A SIMPLE DYNAMIC MODEL OF DRILLING FOR CONTROL...

Glenn-Ole Kaasa

Research Centre Porsgrunn
Hydro Oil and Energy, N-3908 Porsgrunn, Norway
Email: glenn-ole.kaasa@hydro.com

Abstract: A simple dynamic model of well drilling suited for control design...

Keywords: Drilling, Model, Control

INTRODUCTION

Background. During well drilling, a drilling fluid (mud) is pumped into the drill string topside and through the drill bit at the bottomhole of the well. The mud then transports cuttings in the annulus side of the well (*i.e.* in the wellbore outside the drill string) up to the drill rig, where a choke valve and a backpressure pump is used to control the annular pressure [1]. See Figure 1 for a schematic overview of the system.

A main objective is to precisely control the annular pressure profile troughout the wellbore continuously while drilling, *i.e.* to maintain the annular pressure in the well above the pore or collapse pressure and below the fracture or sticking pressure. Basically, this amounts to stabilize the downhole annular pressure at a critical depth within its margins, *i.e.* either at a particular depth where the pressure margins are small, or at the drillbit where conditions are the most uncertain. Basically, two strategies for closed-loop control of the choke are used:

Indirect topside control The bottomhole pressure is indirectly stabilized by applying feedback control to stabilize the topside annulus pressure instead, where the pressure setpoint corresponding to a desired bottomhole pressure is calculated online using a steady-state model. This strategy is the most common and straightforward mainly due to the availability of high-frequency and robust topside pressure measurements.

Direct bottomhole control The bottomhole pressure at the critical depth is stabilized at a desired setpoint directly. Even though a bottomhole measurement usually exists (at least near the bit), an estimate of the pressure is needed between samples because to the transfer rate of the measurement is usually slow, or for additional safety because the sensor itself may be unreliable (since it is exposed to a more hazardous environment).

State-of-the-art solutions typically employ conventional PI control applied to the choke, using one of the above strategies. There are significant drawbacks with both strategies. In both cases, the PI controller relies heavily on integral action to balance the pressure drop caused by friction, which is significant, and the proportional feedback gain must be low to prevent generating pressure pulses by fast changes in the control input. As a result, the control system based on conventional PI control will react slow to fast pressure changes,

[1] Various system configurations and special equipment, other than choke and pump, exists to enable control of the annular pressure. These are not considered in this work.

which *e.g.* results from movements of the drill string. In other words, the disturbance attenuating properties of the controller in these cases are relatively poor.

Another drawback, is the uncertainty in the modelled bottomhole pressure, due to uncertainties in the friction and mud compressibility parameters in both the drill string and annulus. Typically, the model is calibrated by tuning these parameters to fit the measured bottomhole pressure. This is typically a computation routine that is initiated manually.

Litterature review. ...

Motivation. There are significant potentials to improve existing algorithms, either the control law itself, or the observer used to estimate the critical downhole pressure. Model-based control enables improved compensation of pressure fluctuations during particular critical drilling operations, such as *e.g.* operations involving movements of the drill string where the resulting pressure change is straightforward to predict by a model. Also, by using model-based compensation with adaptation of uncertain parameters rather than integral action in the controller, one typically enable faster reaction to changes in setpoints and disturbances (such as *e.g.* a blow-out). An example is model-based compensation of friction by a feedforward term with online adaptation of the friction cooefficient. Furthermore, model-based coordinated control of both the choke and the backpressure pump improves controllability and thus performance in case of shut-down and start-up of the mud pump, *e.g.* during pipe connections. Precise and robust estimation of the downhole pressure during drilling allows for reduced pressure margins, and operations closer to balance. Online adaptation of model parameters is a way of extracting more information from the system, which can be used as fault detection.

Some of the main motivating factors for improved drilling performance, *i.e.*, being able to control the annular downhole pressure more precisely and reduce pressure fluctuations during critical operations and disturbances, are summarized by:

- Reduced formation damage, which is a significant problem in case of unstable formations and borehole stability problems.
- Improved rate of penetration (increased oil recovery and production rate) due to reduced skin (cleaner wells).
- Being able to reach "undrillable reserves" where pressure margins are small, like mature fields with depleted reservoirs, or deep-water wells.

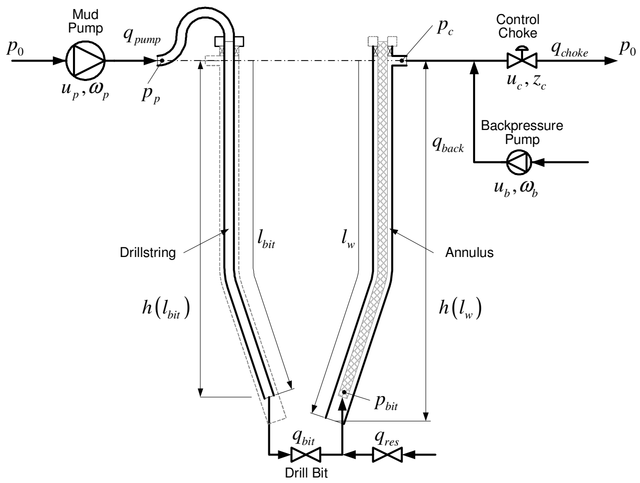

Fig. 1. A simplified schematical drawing of the drilling system.

- Faster drilling operations, *e.g.* improved drilling rates, reduced non-productivity time, reduced downtime (*e.g.* due to kick incidents).
- Reduced loss of circulation mud.

In addition, improved pressure control is a tool to mitigate drilling hazards by ensuring consistent control, and improved

- Disturbance attenuation and robustness
- Downhole pressure estimation
- Fault detection.

Contribution. The main contribution of this paper is the derivation of a simple dynamic model of the system that can be used for observer and model-based control design.

Overview of note...

1. SYSTEM/PROCESS DESCRIPTION

System. A simplified schematic view of the system is given in Figure 1.

2. FLUID FLOW FUNDAMENTALS

In this note we derive a model based on the assumption that the drilling fluid (mud) can be treated as a hydraulic fluid, *i.e.* a liquid. The discrepancy resulting from this simplification will be discussed.

In addition to the viscosity μ being a function of pressure and temperature, the flow of a viscous fluid is completely described by the following four fundamental equations[2].

Equation of State The density as a function of pressure and temperature, $\rho = \rho(p, T)$.

[2] The derivation and description follows Merrrit (1967) to some extent, with details supplemented by White (1994).

Equation of Continuity The mass balance describing the conservation of mass of the fluid, mainly related to the pressure p.

Equation of Momentum The force balance, or Newton's 2nd law of motion of the fluid. Mainly describing the velocity v, or equivalently, the flow rate of the fluid, as a function of pressure.

Conservation of Energy Law of conservation of energy, or the first law of thermodynamics, describing the temperature T.

2.1 Basic assumptions

We make the following assumptions:

(1) We assume turbulent flow, *i.e.* $\mathrm{Re} > \mathrm{Re}_{crit}$, where Reynold's number is defined as

$$\mathrm{Re} = \frac{\rho v D}{\mu}.$$

The range of transition from laminar to turbulent is $\mathrm{Re}_{crit} \in [2000, 4000]$, often taken as $\mathrm{Re}_{crit} = 2300$ White (1994).

(2) We assume that all flow is one-dimensional along the main flow path (drill string, or annulus), *i.e.* time-averaging the fluctuations due to turbulence, and neglecting the momentum effects of cross- or swirl flow due to rotation of the drill string. Note, however, that the resulting friction increase due to swirl flow, and fluctuations of turbulence, *etc.* is not neglected, but is accounted for by an increased friction coefficient.

(3) To simplify further, we assume homogeneous cross sectional conditions, *i.e.* averaging properties over the cross section of the flow.

(4) We assume that density effects in the flow are negligible so that we can treat the flow as incompressible and take density as constant in the momentum equation[3]: $\rho = \rho_0$. Note, however, that we will take compressibility, or the spring effect of the liquid, to account in the continuity equation.

(5) We assume that changes in temperature has negligible effect on the flow, and take temperatures as constant. Even though significant temperature gradients may exist, this is justified because the thermal expansion coefficients for liquids are small, thus density changes due to temperature changes are negligible.

[3] Density effects in the flow does not become significant before the flow velocity approaches the speed of sound of the fluid. In particular, for Mach numbers less than 0.3, the flow is generally termed *incompressible* White (1994).

2.2 Viscosity

The primary effect of the viscosity is on frictional losses in the flow. The viscosity of liquids increases with temperature and pressure, where the dependence on pressure is negligible compared to dependence on temperature. Since temperature is slowly varying, the resulting changes in frictional losses due to viscosity, are slowly varying and are treated as constant.

2.3 The equation of state

Since the changes in density as a function of pressure and temperature are small for a liquid, it is normal to use the linearized equation of state

$$\rho = \rho_0 + \left.\frac{\partial \rho}{\partial p}\right|_{T_0} (p - p_0) + \left.\frac{\partial \rho}{\partial T}\right|_{p_0} (T - T_0)\,,$$

where ρ_0, p_0, and T_0 are standard conditions, or the reference point the density is linearized around. In accordance with assumption 5 above, we assume that the density changes due to temperature is negligible such that the last term can be omitted.

For liquids it is normal to define the *bulk modulus* β of the fluid

$$\beta \triangleq \rho_0 \left.\frac{\partial p}{\partial \rho}\right|_{T_0} = -V_0 \left.\frac{\partial p}{\partial V}\right|_{T_0}, \qquad (1)$$

which is related to the speed of sound c according to

$$c = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\frac{\beta}{\rho}}.$$

The equation of state becomes

$$\rho = \rho_0 + \frac{\rho_0}{\beta} (p - p_0)\,, \qquad (2)$$

or in differential form

$$d\rho = \frac{\rho}{\beta} dp. \qquad (3)$$

Remark 1. Often, the reference point is taken at zero pressure such that the equation of state reduces to

$$\rho = \rho_0 + \frac{\rho_0}{\beta_0} p,$$

and ρ_0 and β_0 are the density and bulk modulus at atmospheric conditions $p_0 = 0$ (relative pressure).

Remark 2. The bulk modulus is a measure of the compressibility, or stiffness, of the liquid, and is thus important with respect to dynamic properties of the system. Liquids are in many cases treated as incompressible because the bulk modulus is very high. For example for pure water, the bulk modulus is approximately $1.5 \cdot 10^6$. However, the bulk modulus decreases sharply with small amounts of air entrained in the liquid (or due mechanical compliance of hoses), such that the effective bulk modulus is usually significantly lower Merrrit (1967).

Remark 3. The effective or total bulk modulus can be obtained experimentally from (1) by measuring the resulting pressure increase Δp when compressing a container of initial volume V_0 by ΔV:

$$\beta_e = \frac{V_0 \Delta p}{\Delta V}.$$

2.4 The continuity equation

For one-dimensional flow with cross sectional area A, the differential equation of continuity becomes

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0, \tag{4}$$

which integrated over a deformable control volume with length L, becomes

$$\frac{d}{dt}\left(\int_0^L \rho A dx\right) = \sum_i w_{in,i} - \sum_i w_{out,i}, \tag{5}$$

where $w_{in,i}$ and $w_{out,i}$ are the inlet and outlet flow respectively, where the integral term is the total mass in the control volume

$$m = \int_0^L \rho A dx.$$

2.5 The momentum equation

Applying the above simplifications, the differential equation of momentum reduces (from the full Navier-Stokes equations) to the one-dimensional equation

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial x} - \frac{1}{A}\frac{\partial F}{\partial x} + \rho g \frac{\partial h}{\partial x}, \tag{6}$$

which is much simpler, but still relatively accurate with respect to averaged flow variables. Here, x is the length coordinate along the flow path, $v = dx/dt$ the velocity of the flow, A is the cross sectional area of the flow, F is the friction force acting on the flow, and h is the depth of the pipe/flow path, positive direction defined downwards. See (White, 1994, p. 304).

Remark 4. Note that F is a lumped force that accounts for all frictional losses due to viscous dissipation, turbulence, swirl flow, and non-ideal flow conditions causeed by restrictions, bends, *etc.*

2.6 The energy equation

We assume constant temperatures, which eliminates the need for the energy equation.

3. MODEL DERIVATION

Our objective is to derive a mechanistic model which can be used for model-based control and observer design. Since the model should not be more detailed than required, we pursue to obtain a reduced-order model that captures only the dominant phenomena of the system, *i.e.* we neglect fast dynamics (and treat unmodelled dynamics as “fast parasitics” in the design). We may also lump together phenomena/parameters with similar effect, and treat slowly varying parameters as constants, assuming this can be handled by online adaptation in a controller, or observer.

3.1 Frictional flow losses

The frictional losses for turbulent incompressible flow can be described by simple empirical equations for the pressure drop as a function of flow rate, where various frictional effects are lumped together in empirical flow coefficients. For special cases, these equations can be obtained by integration of the simple one-dimensional momentum equation (6), letting the various frictional effects be lumped together by the empirical flow coefficients in the friction gradient $\partial F/\partial x$. We review the relevant of these equations below, in order to relate the empirical flow coefficients to the friction gradient.

3.1.1. Pipe flow For pipe flow, the friction loss term can be given as

$$\frac{\partial F}{\partial x} = S\tau_w, \tag{7}$$

where S is the total perimeter of the cross sectional area, and τ_w is the wall shear stress such that the friction force on the volume element Adx is $\tau_w S dx$. For pipe flow, the wall shear stress can be determined by the dimensionless *Darcy's friction factor* f, through the relation

$$\tau_w = f\frac{1}{4}\frac{\rho}{2}v^2. \tag{8}$$

Neglecting the acceleration and gravity term in (6), the pressure gradient becomes

$$\frac{\partial p}{\partial x} = f\frac{S}{A}\frac{1}{4}\frac{\rho}{2}v^2,$$

Hence, for a horizontal pipe with diameter D and length L, the steady-state pressure drop for incompressible, turbulent flow is obtained by integration to be

$$p_1 - p_2 = f\frac{L}{D}\frac{\rho}{2}v^2. \tag{9}$$

Remark 5. Usually, a reasonable estimate of the friction factor for smooth pipes is given by

$$f = 0.316\,\mathrm{Re}^{-\frac{1}{4}} \tag{10}$$

developed by Blasius, which applies for Reynolds numbers less than 10^5. However, since the drilling mud is a non-newtonial fluid, this estimate may be very crude.

3.1.2. Minor losses In addition to friction along the pipe, there are additional so-called minor losses due to bends, section changes and other flow restrictions. For incompressible flow, the equation for these pressure drops (which need not be minor at all), takes the same form as for pipe flow

$$p_1 - p_2 = K \frac{\rho}{2} v^2 \tag{11}$$

where K is an empirical loss coefficient.

Remark 6. For example, for a sudden expansion, *i.e.* flow into a large reservoir, all kinetic energy is lost due to viscous dissipation, and it is straightforward to show that $K = 1$.

3.1.3. Orifice flow A special case of minor loss, is turbulent flow through an orifice or restriction where the minimum cross-sectional area A at the throat is much smaller than the upstream area. Then, the flow velocity v is given by the equation

$$v = C_d \sqrt{\frac{2}{\rho}\left(p_1 - p_2\right)}.$$

where C_d is the discharge coefficient which accounts for an additional flow contraction (known as *vena contracta*, which occurs typically half a diameter downstream the throat of the orifice), plus additional frictional losses. The equation for the volume flow rate q through the restriction, is thus given as

$$q = AC_d \sqrt{\frac{2}{\rho}\left(p_1 - p_2\right)}. \tag{12}$$

Remark 7. Note that pressure p_2 refers to the pressure at *vena contracta* just downstream the minimum restriction area A. For valves and similar restrictions, there is typically a sudden expansion downstream the restriction such that the kinetic energy is typically lost due to viscous dissipation and the upstream pressure p_1 is not recovered. This means that (13) is usually a good model of the flow rate even if p_2 is replaced with a pressure measurement further downstream the restriction.

Rearranging, the pressure loss is given by

$$p_1 - p_2 = \frac{1}{C_d^2} \frac{\rho}{2} v^2$$

Comparing with (11) for minor losses, we see that this corresponds to a loss coefficient

$$K = \frac{1}{C_d^2}. \tag{13}$$

Remark 8. For an ideal restriction with no losses, the discharge coefficient becomes $C_d = 1$. In practice, due to losses, the discharge coefficient will be in the range $0.80 - 0.90$ for orifices with rounded edges, *e.g.* $C_d \in [0.92, 0.98]$ for a venturi nozzle, and in the range $0.60 - 0.70$ for orifices with sharp edges, *e.g.* $C_d \in [0.60, 0.65]$ for a thin-plate orifice (White, 1994, page 364–366).

3.1.4. Friction gradient The friction gradient $\partial F / \partial x$ consists of both pipe flow and minor losses. The friction gradient for pipe flow losses is given in terms of the friction factor f according to (7)–(8)

$$\frac{\partial F}{\partial x} = S \frac{1}{4} f \frac{\rho}{2} v^2. \tag{14}$$

The minor losses given by given by (11) with loss coefficient K over a length ΔL, can be related to the friction gradient $\partial F / \partial x$ according to

$$\frac{\partial F}{\partial x} = A \frac{\partial K}{\partial x} \frac{\rho}{2} v^2, \tag{15}$$

where the minor loss gradient is simply

$$\frac{\partial K}{\partial x} = \frac{K}{\Delta L}.$$

For a given mass flow w or volume flow q, the pipe flow velocity v is determined by the cross-sectional area $A(x)$ of the pipe according to

$$v = \frac{w}{\rho A(x)} = \frac{q}{A(x)}. \tag{16}$$

Summarizing, the total friction gradient of pipe flow and minor losses can be expressed as

$$\begin{aligned} \frac{\partial F}{\partial x} = & \frac{1}{4} f S(x) \frac{\rho}{2} \left(\frac{q}{A(x)}\right)^2 \\ & + \frac{\partial K}{\partial x} A(x) \frac{\rho}{2} \left(\frac{q}{A(x)}\right)^2, \end{aligned} \tag{17}$$

where $A(x)$ and $S(x)$ are the cross sectional area and perimeter of the flow, and $\partial K / \partial x$ the minor loss gradient along the flow path.

3.2 Steady-state pressure profile

Since changes in the flow rate are usually slow compared to the acceleration of the fluid, the pressure distribution along the flow path is dominated by its steady-state characteristics. The steady-state characteristics of the flow is obtained by neglecting the acceleration term in the momentum equation and integrating along the flow path.

Neglecting the acceleration term in (6), the pressure gradient is given by

$$\frac{dp}{dx} = -\frac{1}{A(x)} \frac{\partial F}{\partial x} + \rho g \frac{\partial h}{\partial x}.$$

Substituting (17) for the friction gradient $\partial F/\partial x$, and taking the density as constant $\rho = \rho_0$ (according to assumption 4), we can write

$$\begin{aligned} dp = & -\frac{1}{4} f \frac{S(x)}{A(x)} \frac{\rho_0}{2} \left(\frac{q}{A(x)} \right)^2 dx \\ & -\frac{\partial K}{\partial x} \frac{\rho_0}{2} \left(\frac{q}{A(x)} \right)^2 dx \\ & +\rho_0 g \partial h. \end{aligned}$$

Integrating the pressure along the flow path x,

$$\begin{aligned} \int_{p(0)}^{p^*(l)} dp = & \, p^*(l) - p(0) \\ = & -\int_0^l \left(\frac{1}{4} f \frac{S(x)}{A(x)} + \frac{\partial K}{\partial x} \right) \frac{\rho_0}{2} \left(\frac{q}{A(x)} \right)^2 dx \\ & + \int_{h(0)}^{h(l)} \rho_0 g \partial h. \end{aligned}$$

The steady-state pressure profile of the flow, $p^*(l)$, can thus be given as

$$\begin{aligned} p^*(l) = & \, p(0) - [B_0(l) + f B_1(l)] \frac{\rho_0}{2} q^2 \\ & + \rho_0 g [h(l) - h(0)], \end{aligned} \tag{18}$$

where the frictional loss coefficients B_0 and B_1 are defined by

$$B_0(l) \triangleq \int_0^l \frac{\partial K}{\partial x} \frac{1}{A(x)^2} dx \tag{19}$$

$$B_1(l) \triangleq \int_0^l \frac{1}{4} \frac{S(x)}{A(x)^3} dx. \tag{20}$$

3.3 Complete PDE model

3.3.1. Pressure dynamics From the differential continuity equation (4) we get

$$\begin{aligned} A dx \frac{\partial \rho}{\partial t} &= -A(\rho v) = -dw \\ & \Downarrow \\ \frac{\partial \rho}{\partial t} &= -\frac{1}{A} \frac{dw}{dx}. \end{aligned}$$

Substituting with the equation of state (3),

$$d\rho = \frac{\rho}{\beta} dp,$$

and rewriting in terms of volume flow according to $w = \rho q$, we obtain

$$\frac{\partial p}{\partial t} = -\frac{\beta}{A} \frac{dq}{dx}. \tag{21}$$

3.3.2. Flow dynamics In terms of volume flow $q = \rho A$, the momentum equation can written as

$$\rho \frac{\partial q}{\partial t} = -A \frac{\partial p}{\partial x} - \frac{\partial F}{\partial x} + A \rho g \frac{\partial h}{\partial x}.$$

The complete PDE model in differential form is given as

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\frac{\beta}{A} \frac{dq}{dx} \\ \rho \frac{\partial q}{\partial t} &= -A \frac{\partial p}{\partial x} - \frac{\partial F}{\partial x} + A \rho g \frac{\partial h}{\partial x}. \end{aligned}$$

3.4 Simplified ODE model

For model-based control, an even simpler ODE model is desirable.

3.4.1. Simplified pressure dynamics To derive a simplified equation describing the pressure dynamics at a point, it is conventient to represent the mass m in the form

$$m = \bar{\rho} V,$$

where $\bar{\rho}$ is the average density, defined by

$$\bar{\rho} \triangleq \frac{1}{V} \int_0^L \rho A dx.$$

The left-hand side of the continuity equation (5) can then expressed in the form

$$\begin{aligned} \frac{d}{dt} \left(\int_0^L \rho A dx \right) &= \frac{d}{dt} (\bar{\rho} V) \\ &= V \frac{d\bar{\rho}}{dt} + \bar{\rho} \frac{dV}{dt}. \end{aligned}$$

To change from density to pressure as our main variable, we use (3)

$$d\bar{\rho} = \frac{\bar{\rho}}{\beta} d\bar{p}.$$

Substituting this, gives

$$\begin{aligned} \frac{d}{dt} \left(\int_0^L \rho A dx \right) &= V \frac{d\bar{\rho}}{dt} + \bar{\rho} \frac{dV}{dt} \\ &= V \frac{\bar{\rho}}{\beta} \frac{d\bar{p}}{dt} + \bar{\rho} \frac{dV}{dt}. \end{aligned}$$

Replacing the left-hand side integral term in (5) with its right-hand side (dropping the summation operators for breviety of notation), gives

$$\begin{aligned} \bar{\rho} \left(\frac{V}{\beta} \frac{d\bar{p}}{dt} + \frac{dV}{dt} \right) &= \frac{d}{dt} \left(\int_0^L \rho A dx \right) \\ &= w_{in} - w_{out} \\ & \Downarrow \\ \frac{V}{\beta} \frac{d\bar{p}}{dt} + \frac{dV}{dt} &= \frac{1}{\bar{\rho}} (w_{in} - w_{out}). \end{aligned}$$

We further assume that the inlet and outlet mass flows be given in terms of the average density according to

$$w = \bar{\rho} q.$$

The differential equation describing the dynamics of the average pressure then simplifies to

$$\frac{V}{\beta}\frac{d\bar{p}}{dt} + \dot{V} = q_{in} - q_{out}, \tag{22}$$

where $\dot{V}$ denotes the time-derivative of the control volume, and q_{in} and q_{out} are the inlet and outlet volume flows, respectively.

3.4.2. Simplified flow dynamics To obtain a simplified model describing the dynamics of the flow rate we consider the average flow rate $\bar{q}$ through a pipe. Since the acceleration term $d\bar{v}/dt$ for the average flow $\bar{q}$ is constant along the flow path, the momentum equation (6) can be written as

$$\rho dx \frac{d\bar{v}}{dt} = -\frac{\partial p}{\partial x}dx - \frac{1}{A}\frac{\partial F}{\partial x}dx + \rho g \frac{\partial h}{\partial x}dx.$$

Inserting $\bar{q} = A\bar{v}$, and taking density as constant $\rho = \rho_0$, we can write

$$\frac{\rho_0}{A(x)}dx\frac{d\bar{q}}{dt} = -\partial p - \frac{1}{A(x)}\frac{\partial F}{\partial x}dx + \rho_0 g \partial h.$$

Integration along the flow path x, gives

$$\begin{aligned}\int_0^l \frac{\rho_0}{A(x)}dx\frac{d\bar{q}}{dt} = &-\int_{p(0)}^{p(l)} \partial p \\ &-\int_0^l \frac{1}{A(x)}\frac{\partial F}{\partial x}dx \\ &+\int_{h(0)}^{h(l)} \rho g \partial h.\end{aligned}$$

Inserting (17) for the friction gradient $\partial F/\partial x$, the simplified flow dynamics can thus be expressed in the form

$$\begin{aligned}\frac{\rho_0 l}{\bar{A}}\frac{d\bar{q}}{dt} = &\, p(0) - p(l) \\ &- \left[B_0(l) + f B_1(l)\right]\frac{\rho_0}{2}\bar{q}^2 \\ &+ \rho_0 g\left[h(l) - h(0)\right],\end{aligned} \tag{23}$$

where $\bar{A}$ is the average cross sectional area of the pipe, defined by

$$\bar{A} \triangleq \frac{1}{l}\int_0^l A(x)\,dx \tag{24}$$

and the friction coefficients B_0 and B_1 as

$$B_0(l) \triangleq \int_0^l \frac{\partial K}{\partial x}\frac{1}{A(x)^2}dx \tag{25}$$

$$B_1(l) \triangleq \int_0^l \frac{1}{4}\frac{S(x)}{A(x)^3}dx. \tag{26}$$

Remark 9. Note that gravity term in (23) is positive as long as the volume flow $\bar{q}$ is defined as positive in the same direction as the depth h.

3.5 Drill string

We assume that the topside mud pump pressure p_{pump} can be described by (22), and that the flow q_d through the drill string can be described by (23)–(26).

Pressure dynamics. The mud pump pressure is thus given by

$$\frac{V_d}{\beta_d}\dot{p}_p = q_{pump} - q_{bit}, \tag{27}$$

where q_{pump} is the volume flow rates through the mud pump, $q_{bit} = q_d$ is the flow through the drill bit given by (30), and the parameters V_d and β_d, are the volume and the bulk modulus of the drill string mud, respectively. Note that V_d is constant between each pipe connection, hence we have taken $\dot{V}_d = 0$.

Mud pump flow. The pump flow $q_{pump}(\omega_p)$ is given by the pump speed ω_p according to

$$q_{pump} = N_p V_p 2\pi\omega_p, \tag{28}$$

where $\omega_p\,\mathrm{rad/s}$ is the rotational speed of the pump given by (29), and the parameters N_p and V_p are the number of pistons and volume per stroke per piston, respectively.

Mud pump dynamics. We let the dynamics of the pump speed ω_p be given by

$$\tau_p\dot{\omega}_p = -\omega_p + K_{pump}u_p, \tag{29}$$

where $u_p \in [0,1]$ is the control input, τ_p the time constant, and K_{pump} the steady-state input gain of the mud pump.

Flow dynamics. The volume flow through the drill string $q_d = q_{bit}$ is desribed by

$$\begin{aligned}\frac{\rho_{d0}L_{dN}}{\bar{A}_d}\dot{q}_d = &\, p_p - p_{bit} \\ &- \left(B_{d0} + f_d B_{d1}\right)\frac{\rho_{d0}}{2}q_d^2 \\ &+ \rho_{d0} g h_{bit},\end{aligned} \tag{30}$$

where p_{bit} is the bit pressure at the annulus side at location l_{bit} at the depth $h_{bit} = h(l_{bit})$. The parameter L_{dN} is the total length, ρ_{d0} a constant average density, and $\bar{A}_d$ the average cross sectional area of the drill string, given by

$$\bar{A}_d = \frac{1}{L_{dN}}\int_0^{L_{dN}} A_d(x)\,dx, \tag{31}$$

where $A_d(x)$ is the cross sectional area at location x. The total minor losses along the drill string is given by

$$B_{d0} = \int_0^{L_{dN}} \frac{\partial K_d}{\partial x} \frac{1}{A_d(x)^2} dx, \tag{32}$$

where $\partial K_d / \partial x$ is the minor loss gradient at location x of the drill string. The pipe friction losses are given by the friction factor f_d and the coefficient

$$B_{d1} = \int_0^{L_{dN}} \frac{1}{4} \frac{S_d(x)}{A_d(x)^3} dx, \tag{33}$$

where $S_d(x)$ is the perimeter of the total cross sectional friction surface of the drill string.

3.6 Annulus

In a similar manner as for the drill string, we let the annular choke pressure p_c be described by (22), and the flow q_a through the annulus by (23)–(26).

Pressure dynamics. The choke pressure p_c is thus given by

$$\frac{V_a}{\beta_a} \dot{p}_c + \dot{V}_a = q_{bit} + q_{res} + q_{back} - q_{choke}, \tag{34}$$

where q_{bit} is the volume flow rate through the bit, q_{res} a disturbance reservoir influx (which should be zero under normal operation), q_{back} the flow from the back pressure pump, and q_c the flow rate through the choke valve. The parameter β_a is the effective bulk modulus of the annulus mud.

The choke flow q_c is modelled by the orifice equation (12) as

$$q_{choke} = K_c z_c \sqrt{\frac{2}{\rho_0}(d - p_0)}, \tag{35}$$

where $z_c \in [0, 1]$ is the normalized choke opening given by (**??**), p_0 the pressure downstream the valve, and the parameter K_c the valve flow constant. The flow constant K_c can be given by

$$K_c = A_c C_{d,c},$$

where A_c is the valve opening at fully open valve, and $C_{d,c}$ the corresponding discharge coefficient of the choke valve.

Choke valve dynamics. A simple model of the choke opening z_c is given by

$$\tau_c \dot{z}_c = -z_c + u_c, \tag{36}$$

where $u_c \in [0, 1]$ is the normalized control input, and τ_c the time constant of the dynamics.

Backpressure flow. The backpressure flow $q_{back}(\omega_b)$ is given by the pump speed ω_b according to

$$q_{back} = N_b V_b 2\pi \omega_b, \tag{37}$$

where ω_b rad/s is the rotational speed of the pump given by (29), and the parameters N_b and V_b are the number of pistons and volume per stroke per piston, respectively.

Backpressure dynamics. We let the dynamics of the back pressure pump speed ω_{back} be given by

$$\tau_b \dot{\omega}_b = -\omega_b + K_{back} u_b, \tag{38}$$

where $u_{back} \in [0, 1]$ is the control input, τ_b the time constant, and K_{back} the input gain of the back pressure pump.

Annulus and well volume. The volume $V_a(l_{bit}, l_w)$ in the annulus side of the well, depends on both the position of the bit l_{bit} and the length of the well l_w, and can be given by

$$V_a = A_a l_{bit} + A_w \max(l_w - l_{bit}, 0), \tag{39}$$

where $A_a(l_{bit})$ and $A_w(l_{bit})$ are the cross sectional area of the annulus (around the drill string) and the well at the drill bit, l_{bit}. The length l_w of the drilled well can be described by

$$\frac{dl_w}{dt} = \begin{cases} \dot{l}_{bit} & , \ l_{bit} = l_w \wedge \dot{l}_{bit} \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases} \tag{40}$$

Flow dynamics. The volume flow $q_a = q_{bit} + q_{res}$ through the annulus is governed by

$$\begin{aligned} \frac{\rho_{a0} l_{bit}}{\bar{A}_a} \dot{q}_a = & \, p_{bit} - p_c \\ & - (B_{a0} + f_a B_{d1}) \frac{\rho_{a0}}{2} q_a^2 \\ & + \rho_{d0} g h_{bit}, \end{aligned} \tag{41}$$

where p_c is the pressure downstream the choke valve, ρ_{a0} a constant average density, and the parameter $\bar{A}_a$ the average cross sectional area of annulus, given by

$$\bar{A}_a = \frac{1}{l_{bit}} \int_0^{l_{bit}} A_a(x)\, dx, \tag{42}$$

where $A_a(x)$ is the cross sectional area at location x. The total minor losses along the annulus is given by the loss coefficient

$$B_{a0} = \int_0^{l_{bit}} \frac{\partial K_a}{\partial x} \frac{1}{A_a(x)^2} dx, \tag{43}$$

where $\partial K_a / \partial x$ is the minor loss gradient at location x of the annulus. The pipe friction losses are given by the friction factor f_a and the coefficient

$$B_{a1} = \int_0^{l_{bit}} \frac{1}{4} \frac{S_a(x)}{A_a(x)^3} dx, \tag{44}$$

where $S_a(x)$ is the perimeter of the total cross sectional friction surface of the drill string.

Remark 10. Note that the parameters $\bar{A}_a(l_{bit})$, $B_{a0}(l_{bit})$ and $B_{a1}(l_{bit})$ depends on the location l_{bit} of the drill bit. Typically, the annulus diameter will be piecewise constant along the flow path, which means that $\bar{A}_a$ and B_{a0} will also be constant, while $B_{a1}(l_{bit})$ will depend linearly on l_{bit} within a section with constant annulus diameter.

4. DESIGN MODEL

The equations derived in Section 3 constitute the simplified model of the drilling dynamics, which we will use in the design of an observer and model-based control, thus it is referred to as the design model.

The pump pressure p_p and choke pressure p_c are given by (27) and (34), while the dynamics of q_{bit} is described by both the flow dynamics (30) of the drill string and of the annulus (41). Combining both equations, the downhole bit pressure p_{bit} is cancelled, and the flow q_{bit} is described by a single equation.

Summarizing, the main dynamics of the drilling system can be described by

$$\begin{aligned}
\frac{V_d}{\beta_d}\,\dot{p}_p &= q_{pump} - q_{bit} \\
\frac{V_a}{\beta_a}\,\dot{p}_c &= -\dot{V}_a + q_{bit} + q_{res} \\
&\quad + q_{back} - q_{choke} \\
\left[M_d + M_a\right]\dot{q}_{bit} &= p_p - p_c \\
&\quad - F_d q_{bit}^2 - F_a\left(q_{bit} + q_{res}\right)^2 \\
&\quad + \left(\rho_{d0} - \rho_{a0}\right) g h_{bit},
\end{aligned} \tag{45}$$

where states p_p and p_c are the inlet mud pump and outlet choke pressure, q_{bit} the flow through the drill bit, and $q_{res}(t)$ the reservoir influx, which is an unknown disturbance that should be zero under normal conditions.

The main controlled quantity is the choke flow $q_{choke}(p_c, z_c)$, which is given by (35),

$$q_{choke} = K_c z_c \sqrt{\frac{2}{\rho_0}\left(p_c - p_0\right)}.$$

The choke opening z_c is given by the actuator dynamics (36), which is typically negligible fast compared to the remaining system such that for control design, z_c can be taken as

$$z_c = u_c.$$

The mud pump flow $q_{pump}(\omega_p)$ and back pressure flow $q_{back}(\omega_b)$ are usually treated as known inputs, assuming negligible leakage such that the pump characteristics (28) and (37) provide accurate estimates of the flows given the pump speeds ω_p and ω_b are known.

Remark 11. In existing drilling systems, the pump speeds ω_p and ω_b are either controlled manually, or by decentralized control loops with manual set points. Note, however, that it may be advantageous to control $q_{pump}(\omega_p)$ and $q_{back}(\omega_b)$ actively, coordinated with the choke flow $q_{choke}(p_c, z_c)$, in order to improve the disturbance attenuating properties of the drilling system.

The annulus volume $V_a(l_{bit}, l_w)$ is given by

$$V_a = A_a l_{bit} + A_w \max\left(l_w - l_{bit}, 0\right)$$

where l_w is the length of the well which can be modelled according to (40), and which is indirectly measured during drilling via the location of the drill bit, l_{bit}.

The true vertical depth of the drill bit, h_{bit}, is given indirectly by the geometry of the well path as a function of the location of the drill bit according to $h_{bit} = h(l_{bit})$.

In the flow dynamics, the mass and friction coefficients for the drill string are defined as

$$M_d \triangleq \frac{\rho_{d0} L_{dN}}{\bar{A}_d} \tag{46}$$

$$F_d \triangleq \left(B_{d0} + f_d B_{d1}\right)\frac{\rho_{d0}}{2}, \tag{47}$$

where the average cross sectional area $\bar{A}_d$ is given by (31), and the friction coefficients B_{d0} and B_{d1} by (32) and (33). Similarly, the coefficients $M_a(l_{bit})$ and $F_a(l_{bit})$ for the annulus flow, are defined as

$$M_a \triangleq \frac{\rho_{a0} l_{bit}}{\bar{A}_a(l_{bit})} \tag{48}$$

$$F_a \triangleq \left[B_{a0}(l_{bit}) + f_a B_{a1}(l_{bit})\right]\frac{\rho_{a0}}{2}, \tag{49}$$

where $\bar{A}_a(l_{bit})$ are given by (42), and $B_{a0}(l_{bit})$ and $B_{a1}(l_{bit})$ by (43) and (44).

4.1 Annular pressure profile

The main variable of interest is the annular downhole pressure p_a at critical locations l along the well. From (23), we obtain the annular downhole pressure as a function of location l according to

$$p_a(l) = p_c + M_a(l)\,\dot{q}_{bit} + F_a(l)\,q_{bit}^2 + \rho_{a0} g h(l). \tag{50}$$

We see that in the case $p_{bit} = p_a(l_{bit})$, (50) becomes identical to the flow dynamics of the annulus (41).

4.2 Measurements

Assuming conventional instrumentation for the system (45), we make the following assumptions regarding available measurements.

Topside measurements. The downstream choke pressure p_0 is assumed to be equal to atmospheric, *i.e.* constant.

The topside pressures p_p and p_c are measured with negligible sampling effects and sensor dynamics, such that the measurement can be given according to

$$p_{p,m} = p_p + w_p$$
$$p_{b,m} = p_b + w_b,$$

where $p_{p,m}$ and $p_{b,m}$ are measurements of p_p and p_b, respectively, and w_p and w_b are the corresponding measurement noise of the sensors.

The pump speeds ω_p and ω_b used to determine the flow rate $q_{pump}(\omega_p)$ and $q_{back}(\omega_b)$ are usually not actually measured, but given by the inputs u_p and u_b according to

$$\omega_{p,m} = K_{pump} u_p$$
$$\omega_{b,m} = K_{back} u_b$$

neglecting the pump dynamics (29) and (38).

The drill string is pulled down by its own weight and is positioned in desired position by the topside hook, where l_{bit} is given indirectly by the position of the hook, and the total length of the drill string. A measurement of l_{bit} is usually taken as

$$l_{bit,m} = l_{hook} + L_{dN},$$

where l_{hook} is the position of the hook, defined in accordance with the well coordinates such that $l_{hook} = 0$ just upstream the choke, and L_{dN} is the nominal (unstrained) length of the drill string. In practise, the drill string is strained due to its own weight. The measurement of l_{bit} is thus hampered with an uncertainty according to

$$l_{bit,m} = l_{bit} + \Delta l$$

where the elastic deformation $\Delta l(t) = \bar{\sigma}(t) L_{dN}/E$ of the drill string can be regarded as an unknown disturbance where the average stress $\bar{\sigma}(t)$ in the drill string is unknown.

We assume that the geometry of the well path is accurately known, such that the true vertical depth h_{bit} of the drill bit is a known function of l_{bit} according to $h_{bit} = h(l_{bit})$, and its uncertainty is given by the uncertainty in the measurement $l_{bit,m}$.

Downhole measurements. Conventionally, downhole measurements are transferred from the drill bit by frequency modulated pressure pulses through the mud. Since a lot of addional information (like seismic measurements, *etc.*) are also transmitted, this results in slow sampling and a significant time-delay of downhole measurements as only a few bit can be transferred per second.

The downhole measurement $p_{a,m}$ of the bit pressure p_{bit}, is sampled with sampling period T_s at every time instant $t_k \in [0, T_s, 2T_s, \cdots]$ using zero-order hold between samples, and can thus be given as

$$p_{a,m}(t_k) = p_a(t_k - T_s) + w_a(t_k - T_s),$$

where w_a is measurement noise (which is typically neglected).

Remark 12. In general, downhole measurements must be regarded as uncertain because downhole sensors are exposed to a hazardous environment, and loss of samples can occur due to disturbances of the frequency modulated pressure pulses.

4.3 Wired drill pipe and additional instrumentation

Additional topside measurements. A Coriolis measurement device can be mounted at the mud pump outlet to provide measurements of mass flow $w_{pump} = \rho_{d0} q_{pump}$ and density ρ_{d0}. ...

Wired drill pipe. ...provide high-frequency downhole measurements of pressures at several locations along the drill string...

4.4 Model uncertainties and discrepancies

Several simplifications and assumptions are applied to arrive at the simplified design model (45), and it is important to be aware of the resulting discrepancies in the model and when they may be significant.

Parametric uncertainties. Are discussed below.

Pump leakage. Due to leakage in the real displacement pumps, actual pump flow q_{pump} and q_{back} will be slightly lower than estimated by the models.

Non-newtonian friction. The drilling mud is a non-newtonian liquid which exhibits a Bingham plastic-like behavoir when the mud flow approaches zero velocity, which results in a stiction-like error in the modelled friction force.

4.5 Parametric uncertainties

Several parameters in the design model are highly uncertain, either because they are impossible to determine before actually drilling the well, or they are simply slowly time-varying. These are important uncertainties that must be handled effectively in an observer or controller design.

Frictional losses. In the design model (45), the most important uncertainties are related to frictional losses. In particular, the main uncertainty is related to the friction coefficients f_d and f_a, which may vary due to changes in temperature, mud properties, *etc.* There is also, initially, a significant parametric uncertainty related to minor losses, given by the lumped minor loss coefficients B_{d0} and B_{a0}.

Compressibility. The bulk modulus β_d and β_a are measures of the compressibility of the mud in the drill string and annulus, respectively, and is ideally an inherent property of the mud which is known and identical for the drill string and annulus. However, the effective bulk modulus depend strongly on the amount of entrapped gas in the mud, which again will depend on the conditions in the well. Under steady conditions (with no influx of gas from the reservoir), the bulk modulus of the mud will stabilise at some constant values, typically such that $\beta_d \approx \beta_a$. However, in the case of influxes of gas from the reservoir (even for very small amounts), the bulk modulus β_a of the annulus, may exhibit a significant reduction due to small amount of entrapped gas in the annulus. Consequently, the annululs bulk modulus β_a may be uncertain, and an estimate of β_a may serve as a measure, or indicator, for influx of gas.

Density. The average mud densities ρ_{d0} and ρ_{a0} will typically vary slightly with the pressure levels in the drill string and annulus, and will therefore typically increase slightly with the length of the well. When drilling, it is expected that the density ρ_{a0}, which initially is approximately equal to ρ_{d0}, will change slightly according to the properties of the rock which is being digged out from the bottom of the well, or due to reservoir influx q_{res}. The main effect of the density, is that it determine the mass coefficients M_d and M_a and consequently the time-constant of the flow dynamics. However, since the flow dynamics is derived based on a crude assumption that the mud is simply a rigid mass which is accelerated by the inlet and outlet pressures, it may be benefitial to view the mass coefficients as uncertain lumped parameters which can be estimated to fit the actual flow dynamics of the system.

Reservoir influx. The most important disturbance in the system is a non-zero reservoir influx $q_{res}(t)$ (or outflow if negative, *i.e.* loss of mud), which is is utmost important to detect in order to handle it by the right measures. An alternative, is to view q_{res} as a constant (or slow varying) disturbance, which is estimated in the observer or controller.

Remark 13. In general, conditions in the annulus are to a lesser extent known, such that geometry, surface conditions and mud properties are more uncertain than in the drill string.

5. IMPLEMENTATION

To implement the flow dynamics or pressure profile, all terms on the right-hand side of the momentum equation (6) must be parametrized as functions of the length coordinate x along the flow path.

Well properties. The coordinates of the well path is given by length and depth vectors,

$$\mathbf{L}_w = \left[L_{w1}\ L_{w2}\ \cdots\ L_{wN}\right]^T \tag{51}$$

$$\mathbf{H}_w = \left[H_{w1}\ H_{w2}\ \cdots\ H_{wN}\right]^T \tag{52}$$

such that the depth $h(x)$ at location x is given by interpolation according to

$$h(x) = H_{wk} + \frac{H_{w(k+1)} - H_{wk}}{L_{w(k+1)} - L_{wk}}\left(x - L_{wk}\right), \quad x \in \left[L_{wk}, L_{w(k+1)}\right\rangle, \tag{53}$$

where the pipe inclination is given as

$$\frac{\partial h(x)}{\partial x} = \frac{H_{k+1} - H_k}{L_{k+1} - L_k}, \quad x \in [L_k, L_{k+1}\rangle. \tag{54}$$

Likewise, the cross sectional diameter $D_w(x)$ of the well is piecewise constant function of x, and is therefore conveniently defined by $\mathbf{L}_w$ and

$$\mathbf{D}_w = \left[D_{w1}\ D_{w2}\ \cdots\ D_{wN}\right]^T \tag{55}$$

according to

$$D_w(x) = D_{wk}\chi\left[x \le l_w\right], \quad x \in \left[L_{wk}, L_{w(k+1)}\right\rangle. \tag{56}$$

Here l_w is the currently drilled length of the well, and $\chi[X]$ is the indicator function of the event X (which is 1 if true, otherwise 0) such that the diameter is zero for $x > l_w$.

Drill string properties. Like the annulus diameter, the cross sectional inner and outer (annulus) diameter along the drill string are piecewise constant, defined by the vectors

$$\mathbf{L}_d = \left[L_{d1}\ L_{d2}\ \cdots\ L_{dN}\right]^T \tag{57}$$

$$\mathbf{D}_d = \left[D_{d1}\ D_{d2}\ \cdots\ D_{dN}\right]^T \tag{58}$$

$$\mathbf{D}_a = \left[D_{a1}\ D_{a2}\ \cdots\ D_{aN}\right]^T, \tag{59}$$

such that D_{dk} and D_{ak} are the diameters corresponding to section $[L_{dk}, L_{d(k+1)}]$. The cross sectional area $A_d(x)$ and perimeter surface $S_d(x)$ of the drill string can thus be given as

$$A_d(x) = \frac{\pi}{4}D_{dk}^2, \quad x \in \left[L_{dk}, L_{d(k+1)}\right\rangle \tag{60}$$

$$S_d(x) = \pi D_{dk}, \quad x \in \left[L_{dk}, L_{d(k+1)}\right\rangle. \tag{61}$$

To implement minor losses along the drill string, it is convenient to define the vectors

$$\mathbf{K}_d = \begin{bmatrix} K_{d1} & K_{d2} & \cdots & K_{dN} \end{bmatrix}^T \tag{62}$$

$$\Delta\mathbf{L}_d = \begin{bmatrix} \Delta L_{d1} & \Delta L_{d2} & \cdots & \Delta L_{dN} \end{bmatrix}^T \tag{63}$$

where $\mathbf{K}_d$ and $\mathbf{\Delta L}_d$ are the loss coefficients and corresponding loss length at each location in $\mathbf{L}_d$. The minor loss gradient of the drill string can be given as

$$\frac{\partial K_d}{\partial x} = \mathbf{K}_d^T \boldsymbol{\phi}_d(x), \tag{64}$$

where the basis functions $\phi_i(x)$ in the regressor vector

$$\boldsymbol{\phi}_d(x) = \begin{bmatrix} \phi_{d1}(x) & \phi_{d2}(x) & \cdots & \phi_{dN}(x) \end{bmatrix}^T,$$

are defined according to

$$\phi_i(x) \triangleq \begin{cases} \dfrac{1}{\Delta L_i}, & x \in \left[L_i - \dfrac{\Delta L_i}{2}, L_i + \dfrac{\Delta L_i}{2}\right\rangle \\ 0, & \text{otherwise.} \end{cases} \tag{65}$$

Annulus properties. We let the location coordinate x be defined in accordance with the well coordinates, *i.e.*, with $x = 0$ just upstream the choke. Since the total length of the drill string from the mud pump to the drill bit is L_{dN}, and the length of drill string inserted into the well is l_{bit}, the x coordinate for the drill string must be shifted $L_{dN} - l_{bit}$ compared to the well. The cross sectional area $A_a(x)$ and perimeter surface $S_a(x)$ can then be given as

$$A_a = \frac{\pi}{4}\left(D_w(x)^2 - D_a(L_{dN} - l_{bit} + x)^2\right) \tag{66}$$

$$S_a = \pi\left(D_w(x) + D_a(L_{dN} - l_{bit} + x)\right), \tag{67}$$

where $x \in [0, l_{bit}]$.

The minor losses along the annulus are either losses related to the geometry of the well bore, or losses caused by flow obstructions attached to the drill string (typically, measurement devices). The first type of losses occurs at fixed locations along the well, whereas the location of the latter are fixed to the drill string, and thus given by l_{bit}. Letting the minor losses related to the well bore be defined at locations $\mathbf{L}_w$, and minor losses due to drill string obstructions at locations $\mathbf{L}_d$, the resulting total gradient of minor losses along the annulus can be expressed as

$$\frac{\partial K_a}{\partial x} = \mathbf{K}_w^T \boldsymbol{\phi}_w(x) + \mathbf{K}_a^T \boldsymbol{\phi}_a(L_{dN} - l_{bit} + x), \tag{68}$$

where

$$\mathbf{K}_w = \begin{bmatrix} K_{w1} & K_{w2} & \cdots & K_{wN} \end{bmatrix}^T$$

$$\Delta\mathbf{L}_w = \begin{bmatrix} \Delta L_{w1} & \Delta L_{w2} & \cdots & \Delta L_{wN} \end{bmatrix}^T$$

are the loss coefficients and lengths corresponding to each location in $\mathbf{L}_w$ along the well bore, and the regressor vector

$$\boldsymbol{\phi}_w(x) = \begin{bmatrix} \phi_{w1}(x) & \phi_{w2}(x) & \cdots & \phi_{wN}(x) \end{bmatrix}^T$$

is defined according to (65) by $\mathbf{L}_w$ and $\Delta\mathbf{L}_w$. Similarly, the minor losses caused by obstructions along the drill string are defined at each location in $\mathbf{L}_d$ by

$$\mathbf{K}_a = \begin{bmatrix} K_{a1} & K_{a2} & \cdots & K_{aN} \end{bmatrix}^T \tag{69}$$

$$\Delta\mathbf{L}_a = \begin{bmatrix} \Delta L_{a1} & \Delta L_{a2} & \cdots & \Delta L_{aN} \end{bmatrix}^T \tag{70}$$

$$\boldsymbol{\phi}_a(x) = \begin{bmatrix} \phi_{a1}(x) & \phi_{a2}(x) & \cdots & \phi_{aN}(x) \end{bmatrix}^T. \tag{71}$$

Example 14. ### Simple example to illustrate a typical well... ###

6. CASES

Consider the case when the

Operations

6.1 Case 1: Pipe connection

Critical drilling operations and disturbances

Pump start / stop

6.2 Case 2: Drill string movements

Surge and swab, heave, cleaning, tripping

Disturbances

6.3 Case 3: Sudden change in bottomhole pressure

To analyse the robustness and fault detection properties of an observer (or control system) with respect to sudden changes in the bottomhole pressure, we create a simplified case of drilling into an isolated reservoir zone.

Let the isolated reservoir zone be modelled simply as a chamber with volume V_{res} according to

$$\frac{V_{res}}{\beta_{res}}\,\dot{p}_{res} = q_{res}, \tag{72}$$

where the reservoir flow $q_{res}(p_{res}, p_{bit})$ is given simply as

$$q_{res} = K_{res}(p_{res} - p_{bit})\,\chi[t \geq T_{res}], \tag{73}$$

where T_{res} is the time the drill bit opens the zone, and K_{res} is the flow coefficient.

With the initial reservoir pressure $p_{res}(0)$ set lower than p_{bit}, we emulate a case similar to that of drilling into a depleated reservoir section, resulting in a sudden loss of mud and pressure drop in the annulus.

Similarly, with $p_{res}(0) > p_{bit}$, we emulate the case of drilling into a high-pressure reservoir section, resulting in a sudden pressure increase.

Table 1. Parameters used...

Parameter	**Unit**	**Value**
λ_o	—	50
k_{12}	—	50
k_{23}	—	1

6.4 Case 4: Gas kick

Gas influx

Example 15. ### .... The parameters used in the model are given in Table 1.

7. CONCLUSIONS

(Chapter head:)*

Bibliography

Herbert E. Merrrit. *Hydraulic Control Systems.* John Wiley & Sons, U.S., 1967.

Frank M White. *Fluid Mechanics.* McGraw-Hill, 3. edition, 1994.