Note on Passive Adaptive Observer for MPD

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Abstract

Observer for estimation of the downhole flowrate and pressure with a passive identifier for estimation of friction and density.

1 Model

1.1 Design model

We base our design on the following simplified dynamical model of the system

$$\frac{V_d}{\beta_d}\,\dot{p}_p = q_p - q \tag{1a}$$

$$\frac{V_a}{\beta_a}\,\dot{p}_c + \dot{V}_a = q - q_c \tag{1b}$$

$$M\dot{q} = p_p - p_c - F\left(q, \theta\right) + \Delta\rho g h, \tag{1c}$$

where $F\left(q\right)$ is the total friction loss through the system, and $\Delta G = \Delta\rho g h$ is the difference in gravitational (hydrostatic) pressure between the drillstring and annulus caused by differences in the mean densities $\bar{\rho}_a$ and $\bar{\rho}_d$. The frictional pressure drop is typically a nonlinear function of the flow q with nonlinear friction parameters θ.

Example 1 *As an example, the friction can be modelled such as*

$$F\left(q, \mathbf{a}\right) = a_0 + a_1 q + a_2 q^2, \quad \forall q > 0,$$

with friction parameters

$$\mathbf{a} = \begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}^T.$$

...

The downhole bit pressure can be given as

$$p_{bit} = \begin{cases} p_c + F_a\left(q\right) + G_a(h_{bit}) + M_a\dot{q} \\ p_p + F_d\left(q\right) + G_d(h_{bit}) - M_d\dot{q} \end{cases},$$

with the hydrostatic pressure term

$$\begin{aligned} G_i(h) &= \bar{\rho}_i h \\ &= \beta_i\left(e^{\beta_i^{-1}\rho_i g h} - 1\right), \quad i = \{d, a\}, \end{aligned}$$

based on a linearized equation of state for the liquid...

2 Adaptive observer design

2.1 Parametrization

Consider first the case with the simple quadratic friction model

$$F(q, F_0) = F_0 |q| q, \tag{2}$$

where $F_0 > 0$ is typically an unknown friction coefficient, and the remaining parameters are known. The flow dynamics of (1c) can then be written

$$\dot{q} = \frac{1}{M}(p_p - p_c) - \frac{F_0}{M}|q| q + \frac{\Delta\rho}{M} gh.$$

Defining

$$\begin{aligned} f_0(t) &\triangleq \frac{1}{M}(p_p(t) - p_c(t)) + \frac{\Delta\rho}{M} gh(t) \\ \theta &\triangleq \frac{F_0}{M}, \quad \phi(q) \triangleq -|q| q \end{aligned}$$

the flow dynamics can be expressed in the compact form

$$\dot{q} = f_0(t) + \boldsymbol{\theta}^T \boldsymbol{\phi}(t, q), \tag{3}$$

where $f_0(t)$ denotes the known, time-varying part of the dynamics, and $\boldsymbol{\theta}^T \boldsymbol{\phi}(t, q) = \theta\phi(q)$ denotes the unknown part, written in a generalized, time-varying vector form.

Remark 2 *Note that it is straightforward to rewrite the unknown part of the flow dynamics to include more unknown parameters. For example, if in addition M and $\Delta\rho$ are unknown, the flow dynamics can be expressed simply as*

$$\dot{q} = \boldsymbol{\theta}^T \boldsymbol{\phi}(t, q), \tag{4}$$

with

$$\boldsymbol{\theta} = \begin{bmatrix} \frac{1}{M} \\ \frac{F_0}{M} \\ \frac{\Delta\rho}{M} \end{bmatrix}, \quad \boldsymbol{\phi}(t, q) \triangleq \begin{bmatrix} p_p(t) - p_c(t) \\ -|q| q \\ gh(t) \end{bmatrix}. \tag{5}$$

2.2 Observer design

The objective is to estimate the unmeasured flow q and the uncertain parameter θ. To design our basic reduced-order observer for q, introduce the change of coordinate

$$\xi \triangleq q + l_p p_p. \tag{6}$$

The time-derivative of ξ gives

$$\begin{aligned} \dot{\xi} &= \dot{q} + l_p \dot{p}_p \\ &= f_0(t) + \boldsymbol{\theta}^T \boldsymbol{\phi}(t, q) + l_p \frac{\beta_d}{V_d}(q_p - q), \end{aligned}$$

Let an estimate of q be given by

$$\begin{aligned}\dot{\hat{\xi}} &= f_0(t) + \hat{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q}) + l_p \frac{\beta_d}{V_d}(q_p - \hat{q}) && (7)\\ \hat{q} &= \hat{\xi} - l_p p_p, && (8)\end{aligned}$$

In the absence of parameter errors, this provides exponential estimates of the unmeasured q, and input-to-state stability (ISS) with respect to parameter errors $\tilde{\boldsymbol{\theta}}$ if the regressor $\phi_f(\hat{q})$ of the friction model is monotonic in $\hat{q}$ and bounded.

Proof (Outline). The estimation error

$$\tilde{q} = q - \hat{q} = \tilde{\xi}$$

is governed by

$$\begin{aligned}\dot{\tilde{q}} &= \dot{\tilde{\xi}}\\ &= \boldsymbol{\theta}^T \boldsymbol{\phi}(t,q) + l_p \frac{\beta_d}{V_d}(q_p - q)\\ &\quad - \left(\hat{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q}) + l_p \frac{\beta_d}{V_d}(q_p - \hat{q})\right)\\ &\quad + \boldsymbol{\theta}^T \boldsymbol{\phi}(t,\hat{q}) - \boldsymbol{\theta}^T \boldsymbol{\phi}(t,\hat{q})\\ &= -l_p \frac{\beta_d}{V_d}\tilde{q} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q}) + \boldsymbol{\theta}^T(\boldsymbol{\phi}(t,q) - \boldsymbol{\phi}(t,\hat{q})).\end{aligned}$$

Noting that

$$\boldsymbol{\theta}^T(\boldsymbol{\phi}(t,q) - \boldsymbol{\phi}(t,\hat{q})) = \theta[\phi(q) - \phi(\hat{q})],$$

and using the Mean value theorem, the error dynamics can be written in the linear, time-varying form

$$\dot{\tilde{q}} = -l(q,\hat{q})\tilde{q} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q}), \tag{9}$$

where

$$l(q,\hat{q}) \triangleq l_p \frac{\beta_d}{V_d} - k(q,\hat{q}),$$

and

$$\begin{aligned}k(q,\hat{q}) &\triangleq \theta \left.\frac{\partial \phi(\bar{q})}{\partial q}\right|_{\bar{q}\in[\min(q,\hat{q}),\max(q,\hat{q})]}\\ &= -2\theta|\bar{q}|.\end{aligned}$$

Notice that since $\phi(q)$ is monotonically decreasing in q, $l_p \geq 0$ implies that the nonlinear gain $l(q,\hat{q}) > 0$. More precisely, the nonlinear gain $l(q,\hat{q}) = l_p \frac{\beta_d}{V_d} - k(q,\hat{q})$ will be strictly positive for $\forall l_p > \inf k(q,\hat{q})$.

The stability properties of the $\tilde{q}$–system can then be establised by considering the function

$$V(\tilde{q}) = \frac{1}{2}\tilde{q}^2.$$

The time-derivative of $V(\tilde{q})$ is

$$\begin{aligned}\dot{V} &= \tilde{q}\left(-l(q,\hat{q})\tilde{q} + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q})\right) \\ &= -l(q,\hat{q})\tilde{q}^2 + \tilde{q}\tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q}),\end{aligned}$$

which is negative if

$$|\tilde{q}| > \frac{\tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q})}{l(q,\hat{q})}.$$

Hence, ISS with $\tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}$ as input and $\tilde{q}$ as output is established. ■

Remark 3 *A more flexible observer with weighted injection from both measurements p_p and p_c, is obtained by the choice $\xi \triangleq q + l_p p_p + l_c p_c$. However, this alternative design is not persued in this note.*

In the following, we will extend the observer design with different parameter identifiers to estimate the unknown parameters in $\boldsymbol{\theta}$.

2.3 Stamnes identifier

Passive parameter identifier based on coordinate transformation...See [Stamnes08].

2.4 Passive identifier driven by q

Since $k(q,\hat{q}) < 0$ in (9), the error $\tilde{q}$ is strictly passive from input $\tilde{\boldsymbol{\theta}}$ to output $\boldsymbol{\phi}\tilde{q}$:

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}\tilde{q}^2\right) &= -l(q,\hat{q})\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q})\tilde{q} \\ &\leq -l_p \frac{\beta_d}{V_d}\tilde{q}^2 + \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,\hat{q})\tilde{q}.\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}\int_0^t \tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}\tilde{q} &\geq \int_0^t \frac{d}{d\tau}\left(\frac{1}{2}\tilde{q}(\tau)^2\right) d\tau + l_p \int_0^t \tilde{q}^2 d\tau \\ &= \left[\frac{1}{2}\tilde{q}(\tau)^2\right]_0^t + l_p \frac{\beta_d}{V_d}\int_0^t \tilde{q}^2 d\tau.\end{aligned}$$

Since the system is strictly passive, the adaptation law

$$\dot{\hat{\boldsymbol{\theta}}} = -\dot{\tilde{\boldsymbol{\theta}}} = \Gamma \boldsymbol{\phi}(t,\hat{q})\tilde{q}, \tag{10}$$

would make the resulting error system strictly passive, thus ensuring that $\tilde{q}$ converges to zero.

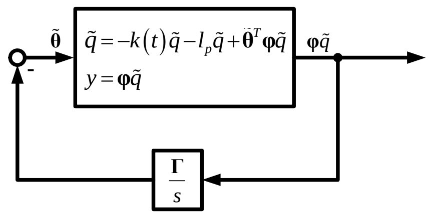

Figure 1: Negative feedback connection of the strictly passive $\tilde{q}$–system and the passive identifier Γ/s.

The estimation error $\tilde{q} = q - \hat{q}$ is not directly known, however, we have from (**??**) that

$$\begin{aligned} \dot{p}_p &= \frac{\beta_d}{V_d} q_p - \frac{\beta_d}{V_d} q \\ &\Downarrow \\ q &= q_p - \frac{V_d}{\beta_d} \dot{p}_p, \end{aligned}$$

which gives

$$\tilde{q} = q_p - \frac{V_d}{\beta_d} \dot{p}_p - \hat{q}. \tag{11}$$

The passive identifier can thus be implemented as

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}} &= \Gamma \boldsymbol{\phi}(t, \hat{q}) \tilde{q} \\ &= \Gamma \boldsymbol{\phi}(t, \hat{q}) \left(q_p - \frac{V_d}{\beta_d} \frac{dp_p}{dt} - \hat{q} \right). \end{aligned} \tag{12}$$

Remark 4 *The integral of $\boldsymbol{\phi}(t) \dot{p}_p(t)$ is implementable if $p_p(t)$ is known:*

$$\int_0^t \boldsymbol{\phi}(\tau) \frac{dp}{d\tau} d\tau = \int_{p(0)}^{p(t)} \boldsymbol{\phi}(\tau(p))\, dp.$$

2.5 Parameter identifier driven by filtered q

Alternatively, we may let the parameter identifier be driven by the low-pass filtered estimation error

$$\tilde{q}_f = \frac{1}{\tau_f s + 1} \tilde{q}, \tag{13}$$

simply by modifying the identifier according to

$$
\begin{aligned}
\dot{\hat{\boldsymbol{\theta}}} &= -\Gamma \boldsymbol{\phi}(t, \hat{q}) \tilde{q}_f \\
&= -\Gamma \boldsymbol{\phi}(t, \hat{q}) \frac{1}{\tau_f s + 1} \tilde{q} \\
&= -\Gamma \boldsymbol{\phi}(t, \hat{q}) \frac{1}{\tau_f s + 1} (q_p - \hat{q}) \\
&\quad -\Gamma \boldsymbol{\phi}(t, \hat{q}) \frac{s}{\tau_f s + 1} p_p, \qquad (14)
\end{aligned}
$$

which is causal, thus implementable. The identifier can be implemented in state-space form as

$$
\dot{\hat{\boldsymbol{\theta}}} = -\Gamma \boldsymbol{\phi}(t, \hat{q}) \frac{1}{\tau_f} \left(x_f - \frac{V_d}{\beta_d} p_p \right) \qquad (15)
$$

$$
\dot{x}_f = -\frac{1}{\tau_f} x_f + \frac{V_d}{\tau_f \beta_d} p_p + q_p - \hat{q}. \qquad (16)
$$

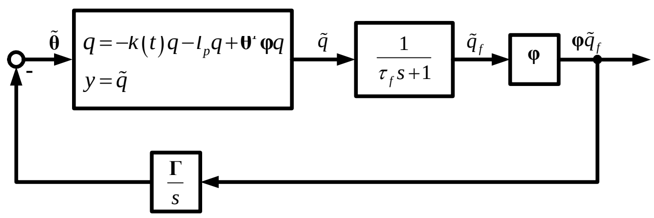

Figure 2: The negative feedback connection of the augmented $\tilde{q}$–system and the passive identifier Γ/s.

2.6 Grip identifier driven by q

An alternative parameter identifier can be derived based on the results in [Grip09#]. The design is based on q to be known, and is performed in a two-step design procedure, which we will outline below. Note that this approach also may be designed to enable estimation of unknown parameters that appear nonlinearly in the system.

Start by rewriting the system (3) as

$$
\dot{q} = f_0(t) + \psi \qquad (17)
$$

by defining

$$
\psi \triangleq \boldsymbol{\phi}(t, q)^T \boldsymbol{\theta}.
$$

2.6.1 Virtual adaptation law for $\boldsymbol{\theta}$

First step is to design a virtual adaptation law for $\hat{\boldsymbol{\theta}}$, which would be exponential convergent if ψ was known. Assuming ψ known, we could use the gradient adaptation law

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\left(\psi - \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}}\right), \tag{18}$$

where $\boldsymbol{\Gamma} > \mathbf{0}$ is a constant adaptation gain matrix. The parameter estimation error $\tilde{\boldsymbol{\theta}}$ would then satisfy

$$\begin{aligned}
\dot{\tilde{\boldsymbol{\theta}}} &= -\boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\left(\psi - \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}}\right) \\
&= -\boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\left(\boldsymbol{\phi}(t,q)^T \boldsymbol{\theta} - \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}}\right) \\
&= -\boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\,\boldsymbol{\phi}(t,q)^T \tilde{\boldsymbol{\theta}}.
\end{aligned} \tag{19}$$

In the scalar case, stability properties can be established by considering the function

$$U\left(\tilde{\boldsymbol{\theta}}\right) = \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \Gamma^{-1}\tilde{\boldsymbol{\theta}}, \tag{20}$$

whose time-derivative is

$$\dot{U} = -\tilde{\boldsymbol{\theta}}^T \boldsymbol{\phi}(t,q)\,\boldsymbol{\phi}(t,q)^T \tilde{\boldsymbol{\theta}}.$$

With $\tilde{\boldsymbol{\theta}} = \tilde{\theta}$ and $\boldsymbol{\phi}(t,q) = \phi(q) = -|q|\,q$, we get

$$\begin{aligned}
\dot{U} &= -\tilde{\theta}^2 \phi(q)^2 \\
&= -\tilde{\theta}^2 \left(-|q|\,q\right)^2 \\
&= -\tilde{\theta}^2 q^4,
\end{aligned}$$

which obviously is strictly negative for $\forall \triangleq q > 0$. Thus, provided $q \neq 0$, we would get exponential convergent estimates of $\hat{\theta}$.

Remark 5 *Extention to the case with more unknown parameters is straightforward, such as parametrization of friction in several parameters, or having an unknown density $\Delta\rho$. With more unknown parameters, exponential convergence is still possible by using data over longer periods of time, provided we have sufficient excitation. Proving exponential stability in this case, involves a more complicated Lyapunov function, and typically incorporates the persistancy-of-excitation condition*

$$\int_t^{t+T} \boldsymbol{\phi}(\tau, q(\tau))\,\boldsymbol{\phi}(\tau, q(\tau))^T d\tau \geq \varepsilon \mathbf{I}.$$

Since ψ is not known, the adaptation law for $\hat{\boldsymbol{\theta}}$ is implemented with an estimate $\hat{\psi}$ of ψ, according to

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\left(\hat{\psi} - \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}}\right). \tag{21}$$

Replacing $\hat{\psi} = \psi - \tilde{\psi}$, gives the parameter error dynamics

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}\boldsymbol{\phi}(t,\hat{q})\,\boldsymbol{\phi}(t,\hat{q})^T \tilde{\boldsymbol{\theta}} - \boldsymbol{\Gamma}\boldsymbol{\phi}(t,q)\,\tilde{\boldsymbol{\psi}}. \tag{22}$$

2.6.2 Estimator for $\hat{\psi}$

Next step is to design an exponentially convergent estimate of $\hat{\psi}$. This is obtained by introducing the state predictor

$$\dot{\hat{q}} = f_0(t) + \hat{\psi} + \gamma_0^{-1} \boldsymbol{\phi}(t,q)^T \dot{\hat{\boldsymbol{\theta}}}, \tag{23}$$

and let an estimate of ψ be given by

$$\hat{\psi} = \gamma_0 (q - \hat{q}) + \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}}, \tag{24}$$

where $\gamma_0 > 0$ is a tunable feedback gain. Notice that the derivative of $\hat{\psi}$ becomes

$$\begin{aligned} \dot{\hat{\psi}} &= \gamma_0 (f_0(t) + \psi) \\ &\quad -\gamma_0 \left(f_0(t) + \hat{\psi} + \gamma_0^{-1} \boldsymbol{\phi}^T \dot{\hat{\boldsymbol{\theta}}} \right) \\ &\quad + \dot{\boldsymbol{\phi}}^T \hat{\boldsymbol{\theta}} + \boldsymbol{\phi}^T \dot{\hat{\boldsymbol{\theta}}} \\ &= \gamma_0 \left(\psi - \hat{\psi} \right) + \dot{\boldsymbol{\phi}}^T \hat{\boldsymbol{\theta}}, \end{aligned}$$

and since

$$\dot{\psi} = \dot{\boldsymbol{\phi}}^T \boldsymbol{\theta},$$

the resulting estimation error will satisfy

$$\dot{\tilde{\psi}} = -\gamma_0 \tilde{\psi} + \dot{\boldsymbol{\phi}}^T \tilde{\boldsymbol{\theta}}.$$

The Grip estimator, given by (21) and (23)–(24) can be given as

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}} &= \gamma_0 \boldsymbol{\Gamma} \boldsymbol{\phi}(t,q)(q - \hat{q}) & (25) \\ \dot{\hat{q}} &= f_0(t) + \gamma_0 (q - \hat{q}) + \boldsymbol{\phi}(t,q)^T \hat{\boldsymbol{\theta}} + \gamma_0^{-1} \boldsymbol{\phi}(t,q)^T \dot{\hat{\boldsymbol{\theta}}}. & (26) \end{aligned}$$

The estimator is governed by the error dynamics

$$\begin{aligned} \dot{\tilde{\boldsymbol{\theta}}} &= -\boldsymbol{\Gamma} \boldsymbol{\phi}(t,\hat{q}) \boldsymbol{\phi}(t,\hat{q})^T \tilde{\boldsymbol{\theta}} - \boldsymbol{\Gamma} \boldsymbol{\phi}(t,q) \tilde{\boldsymbol{\psi}} \\ \dot{\tilde{\psi}} &= -\gamma_0 \tilde{\psi} + \dot{\boldsymbol{\phi}}^T \tilde{\boldsymbol{\theta}}, \end{aligned} \tag{27}$$

which can be shown to be exponentially stable for some $\gamma_0 > 0$ and $\boldsymbol{\Gamma} > \mathbf{0}$, provided $\dot{\boldsymbol{\phi}}$ and $\boldsymbol{\phi}$ are bounded and

$$\int_t^{t+T} \boldsymbol{\phi}(\tau, q(\tau)) \boldsymbol{\phi}(\tau, q(\tau))^T d\tau \geq \varepsilon \mathbf{I}.$$

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