Note on Passive Adaptive Observer for MPD

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Abstract

Observer for estimation of the downhole flowrate and pressure with a passive identifier for estimation of friction and density.

1 Model

Consider the following dynamical system

$$\rho_0 \frac{V_d}{\beta_d} \dot{p}_p = \rho_p q_p - \rho_{bit} q \tag{1a}$$

$$\rho_0 \frac{V_a}{\beta_a} \dot{p}_c = -\rho_0 \dot{V}_a + \rho_{bit} q - \rho_c q_c \tag{1b}$$

$$M\dot{q} = p_p - p_c - F_d \left| q \right| q - F_a q + \Delta\rho g h_{bit} \tag{1c}$$

where the downhole pressure is given as

$$p_{bit} = p_c + M_a \dot{q} + F_a q + \bar{\rho} g h_{bit},$$

$$\bar{\rho} g h_{bit} = \beta_a \left(e^{\frac{1}{\beta_a} \rho_a g h_{bit}} - 1 \right)$$

...densities $\rho_p \triangleq \rho(p_p)$, $\rho_{bit} \triangleq \rho(p_{bit})$, are given according the linearized equation of state as

$$\rho(p) = \rho_0 + \frac{\rho_0}{\beta} p \tag{2}$$

1.1 Design model

For observer design vi use the simplified model

$$\frac{V_d}{\beta_d} \dot{p}_p = q_p - q \tag{3a}$$

$$M\dot{q} = p_p - p_c - (F_d + F_a) \left| q \right| q + \Delta\rho g h_{bit}, \tag{3b}$$

with the downhole pressure at the bit given by

$$p_{bit} = M\dot{q} + p_c + F_a q + \bar{\rho}_a g h_{bit}. \tag{4}$$

...approximated by

$$p_{bit} = p_c + F_a q + \bar{\rho}_a g h_{bit}.$$

2 Observer

System

$$\xi \triangleq q + l_p p_p \tag{5}$$

$$\begin{aligned}
\dot{\xi} &= \dot{q} + l_p \dot{p}_p \\
&= \frac{1}{M}\left[p_p - p_c - (F_d + F_a)\,|q|\,q + \Delta\rho g h\right] + l_p \frac{\beta_d}{V_d} q_p - l_p q \\
&= \frac{1}{M}(p_p - p_c) - \theta_q f(q) + \theta_\rho g(h) + l_p \frac{\beta_d}{V_d} q_p - l_p q,
\end{aligned}$$

where

$$f(q) \triangleq |q|\,q \tag{6}$$

$$\theta_q \triangleq \frac{F_d + F_a}{M}, \tag{7}$$

$$g(h) \triangleq \frac{gh}{M} \tag{8}$$

$$\theta_\rho \triangleq \Delta\rho. \tag{9}$$

Let an estimate of q be given by

$$\dot{\hat{\xi}} = \frac{1}{M}(p_p - p_c) - \hat{\theta}_q f(\hat{q}) + \hat{\theta}_\rho g(h) + l_p \frac{\beta_d}{V_d} q_p - l_p \hat{q} \tag{10}$$

$$\hat{q} = \hat{\xi} - l_p p_p, \tag{11}$$

Note that the estimation error

$$\tilde{q} = q - \hat{q} = \tilde{\xi}$$

is governed by

$$\begin{aligned}
\dot{\tilde{q}} &= \dot{\tilde{\xi}} \\
&= -\theta_q f(q) + \theta_\rho g(h) - l_p q \\
&\quad - \left(-\hat{\theta}_q f(\hat{q}) + \hat{\theta}_\rho g(h) - l_p \hat{q}\right) \\
&= -\theta_q\left[f(q) - f(\hat{q})\right] - \tilde{\theta}_q f(\hat{q}) + \tilde{\theta}_\rho g(h) - l_p \tilde{q} \\
&= -\theta_q\left[f(q) - f(\hat{q})\right] - l_p \tilde{q} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t),
\end{aligned}$$

$$\tilde{\boldsymbol{\theta}} \triangleq \begin{bmatrix} \tilde{\theta}_q \\ \tilde{\theta}_\rho \end{bmatrix}, \quad \boldsymbol{\phi}(\hat{q}, h) \triangleq \begin{bmatrix} f(\hat{q}) \\ g(h) \end{bmatrix}.$$

By using the Mean value theorem, the error dynamics can be written as

$$\dot{\tilde{q}} = -k(t)\,\tilde{q} - l_p \tilde{q} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}, \tag{12}$$

where

$$k(t) \triangleq \theta_q \left.\frac{\partial f(\bar{q})}{\partial q}\right|_{\bar{q} \in [\min(q,\hat{q}), \max(q,\hat{q})]}$$

2.1 Passive identifier

It is straightforward to show that the error $\tilde{q}$ is strictly passive from input $\tilde{\boldsymbol{\theta}}$ to output $\boldsymbol{\phi}\tilde{q}$:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{1}{2}\tilde{q}^2\right) &= -\left(l_p + k\left(t\right)\right)\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\leq -l_p\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q}.
\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}
\int_0^t \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} &\geq \int_0^t \frac{d}{d\tau}\left(\frac{1}{2}\tilde{q}\left(\tau\right)^2\right)d\tau + l_p\int_0^t \tilde{q}^2 d\tau \\
&= \left[\frac{1}{2}\tilde{q}\left(\tau\right)^2\right]_0^t + l_p\int_0^t \tilde{q}^2 d\tau.
\end{aligned}$$

Since the system is strictly passive, the passive adaptation law

$$\dot{\hat{\boldsymbol{\theta}}} = -\dot{\tilde{\boldsymbol{\theta}}} = \Gamma\boldsymbol{\phi}\left(\hat{q}, h\right)\tilde{q} \tag{13}$$

would make the resulting error system strictly passive, thus ensuring that $\tilde{q}$ converges to zero. The estimation error $\tilde{q} = q - \hat{q}$ is not directly known,

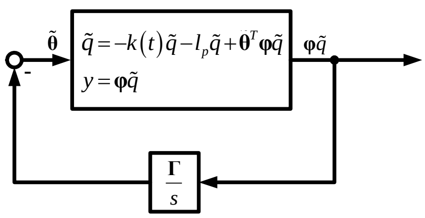

Figure 1: Negative feedback connection of the strictly passive $\tilde{q}$–system and the passive identifier Γ/s.

however, we have from (3a) that

$$\begin{aligned}
\dot{p}_p &= \frac{\beta_d}{V_d}q_p - \frac{\beta_d}{V_d}q \\
&\Downarrow \\
q &= q_p - \frac{V_d}{\beta_d}\dot{p}_p,
\end{aligned}$$

which gives

$$\tilde{q} = q_p - \frac{V_d}{\beta_d}\dot{p}_p - \hat{q}. \tag{14}$$

The passive identifier can thus implemented as

$$\begin{aligned}
\dot{\hat{\boldsymbol{\theta}}} &= \Gamma\boldsymbol{\phi}\left(\hat{q}, h\right)\tilde{q} \\
&= \Gamma\boldsymbol{\phi}\left(\hat{q}, h\right)\left(q_p - \frac{V_d}{\beta_d}\frac{dp_p}{dt} - \hat{q}\right).
\end{aligned} \tag{15}$$

Remark 1 *The integral of $\boldsymbol{\phi}(t)\,\dot{p}_p(t)$ is implementable if $p_p(t)$ is known:*

$$\int_0^t \boldsymbol{\phi}(\tau)\frac{dp}{d\tau}d\tau = \int_{p(0)}^{p(t)} \boldsymbol{\phi}(\tau(p))\,dp.$$

The parameter estimate is thus given by

$$\hat{\boldsymbol{\theta}}(t) = \Gamma\int_0^t \boldsymbol{\phi}(\hat{q}(\tau), h(\tau))\left(q_p(\tau) - \frac{V_d}{\beta_d}\frac{dp(\tau)}{d\tau} - \hat{q}(\tau)\right)d\tau.$$

2.2 Identifier driven by filtered estimation error

We want the identifier to be driven by the low-pass filtered estimation error

$$\tilde{q}_f = \frac{1}{\tau_f s+1}\tilde{q}, \tag{16}$$

and modify the identifier according to

$$\begin{aligned}
\dot{\hat{\boldsymbol{\theta}}} &= -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\tilde{q}_f \\
&= -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\frac{1}{\tau_f s+1}\tilde{q} \\
&= -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\frac{1}{\tau_f s+1}(q_p-\hat{q}) \\
&\quad -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\frac{s}{\tau_f s+1}p_p,
\end{aligned} \tag{17}$$

which is causal, thus implementable. The identifier can be implemented in state-space form as

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\frac{1}{\tau_f}\left(x_f - \frac{V_d}{\beta_d}p_p\right) \tag{18}$$

$$\dot{x}_f = -\frac{1}{\tau_f}x_f + \frac{V_d}{\tau_f\beta_d}p_p + q_p - \hat{q}, \tag{19}$$

more transparently written as

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma\boldsymbol{\phi}(\hat{q},h)\,\tilde{q}_f \tag{20}$$

$$\dot{x}_f = -\tilde{q}_f + q_p - \hat{q} \tag{21}$$

$$\tilde{q}_f = \frac{1}{\tau_f}\left(x_f - \frac{V_d}{\beta_d}p_p\right). \tag{22}$$

To establish the passivity properties of the $(\tilde{q},\tilde{q}_f)$–system, consider the function

$$S(\tilde{q},\tilde{q}_f) = \frac{1}{2}\tilde{q}^2 + \frac{l_p\tau_f}{2}\tilde{q}_f^2. \tag{23}$$

The time-derivative satisfies

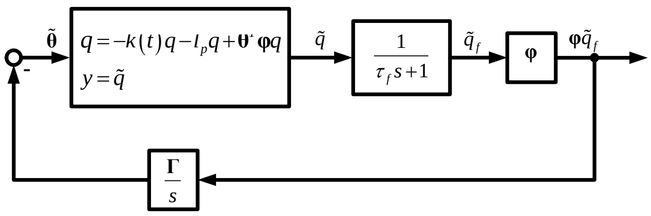

Figure 2: The negative feedback connection of the augmented $\tilde{q}$–system and the passive identifier Γ/s.

$$\begin{aligned}
\frac{dS}{dt} &= -\left(l_p + k\left(t\right)\right)\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\quad -l_p\tilde{q}_f^2 + l_p\tilde{q}\tilde{q}_f \\
&\leq -l_p\tilde{q}^2 - l_p\tilde{q}_f^2 + l_p\tilde{q}\tilde{q}_f + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&= -l_p\left(\tilde{q}^2 + \tilde{q}_f^2 - \tilde{q}\tilde{q}_f\right) + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q}.
\end{aligned} \tag{24}$$

Using the inequality $xy \leq \frac{1}{2}x^2 + \frac{1}{2}y^2$ (completion of squares), we can write

$$\begin{aligned}
\frac{dS}{dt} &\leq -l_p\left(\tilde{q}^2 + \tilde{q}_f^2 - \tilde{q}\tilde{q}_f\right) + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\leq -\frac{l_p}{2}\left(\tilde{q}^2 + \tilde{q}_f^2\right) + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q}.
\end{aligned} \tag{25}$$

Hence, we have established the $(\tilde{q}, \tilde{q}_f)$–system is strictly passive from $\tilde{\boldsymbol{\theta}}$ to $\boldsymbol{\phi}\tilde{q}$. The stability of the complete $\left(\tilde{q}, \tilde{q}_f, \tilde{\boldsymbol{\theta}}\right)$–system is established by the Lyapunov function

$$V\left(\tilde{q}, \tilde{q}_f, \tilde{\boldsymbol{\theta}}\right) = \frac{1}{2}\tilde{q}^2 + \frac{l_p\tau_f}{2}\tilde{q}_f^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}, \tag{26}$$

which the derivative satisfies

$$\begin{aligned}
\dot{V} &= -\left(l_p + k\left(t\right)\right)\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\quad -l_p\tilde{q}_f^2 + l_p\tilde{q}\tilde{q}_f \\
&\quad -\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} \\
&\leq -\frac{l_p}{2}\left(\tilde{q}^2 + \tilde{q}_f^2\right) + \tilde{\boldsymbol{\theta}}^T\left(\boldsymbol{\phi}\tilde{q} - \boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}\right)
\end{aligned} \tag{27}$$

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Consider the error dynamics

$$
\begin{aligned}
\dot{\tilde{q}} &= -k(t)\,\tilde{q} - l_p\tilde{q} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi} \\
\dot{\tilde{q}}_f &= -\frac{1}{\tau_f}\tilde{q}_f + \frac{1}{\tau_f}\tilde{q} \\
\dot{\tilde{\boldsymbol{\theta}}} &= -\boldsymbol{\Gamma}\boldsymbol{\phi}(\hat{q}, h)\,\tilde{q}_f
\end{aligned}
$$

Investigate the stability of the complete $\left(\tilde{q}, \tilde{q}_f, \tilde{\boldsymbol{\theta}}\right)$–system using the function

$$
V\left(\tilde{q}, \tilde{q}_f, \tilde{\boldsymbol{\theta}}\right) = \frac{1}{2}\tilde{q}^2 + \frac{l_p\tau_f}{2}\tilde{q}_f^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} - \tilde{q}\mathbf{c}^T\tilde{\boldsymbol{\theta}}.
$$

The derivative satisfies

$$
\begin{aligned}
\dot{V} &= -\left(l_p + k(t)\right)\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\quad -l_p\tilde{q}_f^2 + l_p\tilde{q}\tilde{q}_f \\
&\quad -\tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q}_f + \tilde{q}\mathbf{c}^T\boldsymbol{\Gamma}\boldsymbol{\phi}\tilde{q}_f \\
&\quad -\left(-k(t)\,\tilde{q} - l_p\tilde{q} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\right)\mathbf{c}^T\tilde{\boldsymbol{\theta}} \\
&\leq -\frac{l_p}{2}\left(\tilde{q}^2 + \tilde{q}_f^2\right) + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q} \\
&\quad -\tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\tilde{q}_f + \tilde{q}\mathbf{c}^T\boldsymbol{\Gamma}\boldsymbol{\phi}\tilde{q}_f \\
&\quad -\left(-k(t)\,\tilde{q} - l_p\tilde{q} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\phi}\right)\mathbf{c}^T\tilde{\boldsymbol{\theta}}
\end{aligned}
$$

###

$$
\begin{aligned}
\dot{x}_1 &= -a_1x_1 + \phi x_2 \\
\dot{x}_2 &= -\phi x_3 \\
\dot{x}_3 &= -a_3x_3 + x_1.
\end{aligned}
$$

References