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Authors
Daniel Cantero
Therese Arvidsson
Eugene OBrien
Raid Karoumi

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Train-track-bridge modelling and review of parameters

Daniel Cantero[1,2]

Therese Arvidsson[1]

Eugene OBrien[2,3]

Raid Karoumi[1]

[1]Civil and Architectural Engineering, KTH Royal Institute of Technology,10044 Stockholm, Sweden

[2]Roughan & O'Donovan Innovative Solutions, Arena House, Arena Road, Dublin 18, Ireland

[3]School of Civil, Structural and Environmental Engineering, University College Dublin, Dublin 4, Ireland

Emails

Daniel Cantero: canterolauer@gmail.com
Therese Arvidsson: therese.arvidsson@byv.kth.se
Eugene OBrien: eugene.obrien@ucd.ie
Raid Karoumi: raid.karoumi@byv.kth.se

Corresponding Author full address

Daniel Cantero

e-mail: canterolauer@gmail.com

Telephone: 0046 8 790 7957

Postal Address: Brinellvägen 23, 10044 Stockholm, Sweden

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Train-track-bridge modelling and review of parameters

This paper gathers into one single document all the necessary information to construct a model to calculate the coupled dynamic response of train–track–bridge systems. Each component of the model is presented in detail together with a review of possible sources for the parameter values, including a collection of vehicle models, a variety of track configurations and general railway bridge properties. Descriptions of the most important track irregularity representations are also included. The presented model is implemented in Matlab and validated against a commercially available finite element package for a range of speeds, paying particular attention to a resonant speed. Finally, the potential of the described model is illustrated with two numerical studies that address interesting aspects of train and bridge dynamic responses. In particular, the effect of the presence of a vehicle on the bridge's fundamental frequency is studied, as well as the influence of the wavelength of the rail irregularities on the dynamic effects of the bridge and the vehicle.

Keywords: railway, bridge, dynamics, irregularity, wavelength, frequency

1. Introduction

Modelling the dynamic behaviour of bridges due to traversing traffic is a complex problem. Not only are correct representations of vehicle and bridge systems needed, but also the interaction between these systems has to be taken into account. The magnitude of dynamic effects varies greatly depending on the length of the bridge and the type of traffic loading. In railway bridges these effects can be considerably greater than for their road counterparts. In particular, for high-speed railway lines, the dynamic effects are major contributors to the total load effects developed in the structure during a vehicle passage. The repeated loading of the bridge due to a regular load configuration (long trains composed of wagons with identical axle spacings) combined with its traversing speed introduces loading frequencies that can lead to resonant behaviour in the bridge. This is a fundamental difference from road bridges, which are loaded by traffic with a mixture of axle configurations and separated by arbitrary gaps between vehicles. Even multiple vehicle events in road bridges do not produce resonant behaviour to the same extent as in railway bridges.

According to the European design codes (Eurocodes, 2003) a numerical simulation is needed when the bridge's fundamental frequency is outside of certain frequency bounds or if the speed of the traversing vehicle exceeds 200 km/h. In these cases, the designer is required to check the effect of the design code load model HSLM (High Speed train Load Model) or, as an alternative, real trains with axle loads and spacings should be specified for each individual project. There are many limiting design criteria, but in practice the vertical deck acceleration criterion will, in most cases, be the decisive factor (Zacher & Baeßler, 2009; UIC, 2009). The Eurocode (2003) limits the maximum deck acceleration for ballasted tracks to 3.5 m/s^2 to avoid ballast instability. In this context, there is a lack of codes dedicated to the assessment of existing structures

(Johansson, Ní Nualláin, Pacoste & Andersson, 2014) with alternative checks of the dynamic behaviour of the bridge.

As a result, dynamic analysis of railway bridges due to traversing trains is often required. The consideration of Vehicle–Bridge Interaction (VBI) and the load distribution provided by the track can lead to reduced bridge responses, and can thus be advantageous, for example, when assessing the capacity of existing bridges. However, there is a lack of commercially available software that is able to calculate the bridge's dynamic response while correctly accounting for the interaction between vehicle and structure. Only some software packages allow for the calculation of simple moving loads. Thus, there is the need for fast and reliable numerical models that consider the VBI. This paper gathers into one document all the necessary information needed to construct such a model. A Finite Element (FE) planar model that incorporates the behaviour of the train, ballasted track and bridge is presented together with a comprehensive list of sources for particular values of the model parameters for each of the subsystems considered. The vehicle is described as a combination of lumped masses, rigid bars, springs and dashpots. The track is modelled as a beam (that represents the rail) resting on periodically spaced sprung mass systems (that represent pads, sleepers, ballast and sub-ballast). The bridge is modelled as a beam. The coupling of each subsystem into one model correctly accounts for the interaction between the vehicle and the infrastructure. The irregularities of the railway track are also considered. This is termed Train–Track–Bridge (TTB) model in this document.

Compared to the simple moving load models, the model presented here gives more accurate results because it includes the VBI. It has been shown in (Doménech, Museros & Martínez-Rodrigo, 2014a) that moving load models generally overestimate the dynamic effects, thus VBI should be included when more realistic calculations are required. The importance of VBI is generally higher for short-span bridges (< 30 m) but is in fact related to a number of relations between the train and the bridge (Arvidsson & Karoumi, 2014). The greatest difference in results between a moving load model and a VBI model is seen for high mass ratios and bogie frequencies close to the bridge fundamental frequency. In addition, including the vehicle behaviour in the model provides an estimate of the vehicle response, which may be used to evaluate passenger comfort, and the wheel–rail forces which may be used to evaluate running safety (Doménech, Museros & Martínez-Rodrigo, 2014a). Furthermore, the consideration of the rail–sleeper–ballast system in the TTB model distributes the load of each axle of the train along the bridge. This load distribution is particularly important for short span bridges as investigated by a number of studies (ERRI, 1999; Museros, Romero, Poy & Alarcón, 2002; Johansson et al., 2011; Axelsson, Syk, Ülker-Kaustell & Battini, 2014), which consistently show that the dynamic response significantly reduces for bridges with high fundamental frequency. The Eurocode (2003) suggests the use of load distribution patterns and recommends it for bridges with a span of less than 10 m, which avoids the definition of a track model. However, there are advantages in introducing a track model, as is done in the model presented in this paper. The approximate description of the track provides a more realistic load distribution compared to the simple load distribution patterns suggested by the Eurocode (2003), as well as it adds the ability of the track to filter high-frequency vibrations (Rebelo, Simões da Silva, Rigueiro & Pircher).

There are many levels of modelling complexity when considering trains travelling over bridges, depending on the assumptions made and the simplifications considered. Once the model is defined, the engineer is faced with the problem of selecting the correct model parameters. These values should ideally be derived from structural measurements and material experimentation, which are generally difficult to obtain. Therefore, the presented model is a trade-off between accuracy, model complexity, number of parameters and computational cost. Commercial FE software packages have the advantage of bypassing limitations in geometry and type of FE idealisation as well as the manual re-derivation of the system equations for the multi-body vehicle models in case of changed vehicle type. However, a specific purpose model can prove valuable for maximum computational efficiency. The TTB model presented in this paper is shown to be more computationally efficient than a corresponding implementation in ABAQUS, which is an essential feature when it comes to the ability to perform parametric studies.

A planar model is certainly less accurate than more complex 3D models. Good examples of 3D models can be found in (Yang, Yau & Wu, 2004; Zhang, Xia & Guo, 2008; Dinh, Kim & Warnitchai, 2009; Zhang, 2010) amongst others. These 3D models provide the lateral response of the infrastructure, which is important for the analysis of wind (Xia, Guo, Zhang & Sun, 2008) or earthquake loading (Yang, Yau & Wu, 2004) and also for traffic loading in curved bridges (Yang, Yau & Wu, 2004). However, 3D models are computationally very expensive and require the definition of many additional parameters, for which the exact numerical values are difficult to determine. Furthermore, it is known that a planar representation provides valid results when there are no significant 3D effects. Thus, the TTB model is valid for the analysis of bridges having beam-like behaviour.

The content of this paper has been divided into three distinct parts: model description, validation and numerical studies. First, the model description presents the sub-models for each of the TTB system components and provides a review of possible sources for the model parameters. Next, the TTB model is validated by direct comparison to the results of a commercial FE package. The model is then used to perform two different numerical studies that highlight important aspects of the VBI that are not often discussed in the literature. The first one investigates the change of the bridge's fundamental frequency during a train passage. The second study examines the influence of different track irregularity wavelength ranges on the responses of the bridge and the vehicle.

2. Model description

This section provides a detailed description of each component of the TTB model. They are presented in separate subsections as follows: vehicle, track irregularity, track and bridge. In addition, there is a comprehensive list of possible sources for each of the model parameters. The coupling of all the components together in the TTB model is summarized in an additional section followed by different alternatives for numerical solvers.

2.1 Vehicle

The vehicle model consists of a combination of lumped masses, rigid bars, springs and dashpots as presented in Figure 1. Each wheel, represented as a mass (m_w), is connected to the bogie by a primary suspension made of a spring (k_p) and a viscous damper (c_p) in parallel. The bogies are modelled as rigid bars with mass (m_b) and moment of inertia (I_b). Similarly, the main body of the vehicle is represented as a rigid bar with mass (m) and moment of inertia (I_v). The secondary suspension links the bogies and the main body together by means of springs (k_s) and viscous dampers (c_s). The geometrical configuration of the vehicle is defined mainly by the distance between the bogie centres (L_v) and the separations between wheels in each bogie (L_b). Finally, the distances from the first bogie to the front of the vehicle (L_f) and from the second bogie to the back of the vehicle (L_b) are important when defining a convoy of trains made of multiple wagons and locomotives. The total number of Degrees Of Freedom (DOF) of this multibody system is ten and corresponds to seven vertical displacements for the masses and three rotations of the rigid bars. Assuming small rotations, we can neglect the nonlinear geometric relations and quadratic terms leading to a linearized system of equations of motion (Nguyen, Goicolea & Galbadon, 2014). This model has been extensively reported in the literature and the equations of motion can be found in many sources: (Lei & Noda, 2002; Sun & Dhanasekar, 2002; Wu & Yang, 2003; Azimi, 2011; Ferrara, 2013), amongst others.

(Suggested location for Figure 1)

Figure 1: Sketch of vehicle model.

A train set can be defined as a succession of individual vehicles. By providing the correct dimensions and properties for each vehicle, it is possible to model locomotives and wagons, for passenger trains or freight traffic. The actual geometric and mechanical properties depend on the particular vehicle to be modelled. However, selecting the correct properties for a vehicle model is not a trivial issue. Geometric properties are generally accessible from various sources. The masses of the vehicle components can be estimated with a certain degree of confidence from maximum permitted axle loads and assumptions on passenger occupancy or freight load or can be weighed using weigh-in-motion systems (Sekula & Kolakowski, 2012). However, it is more difficult to define the properties of the suspension systems since they are not generally publically available.

The authors have compiled a list of sources (Table 1) where geometrical and mechanical properties of various types of vehicle can be found. Amongst the vehicles included are various high-speed trains and also some passenger vehicles for conventional speeds. It is noteworthy that the number of references that provide the properties of freight trains is very sparse, and Table 1 provides only two, namely the Swedish Steel Arrow and a bulk cement wagon from Irish Rail. Finally, the list also includes the properties of the Manchester benchmark which was defined to assess the suitability of various software packages for investigating vehicle dynamic behaviour. It is acknowledged that real train vehicles are very complex systems that have non-linear suspensions (Chu, Garg & Bhatti, 1985; Zhai, Wang & Cai, 2009; Bruni, Vinolas, Berg, Polach & Stichel, 2011). However, for the purpose of infrastructure assessment, simpler planar models like that presented here are generally accepted.

Table 1: Sources of vehicle model parameters.
(Suggested location for Table 1)

2.2 Track irregularities

Track irregularities are deviations from the ideal perfectly smooth track geometry. As the unsprung axle masses travel over the irregular profile, variations in the wheel–rail forces arise, providing an additional excitation of the TTB system. Random track irregularities are often idealized as stationary random processes, described by Power Spectral Density (PSD) functions. Various different PSD functions are used by different railway authorities, generally derived from field measurements. The PSD functions describe the severity of the irregularities as a function of the spatial frequency $\omega = 1/\lambda$ (where λ is the wavelength) or the circular spatial frequency $\Omega = 2\pi\omega$. Three of the most commonly used PSD functions in the railway-infrastructure research field are presented in Figure 2a and their formulations are provided in the following list:

- The SNCF (Société Nationale des Chemins de Fer français) spectrum is defined by Eq. (1) where ω_r = 0.0489 m^{-1} and A = 550 m^2/m^{-1} for poor track and A = 160 m^2/m^{-1} for good track. Eq. (1) can be used to describe wavelengths in the range, 2–40 m (UIC, 1971; Frýba, 1996; Berawi, 2013; Rocha, Henriques & Calçada, 2014).

$$S(\omega) = \frac{A}{\left(1 + \frac{\omega}{\omega_\mathrm{r}}\right)^3} \qquad \text{Eq. (1)}$$

- The German track spectrum is defined in Eq. (2), where Ω_r = 0.0206 rad/m and Ω_c = 0.8246 rad/m. The quality of the track is defined by the parameter A_p that ranges from $A_\mathrm{p} = 4.032\times10^{-7}$ m^2/(rad/m) for good track to $A_\mathrm{p} = 10.80\times10^{-7}$ m^2/(rad/m) for poor track (Guo, Xia, De Roeck & Liu, 2012; Berawi , 2013).

$$S(\Omega) = \frac{A_\mathrm{p}\Omega_\mathrm{c}^2}{(\Omega^2 + \Omega_\mathrm{r}^2)(\Omega^2 + \Omega_\mathrm{c}^2)} \qquad \text{Eq. (2)}$$

- The FRA (Federal Railroad Administration) spectrum is defined by Eq. (3), where $\omega_1 = 23.294\times10^{-3}$ m^{-1} and $\omega_2 = 13.123\times10^{-2}$ m^{-1}, and can be used to describe wavelengths in the range, 1.5–305 m. The scale factor A_v is used to define track class with $A_\mathrm{v} = 15.53\times10^{-8}$ m^2/m^{-1} for Class 1 and $A_\mathrm{v} = 0.98\times10^{-8}$ m^2/m^{-1} for class 6 (Hamid, Rasmussen, Baluja & Yang, 1983; Garg & Dukkipati, 1984; Frýba, 1996). Alternative formulations of the FRA spectrum can be found in (Wu &Yang, 2003; Dinh, Kim & Warnitchai, 2009; Berawi, 2013; Ferrara, 2013).

$$S(\omega) = \frac{A_v \omega_2^2 (\omega^2 + \omega_1^2)}{\omega^4 (\omega^2 + \omega_2^2)} \quad \text{Eq. (3)}$$

Other PSD functions that have been used in recent studies are the Chinese PSD (Lua, Kennedy, Williams & Lin, 2008; Zhai, Wang & Cai, 2009; Berawi, 2013) and the ISO 3095 (2013) in (Berggren, Li & Spännar, 2008; Ferrara, 2013). Frýba (1996) presents a collection of additional PSD functions together with a mathematical formulation that describes typical local isolated irregularities such as rail joints, hanging sleepers or bridge abutments.

The Eurocode EN 13848-5 (2010) prescribes limits for the maximum allowed track irregularities for running safety and track maintenance. It defines values for the range of wavelengths, 3–150 m subdivided into three wavelength ranges, namely D1 (3–25 m), D2 (25–70 m) and D3 (70–150 m). Limits are given for standard deviations over a defined length, as well as for isolated defects. However, no guidance is given in the Eurocode on what wavelength ranges should be adopted for the numerical analysis of bridges. In Section 4.2, the effect on the vehicle and the bridge of each of the wavelength ranges is analysed.

For time domain analyses, realisations of the PSD functions are obtained by performing inverse Fourier transforms with random phases. Detailed explanations on how to generate profile realisations from a PSD can be found for instance in (Claus & Schiehlen, 1998; Dinh, Kim & Warnitchai, 2009; Nguyen, 2013). Examples of profile realisations from the German track spectra containing wavelengths, 3–25 m, 3–70 m and 3–150 m are shown in Figure 2b.

a) b)

(Suggested location for Figure 2a) (Suggested location for Figure 2b)

Figure 2: a) Common PSD functions; b) a realisation of the German (good) track spectra considering various wavelength ranges.

2.3 Track

Ballasted railway tracks generally consist of rails, pads, sleepers, ballast and sub-ballast. It is possible to represent the track with various levels of complexity that can essentially be classified according to the number of sprung layers considered (Arvidsson & Karoumi, 2014). There are interesting examples of two-layer analyses in (Lei & Noda, 2002; Berggren, Li & Spännar, 2008; Savini, 2010; Kouroussis, Verlinden & Conti, 2014) and even four-layer models in (Sun & Dhanasekar, 2009; Nguyen, 2013). The model presented here is a three-layer system, which is recommended by the UIC (2009) to check the design requirements of railway bridges due to the dynamic interaction between train, track and bridge.

In the TTB model the track is modelled as a beam resting on evenly spaced sprung mass systems as illustrated in Figure 3. In particular, the rail is modelled as an Euler-Bernoulli beam using an FE discretization. The behaviour of the beam is defined by four parameters, namely the Young's modulus (E_r), sectional area (A_r), second moment of area (I_r) and its mass per unit length (μ_r). The sleepers are represented as masses (m_s) separated by a regular spacing (L_s) and connected to the rail by a spring and dashpot system with stiffness (k_p) and viscous damping (c_p) that represents the pad. The ballast is represented as a lumped mass (m_{ba}) that interacts with the sleeper by means of a spring (k_{ba}) and a dashpot (c_{ba}). Finally, the third sprung layer represents the dynamic properties of the sub-ballast with another spring and dashpot system in parallel with stiffness (k_{sb}) and viscous damping (c_{sb}) respectively.

(Suggested location for Figure 3)

Figure 3: Sketch of track model.

In reality the track is a complex 3D system that combines granular and fine-grained materials interacting continuously. It is not straightforward to find the equivalent properties of such a system to be modelled using the three-layer track model. However, (Zhai, Wang & Lin, 2004) suggest a clear methodology to find the correct equivalent model parameters. Nevertheless, it was found that the particular values to be used differ significantly, depending on the source. Table 2 shows a compilation of model properties found in the literature.

Table 2: Mechanical properties of three-layer track models.
(Suggested location for Table 2)

In contrast, the properties of the rail are better defined because it is a standard component. The majority of numerical studies model the UIC60 rail, because it is the most common profile used in high-speed railway lines. For completeness, the properties are provided in Table 3 that can be found in (ERRI, 1999). Note that the properties in Table 3 are for one single rail. In order to include both rails in the planar TTB model, the mechanical properties need to be doubled.

Table 3: Mechanical properties of rail.
(Suggested location for Table 3)

2.4 Bridge

The bridge is represented in the TTB model as an Euler-Bernoulli beam with elastic supports, which have vertical and rotational stiffness as shown in Figure 4. This configuration allows for the definition of a variety of bridge supports, from simply supported to fixed-fixed. The behaviour of the beam of length (L) is described by four parameters, namely the material's elastic modulus (E), cross-sectional area (A), second moment of area (I) and mass per unit length (μ). At the supports there is vertical stiffness (k_v) and rotational stiffness (k_r). The bridge has been implemented using an FE formulation using beam elements. The elemental matrices have been described in many publications and the reader can refer to (Kwon & Bang, 1997; Zienkiewicz & Taylor, 2000; Yang, Yau & Wu, 2004), amongst many others. The arrangement of these matrices into global form provides the set of equations of motion that describe the bridge's dynamic behaviour.

(Suggested location for Figure 4)

Figure 4: Sketch of bridge model.

Once the model of the bridge is defined it is paramount to select the correct model parameters. There are many examples in the literature that provide the properties of particular bridges. However, it is difficult to find properties of large bridge catalogues to describe railway bridges in general. This is why a compendium of bridge properties from various references is provided here in order to be able to describe the dynamic behaviour of bridges in general. This information is useful to draw general conclusions and provide recommendations on the design and assessment of railway bridges. Thus, this information has to be taken with caution when used for the analysis of one particular structure. In that case the engineer should refer to the nominal properties of the specific bridge under consideration.

The fundamental frequency (n_o) of a bridge is generally expressed in terms of span. In particular for railway bridges, the Eurocode (2003) provides upper (curve 1) and lower (curve 2) limits, as seen in Figure 5, which mark the typical bounds of n_o. When the fundamental frequency of the bridge under consideration is outside these bounds, additional checks and detailed numerical analysis are required in order to comply with the code. The study of a large railway bridge stock in Sweden in (Johanson et al., 2010), led to the definition of various interpolation curves depending on the bridge typology. Curve 3 in Figure 5 shows the resulting curve for concrete bridges. Also (Frýba, 1996) provides a comprehensive list of railway bridge fundamental frequencies and curve 4 in Figure 5 is the suggested curve for concrete bridges.

(Suggested location for Figure 5)

Figure 5: Fundamental frequency of railway bridges, 1) Upper bound (Eurocode, 2003), 2) Lower bound (Eurocode, 2003), 3) Concrete bridges (Johanson et al., 2010), 4) Concrete bridges (Frýba, 1996).

To completely describe the dynamic behaviour of a bridge it is necessary to know something more about the structure. For a full description either the mass of the structure (μ) or its bending stiffness ($E\ I$) is required. There is an evident lack of sources that provide these properties for railway bridges in general. Only for the mass per unit length of the bridge can some useful sources be found. In (Doménech, 2014) a study of a large catalogue of isostatic bridges for high speed lines was done, which allows for the definition of upper (curve 1) and lower (curve 2) limits (Figure 6). Also (Johanson et al., 2010) provides interpolating curves to define the bridge mass of various types of bridge. Curve 3 in Figure 6 shows the resulting curve for single track concrete bridges.

(Suggested location for Figure 6)

Figure 6: Mass of bridges per unit length, 1) Upper limit (Doménech, 2014), 2) Lower limit (Doménech, 2014), 3) Single track concrete bridges (Johanson et al., 2010).

Finally, some energy dissipation has been modelled in the TTB system as Rayleigh damping, which assumes that the damping matrix is proportional to a linear combination of the mass and stiffness matrices of the bridge (Zienkiewicz & Taylor, 2000). The Eurocode (2003) (Figure 7) provides the design damping for various types of bridge as a function of span. These lines represent the lower bound of a series of measured damping ratios for a wide range of bridges. Therefore, it is a source of conservatism to assume these damping values since, in general, higher damping is observed.

(Suggested location for Figure 7)

Figure 7: Bridge damping according to (Eurocode, 2003) for bridge type: 1) Steel and composite, 2) Pre-stressed concrete, 3) Concrete.

2.5 Coupling of systems

All the presented subsystems are combined together into one TTB model. An overview of the complete model is given in Figure 8 and can be summarized as follows. A train convoy is made up of several consecutive individual vehicles. The vehicles move on a rail that is resting over three layers of spring and dashpot systems that represent the pad, ballast and sub-ballast. The track is then resting either on the bridge or on a stiff foundation. The track is sufficiently long to ensure that the vehicle does not reach the bridge until after it has achieved dynamic equilibrium. It is also sufficiently long for the vehicle to leave the bridge completely.

(Suggested location for Figure 8)

Figure 8: Overview of TTB model.

As mentioned in previous sections, each of the subsystems is defined by a set of equations of motion. These second order differential equations can be represented in a general matrix form in terms of the mass (**M**), damping (**C**) and stiffness (**K**) matrices together with the vector of external forces (**F**) and solved for the vector of DOF (**X**). The coupled equations of motion of the complete model can be expressed in terms of block matrices (Eq. (4)) adopting the subscripts V, T and B to indicate vehicle, track and bridge subsystems respectively. The coupling of the subsystems is expressed mathematically with additional off-diagonal block matrices in Eq. (4). These coupling terms depend on the shape function of the beam element and the mechanical properties of the system. For a detailed derivation of these mathematical expressions, the reader is referred to (Lou, 2007).

$$\begin{pmatrix} M_{\mathrm{V}} & 0 & 0 \\ 0 & M_{\mathrm{T}} & 0 \\ 0 & 0 & M_{\mathrm{B}} \end{pmatrix} \begin{Bmatrix} \ddot{X}_{\mathrm{V}} \\ \ddot{X}_{\mathrm{T}} \\ \ddot{X}_{\mathrm{B}} \end{Bmatrix} + \begin{pmatrix} C_{\mathrm{V}} & C_{\mathrm{V,T}} & 0 \\ C_{\mathrm{T,V}} & C_{\mathrm{T}} & C_{\mathrm{T,B}} \\ 0 & C_{\mathrm{B,T}} & C_{\mathrm{B}} \end{pmatrix} \begin{Bmatrix} \dot{X}_{\mathrm{V}} \\ \dot{X}_{\mathrm{T}} \\ \dot{X}_{\mathrm{B}} \end{Bmatrix} + \begin{pmatrix} K_{\mathrm{V}} & K_{\mathrm{V,T}} & 0 \\ K_{\mathrm{T,V}} & K_{\mathrm{T}} & K_{\mathrm{T,B}} \\ 0 & K_{\mathrm{B,T}} & K_{\mathrm{B}} \end{pmatrix} \begin{Bmatrix} X_{\mathrm{V}} \\ X_{\mathrm{T}} \\ X_{\mathrm{B}} \end{Bmatrix} = \begin{Bmatrix} F_{\mathrm{V}} \\ F_{\mathrm{T}} \\ F_{\mathrm{B}} \end{Bmatrix} \qquad \text{Eq. (4)}$$

The coupling terms between the vehicle and the track depend on the vehicle's position; thus these terms are time dependent and need to be recalculated for every time step. On the other hand, the coupling terms between track and bridge remain constant, since there is no change in their configuration during one simulation. It is important to note also that when the vehicle and track are linked together, some of the DOF's of the vehicle are merged with those of the track. The DOF's of the wheels are no longer free to move; they are now combined together with the DOF of the rail. As a result, the mass matrix of the track ($\mathbf{M}_{\mathrm{T}}$) needs to be updated as well with the additional masses of the wheels at every time step. This is a commonly adopted coupling procedure, used in e.g.

(Lin & Trethewey, 1990; Lou, 2007). Alternatively, a contact spring can be introduced to simulate the wheel–rail contact (Rocha, Henriques & Calçada, 2014; Dinh, Kim & Warnitchai, 2009). In (Antolín, Zhang, Goicolea, Xia, Astiz, & Oliva, 2013) the rigid contact assumption is compared to both linear and non-linear Hertzian contact springs. It is concluded that the vertical vehicle–bridge dynamics can be studied using rigid contact, while the lateral dynamics requires the consideration of a wheel–rail contact element. They demonstrate that, due to the non-suspended wheel mass running over the uneven track profile, the vertical bridge deck acceleration has a slightly larger high frequency content using rigid contact than using Hertzian contact.

The external force vector (**F**) includes the contributions due to gravity and the excitation due to the rail irregularities. This is one advantage of treating the problem as a coupled system; it is not necessary to calculate the contact forces between vehicle and track since they are inherent in the formulation as internal forces. This is an important point since the contact forces consist of the combined contribution of the gravity loading and dynamic effects. Amongst the dynamic effects are the inertial, centripetal and Coriolis forces that develop as the wheel masses move over a flexible beam, in this case the rail. Centripetal forces relate to the curvature of deflection of the beam, while Coriolis forces relate to the rate of inclination of the beam. The centripetal and Coriolis forces can be neglected for a mass moving over a comparatively stiff and massive beam (Michaltsos, 2001; Yang, Yau & Wu, 2004). The curvature and rate of inclination of the beam are then small, making the contribution from the centripetal and Coriolis forces much smaller than those from the inertial forces. This is generally the case for models where the wheel loads are assumed to be moving directly on the smooth bridge beam. However, in the presence of a track with an irregular profile, the centripetal and Coriolis forces have to be considered in the external force vector and in the coupling terms between the vehicle and the track, as shown by (Lou & Au, 2013). For an exact formulation of these terms, the reader is referred again to (Lou, 2007).

2.6 Numerical solution

The coupled equations of motion need to be solved by numerical integration. There are many different methods available, and the reader can find in-depth discussion and descriptions of the algorithms in (Wilson, 1995; Xie, 1996; Zienkiewicz & Taylor, 2000). One method in particular is often used in the field of VBI, namely the Newmark-β method, which can also be found in (Lei & Noda, 2002; Yang, Yau & Wu, 2004; Dinh, Kim & Warnitchai, 2009). It is an unconditionally stable numerical scheme, which means that convergence of the solver is assured, regardless of the size of the chosen time step. However, stability does not ensure accuracy. The time step has to be sufficiently small in order to correctly calculate the dynamic effects up to the desired maximum frequency. It is generally recommended (Zienkiewicz & Taylor, 2000) to set the step size to ten times the maximum frequency of interest.

The implementation of the TTB model and its numerical integration was done in Matlab (2013). The time-variant system matrices have to be updated for every time step. This is achieved in an efficient manner by generating, only once, the uncoupled terms of the equations of motion. The coupling terms are recalculated for every time step and the global system matrices are then assembled as a combination of coupled and uncoupled terms. The total number of DOF's varies with the desired configuration and grows with the number of wagons considered or the desired length of track. Thus the final system matrices can be large, but many of the matrix elements are zero. Matlab allows for efficient management of these sparse matrices with enhanced and faster algorithms for matrix handling, multiplication and factorization.

3. Validation

The TTB model has been validated numerically and the results are presented in this section. As a preliminary check, the vehicle model alone was first validated, using the results from the Manchester benchmark (Iwnick, 1998). It showed good agreement with natural frequencies and modes of vibration. However, validating the whole model proved to be more difficult. There is no commercially available software that readily preforms VBI analysis. For this reason, a model including the train, track and bridge was developed in the multi-purpose FE software ABAQUS (2011) and additional routines were implemented to couple the moving vehicle with the rest of the model.

The ABAQUS model is equivalent to the TTB model, with the exception of the wheel–rail contact formulation. In ABAQUS a sliding surface-to-surface contact definition was used between the vehicle wheel nodes and the rail nodes, with no loss of contact allowed (Saleeb & Kumar ,2011; Arvidsson, Karoumi & Pacoste, 2014). The model is solved using the Hilber-Hughes-Taylor integration method with numerical damping, as opposed to the undamped Newmark method used in the TTB model. The required CPU time for the ABAQUS model is more than 20 times that of the TTB model.

For the numerical validation, the Skidträsk Bridge in Sweden (Ülker-Kaustell & Karoumi, 2012) is adopted. This 36 m simply supported composite steel/concrete bridge has bending stiffness, EI = 172.2 MNm^2, mass per unit length, μ = 15575 kg/m (excluding the track structure) and damping ratio, ζ = 0.5 %. The track structure has properties according to the ballasted track described in (Zhai, Wang & Lin, 2004) so that the fundamental frequency of the bridge, including the track structure, is 3.86 Hz. The vehicle consists of the ICE 2 train in a 12 passenger carriage configuration with a front and rear power car, with properties according to (Doménech, Museros &

Martínez-Rodrigo, 2014b). The rail profile is assumed to be smooth and a track length of 30 m is considered before the bridge. The TTB model is discretised with a mesh size of 0.6 m for the bridge, 0.2 m for the rail, and a time step of 0.0005 s. Rayleigh damping coefficients are chosen so as to achieve the damping ratio of 0.5 % at the first and second bridge modes.

The results of the ABAQUS model were compared to the results provided by the TTB model. Both models have been studied for a wide range of speeds and the maximum bridge deck accelerations compared. Then, the time history responses of vehicle and bridge are inspected at the particular speeds that result in resonant behaviour of the bridge. There was good agreement between the results which validates the TTB model.

3.1 Range of speeds

The bridge mid-span acceleration in the speed range 100–400 km/h is shown in Figure 9. As can be seen the agreement between the TTB and ABAQUS results is very good. The marginally lower accelerations that are obtained at resonance from the ABAQUS model can be attributed to the numerical damping associated with the integration scheme.

(Suggested location for Figure 9)

Figure 9: Bridge deck mid-span vertical acceleration.

For comparison, a moving load analysis using the TTB model is also included in Figure 9, i.e., an analysis where VBI is not considered. For this particular example, the effect of including VBI is considerable. This is due to the closeness of the bogie frequency to the bridge frequency, and the relatively high bogie–bridge mass ratio and bridge–carriage length ratio (Arvidsson & Karoumi, 2014; Doménech, Museros & Martínez-Rodrigo, 2014b).

3.2 Resonant behaviour

The system behaviour at resonant speeds is of great interest. Regularly spaced axles produce repetitive loading on the structure that at certain speeds lead to significantly greater dynamic effects. In particular, for the 36 m bridge traversed by the ICE 2 train, the resonant speed is 365 km/h, as can be seen in Figure 9. Time histories for the bridge mid-span acceleration and the car body acceleration for the last passenger carriage are shown in Figure 10. Both models provide almost identical responses at the critical speed, not only for the bridge but also for the vehicle.

a) b)

(Suggested location for Figure 10a) (Suggested location for Figure 10b)

Figure 10: For the resonant train speed of 365 km/h: a) bridge deck mid-span vertical acceleration; b) car body vertical acceleration of the last ICE 2 passenger carriage.

4. Numerical studies

The TTB model is powerful, versatile and efficient software that can be used to study the dynamic behaviour of TTB systems. It allows the user to study the response of the vehicle and the infrastructure for a wide range of model configurations. In this section, the model is used to perform two numerical studies. First, the change of the bridge's fundamental frequency is investigated for various locations of a locomotive on the structure. The second study examines the influence of track irregularity wavelength ranges on the responses of the bridge and the vehicle.

4.1 Change of fundamental frequency

It is known that the fundamental frequency of a bridge changes when additional masses are located on it, as is the case during the traversing of a train. It has been reported in (Liu, Reynders, De Roeck & Lombaert, 2009) that the frequencies of a particular bridge are different when it is in forced, as opposed to free, vibration. The extent of the variation in frequency depends on the position of the vehicle and the (vehicle to bridge) mass ratio. This had been explained for the case of a moving mass by Frýba (1996, 1999) who provides an approximate analytical expression. More recently there have been new analytical (Yang, Cheng & Chang, 2013) and numerical (Cantero and OBrien, 2013) studies on the change of the bridge's fundamental frequency that consider simple 1-DOF sprung vehicles. However, the available studies deal only with very simple vehicle configurations (moving masses or 1-DOF vehicles). With the TTB model it is possible to study the change of the system frequencies of more complex and realistic vehicle–track–bridge configurations.

The change in the bridge's fundamental frequency is studied for the particular case where big changes in frequency can be expected, that is, for the situation with a high mass ratio. The locomotive of the Steel Arrow (Arvidsson & Karoumi,2014) (an 84 t vehicle) is modelled traversing the 36 m single track Skidträsk Bridge from Section 3 (with an approximate total mass of 612 t, including the track). This situation features a high mass ratio because it consists of a heavy vehicle and a relatively light bridge, a combination that gives a mass ratio of 0.14. This has been modelled using the TTB model for various locations of the vehicle and the change in the fundamental frequency of the bridge is shown in Figure 11, where the vehicle position refers to the position of the first axle with respect to the first support of the bridge. The results show clearly that the fundamental frequency of the bridge changes with the position of the vehicle on the structure. The maximum difference in beam frequencies is found when the centre of gravity of the vehicle is exactly at mid-span. In that case the frequency of the bridge is 3.743 Hz while the original frequency of the bridge without the vehicle is 3.865 Hz, a variation of -3.16 %. Note that the rest of the system frequencies also change but to a lesser extent. Compared to the approximate expression for the moving mass case in (Frýba, 1996), the actual variation in frequencies is much smaller. The approximate expression gives a fundamental frequency when the vehicle is at mid-span of 3.386 Hz, a difference of -12.40 % with respect to the original value. This is because the expression assumes that the entire weight of the vehicle is concentrated into a single load, which is then resolved into a Fourier series and added to the self-weight of the bridge.

(Suggested location of Figure 11)

Figure 11: Bridge's fundamental frequency as a function of the vehicle's position. No vehicle (dotted), coupled vehicle–track–bridge (solid), approximate expression (dashed) (Frýba, 1996).

Figure 12 shows the maximum change in the bridge fundamental frequency for a range of mass ratios. The mass of a locomotive located at the centre of the bridge is changed in order to achieve the indicated mass ratio. Again, the results from the numerical TTB model and the approximate analytical solution in (Frýba, 1996) are directly compared. The results clearly show that the approximate analytical expression of a concentrated mass significantly overestimates the effect of the vehicle on the bridge frequency.

(Suggested location for Figure 12)

Figure 12: Maximum change in bridge fundamental frequency. Coupled vehicle–track–bridge (solid), approximate expression (dashed) (Frýba, 1996)

However, it is found that the study of the bridge's fundamental frequency becomes a rather complex problem when complicated vehicle models are considered like in the TTB model. The change in frequency does not depend only on the location and mass ratios, but also on frequency ratios between vehicle and bridge. For this reason, in Figure 12, the moments of inertia and suspension stiffness properties of the vehicle have been adapted in order to maintain constant frequency ratios between systems.

The results presented in this section clearly show that changes in a bridge's fundamental frequency do occur and can be of significant magnitude. This is a fact that is generally overseen when engineers study the dynamic responses of structures in forced vibration. As in the example in Figure 11 a change of -3.16 % in frequency could lead to the wrong conclusion that the structure is damaged with a stiffness reduction of 10 % approximately. It is acknowledged that a better understanding of this phenomenon is needed. A detailed analysis of the bridge's fundamental frequency in terms of mass and frequency ratios using TTB model will address this issue in future studies, but is out of the scope of this document.

4.2 Track irregularity and wavelength

It is well-known in railway vehicle dynamics that the ride comfort is affected by track irregularities, and that also longer wavelengths could significantly affect the car body response. However, which wavelengths are most important for the bridge response is not often discussed. An example on the effect of different wavelength ranges on bridge displacement was given by (Dinh, Kim & Warnitchai, 2009). In this section, the effect on bridge deck acceleration, as well as on vehicle response and wheel–rail contact forces, will be exemplified. Three ranges were considered for the track profile, namely 3–25 m, 3–70 m and 3–150 m, which were chosen according to the Eurocode EN 13848-5 (2010) that defines wavelength ranges D1 (3–25 m), D2 (25–70 m) and D3 (70–150 m). The German PSD function was adopted and scaled to obtain a track quality class A for design speeds 230–300 km/h according to EN 13848-6 (2014), which corresponds to a standard deviation of 0.4 mm within the wavelength range, D1. Realisations of this profile have been included in the same TTB configuration as used in the validation section, i.e. the ICE 2 vehicle traversing a simply supported 36 m span bridge. The only differences are the length of the track and the additional irregularities. Here a 300 m long track section is considered at the approach to allow the vehicle to reach dynamic equilibrium, as well as a 50 m track section after the bridge.

The results presented in Figure 13 and Figure 14 provide the statistical summary of the analysis on 15 profile realisations in the range 100–400 km/h. The solid lines correspond to the average result, whereas the shaded areas indicate the dispersion of values in terms of one standard deviation above and below the mean. From the results it can be seen that the bridge deck acceleration, presented in Figure 13a, increases slightly with a D1 profile as compared to the smooth profile (no irregularity). The addition of longer wavelengths (D2) has a small effect on the bridge deck acceleration and the addition of even longer wavelengths (D3) has no discernible effect. Moreover, it can be observed that the standard deviation of the acceleration increases at resonance.

a) (Suggested location for Figure 13a)

b) (Suggested location for Figure 13b)

Figure 13: Effect of track irregularity wavelengths 3–25 m, 3–70 m, 3–150 m and a smooth profile. Mean value (solid line) and standard deviation (shaded area) of 15 profile realisations, a) bridge deck mid-span vertical acceleration, b) minimum and maximum wheel–rail contact forces for the fourth wheel of the last passenger carriage.

The maximum and minimum wheel–rail forces are plotted in Figure 13b. It can be seen that the presence of track irregularities D1 has a significant effect on the wheel–rail forces, while the addition of longer wavelengths (D2 and D3) has a negligible effect. This is unsurprising as the majority of the energy content in the contact forces originate from higher frequencies and it is thus affected mostly by the shorter wavelengths, D1. It is important to note that TTB uses a rigid contact model between wheel and rail. Thus, the high-frequency energy content of the calculated contact force is not as accurate as with other more complex contact representations. If a detailed analysis of the contact force is required a more realistic contact model should be used. However, for the analysis of bridge responses the use of the rigid contact model is sufficiently accurate.

The vertical car body acceleration provides an estimate of the passenger comfort. The vertical acceleration of the last passenger carriage of the ICE 2 is plotted in Figure 14a. It can be observed that the car body acceleration increases only slightly with the introduction of D1 track irregularities, as compared to a smooth profile. On the other hand, the addition of wavelength ranges D2 and D3 has a considerable effect, especially so for the longest wavelengths (D3). The car body bounce natural frequency is 0.64 Hz. Thus, the car body frequency lies within the range of the frequencies induced by the D3 wavelengths in the speed range 100–400 km/h (0.2–1.6 Hz). A comparison between the car body acceleration in Figure 14a and the bogie acceleration in Figure 14b shows that the vehicle suspension system effectively filters the high frequencies (short wavelengths), while the lower frequencies close to the vertical car body frequency are not as effectively filtered.

a) b)

(Suggested location for Figure 14a) (Suggested location for Figure 14b)

Figure 14: Effect of track irregularity wavelengths 3–25 m, 3–70 m, 3–150 m and a smooth profile. Mean value (solid line) and standard deviation (shaded area) of 10 profile realisations, a) car body vertical acceleration, b) bogie vertical acceleration of the first bogie of the last passenger carriage.

The results from this example supports the conclusion that the shorter wavelengths are important for safety analyses (wheel–rail forces), while the longer wavelengths (D3) are more related to vehicle ride quality (EN 13848-5, 2010). It should be stressed, though, that the frequencies induced from each wavelength range increase with speed. Therefore, for high speeds the effects on safety measures from the longer wavelengths may still be important.

5. Conclusions

This paper has presented in detail a Matlab model to simulate the dynamic interaction that occurs between a train and the infrastructure (track and bridge). The paper provides a review of sources for model parameters, track irregularity representations and numerical solvers. The model has been validated against ABAQUS showing that it is accurate, versatile and efficient.

The TTB model was then used to perform numerical studies on two aspects of the train-bridge interaction problem. The results clearly show that changes in the bridge's fundamental frequency occur during train passages and that they can be of significant magnitude. These changes are due to the sole presence of a vehicle on the bridge. Furthermore, an analysis of the influence of various wavelength ranges confirms that shorter wavelengths are important for safety analyses (wheel–rail forces) and structural assessment, while the longer wavelengths are more relevant to vehicle ride quality.

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