

Backstepping on the Euler approximate model for stabilization of sampled-data nonlinear systems

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Abstract

Two integrator backstepping designs are presented for digitally controlled continuous-time plants in special form. The controller designs are based on the Euler approximate discrete-time model of the plant and the obtained control algorithms are novel. The two control laws yield, respectively, semiglobal-practical stabilization and global asymptotic stabilization of the Euler model. Both designs achieve semiglobal-practical stabilization (in the sampling period that is regarded as a design parameter) of the closed loop sampled-data system. A simulation example illustrates that the obtained controllers may be superior to backstepping controllers based on the continuous-time plant model that are implemented digitally.

1 Introduction

The backstepping techniques have attracted a lot of attention in the last ten years, leading to systematic controller designs for several important classes of nonlinear control systems, such as strict feedback systems (see [4] and references therein). While backstepping is very well understood for the case when the plant and controller are both continuous-time systems, the case when a continuous-time plant needs to be controlled via a digital controller is much less understood. Two different controller design approaches have been suggested in the literature for this situation. In the first approach (continuous-time backstepping), a continuous-time controller is designed for the continuous-time plant completely ignoring sampling (using, for instance the tools from [4]) and then the controller is discretized and implemented using sample and hold devices (see for instance [12]). A shortcoming of this approach is that the sampling is completely ignored at the controller design stage; hence, it is reasonable to

expect that other approaches that take sampling into account would yield much better results.

In the second approach (discrete-time backstepping), a discrete-time controller is designed for the exact discrete-time model of the plant, which is in strict feedback form (see [3, 5, 6, 7, 17, 18, 19, 20, 21]). In this case, *it is assumed* that the exact discrete-time model of the plant is known and it has a feedback structure that is amenable to backstepping. However, both of these assumptions are unrealistic in the case when a continuous-time plant that has strict feedback structure needs to be controlled via a digital controller. Indeed, in this case the exact discrete-time plant model is typically unknown since we need to solve explicitly a nonlinear differential equation over one sampling interval. Moreover, even if the exact discrete-time model of the plant could be found, the model would usually not have the strict feedback structure that is needed in backstepping designs. In other words, the sampling destroys the strict feedback structure.

Recently the authors proposed a framework for digital controller design based on approximate discrete-time models of the plant (see [10]). General conditions on the controller, discrete-time approximate plant model and the continuous-time plant model were given, in [10], which guarantee that if a controller stabilizes an approximate discrete-time plant model then for sufficiently small sampling periods the same controller will stabilize the exact discrete-time plant model in a semiglobal-practical sense. We note that controller design was not addressed in [10]. Several adaptive control schemes based on the Euler approximate model for several classes of systems can be found in [1, 2, 8].

In this paper we present several backstepping designs based on the Euler approximate discrete-time model of a continuous-time plant that is in strict feedback form. Motivation for doing this comes from the following: (i) The Euler approximate discrete-time model preserves the strict feedback structure of the continuous-time plant. Hence, the strict feedback assumption of the Euler approximate model is justified. (ii) We obtain completely new control algorithms in this

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way. (iii) The backstepping controllers based on the Euler approximate plant model may outperform discretized continuous-time backstepping controllers (see the example in the last section). (iv) Not every backstepping controller based on the Euler approximate plant model stabilizes the sampled-data plant (for instance, see the triple integrator example in [13]). Hence, if an approximate discrete-time model is used in backstepping, a careful investigation is needed.

Our first design achieves semiglobal-practical asymptotic (SPA) stabilization of the Euler model whereas the second design achieves global stabilization of the Euler model. Both designs achieve semiglobal-practical stabilization of the closed-loop sampled-data system in the sampling period that is assumed to be a design parameter. For space reasons proofs of all results are omitted.

2 Preliminaries

A function $\gamma : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is of class- \mathcal{K} ($\gamma \in \mathcal{K}$) if it is continuous, strictly increasing and $\gamma(0) = 0$. It is of class- \mathcal{K}_∞ if it is of class- \mathcal{K} and is unbounded. Functions of class- \mathcal{K}_∞ are invertible. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is of class- \mathcal{KL} if $\beta(\cdot, t)$ is of class- \mathcal{K} for each $t \geq 0$ and $\beta(s, \cdot)$ is decreasing to zero for each $s > 0$. Sets of real and natural numbers are respectively denoted as \mathbb{R} and \mathbb{N} . In order to simplify calculations we use ∞ -norm for vectors, that is $|x| = \max_i |x_i|$.

We initially consider continuous-time plants of the following form:

$$\begin{aligned}\dot{\eta} &= f(\eta) + g(\eta)\xi \\ \dot{\xi} &= u.\end{aligned}\quad (1)$$

For simplicity, it is assumed that $\eta \in \mathbb{R}^n, \xi \in \mathbb{R}, u \in \mathbb{R}$, $f(0) = 0$ and f, g are differentiable sufficiently many times. The system (1) is between a sampler and zero order hold. In other words, the control is taken to be a piecewise constant signal $u(t) = u(kT) =: u(k), \forall t \in [kT, (k+1)T[, k \in \mathbb{N}$ where $T > 0$ is a sampling period. We also assume that the state measurements $\eta(k) := \eta(kT)$ and $\xi(k) := \xi(kT)$ are available at sampling instants $kT, k \in \mathbb{N}$. The sampling period T is assumed to be a design parameter which can be arbitrarily assigned. We consider the difference equations corresponding to the exact plant model and its Euler approximation respectively:

$$\begin{aligned}x(k+1) &= F_T^e(x(k), u(k)) \\ x(k+1) &= F_T^{Euler}(x(k), u(k)),\end{aligned}\quad (2)$$

where we used the notation $x := (\eta^T \ \xi^T)^T$ and

$$F_T^{Euler}(x, u) := \begin{pmatrix} \eta + T(f(\eta) + g(\eta)\xi) \\ \xi + Tu \end{pmatrix}. \quad (4)$$

Note that both models (2) and (3) are parameterized with the sampling period T . We emphasize that F_T^e is not known in most cases. Moreover, even when F_T^e is known, it usually does not have the structure of (1). On the other hand, F_T^{Euler} always has the structure like (1). Note that F_T^{Euler} is defined globally if the functions f and g in (1) are defined globally. On the other hand, we will think of F_T^e as being defined globally for all small T even though the initial value problem (1) may exhibit finite escape times. We do this by defining F_T^e arbitrarily for pairs $(x(k), u(k))$ corresponding to finite escapes and noting that such points correspond only to points of arbitrarily large norm as $T \rightarrow 0$, at least when f and g are locally bounded. So, the behavior of F_T^e will reflect the behavior of (1) as long as $(x(k), u(k))$ remains bounded with a bound that is allowed to grow as $T \rightarrow 0$. This is consistent with our main results that guarantee SPA stability. In general, one needs to use small sampling periods T since the approximate plant model is a good approximation of the exact model typically only for small T . It is clear then that we need to be able to obtain a controller $u_T(x)$ based on the Euler model, which is, in general, parameterized by T and which is defined for all small T . We will use the following definitions:

Definition 1 We say that the family of controllers u_T semiglobally-practically asymptotically (SPA) stabilizes F_T if there exists $\beta \in \mathcal{KL}$ such that for any pair of strictly positive real numbers (D, ν) there exists $T^* > 0$ such that for each $T \in (0, T^*)$ the solutions of $x(k+1) = F_T(x(k), u_T(x(k)))$ satisfy:

$$|x(k, x(0))| \leq \beta(|x(0)|, kT) + \nu, \quad k \geq 0, \quad (5)$$

whenever $|x(0)| \leq D$. ■

Definition 2 Let $\hat{T} > 0$ be given and for each $T \in (0, \hat{T})$ let functions $V_T : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ and $u_T : \mathbb{R}^n \rightarrow \mathbb{R}$ be defined. We say that the pair (u_T, V_T) is a semiglobally-practically asymptotically (SPA) stabilizing pair for F_T if there exist $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ such that for any pair of strictly positive real numbers (Δ, δ) there exists a triple of strictly positive real numbers (T^*, L, M) ,

with $T^* \leq \hat{T}$, such that for all $x, z \in \mathbb{R}^n$ with $\max\{|x|, |z|\} \leq \Delta$, and $T \in (0, T^*)$ we have:

$$\begin{aligned} \alpha_1(|x|) \leq V_T(x) &\leq \alpha_2(|x|) \\ \Delta V_T &\leq -T\alpha_3(|x|) + T\delta \\ |V_T(x) - V_T(z)| &\leq L|x - z| \\ |u_T(x)| &\leq M, \end{aligned} \quad (6)$$

where $\Delta V_T := V_T(F_T(x, u_T(x))) - V_T(x)$. Moreover, if there exists $T^{**} > 0$ such that the above conditions with $\delta = 0$, hold for all $x \in \mathbb{R}^n$ and all $T \in (0, T^{**})$ then we say that the pair (u_T, V_T) is a globally asymptotically (GA) stabilizing pair for F_T . ■

A direct consequence of Definition 2 is:

Lemma 1 (u_T, V_T) is a GA stabilizing pair for $F_T \implies (u_T, V_T)$ is a SPA stabilizing pair for F_T . ■

The proofs of the following two results come directly from [10], in particular, from the proof of [10, Theorem 2], the definition of one-step consistency [10, Definition 1] and sufficient conditions for one-step consistency [10, Lemma 1].

Theorem 1 (u_T, V_T) is a SPA stabilizing pair for $F_T^{Euler} \implies (u_T, V_T)$ is a SPA stabilizing pair for F_T^c . ■

Theorem 2 (u_T, V_T) is a SPA stabilizing pair for $F_T \implies u_T$ SPA stabilizes F_T . ■

Hence, if we can find a pair (u_T, V_T) that is a GA or SPA stabilizing pair for F_T^{Euler} , then the controller u_T will SPA stabilize the exact model F_T^c . In the sequel we present such designs.

3 Integrator backstepping

Design of SPA and GA control laws is hard for general nonlinear systems. However, if the system has the special form (1) these control laws can be systematically derived. We discuss one such situation next. First, we design a SPA stabilizing pair (u_T, V_T) for F_T^{Euler} of (1) and then a GA stabilizing pair for F_T^{Euler} of (1). In our approach, there is no loss of generality to consider $\xi = u$ in (1) instead of $\xi = h(\eta, \xi) + k(\eta, \xi)v$, where $k(\eta, \xi) \neq 0, \forall \eta, \xi$. Indeed, if that is the case then we can use a preliminary control $v = \frac{u - h(\eta, \xi)}{k(\eta, \xi)}$ to obtain the Euler model structure as in (7), (8) that is needed in our approach. Note, however, that the exact discrete-time model or higher order approximate models would in general depend on k and h even after the preliminary control is applied.

3.1 SPA stabilizing pair for Euler

In this sections we use the Euler approximate model of (1):

$$\begin{aligned} \eta(k+1) &= \eta(k) + T \left(f(\eta(k)) \right. \\ &\quad \left. + g(\eta(k))\xi(k) \right) \\ \xi(k+1) &= \xi(k) + Tu(k) \end{aligned} \quad (7)$$

to design controllers that SPA stabilize the exact discrete-time model of (1).

The main result of this section is stated next.

Theorem 3 Consider the Euler approximate model (7), (8). Suppose that there exists $\hat{T} > 0$ and a pair (ϕ_T, W_T) that is defined for all $T \in (0, \hat{T})$ and that is a SPA stabilizing pair for the subsystem (7), with $\xi \in \mathbb{R}$ regarded as its control. Moreover, suppose that the pair (ϕ_T, W_T) has the following properties:

1. ϕ_T and W_T are continuously differentiable for any $T \in (0, \hat{T})$;
2. there exists $\tilde{\varphi} \in \mathcal{K}_\infty$ such that

$$|\phi_T(\eta)| \leq \tilde{\varphi}(|\eta|). \quad (9)$$

3. for any $\tilde{\Delta} > 0$ there exists a pair of strictly positive numbers (\tilde{T}, \tilde{M}) such that for all $T \in (0, \tilde{T})$ and $|\eta| \leq \tilde{\Delta}$ we have

$$\max \left\{ \left| \frac{\partial W_T}{\partial \eta} \right|, \left| \frac{\partial \phi_T}{\partial \eta} \right| \right\} \leq \tilde{M} \quad (10)$$

Then there exists a SPA stabilizing pair (u_T, V_T) for the Euler model (7), (8). In particular, we can take:

$$u_T(x) = -c(\xi - \phi_T(\eta)) - \frac{\widetilde{\Delta W}_T}{T} + \frac{\Delta \phi_T}{T} \quad (11)$$

where $c > 0$ is arbitrary,

$$\Delta \phi_T := \phi_T(\eta + T(f + g\xi)) - \phi_T(\eta) \quad (12)$$

$$\widetilde{\Delta W}_T = \begin{cases} \frac{\Delta W_T}{(\xi - \phi_T(\eta))}, & \xi \neq \phi_T(\eta) \\ T \frac{\partial W_T}{\partial \eta}(\eta + T(f + g\xi)) \\ + g(\eta)\xi)g(\eta), & \xi = \phi_T(\eta) \end{cases} \quad (13)$$

$$\begin{aligned} \overline{\Delta W}_T &:= W_T(\eta + T(f + g\xi)) \\ &\quad - W_T(\eta + T(f + g\phi_T)) \end{aligned} \quad (14)$$

and $V_T(x) = W_T(\eta) + \frac{1}{2}(\xi - \phi_T(\eta))^2$. ■

Remark 1 Note that since W_T is continuously differentiable and ϕ_T is continuous, the control law u_T is continuous. Moreover, note that $(\xi - \phi_T(\eta)) \cdot \widetilde{\Delta W}_T = \overline{\Delta W}_T$ holds for all $x \in \mathbb{R}^{n+1}$. ■

Remark 2 A continuous-time counterpart of the control law (11) can be found, for instance, in Lemma 2.8 in [4] and it takes the following form:

$$u(x) = -c(\xi - \phi(\eta)) + \frac{\partial \phi}{\partial \eta}(\eta)[f(\eta) + g(\eta)\xi] - \frac{\partial W}{\partial \eta}(\eta)g(\eta), \quad (15)$$

where ϕ is the control law that stabilizes the η subsystem and W is the Lyapunov function for the η subsystem with the control law ϕ . We note that although (11) and (15) are similar, they are in general different. More importantly, we will show by simulations in the last section that (11) may outperform (15) when both controllers are implemented digitally. ■

3.2 Building a GA stabilizing pair for the Euler model

In this section we build a GA stabilizing pair for the Euler approximate model of (1) and the main result is stated in Theorem 4. Before stating the main result we state Lemma 2 that is instrumental in proving Theorem 4 and that is inspired by the results in [15]. All proofs are omitted for space reasons.

Lemma 2 Consider the system:

$$\eta(k+1) = \eta(k) + T(f(\eta(k)) + g(\eta(k))u(k)) \quad (16)$$

where $\eta \in \mathbb{R}^n, u \in \mathbb{R}, f(0) = 0$ and f, g are continuous. Suppose that there exist functions $\alpha_1, \alpha_2, \alpha_3, \tilde{\varphi}_1, \tilde{\varphi}_2 \in \mathcal{K}_\infty, c_1 \geq 0, T^* > 0$ and for all $T \in (0, T^*)$ there exist a function $\phi_T : \mathbb{R}^n \rightarrow \mathbb{R}$ and a continuously differentiable function $W_T : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ such that for all $\eta \in \mathbb{R}^n$ and all $T \in (0, T^*)$ we have

$$\Delta W_T \leq -T\alpha_3(|\eta|) \quad (17)$$

$$\begin{aligned} |g(\eta)\phi_T(\eta)| &\leq \tilde{\varphi}_1(|\eta|) \\ \left| \frac{\partial W_T}{\partial \eta}(\eta) \right| &\leq c_1 + \tilde{\varphi}_2(|\eta|), \end{aligned} \quad (18)$$

where $\Delta W_T := W_T(\eta + T(f(\eta) + g(\eta)\phi_T(\eta))) - W_T(\eta)$. Then there exist $\gamma \in \mathcal{K}_\infty$, (smooth) function $\rho : \mathbb{R}_{\geq 0} \rightarrow (0, 1]$ and a family of control laws:

$$\tilde{u}_T(\eta, \zeta) = \phi_T(\eta) + \rho(\eta)\zeta \quad (19)$$

such that for all $\eta \in \mathbb{R}^n, \zeta \in \mathbb{R}$ and $T \in (0, T^*)$ we have:

$$\begin{aligned} W_T(\eta + T(f(\eta) + g(\eta)\tilde{u}_T(\eta, \zeta))) - W_T(\eta) \\ \leq -\frac{T}{2}\alpha_3(|\eta|) + T\gamma(|\zeta|). \end{aligned} \quad (20)$$

Theorem 4 Consider the Euler approximate model (7), (8). Suppose that there exists $\hat{T} > 0$ and a pair (ϕ_T, W_T) that is defined for all $T \in (0, \hat{T})$ and that is GA stabilizing for the subsystem (7), with $\xi \in \mathbb{R}$ regarded as its control. Let ρ come from Lemma 2. Moreover, suppose that the pair (ϕ_T, W_T) has the following properties:

1. ϕ_T and W_T are continuously differentiable for any $T \in (0, \hat{T})$;
2. there exists $\tilde{\varphi} \in \mathcal{K}_\infty$ such that for all $\eta \in \mathbb{R}^n$ and all $T \in (0, \hat{T})$ we have

$$|\phi_T(\eta)| \leq \tilde{\varphi}(|\eta|). \quad (21)$$

3. for any $\tilde{\Delta} > 0$ there exists a pair of strictly positive numbers (\tilde{T}, \tilde{M}) such that for all $T \in (0, \tilde{T})$ and $|\eta| \leq \tilde{\Delta}$ we have

$$\max \left\{ \left| \frac{\partial W_T}{\partial \eta} \right|, \left| \frac{\partial \phi_T}{\partial \eta} \right| \right\} \leq \tilde{M} \quad (22)$$

Then, there exists a GA stabilizing pair (u_T, V_T) for the Euler model (7), (8). In particular, the family of control laws can be taken to be:

$$u_T(x) = \frac{\Delta \phi_T}{T} + \frac{\Delta \rho}{T} \zeta - c\rho(|\eta + T(f + g\xi)|)\zeta, \quad (23)$$

where $c > 0$ is arbitrary and

$$\Delta \phi_T = \phi_T(\eta + T(f + g\xi)) - \phi_T(\eta) \quad (24)$$

$$\Delta \rho = \rho(|\eta + T(f + g\xi)|) - \rho(|\eta|) \quad (25)$$

$$\zeta = \frac{\xi - \phi_T(\eta)}{\rho(|\eta|)} \quad (26)$$

and there exist two smooth functions $\tilde{\theta}_1, \tilde{\theta}_2 \in \mathcal{K}_\infty$ such that we can take V_T to be:

$$V_T(x) = \tilde{\theta}_1(W_T(\eta)) + \tilde{\theta}_2 \left(\frac{1}{2} \frac{(\xi - \phi_T(\eta))^2}{\rho^2(|\eta|)} \right). \quad (27)$$

4 Example

Consider the continuous-time plant:

$$\begin{aligned} \dot{\eta} &= \eta^2 + \xi \\ \dot{\xi} &= u. \end{aligned} \quad (28)$$

First we design the continuous-time backstepping controller based on (28). Note that the first

subsystem can be stabilized with the “control” $\phi(\eta) = -\eta^2 - \eta$. This is verified using the Lyapunov function $W(\eta) = \frac{1}{2}\eta^2$. Using this information and applying (15) with $c = 1$ (see also Lemma 2.8 in [4]), we obtain:

$$u^{ct}(\eta, \xi) = -2\eta - \eta^2 - \xi - (2\eta + 1)(\xi + \eta^2). \quad (29)$$

Assume now that the plant (28) is between a sampler and a zero order hold and consider its Euler approximate model:

$$\begin{aligned} \eta(k+1) &= \eta(k) + T(\eta^2(k) + \xi(k)) \\ \xi(k+1) &= \xi(k) + Tu(k). \end{aligned} \quad (30)$$

Again, the control law $\phi(\eta) = -\eta^2 - \eta$ and the Lyapunov function $W(\eta) = \frac{1}{2}\eta^2$ are a GA stabilizing pair for the η -subsystem of (30). Using (11) with $c = 1$ in Theorem 3, we obtained the controller:

$$u_T^{Euler}(\eta, \xi) = u^{ct}(\eta, \xi) - T[0.5\eta^2 + 0.5\xi - 0.5\eta + (\xi + \eta^2)^2]. \quad (31)$$

Note that the term $-T[0.5\eta^2 + 0.5\xi - 0.5\eta + (\xi + \eta^2)^2]$ can be regarded as a modification of the controller (29). Moreover, for $T = 0$ we have that $u_0^{Euler}(\eta, \xi) = u^{ct}(\eta, \xi)$. We have compared the performance of the sampled-data systems with the two different controllers and have observed that u_T^{Euler} consistently yielded at least 4 times larger domain of attraction than u^{ct} for all tested sampling periods. In particular, Figures 1 and 2 show respectively trajectories with the $u^{ct}(\eta, \xi)$ and $u_T^{Euler}(\eta, \xi)$ starting from the same initial condition and with the same sampling period. While the trajectory with $u^{ct}(\eta, \xi)$ escapes in finite time, the trajectory with $u_T^{Euler}(\eta, \xi)$ is bounded and it converges to the origin. Estimates of domains of attraction (DOA) with the two controllers for the sampling period $T = 0.5$ sec are given in Figure 3. The DOAs were obtained using simulations in SIMULINK in the following manner (we have used the following simulation parameters: variable step size; ode45; relative tolerance 10^{-9} , absolute tolerance “auto”; max step size “auto”; initial step size “auto”). We have picked a number of rays starting at the origin and simulated the system with u^{ct} for a set of initial conditions on these rays where we increased/decreased the x coordinate by 0.1 in each simulation. This process was stopped when we obtained an unbounded trajectory and the initial condition from

the previous simulation was used as a point on the boundary of DOA indicated by “*” on Figure 3. DOA with u_T^{Euler} was not estimated with such an accuracy since we have simply multiplied the coordinates of the obtained points on the boundary of DOA with u^{ct} by 4 and then verified that trajectories from these points still converged for the system with u_T^{Euler} . These points are denoted by “Δ” in Figure 3. Hence, DOA for the system with u_T^{Euler} may be much larger than the estimate given in Figure 3. The reason for not estimating the DOA with u_T^{Euler} more accurately is that the simulations were taking a very long time for large initial conditions. Both systems were then extensively simulated for initial conditions that were chosen randomly within DOAs in Figure 3 to verify that there are no unbounded trajectories from initial states in these regions.

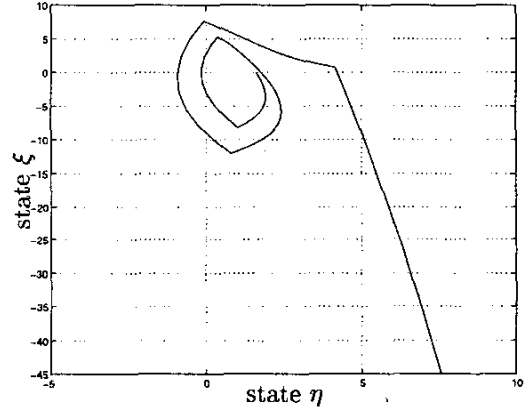


Figure 1: Simulation with $u^{ct}(\eta, \xi)$, $x_0 = [1.6 \ 0]^T$ and $T = 0.5$ s.

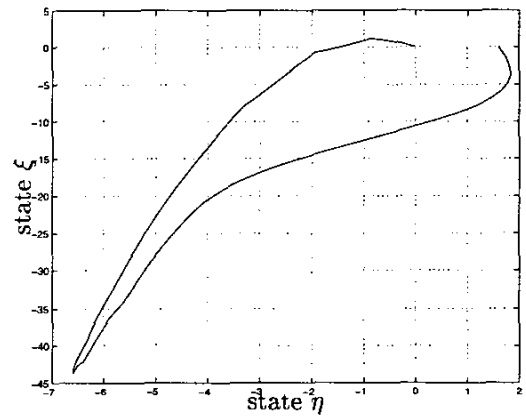


Figure 2: Simulation with $u_T^{Euler}(\eta, \xi)$, $x_0 = [1.6 \ 0]^T$ and $T = 0.5$ s.

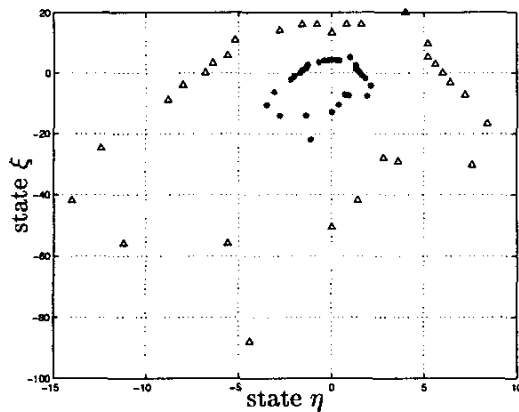


Figure 3: Estimates of domains of attraction with $T = 0.5$ sec.

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