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Stabilization of slugging by sliding mode control

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Problem Description

Se vedlegg.

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Note

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1 Project description

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1.1 Background

In oil production from mature fields, unstable multiphase flow from wells, known as slugging, is an increasing problem. Slugging often leads to reduced production as the well must be choked down for the downstream processing equipment on the platforms to be able to handle the resulting variations in liquid and gas flow rates.

Active control of the production choke at the platform/well head can be used to stabilize or reduce instabilities in the flow from the wells. Conventionally, this is done by applying conventional PI control to a measured downhole pressure to stabilize this pressure at a specified setpoint, thus stabilizing the flow. For wells, however, PI control is often insufficient:

Either it is not robust, it requires frequent re-tuning, or it does not achieve proper stabilization at all.

Using the derivatives of the pressure up to the order of the relative degree of the system, a sliding mode controller may be designed to stabilize the downhole well pressure. Furthermore, by designing the controller assuming either the rate or acceleration of the choke opening as input, we obtain a continuous choke control input.

1.2 Objective

The objective of the project is to design, implement, and evaluate sliding mode control applied to stabilize slugging using only the downhole pressure as measurement. The pros and cons of the resulting output-feedback sliding mode controller is to be evaluated against the conventional PI controller, and possible other controllers.

1.3 Main tasks

The project involves the following main tasks:

- Implement a high fidelity simulation model of severe riser slugging in Matlab.
  - Consider the case of riser slugging from a well
    - Well with low point – *i.e.* terrain-induced slugging
    - Two-phase air-water flow .
  - Fit model to experimental data from StatoilHydros test rig in Porsgrunn.
  - The model should include:
    - Simple reservoir model
    - Multiphase flow in riser using OLGA
    - Choke flow rate using the multiphase choke model by Fjalestad
    - Choke actuator dynamics and constraints
    - Control system with zero-order hold and sampling effects
    - Measurement noise and possibly sensor dynamics
- Design and implement a differentior for estimation of the time-derivatives of the pressure.
  - Perform a literature review on methods available for online differentiation
  - Implement and compare a few selected methods, in addition to
    - High-gain observer (Khalil)
    - Sliding mode observer (Levine)
    - Discuss performance with respect to measurement noise, sample time and CPU requirements
- Design and implement a sliding mode controller
  - Compare designs with rate $\dot{u}$ versus acceleration $\ddot{u}$ as input.
  - Discuss the properties of the controllers
- Perform a thorough simulation study of controller performance
  - Implement a conventional PI as reference controller
  - Analyse and discuss the properties of the output-feedback sliding mode controller

More details on the different tasks are described in the following sections.

2 Simulation model

The simulation model is to be used for evaluation and analysis of controller robustness and performance, and should be as realistic as possible in all relevant aspects of riser slugging for a well.

In particular, the model should include the following:

- Multi-phase pressure and flow dynamics
  - Use OLGA to simulate two-phase air-water flow
  - Set up the model with the test rig as base case
  - Fit model roughly to experimental data
- Choke model
  - Model choke flow rate using two-phase flow model
  - Model actuator dynamics including physical restrictions
- Reservoir model
  - Start by considering the following simple models
    - Constant mass feed rate of gas and liquid
    - Simple PI model
- Control system and measurements
  - Control system with zero-order hold and sampling effects
  - Measurement noise and possibly sensor dynamics

2.1 Simulation structure

Implement the simulation model in Matlab and avoid using Simulink.

Implement each dynamical subsystem as functions that can be run every sample from a for-loop, i.e., each dynamical subsystem is solved one sample ahead (using an ODE solver) every function call.

This is a modular and flexible simulation structure that enables each subsystem to be modelled separately.

2.2 OLGA model

The multiphase flow dynamics should be implemented using OLGA. It shall be possible to implement OLGA as a function in Matlab that can be called every sample, to be simulated one sample ahead. That is, set up the OLGA model such that it works as function that can be called by an m-script in Matlab, with inputs and outputs.

Inputs are typically the reservoir volumetric influx of gas $q_{g,res}$ and liquid $q_{l,res}$ and corresponding densities $\rho_{g,res}$ and $\rho_{l,res}$, respectively, and the topside flows $q_{g,choke}$ and $q_{l,choke}$ through the choke with densities $\rho_{g,choke}$ and $\rho_{l,choke}$.

Outputs are typically pressures, densities and liquid hold up fraction in the pipeline from the reservior to the topside choke.

Set up the model roughly with the test rig as base case:

- Hestetun07 - Description of the Multi-Phase and Separator Control Test Rig (MaSCoT).doc

Fit model roughly to experimental data

- Alstad07 - Simulation data for simplified model of unstable flow.doc

2.3 Choke model

2.3.1 Choke flow rate

Implement the Fjalestad two-phase model for the flow rate through the choke. See script in the end of the paper.

Model choke flow rate using two-phase flow model

- Fjalestad07 - Simple equation for two-phase flow rate prediction through chokes.pdf
- Fjalestad07b - Simple multi phase choke equations.ppt

2.3.2 Actuator dynamics and constraints

Assume the actuator dynamics is governed by a slightly underdamped 2nd-order dynamics according to

$$z_c = \frac{\omega_c^2}{s^2 + 2\zeta_c \omega_c s + \omega_c^2} u_c \,, \tag{2.1}$$

where $z_c \in [0,1]$ is the relative choke opening, $u_c \in [0,1]$ is the control input, the parameter $\zeta_c = 0.7$ the damping coefficient, and ω_c the resonance frequency of the actuator dynamics.

Also implement a rate limiter on z_c such that

$$\dot{z}_c \in [-R, R],$$

where R is the maximum opening/closing rate of the valve actuator.

3 Observer for estimation of output-derivatives

4 Stabilization by sliding mode control

Compare designs with

- Rate $r = \dot{u}$, versus
- Acceleration $a = \ddot{u}$

as control input for the sliding mode design.

5 Simulation study

The main objective of the simulation study is to perform a thorough analysis of the performance and robustness of the sliding mode controller.

- The conventional PI controller is to be implemented as reference and comparison of robustness and performance with the sliding mode controller.

In particular, it is important to consider

- Unmodelled valve dynamics
- Control input saturation
- Measurement noise
- Disturbances like changes in reservoir, e.g. in reservoir influx.

Also analyse the performance of the controllers in the following situations

- Changes in the reservoir, such as changes in flow rate and reservoir pressure
- …

Preface

This master thesis is submitted as a part of the requirements for the M.Sc degree within the field of Engineering Cybernetics at the Norwegian University of Science and Technology (NTNU). The thesis is a continuation of the work in the project [48]. The thesis has been written in co-operation with StatoilHydro's Research Centre in Porsgrunn. The contributions of this thesis are within the area of nonlinear control of riser slugging. The thesis can be looked on as a preparatory study for checking out if sliding mode, a specific VSCS (variable structure control system), could be a robust and beneficial approach for stabilization of unstable multiphase flow, or slugging, from wells (and transport lines).

A special tank to my supervisor at StatoilHydro, Dr.ing. Glenn-Ole Kaasa, who gave me the opportunity to work with this subject. I am grateful for his valuable help. Especially for spending quite a lot of time discussing the challenges and results by mail and during meetings. I am also very thankful for all the help I have gotten from Professor Ole Morten Alamo, my supervisor at the Department of Engineering Cybernetics. I am very obliged with both their patience with me in what turned out to be a very long process of working.

I will also give tanks to Marianne Ingeborg Karlsen for the invaluable support when the proceeding of this thesis was on the most difficult stages, and of course to my parents for believing in me.

Summary

The objective of this master thesis is to design, implement, and evaluate sliding mode control (SMC) applied to stabilize slugging using only the downhole pressure as measurement. The pros and cons of the resulting output-feedback sliding mode controller are to be evaluated against the conventional PI controller.

The thesis is based on, and a continuation of the work and conclusions of my project thesis [48], where the conclusion was that the SMC might have a significant potential for increased oil production and recovery. The clear limitation was however the uncertainty regarding the validity of the van Der Pol model used, and the fact that the SMC was provided the real time-derivative states[1].

Therefore the original main tasks of this thesis was to implement a high fidelity simulation model of severe riser slugging, and to design (and test) differentiators with the purpose of evaluating the output-feedback performance of the SMC. As it turn out that I was not able to achieve the task of stabilizing the pressure with SMC on the chosen OLGA model, the focus of this thesis has in agreement with my supervisors been changed quite a lot from the task requested in the project description. Possible reasons for the lack of results, and the chosen focus of the thesis is presented in the introduction chapter. The thereby chosen focus became the task of designing testing differentiators for the SMC, but with testing on the van Der Pol model.

Before presenting and arguing for this change of focus in the section about task and limitation, the introduction chapter starts by giving a brief overview of the environment or setting the controlling challenge is a part of, and follows up by presenting the work and conclusion of that project thesis. In the end of the chapter, the structure of the thesis is shortly listed.

In chapter 2, the introduction is followed up by giving a further insight to the slug problematic. An overview of the historical development, and a description of some research within the field are provided. The last part of the chapter gives a quite thorough description on the riser slugging phenomenon.

Thereby the sliding mode controller (SMC) is presented in chapter 3. The challenge concerning chattering is also discussed, and the approach of

[1]During measurement noise testing, the noise was separately added to each state.

using boundary layer to suppress the chattering is introduced.

In chapter 4, the empirical van der Pol based model is derived, and the model is augmented to use the valve rate as control input. Chapter 5 gives a discussion on why the SMC should be performed on the choke rate and not directly on the choke opening. This section also designs the SMC that will be used for testing, and the needed time-derivatives for testing the SMC are derived.

The chosen main focus of this thesis is to evaluate if there is possible to design observers (differentiators) that meets the requirements for the designed SMC to stabilize slugging. The tests are performed on the van Der Pol model. This task is introduced through chapter 7, that present general observer theory, theory about using observers combined with SMC, and finally the two observers chosen for further testing; the high gain observer (HGO) and the robust high-order sliding mode differentiator (RHOSMD).

The first stage in evaluating the HGO and RHOSMD is open loop testing, and is described in chapter 8. The observers perform well for the ideal case of no disturbance, but in the presence of measurement noise the conclusion is that estimations of the higher order time-derivatives do likely not meet the very demanding requirements of the SMC. The biggest problem is probably the time-delay of the estimation, but the correctness of the amplitude might also be a problem.

In chapter 9 the observers was tested further in a SMC controlled closed loop system to get a more precise indication on how well the observers are fitted for their intended task of providing the SMC with the required estimations. As for the open loop tests, the '*isolated*' estimation performance of the required states was considered, but the main focus was the performance of the output-feedback SMC compared to the performance of manual choking, the PI controller, and the performance of the state-feedback SMC. This chapter confirmed the assumption from the open loop testing that in presence of measurement noise, the observers is not able to meet the demanding requirements of the designed SMC. For the theoretical ideal case of no disturbance the results is very good, especially for the HGO. In the case of measurement noise, the RHOSMD perform slightly better.

In both chapter 8 and chapter 9, tuning is considered and discussed. However, since the conclusions of the observer testing is negative, the SMC is not tested further (regarding ΔT_c, continuous p_{ref}-*changes*, choke conditions, etc.). The results will be negative for output-feedback testing, and a quite extensive state-feedback testing of the SMC is performed in [48].

The main results and conclusions throughout the thesis, are presented in chapter 10. The chapter also contain a short discussion where it is concluded that the alternative designs, SMC with direct choke rate control and SMC of the choke acceleration, will not be a solution on the state-feedback SMC problems described in this thesis. There is also a short discussion regarding CPU and system requirements for the observers and the controller. At the very end, further work is discussed.

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Nomenclature

List of symbols

a_1	Frequency or stiffness of slug model
a_2	local '*degree of the stability/instability*' at fixed valve opening z
b_0,b_1	Positive constants for modelling β
c_0,c_1	Positive constants for modelling ζ
B	Boundary layer (area)
g	Acceleration due to gravity
H	Height of riser
K	SMC-gain
K_c	Flow constant of the valve
K_f	Frictional constant of the riser
p	Model pressure
p_0	Pressure downside the valve
p_{bifur}	'*Bifurcation level*' – where p_{eq} goes from stable to unstable
p_B	Pressure in the riser base
p_{diff}	$p - p_{eq}$
p_{eq}	Pressure of equilibrium at current z
p_{ref}	Reference pressure (for control)
p_{res}	Pressure in the reservoir
p_T	Pressure upside the valve
q	Model state: proportional to Δp_C at steady-state w_{res}
s	The sliding variable – the base for the sliding surface and the virtual control law
T_{valve}	The *valve rate constant* – the (minimum) time in seconds the valve uses to change its opening by 60 percent
T_s	The *sample time* of each OLGA iteration
T_{samp}	The *sample time* of the controller in seconds
w	pressure rate (model)
w_g	Gas flow into riser base
w_{res}	Reservoir influx
z	Relative valve opening
z_0	Initial relative valve opening
z_{0power}	The valve opening that applies no power to α
z_{eff}	z-value that would produce current state q if z was constant
z_{min}	Minimum relative valve opening
z_{max}	Maximum relative valve opening

α	The virtual sliding mode control law (depending on s)
$\bar{\alpha}_l(\cdot)$	mean liquid fraction function
β	Equilibrium pressure (slug model)
δ	Represents control input in model
Δp_C	Pressure drop in the valve
Δp_g	Gravitational pressure at riser base (static head)
Δp_F	Frictional drop in riser
ΔT_o	Observer sampling time (sec.)
ΔT_o	Controller sampling time (sec.)
ϵ	Boundary layer width
η	Strictly positive constant
λ	Tuning variable for s
λ^*	Eigenvalues of linearized model
Φ	Boundary layer thickness
$\bar{\rho}$	Average density in riser
ρ_g	Gas density
ρ_l	Liquid density
ρ_T	Average density upside the valve
ζ	Amplitude of oscillation of fixed valve opening z (model)

List of abbreviation

FOHGO	Full Order High Gain Observer
HGO	High Gain Observer
RHOSMD	Robust High-Order Sliding Mode Differentiator
SM	Sliding Mode
SMC	Sliding Mode Controller OR Sliding Mode Control
VSC	Variable Structure Control
VSCS	Variable Structure Control System
VSS	Variable Structure System

Chapter 1

Introduction

The contributions of this thesis are within the area of nonlinear control of riser slugging. The thesis can be looked on as a preparatory study for checking out if sliding mode, a specific *variable structure control system* (VSCS), could be a robust and beneficial approach for stabilization of riser slugging.

This introduction chapter will start with a brief overview of the environment or setting the controlling challenge is a part. This master thesis is based on the work of the project thesis [48]. Therefore the chapter will continue with presenting the work and conclusion of that project, before presenting the tasks and limitations of this thesis. Due to lack of results in the early stages of the work, the focus of this thesis has in agreement with my supervisors been changed quite a lot from the task requested in the project description. Possible reasons for the lack of results, and the chosen focus is presented in the section about task and limitation. In the end of the chapter, the structure or the thesis is presented.

1.1 Brief presentation

In an attempt to utilize smaller and less valuable fields, oil production in an oceanic environment involves getting oil and gas from several fields that are connected as tie-ins to a separator on a production platform. Both because of these more marginal and complex fields, and that the bigger field is in a tail production phase[1], more and more of the production consist of multiphase flow - a mixture of gas, oil and water. Transporting multiphase flow is a complicated task for the oil and gas industry. One major problem with transporting oil, gas and water in the same pipeline over long distances is the possibility for introducing a flow regime called slug flow - characterized by its unstable flow behaviour. Slugging occurs when liquid accumulates in a part of the pipe and blocks the inflow of gas. The pressure will eventually

[1] The production phases, and the background of oil production is described in more detail in the project thesis [48].

rise and cause the slug to 'break' loose and flow forward, until the slug starts to build up again. Slugging may be divided into four different types [2]: Hydrodynamic slugging, Riser slugging, Terrain slugging, and Transient slugging. Riser slugging (caused by liquid accumulating at the bottom of the riser), possibly combined with or initiated by terrain slugging is the most serious for oil/water-dominated systems, and the focus and concern of this thesis. Riser slugging may generate large pressure and flow oscillations. For the most serious cases the riser slugs can fill up the entire riser and be several hundred meters long. This may cause poor separation or in the worst case even flooding of the separator. The large pressure and flow oscillations also give rise to unnecessary wear and tear on the process equipment. Hence, riser slugging must be avoided in pipeline-riser systems.

Minimizing the consequences slugging imply is therefore an important task in the oil industry. Historically slugs have been coped with in two ways. Change of design or change of operation conditions. A more recently adopted way of avoiding slugging is to use control methods.

1.2 The project thesis

This master thesis is based on the work of my project thesis [48]. Both theses are written in co-operation with StatoilHydro's Research Centre in Porsgrunn. This section presents the work and conclusions of the project thesis. The aim of the project was to investigating the potential of the sliding mode controller (SMC) to suppress the sluggish behaviour present in a pipeline from unstable oil wells. This by testing the performance of the SMC on an empirical van der Pol based model of the upside riser pressure variations during slugging in the unstable oil well. The idea is that SMC might be able to stabilize the pressure at a lower pressure than by choking the valve or using PI or PD controllers.

In the project thesis the SMC was tested on a broad combination of cases. One instance was testing under different degree of measurement noise (ideal, low and high). Another was if the controller was started from a high stabilized pressure, or when the system was in a slugging flow regime. A third instance was the controller sample time. The controller was tested for one quite fast sample rate, one sample rate that is commonly used in industrial control system today, and a third sample rate that is faster than the commonly used sample rate – but that might be possible to implement. Combining these instances gave 18 cases the controller was tested for. In each case the SMC-performance for different maximum choke valve rates was tested. In addition, there were performed tests where the reference pressure was changed stepwise or continuous against the desired level. The

[2] Shortly described in [48].

thesis also included the performance of the PI controller on the system, to clarify the possible potential of the SMC.

As a part of the thesis there is also done a quite thorough qualitative analysis of how the slugging system described by the designed van Der Pol model can be controlled in the unstable region (see appendix A)[3]. The discussion is valid on this specific empirical model and not a real system, but as the bifurcation plot and oscillations of a genuine system can be fitted well, it is reasonable to assume that the analysis points out important aspect of controlling on a real system. The examination was divided in two; the second part discuss how the SMC can locate the stabilization point (correct system states and desired valve opening), while the first part is about how the controller can maintain to stabilize the system while on this point. It was concluded that the SMC achieve to do so by acting as a four level control hierarchy. An important aspect with this section is that it shows that while in the unstable system area, there is very little room for the SMC to do errors or be inaccurate. Therefore, the SMC has to meet very demanding requirements about *fast* and *correct* response to changes in both the pressure, and the 1st. to 3rd. order time-derivatives, again requiring very accurate and little time delayed knowledge of these states.

From the numerous tests of the thesis it was concluded that SMC might have a significant potential for increased oil production and recovery. It was stated that noise was a challenge, but that medium measurement noise could be handled. In the view that the SMC could stabilize in the unstable system region, something that the PI controller was not able to, it was considered quite robust. Further, it was stated that the rate of the production valves today should be good enough to explore the potential of the SMC. However the controller sample rate of today's system might heavily curtail the potential of the SMC. The recommendation was therefore that the SMC, and how it could be implemented should be investigated further in a rigorous multiphase flow simulator (such as OLGA), and thereafter through experiments. A clear limitation to the value of conclusions of the project thesis is that the needed time-derivative states (1st. to 3rd. order) are made directly available for the SMC (with possible added noise disturbance). There is a considerable uncertainty in how well these states can be estimated in the presence of measurement noise. Another limitation is the uncertainty of how good fitted the van Der Pol model is to evaluate the potential for a controller scheme to suppress slugging.

[3]The qualitative analysis of system controllability is added as an appendix as it is often referred to in the arguing of this master thesis.

1.3 Limitation of scope

Based on the conclusions of the project thesis, the objective of this master thesis is to design, implement, and evaluate sliding mode control applied to stabilize slugging using only the downhole pressure as measurement. The pros and cons of the resulting output-feedback sliding mode controller are to be evaluated against the conventional PI controller.

1.3.1 Regarding OLGA

I was requested to implement a high fidelity simulation model (like OLGA) of severe riser slugging in Matlab. The model should consider the case of riser slugging from a well with low point – i.e. terrain-induced slugging, when the multiphase flow is a two-phase air-water flow. The model should be fitted to experimental data from StatoilHydro's test rig in Porsgrunn, and the model should include:

- An simple reservoir model.
- Multiphase flow in riser using OLGA.
- Choke flow using the multiphase choke flow model by Fjalestad.
- Choke actuator dynamics and constraints.
- Control system with zero-order hold and sampling effects.
- Measurement noise and sensor dynamics.

Project work with OLGA

In the beginning of the work with the master, I implemented a simulation system in Matlab after the above descriptions, but with some limitations:

1. The reservoir was at this point modeled with constant air and water flow.
2. In agreement with my supervisors, the Fjalestad model [19] (an internal StatoilHydro article) was not to be implemented, as it proved to be difficult to modify the chosen OLGA model to deal with the flows through the choke valve as inputs.
3. Modelling of the choke valve dynamics, was at this point limited to setting the maximum choke rate.
4. The sensor dynamics was not included.

The clear intention was to later modify the system to better meet requested requirements. The OLGA model I decided to use is designed by Kjetil Ellingsen, and described in the internal report [17]. The model is fitted to experimental data from StatoilHydro's test rig[4] in Porsgrunn. As the OLGA model do not perfectly reproduce the behaviours of the test rig, Einar Hauge discuss possible tunings of the OLGA model in his master thesis [22], but his conclusions was to not change any parameters[5].

Thereafter, I implemented the differentiators[6]; High-Gain observer and Robust High-Order Sliding Mode Differentiator for estimation of the time-derivatives of the pressure and a designed sliding mode controller (SMC), as requested. However, after some time of tuning both the differentiators and the SMC, I had to conclude that I was not able to achieve SMC-stabilization of the pressure (and thereby avoid slugging) in the OLGA model. It was therefore in corporation with my supervisors concluded that I instead should focus on investigating the observer performance through the van Der Pol model. The structure of this testing will be presented after a brief discussion on likely or possible reasons for my problems with SMC in the OLGA model.

Possible challenges with OLGA simulations

As the van Der Pol is a simple known mathematical model, it is possible to do exact calculations of all the needed time-derivatives of the pressure. In the project thesis this was used to isolated check the performance of the SMC. Likewise it is possible to isolated check the performance of the observers. With knowledge of the isolated performance, it is easier to design a well working combined observer controller system, and do good evaluations of the overall system.

A clear challenge with the OLGA system is that the model does not provide any direct and exact knowledge about the time-derivatives, and thereby make it difficult to separate the tuning and evaluation process of the observer and controller. It might simply be difficult to get the control system to work well, as you might not really know what the problem is.

In my quest to try to get an idea of how the observers is performing, and thereby if it is the observers, the SMC or maybe both that is not working as intended, I attempted to estimate the required time-derivatives by the

[4]The Multi-Phase and Separator Control Test Rig (MaSCoT) is a pilot scale test facility for unstable flow and separator control, and is described in the internal report [28]. The experimental rig can be used to study unstable flow under semi-realistic conditions and is ideal for development and testing of new control solutions for anti-slug and separator control. The test rig includes a 3 inch, 100 meter long pipe section and a three-phase separator with pressure and level control.

[5]Hauge pointed out that the tuning parameters should be the *'discharge coefficient'* (CD) or the *'interfacial friction parameter'* (LAM_LGI). I considered to check his work, but could not do so because of license limitation.

[6]Differentiators is a type of observers. Observer is the mainly used term in this thesis.

simple Euler algorithms:

$$
\begin{aligned}
w_{n+1} &= \frac{p_{n+1} - p_n}{\Delta T_s} \\
w2_{n+1} &= \frac{w_{n+1} - w_n}{\Delta T_s} \\
w3_{n+1} &= \frac{w2_{n+1} - w2_n}{\Delta T_s}
\end{aligned}
$$

where w,w2,w3 is the 1st. to 3rd. order time-derivatives of the pressure, and ΔT_s is the sample time of each OLGA iteration. This estimation can not be used as a real observer as it do not contain any sort of filter, and therefore do not tolerate any noise. However, given fast sampling, a noise free signal that is *'sufficiently smooth'*[7], it will give a quite exact prediction of the real states.

The problem in the case of the OLGA simulation is that the pressure is clearly not sufficiently smooth. Thereby the Euler estimations experienced noise like oscillations of a significant amplitude, dependent on the sampling time ΔT_s. Thereby, it is obvious that the Euler estimation can not give a good indication of the performance of the observers. I have also attempted to control the SMC with use of the Euler estimations, but not with any better results.

Other then the problems with tuning the observers and evaluation the performance, it could also be that the designed SMC simply do not perform as good on the Olga model as the van Der Pol model. One reason for this could be that their dynamics is simply too different.

Other, then this presentation, I have not been able to do an analysis of what is the problem, regarding the simulations of SMC with the OLGA model. I will therefore not do any further presentation of my OLGA result in this thesis, but concentrate on the van Der Pol analysis of the observers.

1.3.2 Regarding observer testing

As I did not achieve desired results with the Olga model, the thesis will instead concentrate on testing of the high-gain observer and the robust high-order sliding mode differentiator through the use of the van Der Pol model presented in chapter 4. The theory behind the observers will be presented in chapter 7. Then in chapter 8 the observers is open loop tested for a slugging van Der Pol model. This to do an isolated evaluation on how well the observers are able to estimate the pressure and the required time-derivatives when the model is in a slugging modus. In chapter 9 the observers will be

[7]By *'sufficiently smooth'*, I here mean that the time-derivatives of interest (1st. to 3rd. order) are defined and continuous.

tested for a SMC closed loop system, to get a more precise indication on how well the observers are fitted for their intended task of providing the SMC with the required estimations. It is considered how well the required states is estimated, but the main evaluation is simply be how well the SMC is performing compared to the performance of manual choking the PI controller, and the performance of the SMC when provided exact states. In these chapters the performance in bough ideal and noise inflicted pressure measurement is considered. The observer sampling time, and tuning is also considered and discussed.

1.4 The structure of the report

Chapter 2 will give a further introduction to the slug problematic. An overview of the historical development, and a description of some research within the field are provided. The last part of the chapter gives a quite thorough description on the riser slugging phenomenon.

Chapter 3 presents the theory behind the sliding mode controller (SMC), included the general form of the design we desire to use. The challenge concerning chattering is also discussed, and the approach of using boundary layer to suppress the chattering is introduced.

Chapter 4 derives the empirical van der Pol based model, and the model is augmented to use the valve rate as control input.

Chapter 5 gives a discussion on why the SMC should be performed on the valve rate and not directly on the valve opening. This section also designs the SMC that will be used for testing, and the needed derivatives for testing the SMC are derived.

Chapter 6 is about the performance of the PI controller on the system. This section is added to clarify the performance of the SMC using the observers.

Chapter 7 presents general observer theory, theory about using observers combined with SMC, and presents the high gain observer (HGO) and the robust high-order sliding mode differentiator (RHOSMD).

Chapter 8 describes the open loop tested testing of the HGO and RHOSMD for a slugging van Der Pol system (see 1.3.2).

Chapter 9 contains the testing of the HGO and RHOSMD for the SMC closed loop van Der Pol system (see 1.3.2).

Chapter 10 presents the main results and conclusions through all the chapters, and at the very end, further work is discussed.

Chapter 2

Background

Slug flow has historically been coped with by either change of design or operational changes. In [24], Havre and Dalsmo distinguished the methods for handling slugs into design changes, operational changes and procedures, and control methods. Design changes can be to increase the buffer capacity of the separators at the riser outlet. This approach is however costly and also unwanted as it do not get rid of the slugging problematic. Operational changes can be to bringing the well outside the slug flow regime by increasing the bottom hole pressure through the topside choke valve. Decreasing the choke opening has the disadvantage of lower production and thereby reduced income.

2.1 Historical development

Experiments with slug control were run in small scale already in 1979 [50][1] and in a larger scale in the late 80's [26]. In this experiment, the topside choke was actively used to control the riser foot pressure in a tow-phase flow line with a simple feedback PI control scheme. Several companies have developed similar solutions for slug control. Shell has a patented slug control system [29] where a mini-separator with flow control is used for stabilization, and one of the first known industrial implementations of a slug controller known to the public was archived by Total at the Dunbar pipeline. This is a 16 inch multiphase pipeline connecting the Dunbar field with the Alwyn platform at the British side of the North Sea. ABB have also patented a similar slug control systems [23]. The first StatoilHydro slug control installation was completed in April 2001 at the Heidrun oil platform in the Norwegian Sea [54]. Two satellite fields from the Heidrun Northern Flank are connected to the Heidrun oil platform with 4 km and 7 km long multiphase flow lines.

[1] In the experiment pressure measurement upstream the riser in addition to a flow measurement of the fluid through the riser was used. Control was based on an algorithm where the choke valve was adjusted automatically to cope with the slug flow.

Severe slugging in the riser was experienced and a slug control system was installed to suppress this. The slug control suppressed the slugging and stabilized the flow, and the flow line pressure was reduced significantly.

2.2 Research

However, no common agreement on proper solutions for dealing with slugging has yet been worked out, but numerous scientists and research environments are and have been working on numerous models and control systems. As the control task of slugging has proven to be very complex and thereby challenging for the PI controller, quite a lot of effort are putted into try to develop advance and nonlinear control schemes that hopefully could be both more robust and with the ability to control on a lower pressure – and thereby increase the recovery rate. Norway plays an important role in developing new technology for oil production and is clearly interested in finding better solutions to the riser slugging problem. Different departments at the Norwegian University of Science and Technology in Trondheim have done some work on this problem. Storkaas [56, 57, 58], Skogestad [57, 58] and Bårdsen [11] have all done some specific work in modelling and controlling slugging. Additionally the Department of Energy and Process Engineering is doing some work on analysing multiphase flows, (see [3]) while the Department of Engineering Cybernetics has focused on control applications, e.g., Sagatun [49].

Another important institution in Trondheim is SINTEF, which possesses a multiphase laboratory for experiments of these phenomena. Most of the big oil companies in Norway also work in this area, and ABB has some important research on controlling unstable wells. (see [8], which also refers to more articles).

Several research environments can be found in the USA, e.g., Cambridge University where they have done some studies on multiphase flow (see [12]). It is possible to find a few patents from the U.S. market by searching the internet (see [2] and [1]), although the results of these solutions are unknown.

A few important articles have been published on this topic from France. In collaboration with an Indonesian scientist a French article [27] describes the slugging problem carefully. A similar problem with multiphase flow is described by looking at gas-lifts, see Sinegre [53].

Even though there are many people working on this topic, there has not been made too many proper solutions. The surroundings at where slugging appears are often difficult too handle, in terms of getting good measurements. Quite a few functional solutions have been implemented, but they suffer lack of robustness and opportunities to handle small differences from one place to another. They consequently need a lot of tuning and adjustment to adapt to new oil production facilities.

2.3 The riser slug phenomena

Riser induced slug flow may occur in pipelines with multiphase flow. These multiphase pipelines can be found offshore between a production platform and subsea wells (or to a wellhead platform), and is an increasing phenomena in StatoilHydro operated fields [21]. Slug flow is an unstable oscillating flow regime which often has a period in a matter of hours [2]. Slugging appears in the riser base when liquid accumulates. This accumulation is mainly caused by the gravity which pulls the liquid toward the lowest point, where it eventually will block the pipe. The liquid will be pushed up in the riser as downstream flow from the inlet increases the pressure on the slug. When the pressure exceeds a certain limit, the liquid will shoot up in the riser against the platform valve. Additionally the gas flow will penetrate the liquid which leads to extra entrainment. Finally gas and liquid in the riser will flow out through the top valve. As the pressure drop over the riser decreases, the speed will decrease until some of the liquid begins to flow back against the riser base. Liquid will then accumulate again and the system goes into an oscillatory and unstable cycle.

The geometry of the pipeline, which allows for accumulation of liquid (oil/water) at the riser bottom, is first of all the reason for riser induced slug flow. In addition to this is often at low liquid and gas rates an impotent factor. This is understandable from the explanation above and from the fact that a higher liquid and gas rates would mean more kinetic energy in the system, which would be enough for the liquid to be forced up the riser without falling back causing the liquid accumulation. Having this in mind, it is understandable that slugging is an increasing phenomenon in StatoilHydro operated fields. More and more oil and gas fields are going into tail-end production, decreasing the reservoir pressure, and thereby causing a lower velocity rate for the multiphase flow. The gas to oil ratio has also a significant impact on the existence of slug flow, as gas has a much lower density than the oil phase. More gas (and water) as a part of the oil production is also a tail-end production problem. In addition to this, the choke valve opening has also an important on the sluggish dynamics. This will be evident from the following discussion.

Figure 2.1 illustrates the slugging process given in four steps. First; liquid is accumulated in the riser low-point due to gravity. If the velocity is low enough the liquid will block the gas coming from the feed-pipe. Now, if the pressure difference over the riser is smaller than the hydrostatic head the slug will continue to grow. The low point pressure will eventually be large enough to push gas trough the riser. This will cause a pressure drop in the

[2]Note that the slug frequency of the van Der Pol model that will be used in this thesis has a slug period in matters of minutes. This as this work has been intended to be a preparatory study before testing on the multiphase pilot rig at StatoilHydro's Research Centre in Porsgrunn, which is of a much smaller scale than a pipeline-riser system offshore.

riser low point and the gas velocity in the riser increases. This also increases the flow rate through the riser and the slug is blown out of the valve. Liquid then starts to fall back in the riser; creating a new slug.

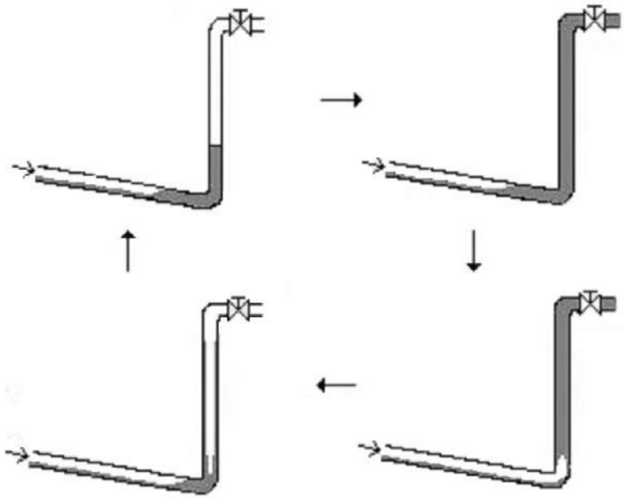

Figure 2.1: *Riser-slugging illustration. Recourse: [57]*

This slugging is as shown in [57] a stable limit cycle and can only be reduced by choking the valve at top-side. Figure 2.2 shows an example (from [57]) created by simulating a given riser-slugging process utilizing OLGA; a fluid simulation tool. As can be seen from the figure; the process is stable with a valve opening from $z = 0$ to $z \approx 13\%$. For larger values the pressure starts to oscillate between *6 bar* and *72 bar*.

Traditionally these instability problems have been solved by decreasing the valve opening at the platform and with that increase the pressure drop over this valve. To see the effect of the valve opening we can use a few simplified expressions, based on the pressure in the riser base, P_B (see the pressures in Figure 2.3). The gas flow into the riser base, w_g, is proportional to the pressure drop from the reservoir, P_{res} (which is assumed to be slow varying), to the riser base, P_B, i.e.,

$$w_g \propto (P_{res} - P_B) \tag{2.1}$$

Further the riser base pressure consists of two main parts; the gravitational pressure from the liquid in the riser, and the pressure at the top valve:

$$P_B = \Delta P_g + P_T \tag{2.2}$$

where the gravitational pressure (static head), ΔP_g, can be estimated as

$$\Delta P_g = \overline{\alpha}_l(w_g)\rho_l g H$$

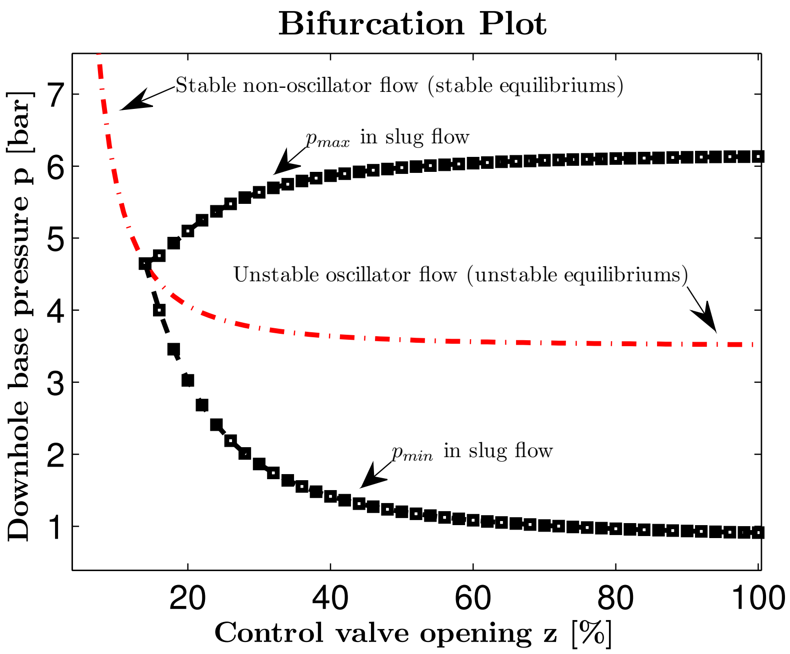

Figure 2.2: *Pressure as a function of valve opening*

where $\overline{\alpha}_l(w_g)$ denotes the mean liquid fraction in the riser as a function of gas flow. An increasing gas flow into the riser will clearly decrease $\overline{\alpha}_l(w_g)$ and thereby decrease ΔP_g. Further this decrease will make the riser base pressure fall (Equation 2.2), and increase the incoming gas flow even more (Equation 2.1). As a consequence this term makes an accelerating unstable rise in incoming gas flow.

A simplified expression can also be derived for the pressure at the valve, P_T, by using a simplified valve equation. This valve equation is originally derived for liquid flow, but gives a satisfactory approach to this multiphase flow.

$$w_g = K_1 z \sqrt{\rho_T (P_T - P_O)}$$

this gives

$$P_T = P_O + K_2 \left(\frac{w_g}{z}\right)^2$$

The pressure at the downside of the valve, P_O, can be seen as a slow varying term. As distinct from ΔP_g this expression will increase by an increased gas flow, which again reduces the gas flow, according to Equation 2.1. Consequently this will stabilize the riser base pressure. By summarizing we can see that the riser base pressure, given in Equation 2.2, is affected by two main terms which tend to influence the system stability in different

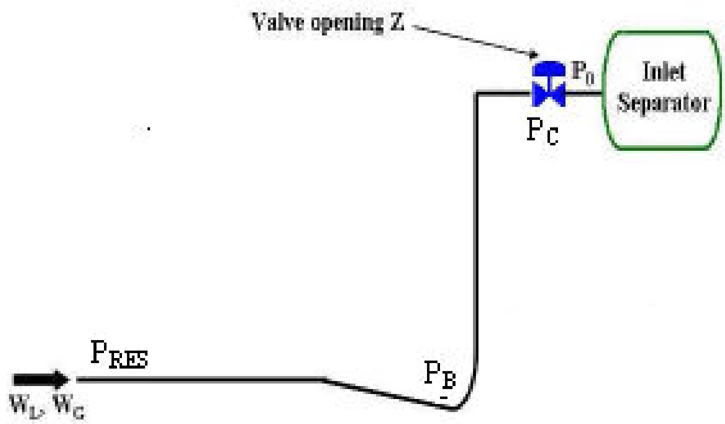

Figure 2.3: *Simplified representation of riser pressures*

directions. The dominating part of P_B will decide whether riser slugging will appear (instability from gravitational pressure) or not (stability from valve pressure difference). From the outside there is one parameter that we can adjust directly to control the system behaviours, that is the valve opening z. By decreasing this opening, the pressure over the top valve will increase and affect the system in a stabilizing manner.

Chapter 3

Sliding mode

Sliding mode control (SMC) is a nonlinear control strategy, recognised as an efficient tool to design robust controllers for complex high-order nonlinear dynamic plant operation under uncertain conditions. The approach of SMC is to transform a higher-order system into a first-order system. In this way, a simple straightforward and robust control algorithm can be applied. The controller use '*brute force*' to ensure stability so a model of the system is not required, but the degree of the system has to be known and it has to ensured that the applied force is large enough by using Lyapunov theory. The Lyapunov theory ensures that the controller stabilizes the nonlinear system in finite time.

Typically SMC might experience problems with so called chattering. If a model of the system with known bounds on the uncertainties exists, this model and its uncertainty bounds can be used to divide the control into one continuous feedback and one discontinues switching components. This approach can reduce the total extend of power needed to ensure stability and as a consequence, the problem with chattering is normally also reduced. Another way to suppress the chattering problem is to introduce a boundary layer where the force is reduced it the state is close to the so called sliding surface (close to the temporary desired state).

3.1 The motivation behind sliding control

In the formulation of a control problem it is standard procedure to describe the system through a mathematical model. The modeling is of key importance for the designer both in the design of the controller, and in the analysis of the system with and without the controller. However there will typically be discrepancies between the created model and the actual system. The modeling inaccuracies can be classified into two kinds: *structured* (parametric) and *unstructured* (unmodelled dynamics). The first kind corresponds to inaccuracies in the terms included in the model, while the other corresponds to inaccuracies in the model. The engineer must ensure that controllers have the ability to produce the required performance despite such mismatches. This has lead to an intense interest in the development of so-called robust control methods. One particular approach to robust controller design is the variable structure control methodology.

The characteristics of variable structure control systems (VSCS) are a suite of feedback control laws and a decision rule. The decision rule, termed the switching function, has as its input some measure of the current system behaviour and produces as an output the particular feedback controller that should be used at that instant in time.

SMC is a particular type of VSCS and is recognised as an efficient tool to design robust controllers for complex high-order nonlinear dynamic plant operating under uncertain conditions. The research in this area was initiated in the former Soviet Union in the 1960's, and the SMC methodology has subsequently received much more attention from the international control community within the last two decades.

The major advantage of SM is low sensitivity to plant parameter variations and disturbance which eliminates the necessity of exact modeling. SMC enables the decoupling of the overall system motion into independent partial components of lower dimension and, as a result, reduces the complexity of feedback design [60].

SMC implies that control actions are discontinuous state functions which may easily be implemented by conventional power converters with '*on/off*' as the only admissible operation mode. Due to these properties, the intensity of the research at many scientific centres of industry and universities is maintained at a high level, and SMC has been proved to be applicable to a wide rage of problems in robotics, electric drives and generators, process control, vehicle and motion control.

3.2 The switching control law

SMC is a type of VSC where the dynamics of a nonlinear system is altered via application of high-frequency switching control. This is a state feedback control scheme where the feedback is not a continuous function of time since the gains in each feedback path switches between two values according to the control rule. The purpose of the switching control law is to drive the nonlinear plant's state trajectory onto a prespecified (user-chosen) surface or manifold[1]. When on the surface the controller will ensure that the state trajectory stays on this surface for subsequent time. To manage this, when

[1]Manifold is the formal correct term in the n-th order space, but the theory is generally explained through the concept of the surface. This thesis will therefore follow the common practice of using the term surface even when we are dealing with a subset in a higher order space - a so called manifold

the plant state trajectory is '*above*' the surface a feedback gain is impressed to drive the state trajectory back to the surface, and when the trajectory drops '*below*' the surface, the '*opposite*' gain is impressed. Because of this control purpose, the surface is also often referred to as the switching surface. Further, the surface is designed in such a way that when the state trajectory stays on the surface, the principles of standard linear control theory will ensure that the desired state are reached in exponential time, and therefore the manifold is also referred to as the sliding surface. The *reaching phase* and the *sliding phase* of the state trajectory motion is illustrated in figure 3.1.

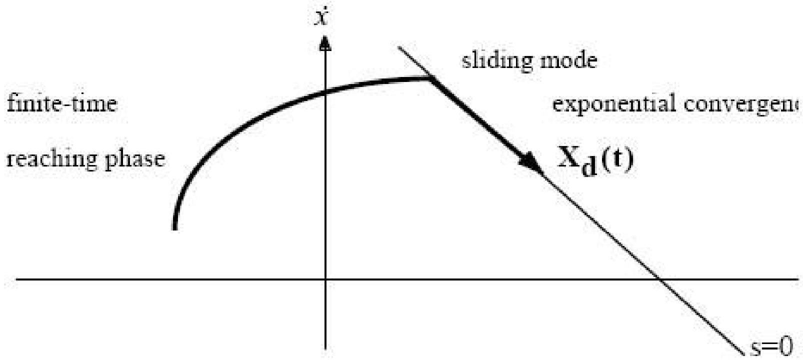

Figure 3.1: *Graphical illustration of a sliding mode controlled system. Resource: [55].*

To ensure that the sliding surface is stable and reached in finite-time, Lyapunov theory is used in the design of the switching control law. Lyapunov theory is essential in nonlinear control theory and is used to determine the stability of a certain fixed point (equilibrium point) in a dynamical system or autonomous differential equation. The principles of Lyapunov stability is as following:

1. Define a positive definite[2] continuously differentiable scalar function V in a domain D that contains the origin.

2. Show that for all points in the domain D there will be a motion (derivative) against new points with a lower function values for V.

3. Consequently $V(x) \rightarrow 0$, and thereby the equilibrium point will be stable and will be reached in finite time.

[2] V(x) is said to be positive definite if V(0) = 0 and $V(x) > 0$ for all x.

For future reading about Lyapunov theory [34] or [55] can be recommended. In the following it will be assume that the reader has basic knowledge of Lyapunov theory.

3.3 Deriving the sliding surface

The sliding surface is designed in such a way that standard linear theory will ensure that when the states trajectory is on the surface, the desired state will be reached in exponential time. Therefore the surface itself can be looked on as a passive controller. This section will introduce one standard way to design a surface that will achieve this property.
Consider a general n-th order dynamic system that needs to be controlled:

$$x^{(n)}(t) = f(\mathbf{x}, t) + b(\mathbf{x}, t)u \tag{3.1}$$

where $x(t)$ is the scalar output of interest (in this thesis the pressure at the bottom of the riser), $\mathbf{x} = [x\ \dot{x}\ ...\ x^{(n-1)}]^T$, and $u(t)$ is the control input (*here: the normalized production chock opening*). The superscript n of $x(t)$ is the order of the differentiation equation. Generally a system with a high order is more challenging to control because it contains more integrators. The effect of the integrators is that the (controlled) input has less immediate effect and worse, that the system has a higher degree of memory of past inputs.

The strength of SM-control is the robustness against system uncertainties. It can therefore be assumed that neither $f(\mathbf{x}, t)$ or the control gain $b(\mathbf{x}, t)$ is exactly known, but the extent of uncertainty is known. For $f(\mathbf{x}, t)$ the extent of imprecision is upper bounded by a known continuous function of $\mathbf{x}$ and t. The control gain $b(\mathbf{x}, t)$ is not exactly known, but is of known sign and is bounded by a known, continuous functions of $\mathbf{x}$ and t.

For the SMC purpose a sliding variable $\mathbf{s}$ that depends on the system described by (3.1) is defined. The sliding surface is then defined by $s(\mathbf{x}, t) = 0$. A requirement for the sliding surface is that it has a passive control property, where the desired state is reached in exponentially time. Therefore the sliding variable $s(\mathbf{x}, t)$ has to be defined in such a way that this requirement is guaranteed to be fulfilled. This thesis uses the definition of the sliding variable given in [55]. Thereby it is defined as

$$s = (\frac{d}{dx} + \lambda)^{(n-1)}\tilde{x} \tag{3.2}$$

Here, λ is a strictly positive constant describing the placement of the poles and thereby the bandwidth of the system while on the surface. Further $\tilde{x} = x - x_d$ is the error in the output, where x_d is the desired output.

Using this, if we then define

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x_d} = [\tilde{x}\ \dot{\tilde{x}}\ ...\ \tilde{x}^{(n-1)}]^T$$

equation (3.2) can also be written as

$$s = \mathbf{A}\tilde{\mathbf{x}}$$

where the matrix $\mathbf{A}$ is negative definite; ensuring the stability of the system. The fact that $\mathbf{A}\tilde{\mathbf{x}} = 0$ is exponential stable, reveal that the problem of tracking $\mathbf{x} \equiv \mathbf{x_d}$ is equivalent of that of remaining on the surface s(t) for all $t > 0$. Thereby, the problem of tracking the n-dimensional vector $\mathbf{x_d}$ can be reduced to that of keeping the scalar quantity s at zero. Since s contains $\tilde{x}^{(n-1)}$, we only need to differentiate s once for the input u to appear, so the original n^{th} order tracking problem is now replaced by a 1^{st}-order stabilization problem in s.

This control transformation is valid since bounds on s can be directly translated into bounds on the tracking error vector $\tilde{\mathbf{x}}$. This means that the scalar s represent a true measure of tracking performance. Assuming $\tilde{\mathbf{x}}(0) = \mathbf{0}$ the corresponding performance is described by the transformation

$$\forall t \geq 0, |s(t)| \leq \Phi \quad \Rightarrow \quad \forall t \geq 0, |\tilde{x}^{(i)}(t)| \leq (2\lambda)^i \epsilon \qquad \epsilon = \Phi/\lambda^{(n-1)} \qquad i = 0, ..., n-1 \tag{3.3}$$

The simplified 1^{st}-order problem of keeping the scalar s at zero can now be achieved by choosing the control law u of equation (3.1) such that when not on the surface $s = 0$

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{3.4}$$

where η is a strictly positive constant [55]. This condition states that the square of the '*distance*' to sliding surface decreases along all system trajectories. Thus, the sliding surface will be reached in finite time and once on the surface the system trajectory will remind at the surface. Thereby the passive control properties of the surface will assure that the desired state will be reached. This implies that the active and passive control will work together and continually apply tracking of a time varying desired state.

3.4 Chattering and boundary layer

Ideal sliding mode exists only when the state trajectory stays on the sliding surface for all future after the reaching time. To manage this, infinitely fast switching may be required. However, in real time system the switching controller has imperfections which limit switching to a finite frequency. Thereby the state trajectory will not be able to change '*direction*' at the surface and will thereby drift outside the surface. This will result in oscillations within a neighbourhood of the switching surface. This oscillation is known as chattering, and is illustrated on figure 3.4. The subject of chattering is

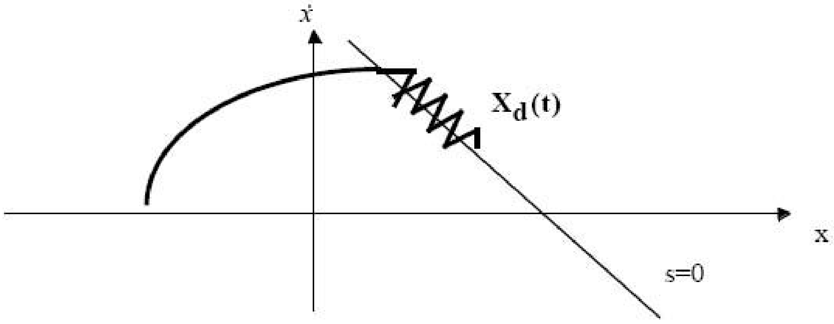

Figure 3.2: *Chattering as a result of imperfect control switching*

of great importance when we intend to use the theory on real applications. The chattering oscillations are know to result in low control accuracy, high heat losses in electrical power circuits, and high wear of moving mechanical parts. Further, the high-frequency activity of the controller might awake high-frequency dynamics neglected in the modelling of the system. Therefore the robustness or the problem of correspondence between an ideal sliding mode and real-life processes in the presence of un-modelled dynamics should be analysed. These phenomena have been considered as serious obstacles for the application of SMC in many papers and discussions [60].

Since chattering has so many undesirable effects it has to be reduced as much as possible. One evident way of doing so is to reduce the magnitude/gain of the controller. This because the controller will have a reduced effect on the system, so the '*speed*' of the chattering, and thereby how fare the state will be able to drive away from the surface before it will be driven back by the next sample will be reduced. The challenge her is that the gain must be large enough to ensure Lyapunov stability. One way of reducing the gain, and still ensure that the Lyapunov stability holds, is to smooth out the control discontinuity in a thin boundary layer neighbouring the switching surface

$$B(t) = \{x, \|s(x;t)\| \leq \Phi\} \qquad \Phi > 0 \tag{3.5}$$

where Φ is the boundary layer thickness, and $\epsilon = \Phi/\lambda^{n-1}$ is the boundary layer width. This means that outside the boundary layer $B(t)$, the control law is chosen as before, which guarantee that the boundary layer is attractive, and hence invariant; all trajectories starting inside $B(t = 0)$ remain inside B(t) for all $t \geq 0$; and u is interpolated inside B(t). For example in a linear way by letting the applied power depend on $\frac{s}{\Phi}$ inside $B(t)$, but quadratic, exponential or other approaches might also be considered.

The use of boundary layer leads to tracking within a guaranteed precision ϵ (rather than perfect racking), and more generally guarantees that for all trajectories starting inside $B(t=0)$

$$\forall t \geq 0, \left\| \tilde{x}^{(i)}(t) \right\| \leq (2\lambda)^i \epsilon \qquad i = 0, ..., n-1 \tag{3.6}$$

Another approach is to apply model knowledge and add a continuous part to the controller. Thereby the discontinuous gain can be reduced since much of the dynamics is suppressed by the continuous parts, and this will in general give the important result that the total control energy is reduced.

3.5 Discontinuous SMC[3]

This section will introduce the design of a purely discontinuous SMC, and will thereby derive a gain for this controller that will ensure that sliding condition (3.4) is fulfilled for a general n^{th}-order system described by 3.1 . By Lyapunov theory it is then assured that the sliding surface will be reached in finite time, and further the properties of this surface will assure that the desired state is reached. In the next section the same derivation will be done for a SMC where a model based continuous part is added. The reason for the approach is that the totally gain might be reduced (while sliding condition is still fulfilled), and thereby the problems with chattering might be reduced.

For simplicity the design and derivation will be done for the following second order equation. However, the design principles and derivation for a general n^{th} order system will be the same. The only difference is that the sliding surface corresponding to the n^{th} order will be used, and that the equations will have a higher complexity.

Consider the second-order system

$$\ddot{x} = f + bu \tag{3.7}$$

where u is the control input, x is the scalar output of interest and f is some unknown dynamics. Further b is some unknown control gain (possible time-varying and state-dependent), but with known lower bound

$$b \geq b_{min} > 0 \tag{3.8}$$

In order to have the system track $x_d(t)$ the sliding variable is defined according to (3.2). This gives the formulation

$$s = (\frac{d}{dx} + \lambda)\tilde{x} = \dot{\tilde{x}} + \lambda x \tag{3.9}$$

The derivative of the sliding variable is then

$$\begin{aligned} \dot{s} &= \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}} \\ &= f + bu - \ddot{x}_d + \lambda\dot{\tilde{x}} \\ &= A + bu \end{aligned} \tag{3.10}$$

[3]Main recourse: [55]

where

$$A = f - \ddot{x}_d + \lambda\dot{\tilde{x}} \tag{3.11}$$

The control output can then be defined as

$$u = -ksign(s) \tag{3.12}$$

Using Lyapunov theory can prove that by choosing $k = k(x, \dot{x})$ large enough, the sliding surface will be reached in finite time. Indeed, from (3.7) - (3.10)

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}s^2 &= s\dot{s} \\ &= [A - bksign(s)]\, s \\ &= A \cdot s - bk|s| \end{aligned} \tag{3.13}$$

By choosing

$$k = \frac{|A|_{max} + \eta}{b_{min}} \tag{3.14}$$

3.13 can then be rephrased as

$$\frac{1}{2}\frac{d}{dt}s^2 = \overbrace{A \cdot s - \underbrace{\frac{b}{b_{min}}}_{\geq 1} |A|_{max}|s|}^{\leq 0} - \underbrace{\frac{b}{b_{min}}}_{\geq 1} \eta\|s| \leq -\eta|s| \tag{3.15}$$

and it is thereby guaranteed that the sliding surface is reached in finite time, and with ideal sampling the trajectory will continue to stay on the sliding surface.

Chapter 4

Modelling[1]

This chapter elaborate the model that is developed to test the sliding mode controller's ability to suppress severe riser slugging. The simple empirical model describes the qualitative behaviour of the downhole pressure during severe riser slugging. By using this model for testing, the thesis will test the potential of sliding mode control applied for stabilization of unstable flow in oil wells.

4.1 About modelling

A common approach for modelling of a system is to study and describe physical phenomena. For purely simulation purposes, there goal is to get an as exact model as possible, and therefore an elaborate study of phenomena in the process is regular. The result is an extensive model that describes how these phenomena interacts with each other and inflicts the process states of interest. However such models will in general be too complicated for control purposes. They are to difficult (or impossible) to analyse and achieve the needed understanding to do good and reliable design choice. The controllers will be too complicated and to difficult to follow, and the needed calculations might also be too time consuming. Therefore a model for control purposes should only contain the significant phenomena in the frequency domain of interest. Phenomena that is only significant in a range that is higher then the controller is not of interest for the controller as the controller is not able to follow them, so from the controllers point of view these phenomena is high frequency noise. A phenomenon that has a much slower response then the control area of interest is neither of interest as they can be treated as a '*constant*' bias.

Describing the significant physical phenomena is not the only approach for creating a model of a system. In lack of a physical based model that is both simple and exact enough this thesis will instead use a empirical

[1]The derivation of 4.1 - 4.6 is heavily based on [32].

model that follow the main quantitative behaviours of a sluggish system. The empirical model is based on the observation that the downhole pressure during severe riser slugging has an oscillating behaviour with the property of a *limit cycle*. Therefore a slightly modified *van der Pol* equation is used as a base for the creation of the model. The principle of the van der Pol equation can be illustrated through the phase plot of figure 4.1. The model will be able to roughly fit the oscillation frequency and the bifurcation plot of a real slug. Therefore it is reasonable to assume that the model will describe the main properties of a sluggish system, and thereby give a reasonable indication of a controller's ability to suppress slugging in a real system.

$$\dot{p} = w \tag{4.1}$$

$$\dot{w} = a_1(\beta - p) + a_2(\zeta - w^2)w \tag{4.2}$$

In these equations the states p and w are the downhole pressure in the riser and its time derivative. The coefficients can be explained as follows:

- β : equilibrium pressure
- a_1 : frequency or stiffness of the system
- a_2 , ζ : local '*degree of the stability/instability*' and amplitude of the oscillation of a fixed valve opening z

4.2 The equilibrium downhole pressure

From equations (4.1) - (4.2) it is trivial to verify that the equilibrium point (p^*, w^*) is given by

$$\begin{bmatrix} p^* \\ w^* \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \end{bmatrix} \tag{4.3}$$

This points out that β is the equilibrium downhole pressure or as defined above - the steady state pressure. From physical considerations, section 2.3 points out that the equilibrium downhole pressure $p^* = \beta$ is given by

$$\beta = \bar{\rho} g H + \Delta p_F + \Delta p_C + p_0 \tag{4.4}$$

In this equation $\bar{\rho} g H$ is the static head with $\bar{\rho}$ being the average density in the riser, Δp_f is the frictional pressure drop, Δp_c is the pressure drop over the production valve, and p_0 is the pressure downstream the valve. When the hole system is in steady state, the differential pressure over the production valve depends directly on the reservoir influx w_{res} and is given by its flow characteristics according to

$$\Delta p_c(w_{res}) = \frac{w_{res}^2}{(K_z z)^2 \rho_T} \tag{4.5}$$

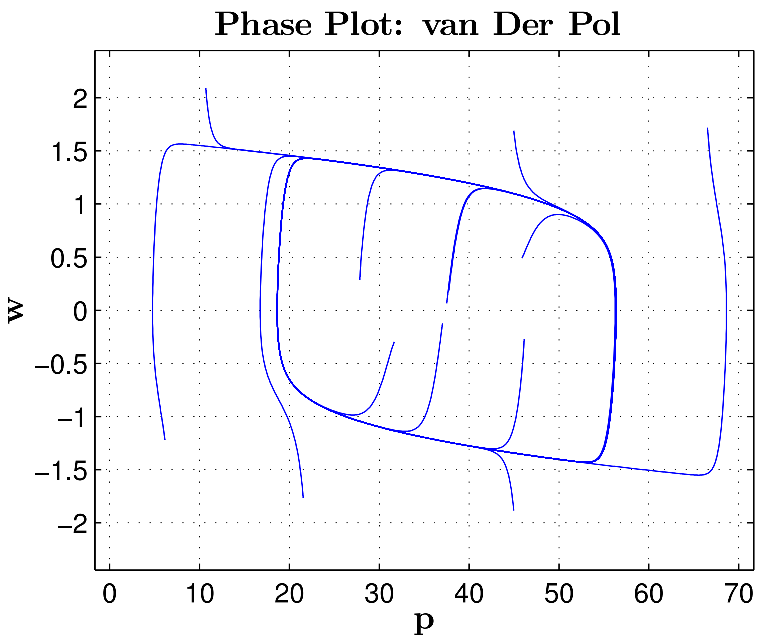

Figure 4.1: P*Phase plot of the van der Pol equation. The illustration is created by the model this chapter is deriving.*

Here ρ_T is the density upstream the valve, z the valve opening, and K_z the flow constant for the valve. Furthermore, the average density $\bar{p}(w_{res})$ is a decreasing function of w_{res} as the liquid hold-up will increase with more energy into the system. The frictional pressure drop $\Delta p_f(w_{res})$ is as (4.5), a increasing function of w_{res} according to

$$\Delta p_f(w_{res}) = K_f w_{res}^2 \tag{4.6}$$

By doing the simplification to assume that the influx w_{res} stays constant, the equation for β can be given in the lumped form

$$\beta(q) = b_0 + b_1 q \tag{4.7}$$

where b_0 and b_1 are positive constants, and q is proportional to the differential pressure Δp_c at steady-state flow w_{res}. In figure 4.2, the stable equilibrium pressure β is plotted as a function of the valve opening.

4.3 Frequency or stiffness of the system

To understand how the different parameters in the system given by equations (4.1) - (4.2) are related to properties of the system, it is convenient to

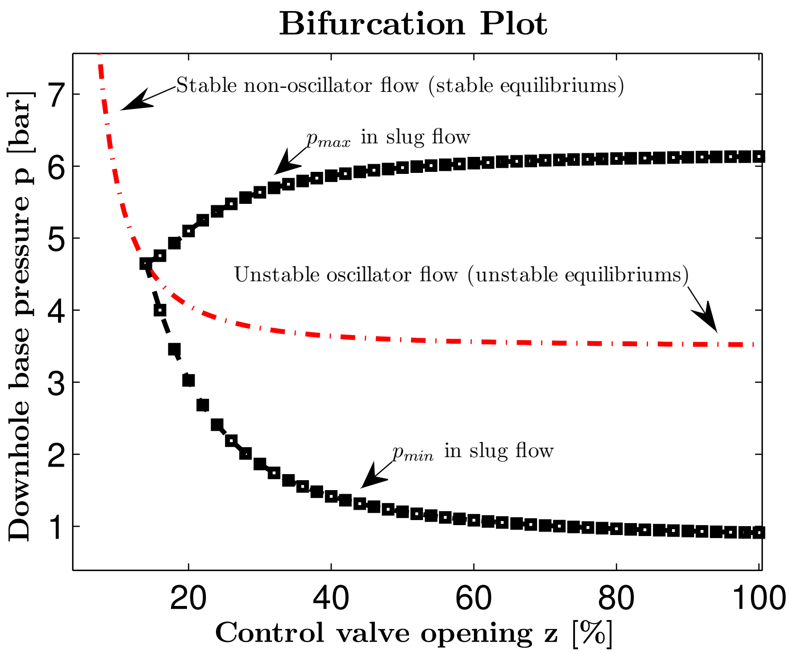

Figure 4.2: *Pressure as a function of chock opening.*

linearize the system to get

$$\dot{\Delta p} = \Delta\omega \tag{4.8}$$

$$\dot{\Delta w} = -a_1\Delta p + a_2\zeta\Delta\omega \tag{4.9}$$

Laplace transformation of the system gives

$$s^2\Delta p - sp(0) - w(0) = -a_1\Delta p + sa_2\zeta\Delta p - a_2\zeta\Delta p(0) \tag{4.10}$$

If the initialization is done when Δp is zero, $\omega(0)$ can be defined to some constant k, witch give the transfer function

$$\Delta p = \frac{k}{s^2 - sa_2\zeta + a_1} \tag{4.11}$$

By comparing with the transfer function and the frequency response (bode diagram) of a general second order oscillating system, it is clear that the parameter a_1 determine the cut-off frequency of the system. The cut-off frequency represent a boundary in a system's frequency response at which energy entering the system begins to be attenuated or reflected instead of transmitted; and thereby the stiffness of the system.

4.4 Local degree of stability/instability

By again comparing equation (4.11) with the general second order oscillating system, it can be seen that the parameters a_2 and ζ determine the gain at the resonance frequency. Therefore it is evident that they are related to the amplitude of oscillation and stability properties of the fixed point.

Future, the eigenvalues of the linearized system (4.8) - (4.9) are

$$\lambda^* = \frac{a_2\zeta \pm \sqrt{a_2^2\zeta^2 - 4a_1}}{2} \tag{4.12}$$

which means that (assuming $a_1 > 0$ and $a_2 > 0$) the following listed ζ-values gives the following properties

- $\zeta < 0$, equilibrium point/system is stable
- $\zeta = 0$, bifurcation point
- $\zeta > 0$, equilibrium point/system is unstable

The control valve opening will influence the value of ζ and thereby witch of these three conditions the system is influenced by, as illustrated in figure 4.2.

With the assumption that the flow rates of liquid and gas from the reservoir is constant, ζ can be given in the lumped form [enkel lineær ligning med ventiltrykkfallet som oppfyller kvavene til listen over]

$$\zeta(q) = c_0 + c_1 q \tag{4.13}$$

where c_0/c_1 denotes the bifurcation point and c_0, c_1 are positive constants. (4.13) is created by fitting a linear assumption to the requirements for ζ listed above.

4.5 Transportation delay

The control input acts on the dynamics of (4.1)-(4.2) through the differential pressure over the production valve. The pressure changes are transported through the well at the speed of sound, and therefore, since the riser is several hundred meter (or even kilometres), a significant time-lag is expected between application of the control signal to the valve and seeing the effect in (4.1)-(4.2). The effects of the differential pressure over the production on (4.1)-(4.2) is represented by the variable q, and the time-lag is modelled as

$$\dot{q} = -\frac{1}{\tau}q + \frac{1}{\tau}\delta \tag{4.14}$$

where δ represents the control input and is a strictly decreasing function of the normalized production valve opening $z \in [0, 1]$. The relationship between z and δ is modelled as

$$\delta(z) = \frac{K_q}{u^2} \qquad z \in [0, 1] \tag{4.15}$$

4.6 Simplified model of riser slugging

Based on (4.7) and (4.13), the system dynamics (4.1)-(4.2) and (4.14) can be assembled into

$$\begin{aligned} \dot{p} &= w && (4.16)\\ \dot{w} &= -a_1 p + h(w) + g(w)q + a_1 b_0 && (4.17)\\ \dot{q} &= -\frac{1}{\tau}q + \frac{1}{\tau}\delta(z) && (4.18)\\ \delta(z) &= \frac{K_q}{z^2} \qquad z \in [0,1] && (4.19) \end{aligned}$$

where the functions h and g are defined as

$$\begin{aligned} h(w) &= a_2 c_0 w - a_2 w^3 \\ &= h_0 w - h_1 w^3 \end{aligned} \tag{4.20}$$

$$\begin{aligned} g(w) &= a_1 b_1 - a_2 c_1 w \\ &= g_0 - g_1 w \end{aligned} \tag{4.21}$$

The positive parameters a_i, b_i and c_i $(i = 1, 2)$in the model are empirical parameters that are adjusted to produce the right behaviour of the downhole pressure p. The system (4.16)-(4.17) can capture some of the qualitative properties in the downhole pressure during riser slugging.

Decreasing control gain: For riser slugging control the static gain is decreasing with the valve opening.

Bifurcation: At a certain valve opening (c_0/c_1) the steady-state response of the downhole pressure goes from a stable point to a stable limit cycle (see Figure 4.2).

Time lag: Transportation delay between changes in the valve opening to the resulting change in the downhole pressure p as the pressure change is transported thought the pipe with the speed of sound. This delay is modelled as a simple 1st-order lag.

4.7 Augmented system

For real applications it will be preferable to design a sliding mode controller with continuous control input. This can be achieved be introduce the rate of change r of the valve opening z as the control input, thus, augmenting the system model with the valve opening z as a new state:

$$\dot{p} = w \tag{4.22}$$
$$\dot{w} = -a_1 p + h(w) + g(w)q + a_1 b_0 \tag{4.23}$$
$$\dot{q} = -\frac{1}{\tau}q + \frac{K_q}{\tau}z^{-2} \tag{4.24}$$
$$\dot{z} = r \tag{4.25}$$

where,

$$\begin{aligned} h(w) &= a_2 c_0 w - a_2 w^3 \\ &= h_0 w - h_1 w^3 \end{aligned} \tag{4.26}$$

$$\begin{aligned} g(w) &= a_1 b_1 - a_2 c_1 w \\ &= g_0 - g_1 w \end{aligned} \tag{4.27}$$

The sliding mode control law $r = \alpha(\cdot)$ will then have to be designed for the system (4.22) - (4.25) with the rate of change r as the control input. The actual control input $z = u$ applied to the actual system (4.16)-(4.19) is given by its integral according to

$$z = \int_0^t \alpha \, dt \tag{4.28}$$

where the sliding mode control law is given as a function of the output p and its derivatives according to $\alpha(s)$ [31].

Implementation remark

Note that the integrated control law should be implemented with projection so that integration is stopped when the bounds of z are reached, thus ensuring $z \in (z_{min}, 1)$. in the scalar case, the discontinuous projection operator is defined as

$$P(f, x) = P(f, x, x_{lb}, x_{ub}) = \begin{cases} 0, & x \geq x_{lb} \wedge f > 0 \\ 0, & x \leq x_{lb} \wedge f > 0 \\ f, & \text{else} \end{cases} \tag{4.29}$$

Using the above discontinues projection operator, the controller should thus be implemented according to

$$z = \int_0^t P(\alpha, u, z_{min}, 1) \, dt \tag{4.30}$$

4.8 Used model parameters

In the simulations of this thesis the empirical augmented model is used with the parameters: $a_1 = 0.025$, $a_2 = 50$, $b_0 = 3.5$,$b_1 = 5$,$c_0 = 0.02$ and $c_1 = 0.1$. This makes: $h_0 = 1$, $h_1 = 50$, $g_0 = 0.125$ and $g_1 = 5$. Further, $\tau = 1$ sets the simulated transport delay constant to 1 second.

Note: it can be remarked that for numerical reasons $m = 10p$ is simulated, and p is derived form m. From (4.1), (4.2), (4.7) and (4.13) it was clear that the following parameter justifications was necessary: $a_2(m) = a_2/100 = 0.5$, $b_0(m) = 10b_0 = 35$, $b_1(m) = 10b_1 = 50$, $c_0(m) = 100c_0 = 2$, $c_1(m) = 100c_1 = 10$, $h_1(m) = h_1/100 = 0.5$, and $g_0(m) = 10g_0 = 1.25$.

Chapter 5

Controller design

The objective of this chapter is to outline and describing the design of the sliding mode controller (SMC)that will be tested in this thesis. More specific; the general design outline form the sliding mode introduction chapter will be used to specify a controller for the empirical model of riser slugging described in the previous chapter. One main strategies or scheme is neither letting virtual SMC be based directly on the simplified model ((4.16)-(4.19)) and have *direct control of the valve opening*. Another, using the augmented model ((4.22)-(4.25)), letting the virtual SMC *control of the valve opening/closing rate*. In the research of the project thesis [48], both schemes of control was tested and control of the valve opening/closing rate was concluded to concluded to be the best choose, and is therefore the control approach used in this thesis. It was concluded, through both the analysis and the research of the project thesis, that the schemes of virtual controlling the valve opening/closing rate was the best approach[1] and is therefore the chosen control scheme of this master thesis.

5.1 Derivation: SMC of the choke

From equation (4.22)-(4.25) it is clear that the relative order (number of integrators between z and p is four. Using definition 3.2; the SMC law α for the augmented system is given as

$$s = \left(\frac{d}{dx} + \lambda\right)^{(3)} \tilde{x} = \tilde{x}^{(3)} + 3\lambda\ddot{\tilde{x}} + 3\lambda^2\dot{\tilde{x}} + \lambda_c^3\tilde{x} \tag{5.1}$$

Here $\tilde{x} = p - p_{ref}$ is the error between the actual pressure and the set point of the controller, and λ is a design constant that can be viewed as the bandwidth of the closed-loop SMC. By combining the virtual control law α

[1]It was actually concluded in the project thesis that the direct control approach could actually not work without extensive system knowledge - as our controller will not have access to.

with the equations 4.29 and 4.30, the actual control u applied to the input z of the real system ((4.16)-(4.19)) is given by

$$\dot{u} = P(\alpha(s(p, \dot{p}, \ddot{p}, p^{(3)})), u, z_{min}, 1)\, dt \tag{5.2}$$

initialized according to $u(t = 0) = u_0$ and α is given as

$$\alpha = K sign(s), \tag{5.3}$$

where K is the gain created by the valve rate.

5.2 Boundary layer

As discussed in 3.4, for practical system it is necessary to supress chattering and '*calm down*' the system. On way to do so, is to use a boundary layer defined as

$$B(t) = \{x, \|s(x;t)\| \leq \Phi\} \qquad \Phi > 0, \tag{5.4}$$

and then redefine the controller α as

$$\alpha = \begin{cases} K sign(s), & s \notin B(t) \\ K\frac{s}{\Phi}, & s \in B(t) \end{cases} \tag{5.5}$$

5.3 Finding the needed derivatives

To find the derivatives needed in the sliding variable, an observer or a (numerical) differator has to be used.

For simulation purposes, the second- and third-order derivative of $\tilde{x}$ can be derived from the model:

$$\begin{aligned} w_2 &= \ddot{p} = \ddot{\tilde{x}} \\ &= -a_1 p + h(w) + g(w)q + a_1 b_0 \\ w_3 &= p^{(3)} = \tilde{x}^{(3)} \\ &= -a_1 w + h(\dot{w}) + g(\dot{w})q + g(w)\dot{q} \\ &= -a_1 w + h(\dot{w}) + g(\dot{w})q + g(w)\left(-\frac{1}{\tau}q + \frac{K_q}{\tau}z^{-2}\right) \\ &= f + bz^{-2} \end{aligned} \tag{5.6}$$

where f and b are defined as

$$\begin{aligned} f &= -a_1 w + h(\dot{w}) + g(\dot{w})q - g(w)\frac{1}{\tau}q \\ b &= g(w)\frac{K_q}{\tau} \end{aligned} \tag{5.7}$$

future, the functions h, g, $\dot{f}$ and $\dot{g}$ is defined by or derived from (4.26)-(4.27) to be

$$\begin{aligned} h(w) &= h_0 w - h_1 w^3 \\ h(\dot{w}) &= h_0 w_2 - 3h_1 w^2 w_2 \\ g(w) &= g_0 - g_1 w \\ g(\dot{w}) &= -g_1 w_2 \end{aligned} \tag{5.8}$$

Chapter 6

PI controller

There exist several conventional and known possible ways to use active control to eliminate slug flow. All of them can be highly economical beneficial compared to the old fashion way of solving the slug flow problem by choking down the valve – introducing high pressure and reduced production. This chapter will test out how a PI controller will perform on the simple van Der Pol model described in chapter 4. PI control is today the conventional active control method for stabilization of slugging.

In the following three chapters, the High-Gain observer (HGO) and the Robust High-Order Sliding Mode Differentiator (RHOSMD) will be introduced and evaluated on the van Der Pol model. The evaluation of chapter 9 will be how well the SMC perform when using the observers. The criteria then simply that if the SMC performance is good, the observer is fitted for usage together with the SMC and visa versa. For the evaluation to be qualitative, it will be useful to compare the SMC performance with the performance of the PI controller.

The PI controller will be tested for the ideal case of no disturbance, and for the case of small measurement noise with a normal derivation of 0.5% or 30.5 mbar, as the reference is set to 6.1 bars – the maximal pressure under the slugging achieved by the model when the choke valve is fully opened. I have chosen to not use any kind of filtering of measurement noise, but it should be possible to improve the result quite a lot with a fitted filter.

6.1 Designing the PI controller

There is many way to design PI control structure for slug flow. Many different measurements and measurements points can be designed. One example is the control structure that has been implemented offshore at Heidrum that use bottom whole pressure and topside flow [21]. The concern of this thesis is however limited to the case where the only measurement available or (at least) used – is the bottom hole pressure. Then a standard way of expressing the control law u_{PI} is given of the form:

$$u_{PI} = u_I - K_p(p - p_{ref}) \tag{6.1}$$

where u_I is the bias for a given pressure set-point p_{ref}, generated by slow internal action according to

$$\dot{u_I} = \frac{K_i}{T_i}(p - p_{ref}) \tag{6.2}$$

[32] explore the potential of this controller on the model used in this article ((4.16)-(4.19)) by linearization the closed loop dynamics. The Jocabian matrix is then found to be

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -a_1 & h_0 - g_1 q_{ref} & g_0 \\ -\frac{K_p}{\tau}\delta'(u_I) & 0 & -\frac{1}{\tau} \end{bmatrix} \tag{6.3}$$

this gives the characteristic equation

$$\lambda^3 + \left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)\lambda^2 + \frac{1}{\tau}(\tau a_1 - h_0 + g_1 q_{ref})\lambda + \frac{g_0}{\tau}\delta'(u_I)K_p + \frac{a_1}{\tau} = 0 \tag{6.4}$$

By using the Hurwitz criterion, [32] states that it turns out that local exponential stability can be achieved by PI control if

$$p_{ref} > b_0 + \frac{h_0 g_0}{a_1 g_1} - min\left\{\frac{g_0 \tau}{g_1}, \frac{g_0}{g_1 \tau a_1}\right\} \tag{6.5}$$

However my simulations indicates that the condition is

$$p_{ref} > b_0 + \frac{h_0 g_0}{a_1 g_1} + min\left\{\frac{g_0 \tau}{g_1}, \frac{g_0}{g_1 \tau a_1}\right\} \tag{6.6}$$

My opinion[1] is that this equation has to be the correct answer. Supporting my point of view is that if 6.5 was to be correct, increasing τ (and thereby greater time-lag) will result in a lower limit for the performance of the PI controller.

6.2 Performance from stabilized pressure

The van Der Pol system parameters used in the simulations are given in 4.8 Using these system parameters, equation 6.6 gives that the PI controller can be stabilized for $p_{ref} > 4.525$. This is actually in the stable region of the system, above the '*bifurcation level*'[2] that is 4.5 bars for this system.

[1] At the stage of the thesis, when dealing with this issue, time was a critical factor, so I choose not to spend time recalculating the equations.

[2] The *bifurcation level*' is the limit between the stable and unstable region.

It is thereby also clear that the PI controller is performing quit bad on the van Der Pol system, as it is actually possible stabilize the pressure with manual chocking down to the bifurcation level (if system is not already choking, the system must first be stabilized at a higher level).

K_p	K_i	T_i
0.002	.005	80

Table 6.1: *The PI parameters used in this thesis.*

As the PI controller is performing in the stable region, a quite slow tuning is obvious an advantage, as no fast action is required. The PI parameter setting used in this thesis is given be table 6.1. By using such a slow tuning, the controller action becomes quite good for all controller sample rates ($T_{samp} = \{0.1, 0.4, 1\}$), against all valve rates of interest ($T_{valve} \in [1, 30]$), and for the ideal case and all noise levels (*low, medium and high*). This is illustrated through figure 6.1 and figure 6.2. Figure 6.1 shows that in the idle case the low and the fast possible valve rate gave the same performance. The reason for this is that the PI controller is so slowly tuned that in general is change the coke slower then the slowest maximum choke rate ($T_{valve} = 30$). Figure 6.2 demonstrates simulations at the worst case scenario for high measurement noise and low sample rate for different valve rates, and shows that the performance is quite good used in the worst case. The reason is that the controller is acting so slow that the action between each controller sample is not so great even for long sample times, and that wrong action based on measurement error is limited. Tests show that if the PI controller is tuned to be faster, the performance is reduced. Further it should be noted that for the simulation time of the figures, it might seems like it is possible to stabilize the pressure below the limit given by equation 6.6. To reveal marginal instabilities quite longer simulation times has to be used. For the controller settings used here, the controller turned out to become unstable after roughly 26000 seconds when $p_{ref} = 4.524$.

At a reference pressure level of 4.48 the controller is unstable as shown on figure 6.3. However, the instability is limited, but as the p_{ref} is lowered, the degree of instability increase rapidly. This is in the figure illustrated by $p_{ref} = 4.2$ (*blue line*). The simulation of the figure uses $T_{samp} = 0.1$ and $T_{valve} = 1$, and the same parameters as above is used. But no tuning, sample time or valve rate is able to stabilize at this level. The reason for this is given by equation 6.6.

As for the SMC it would (or course) be possible to use continuous or stepwise change in the reference pressure toward a desired pressure. However, for the case of the PI controller there would be little or no point in doing so. This because the PI controller is able to perform a very calm control, and (again of course), my tests concluded that the performance of the PI

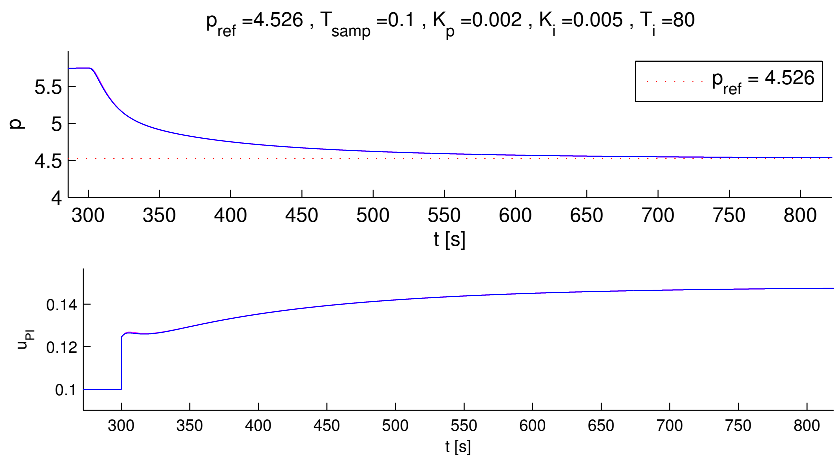

Figure 6.1: *PI controller at idle conditions: Here* $T_{samp} = 0.1$ *and* $T_{valve} = 1$ *(blue line) and* $T_{valve} = 30$ *(mauve line)*

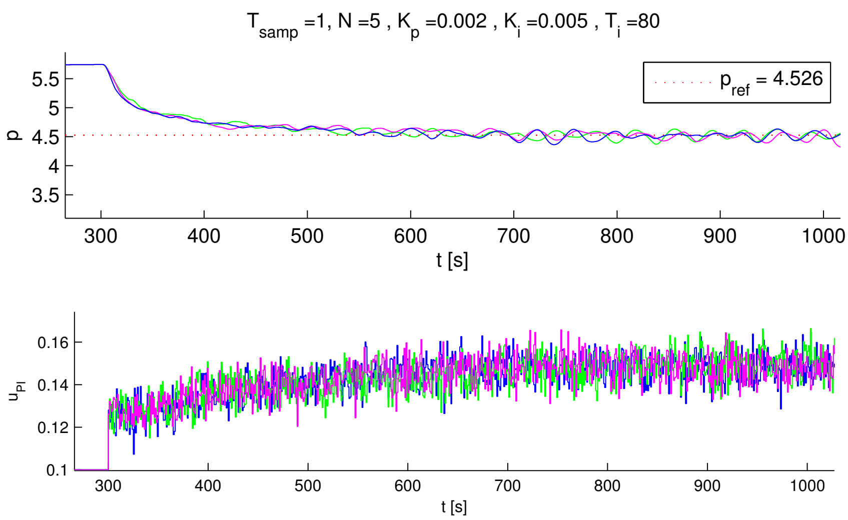

Figure 6.2: *PI controller influenced by noise: simulations done with high measurement noise (noise* = 5%*), with* $T_{samp} = 1$ *and the valve rates;* $T_{valve} = 1$ *(blue line),* $T_{valve} = 10$ *(mauve line) and* $T_{valve} = 30$ *(green line)*

controller was still limited by equation 6.6. For these two reasons I have chosen not to present the continuous changing control scheme her.

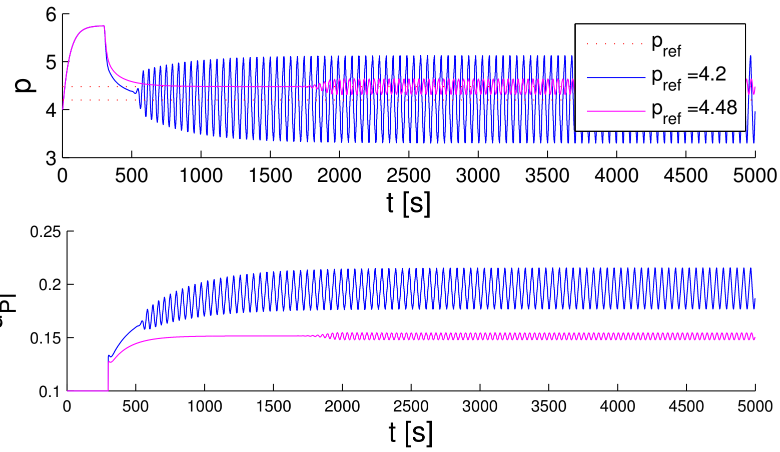

Figure 6.3: *PI controller in unstable area:* $T_{samp} = 0.1$ *and* $T_{valve} = 1$

6.3 Performance from slugging

In last section we discussed how the PI controller performs when it starts form a stabilized pressure level. The natural next question is how the controller will perform when started while the system is slugging. Investigating this question, we must have in mind that the performance of the controller might depend on the state of the system when it starts, and the initial valve opening. Therefore I have chosen to investigate the eight cases that become out of combining that the controller can start early or late in up-building or flushing of the slug, and initial valve opening of 20% or fully opened. For the SMC to suppress slugging and stabilize at a certain pressure level, the requirement is that all eight cases should be stabilized.

If figure 6.5 it is given an example where the PI controller stabilize a system that is slugging from the four cases where the valve is fully opened at the controller start-up. These four cases are covered by letting the controller starts after 120 sec., 145 sec. , 170 sec. and 185 sec.

It should be noted that the PI controller used her has the modified tuning: $K_p = 0.002$, $K_i = 0.005$ and $T_i = 80$. The tuning used in last section would also stabilize the slugging, but then with very poor damping of the slugging with the result of using long time to calm down the system. However, after stabilizing of the slugging the parameter setting of last section would let the controller perform a little better than the parameter used here.

Further test shows that also in the case where the controller starts while the system is slugging, the PI controller action becomes quite good[3] for all controller sample rates ($T_{samp} = \{0.1, 0.4, 1\}$) against all valve rates of interest ($T_{valve} \in [1, 30]$), and this for the ideal case and all noise levels (*low,*

[3]All eight cases is stabilized

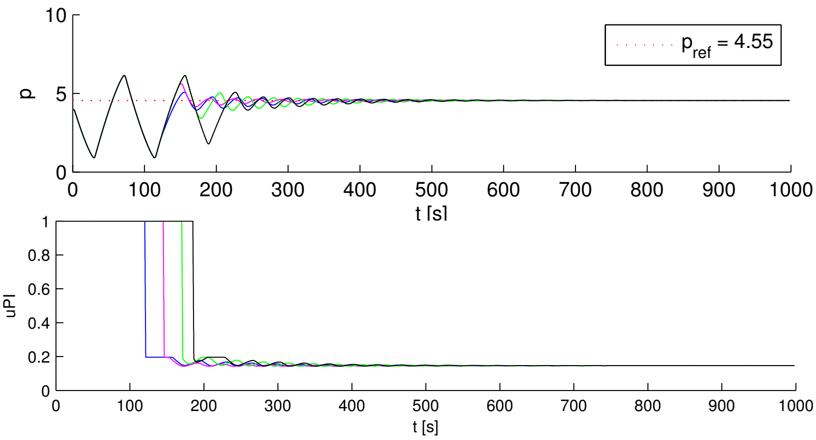

Figure 6.4: *The figure illustrate the four cases of slugging where the valve is fully opened. The fundamental simulation setup is:* $T_{samp} = 0.1$*, noise = 0 and valve time constant is 1 second.*

medium and high). Figure 6.5 illustrate the stabilization for one of the required eight cases for the worst scenario of when there are high measurement noise influence ($noise = 5\%$) and slow controller sampling time ($T_{samp} = 1$) for different maximum choke rates.

The overall conclusion about the PI controller is that the lowest pressure it can stabilize the system at is quite high, but above this level of pressure the PI controller is very robust while testing on this system.

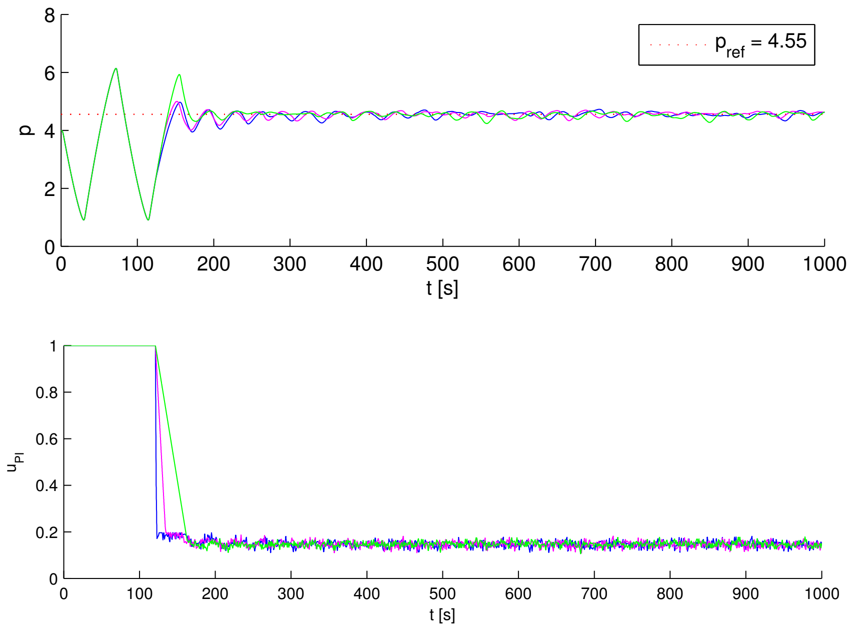

Figure 6.5: *PI controller influenced by noise on slugging system: simulations done with high measurement noise (noise = 5%), with* $T_{samp} = 1$ *and the valve rates;* $T_{valve} = 1$ *(blue line),* $T_{valve} = 10$ *(mauve line) and* $T_{valve} = 30$ *(green line)*

Chapter 7

Observer Theory

7.1 Introduction

There exists a vast literature on the design of robust state feedback controllers for uncertain dynamic systems. Typical techniques are high gain feedback control [42], Lyapunov Min-Max control [13], and Variable Structure System (VSS) with Sliding Mode Control (SMC). These control strategies assume that states of the system are available for the output feedback. However, in practical problems it is either not possible to measure all state feedback, or they are by chose not measured of technical or economic reasons. Therefore it is important to extend these techniques to output feedback. In a few special cases these techniques can be modified to produce an output feedback controller. However, in more general cases, it is necessary to use dynamic compensation to extend state feedback designs to output feedback. One forms of dynamic compensation it to use observers that asymptotically estimate the state from output measurements.

For SMC, the concern of this thesis, there exist a few studies for output feedback VSC. Some of them (like [16, 25, 61]) could manage without an observer, because they are limited to relative degree one systems. Other papers (like [10, 15]) studied systems of relative degree higher than one, but limited them self to linear systems and assumed perfect knowledge of state model. Thereby, standard observer design techniques could be used. However, when we want to deal with higher order nonlinear systems with uncertainty or unknown state models, the available observer techniques are greatly reduced. In this thesis I will consider the high-gain observer, which is a special form of the polplacement technique and the more complex arbitrary-ordered sliding mode differentiator which is based on the high order sliding mode technique - an extension of standard sliding mode. From my literature review, variations of these observer types are generally what seem to be used in combination with SMC. There are in general very few approaches that are not based on model knowledge, and might meet the very demanding requirements of fast estimation and high acceptance for noise. To other possible methods is the extended Kalman filter and the Alpha-Beta filter, which is based on Kalman filter, but I have found no literature on either of these methods in combination with SMC.

The main difficulty we are facing with all design techniques is the obvious differentiation sensitivity to input noises. Indeed small high-frequency noises can practically destroy any time-derivative. Thus practical differentiation is a trade-off between the exact differentiation and the noise rejection.

7.2 High-Gain observer[1]

High-gain observers have in the latest decades played an important part in the design of output feedback control of nonlinear system. The popularity of HGO is caused by the known robustness, and that they works for a wide class of nonlinear systems and guarantees that the output feedback controller recovers the performance of the state feedback controller when the observer gain is sufficiently high [33]. The high-gain observer is basically an approximate differentiator, and fundamental properties with high gain systems is their relationship with singularly perturbed systems[2].

Theoretical, the high-gain observers provide for exact estimation when the gains tend to infinitely. However, this is not achievable in practice as the observers experience the following four challenges and drawbacks:

1) *Noise*
2) *Sampling*
3) *The peaking effect*
4) *The separation problem*

Noise: The observer's sensitivity to small high-frequency grows infinitely with the gain. Thereby, a finite gain has to be chosen as a trade-off between exact estimation and noise rejection, resulting in an observer with finite bandwidth.

Sampling: The ratio between the gain and the sampling period of the implemented observer is critical for the performance. A too high gain will result in a too great ratio, which will destroy the fast sampling property and may lead to instability of the discretizated observer. A to low ratio will on the other hand destroy the high-gain property of the observer. This will be further discussed in subsection 7.2.2.

[1] This section is heavily based on [14].

[2] The following derivation is based on a basic understanding of both standard perturbed systems and singularly perturbed systems. For a introduction, [34] could be a good start as it describes perturbed systems, singularly perturbed systems, high-gain observers and other basic nonlinear system theory that this thesis is based on.

The peaking effect: As demonstrated in [18], the use of high-gain observers for systems of relative degree higher than one results in peaking of state variables as well as shrinking of the region of attraction. Fortunately, [18] also introduced a method[3] to deal with these undesirable properties by saturating the control outside a compact region of interest.

The separation problem: A challenge in validating observer feedback applications is the so-called separation problem. The separation principle means that a controller and an observer can be designed separately, so that the combined observer-controller output feedback preserve the main features of the controller with the full state available. In [18] it was proved, that given that the system is linear, that the separation principle is valid for a HGO when a Lipschitz continuous globally bounded state feedback controller is designed to stabilize the origin. Later, Atassi and Khalil [5, 6] proved a nonlinear separation principle for the stabilization of nonlinear systems using high-gain observers, but the condition was still that the feedback controller is Lipschitz continuous globally bounded. [40] state that his important result is true in spite of the non-exactness of high-gain observes with any fixed finite gain values. The qualitative explanation is that the output time-derivatives of all orders vanish during the smooth-feedback stabilization at equilibrium. Thus, the frequency of the signal to be differentiated also vanishes, and the differentiator provides for asymptotically exact time-derivatives.

However, the Lipschitz property of the globally bounded feedback control plays a crucial rule in the stability analysis. Since SMC is discontinuous in the state variables, it is not Lipschitz continuous. Therefore the proof that the separation principle used in [18] will stabilize the closed loop system is not valid for the SMC, or any other discontinuous feedback controllers.

Therefore another approach is used in [43, 44, 45, 47]. The approach used is to design the high-gain observer first, and then the SMC is designed as a function of the state estimates to ensure attractively of the sliding manifold. In both [43, 47], the attractively of the manifold is achieved by designing the control to cancel or dominate peaking terms. Hence the performance of the closed loop system suffers from the peaking phenomenon. This is avoided in [44, 45] by using a slightly different scheme where the globally bounded control is designed by requiring the attractively of the manifold to hold $\forall t \geq T \geq 0$ for arbitrary small T, rather than for all $t \geq 0$.

It is important to be aware of that the separation problem is first of all a problem concerning qualitative analysis and giving general guarantees for stability. For a specifically case the separation problem might not be a concern of interest as the feedback system might work fine anyway. However,

[3] The method of using globally bounded controllers to overcome the peaking phenomenon and the associated shrinking of the regions of attraction has since become a popular technique adopted by several researchers in their work. See for example [30, 33, 41, 59] and the references therein.

without solving the separation problem, nothing can be guaranteed in the general case.

7.2.1 Continuous-time design

Choose of approach

The approaches of [43, 44, 45, 47], where the separation problem is avoided by designing the HGO and the SMC '*as a hole*', by evaluate each other in the design process, is considered outside the scope of this thesis. These solutions give a very complex design, and to use the advantage of the design, and fulfil the requirements for guaranteed feedback stability, the properties of the system have to be quite good known. Preferable a quite good nominal system model should exist, and a limitation on its errors has to be known.

In this thesis the idea is to design a simple but robust SMC without a nominal model and with the relative degree of the system as the only required system knowledge. Therefore the HGO (and SMC) will be designed with no consideration of each other, even though the separation principle can not be validated. Any errors created by the separation problem may be considered as model error. The SMC in general is designed to be very robust against model errors, and the SMC of this thesis is intended to cope with no model knowledge at all. Therefore the separation problem is not likely to create any real challenges, other than that the result has to be validated through qualitative analysis, but testing that is not valid for the general case.

Evaluation system description

The HGO is really a differentiator in the sense that the observer is able to estimate the states of a chain of the first to $n-th$ time-derivative of a measurable state. If the system is on a other form where the states of interest is not directly part of such a chain, the HGO method can still be used it the states is mapped into such a chain by doing a transformation. The description of [44] or [45] gives an example of one such a mapping by transformation. Let us consider a class of singel-input-singel-output(MIMO) systems were the states needed to be observed are part of such a chain. The class of systems is represented by[4]

$$\dot{x} = A_c x + B_c \phi(x, z, u) \triangleq f(x, z, u) \tag{7.1}$$
$$\dot{z} = \psi(x, z, u) \tag{7.2}$$
$$y = C_c x \tag{7.3}$$
$$\zeta = \Theta(x, z) \tag{7.4}$$

[4]Following the steps of [34] it is straightforward to extend the results to multiple-input-multiple-output (MIMO) nonlinear systems.

In the system, $u \in \mathcal{R}$ is the control input, $y \in \mathcal{R}$ and $\psi \in \mathcal{R}^s$ are measured outputs, and $x \in \mathcal{R}^s$ and $z \in \mathcal{R}^l$ constitute the state vector. The $r \times r$ matrix A_c, the $r \times 1$ matrix B_c, and the $1 \times r$ matrix C_c, is given by

$$A_c = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$$Bc = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} C_c = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

It can be noticed from A_c that the states x are the chain of r integrators, from B_c it is clear that the input to the integrator chain is described by the nonlinear function ϕ, and from C_c that the outputs for the chain is the measurement y. In addition it can be noticed that z is a set of l additional states described by the nonlinear function ψ. Bought ϕ and ψ might depend on all states (x,z) and the input u. ζ is a set of additional measurements described by the state dependent function Θ.

Further, is *assumption 1* a necessity for the following derivation of the high-gain observer for feedback control.

Assumption 1: The functions $\phi : \mathcal{R}^r \times \mathcal{R}^l \times \mathcal{R} \to \mathcal{R}$ and $\psi : \mathcal{R}^r \times \mathcal{R}^l \times \mathcal{R} \to \mathcal{R}^l$ are locally Lipschitz in their arguments over the domain of interest. In addition, $\phi(0, 0, 0) = 0$, $\psi(0, 0, 0) = 0$ and $\Theta(0, 0) = 0$.

Partial State Feedback Control

Given the system described by (7.1)-(7.4), we desire to use feedback control to stabilize the origin of the closed loop systems using only the measured outputs y and ζ. This is to be achieved by a two-step approach. The first part is to design a partial state feedback control that uses the states x and measurements ζ. Then a high-gain observer is designed to estimate x from y. Let us define the state feedback control by the form

$$u = \gamma(x, \zeta) \tag{7.5}$$

If the *separation principle* was to be valid, the following three properties of assumption 2 would be a necessity for the state feedback design.

Assumption 2:

1) $\gamma(0,0) = 0$;
2) γ is locally Lipschitz in its arguments over the domain of interest and globally bounded in x;
3) The origin $(x = 0, z = 0)$ is an asymptotically stable equilibrium point of the closed-loop system.

As our feedback design, the SMC, is discontinuous, this requirement is not fulfilled, and the separation principle is not valid. But as has been discussed in the beginning of this subsection about Continuous-time design for HGO, we will still use the separation principle in the designing process, and it is not expected that this will create any problem for our system. But this can not bee proven. We can only justify this assumption through testing.

Observer Design

The high-gain observer of system (7.1) - (7.4) is given by

$$\dot{\hat{x}} = A_c\hat{x} + B_c\phi_0(x,z,u) + H(y - C_c\hat{x}) \tag{7.6}$$

where $\phi_0(x,z,u)$ is a nominal model of the nonlinear function $\phi(x,z,u)$. If such a nominal model is not available, we can take $\phi_0 = 0$, which results in a linear observer. Requirements for ϕ_0 is locally Lipschitz in the domain of interest, globally bounded in x, and that $\phi_0(0,0,0) = 0$. The output feedback controller is then given as

$$u = \gamma(\hat{x}, \zeta) \tag{7.7}$$

The estimation error satisfies the equation

$$\dot{e} = (A_c - HC_c)e + B_c\Delta(c,z,e,t) \tag{7.8}$$

where

$$\Delta(c,z,e,t) = \phi(x,z,u) - \phi_0(x,z,u)$$

It is shown in [5] that if the separation principle is valid, the output feedback controller will recover the performance achieved under state feedback. The observer gain H is chosen as

$$H^T = \begin{bmatrix} \frac{\alpha_1}{\epsilon} & \frac{\alpha_2}{\epsilon^2} & \cdots & \cdots & \frac{\alpha_r}{\epsilon^r} \end{bmatrix} \tag{7.9}$$

where ϵ is a small positive parameter and the positive constants α_i are chosen such that the roots of

$$s^r + \alpha_1 s^{r-1} + \cdots + \alpha_{r-1}s + \alpha_r = 0 \tag{7.10}$$

have negative real parts. By setting $\epsilon << 1$, this choice of H suppresses the disturbance Δ to an arbitrarily small value, and since the eigenvalues of $(A_c - HC_C)$ is assigned to $1/\epsilon$ times the roots of (7.10), the observer error equation (7.8) decays rapidly towards zero. The fault of the high-gain method is that for a short transient period of the form $[0, T_1(\epsilon)]$ where $T_1(\epsilon)$ tends to zero as ϵ tends to zero, it can be derived that the order of the estimation error is $O(1/\epsilon)$, and there the estimate $\hat{x}$ may exhibit an transient response with the impulse-like from $(k/\epsilon^{\delta})e^{-at/\epsilon}$ for some positive constants a and δ. If the peaking brings the state outside the region of attraction, it could destabilize the system. Fortunately, the peaking phenomenon can be overcome by saturating the control γ outside a compact region of interest of x in order to create a buffer that protects the plant form peaking.

7.2.2 Discrete-Time Implementation

For practical usage the continuous observer described above, has to be discredited for digitally implementation. For this purpose, let us consider a zero-order-hold where u is held constant in between the (uniformly spaced) sampling points of interval T. We now need to restrict the ratio T/ϵ such that $0 < r_1 < T/\epsilon < r_2 < inf$. This because the limit $T/\epsilon \to 0$ would destroy the high-gain property of the observer while the limit $T/\epsilon \to 0$ would destroy the fast sampling property and may lead to instability of the discredited observer. This restriction may be implemented by letting $T = \alpha\epsilon$ where $\alpha \in [r_1, r_2]$ may be a constant or a variable. The signals at the kth sampling point are denoted by $x(k)$, $\hat{x}(k)$, u(k), etc., and the control is implemented by

$$u(k) = \gamma(\hat{x}(k), \zeta(k)) \tag{7.11}$$

We start by scaling the observer variables to avoid inherent ill-conditioning of the realization (7.6) when ϵ is very small. Let

$$\varphi = D\hat{x} \tag{7.12}$$

where

$$D = diag \begin{bmatrix} 1 & \epsilon & \cdots & \cdots & \epsilon^{r-1} \end{bmatrix} \tag{7.13}$$

to obtain

$$\dot{\varphi} = \frac{1}{\epsilon}[A_0\varphi + H_o y + \epsilon^r B_c \phi_0(D^{-1}\varphi, \zeta, u)] \tag{7.14}$$

$$\hat{x} = D^{-1}\varphi \tag{7.15}$$

where

$$A_0 = \epsilon D(A_c - HC_c)D^{-1} = A_c - H_oC_o$$
$$H_o^T = \begin{bmatrix}\alpha_1 & \alpha_2 & \cdots & \cdots & \alpha_r\end{bmatrix}$$

The characteristic equation of A_0 is (7.10). The effect of peaking is now contained in the output equation $\hat{x} = D^{-1}\varphi$ and is overcome by the fact that $\hat{x}$ enters all equations through functions which are globally bounded in $\hat{x}$. This makes it much easier to discretizate the equation. Depending on whether or not the nominal function is zero, the HGO (7.6) can be linear or nonlinear. As we are not going to use a nominal model ϕ_0, our high-gain observer is linear, and can thereby discretizated by using many different methods. The observer is thereby implemented in discrete time by

$$q(k+1) = A_{do}q(k) + B_{do}y(k) \quad (7.16)$$
$$\hat{x} = C_{do}q(k) + D_{do}y(k) \quad (7.17)$$

where A_{do}, B_{do}, C_{do}, D_{do} depends on the discretization method used. As it in [14] is concluded that bilinear transformation method (BT), this is our method of choice, and we thereby get

$$A_{do} = (I + \frac{\alpha}{2}A_o)(I - \frac{\alpha}{2}A_o)^{-1} \triangleq N_2M_2$$
$$B_{do} = \alpha M_2 H_o$$
$$C_{do} = D^{-1}M_2$$
$$D_{do} = \frac{\alpha}{2}C_{do}H_o$$

7.3 Robust Higher-order Sliding Mode Differentiator

A differentiator is an observer that estimate the time derivatives [5] of a state; normally with a measurement of that state as input. The main difficulty is the obvious differentiation sensitivity to input noises. In particular, small high-frequency noises can be a huge problem and destroy any derivatives. Therefore, practical differentiation has to be a trade-off between the exact differentiation and the noise the noise rejection. This as one obviously can not reliably distinguish between the noise and the basic signal. The traditional approach to the problem is to assume that the noise is the high-frequency component of the signal. Numerous linear filtration methods are based on this approach [36, 46].

[5] From zero-order to a given arbitrary-order

In particular, the popular high-gain observer (HGO) presented in previous section provide for exact derivatives when the gain tends to infinity. Unfortunately, at the same time the sensitivity of the HGO to small high-frequency noises also grows infinitely. To cope with this the HGO has to set a finite gain value that gives it a finite bandwidth. This and other challenges (concerning the HOG) as *sampling*, *the peaking effect*, and *the separation problem* is discussed in section 7.2 above.

In [40, 39] by Levant, it is pointed out that there are several problems with using the HGO in combination with discontinuous controllers like SMC. For once, the SMC will loose its exactness when a HGO is use. One important aspect her is that the closer to the sliding surface the state gets, the higher gain is needed to produce a good time-derivative estimation of the chattering coordinates [9]. Another problem is that the separation principle is not valid, and therefore the stability and performance and stability of the closed loop system can not be proved and assumed in the general case.

Therefore, Levant present a so called arbitrary-order robust exact finite-time-convergent differentiator in [40] and [39]. From [38] define that a differentiator is called exact on some input if the output coincides with its time-derivative, and that it is considered robust if it can handle small input noises. A first order version of the robust exact finite-time-convergent differentiator is presented in the papers [20, 35, 38, 51, 52]. Such differentiators are based on second-order sliding modes [6] [7, 37]. Under the condition that the second order time-derivative of the unknown basic signal is bounded, the design used in [38] is proved to feature the best possible asymptotic in the presence of small measurement noise. The accuracy for the differentiator is proportional to $\varepsilon^{1/2}$, where ε is the maximal noise magnitude. By successively implementing n such differentiators the nth order differentiation accuracy of $\varepsilon^{2^{-n}}$ could be achieved. However, in [38], it is also proved that when then the Lipschitz constant [7] of the nth time-derivative unknown *'clear-of-noise'* signal is bounded by a given constant L, the best possible differentiation accuracy of the ith time-derivative is proportional to $L^{i(n+1)}\varepsilon^{(n+1-i)/(n+1)}$, $i = 0, 1, ..., n$. Based on this knowledge, [39] present an arbitrary-order design that is fitted to the desired order of differentiation. The drawback of the design is that if the controller is to operate in a large region, then L might have to be chosen quite high to provide for exact differentiation in the whole operation region. The problem with this is that when L is excessively large, the sensitivity of the differentiator to noise grows. The consequence of this is that the differentiator output contains redundant chattering when differentiating slow signals. Thereby, granting the differentiator satisfactory performance at the boundary of the operation region, causes performance

[6] Higher order sliding mode control is an approach recommended by Levant for uncertain systems of higher relative degree, but is outside the scope of this thesis.

[7] The Lipschitz constant L is given by $\|f(t,x) - f(t,y)\| \leq L\|x-y\|$, $\forall (t,x),(t,y)$ in some neighbourhood of the starting condition (t_0, x_0) [34].

degradation in the middle of the region. Therefore the controller proposed in [40] is based on a variable Lipschitz constant L(t), which make it possible to grant high performance in the whole region of interest.

7.3.1 The differentiator structure

In this thesis we choose to choose to present and use the arbitrary-order robust exact finite-time-convergent differentiator design of [40]. The final proposed design of the paper is:

$$\begin{aligned}
\dot{z}_0 &= v_0,\ v_0 = -\lambda_k L(t)^{1/(k+1)}|z_0 - f(t)|^{k/(k+1)} sign(z_0 - f(t)) + z_1 \\
\dot{z}_1 &= v_1,\ v_1 = -\lambda_{k-1} L(t)^{1/k}|z_0 - v_0|^{(k-1)/k} sign(z_0 - v_0) + z_2 \\
&\cdots \\
\dot{z}_{k-1} &= v_{k-1},\ v_{k-1} = -\lambda_1 L(t)^{1/2}|z_{k-1} - v_{k-2}|^{1/2} sign(z_{k-1} - v_{k-2}) + z_k \\
\dot{z}_k &= -\lambda_0 L(t) sign(z_k - v_{k-1})
\end{aligned} \tag{7.18}$$

where the input signal is f(t) and its kth time-derivative is z_k. Parameters $\lambda_0,\ \lambda_1, ...,\ \lambda_k$ are chosen such that the differentiator is finite-time stable [8] with $L \equiv 1$. It is assumed that the kth time-derivative has a known local time varying Lipschitz constant $L(t) \geq 0$.

7.3.2 Tuning

In [39, 40], Levant proposes the parameters $\lambda_0 = 1.1,\ \lambda_1 = 1.5,\ \lambda_2 = 2,\ \lambda_3 = 3,\ \lambda_4 = 5,\ \lambda_5 = 8,$, for all $k \leq 5$. This parameter suggestion is a good basis, but not necessary the fixed best choice, so some tuning on these parameters could be recommended. The differentiator of equation (7.18) have a recursive structure. Therefore once one have the parameters $\lambda_0,\ \lambda_1, ...,\ \lambda_{k-1}$ are chosen properly for the $(k-1)$th order differentiator with the Lipschitz constant L, only one parameter λ_k is needed to be tuned for the kth order differentiator with the same Lipschitz constant. Any $\lambda_0 > L$ can be used to start this process. This recursive structure of the differentiators should be used if one desire to tune the λ-values.

Once one have found a λ-structure that seems to be good, L(t) should be the natural tuning value in the continuation. In this thesis the natural choice should be $L(t) = L$, where L is a constant. This as the observer does not know the structure of the system. An alternative could be to let $L(t)$ depend on the *sliding variable*.

[8] There exist such functions $\delta(t) > 0$ and $T(t) > 0$ that any solution of (7.18) that satisfy conditions $|z_i(t_0) - f_0^{(i)}(t_0)| \leq \delta(t_0)$, $i = 0, 1, ..., k$, will also satisfy the equation $z_i = f_0^{(i)}(t)$, $i = 0, 1, ..., k$ for any $t \geq t_0 + T(t_0)$. This is theorem 1 in [40].

Chapter 8

Evaluation of observers - open loop

This section tests the High-Gain observer (HGO) and the Robust High-Order Sliding Mode Differentiators (RHOSMD) ability to estimate the pressure and the first to third order time-derivatives under a slugging process. For this purpose we will use the simple van Der Pol model described in chapter 4. The model will be in an open-loop state, and the choke opening will be set to fully open (100%).

As the SMC designed in chapter 5 require (good) estimates of the pressure and the 1st. to 3rd. order time-derivative. This chapter will describe a design of the observers that can estimate four states. Thereafter, the observer's ability to do accurate estimates of these states will be considered. The estimation of the 3rd. order derivative will be given the most concern, as the difficulty of the estimation (naturally) increases with the order of the derivative. Therefore for analysis of observer performance, the 3rd. and partly the 2nd. order derivatives are the factors of most interest.

The observers will be tested for the ideal case of no disturbance, and for the case of small measurement noise. The noise added will have a normal derivation of 0.5% or 30.5 mbar, as the reference is set to 6.1 bar - the maximal pressure under the slugging achieved by the model when the choke valve is fully opened.

8.1 High-Gain observer

The High-Gain observer that will be tested in this thesis is of so called full order (FOHG). This implies that the pressure is estimated, and used in the observer as a hole, even though the pressure is directly measured.

8.1.1 The design

It would be possible but in this case not of any interest to rewrite the van Der Pol model of chapter 4 to fit the nonlinear MIMO model (7.1)-(7.1), used for derivation of the HGO in chapter 7. As we are interested in the pressure and the first to third order derivative of the pressure, we have that $x^T = [x_1\ x_2\ x_3\ x_4]$. By combining a nonlinear MIMO model structure of the van Der Pol model, the HGO equation (7.6) presented in the same section as the MIMO model, and the fact that we do not desire to use a nominal model (ϕ_0); we get the following continuous HGO equation:

$$\dot{\hat{x}} = A_c\hat{x} + H(y - C_c\hat{x}) \tag{8.1}$$

where

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

and where

$$H^T = \begin{bmatrix} \frac{\alpha_1}{\epsilon} & \frac{\alpha_2}{\epsilon^2} & \frac{\alpha_3}{\epsilon^3} & \frac{\alpha_4}{\epsilon^4} \end{bmatrix} \tag{8.2}$$

.

Discrete-Time Implementation

For implementation, the High-Gain observer has to be discretizated in some form. In chapter 7 about observer theory, it was concluded that *bilinear transformation method (BT)* is the best way to implement the HGO. By following the derivation of 7.2.2, it is clear that the observer can be implemented as the system:

$$q(k+1) = A_{do}q(k) + B_{do}y(k) \tag{8.3}$$

$$\hat{x} = C_{do}q(k) + D_{do}y(k) \tag{8.4}$$

where

$$\begin{aligned} A_{do} &= (I + \frac{\alpha}{2}A_0)(I - \frac{\alpha}{2}A_0)^{-1} \triangleq N_2M_2 \\ B_{do} &= \alpha M_2 H_0 \\ C_{do} &= D^{-1}M_2 \\ D_{do} &= \frac{\alpha}{2}C_{do}H_0 \end{aligned}$$

In these matrixes α is defined by $\Delta T_o = \alpha\epsilon$, where ΔT_o is the observer sampling time. A_o and H_o is given by

$$A_o = \begin{bmatrix} -\alpha_1 & \epsilon & 0 & 0 \\ -\alpha_2 & 0 & \epsilon^2 & 0 \\ -\alpha_3 & 0 & 0 & \epsilon^3 \\ -\alpha_4 & 0 & 0 & 0 \end{bmatrix}$$

$$H_o^T = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix}$$

8.1.2 Estimation under ideal conditions

Open-loop simulation with HGO under ideal condition confirms the statement of the HGO-theory (chapter 7) that a '*low*' ϵ-value [1] is of importance for accurate estimation. The HGO-theory further states that if the rate $\frac{\Delta T_o}{\epsilon} \rightarrow \infty$ [2], the fast sampling rate property of the observer will be destroyed. This is clearly a problem when the observer sampling time is as high as 100 ms. To ensure that the observer settles within reasonable time, the ϵ-value had to be chosen to high to give optimal estimation of the 3rd. order derivative (and partly 2nd.) of the pressure. By using $\Delta T_o = 10\ ms$ or lower, estimation of all four states would be close to perfect.

For the HGO to be stable the poles of the observer feedback loop has to be negative (see equation (7.10) and the following theory). This would be archived by choosing the ϵ- and α-parameters, according to sample times as in the sets of table 8.1. The sets will place the four poles at $-\frac{1}{\epsilon}$. The reason that a higher ϵ is used for $\Delta T_o = 100\ ms$, is the $\frac{\Delta T_o}{\epsilon} \rightarrow \infty$ effect described above. Figure 8.1 illustrate the estimation performance for the pressure

ΔT_o (ms)	ϵ	α_1	α_2	α_3	α_4
100	.005	4	6	4	1
10	.0001	4	6	4	1
1	.0001	4	6	4	1

Table 8.1: *The HGO parameters for the ideal case in open loop. The sets will place the four poles of the observer feedback loop at* $-\frac{1}{\epsilon}$.

and 1st. to 3rd. order derivatives using different sampling times with the parameter sets of table 8.1. It is evident that faster sampling improves the settling time of the controller. Here, one should be aware that the reason that $\Delta T_o = 10\ ms$ and $\Delta T_o = 100\ ms$ has about the same settling time is because the significant difference in the ϵ-value. As stated in the introduction of the

[1] The ϵ-value is the main gain factor of the HGO. See general HGO theory or chapter 7.

[2] ΔT_o is the sampling time of the observer.

chapter; for analysis of observer performance, the 2nd and 3rd derivatives are the factors of interest. Therefore a more detailed picture of these is given in figure 8.2.

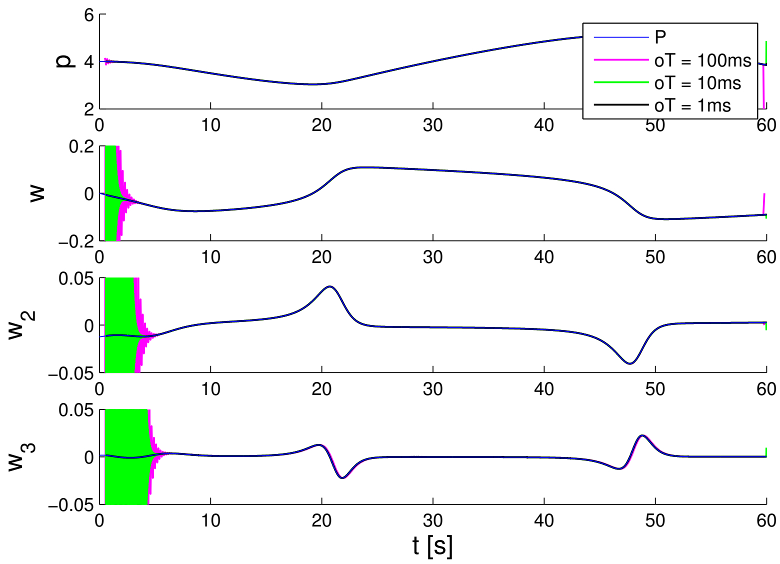

Figure 8.1: *HGO ideal performance: Estimation of the pressure and its 1st. to 3rd. order time-derivatives when* ΔT_o *is* 1, 10, *and* 100 ms. *For* $\Delta T_o =$ 100ms, *the observer uses an* ϵ*-value that is 50 times higher than for the other* ΔT_o. *rates. The real states are the blue lines.*

Considering the results presented in these figures, it would be reasonable to expect that in the ideal case of no noise, the performance of the HGO is good enough for use in SMC-stabilization.

8.1.3 Estimation in the case of measurement noise

A huge drawback for the HGO is that reducing the ϵ-value create a significant increase in the sensibility to noise. With the parameter settings used for the ideal case, even a very low measurement noise will *'drown'* the signal itself. Therefore the reduction of ϵ would have to be limited with the cost of a poorer estimation - containing a significant time delay.

As stated in the chapter introduction, the noise added will have a normal derivation of 0.5%.

Choice of HGO-parameters

During the tuning of the HGO it was found that with the presence of small measurement noise, it was impossible to achieve good estimations of the

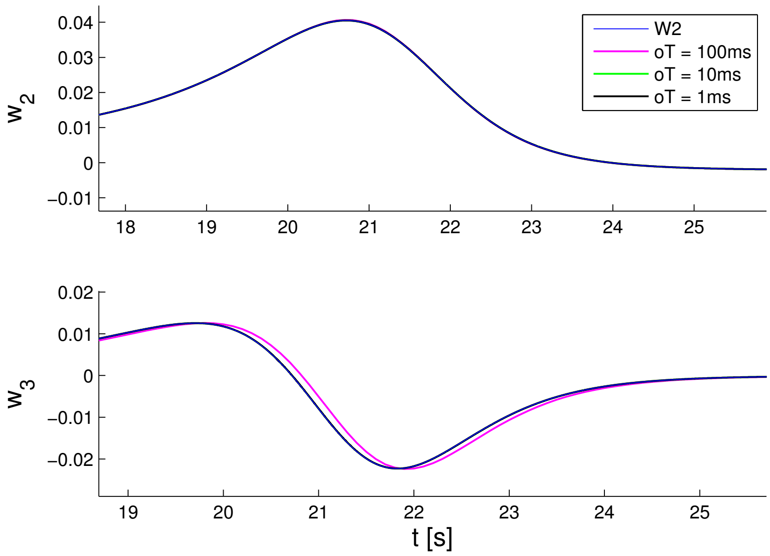

Figure 8.2: *The figure contains a focus of the 2nd. and 3rd. derivatives of figure 8.1. The real states are the blue lines.*

3rd. order derivative, and the estimation of the 2nd. order derivative was neither not too good. To clarify this, the very low observer sampling time of 0.1 ms is used for the following simulations concerning the discussion of setting HGO-parameters. After this discussion, the effect of using high (more realistic achievable) sampling time will be evaluated.

While working with the tuning of the HGO it became clear that just tuning the ϵ-parameter was not good enough. The α-parameters also had to be tuned to create a significant difference in the magnitude of the poles of the observer feedback loop. The likely reason for this is the need to suppress the high derivatives of the noise without doing the estimation of all states to slow. The α-parameters of *set 3* in table 8.2 was found to be a pretty good choice for noise of magnitude 0.5%.

Tuning notes: The following list gives a tuning description I found to be convenient to use:

1. Use the α-parameters set above.

2. Find the ϵ-parameter that give the best trade-off between correct estimation (low time delay), and noise reduction for the 3rd. order derivative.

3. Tune the fourth pole to fit the amplitude of the estimation of 3rd. order derivative. α_4 can be chosen instead for the fourth pole, as it is

simpler and works approximately as good as tuning the pole).

4. Look at the 2nd. order derivative . Consider to tune the third pole (or just α_3).

Illustrating parameter effects: The 3 parameter-sets of table 8.2 will be used to demonstrate that ϵ-value (because of the noise) has to be chosen so high that the estimation of the 3rd.(and partly 2nd.) order derivative is significant time delayed, and in general not good. The sets also illustrate the effect of tuning the fourth pole. The results are shown in figure 8.4. As

Set	ϵ	α_1	α_2	α_3	α_4	p_1	p_2	p_3	p_4
1	.040	1.3	.33	.028	.0008	-1	.15	-.1	-.05
2	.040	1.25	.27	.016	.000045	-1	-.15	-.1	-.003
3	.090	1.3	.33	.028	.0008	-1	.15	-.1	-.05

Table 8.2: *HGO parameters used for illustrating tuning during noise in the open loop.* p_1 - p_4 *is the four corresponding poles. Multiplied with* $\frac{1}{\epsilon}$, *these are the poles of the observer feedback loop.*

the ϵ-value of *set* 1 is relative high, but the noise infliction is still significant, it is clear that the ϵ-value has to be chosen quite high to suppress the noise. At the point where the noise suppression is good enough, the estimation is considerable time delayed, and the magnitude and *'form'* of the estimation is neither too good. *Set* 2 is added to illustrate that the noise (of course) can be suppressed through increasing the lowest pole with the α-parameters (instead of through ϵ), but that the result will be much poorer as this destroyed the magnitude and form even more. Figure 8.4, shows a similar results for estimations of the 2nd. order derivative, but the HGO estimation is (not surprisingly) a lot better.

HGO sampling time

The HGO-parameters for *set* 3 of table 8.2 is the best found parameters found for 0.5% noise, when the sampling time is as low as 0.1 ms. However, this sampling time is likely way to low to be implemented in a real system. If the sampling time is increased, the noise present in the estimation of the 3rd. order derivative (and the other states), is significant. This is illustrated in figure 8.5, by using the parameters of set 3 for the ΔT_o rates 0.1, 1, and 10 ms.

Further testing show that the noise can be suppressed by increasing the ϵ-parameter, and do a corresponding tuning of the smallest pole through the α-parameters. But this at a price of an even greater time delay for the estimation. A possible solution to suppress the noise for the higher sampling

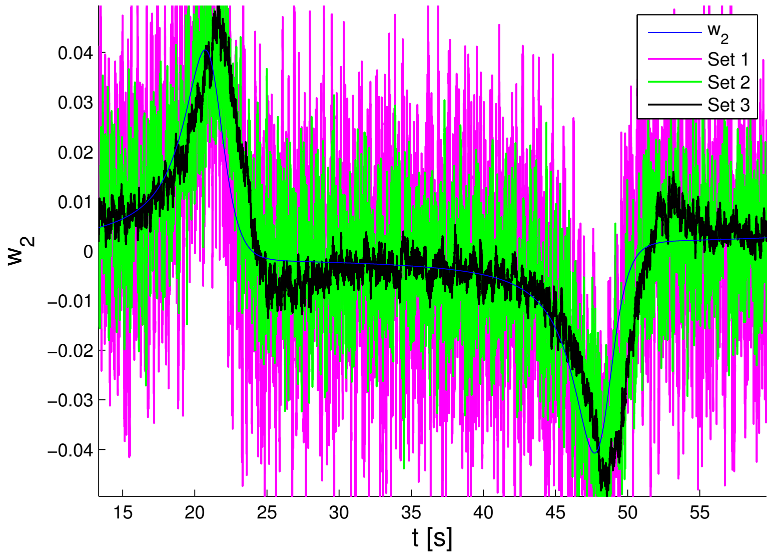

Figure 8.3: *Tuning illustration: Tracking of 2nd. order time-derivative with different* ϵ *(*$\Delta T_o = 0.1ms$*).*

times would be to increase theϵ-value further, but this with the critical cost of an even more significant time delay.

Overview

With the presence of low measurement noise of 0.5%, the best found HGO-parameters are given by *set 3* in table 8.2. The HGO performance of estimating the pressure, and 1st. to 3rd. order derivatives, with the low $\Delta T_o = 0.1\ ms$, is illustrated in figure 8.6.

Conclusion

From the project thesis [48], it is known that for a SMC to be able to stabilize the pressure, the SMC has to meet very demanding requirements about *fast* and *correct* response to changes in both the pressure, and the 1st. to 3rd. order derivatives (see A for a derivation). It is thereby considerable reason to fear that in the presence of noise disturbances (as there will be in a real case), the SMC can not stabilize the pressure, if the described HGO is used for estimation. This as an ϵ-value that will make the observer fast enough, also will add to much noise to the estimations of the higher order derivatives. The vanity of this assumption will be checked in chapter 9 about observer performance in a closed loop (active SMC).

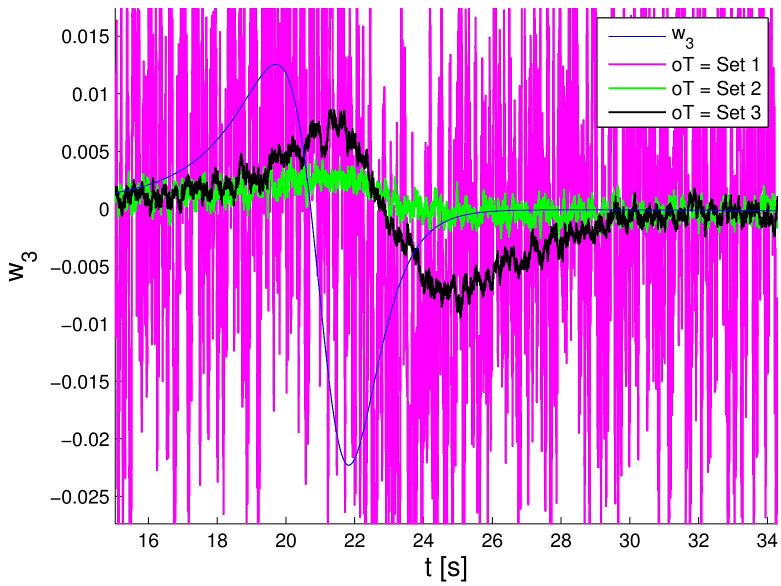

Figure 8.4: *Tuning illustration: Tracking of 3rd. order derivative with different* ϵ *(*$\Delta T_o = 0.1ms$*).*

8.2 Robust Higher-order Sliding Mode Differentiator

As we desire to estimate the bottom hole pressure and the 1st. to 3rd. order time-derivatives, the robust exact finite-time-convergent high-order sliding mode differentiator used in this thesis will be of 3th-order. The design is based directly of article [40] by Levant, and is briefly discussed in chapter 7 of this thesis.

8.2.1 The design

Using the presented theory, the RHOSMD of 3th-order will have the design:

$$\begin{aligned}
\dot{z}_0 &= v_0,\ v_0 = -\lambda_3 L(t)^{1/4}|z_0 - f(t)|^{3/4} sign(z_0 - f(t)) + z_1 \\
\dot{z}_1 &= v_1,\ v_1 = -\lambda_2 L(t)^{1/3}|z_0 - v_0|^{(2)/3} sign(z_0 - v_0) + z_2 \\
\dot{z}_2 &= v_2,\ v_2 = -\lambda_1 L(t)^{1/2}|z_2 - v_2|^{1/2} sign(z_2 - v_1) + z_3 \\
\dot{z}_3 &= -\lambda_0 L(t) sign(z_3 - v_2)
\end{aligned} \tag{8.5}$$

where the input signal is f(t) and parameters $\lambda_0,\ \lambda_1, ...,\ \lambda_k$ are chosen such that the differentiator is finite-time stable with $L \equiv 1$. Further, is it assumed

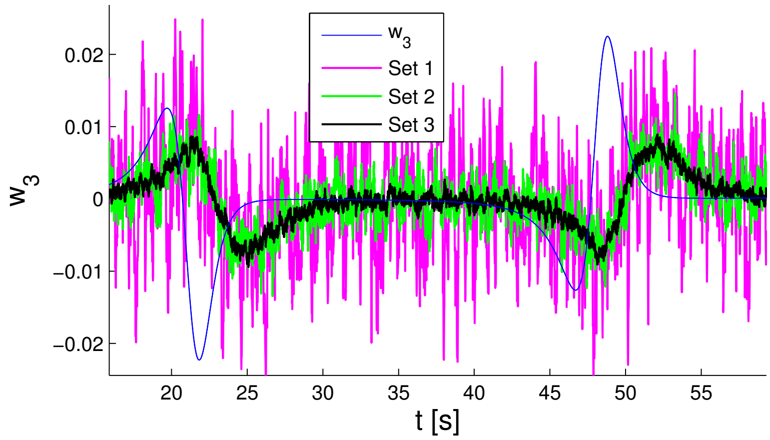

Figure 8.5: *Sampling illustration: Tracking of 3rd. order time-derivative during different observer sampling time, but equal parameters.* ($\Delta T_o = \{0.1, 1, 10\}$ *ms*)

that the kth derivative has a known local time varying Lipschitz constant $L(t) \geq 0$.

The parameter suggestion $\lambda_0 = 1.1,\ \lambda_1 = 1.5,\ \lambda_2 = 2,\ \lambda_3 = 3$ given by Levant in [39, 40], will be considered as a guideline, but starting out with this suggestion the parameters will also be tuned through the recursive procedure described in the observer theory chapter. When a tuning is considered good or optimal through several case tests where $L \equiv 1$ and the observer sampling time is very low, this parameter setting will be viewed on as the optimal of the system of this thesis, and fixed.

Further tuning for the different simulation-cases presented in this thesis will be through L(t). As we do not have system knowledge, we will set $L(t) = L$, where L is a constant.

8.2.2 Estimation under ideal conditions

The λ-parameters was open-loop tuned[3] with the low sampling time $\Delta T_o = 0.1\ ms$ and with $L \equiv 1$. The conclusion was a parameter set slightly different from that suggested by Levant. Thereafter simulations for slower observer sampling times were carried out. Here, it was concluded that L should be reduced considerable because of chattering. The table below lists the suggested and used parameter sets. The first set is for $\Delta T_o = 0.1\ ms$, and the second for when ΔT_o is 1, 10 and 100 ms. It can be noted that independent of the sampling time, the chosen L of *set* 2 was the lowest possible choice possible without creating very poor estimation. The existence of such a

[3]Through the ideal case of no noise and through the case of 0.5% noise.

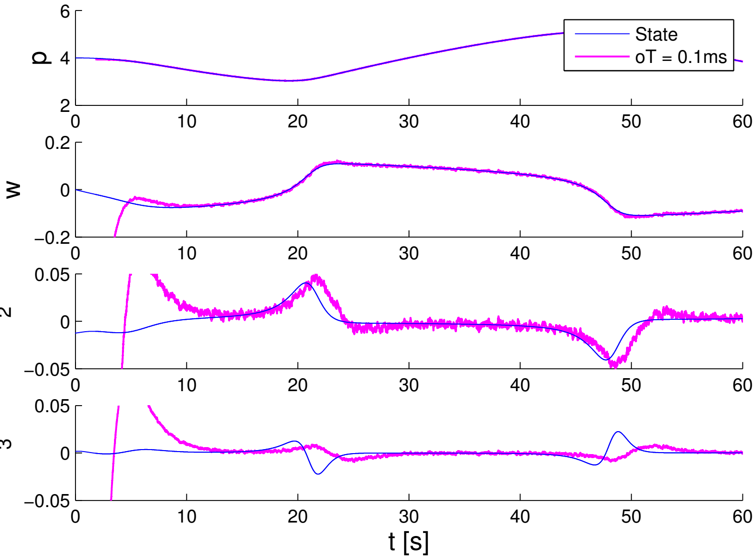

Figure 8.6: *HGO noise performance: Estimation of the pressure and its 1st. to 3rd. order derivatives during the present of 0.5% measurement noise for* $\Delta T_o = 0.1ms$.

lower limit for L coincides with the theory (chapter 7). A drawback with the suggested low L-value is a large settling time. Another factor is that SMC might generate faster changes in the states, and therefore demand a faster observer then what is required her. As a result the L-value would likely have to be higher under control.

Set	L	λ_0	λ_1	λ_2	λ_3
1	1	1.1	2	3	5
2	.03	1.1	2	3	5

Table 8.3: *The RHOSMD-parameters for the ideal case in open loop. The sets will place the four poles of the observer feedback loop at* $-\frac{1}{\epsilon}$.

In figure 8.7 the estimation performance for the pressure and the 1st. to 3rd. order time-derivatives for $\Delta T_o = 10\ ms$ and $\Delta T_o = 100\ ms$ is illustrate. A more detailed picture of the 2nd. and 3rd. order time-derivatives are given in figure 8.8. From the figures it is clear that the sampling time $\Delta T_o = 100\ ms$ give an approximate, but not good estimation of the 3rd. order derivative. However, when ΔT_o is 10 ms, a very good estimation is given of all the states. Using a lower sampling time would than this will reduce the amplitude of the estimation error considerably, but the estimation should be good enough for $\Delta T_o = 10\ ms$. This is quite logical as the differentiator is chattering because of its nonlinear structure. From the results of the simulations, it is clear that the RHOSMD perform quite well under ideal condition, but not as well as the HGO. It can not however be concluded that the HGO is a better choice than the RHOSMD, as it is the performance under the presence of measurement noise that is of importance. This will be considered in the following subsection.

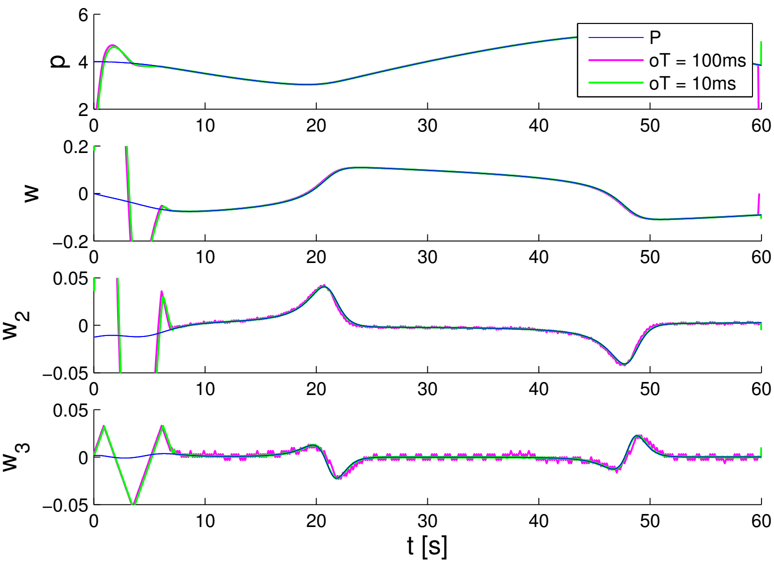

Figure 8.7: *RHOSMD performance in ideal case: Estimations of the pressure and its 1st. to 3rd. order time-derivatives when ΔT_o is 10 and 100 ms. The blue lines are the real states.*

8.2.3 Estimation in the case of measurement noise

During the simulations it became (logical) evident that a quite high L-value gave the estimate lower time delay, but with the cost of increasing the sensibility to measurement noise. Therefore this subsection will start with a discussion on the choice of L when ΔT_o is 0.1 ms - ergo when we can archive the best possible performance. The subsection will then continue by looking on the best possible performance when the differentiator has higher sampling time. At the end, the performance of the RHOSMD under the influence of measurement noise will be compared with the performance of the HGO. This will be done for both the very fast $\Delta T_o = 0.1\ ms$ and for the more realistic $\Delta T_o = 10\ ms$.

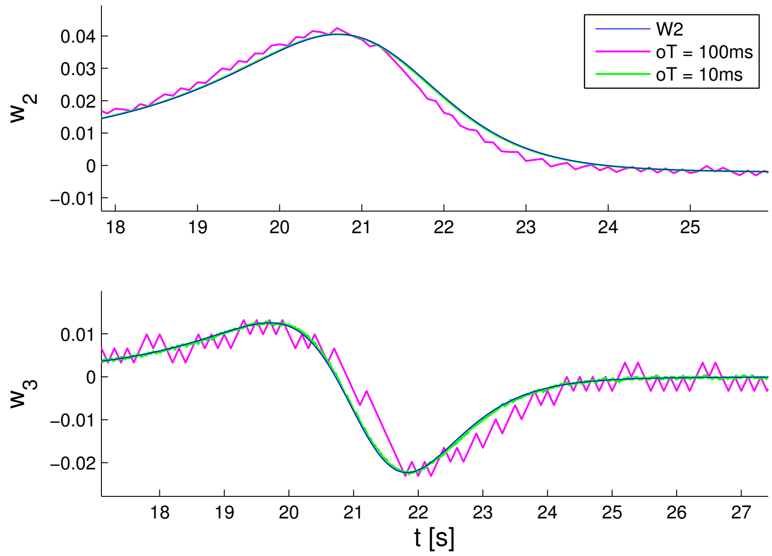

Figure 8.8: *Focus of 2nd. and 3rd. order time-derivatives of figure 8.7. The blue lines are the 'real states'.*

Best possible performance

The λ-parameter sets suggested by Levant, and the set suggested be me, has been tested against other possible parameter sets for the case of 0.5% noise under several different observer sampling times. The conclusion was that the set suggested by me proved to be the best parameter combination during limited measurement noise – independent of the sampling times . The figures 8.9 and 8.10 gives the open loop estimation result of this parameter set with the three Lipschitz constants L is 0.1, 0.2, and .5 when ΔT_o is 0.1 ms. From these figures (and other simulations), I conclude that $L = 0.2$ is the best choice when ΔT_o is 0.1 ms. Using $L = 0.5$ would slightly reduce the estimation time delay, but I consider the cost of increased noise-sensibility to great. With $L = 0.2$, the noise infliction is of an acceptable magnitude, not making it worth reducing L further.

Compare sampling time

Figure 8.11 and 8.12 compare the RHOSMD performance of $\Delta T_o = 0.1$ ms, $\Delta T_o = 1$ ms and $\Delta T_o = 10$ ms. When considering the best performance for each ΔT_o, I do the same evaluations as above, and my conclusion is that the *'magnitude'* and *'form'* of the estimations is not to bad, but the time delay

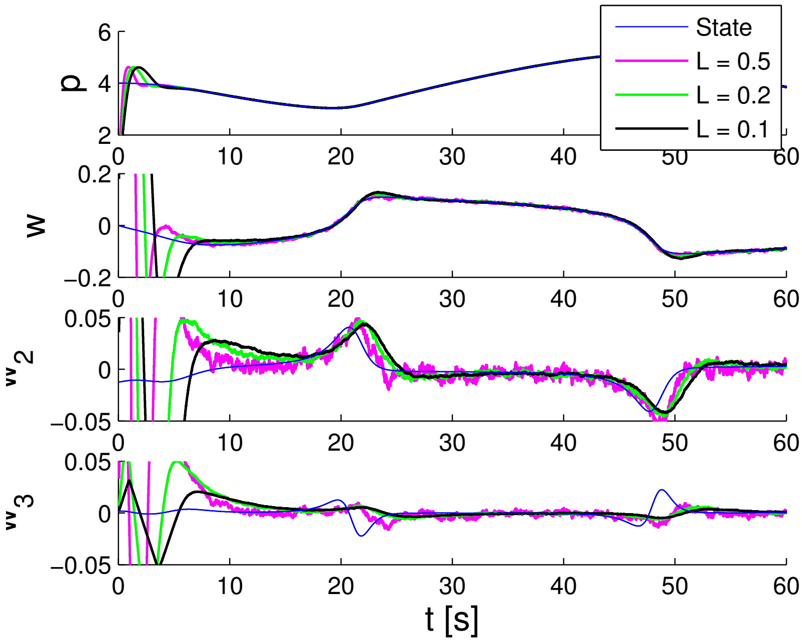

Figure 8.9: *RHOSMD performance with fast ΔT_o: Estimations of the pressure and its 1st. to 3rd. order time-derivatives during* 0.5 *ms measurement noise when ΔT_o is* 0.1 *ms, and L is* 0.1, .2 *and* .5.

is considerable. Especially when $\Delta T_o = 0.1\ ms$ is as high as $10\ ms$[4], but I fear that the results for the faster sampling times is neither good enough to be used successfully together with the SMC.

Compare with HGO

For the ideal case the HGO proved to perform better then the RHOSMD. The more interesting question is how they perform compeered with each other in the presence of measurement noise. In figure 8.13 the observers are compeered in the measurement noise case for the very fast observer sampling rate 0.1 ms, and figure 8.14 for the still quite fast, but possible more realistic achievable sample rate 10 ms. What is clear from the figures that the performance is (very surprisingly) equal. Maybe the RHOSMD is slightly better, but this can not be clearly concluded, and the difference is insignificant.
In the presented simulations the parameters of set 3 in table 8.2 was used for the HGO. For the RHOSMD $L = .2$ was used for $\Delta T_o = 0.1\ ms$, and $L = .005$ was used for $\Delta T_o = 10\ ms$. The λ-parameters was chosen as earlier described.

[4] In the conclusion chapter I will argue that this is the fastest realistic implemental ΔT_o.

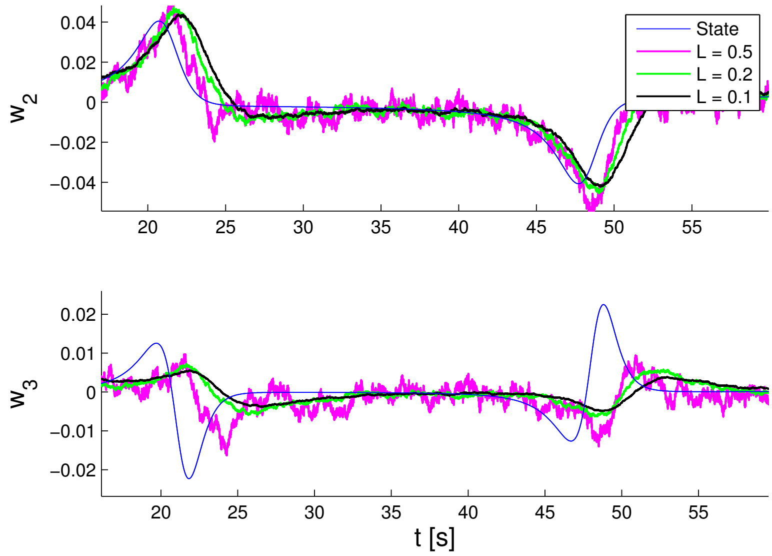

Figure 8.10: *The figure contains a focus of the 2nd. and 3rd. order time-derivatives of figure 8.9.*

Conclusion

From the simulations above to HGO and the RHOSMD seems to have the same performance. However it can not for certain be concluded that this is the case for a closed loop SMC controlled system. In addition, this chapter indicates that under the influence of noise, neither of the observers will meet the performance demands for the SMC to stabilize the pressure. But from the viewpoint of this thesis, there are no good (or clear) alternative observers. Therefore, to see if not maybe at lest one of the observer might beat the assumption from this chapter, it becomes important to check the performance of both observer methods in the closed loop SMC controlled system. This will be the task of the next chapter.

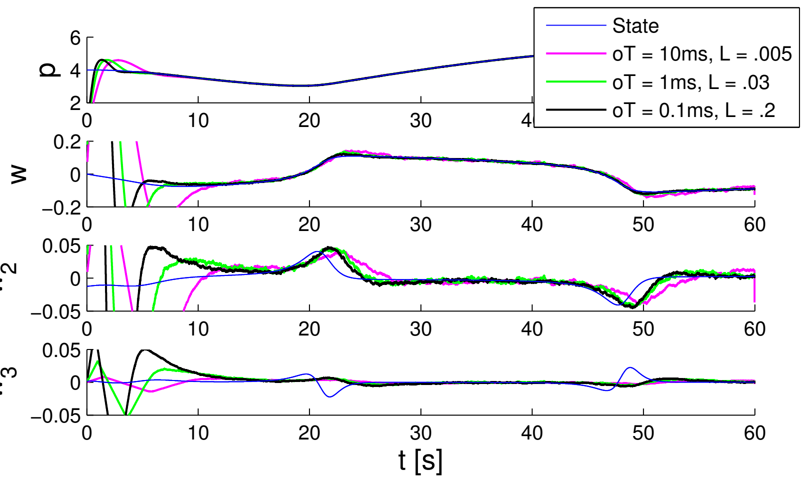

Figure 8.11: *ΔT_o influence on RHOSMD performance for measurement noise case in open-loop: Estimation of pressure and its 1st. to 3rd. order time-derivatives during* 0.5 *ms noise for* $\Delta T_o = \{.1, 1, 10\}$ *ms with different and fitted L.*

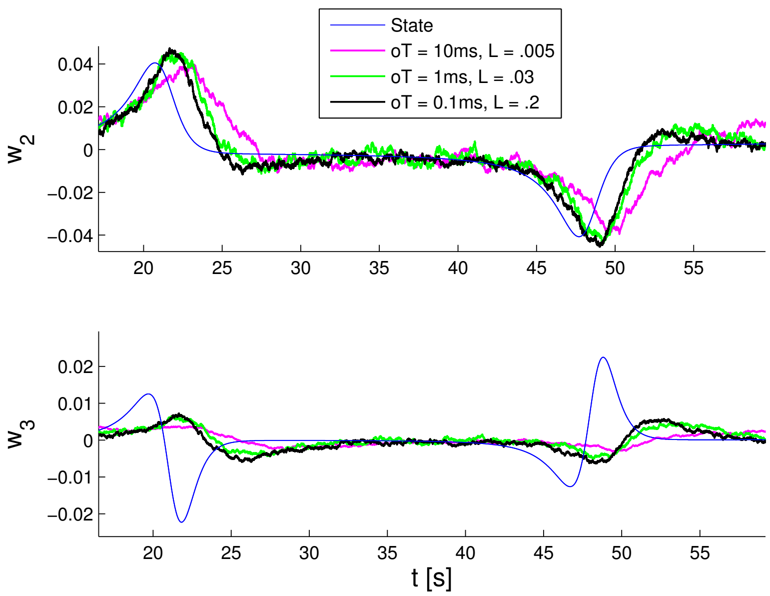

Figure 8.12: *Focus of the 2nd. and 3rd. time-derivatives of figure 8.9.*

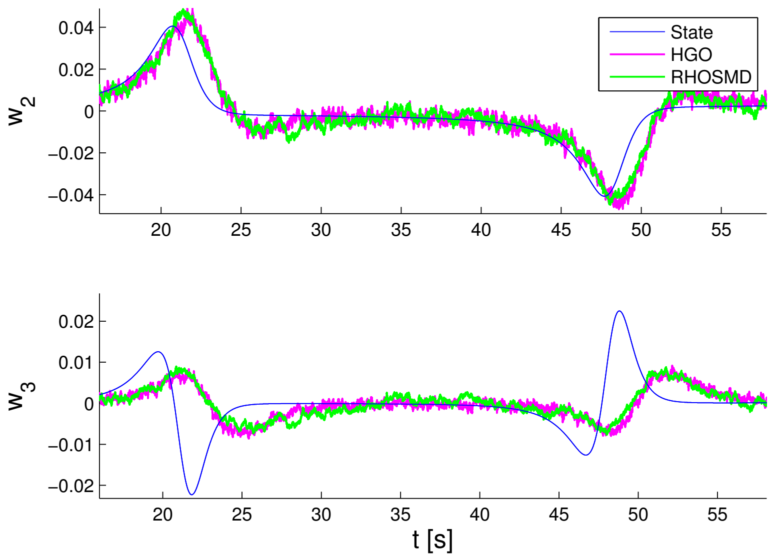

Figure 8.13: *The figure compare the HGO and RHOSMD when the sampling time is* 0.1 *ms, through a focus of the 2nd. and 3rd. order time-derivatives.*

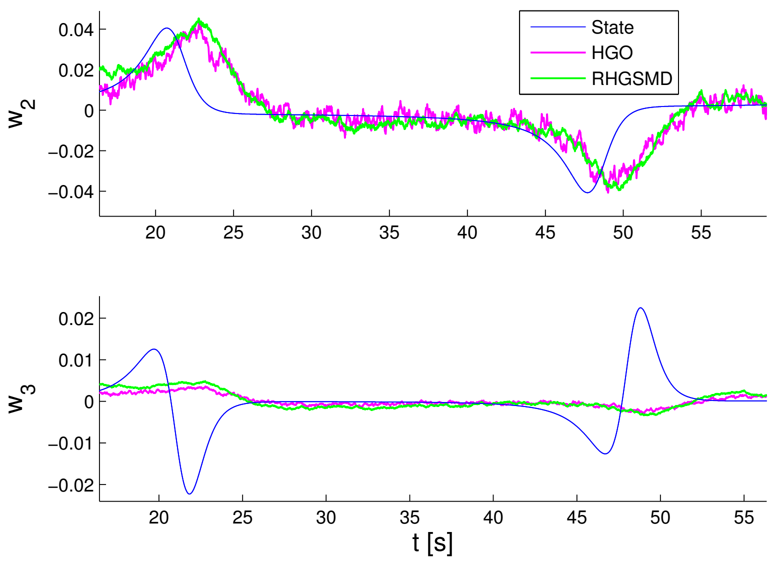

Figure 8.14: *The figure compare the HGO and RHOSMD when the sampling time is* 10 *ms, through a focus of the 2nd. and 3rd. order time-derivatives.*

Chapter 9

Evaluation of observers - closed loop

In the previous chapter, the performance of the High-Gain observer and the Robust High-Order Sliding Mode Differentiators was evaluated in an Open-Loop slugging system based on a simple van Der Pol based model. In this chapter the performance evaluation of the HGO and the RHOSMD will be followed up by closing the system-loop through use of the Sliding Mode Controller introduced in chapter 3, and described in chapter 5. The SMC-parameter setup used in this chapter will in all presented cases stabilize the pressure (at approximate the desired reference pressure) if the *'real' states*[1] are known. The HGO and RHOSMD used is introduced in chapter 7, and described in the previous chapter.

The observers will be tested for the ideal case of no disturbance, and for the case of small measurement noise. The observers will evaluated after how well the SMC perform when they are used. The performance will be compared with that of the PI. The noise added will have a normal derivation of 0.5% or 30.5 mbar.

This section is about testing the performance of the HGO and RHOSMD as observers for SMC and not the performance of the SMC itself. Therefore the sampling time of the SMC will by $100ms$ during all simulations. This as the project thesis [48] considered $100ms$ as a fast and well function for SMC.

It can also be noted that the van Der Pol system parameters used are given in 4.8.

9.1 High-Gain observer

The HGO that will be tested in this thesis is of so called full order (FOHGO). This implies that the pressure is estimated, and used in the observer as a

[1] Pressure and 1st. to 3rd. order time-derivatives

hole, even though the pressure is directly measured. The design of the HGO is described in previous chapter.

9.1.1 Estimation under ideal conditions

For the van Der Pol based model, the project thesis [48] concluded that given the exact knowledge of the desired states under ideal conditions[2], SMC-stabilization was possible all the way down to $p_{ref} = 3.5225$ - the theoretical limitation of pressure stabilization for the given system model[3]. Simulations show that by employing HGO with $\Delta T_o = 100\ ms$, and by use the first parameter sets[4] of table 9.1, stabilization can be done down to $p_{ref} = 3.6$. In ΔT_o is 10 ms or lower, the stabilization is achievable even down to $p_{ref} = 3.53$, with the use of parameter set 2. Figure 9.1 illustrate

Set	ϵ	α_1	α_2	α_3	α_4
1	.05	4	6	4	1
2	.001	4	6	4	1

Table 9.1: *HGO parameters for SMC in ideal case.*

the HGO estimation (and real states), of the pressure, and the 1st. to 3rd. order time-derivatives during SMC-stabilization for the p_{ref}-limits described above. $\Delta T_o = 10\ ms$ is used, as it is considered fast but realistic achievable, as will be discussed in the conclusions chapter. The parameters of the SMC used under these simulations is given in table 9.2, where λ is the tuning parameter of the SMC, and T_{valve} is the *choke rate constant*[5]. Figure 9.2 show that the HOG estimate of the 2nd. and 3rd. order time-derivative is very good for $\Delta T_o = 10ms$ or lower(even through fast oscillations) in the ideal case.

λ	T_{valve}
.3	10

Table 9.2: *The SMC parameters used in this chapter.*

Note 1: *In the simulations the discussion above is based on, the choke is 10% opened when the controller starts (system is in stable state). Further test show that if the controller is designed such that it with the knowledge of the 'real' states can stabilize the system (pressure) when started from a*

[2]No measurement noise or other types of disturbance.

[3]With the van Der Pol model parameters described in subsec:parameters.

[4]For information about why these sets are chosen and used as they are, see 8.1.2.

[5]The *choke rate constant* is in this thesis defined as the (minimum) time in seconds the choke uses to change its opening 60%.

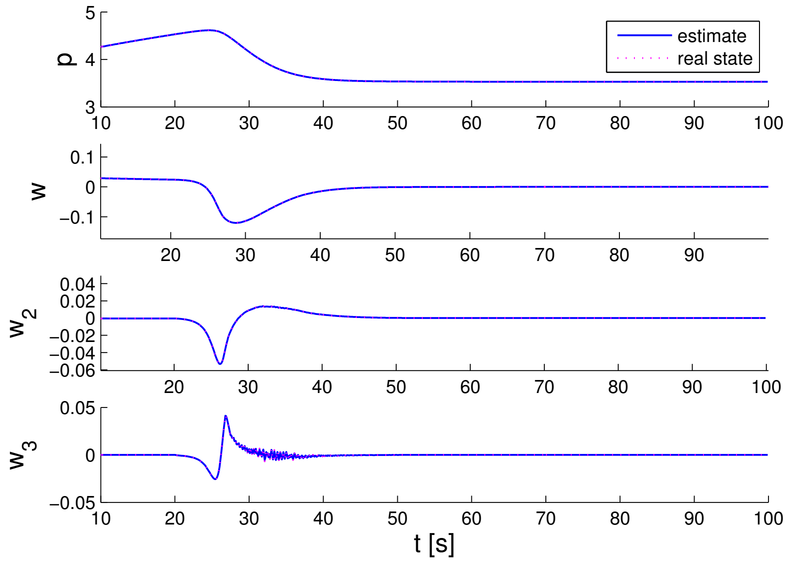

Figure 9.1: *SMC-stabilization with the use of HGO: Estimations and real states for the pressure and the 1st. to 3rd. order time-derivatives when the observer sampling rate is 10ms.*

slugging system; then, in ideal case, the SMC will also be able to stabilize the system with the use of HGO.

Note 2: *It was in the project thesis shown that when the controller was started from a slugging system, the lower pressure limit for SMC-stabilization (directly) would be marginally high then for the case of starting from a stable system. This can of course be fixed by doing the stabilization to the desired pressure in two or more operations (as also described in the project thesis).*

Note 3: *For the controller to be able to stabilize from 'slugging modus', the upper chock limit (z_{max}), has to be fitted to the desired reference pressure (u_{ref}). Ideally, z_{max} should be slightly higher then u_{ref}* [6]*. For the van Der Pol system model used here, we have that $u_{ref} = \sqrt{(\frac{b_1 K_q}{p_{ref}-b_0})}$. The reason that it is necessary to set z_{max} not to high above u_{ref} is thoroughly discussed in the project thesis (in the chapter about qualitative analysis of the systems controllability). Combining figure 9.3 and 9.4 can give a partly understanding of why it necessary to fit z_{max}, but for a good understanding, analysis of u/p plots and phase plots is required. Simplified one can say that the controller is required to (gradually) reduce the dynamical oscillations of the system.*

[6] u_{ref} is the opening the choke has to be stabilized to (or around) in order to archive stabilization at p_{ref}.

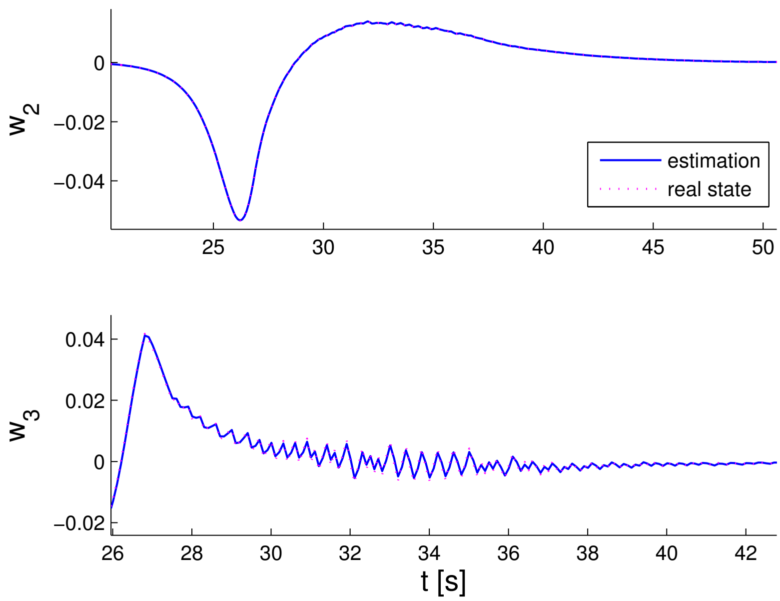

Figure 9.2: *Tracking of the 2nd. and 3rd. order time-derivatives for simulation of figure 9.1.*

Something that is not possible if the choke is opening too much. The reason is that the SMC then will not get access to the 'braking properties' of the system fast enough when required.

9.1.2 Estimation in the case of measurement noise

As discussed in the previous chapter, a huge problem is that reducing the ϵ-value will create a significant increase in the sensibility to noise. With the parameter setting used for the ideal case, even a very low measurement noise will *'drown'* the time-derivative signals. Therefore the ϵ-value would have to be limited with the cost of a poorer estimation, containing a considerable time delay. The conclusion of the previous chapter was that the best possible solution would be a compromise where the estimation of the 3rd. (and partly the 2nd.) order time-derivative is significantly time delayed, and the magnitude and *'form'* is not close to correct. Further, as the SMC demand very *fast* and *correct* response to changes in both the pressure, and the 1st. to 3rd. order time-derivatives, it is not likely that a satisfactory SMC-stabilization is possible with use of the described HGO under the presence of measurement noise. The aim of this subsection is to investigate this further through testing actual SMC by use of HGO estimation.

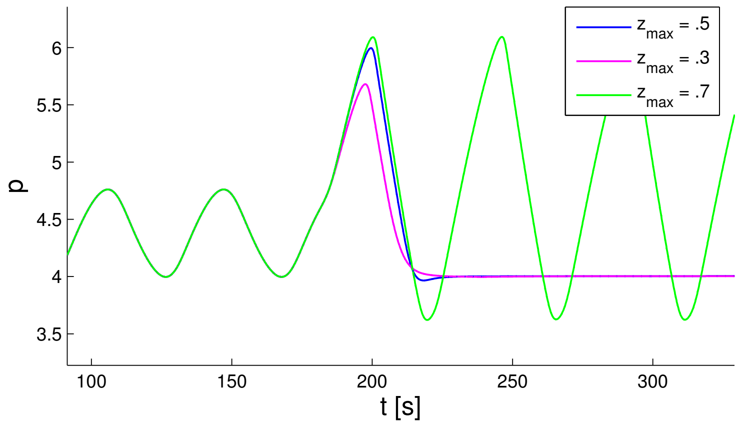

Figure 9.3: *SMC-stabilization of the pressure (with the use of HGO), for* $p_{ref} = 4$*, when the system is in a slugging mode when controller start. The only difference between the three simulations is different* z_{max}*-values.*

Simulations

Figure 9.5 illustrate the best result found through simulations. Here $p_{ref} = 4.3$, and the performance of the SMC is nothing else than poor. The figure also shows that the effects of reducing the observer sampling rate below 10 ms are very limited. Figure 9.6 show the estimate of the 3rd. order time-derivative. From the figure, and from the knowledge of the project thesis that the SMC-stabilization is quite time critical, it is reasonable to conclude that the main problem is the time delay. The observer parameters used for all observers sampling rate are given by table 9.3. where $\frac{p_1}{\epsilon}$ - $\frac{p_4}{\epsilon}$ is the four

ϵ	α_1	α_2	α_3	α_4	p_1	p_2	p_3	p_4
.090	1.3	.33	.028	.0008	-1	.15	-.1	-.05

Table 9.3: *Noise fitted HGO parameters for SMC-stabilization*

poles of the observer feedback loop.

Evaluation

Chapter 6 shows that for this van Der Pol system, the PI controller can calm (good stabilization) the pressure around 4.55 *bar*, even with the presence of high measurement noise. This would clearly be preferred before the results illustrated in figure 9.5. Actually, as discussed in chapter 6, the PI controller performance on the van Der Pol model is not very impressive itself as a stabilization down toward 4.5 *bar* could be archived by manual choking.

Further simulations reviled that increasing the measurement noise from

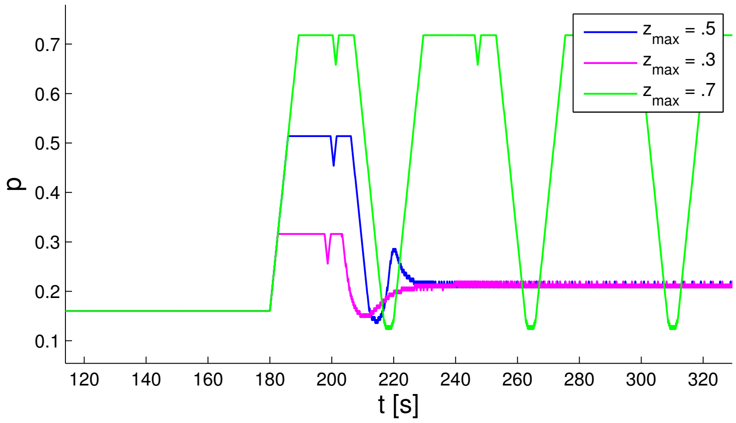

Figure 9.4: *Choke opening during SMC-stabilization (with the use of HGO), for* $p_{ref} = 4$*, in the case where the system is in a slugging mode when controller start. The only difference between the three simulations is different* z_{max}*-values. The figure illustrates the result of the same simulations as figure 9.3.*

0.5% to 5% did not have dramatically consequences on the performance of the SMC. From this, the question if the HGO perform an estimation that is so poor (considering the requirement of the SMC) that it would aquatically be better to receive no estimation at all (simply zero). Testing this reviled that this actually was the case (illustrated in figure 9.7). However, estimation of the 2nd. order time-derivative is a lot better then no estimations. It can be noted that in the simulation of figure 9.7, $\Delta T_o = 0.1\ ms$, but the result is the same for slower sampling times.

The conclusion is thereby that, at least for the van Der Pol model, the designed HGO can not be used as an observer for the designed SMC. As there is validity of the van Der Pol model is not known, it can not be concluded with certainty from this thesis, that the SMC can not archive acceptable performance on the OLGA model and on a real system. However, the problems describe in this thesis, and the fact that I did not succeed in achieving SMC-stabilization on the OLGA model (even in the case of no measurement noise), indicates that it at least not is a trivial case.

9.2 Robust Higher-order Sliding Mode Differentiator

The RHOSMD tested in this section will be of 3th-order. The design is described in previous chapter about open loop estimation, introduced in chapter 7, and based directly on article [40] by Levant.

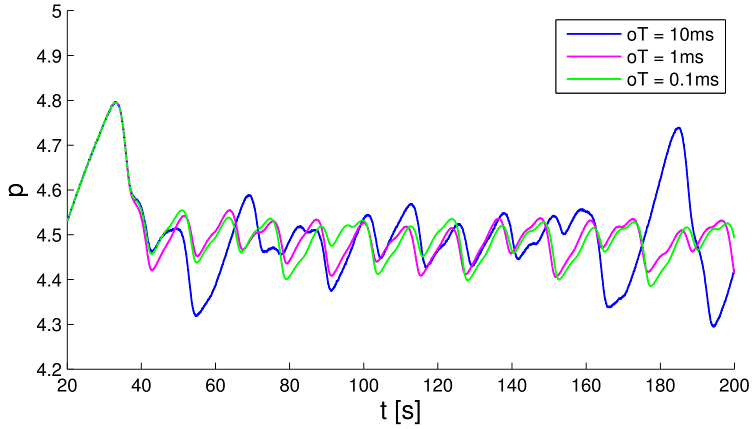

Figure 9.5: *SMC-stabilization of the pressure when* $p_{ref} = 4.3$ *and with use of HGO at the presence of 0.5% measurement noise.*

9.2.1 Estimation under ideal conditions

In previous section, it was stated that given the ideal condition of no noise disturbance, the performance of the HGO was considered to be very good as it was possible to stabilize the pressure at a very low value, not much above the levels possible with exact knowledge of the desired states. If $\Delta T_o = 100\ ms$, the lowest possible SMC-stabilization is $p_{ref} = 3.6$. For $\Delta T_o = 10\ ms$ and lower, the stabilization was achievable even down to $p_{ref} = 3.53$.

Simulations indicate that the performance of RHOSMD is not as good under ideal conditions as the performance of the HGO. For $\Delta T_o = 100\ ms$ the best performance is as high as $p_{ref} = 4.1$. When reducing ΔT_o to $10\ ms$, the best performance was improved to $p_{ref} = 3.7$. By using the quite low $\Delta T_o = 1\ ms$ the achievable performance was $p_{ref} = 3.58$. The RHOSMD-estimation (and real states) for the pressure, and the 1st. to 3rd. order time-derivatives for the best performance for these sample times is given in figure 9.8. The differentiator parameter that was found to be best for all ΔT_o is given in table 9.4. It can be noted that for $\Delta T_o = 1\ ms$, $L = 1$ would give a very accurate estimation of the 3rd. order time-derivative, while $L = .3$ gives a slower and not that accurate estimation, but is still an overall better choice as the pressure stabilization is smoother. The reason is that $L = 1$ creates greater chattering when the pressure (and thereby also the 3rd. order time-derivative) is stabilized. This is illustrated in figure 9.9. The parameters of the SMC used under these simulations are given by table 9.2.

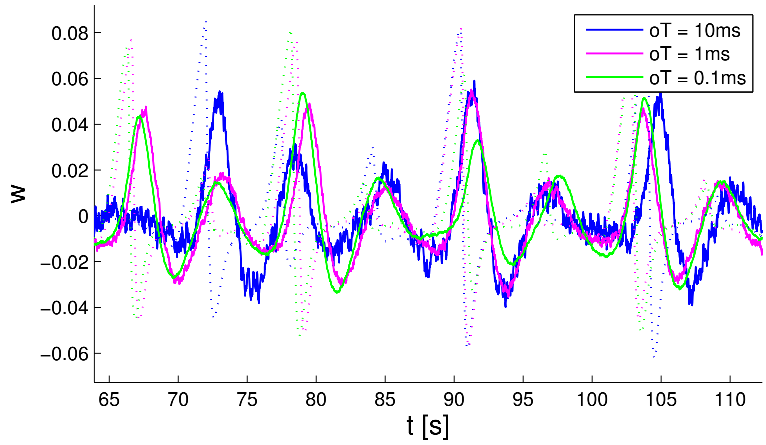

Figure 9.6: *Tracking of the 3rd. order time-derivative during the SMC-stabilization of figure 9.5. The line is the estimate, and the dotted line is the real state.*

L	λ_0	λ_1	λ_2	λ_3
.3	1.1	2	3	5

Table 9.4: *Ideal case RHOSMD parameters for SMC.*

Note 4: *I should in addition be noticed that as for the HGO, the discussion above is based on simulations where the choke is 10 % opened when the controller starts (system is in a stable modus). As for the HGO, if the controller is designed such that it can stabilize the system (pressure), it will also be able to do so with the use of RHOSMD in the ideal case. A requirement for this is that the SMC is designed in such a way that the maximum choke opening* (z_{max}) *is not to large, and fitted to the desired pressure* (p_{ref})*. The reason why this is necessary is discussed in the section about HGO.*

9.2.2 Estimation in the case of measurement noise

The best performance for RHOSMD is found by choosing $p_{ref} = 4.2$, and illustrated in figure 9.10. If compare the performance with that of the HGO by looking at figure 9.5, we see that in the case of measurement noise, the SMC actually perform a little better with the RHOSMD than the HGO. Unfortunately, the indication of last section, that neither of the observers estimate the 3rd. order time-derivative (and likely nor the 2nd.) good enough for SMC-stabilization of the pressure, seems to be correct. The performance is simply not good. Indeed, when ΔT_o is 0.1 ms, the oscillation is to a certain degree limited, and the pressure is generally below the pressure stabilization achievable by the PI controller or by simply manual chocking. However,

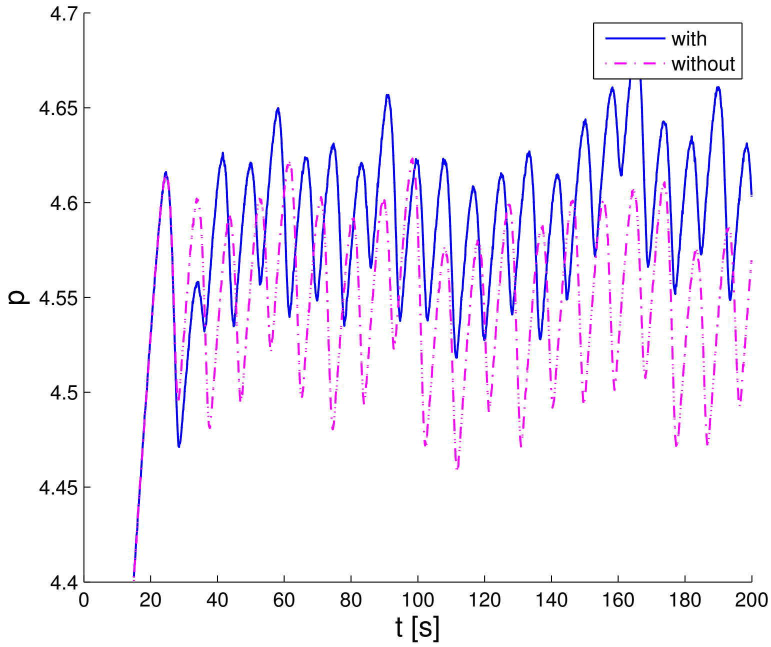

Figure 9.7: *Compare with HGO based SMC-stabilization with normal estimation, with the case of estimating the 3rd. order time-derivative as zero.* $\Delta T_o = 0.1$ *ms and* $p_{ref} = 4.3$.

the fact that there is quite significant e oscillations combined with the very small gain, make this result overall poor. Anyway, 0.1 *ms* is a very fast ΔT_o, that is likely not close to implemental. Figure 9.11 show the estimate of the 3rd. order time-derivative. From this figure, and earlier discussions that the SMC-stabilization is quite time critical, it is reasonable to conclude that the main problem is the time delay. In the simulations my suggested λ-parameters was used, and the used L values are listed in figure 9.11.

As for the HGO, testing reviles that increasing the measurement noise from 0.5% to 5% did not have dramatically consequences on the performance of the SMC, and the performance of the RHOSMD is actually so poor (considering the requirement of the SMC) that it would aquatically be better to receive no estimation at all (simply zero). However, the estimation of the 2nd. order time-derivative is a lot better then no estimations.

Conclusion

The conclusion is thereby as for the HGO that, at least for the van Der Pol model, the designed RHOSMD can not be used as an observer for the designed SMC. The same reservations in the conclusion as for the HGO have

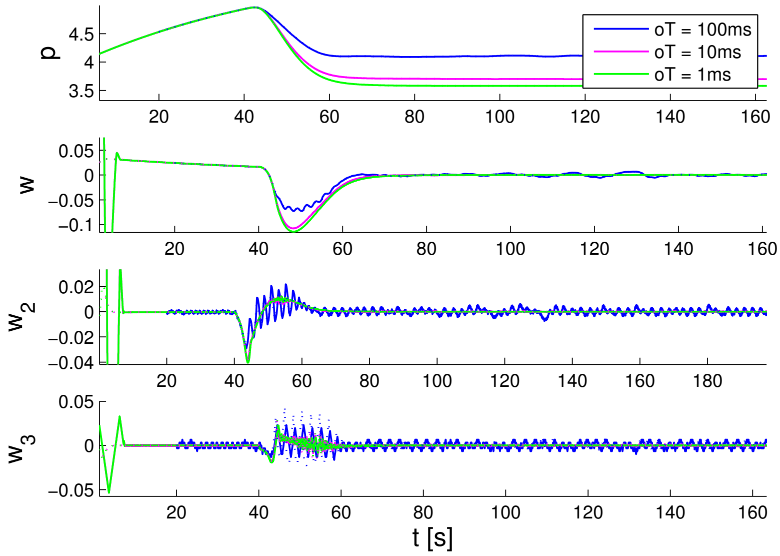

Figure 9.8: *SMC-stabilization with the use of RHOSMD. The figure illustrate the RHOSMD-estimations (and the real states) for the pressure and the 1st. to 3rd. order time-derivatives when ΔT_o is 1, 10 and 100 ms.*

to be taken.

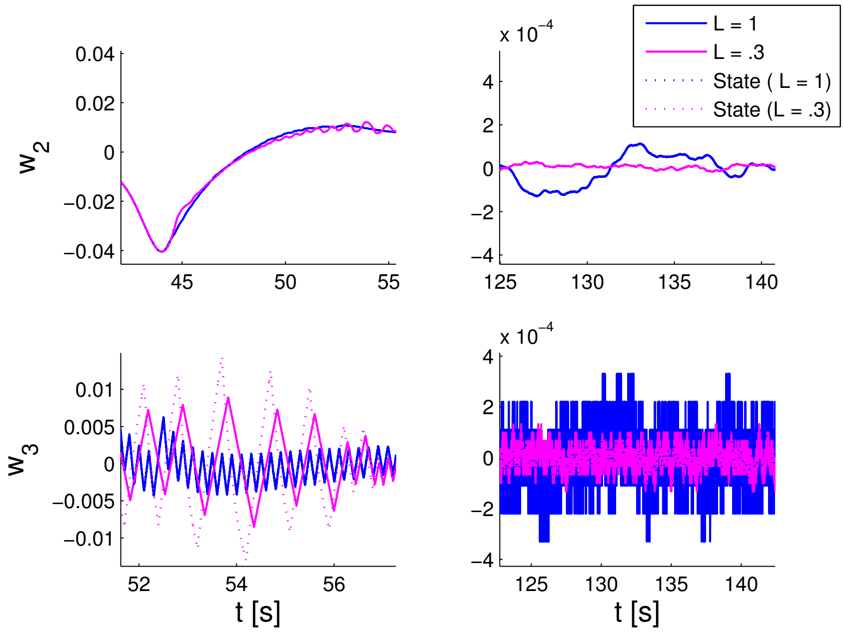

Figure 9.9: *Effect of different L values on RHOSMD in ideal case.* $\Delta T_o = 1$.

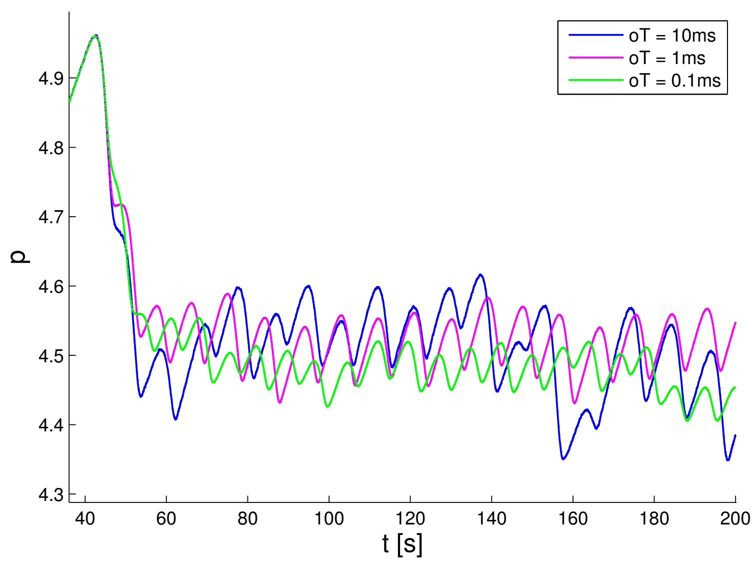

Figure 9.10: *RHOSMD based SMC-stabilization of the pressure for* $p_{ref} = 4.3$ *in case of 0.5% measurement noise.*

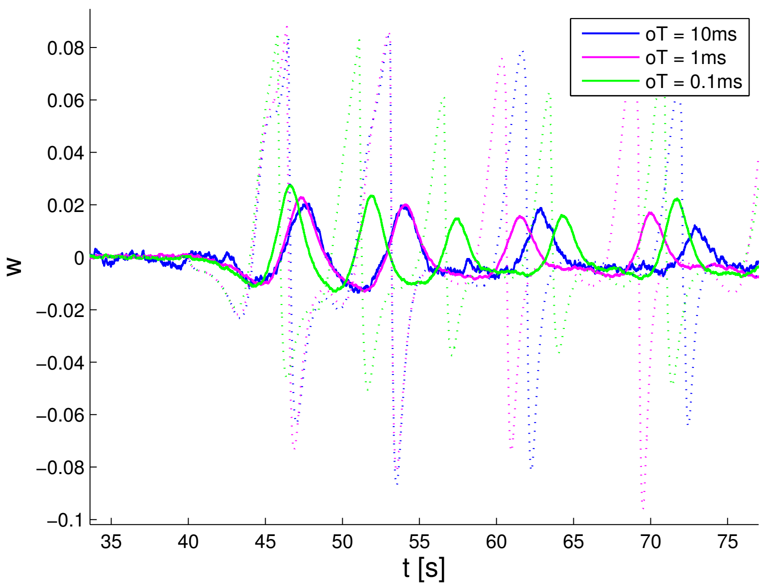

Figure 9.11: Tracking of the 3rd. order time-derivative during SMC-stabilization ($p_{ref} = 4.3$) using RHOSMD at the presence of 0.5% measurement noise. $L = .1$ is chosen for $\Delta T_o = 10\ ms$,$L = .3$ for $\Delta T_o = 1\ ms$ and $L = 1$ for $\Delta T_o = 0.1\ ms$. The solid lines are the estimates, and the dotted lines are the real states.

Chapter 10

Conclusions

In my former project thesis [48], a sliding mode controller (SMC) was designed and state-feedback tested on the van Der Pol model with good results. The objective of this master thesis was to implement a more high fidelity simulation model (OLGA), and design and test different types of differentiators in order for more valuable output-feedback testing of the SMC. The possibility and effect of modifying the SMC from controlling the choke rate to controlling the choke acceleration should also be evaluated.

However, as I did not succeed[1] in performing SMC-stabilization on the implemented OLGA model, the focus of the thesis has been changed to test how well the design differentiators was fitted to the task of performing sufficient good high order time-derivatives estimations required by the designed SMC. The tests were performed on the van Der Pol model.

In this chapter, the aim is to briefly draw a main conclusion of the discussions done in the previous chapters. There is also a short discussion regarding CPU and system requirements for the observers and the controller. At the very end, further work is discussed. The reader is referred to the chapter of interest for further discussion and conclusions.

10.1 Main conclusion

To design an implemental model independent output-feedback SMC for stabilization of riser slugging, would be an achievement of great importance. In conclusion section 10.2 it is argued that a SMC that might stand for the challenge is designed, but unfortunate the testing indicates that it is impossible to design a observer that will meet the very demanding requirements of this SMC (regarding estimation of higher order time-derivatives). Further, in 10.3 it is concluded that neither a SMC with direct choke control

[1]Possible reasons for the lack of results and the chosen focus of the thesis is presented in the introduction chapter.

nor a SMC designed for controlling the choke acceleration might solve this problem.

10.2 The potential and problem of the SMC

Model avoidance: An implemental controller for stabilization of riser slugging that could operate without model knowledge would be of great importance as it has proved very difficult to create a good controller model for this purpose. Further complexity involved with using a model is the fact that the operation conditions is varying from fields to fields, pipe to pipe and is also continous changing within each pipe.

The potential: The sliding mode control strategy is recognized as an efficient tool to design robust controllers under uncertain conditions. What is very interesting and tempting about the SMC in the case of slugging, is that it actually is able to perform quite impressive without model knowledge, as long as the required states[2] itself is known. Medium measurements disturbance added to each state is allowed (see [48]), but the provided estimation signals must not be significantly time-delayed (see chapter 9). Another great advantage with the SMC is the simplicity, and the quite intuitive understandable structure.

Tough requirements: In my project thesis [48] there where performed a qualitative analysis of the controllability of the van Der Pol based system model (as this analysis is important for, and often used in the argumentation of this thesis, it is added in appendix A). The analysis proved very demanding requirements for a controller to be able to stabilize the pressure in the unstable system area. The controller would need to act at an early stage. If not, or if it does an over-reaction or under-reaction, it would soon be physical impossible for the controller to avoid slugging.

The disadvantage: There is a quite true saying that nothing is without a cost. For the designed SMC the cost of not using a model is that the 1st. to 3rd. order time-derivatives would have to be predicted. In the qualitative analysis of controllability it was illustrated that SMC would have to meet very demanding requirements about *fast* and *correct* response to changes in the pressure and all these time-derivative states. Thereby the time-derivative states would have to be estimated quite accurate – with little room for time delay.

[2] For the derived SMC: The pressure and its 1st. to 3rd. order time-derivatives.

The problem: Chapter 8 and chapter 9 indicate that given the realistic case of measurement noise, neither the HGO nor the RHOSMD can provide estimates of the high order time-derivatives with the performance required by the SMC. The main problem is the time-delay caused by the necessary noise filtering implemented in the observers' properties.

Based on the literature review, I doubt that there are any public well known observers that can perform any better than the HGO or RHOSMD in this case, even though the extended Kalman filter or an extension of the Alpha-Beta filter could be considered. There are in general very few approaches that are not based on model knowledge that might meet the very demanding requirements of fast estimation and high acceptance for noise. Variations of the HGO and RHOSMD observer types are, with very few exceptions, the used observers in combination with SMC.

The inference The inference is thereby that, at least for the van Der Pol model, the designed HGO can not be used as an observer for the designed SMC. As the validity of the van Der Pol model is not known, it can not be concluded with certainty from this thesis, that the SMC can not archive acceptable performance on the OLGA model or a real system. However, the problems described in this thesis, and the fact that I did not succeed in achieving SMC-stabilization on the OLGA model indicates that it at least is not a trivial case.

10.3 Choice of implementation scheme for the SMC

The design scheme chosen in this thesis where SMC is controlling the choke rate is derived in chapter 5. Further in this section it is discussed why neither a SMC controlling the choke directly nor a SMC controlling the acceleration of the choke might solve the problems with output-feedback.

10.3.1 Direct choke control SMC

The main reason why not a output-feedback choke rate SMC is not archiving satisfying performance regarding stabilization, is the poor estimation (in the SMC point of view) of the 3rd. order time-derivative of the pressure. Therefore it might seem tempting to derive and test out a SMC that control the choke opening, and thereby would only require estimation of the 1st. and 2nd. order time-derivatives.

Such a design is derived and considered in the project thesis [48]. By using the qualitative analysis of controllability (see appendix A), it was pointed out that it would be impossible for such a design to work without extensive model knowledge. Anyway, the design was also tested with negative result. The problem is that when the system is in a controllable mode, the controller would not know which chock openings applies positive power and opposite. This as the zero-power limit changes with the state of the system. This is a property that with good certainty could be concluded to be valid also for a real slugging system. Therefore to use a SMC design with direct choke control, extensive model knowledge would be required, and then much of the advantage of the SMC potential would be gone.

10.3.2 The chock acceleration SMC

In the project description of this thesis, I was asked to compare the chosen design with a design where the SMC controls the chock acceleration. In the SMC point of view this might prove to be a great idea. Especially in the more realistic case where choke dynamics like stiction and acceleration limitation is taken into account.

However, this would require estimation of the 4th. order time-derivative, a more demanding task than estimating the 3rd. order time-derivative. As too poor estimation of that state is the main reason why satisfying output-feedback choke rate SMC was not archived at the presence of measurement noise, a chock acceleration output-feedback design would simply not succeed. The estimation of the 3rd. order time-derivative was actually so poor that it was better to estimate it as zero.

Therefore I concluded that with the knowledge of this observer limitation, it was of no interest to design and test a SMC that control the chock acceleration.

10.4 CPU and system requirements

10.4.1 For the SMC

In the testing of the thesis, controller sampling time (ΔT_c) was chosen to be 0.1 second. This was in the project thesis considered as a fast and well functioning sampling time, and therefore chosen as the performance as the state-feedback was well known[3] In the project $\Delta T_c = 1\ s$ was considered the realistic implemental today, and $\Delta T_c = 0.4\ s$ a sample rate that might be achievable with some improvements. What limits the implemental sampling time today is how often the choke valve mechanics can handle changes in the rate.

The maximum chock rate used during testing in this thesis was $T_{valve} = 10$ (the choke uses 10 second to open or close 60%). In the project thesis

[3]To test the other (ΔT_c is not of importance as the observers and not the SMC was the main focus of the tests in this master thesis. The results of the higher ΔT_c was not as great, but still quite good (considering the state-feedback), if the measurement noise disturbance was not more than medium.

this was considered as a realistic maximum rate, and the tests indicates that the SMC might perform close to optimal with this maximum choke rate.

It should be added that the simulation of these two thesis is fitted the StatoilHydro test ring in Porsgrunn. The slugging dynamics of a real riser system is much slower, and therefore the requirement to the ΔT_c might also be lower.

10.4.2 For the observers

In the Matlab simulation on my computer, the calculation of each HGO iteration take about 0.15 to 0.3 ms while the RHOSMD iteration take about 0.5 to 1.5 ms. This indicates that if the calculation was the limitation, the real time estimation rate could with ease (with a more efficient program code on a more efficient real time system) by 1 ms or mayby even $0.1ms$. However, in this case, the main limitation factor will be the actual measurement of the bottom hole pressure, combined with the transfer of the measurement data. Therefore I would consider 10 ms the (hopeful) realistic achievable observer rate, and I doubt that this is possible in today's systems. However, as for the ΔT_c the requirement to the ΔT_o might be lower in a real riser system as the slugging dynamic is much slower then for this system that is intended for the StatoilHydro test ring in Porsgrunn.

10.5 Further work

It can not be concluded with certainty from this thesis that the SMC design of controlling the coke rate can not be used for riser slugging stabilization. However, the tests that indicate that the 3rd. order time-derivative can not be estimated well enough for satisfactory SMC, should at least arise questions about if the design is suited for the purpose. It should maybe also call for a review about if there might be other approaches that should be looked further into before the SMC (without a model knowledge).

If it is desirable to continue the work with the SMC, the focus should be on the OLGA model, or another high fidelity model for riser slugging. The problems I experienced should be met by very systematic analysis, and a better method then Euler for predicting the real time-derivative states. If it is desirable to look into other methods for estimation, the extended Kalman filter and an extension of the Alpha-Beta filter should be considered.

Appendix A

Qualitative analysis of system controllability

This appendix will by focusing on the sliding mode controller give a qualitative analysis of how the slugging system, described in the chapter 4 can be controlled in the unstable region. It should be noted that the section is poorly theoretical, and is concerned with the requirements for controlling the model and not the real life slug. However, as the model can be roughly fitted to have the same oscillation frequency and bifurcation plot as a real life slugging system, it is reasonable to assume that the main quantitative behaviors of a sluggish system is pretty well described by the model, and thereby it is reasonable to assume that the behavior outlined in this section also will be representative for the real life system.

The description of how the system must be controlled is divided into two parts; *Locating stabilizing state* and *Local stabilizing.* The discussion on local stabilization will be presented first. This as the qualitative analysis done here will ease and improve the understanding of the subsection about locating the stabilizing point. The subsection about location the stabilization point describes how the SMC is able to handle the states of the system in such a way, that the valve opening that place the unstable equilibrium point is indirectly found during the process of driving the states to their desired levels (In the continuation I will define this valve opening as the desired valve opening z_{ref}). The first subsection, on '*local*' stabilizing is about how the controller can achieve to maintain the state levels very close to their desired values, even though the system is highly unstable at the operating valve opening.

A.1 Local stabilizing

How the controller can stabilize the system locally, can be outlined through the figure A.1. In the first plot of the figure, the state is what I will define as the *effective valve opening* (z_{eff}) and the pressure. The figure illustrate the line of equilibrium points for different valve levels, and how the controller should operate to move the state against the point where the equilibrium point is placed at the desired pressure p_{ref}. I choose to introduce the effective valve opening z_{eff}, as there is a time-lag from the valve to the actually system. As described in section 4.5, this time-lag is modeled by a first order transfer equation, and the effect of the valve on the system is represented through the state q. Thereby, I define z_{eff} as the valve opening that in steady case would give the current state of q.

The second plot of figure A.1 is concerned with the qualitative properties of the system around the equilibrium point of a given z_{eff}, and the *states* are the pressure difference (p_{diff}[1]) from the equilibrium point, and valve rate (w). From the theory of qualitative analysis it is known that the equilibrium point is the point where the nullcline of the state variables intersect [4]. The nullcline of a states variable is the curve where the rate of the variable is zero. By studying the phase diagram of the system for different unstable valve opening, it can be seen that the structure of the system around any equilibrium point is equal, as the nullcline of the pressure difference is always along the x-axis, while the slope of the nullcline for w is roughly the same and by definition goes through the origin since the origin is defined as the equilibrium point. Future, above this nullcline w will increase, under this line w will drop, and as shown at the figure, the nullcline forms a separator line close by. By this equality we can use the same diagram even though z_{eff} change. This as the change of the equilibrium point can be represented as a change in the pressure difference.

If no control is perform the pressure will rapidly increase from a state above the separator line, while rapidly drop from a state below the separator line. Therefore, to control the system, a controller must be able to drive the state from one side of the separator line to the other. Future, the controller should use the nullcline to control the chance in the pressure rate (w_2). This as close to the nullcline the change of w will be low, while further away it will be grater. The following enumerated outlining gives a principal description of how these quantitative properties can be used for local stabilizing.

1. During local stabilizing, if z_{eff} is z_{ref}, the pressure is slightly higher then p_{ref} and w is above zero, this might correspond with state point **A** in figure A.1. Since the state of **A** is above the separator line, w, and thereby the pressure itself will form this point rapidly increase. The only way to avoid this is to move the state to the other side of the separator line. This can be achieved if the controller increase the p_{diff} faster then system would do by itself, which can be done by reducing p_{eq} through increasing z_{eff} and thereby moving the state on the figures from point **A** to point **B**.

[1] $p_{diff} = p - p_{eq}$

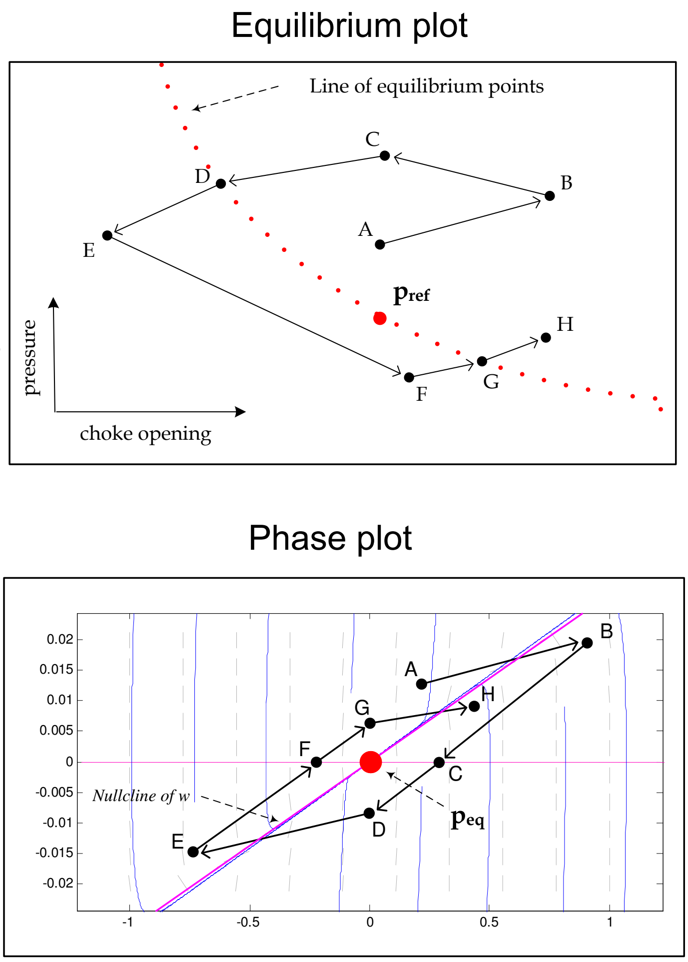

Figure A.1: *The sketch illustrate the u/p plot and the phase plot of required control action.*

2. Now the rate should be reduced to zero in a controlled manner by reducing z_{eff} in such a way that states stay close to the nullcline while moving from point **B** to point **C**.

3. At this point, the increasing of the pressure is stopped, but the nullcline should still be followed until the equilibrium line is crossed in point **D**.

4. At point **D** $p = p_{eq}$. However $p > p_{ref}$ and $w < 0$. Therefore the controller should ensure that the nullcline is followed in negative direction for a while longer before it is crossed in point **E**. From point **E**, the nullcline should be followed in positive direction until $w = 0$ in point **F**.

5. From point **F** it is again time to cross the nullcline by moving toward point **H** (through point **G** where $p = p_{eq}$).

6. Anyhow, if point **G** is more advantageous then point A, the controller is for sure working properly. Point **G** can now be considered as point **A**, and the procedure can be repeated.

One exemple where the SMC perfectly and another where it acts quite well is given through figures A.3 to A.5. These figures are created from actually simulations of how the SMC preform the local stabilization. The simulation starts locally, this to easier compare the result to the principal outline done in the other to figures. However, is the simulation is started for far away, the locally performance will (of course) perform based on the same principles, and look similar.

A.1.1 Why the SMC manage the local stabilization

The sliding variable consist of a weighting between the factors: p,w,w_2 and w_3 (pressure and the first to third order derivative). Thereby the effects of the virtual controller α is directly detected through $\dot{q}$ and q (without the delay of an integrator) as these states has an effect on w_3,w_2. This makes it possible for the SMC to act fast and accurate enough. More about this can be found in chapter 3 about sliding mode. The following list describe roughly how the SMC is able to do the control actions that is listed above, and illustrated by figures A.1. It should be noted that to be able to perform a smooth control as the list and the figure imply, it will be necessary to use boundary action. If not, the controller will drive the state fast back and fourth over the nullcline. The controller could still work, but not as precisely. This will shortly be discussed in the next subsection about the influence of the boundary layer on the local control.

A to B: All the factors tells the controller to increase z and thereby z_{eff}

B to D: with a quite high weighting on the higher order derivatives, the w_2 factor will slightly override the other factors, and contribute to that z_{eff} is gradually reduced at the right speed. w_3 factor will help at stabilizing the states placement relative to the nullcline.

D to E: as the rate gets lower and lower the w factor contributes to increase the rate z_{eff} decrease at, thereby the state will gradually get closer to the nullcline and eventually cross it.

E to G: the w_2 will now ensure that nullcline is followed at the topside, with w_3 as a stabilizing factor.

G to H: the w factor will drive the state closer to and across the nullcline.

From this description, it becomes clear that we indirectly can look at the different factors in the sliding variable as a four level hierarchy of controllers; where the lowest levels is weighted highest (and thereby are preformed first and fastest), because their control actions has to be achieved for the controllers higher in the hierarchy to work properly. From bottom to top of the hierarchy we can describe the controllers and their purpose as following:

Level 4: w_3 contributes to stabilize the distance to the nullcline.

Level 3: w_2 contributes to following of the nullcline.

Level 2: w contributes to not letting the state drift to far away from the equilibrium.

Level 1: p ensures that the system will not stabilize at the wrong equilibrium. Will change z_{eff}, so that the equilibrium the level 2 controller work against become closer and closer to p_{ref}.

Simulations of the SMC (with use of well fitted boundary layers) performing the local stabilizing control as described is shown in figure A.3 to A.5. The values of the sliding variable and it's factors is shown in figure A.2. Development and amplitude of the factors underline the descpription on how the SMC is able to preform the controll, and that it works as a four layer controller. As the boundary line throw light on this, a very brief discussion is given in the following subsection about the influence of the boundary layer.

A.1.2 Influence of the boundary layer

To get a perfectly local stabilizing control as shown for $p_{ref} = 3.5235$ thought figure A.4 and figure A.5, the boundary layer has to be well fitted. If it is too small, full power (valve rate) will be generated, and there will be oscillation as for $p_{ref} = 3.55$ in A.5. In this case, the movement of the equilibrium point is not too large, so the w_3 factor is able to suppress the oscillation and thereby stabilize the lower degree derivatives. These are roughly following the same path as those of the perfect control. On the other hand, a too large boundary layer might make the control unstable, as the power generated might then be to low to control the system. The power will then grow as the

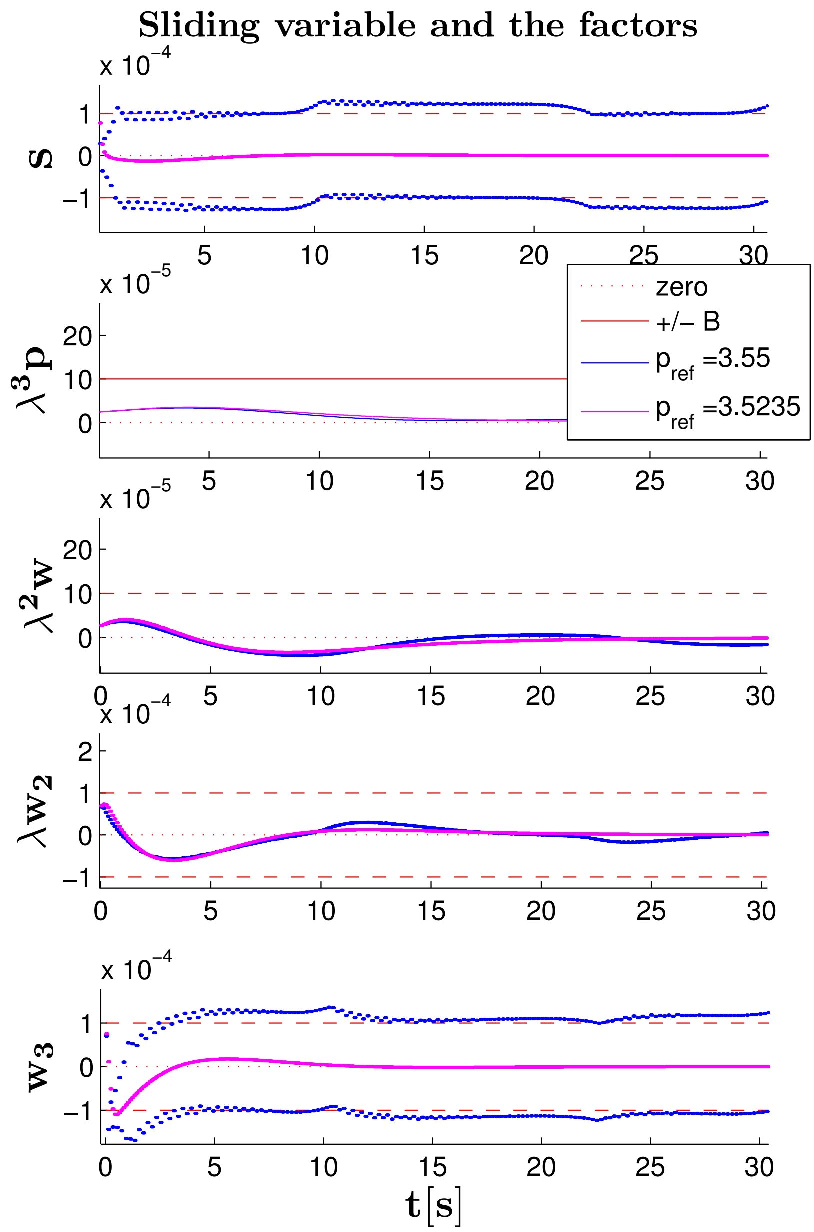

Figure A.2: *The sliding variable and its factors during local stabilization. To increase the overview, the graph uses one dot per controll sample instead of a solid line. The red dashed line marked B is the limit of the boundary layer.*

errors on the sliding variable increase, but as evident from the qualitative analysis, early control is a critical factor, so it might then be too late to gain control again.

As we can see from figure A.5, the sliding variable for $p_{ref} = 3.55$ just pass

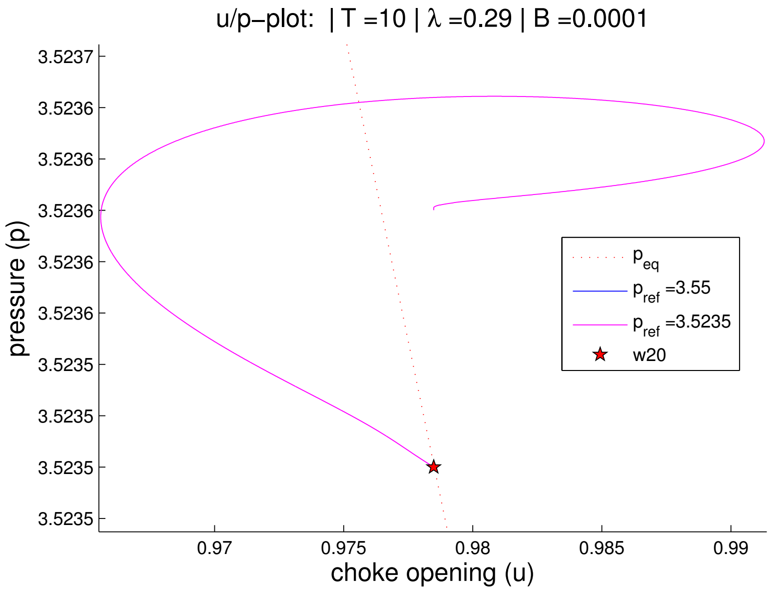

Figure A.3: *Local z_{eff}/p-plot around* $p_{ref} = 3.55$

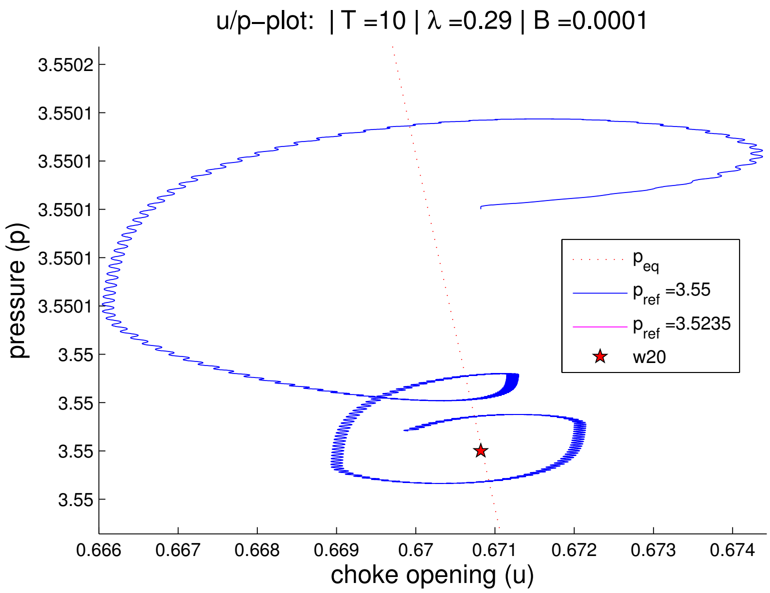

Figure A.4: *Local z_{eff}/p-plot around* $p_{ref} = 3.5235$

the limit of the boundary layer (the red dashed lines in the figure). Therefore a slightly increase of the boundary layer from $1 \cdot 10^{-4}$ to $1.1 \cdot 10^{-4}$ would have gentled the control, and created perfect performance. The reason what the boundary needs to be higher (create more and wither rate damping) is

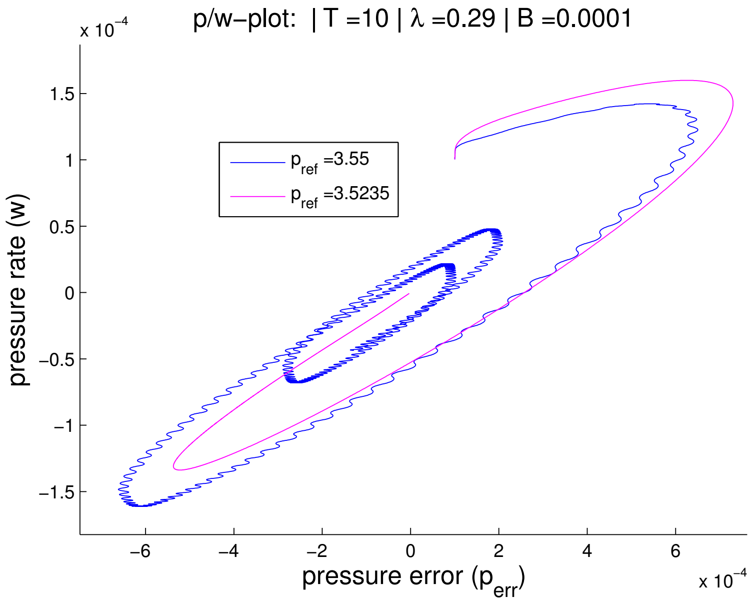

Figure A.5: *The sliding variable and its factors during local stabilization*

that the equilibrium line is steeper her, such that the effect on the movement of the equilibrium by the chance of z_{eff} is higher. As the equilibrium line is very flat for this higher valve openings, this (correctly) indicates that the boundary layer should be much higher when p_{ref} is set on higher levels. So for optimal control the controller actually has to be tuned different for p_{ref} and other changing operating conditions. If you have a roughly idea of the path of the equilibrium line, it should however be easy to create a tuning law for the size of the boundary layer based on the slope of the equilibrium line. Another problem is however that the tuning variable λ likely has to be retuned, but this will be discussed in the next section about how the SMC has to perform to locate the stabilizing point.

What should be clear is that there is not an explicit requirement to use boundary layers. The controller will still work without them as illustrated on figures A.6 to A.8, but not as optimal and economical. As shown on the figure the control becomes more chaotic, and a bias is created. The desired ideal path is not longer followed, but the principle for control is still valid. In reality this might be the local control picture as there is measurement noise, system variations, and the states will not be exactly known, but has to be estimated.

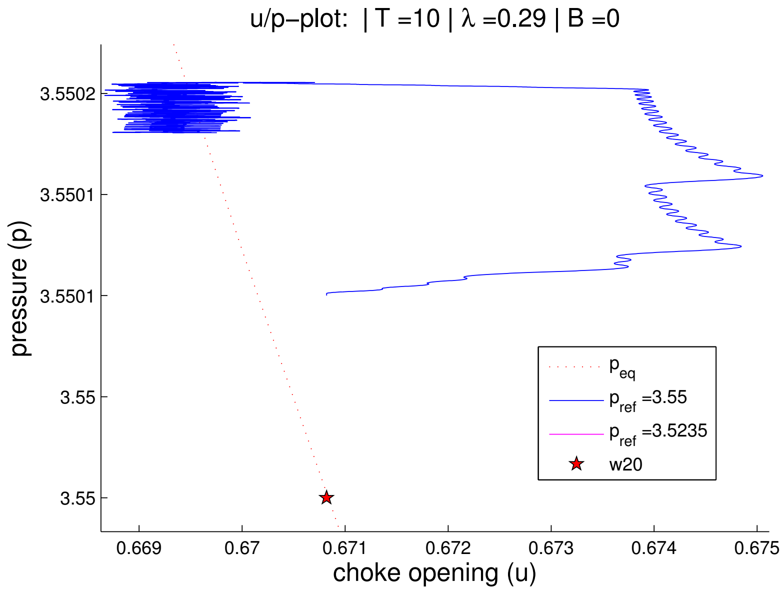

Figure A.6: *z_{eff}/p plot of simulation around $p_{ref} = 3.55$.*

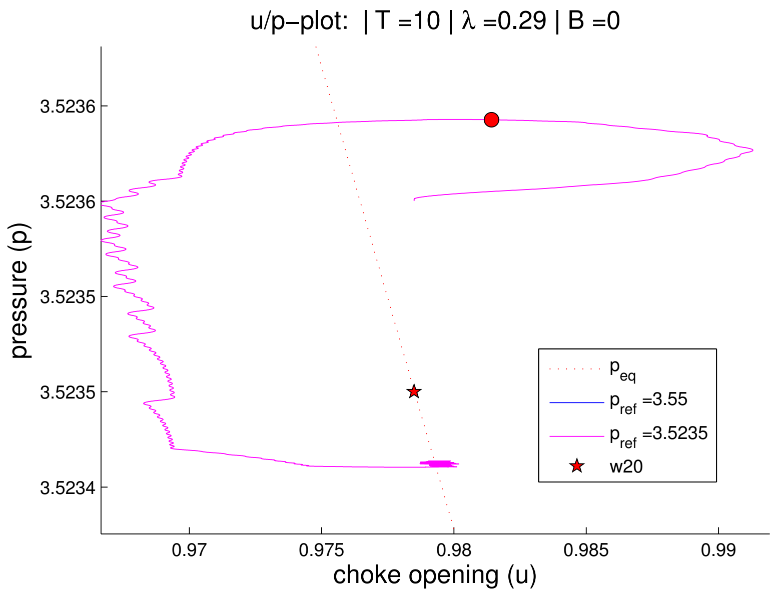

Figure A.7: *z_{eff}/p plot of simulation around $p_{ref} = 3.5235$.*

A.2 Locating stabilizing point

In figure A.9 it is shown an example of how different tuned SMC tries to approach two reference pressure and the corresponding valve opening that

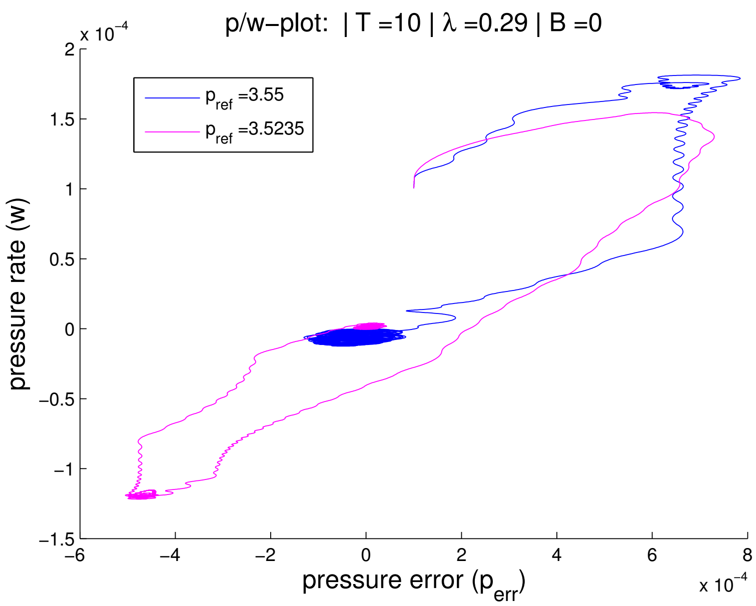

Figure A.8: *The sliding variable and it's factors under simulation of local stabilization*

place the unstable equilibrium point at the reference pressure. What is interesting with this plot is how the controller open and close the valve in its '*search*' to stabilize the system. In the start of the search, the controller with the lowest λ stop increasing the coke opening at an earlier stage because the derivatives is higher weighted, and therefore easier overcome the pressure error. The lower p_{ref} is about the lowest pressure it is possible to stabilize at for the current settings. When the sliding mode discover that it is time to '*brake*' (reduce the negative w-value), the only way to do so is to get close to the bifurcation line[2]. The reason for this is evident from the qualitative discussion in the above section about local stabilization. To break, the controller has to get on the topside of then nullcline[3], and as the negative rate is close two maximum, the only way to do this is to get close to or past the lower '*bifurcation line*' (in horizontal direction).

The action of the SMC to open the valve before the rate is braked down is really unnecessary risk taking without any desirable purpose as the rate the pressure is dropping would have been more than enough without this action. With lower λ-values, the valve would not open as much, and many types of limitation to the controller action could be added to stop this action. However as long as the controller is able to close the valve enough to start braking in time, this part is not that important. It is from the braking the

[2]I have chosen to call the lines that illustrate max/min pressure for the bifurcation lines

[3]Referring to the phase plot with the states p_{diff} and w

really critical part of the control starts. From her there is (especially in the case of a low p_{ref}), now room for errors. The actions of the controller close to the bifurcation line could be compared with dose of a *formula 1* car during a corner. The braking would have to be perfect. Not too soon, not to late, not too much and powerful, and not little – this to be able to make the perfect line out of the corner. Because the point of leaving and the rate at that moment have do be very precise, the SMC has to be very correctly tuned. From the moment the valve start opening again the controller has to follow a small corridor like a plane going inn for landing. Below the equilibrium point line the rate can only be counteracted in the negative direction. For this '*landing process*', the thick dashed blue line (in the figure) shows, for the thin blue line, which pressure level that would have been zero (at the nullcline). The distance from this line illustrate the high extend of braking. If the state moves to fare away from the line, the rate will slow down to fast, resulting in a chain reaction where the level of the line will increase – resulting in even more powerful braking. At this stage it will not be possible to gain control again, a slug has started to build up. Another problem is if the pressure becomes so low, that it is not possible to cross the equilibrium line at low speed. Then it will neither be possible to gain control. This will happen if the bifurcation line is left to early (and to late, but then the above scenario is the main problem). However, with a higher p_{ref} everything becomes much easier as there is room for errors. If the pressure gets to low and the rate is braked down to be very low it is a short change in z_{eff} until the equilibrium line is reached, and the state can be controlled on the upside of the equilibrium line as described in the local stabilisation section.

A.2.1 Continuous change of reference point

As was discussed farther in the project thesis [48] (regarding the performance of the SMC), this way of locating the desired pressure by directly setting of the reference will create a very steep fall against the desired pressure. The result might be problems on a real system where the dynamics is varying and not exactly as for this system. Another problem is that the fast falling rate might scare operators, and not yield their trust. Therefore it would be better to follow the equilibrium line more closely. One evident way of doing so, will be to not directly set the reference point to its desired value, but for a certain time period, let it slide toward the desired level. This is illustrated through figure A.10. In the figure the reference point change from $p_{ref} = 5.5$ to $p_{ref} = 3.55$ over a time period of 800 seconds. That the rate is rather high before it starts braking stiffly while using direct reference setting, and that the rate of the continuous controller is much lower is illustrated in the phase plot of figure A.11.

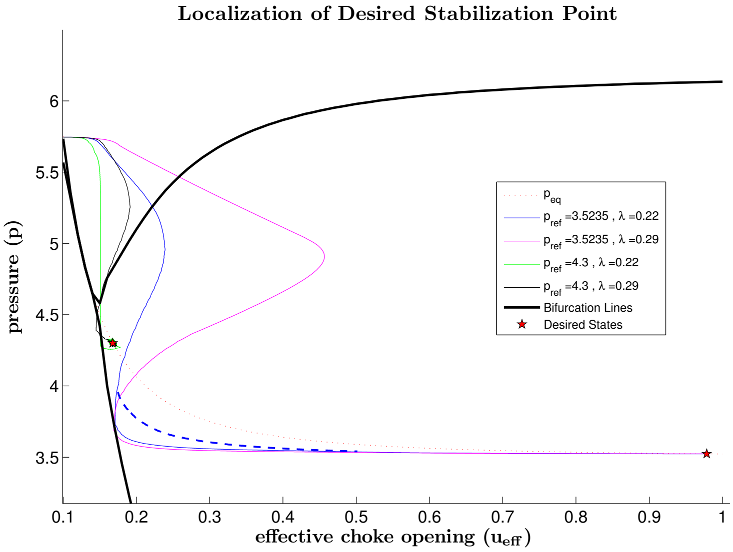

Figure A.9: *The figure shows how the SMC is able to reach to different desired states by the use of two slightly different tuning variables (λ). The thick dashed blue line shows at which pressure level w_2 would have been zero as the control setup for the blue line approaches the decried state. The distance from this line illustrate the high extend of braking.*

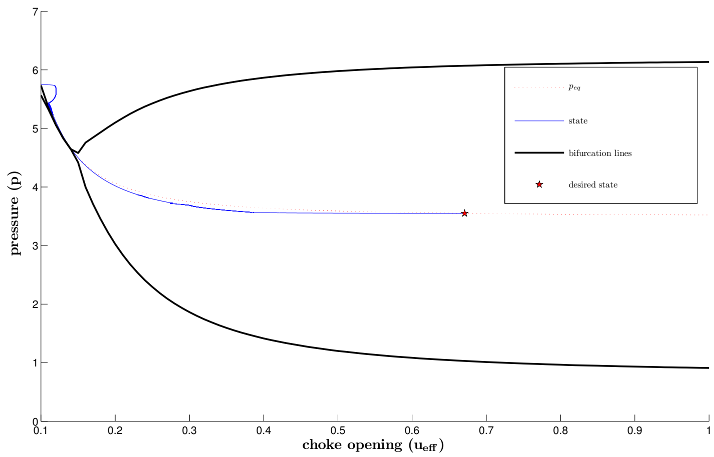

Figure A.10: *The figure shows how the equilibrium line is followed closely when the controller continuous change the reference point from* $p_{ref} = 5.5$ *to* $p_{ref} = 3.55$ *over a time period of 800 seconds.*

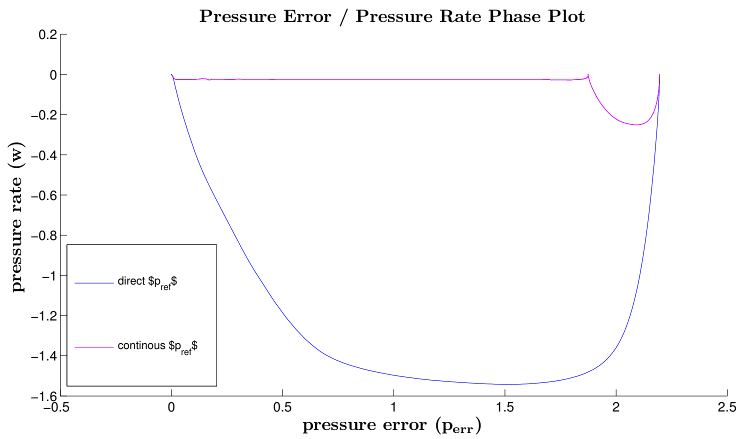

Figure A.11: *The figure compare the rate of pressure for a controller that use direct setting of the reference point to the desired reference, and a controller with the same settings, but uses continuous change the reference point from* $p_{ref} = 5.5$ *to* $p_{ref} = 3.55$ *over a time period of 800 seconds.*

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