, 0.50% and 0.75%, and iii) the upper tail with quantiles 0.90%, 0.95% and 0.99%.

Rating	Model	Rejections
1	EWDKQR	8
2	EWQR	9
3	SA CAViaR	9
4	GARCH-T	11
5	QR	12
6	AS CAViaR	17

Lower tail (36)

Rating	Model	Rejections
1	QR	17
2	EWQR	19
3	AS CAViaR	20
4	EWDKQR	22
5	SA CAViaR	25
6	GARCH-T	31

Mid-region (36)

Rating	Model	Rejections
1	EWQR	13
2	GARCH-T	15
3	SA CAViaR	15
4	QR	19
5	EWDKQR	20
6	AS CAViaR	22

Upper tail (36)

Table 6.6: Total number of test rejections in sections of the distribution. The table displays the total number of test rejections per model at the 5% significance level. The numbers in parentheses give the maximum number of rejections. A high number of rejections indicates poor calibration.

EWQR perform second in the lower tail and best in the upper tail. In the mid-region it performs second best (which is of less interest regarding risk management). In general, it is harder to predict the lower tail than the upper tail. AS CAViaR are performing worst for both tails.

We rate EWQR as the best model overall. This model has the fewest test rejections in total, and shows particularly good performance in the tails. This indicates that this model (although not perfect) is able to account for the changing market dynamics in Germany. The reason that EWQR generally outperform EWDKQR might be due to that the latter suffer from overfitting (EWDKQR requires estimation of an additional parameter).

Conclusion

In this paper, our aim have been to forecast VaR for the German EPEX spot price using various set of fundamentals and state of the art models. VaR model analysis and forecasting for energy commodities remains an under-research area despite the need for energy risk management among producers, consumers, and other participants in this market. We have focused on using fundaments to capture the complex and non-linear response of supply- and demand variables to electricity price. Not only can fundamentals improve forecasting electricity price distributions (which is the main aim of this paper), but also help us in understanding which risk drivers influence most at certain hours and certain quantiles.

We apply state of the art models found to yield good results in other studies of commodity VaR forecasting. These are quantile regression, exponential weighted quantile regression, exponential weighted double kernel quantile regression, GARCH models with skewed t error distributions, and various CAViaR models. We optimize the use of exogenous variables by finding the best models in-sample using the SIC criterion as well as checking whether these models pass some of the out-of-sample tests for each hour and each quantile. This is motivated by evidence in literature that the impact of fundamentals differs across the distribution and between trading periods. Our findings highlight the importance of variable selection, and show that it in many cases is as important as the choice of model.

We investigate hours 3, 8, and 19 in this study using daily prices and fundamental data from the period 01.01.2010 to 31.08.2016. The set of fundamentals we use are the coal price, gas price, oil price, CO_2 allowance price, expected wind infeed, expected solar infeed, expected power plant availability, and expected demand. In general we find that exponential weighted quantile regression is the best model overall based on the total number of test rejections. This model is among the top-performers for all trading periods, and performs particularly well in the outer quantiles. This is also an easy model to implement relative to the other models investigated in this study. Thus we recommend this model together with carefully selecting fundamentals for given hours and quantiles when the aim is to forcast VaR for German electricity prices. This insights can be applied by market participants who seek to determine optimal biddings strategies, trading strategies and risk management in general.

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