Workers' rest allowance and smoothing of the workload in assembly lines

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Ergonomic aspects have a crucial role in manual assembly systems. They impact on the workers' health, final product quality and productivity. For these reasons, there is the necessity to integrate them into the assembly line balancing phase as, whereas, only time and cost variables are considered. In this study, human energy expenditures are considered as ergonomic aspects and we integrate them, for the first time, into the assembly line balancing problem type 2 through the rest allowance evaluation. We consider as an objective function the minimization of the smoothness index. Firstly, a new optimal method based on mixed integer linear programming and a new linearization methodology are proposed. Then, a heuristic approach is introduced. To complete the study, a computational experimentation is presented to validate the mathematical model and to compare the methodologies proposed in terms of computational time, complexity and solution. Additionally, we provide a detailed analysis of the impact that rest allowance evaluation can have on productivity comparing the results obtained, taking into account the rest allowance integration before, during and after the assembly balancing process.

Keywords: assembly line balancing; ergonomics; optimization; rest allowance; smoothness index

1. Introduction

Manual assembly lines are special flow-line production systems which are typically used in the final steps of production of standard products in high quantities (Scholl, 1999). Assembly lines were initially introduced to increase the efficiency in the mass production of standardized products but over the years, with some appropriate changes, they have been widely used also for customized products. A typical manual assembly line consists of several workstations, on which a set of tasks are performed by one or more workers, and transportation systems that move parts and products between the workstations. Furthermore, task times can be deterministic or stochastic (Otto and Scholl, 2011).

The manual assembly process required to obtain the final product represents one of the most important phases of production systems due to its high added value, its contribution to the final product quality and its direct connection with the final market (Dolgui and Proth, 2010). For these reasons, practitioners and academics continuously tend to develop new approaches and to ameliorate the existing ones to improve the efficiency, the productivity and to guarantee the required flexibility. In particular, in the last decades, several research studies have been conducted with the aim to evaluate the workers ergonomic conditions during the assembly process and to define new strategies to integrate the ergonomic measure during the assembly line design phase. The reason is closely linked to the fact that some recent available estimations reveal that about 160 million workers in the world suffer from work-related musculoskeletal disorders (WMSDs) and the decrease of the gross national product due to this problem has been considered to be equal to 3.94% (ILO, 2019).

WMSDs are one of the main causes of productivity reduction in fact, especially in manual assembly systems, workers have to perform repetitive movements with a high level of stress and physical fatigue and awkward postures. In this way, the ergonomic risk, the fatigue level and, consequently, WMSDs among workers increase (Falkenauer E., 2005) causing a reduction of workers' well-being, product quality and efficiency (Otto and Scholl, 2011).

For this reason, several studies have been conducted to incorporate ergonomic estimation methods into assembly line balancing problems (ALBP) to take into account human aspects and working conditions. Generally, in fact, the ergonomics analysis is made by experts only after the ALBP and so, sometimes the required modifications are infeasible due to technological constraints or to the high impact on the company final costs (Battini et al., 2016).

One of the first attempts to integrate ergonomics into ALBP was carried out in the research conducted by Otto and School (2011). After, other studies and models that incorporate traditional ergonomic evaluations and traditional ergonomic indexes have been developed and several mathematical models have been proposed integrating workers' conditions and ergonomic aspects (Otto and Battaia, 2017). However, the majority of them are non-linear with a high computational complexity. Additionally, the ergonomic methods used to evaluate MSDs risks are semi-quantitative indexes, they are not applied to single tasks, they require high ergonomic competences and a lot of time to evaluate them.

In order to provide a method to evaluate the ergonomic level in an industrial context, Battini et al. (2015, 2016) introduced a new methodology to incorporate human aspects in ALBP taking into account human energy expenditure and rest allowance.

For this reason, starting by these two recent works we propose a new model that incorporates rest allowance with the formulation provided by Price (1990) and we evaluate the optimal balancing solution that minimizes the smoothness index. Additionally, due to the non-linearity of the problem, we suggest a new approach to linearize it.

The rest of the study is organized as follows: Section 2 provides an overview of the smoothness index and the role of rest allowance in the assembly systems. In Section 3, the linearization methodology is discussed and compared with the existing method and the model with rest allowance integration is explained. In Section 4, the heuristic methodology is described. In Section 5, a numerical experimental analysis is provided and discussed to demonstrate the validity of the models. The different methodologies are compared, and some guidelines are given. Finally, in Section 6, several conclusions are presented to summarize the contributions of this work and some perspectives will be given.

2. Literature background

2.1.The smoothness-index

As stated in the previous section an assembly line consists of a set of workstations where a set of operations are carried out with the aim to obtain the final product. The decision of which set of tasks must be performed in each workstation is known as the assembly line balancing problem (ALBP) (Scholl, 1999). It is possible to divide ALBP into two main categories: Simple-ALBP (SALBP) and General-ALBP (GALBP) (e.g., Boysen et al, 2008).

In SALBP the objective function can be the minimization of the workstations number (SALBP-1), the minimization of cycle time (SALBP-2) or the minimization of the idle time (SALBP-E). Additionally, in a SALBP-F, the feasibility problem is evaluated for a given number of workstations and a cycle time.

The three objective function described above are the most used. Nevertheless, other objective functions can be used or can be integrated into one of those described above to improve the efficiency or decrease the total cost of the final assembly line (e.g., Battaia and Dolgui, 2013).

One of these supplementary objective functions is the smoothness index (SI) that measures the equality of the distribution of work among the stations in order to have similar workstation processing times (Scholl, 1999).

The SI was introduced and discussed for the first time by Moodie and Young (1965). Even if in the literature there are several formulations to evaluate the SI the most used is defined as follows:

$$\sqrt{\sum_k (c - T_k)^2} \tag{1}$$

Where:

- c is the cycle time;
- T_k is the workstation time.

After the work of Moodie and Young (1965), several heuristics and meta-heuristics models have been developed with the aim to improve the existing ones and to evaluate the SI as a part of multi-objective functions. In particular, a two stage-heuristic algorithm is proposed by Rachamadugu and Talbot (1991) and by Eswaramoorthi et al. (2012) that integrate SI in a multi-objective model with lean perspectives. On the other side, meta-heuristic models (i.e. genetic algorithms, differential evolution algorithms) have been proposed by Nearchou (2008) and Hamta et al. (2013) in which SI is considered in different multi-objective functions.

To the best of our knowledge, in only two cases (Esmaeilbeigi et al. 2015, Azizoğlu et al. 2018) exact approaches are proposed. In particular, Esmaeilbeigi et al. (2015) propose a mixed integer linear programming formulation for the SALBP-E while in the recent work of Azizoğlu et al. (2018) a branch and bound approach to minimize SI in a SALBP-F is proposed.

These last two papers provide also some interesting linearization methodologies. In particular, in Esmaeilbeigi et al. (2015) three linear methods are proposed, the SI is defined considering the idle time as a variable and they assume a range of values that idle time can assume. To further strengthen their formulation additional valid inequalities and auxiliary variables are introduced. On the other side, Azizoğlu et al. (2018) minimize the SI, considering that for a SALBP-F it is equivalent to the minimization of the workstations time as the cycle time is constant between stations. In this way, they linearize only T_k and additional variables and constraints are introduced. However, one of the main limitations of this method is the high number of variables required as underlined also by the authors.

As defined in the literature, there are several reasons to consider SI:

- it establishes the sense of equity among workstations (Rachamadugu and Talbot, 1991);
- it contributes to increasing the final output (Smunt and Perkins, 1985);
- it reduces the breakdown probabilities of machines and consequently, it increases the remaining lifetime of the machines (Otto and Scholl, 2011);
- it increases the chance of reaching the target production rate (Otto and Scholl, 2011);
- it reduces the ergonomic risk (Groover, 2013).

However, even if Groover (2013) underlines the importance to evaluate SI as a method to reduce the ergonomic risk, until now no methods have been proposed to evaluate the impact that ergonomic aspects can have on the smoothness index. Furthermore, when additional non-linear constraints, such as those linked to the ergonomic field, are introduced in the SI formulation the methodologies proposed are not able to give a solution in a reasonable computational time, and so it is necessary to create ad hoc methods and models or adapt the existing ones to the specific problem.

2.2.Ergonomic aspects

Recently, Otto and Battaia (2017) provided a literature review about the integration of ergonomics into ALPs. They evaluate only papers that propose optimization models for the assembly balancing and scheduling and it emerges that the literature on this topic is quite scarce and traditional ergonomic risk indexes (e.g., OCRA, RULA, REBA, NIOSH) are the most used. Baykasoglu et al. (2017) integrates the ergonomic risk in the assembly system design through a systematic approach divided in three phases on which OCRA index is considered to evaluate the ergonomic risk. Recently, Akyol and Baykasoğlu (2019) have proposed a multiple-rule based constructive randomized search approach to solve an assembly line worker assignment and balancing problem (ALWABP) which considers ergonomic risks through OCRA index. However, as underlined by Carnahan et al. (2001) the workers' fatigue level also can negatively impact the productivity and it should be considered during the assembly line design. The workers' fatigue could affect the whole body or only some parts and, for this reason, it can be evaluated with several methods (Konz, 1998).

Recently, Abdous et al. (2018) have proposed a new optimal method based on mixed integer linear programming with consideration of both fatigue and recovery of workers. They introduce in the SALBP the dynamic fatigue through the Ma et al. (2009) model. Recently, El Mouayni et al. (2019) have proposed a simulation-based approach for time allowances assessment in three production system configurations considering also worker's fatigue, learning and reliability aspects.

However, when the whole body is used to execute a task it could be better to evaluate the energy expenditure level as defined by Astrand (1967). Additionally, this type of measure could be evaluated for every single task, despite other methodologies that evaluate macro-activities.

To avoid the decreasing physical level of workers, some models have been created to evaluate the so-called rest allowance (RA) that can be defined as the time needed for adequate rest after the execution of static or dynamic exertion (Rohmert, 1973). In the literature review compiled by Imbeau (2009) five rest allowance formulations are compared and the advantages and disadvantages of each are analyzed. The majority of them evaluate the fatigue level in a non-straightforward way and a lot of parameters are required to calculate RA. Furthermore, the input data required is very difficult to obtain and, consequently, there are limits to practical applications. To the best of our knowledge, only the model proposed by Price (1990) can be used to define in a very simple way the RA required after the execution of tasks that involve the whole body. In his model, the ratio between energy expenditure and the time required to execute a task (called mean working rate, MWR) and the maximum acceptable working level (MAWL) are used.

The model proposed is very simple and at the same time can be very useful for managers and practitioners who want to evaluate the required recovery time to give to workers. Following this formulation, RA occurs only if the MWR exceeds the threshold value, MAWL. Additionally, it can be easily modified if the characteristics of the worker change.

As RA is used to protect the workers' health during the working period, it can be considered as a support of a fixed allowance that takes into account only the workers' personal needs and the basic fatigue.

Considering the application of human energy expenditure in the field of ALBP, Gunther et al. (1983) represent the first attempt to introduce it into assembly balancing problem. In his work the necessity to avoid the assignment of several heavy tasks to the same worker also represents one of the main goals that companies would obtain.

After this work, to the best of our knowledge, Battini et al. (2016) propose the Predetermined Motion Energy System (PMES) and a multi-objective model for a SALBP-2 considering the task times and the task energy expenditures.

On the other side, the RA has been introduced for the first time into ALBP in 2015 when Battini et al. (2015) proposed a comparison between the multi-objective model (Battini et al., 2016) and a single-objective one where energy expenditure has been converted into RA with Rohmert's formulation (1973).

Recently, Finco et al. (2018) have analyzed the impact that human energy expenditure can have on ALBP. In particular, they propose a heuristic procedure and they compare the results obtained when human energy aspects are considered before, during and after the balancing phase. In Tiacci et al (2018) an approach to simultaneously finding solutions for the ALBP and the Rest Time Assignment Problem is proposed.

3. Problem description and notation

3.1. Smoothness-index linearization

3.1.1. SALBP-2-SI formulation

In this paper the objective function is the minimization of the smoothness index as defined in (1). The final model (SALBP-SI-2) can be formally stated as follows:

- m workstations arranged along an assembly line;
- n tasks to obtain the final product;
- deterministic execution time for each task t_i;
- task i can be assigned only if its predecessors have already been assigned;
- each station k has a station load defined as T_k;
- each workstation should complete its assigned tasks within a specific cycle time, c.

The objective function, in a SALBP-SI-2, is to find a feasible line balance that minimizes:

$$SI = \sqrt{\sum_k (c - T_k)^2} \text{ (2)}$$

While the constraints are defined as follows:

$$\sum_{j \in [E_i, L_i]} x_{ij} = 1 \ \forall \, i=1,...,n \text{ (3)}$$

$$T_k \le c \quad \forall \, k = 1,..,m \text{ (4)}$$

$$\sum_{k \in [E_h, L_h]} x_{hk} k \le \sum_{j \in [E_i, L_i]} j x_{ij} \ \forall \, (h,i) \in A \, and \, L_h \ge E_i \text{ (5)}$$

Where:

- $c \in Z^+$;
- $T_k = \sum_{i \in B_k} x_{ik} t_i \quad \forall \, k=1,...,m$;
- $x_{ik} \in \{0;1\} \qquad \forall \, i=1,..,n \ \forall \, k=1,..,m$;

Constraint (3) ensures that each task is assigned to exactly one workstation. Constraint (4) guarantees that the cycle time, c, is not exceeded. Constraint (5) maintains precedence and technological priority among tasks.

In this model, there are two types of decision variables which are the cycle time c, an integer decision variable, and x_{ik}, Boolean variables associated to each task.

As defined by Scholl (1999) the workload smoothing line balancing is strongly NP-hard. If we consider a SALBP-2 and we want to minimize the SI, it is necessary to investigate a method to linearize SI as it is a non-linear function. However, it is obvious that instead of minimizing the SI directly, one can minimize SI^2. Furthermore, it is possible to replace the difference between c and and T_k with an additional variable called idle time (∂_k) and add in the model this following constraint:

$$T_k + \partial_k = c \ \forall\, k = 1,..,m \ (6)$$

that permits to evaluate the idle time for each workstation.

In this way, minimizing equation (2) is equivalent to minimize:

$$SI^2 = \sum_k (\partial_k)^2 \ (7)$$

The quadratic term of equation (7) implies that a major *c* can be a solution to have a lower SI as demonstrated in this following example.

We assume to have a list of 4 tasks to execute in 3 workstations. The precedence graph is illustrated in Figure 1. We consider two values of *c*, respectively, 10 s and 11 s and we evaluate the possible solutions. In this case, for the higher *c* we have found a lower SI (see Table 1). This is due to the quadratic term of ∂ that appears in (1). So, the minimization of *c* does not imply the SI one.

Please insert here Figure 1

Please insert here Table 1.

As in a SALBP-2, *c* is unknown as the task to station assignment, so, the ∂ of each workstation is also unknown, however, the theoretical upper bound presented in Proposition 1 (see Appendix 1) can be used to limit the value that ∂ can assume.

For *m* workstations a lower (resp. upper) bound, LB(*m*) (resp. UB(*m*)), of *c* exist. Under LB(*m*) some tasks could remain unsigned while above UB(*m*) some workstation cannot have tasks. To evaluate UB(*m*) we consider the case in which in a workstation there are the *n-(m-1)* bigger tasks while in the others *m-1* ones there are, in each of them, one of the remaining *m-1* lower tasks. In this case, UB(*m*) is defined as the sum of the *n-(m-1)* bigger tasks. So, if *c* is equal to the UB(*m*) this implies that in at least one workstation only one task is assigned, and its time is at least equal to the minimum task time. Thus, for this workstation the ∂ is equal to Ψ (see Appendix 1). On the other side, ∂ has a minimum value equal to zero since in at least one workstation its total time is equal to the *c*.

As ∂_k is a positive integer variable we can use the Property 1 (see Appendix 1) to linearize the quadratic term of equation (6). We assume that an integer variable can be written using the base 2 formulation. Consequently, ∂_k and its quadratic term becomes a linear function of Boolean variables (see Appendix 1).

In this way, the objective function (7) can be replaced by (8):

$$\min SI^2 = \sum_k \vartheta_k \quad (8)$$

where:

- $\vartheta_k = \partial_k^2$;
- the constraints set (21)-(25) (see Property 1, Appendix 1) is also included in the final model.

Finally, the objective function (8) with constraints (3)-(6), (21)-(25) represent the final linear formulation. Note that m·p real and m·p Boolean additional variables are required.

If this method is compared with the similar approach proposed by Esmaeilbeigi et al. (2015) we can note that the number of additional Boolean variables required to linearize SI with their formulation is equal to m·Ψ that is greater than our method. Furthermore, our methodology can easily adapt to cases where idle time does not assume integer value.

This happens if we want to integrate the worker's fatigue evaluation, through the RA as will be explained in the next sections.

3.1.2 SALBP-2-SI with RA integration before the balancing process

In this case, we assume to know, for each task, the energy expenditure, e_i, expressed in kCal/min. Initially we convert e_i in rest allowance before the optimization process. Considering Price's formulation (Price, 1990) we can define, for each task, the rest allowance, RA, as follows:

$$RA_i = \max\left(0; \frac{\frac{e_i}{t_i} \cdot 60 - 4.3}{4.3 - 1.86}\right) \quad (9)$$

Where:

- t_i are measured in seconds;
- 4.3 represents the MAWL (expressed in kCal/min);
- 1.86 represents the relaxation rate in a standing position (expressed in kCal/min).

In this way, the final time (execution time + recovery time) for each task is equal to:

$$t'_i = t_i(1 + Ra_i) \quad (10)$$

Replacing t_i by t'_i we can apply the same model explained in the previous section. However, ∂_k does not assume only integer value and for this reason a new approach must be used according to Proposition 2 (see Appendix 1).

In this way, by integrating Proposition 2 in Property 1 the model can be easily used to evaluate the minimization of the SI with task times that can assume real values.

The main difference between the method previously proposed is the larger number of Boolean variables required to represents ∂_k which depends on the precision level we want to assure according to the ε value.

Additionally, as in this case ∂_k is defined with a precision equal to ε it is necessary to replace Equation (6) by (11) which means that ∂_k can be evaluated with an accuracy equal to ε.

$$c - T_k + \partial_k \leq \varepsilon \, \forall \, k = 1,..,m \, (11)$$

In conclusion, the final model becomes:

$$\min SI^2 = \sum_k \vartheta_k \; (12)$$

sub to
(2)-(5), (11), (21)-(25).

3.1.3. SALBP-2-SI with RA evaluation after the balancing process

Another way to evaluate the RA required to alleviate the fatigue effort is to evaluate it after the balancing phase considering the following equation:

$$RA_j = \max(0; \frac{\frac{\sum_{i \in Bj} e_i}{\sum_{i \in Bj} t_i} \cdot 60 - 4.3}{4.3 - 1.86}) \qquad \forall j = 1,..,m \, (13)$$

Where B_j represents the set of tasks associated to station j.

The tasks to station assignments are defined using the model explained in Section 3.1.1 and during the post-processing the energy expenditure values are introduced to evaluate, for each workstation, the RA and, finally, the workstation time. Due to the RA integration the *c* could increase with a consequent change of the SI. Therefore, the final solution cannot be the one that minimizes the SI.

3.2. SALBP-2-SI with RA integration

The models proposed in the previous section have some limits. In fact, if RA is evaluated for each task, the final solution can overestimate the RA and, consequently, the non-value production time tends to increase. In fact, the worker rests for a time which does not represent his real recovery necessity. On the contrary, considering the RA evaluation after the balancing phase the recovery can be underestimated.

For this reason, according to Price (1990), it is necessary to evaluate the correct tasks to station allocation considering that for a set of tasks the RA is equal to:

$$RA = \max(0; \frac{\frac{\sum_i e_i}{\sum_i t_i} \cdot 60 - 4.3}{4.3 - 1.86}) \ (14)$$

This means that a task with a low energy expenditure can balance a task with a higher energy expenditure and, consequently, the RA evaluated in this way is lower than the sum of each RA associated to each task.

Considering equation (15) the final execution time for a workstation becomes:

$$T' = \sum_i t_i \cdot (1 + RA) \ (15)$$

These assumptions can now be used to define the new model to evaluate the SI. Considering the model presented in the previous section we are now able to formulate a new approach to integrate RA. The objective function remains the same as in (6) however, the workstation time T_k becomes:

$$T'_k = \sum_{i \in B_k} x_{ik} t_i (1 + R'_k) \quad \forall k = 1,..,m \ (16)$$

Where:

$$R'_k = \max(0; \frac{\frac{\sum_{i \in B_k} x_{ik} e_i}{\sum_{i \in B_k} x_{ik} t_i} \cdot 60\text{-}4.3}{4.3\text{-}1.86}) \ (17)$$

It is possible to note that equation (16) and (17) are not linear, and T'_k (resp. R'_k) are real values. However, using Property 2 (see Appendix 1.) we can convert this set of equations into linear ones.

Finally, considering Property 1 and 2 we are now able to present the final model where RA is evaluated during the balancing phase.

The final model is as follows:

$$\min SI^2 = \sum_k \vartheta_k \quad (18)$$

sub to

(3), (5), (21)-(32) and

$$c - T'_k + \partial_k \leq \varepsilon \; \forall k = 1,..,m \quad (19)$$

Where equation (19) evaluates ∂_k with an accuracy at least equal to ε .

4. Heuristic method

In this Section, we propose a heuristic approach to evaluate the SI with the integration of RA.

As defined in Scholl (1999), ALBPs problems fall into the NP-hard class of combinatorial optimisation problems. For this reason, over the years several heuristic algorithms have been developed (e.g., Becker and Scholl, 2006). Some of them define a procedure to obtain a lower SI (e.g., Moodie and Young, 1965) however they cannot be easily adapted to our problem because station time depends on the RA, which can change according to the time and the energy associated to each task. For this reason, an ad-hoc heuristic is here proposed (see Figure 2). For a given number of tasks, *n*, their time, human energy expenditure and precedence relations are known. Additionally, for a given number of workstations, m, we can evaluate the lower and upper bound according to Table 2. Then, an ordered list (OL) is created to list tasks following the descending order of their time as the aim is to allocate first the task with a longer time. Initially, cycle time, c, is set equal to the maximum value of the lower bound while the idle time, ∂, for each station is equal to c. The first task of the OL is taken and its earliest and latest station are calculated. In particular, the earliest, as well as the latest station, are calculated with and without the integration of RA. If the earliest station is bigger than the latest one c is incremented by 1 and the procedure restarts. Otherwise, for the task chosen the feasible station list (FS) is created considering that each station must have a sufficient ∂, at least equal to the task time with its RA. If the FS is empty, c is incremented by 1 and the procedure restarts. On the contrary, a station is randomly chosen evaluating the not yet assigned predecessors and successors of the task chosen. In particular, if the number of predecessors is greater than that of the successors the probability to choose the latest station of the FS will be higher than the other available station. On the contrary, if the successors time is greater than that of the predecessors, the earliest station belonging to the FS will be selected.

After the selection of the station, its time (T) and energy expenditure (E) are updated and, accordingly the RA is updated as well, considering equation (14). Finally, the station time (T'), with RA integration, is evaluated and ∂ for this station is revised. The task assigned is deleted in the OL and the procedure is repeated until all tasks are assigned to a workstation.

To avoid the violation of precedence constraints the earliest and latest station of each task are updated taking into account the following assumptions:

- The earliest station must be greater or at least equal to the greater station of the previously assigned predecessor tasks;

- The latest station must be lower or at the least equal to the lower station of the already assigned successor tasks.

After the assignment phase, the SI is calculated.

Focusing on the station selection phase these additional assumptions can be useful to better understand the assignment process.

For a generic task, *i*, we know:

- the ratio between the number of successors and predecessors not already assigned, RSP(*i*);
- the set of stations on which task *i* can be assigned, FS(*i*);
- the number of stations on which task *i* can be assigned, Nb_St(FS(*i*)).

and, for each station belonging to FS(*i*), we define its normalized weight according to (20):

$$Norm\_W_j = \frac{(RPS(i))^{(\left\lfloor \frac{Nb\_St(FS(i))}{2} \right\rfloor - \phi)\cdot\lambda}}{\sum_{\phi}(RSP(i))^{(\left\lfloor \frac{Nb\_St(FS(i))}{2} \right\rfloor - \phi)\cdot\lambda}} \quad (20)$$

Where:

- $\phi \in \{0,..,Nb\_St(FS(i))\} \setminus \{\left\lfloor \frac{Nb\_St(FS(i))}{2} \right\rfloor\}$ if the station number is even;
- $\phi \in \{0,..,Nb\_St(FS(i))\text{-}1\}$ otherwise;
- $j \in FS(i)$;
- $\lambda \in]0;1]$;

Please insert here Table 2.

Please insert here Figure 2.

4.1. Illustrative example

In this paragraph a small illustrative example is proposed to better understand in which way the proposed heuristic approach works. In figure 3, precedence graph with time and energy expenditure is illustrated.

Please insert here Figure 3.

We assume to assign tasks to 3 workstations and we assume λ=0.5. According to Table 2, LB(m) (resp. LB'(m)) is equal to 39 seconds (resp. 39 seconds, this means that no rest allowance is required for these tasks). With this first value of *c*, E_i (resp. E'_i) and L_i (resp. L'_i) have been defined for each task (see Table 3, note that tasks are listed by descending order of their time). According to Figure 3, procedure to assign tasks to stations starts with the bigger task time, in this case Task 7 that is assigned to station 3 as it is the only available station. The same procedure is applied for tasks 3 that is assigned to station 1. For task 2 the procedure to assign task to station is more complicated as we must choose between station 1 and 2. In this case, $Norm\_W_1$ and $Norm\_W_2$ are equal to 0.5 as the number of not already assigned tasks is 1 and RPS(2) is 1. This means that when can choose in the same way station 1 or 2, however, as station 2 is empty we assign task 2 to station 2. The procedure continues, task 6 is assigned to station 2 while task 1 is assigned to station 1. All stations have idle time greater than 0 so other tasks can be assigned. For task 9 we can choose between station 2 and 3. In this case the RSP(9) is equal to 0.5 and, applying Equation (20) we obtain $Norm\_W_2 = 0.4142$ and $Norm\_W_3 = 0.5858$ respectively, so we select station 3 for task 9. In the same way, task 4 will be assigned to station 1, task 5 and 10 respectively to station 2 and 3 as idle time is greater than 0. However, the last task, task 8, cannot be assigned to a station as there is not available time in any stations. It is necessary to increase by 1 cycle time obtaining new values of E(i) (resp. E'(i)) and L(i) (resp. L'(i)) as defined in Table 3, in brackets. In this case, applying the same procedure we can assign all tasks to a station. In station 1 tasks 1, 3 and 4 are assigned, in station 2 we have tasks 2, 5, 6 and 8 while in station 3 tasks 7, 9 and 10. Idle time is equal to 2 for station 1 and 3 while it is 0 for station 2.

Please insert here Table 3.

5. Computational experimentation and discussion

To evaluate the performance of the proposed formulations, a computational study, on the benchmark data, is carried out. We solve the optimisation model with software IBM ILOG CPLEX 12.7.1 with default settings (e.g. parallelisation, automatic selection of the optimisation method) while the heuristic approach is coded in C++ language with VisualStudio2017. All the experiments run on a computer i7-6500U Intel Core, 2.5 GHz, and 12.0 GB RAM.

5.2. Comparison with the method from the literature

In this Section, we present the comparison between the linearization methodology proposed in Section 3.1.1 and the one developed by Esmaeilbeigi et al. (2015). As in their work a SALBP-E has been considered we have adapted their formulation for a SALBP-2. In particular, the execution time, the gap, as well as the number of constraints and variables have been compared.

For our analysis we have selected 13 datasets that come from Scholl benchmark dataset. Additionally, we have randomly selected 3 instances for small, medium and large dataset group provided by Otto et al. (2013). Both dataset categories are available in the website https://assembly-line-balancing.de. For each of them we have varied the number of workstations between 5 and 10.

However, due to the fact that these instances contain only the number of tasks, task times and precedence relations, we have generated a random dataset of energy expenditure values. We assume, according to Astrand (1967), that for each task the MWR can assume a value between 2 kCal/min and 10 kCal/min and starting from this assumption we generate, for each dataset, 6 random sets of energy expenditure data assuming a beta distribution with different alpha e beta value according to Table 4. Table 3 includes also the mean energy values and the standard deviation obtained for the alpha and beta value selected.

In this way, we have analysed 87 instances when only task times are used and 792 instances when energy expenditure is introduced. In both cases the execution time limit has been set at 900 seconds.

Please insert here Table 4.

Table 5 presents the final results obtained for the SALBP-SI-2 without the integration of ergonomic aspects. Note that with M1 we have indicated the Esmaeilbeigi et al. (2015) approach while M2 defines ours. Both methods provide equivalent results with a slightly larger time for M2 but a slightly lower gap when Scholl dataset is considered while the execution time for M2 is lower when Otto et al. dataset is considered. In both datasets, M1 needs a lot of variables despite M2 and this is due to the method we have used to linearize the quadratic term of each workstations idle time, using a base 2 notation.

Please insert here Table 5.

The high number of variables required with the Esmaeilbeigi et al. (2015) approach tends to significantly increase when the RA is introduced as illustrated in Table 6. In this way in the maximum time limit we have fixed, the M1 model is never able to give us a solution. On the other side, with M2 the number of variables is lower, and we always obtain a solution even if in some cases it is not the optimal one. Moreover, the time required is greater than that one illustrated in table 4, but this is due to the additional constraints we have considered to introduce RA.

In conclusion, the new proposed approach works well and it outperforms, if real values are considered, the other approaches that the current literature proposes.

Please insert here Table 6.

5.2. Choice of the best way to integrate RA

Another issue of this paper is to define when it is better to evaluate the RA associated with each workstation to have a minimum SI and a minimum cycle time. For this reason, in this section we compare the models explained in Section 3.1.2, 3.1.3, 3.2 and 4.

In Table 7 the computational time is illustrated. Considering the exact methods, the computational time required to obtain a solution is significantly lower when the RA is integrated in the final model after the balancing phase. Still, as it is possible to deduce the heuristic approach perform always better despite the exact approaches, but the solution obtained cannot be the exact one.

Please insert here Table 7.

However, computational time is not the best way to compare the methods. In fact, it is necessary to evaluate and to compare the solutions these methods give us. For this reason, we compare the cycle time and the SI of each instance and we define the relative percentage difference with the solutions obtained with the application of M2 with the RA integration during the balancing phase as it is the exact approach to integrate RA. In Table 8 the mean value of relative percentage differences for both cycle time and smoothness index is given. As we can see the RA integration during the balancing phase provides a better solution both for cycle time and SI than other methods. Moreover, even if the difference in terms of cycle time is limited we cannot say the same for the SI, especially when RA is integrated before or after the balancing phase.

The heuristic approach gives solutions very close to the exact method to integrate RA in terms of cycle time and, generally, it gives a solution with a lower cycle time but a higher SI. This is linked to the approach used to allocate tasks to stations because each time a task is assigned the RA linked to the station chosen is also update. Additionally, this approach stops when all tasks are assigned to a station and so it gives a major importance to the cycle time. As a consequence, the smoothness index could be reduced increasing the cycle time as we have demonstrated is Section 3.1.

However, if we compare the relative percentage difference obtained with the heuristic approach to the other two exact methods, we can note that it is always lower and so it is more performant. So, if it is possible it is better to apply the exact method to integrate the RA during the balancing phase otherwise the heuristic approach, in a lower computational time, is able to provide a solution very close to the exact one.

In conclusion, the RA integration made before or after the balancing phase cannot be applied as they tend to reduce the productivity and the workload among stations is very unbalance. Thus, for some workers the recovery time is overestimated, while the heuristic procedure provides a high SI but in this case the cycle time is very close or lower than M2 and consequently the productivity is higher.

Please insert here Table 8.

Furthermore, if we compare the exact solution obtained with M2 without and with the RA integration we can evaluate the impact of energy expenditure in the cycle time as illustrated in Table 9. Note that in this case we have considered and thus compared only the exact solutions obtained with M2 with RA integration. As illustrated in Table 9, for the higher value of energy expenditure the cycle time tends to increase.

Please insert here Table 9.

6. Conclusion

In this paper we have considered the impact that human energy expenditure can have on the SI for a SALBP-2. Starting with the methodology proposed by Battini et al. (2015) we have converted the energy expenditures into a rest allowance using Price's formulation (Price, 1990) and we have proposed several approaches to introduce it into ALBP.

Because RA can assume real values, the approaches proposed by literature required a high computational time, so we have proposed a new linear approach to minimize the SI in a SALBP-2.

Furthermore, a heuristic approach is proposed to search for a solution when the computational time is limited or the number of tasks to assign to a station is very high.

In conclusion, after lengthy computational experimentation, the introduced model has been demonstrated to allow for active application in industrial systems as data required to apply them is easy to obtain, such as heart rate for example. Additionally, it allows improvement of the performance of the assembly process considering not only productivity aspects but also the workers' well-being.

As future work we propose the integration of workers characteristics since for the moment, we assume that all workers have the same features. In fact, gender, age, sedentary level or health problems can give different energy expenditure values, and, at the same time, the threshold value proposed by Price (Price, 1990) cannot be efficiently applied. Thus, the RA obtained could underestimate or overestimate the necessary recovery time. Additionally, an application of the method proposed here to an industrial context could be a more appropriate way to see benefits for the two main subjects involved: the company and the workers.

References

Abdous, A., Delorme, X., Battini, D., Sgabossa, F., & Berger-Douce, S. (2018). Assembly line balancing problem with consideration of workers fatigue and recovery. *In 20th International Working Seminar on Production Economics*.

Akyol, S. D., & Baykasoğlu, A. (2019). ErgoALWABP: A multiple-rule based constructive randomized search algorithm for solving assembly line worker assignment and balancing problem under ergonomic risk factors. *Journal of Intelligent Manufacturing*, 30(1), 291-302.

Åstrand, I. (1967). Degree of strain during building work as related to individual aerobic work capacity. *Ergonomics*, 10(3), 293-303.

Azizoğlu, M., & Imat, S. (2018). Workload smoothing in simple assembly line balancing. *Computers & Operations Research*, 89, 51-57.

Baykasoglu, A., Tasan, S. O., Tasan, A. S., & Akyol, S. D. (2017). Modeling and solving assembly line design problems by considering human factors with a real-life application. *Human Factors and Ergonomics in Manufacturing & Service Industries*, *27*(2), 96-115.

Battaïa, O., & Dolgui, A. (2013). A taxonomy of line balancing problems and their solution approaches. *International Journal of Production Economics*, 142(2), 259-277.

Battini, D., Delorme, X., Dolgui, A., & Sgarbossa, F. (2015). Assembly line balancing with ergonomics paradigms: two alternative methods. *IFAC-PapersOnLine*, 48(3), 586-591.

Battini, D., Delorme, X., Dolgui, A., Persona, A., & Sgarbossa, F. (2016). Ergonomics in assembly line balancing based on energy expenditure: a multi-objective model. *International Journal of Production Research*, 54(3), 824-845.

Boysen, N., Fliedner, M., & Scholl, A. (2008). Assembly line balancing: Which model to use when?. *International journal of production economics*, 111(2), 509-528.

Carnahan, B. J., Norman, B. A., & Redfern, M. S. (2001). Incorporating physical demand criteria into assembly line balancing. *Iie Transactions*, 33(10), 875-887.

Dolgui, A., & Proth, J. M. (2010). Supply chain engineering: useful methods and techniques. *Springer Science & Business Media*.

El Mouayni, I., Etienne, A., Lux, A., Siadat, A., & Dantan, J. Y. (2019). A simulation-based approach for time allowances assessment during production system design with consideration of worker's fatigue, learning and reliability. *Computers & Industrial Engineering*.

Esmaeilbeigi, R., Naderi, B., & Charkhgard, P. (2015). The type E simple assembly line balancing problem: A mixed integer linear programming formulation. *Computers & Operations Research*, 64, 168-177.

Eswaramoorthi, M., Kathiresan, G. R., Jayasudhan, T. J., Prasad, P. S. S., & Mohanram, P. V. (2012). Flow-index based line balancing: a tool to improve the leanness of assembly line design. *International Journal of Production Research*, 50(12), 3345-3358.

Falkenauer, E. (2005). Line balancing in the real world. *In Proceedings of the international conference on product lifecycle management PLM* (Vol. 5, pp. 360-370).

Finco, S., Battini, D., Delorme, X., Persona, A., & Sgarbossa, F. (2018). Heuristic methods to consider rest allowance into assembly balancing problem. *IFAC-PapersOnLine*, 51(11), 669-674.

Groover, M.P., 2013. Work Systems and the Methods, Measurement and Manage- ment of Work. *Pearson Prentice Hall*, pp. 27– 46.

Gunther, R. E., Johnson, G. D., & Peterson, R. S. (1983). Currently practiced formulations for the assembly line balance problem. *Journal of Operations Management*, 3(4), 209-221.

Hamta, N., Ghomi, S. F., Jolai, F., & Shirazi, M. A. (2013). A hybrid PSO algorithm for a multi-objective assembly line balancing problem with flexible operation times, sequence-dependent setup times and learning effect. *International Journal of Production Economics*, 141(1), 99-111.

Konz, S. (1998). Work/rest: Part II-The scientific basis (knowledge base) for the guide. *International Journal of Industrial Ergonomics*, 22(1-2), 73-99.

Imbeau, D. (2009). Comparison of rest allowance models for static muscular work. *International Journal of Industrial Ergonomics*, 39(1), 73-80.

Ma, L., Chablat, D., Bennis, F., & Zhang, W. (2009). A new simple dynamic muscle fatigue model and its validation. *International Journal of Industrial Ergonomics*, 39(1), 211-220.

Moodie, C. L., and Young, H. H., 1965, A heuristic method of assembly line balancing for assumptions of constant or variable work element times. *Journal of Industrial Engineering*, XVI, 23-29.

Nearchou, A. C. (2008). Multi-objective balancing of assembly lines by population heuristics. *International Journal of Production Research*, 46(8), 2275-2297.

Otto, A., & Battaïa, O. (2017). Reducing physical ergonomic risks at assembly lines by line balancing and job rotation: A survey. *Computers & Industrial Engineering*, 111, 467-480.

Otto, A., & Scholl, A. (2011). Incorporating ergonomic risks into assembly line balancing. *European Journal of Operational Research*, 212(2), 277-286.

Price, A. D. (1990). Calculating relaxation allowances for construction operatives—Part 1: Metabolic cost. *Applied Ergonomics*, 21(4), 311-317.

Rachamadugu, R., & Talbot, B. (1991). Improving the equality of workload assignments in assembly lines. *The International Journal of Production Research*, 29(3), 619-633.

Rohmert, W. (1973). Problems of determination of rest allowances Part 2: Determining rest allowances in different human tasks. *Applied ergonomics*, 4(3), 158-162.

Scholl, A. (1999). Balancing and sequencing of assembly lines (Contributions to Management Science). *Physica*.

Smunt, T. L., & Perkins, W. C. (1985). Stochastic unpaced line design: review and further experimental results. *Journal of Operations Management*, 5(3), 351-373.

Tiacci, L. (2018). The problem of assigning rest times to reduce physical ergonomic risk at assembly lines. *IFAC-PapersOnLine*, 51(11), 692-697.

Appendix 1.

Proposition 1. In the SALBP-2, $\Psi := UB(m) - \min(t_i)$ represents the upper bound, while zero represents the lower bound that ∂_k, the idle time of a generic workstation *k*, can assume. So $\partial_k \in [0, \Psi]$ and it assumes integer values.

Property 1. For each $\partial_k \in [0, \Psi]$, for $p = \left\lceil \log_2(\frac{\Psi}{\varepsilon} + 1) \right\rceil$ and for $\vartheta_k \in R$, $\vartheta_k = \partial_k^{\ 2}$ if and only if there exists a variable $y_{lk} \in \{0,1\}$ and there exists an integer positive variable $q_{lk} \in R^{+p}$ such that the following constraints have to verify simultaneously:

$$\vartheta_k = \varepsilon \sum_{l=1}^{p} 2^{l-1} q_{lk} \; \forall k = 1,..,m \quad (21)$$

$$\partial_k = \varepsilon \sum_{l=1}^{p} 2^{l-1} y_{lk} \;\; \forall k = 1,..,m \quad (22)$$

$$q_{lk} \le y_{lk} \Psi_{it} \;\; \forall k = 1,..,m \; \forall l = 1,..,p \quad (23)$$

$$q_{lk} \le \partial_k \qquad\qquad \forall k = 1,..,m \; \forall l = 1,..,p \quad (24)$$

$$q_{lk} \ge \partial_k - \Psi_{it}(1 - y_{lk}) \;\; \forall k = 1,..,m \; \forall l = 1,..,p \quad (25)$$

Where ε represents a constant value that is equal to 1 if integer values are considered otherwise it is equal to 0.0005 for real values. q_{lk} represents the products $\partial_k y_{lk}$ which is defined through the set of equations (21)-(25).

Equation (21) represents the quadratic term of idle time as a linear function of real positive variables.

Equation (22) represents the idle time as a linear function of Boolean variables, while equations (23)-(25) are required to evaluate the exact value of q_{lk} .

Proposition 2. A real positive variable η_i lower than U_i can be written as $\eta_i = \varepsilon \sum_{w=1}^{W} 2^{w-1} \kappa_w + s$ where s is a real positive variable, lower than ε, κ_w are Boolean variables and $W = \lceil \log_2((U_i / \varepsilon) + 1) \rceil$. If variable s is omitted $\varepsilon \sum_{w=1}^{W} 2^{w-1} \kappa_w$ can be considered as an approximation of η_i with a precision ε. Furthermore, a low value of ε implies a better representation of a real variable and a better approximation if variable s is omitted.

Property 2. The workstation time $T'_k = \sum_{i \in B_k} x_{ik} t_i [1 + \max(0; \frac{\frac{\sum_{i \in B_k} x_{ik} e_i}{\sum_{i \in B_k} x_{ik} t_i} \cdot 60 - 4.3}{4.3 - 1.86})]$, that includes the worker' recovery time, can be linearized using the following additional equations:

$$T'_k = \sum_{i \in B_k} x_{ik} t_i + RA'_k \qquad \forall k = 1,..,m \ (26)$$

$$T'_k \le c \ (27)$$

$$RA_k = \frac{60}{4.3 - 1.86} \sum_{i \in B_k} x_{ik} e_i - \frac{4.3}{4.3 - 1.86} \sum_{i \in B_k} x_{ik} t_i \qquad \forall k = 1,..,m \ (28)$$

$$RA'_k \ge RA_k \ \forall k = 1,..,m \ (29)$$

$$RA'_k \ge 0 \qquad \forall k = 1,..,m \ (30)$$

$$RA'_k \le RA_k - UB(1 - \beta_k) \qquad \forall k = 1,..,m \ (31)$$

$$RA'_k \le 0 + UB \cdot \beta_k \qquad \forall k = 1,..,m \ (32)$$

Where constraint (26) defines the station time while (27) assures that it is lower than *c*. Constraint (28) evaluates the required RA. It can assume also negative values, but constraints set (29)-(32) permits to consider its the real value as the maximum value between zero and the value calculated with (28). UB represents an upper bound of RA'_k and β_k is an additional Boolean variable required to evaluate which value assumes RA'_k

.

Figure 1. Precedence graph

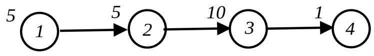


Figure 2. Heuristic process.

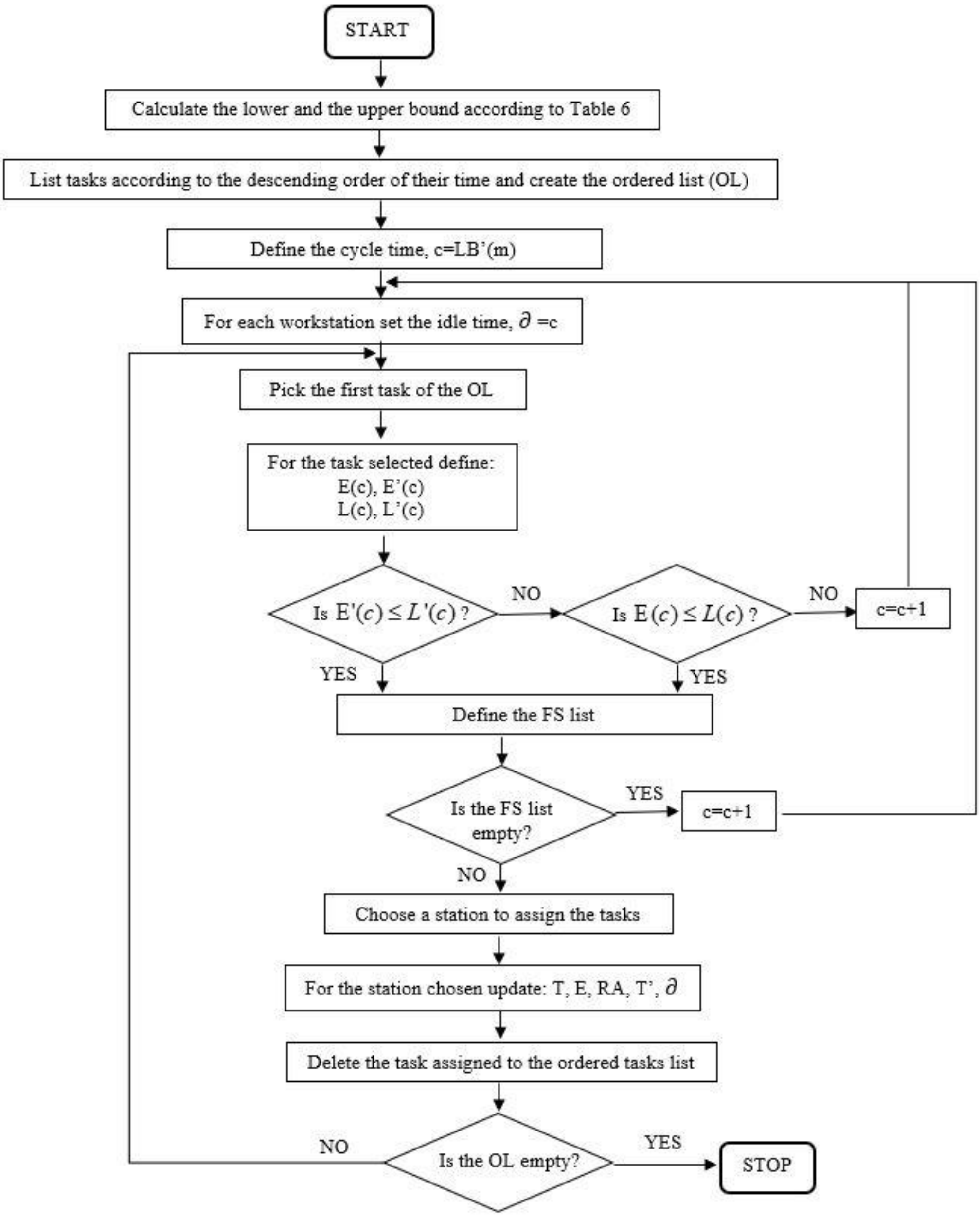
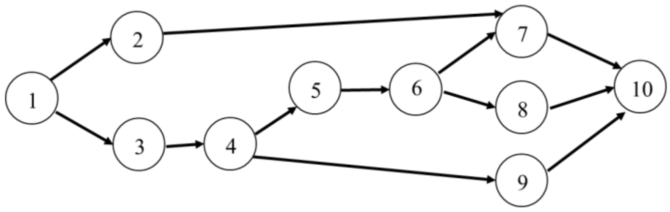


Figure 3. Precedence graph



TASK	TIME [seconds]	ENERGY [kCal]
1	12	0.94
2	14	0.12
3	17	0.74
4	9	0.50
5	8	0.80
6	14	0.62
7	21	0.87
8	4	0.14
9	12	0.96
10	5	0.54

Table 1. SI values for different cycle time.

Station	C=10		C=11	
	Tasks	**Station time**	**Tasks**	**Station time**
1	1, 2	10	1	5
2	3	10	2	5
3	4	1	3, 4	11
SI		**9**		**8.485**

Table 2. Notations and parameters for the implemented heuristic procedure

Notation	Definition
LB(m) (resp. LB'(m))	Lower bound of cycle time with *m* workstations (resp. with RA) $LB(m) = \max\{t_{max}, \left\lceil \frac{\sum_i t_i}{m} \right\rceil\}$ (resp. $LB'(m) = \max\{t_{\max}(1+RA); \left\lceil \frac{\sum_i t_i(1+RA_{tot})}{m} \right\rceil\}$)
UB(m) (resp. UB'(m))	See Section 3.1.1.
E_i (resp. E'_i)	The earliest workstation for task *i* (resp. with RA) i.e. $E_i = \left\lceil \frac{t_i + t_{P_i}}{c} \right\rceil$ (resp. $E'_i = \left\lceil \frac{\sum_{j \in P_i} t_j(1+RA_j) + t_i(1+RA_i)}{c} \right\rceil$)
L_i (resp. L'_i)	The latest workstation for task *i* (resp. with RA) i.e. $L_i = m + 1 - \left\lfloor \frac{t_i + t_{F_i}}{c} \right\rfloor$ (resp. $L'_i = m+1-\left\lfloor \frac{\sum_{j \in F_i} t_j(1+RA_j) + t_i(1+RA_i)}{c} \right\rfloor$)
FS_i (resp. FS'_i)	The set of workstations to which task *i* is feasibly assignable $FS_i = \{E_i, E_i + 1, .., L_i\} \backslash \{stations\ that\ have\ an\ idle\ time\ lower\ than\ the\ task\ time\}$ (resp. $FS_i = \{E'_i, E'_i + 1, .., L'_i\} \backslash \{stations\ that\ have\ an\ idle\ time\ lower\ than\ the\ task\ time\}$)

Table 3. Earliest and Latest station for a cycle time equal to 39 seconds (resp. 40 seconds).

Task	**Time(i)**	**Energy(i)**	**ET(i)**	**RA(i)**	**T-RA(i)**	**E(i)**	**L(i)**	**E'(i)**	**L'(i)**
7	21	0.87	2.49	0.00	21.00	3(3)	3(3)	3(3)	3(3)
3	17	0.74	2.61	0.00	17.00	1(1)	1(1)	1(1)	1(1)
2	14	0.12	0.51	0.00	14.00	1(1)	2(3)	1(1)	2(2)
6	14	0.62	2.66	0.00	14.00	2(2)	2(2)	2(2)	2(2)
1	12	0.74	3.70	0.00	12.00	1(1)	1(1)	1(1)	0(1)
9	12	0.96	4.80	0.20	14.46	2(2)	3(3)	2(2)	3(3)
4	9	0.5	3.33	0.00	9.00	1(1)	2(2)	2(1)	1(2)
5	8	0.8	6.00	0.70	13.57	2(2)	2(2)	2(2)	2(2)
10	5	0.54	6.48	0.89	9.47	3(3)	3(3)	4(3)	3(3)
8	4	0.14	2.10	0.00	4.00	2(2)	3(3)	2(2)	3(3)

Table 4. Alpha e beta value used to create random datasets of energy expenditure

	MWR1	MWR2	MWR3	MWR4	MWR5	MWR6
Alpha	1	1	2	3	2	3
Beta	8	4	8	8	4	4
Energy mean value [kcal/min]	2.842	3.553	3.684	4.317	4.868	5.777
Std Dev [kcal/min]	0.874	1.395	1.086	1.186	1.613	1.503

Table 5. Comparing our approach with that one of Esmaeilbeigi et al. (2015).

Scholl Dataset (1999)	# Tasks	Mean Gap% M1	Mean Gap % M2	Mean Time M1 [s]	Mean Time M2 [s]	Mean # of constraints M1	Mean # of constraints M2	Mean # of variables M1	Mean # of variables M2
Arcus	83	83.33%	31.04%	900.49	659.11	242	609	549074	894
Buxey	29	0.00%	0.00%	1.35	3.02	111	298	2449	369
Gunther	35	0.00%	0.00%	2.08	2.63	126	313	3790	414
Hann	53	0.00%	0.00%	99.65	33.22	181	480	102329	624
Kilbridge	45	0.00%	0.00%	1.54	1.73	153	362	4268	504
Lutz1	32	0.00%	0.00%	24.82	3.50	116	415	95119	466
Lutz2	89	0.00%	0.00%	4.79	4.97	253	440	4255	819
Lutz3	89	0.00%	0.00%	11.75	14.22	253	485	12844	849
Mukherje	94	0.00%	2.30%	11.11	256.22	321	598	31786	916
Sawyer	30	0.00%	0.00%	1.96	2.69	108	295	2489	376
Tonge	70	0.00%	16.67%	41.44	215.26	202	456	26544	721
Warnecke	58	0.00%	0.00%	7.70	20.18	174	406	11548	616
Wee-Mag	75	0.00%	0.00%	8.53	5.99	208	440	11584	744
		6.41%	**3.85%**	**85.94**	**94.06**	**188**	**430**	**66006**	**639**
Otto et al. Dataset (2013)									
Small	20	0.00%	0.00%	27.32	22.19	94	362	32805	354
Medium	50	16.67%	7.40%	337.37	225	142	441	76605	601
Large	100	24.15%	19.80%	441.39	314.70	341	662	262176	943
		13.61%	**9.07%**	**268.68**	**187.30**	**192**	**488**	**123862**	**633**

Table 6. Comparing the two approaches considering RA evaluation during the balancing phase.

Scholl Dataset (1999)	# Tasks	Mean Gap% M1	Mean Gap % M2	Mean Time M1 [s]	Mean Time M2 [s]	Mean # of constraints M1	Mean # of constraints M2	Mean # of variables M1	Mean # of variables M2
Arcus	83	No solution	70.43%	900.00	47.22	249	1040	3614256	836
Buxey	29	No solution	0.00%	900.00	181.35	126	549	643575.3	575
Gunther	35	No solution	4.23%	900.00	33.41	141	605	960287	607
Hann	53	No solution	5.22%	900.00	186.45	177	811	2043621	769
Kilbridge	45	No solution	29.04%	900.00	445.92	168	684	1100029	640
Lutz1	32	No solution	0.00%	900.00	341.71	135	651	749651	700
Lutz2	89	No solution	45.11%	900.00	716.60	268	975	962358.7	681
Lutz3	89	No solution	8.00%	900.00	602.83	268	1002	3280692	723
Mukherje	94	No solution	46.55%	900.00	770.99	336	1060	8412332	821
Sawyer	30	No solution	5.33%	900.00	893.03	123	557	643582.8	574
Tonge	70	No solution	52.56%	900.00	548.19	217	888	7017550	714
Warnecke	58	No solution	72.74%	900.00	246.67	189	801	3091126	691
Wee-Mag	75	No solution	75.00%	900.00	761.24	223	895	2989587	675
			31.86%	**900.00**	**444.28**	**202**	**809**	**2731434**	**693**
Otto et al. Dataset (2013)									
Small	20	No solution	0.00%	900.00	195.26	106	492	625482	496
Medium	50	No solution	8.51%	900.00	294.65	174	742	2994562	639
Large	100	No solution	62.85%	900.00	801.36	384	1125	8648541	884
			23.79%	**900.00**	**430.42**	**221.33**	**786.33**	**408952**	**673**

Table 7. The mean computational time required to obtain a solution

Scholl Dataset (1999)	Mean Time M2 with RA integration during balancing phase [s]	Mean Time M2 with RA integration before balancing phase [s]	Mean Time M2 with RA integration after balancing phase [s]	Mean Time Heuristic approach [s]
Arcus	47.22	821.95	659.11	2.253
Buxey	181.35	56.75	3.02	1.562
Gunther	33.41	217.70	2.63	1.414
Hann	186.45	312.54	33.22	1.984
Kilbridge	445.92	772.13	1.73	1.641
Lutz1	341.71	39.36	3.50	1.246
Lutz2	716.60	425.88	4.97	1.384
Lutz3	602.83	388.32	14.22	1.743
Mukherje	770.99	900.23	256.22	1.947
Sawyer	893.03	228.49	2.69	1.238
Tonge	548.19	560.69	215.26	1.896
Warnecke	246.67	474.17	20.18	1.695
Wee-Mag	761.24	876.72	5.99	1.349
	444.28	**453.84**	**94.06**	**1.642**
Otto et al. Dataset (2013)				
Small	195.26	34.16	22.19	1.124
Medium	294.65	652.21	225	1.871
Large	801.36	796.54	314.70	1.983
	430.42	**494.30**	**187.30**	**1.659**

Table 8. Relative percentage difference between the exact method and the other approaches

Scholl Dataset (1999)	Cycle time relative difference M2 with RA integration before balancing phase [%]	Cycle time relative difference M2 with RA integration after balancing phase [%]	Cycle time relative difference Heuristic approach [%]	SI relative difference M2 with RA integration before balancing phase [%]	SI % relative difference M2 with RA integration after balancing phase [%]	SI % relative difference Heuristic approach [%]
Arcus	15.16%	9.68%	-1.44%	16940.40%	21148.69%	343.34%
Buxey	3.68%	15.93%	-0.93%	656.13%	1478.74%	113.22%
Gunther	3.97%	18.29%	-0.99%	615.21%	1555.89%	165.08%
Hann	1.59%	13.34%	-0.36%	98.59%	288.04%	74.00%
Kilbridge	5.01%	11.96%	-1.30%	30243.08%	19270.45%	606.74%
Lutz1	3.35%	11.11%	-0.59%	142.43%	393.37%	37.59%
Lutz2	5.28%	11.11%	-1.00%	74435.03%	84070.98%	1790.30%
Lutz3	5.48%	12.58%	-0.79%	4130.88%	6842.29%	2129.66%
Mukherje	3.87%	10.56%	-1.62%	532333.60%	387190.12%	12774.86%
Sawyer	3.51%	22.84%	-0.73%	943.25%	3812.14%	140.10%
Tonge	9.29%	20.39%	19.57%	125094.23%	819.83%	643.13%
Warnecke	9.26%	11.50%	-1.21%	342.65%	9495.17%	64.76%
Wee-Mag	5.80%	8.52%	-1.50%	292960.54%	264283.82%	870.40%
	5.79%	**13.68%**	**0.55%**	**82995.08%**	**61588.42%**	**1519.48%**
Otto et al, Dataset (2013)						
Small	2.36%	8.64%	1.08%	485.62%	2674.84%	167.54%
Medium	2.94%	13.52%	-0.74%	185.65%	96485.74%	697.65%
Large	5.41%	14.95%	3.87%	17946.25%	264743.64%	256.34%
	3.57%	**12.37%**	**1.40%**	**6205.84%**	**121301.41%**	**373.84%**

Table 9. The increment of cycle time due to energy expenditure integration.

	Mean increase of cycle time
MWR1	0.14%
MWR2	7.64%
MWR3	3.24%
MWR4	13.15%
MWR5	23.74%
MWR6	68.21%
	19.35%