

# Optimal Power Flow Analysis in Power Dispatch for Distribution Networks

Paolo Pisciella, Maria Teresa Vespucci, Giacomo Viganò, Marco Rossi,  
Diana Moneta

**Abstract** We present two applications of Optimal Power Flow analysis for active and reactive power redispatch in Medium Voltage Distribution Networks and show how this tool can be used to efficiently manage the selection and operation of network resources as well as the definition of a market interface with the Transmission Network. We complement the description of the frameworks with the analysis of a case study for the optimal selection and operation of available devices.

## 1 From centralized to distributed generation

Distributed Generation (DG) employs small-scale technologies to produce electricity in the proximity of the consumption areas [1]. DG technologies are normally constituted of modular and renewable-energy generators. These generators offer lower cost electricity and allow the grid to maintain a higher level of reliability and security. Moreover, DG has a lower environmental impact compared to the usage of traditional large-scale generators.

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Paolo Pisciella  
Department of Industrial Economics and Technology Management, NTNU, Alfred Getz Vei 3,  
7491 Trondheim, Norway  
e-mail: paolo.pisciella@ntnu.no

Maria Teresa Vespucci  
Department of Management, Information and Production Engineering, University of Bergamo, via  
Marconi 5, 24044 Dalmine (BG), Italy  
e-mail: maria-teresa.vespucci@unibg.it

Giacomo Viganò, Marco Rossi, Diana Moneta  
RSE - Ricerca sul Sistema Energetico S.p.A., via Rubattino 54, 20134 Milano, Italy  
e-mail: giacomo.vigano@rse-web.it  
e-mail: marco.rossi@rse-web.it  
e-mail: diana.moneta@rse-web.it

In contrast with the established paradigm of using few large scale plants, located away from centres, and long transmission lines to bring power where load is located, DG utilizes a large number of small plants with capacities ranging from a fraction of a kilowatt [ $kW$ ] to about 10 megawatt [ $MW$ ], whereas conventional power plant can reach capacities exceeding 1000  $MW$  [3].

In the current model large power plants utilize combustion (coal, oil and natural gas) or nuclear to generate large amounts of power that require to be transmitted from the plants to the final consumers, often over long distances. This creates disadvantages, mainly due to inefficiencies and power loss over the lengthy transmission lines, environmental damages where the power lines are built and security related issues. In addition, such kind of set-up involves the by-production of greenhouse gases. Utilization of distributed energy can mitigate the negative impact of these issues, since DG is often produced by small modular energy conversion units, such as solar panels or small wind generators. Consumers who have installed solar panels will contribute more to the grid than they take out, resulting in a win-win situation for both the power grid and the end-user. The two different approaches are displayed in Figure 1.

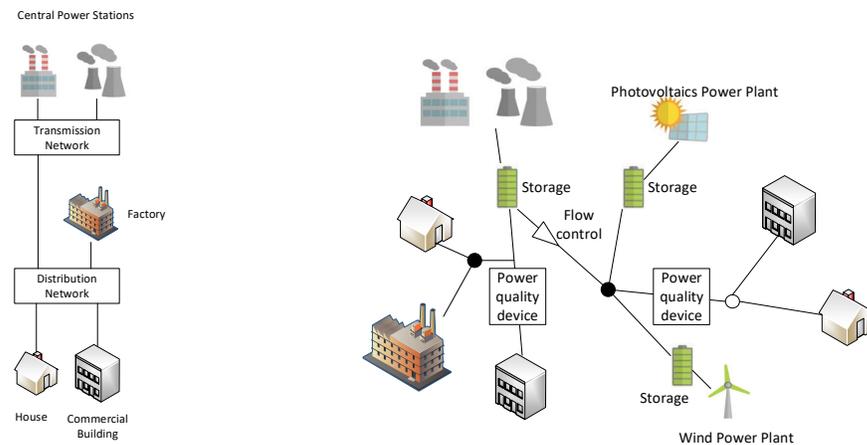


Fig. 1: Different power system paradigms with centralized power plants (a) and Distributed Generation (b) (source: Introduction to Distributed Generation. European Commission Research and Innovation)

Distributed generation takes place on two-levels: the local level and the end-point level. Local level power generation plants are normally site specific. Depending on the peculiarities of the installation site one might have wind turbines, geothermal energy production, photovoltaic systems and hydro-thermal plants. Albeit smaller than the conventional power plants, these plants tend to be more energy/cost efficient and reliable. They also usually have a lower pollutant impact. At the end-point level one finds the energy consumer, which can install similar technology on an individual

base [2]. In this respect, DG technologies can both operate energy production for immediate consumption or they can serve as small contributors to the power grid.

It is also a shared opinion that the distributed process involved in the DG can improve the efficiency of electric power production and distribution through the reduction of the transmission distances, which might save potential losses up to about 9% of the produced power. These losses are mainly due to the aging of the transmission equipment and growing levels of congestion [13, 24]. Moreover, power quality is often characterized by sudden changes in voltages or electrical flows which result from different causes, such as poor switching operations in the network, over- or under-voltages, interruptions, and network disturbances from loads. Costs related to the management of this transmission grid is included in the final consumers' bill. Therefore the use of on-site generated power is expected to bring the end user to obtain a higher level of power quality at lower costs. In addition, the end users will take the new role of prosumers, by simultaneously being consumers and producers of electric power, enabling in this way the possibility to sell the excess production to the grid [13]. Producing directly for the end users also allows to reduce the demand during peak times and minimize the congestion of power on the network.

Another advantage of the DG technologies takes the form of improved reliability for industries that require uninterrupted services. It is estimated that only in the United States, the costs related to power outages and quality disturbances amount to 119 billion USD per year [12], bringing the average per hour cost of a power outage to 6,480,000 USD [21]. Moreover, DG technology improves the security of the grid. The decentralization of power production helps reducing the negative effects of interruptions of services by insulating the grid from failures if a line or a power plant goes down. Thanks to the independence of DG technologies from the grid, these technologies can provide emergency power for a huge number of public services, ranging from hospitals and schools to police stations, military bases and prisons as well as water supply and sewage treatment plants, natural gas transmission and distribution systems, and communications stations. This is a quite effective defensive measure in case of particularly negative events, such as natural disasters or terrorist attacks to parts of the power system [13].

Finally, one of the most important effects of Distributed Generation is related to the environmental safeguard. Large power plants are responsible for large levels of pollution, while a widespread use of DG technology would reduce  $CO_2$  emissions by a substantial extent [15]. Under an economical viewpoint, DG can support the Nations to increase their diversity of energy sources, often free from consumption of fossil fuels. The result is an improved insulation of the economies from price shocks, interruptions and fuel shortages [11].

The cost reduction and the increased availability of distributed power resources are boosting the shift towards a distributed generation paradigm. Nonetheless, this transition is also fostered by the ability to overcome the constraints related to an expansion of the power generation and transmission system. The reduced size of the generation units, the lower capital requirements and the short time needed for their construction establish a particularly good ground for the development of DG systems [22].

These power technologies are today ready to compete with the cost and performance of central power plants. This because today's technology allows these resources to work in a coordinated manner either on-site or remotely. This synchronization of distributed technologies places DG to operate in an integrated energy system alongside with conventional power plants.

## 2 The bottleneck of transmission

Even in the most developed countries, environmental and regulatory barriers often hinder the possibility of expanding the power transmission network. Even when these constraints can be overcome, the high investment levels and the long planning process makes it economically unsuitable. Network constraints constitute a big reason that advocates in favour of the development of distributed power generation. This is even clearer in developing countries, where a less developed transmission and distribution system makes it necessary to call for a DG paradigm for meeting energy needs. The large amount of capital needed to expand and/or strengthen the transmission network is testified by the investments that have been allocated in projects around the world. As a mention, in 2012 US electric utilities invested approximately 20 billion USD on transmission alone. Also Europe needs to increase the transmission capacity in many regions, but difficulties in regulations and high investment costs are often leading to delays or cancellations of projects [5]. Nevertheless, in 2006, the association of European Transmission System Operators indicated that in some countries not a single overhead power line exceeding five kilometers has been built in the last 10 years. The International Energy Agency (IEA) has estimated that 187 billion USD of transmission investments are required in Europe through 2030 [16]. In developing countries such as China and India the main investments are focused on the expansion of the Transmission and Distribution infrastructure. Here, distributed power can provide power to remote areas, currently not reached by the transmission network. Developing countries also face the problem of electricity theft, which cannot be measured directly, but only estimated. This issues can be solved or reduced through the use of DG, locating the production near the end users. It is important to understand that DG must be supported by an appropriate Distribution Network, which must also take care of the integration and coordination of the DG resources. In fact, the widespread installation of generation units might lead to higher levels of active and reactive power losses or voltage deviations due to imbalances between production from renewable sources and load [25]. These voltage fluctuation are subject to continuous variations due to the non-programmable change of output over time of the renewable generators. Voltage fluctuations are, in turn, directly linked to the power quality delivered to customers. Different DG power plant types may have different impacts on distribution networks. For example the power injected into the distribution network from a photovoltaic system might have a different impact to that of a wind turbine [26].

The new paradigm introduced with Smart Grids involves the exploitation of well established technologies and concepts by pooling and coordinating the available resources. This allows for an aggregated response to the increased demand and requirements regarding quality and sustainability of the energy sector [6]. The fast processing and exchange of information between available resources bring even more incentive in developing a common framework to manage the network operations in coordination. This integration of network resources allows for the creation of a more robust and reliable power system which is less hindered by human limitations.

In order to fully exploit this integration it is important that the system is endowed with some degree of intelligence so as to be able to provide responses to sudden changes in the network structure. In other words, it is important to give the system the possibility to automatically balance the generation-load profile, congestion on lines or shift in voltages. In this case, the system's intelligence can be represented by the response of computer programs that employ optimization techniques and support decision making.

In this respect, this chapter will describe the usage of Optimal Power Flow (OPF) analysis for ensuring efficiency in dispatch of power in Distribution Networks with Distributed Generation and Storage Devices. The OPF tool will provide its utility in two applications. The efficient choice of devices for power redispatch and the construction of equivalent capabilities to provide bids from the Distribution Network to the Transmission Network.

### **3 A procedure for the optimal management of medium-voltage AC networks with distributed generation and storage devices**

In this section we describe a two-step solution method for the problem of determining the best control action to take when imbalance between load and generation occurs in a Medium-Voltage Distribution Network. Due to the partially unpredictability of load and of generation stemming from renewable sources, Distribution Networks are subject to continuous imbalances. The Distribution System Operator (DSO) is the agent in charge of rebalancing the supply by using the available resources in the grid. The framework defines the DSO as facing two layers of decisions, with the first layer consisting in the selection of the most suitable devices to be used in the power redispatch and the second layer consisting in determining the optimal usage of the selected resources to perform the power redispatch. The problem is modeled as a Mixed Integer Non Linear Problem (MINLP) where integrality is due to the binary variables for modelling fixed costs related to the selected devices and nonlinearity pertains to the constraints typical of the Optimal Power Flow problem in Alternating Current. The time period, or planning horizon, under consideration is discretized in time units (e.g., one day divided in 24 hours or in 96 quarters of hour) and inter-temporal energy balance constraints are introduced to model storage units. Distribution networks with a large number of nodes and lines give rise to large di-

mensional MINLP models, which in turn require large computational effort for their solution. Bosisio et al. [4] propose to decouple the determination of the devices to be used and the AC power redispatch to account for security constraints (current in lines) and delivered power quality (voltages) for every considered time unit. In this approach an approximate solution of the MINLP problem is obtained by means of a two step procedure, with the first step based on a Mixed Integer Linear Program and the second step taking into account the nonlinear OPF constraints. In this approach the DSO is assumed to have knowledge on the planned output of active and reactive power by the controllable generators and on the forecast for load and power output of non controllable generators. The possible imbalances between load and generation are solved by employing an OPF tool providing the least cost solution for the power redispatch by determining the optimal contribution of internal regulation resources (i.e. directly operated by the DSO), such as On-Load Tap Changers (OLTC) and storage units, and external regulation resources, such as active and reactive power injection/absorption from controllable resources, which are required to modify their production plans in exchange for an economic compensation (for both positive and negative variations of power generation). Costs may be defined either as market prices for the usage of controllable resources or as values for accounting deterioration of internal resources, such as OLTC or storage devices.

The MILP model proposed in [4] for the first step of the procedure is as follows.

### Sets

$\mathcal{N}$	set of nodes, indexed by $i$
$\mathcal{L}$	set of lines, indexed by $l$
$\mathcal{G}$	set of power generators, indexed by $g$
$\mathcal{B}$	set of storage devices, indexed by $b$
$\mathcal{T}$	set of time units, indexed by $t$ , in which the time horizon is divided
$\mathcal{G}^{ND} \subseteq \mathcal{G}$	set of non-dispatchable generators
$\mathcal{G}^{DS} \subseteq \mathcal{G}$	set of dispatchable generators with interruptible production
$\mathcal{G}^D \subseteq \mathcal{G}$	set of dispatchable generators with non interruptible production

### Parameters

$dt$	[ $h$ ]	duration of time unit
$C_g^S$	[ $\text{€}$ ]	fixed cost for interrupting production of generator $g$
$C_g^{DF}$	[ $\text{€}$ ]	fixed cost for modifying scheduled production of generator $g$
$C_g^{DU}$	[ $\text{€}/MWh$ ]	unitary cost for increasing production of generator $g$
$C_g^{DD}$	[ $\text{€}/MWh$ ]	unitary cost for decreasing production of generator $g$
$C_b^{\text{in}}$	[ $\text{€}/MWh$ ]	unitary cost for charging storage device $b$
$C_b^{\text{out}}$	[ $\text{€}/MWh$ ]	unitary cost for discharging storage device $b$

$\hat{p}_{g,t}$	[MW]	scheduled power output of generator $g \in \mathcal{G}^{DS} \cup \mathcal{G}^{DS}$ at time $t$
$P_g^{\max}$	[MW]	maximum power output of dispatchable generator $g$
$P_g^{\min}$	[MW]	minimum power output of dispatchable generator $g$
$L_{g,t}^F$	[MW]	power output of non dispatchable generator $g$ at time $t$
$e_b^{\text{iniz}}$	[MWh]	energy in storage device $b$ at the beginning of $t = 1$
$e_b^{\text{fin}}$	[MWh]	energy required in storage unit $b$ at the end of time horizon
$e_b^{\max}$	[MWh]	capacity of storage device $b$
$p_b^{\max}$	[MW]	maximum charge/discharge of storage device $b$
$\eta_b^h$	[-]	energy loss coefficient of storage device $b$
$\eta_b^{\text{in}}$	[-]	charge loss coefficient of storage device $b$
$\eta_b^{\text{out}}$	[-]	discharge loss coefficient of storage device $b$
$l_{i,t}^F$	[MW]	load at node $i$ at time $t$
$p_{i,t}^F$	[-]	loss rate at node $i$ at time $t$
$\sigma_{i,l,t}$	[-]	Power Transfer Distribution Factor (PTDF) of line $l$ from node $i$ at time $t$
$\bar{f}_{l,t}$	[MW]	maximum power flow on line $l$ at time $t$
$\underline{f}_{l,t}$	[MW]	minimum power flow on line $l$ at time $t$

### Decision Variables

$\gamma_{g,t}$	[0/1]	binary variable (1: production of generator $g$ is interrupted at time $t$ ; 0: otherwise)
$\delta_{g,t}$	[0/1]	binary variable (1: scheduled production of generator $g$ is modified, i.e increased or reduced, at time $t$ ; 0: otherwise)
$u_{g,t}$	[MW]	increase of scheduled production for generator $g$ at time $t$
$d_{g,t}$	[MW]	reduction of scheduled production for generator $g$ at time $t$
$p_{b,t}^{\text{in}}$	[MW]	energy rate from source of storage device $b$ at time $t$
$p_{b,t}^{\text{out}}$	[MW]	energy rate to load of storage device $b$ at time $t$
$e_{b,t}$	[MWh]	energy in storage device $b$ at the end of time $t$
$f_{l,t}$	[MW]	power flow on line $l$ at time $t$

$$\min \quad dt \sum_{t \in \mathcal{T}} \left[ \sum_{g \in \mathcal{G}^D} (C_g^{DF} \delta_{g,t} + C_g^{DU} u_{g,t} + C_g^{DD} d_{g,t}) + \sum_{g \in \mathcal{G}^{DS}} C_g^S (1 - \gamma_{g,t}) + \sum_{b \in \mathcal{B}} (C_b^{\text{in}} p_{b,t}^{\text{in}} + C_b^{\text{out}} p_{b,t}^{\text{out}}) \right] \quad (1)$$

subject to

$$0 \leq u_{g,t} \leq (P_g^{\max} - \hat{p}_{g,t}) \delta_{g,t} \quad g \in \mathcal{G}^D, t \in \mathcal{T} \quad (2)$$

$$0 \leq d_{g,t} \leq (\hat{p}_{g,t} - P_g^{\min}) \delta_{g,t} \quad g \in \mathcal{G}^D, t \in \mathcal{T} \quad (3)$$

$$0 \leq u_{g,t} \leq (P_g^{\max} - \hat{p}_{g,t}) \delta_{g,t} \quad g \in \mathcal{G}^{DS}, t \in \mathcal{T} \quad (4)$$

$$u_{g,t} \leq (P_g^{\max} - \hat{p}_{g,t}) \gamma_{g,t} \quad g \in \mathcal{G}^{DS}, t \in \mathcal{T} \quad (5)$$

$$0 \leq d_{g,t} \leq \hat{p}_{g,t} \delta_{g,t} \quad g \in \mathcal{G}^{DS}, t \in \mathcal{T} \quad (6)$$

$$\hat{p}_{g,t} - \hat{p}_{g,t} \gamma_{g,t} \leq d_{g,t} \leq \hat{p}_{g,t} - P_g^{\min} \gamma_{g,t} \quad g \in \mathcal{G}^{DS}, t \in \mathcal{T} \quad (7)$$

$$e_{b,t} = \left( \eta_b^h e_{b,t-1} + \eta_b^{\text{in}} p_{b,t}^{\text{in}} - \eta_b^{\text{out}} p_{b,t}^{\text{out}} \right) dt \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (8)$$

$$0 \leq e_{b,t} \leq e_b^{\max} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (9)$$

$$e_b^{\text{fin}} \leq e_{b,|\mathcal{T}|} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (10)$$

$$0 \leq p_{b,t}^{\text{in}} \leq p_b^{\max} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (11)$$

$$0 \leq p_{b,t}^{\text{out}} \leq p_b^{\max} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (12)$$

$$\sum_{i \in \mathcal{N}} (1 + l_{i,t}^F) L_{i,t}^F = \sum_{g \in \mathcal{G}^{ND}} p_{g,t}^F + \sum_{g \in \mathcal{G}^D} (\hat{p}_{g,t} + u_{g,t} - d_{g,t}) + \sum_{b \in \mathcal{B}} (p_{b,t}^{\text{out}} - p_{b,t}^{\text{in}}) \quad t \in \mathcal{T} \quad (13)$$

$$f_{l,t} = \sum_{i \in \mathcal{N}} \sigma_{i,l,t} \left[ \sum_{g \in \mathcal{G}^{ND}} p_{g,t}^F + \sum_{g \in \mathcal{G}^D} (\hat{p}_{g,t} + u_{g,t} - d_{g,t}) \right. \quad (14)$$

$$\left. + \sum_{b \in \mathcal{B}} (p_{b,t}^{\text{out}} - p_{b,t}^{\text{in}}) - \sum_{i \in \mathcal{N}} (1 + l_{i,t}^F) L_{i,t}^F \right] \quad l \in \mathcal{L}, t \in \mathcal{T}$$

$$\underline{f}_{l,t} \leq f_{l,t} \leq \bar{f}_{l,t} \quad l \in \mathcal{L}, t \in \mathcal{T} \quad (15)$$

The objective function (1) represents the cost, to be minimized, of the DSO control action. For dispatchable generators with non interruptible production, constraints (2) and (3) guarantee that power output, after variation, is between its minimum and maximum values. For dispatchable generators with interruptible production, constraints (4) to (7) state that power output, after variation, is either between its minimum and maximum values, if  $\gamma_{g,t} = 1$  (i.e. if production is not interrupted), or zero, if  $\gamma_{g,t} = 0$ . Constraints (8) are the intertemporal energy balance constraints of storage device  $b$ , in which losses are taken into account. Constraints (9) impose lower and upper bounds to the energy stored in storage device  $b$  at the end of time

$t$ . Constraints (10) guarantee the required minimum energy in storage device  $b$  at the end of the period under consideration. Constraints (11) impose lower and upper bounds to the energy rate from source and constraints (12) impose lower and upper bounds to the energy rate to load. Constraints (13) are the power balance equations that must hold at every time  $t$ : the power output of both dispatchable and nondispatchable generators plus the net power output of storage devices must equal the sum of loads over all nodes, plus a term that represents the losses in lines, which are taken into account by means of the loss coefficients  $l_{i,t}^F$  associated to nodes. Constraints (14) define the power flows  $f_{l,t}$  on line  $l$  in period  $t$ , which are guaranteed by constraints (15) to be between their lower and upper bounds.

The solution of the MILP problem provides the list of the most efficient devices to be used in the power redispatch. This set of devices is used as data in Step 2 of the procedure, where the AC power redispatch has to be determined, taking into account security constraints (current in lines) and delivered power quality (voltages) for every unit of the discretized time horizon. In order to state the nonlinear model to be used in Step 2, the following sets, parameters and decision variables are defined.

### Sets

$\mathcal{N}$	set of nodes, indexed by $i$
$\mathcal{G}$	set of selected power generators, indexed by $g$
$\mathcal{B}$	set of selected storage devices, indexed by $b$
$\mathcal{T}$	set of time units, indexed by $t$ , in which the time horizon is divided
$\mathcal{L}$	set of ordered pairs of nodes $(i, j)$

The subset  $\mathcal{G}_i \in \mathcal{G}$  contains all generators located at node  $i$ . An element  $(i, j) \in \mathcal{L}$  may represent

- a transformer with On Load Tap Changer either on the primary winding ( $\mathcal{L}_1^{TC} \subset \mathcal{L}$ ) or on the secondary winding ( $\mathcal{L}_2^{TC}$ )
- network lines or fixed ratio transformers  $\mathcal{L} \setminus (\mathcal{L}_1^{TC} \cup \mathcal{L}_2^{TC})$

For every storage device  $b$  a line  $(i(b), j(b))$  is defined, where  $j(b)$  is the network node where the storage device is located and  $i(b)$  is a virtual node defined so as to take into account losses related to charge and discharge operations.

### Parameters

$P_{g,t}^0$	[MW]	scheduled active power output for generator $g$ at time $t$
$Q_{g,t}^0$	[Mvar]	scheduled reactive power output for generator $g$ at time $t$
$c_{g,t}^{\Delta P^+}$	[€/MW]	cost of active power increase of generator $g$ at time $t$
$c_{g,t}^{\Delta P^-}$	[€/MW]	cost of active power decrease of generator $g$ at time $t$
$c_{g,t}^{\Delta Q^+}$	[€/Mvar]	cost of reactive power increase of generator $g$ at time $t$

$c_{g,t}^{\Delta Q-}$	[€/Mvar]	cost of reactive power decrease of generator $g$ at time $t$
$c_{b,t}^{\text{out},P}$	[€/MW]	cost of active power discharge of storage device $b$ at time $t$
$c_{b,t}^{\text{in},P}$	[€/MW]	cost of active power charge of storage device $b$ at time $t$
$c_{b,t}^{\text{out},Q}$	[€/MW]	cost of reactive power discharge of storage device $b$ at time $t$
$c_{b,t}^{\text{in},Q}$	[€/MW]	cost of reactive power charge of storage device $b$ at time $t$
$\bar{P}_g$	[MW]	maximum active power output of generator $g$
$\underline{P}_g$	[MW]	minimum active power output of generator $g$
$\bar{Q}_g$	[Mvar]	maximum reactive power output of generator $g$
$\underline{Q}_g$	[Mvar]	minimum reactive power output of generator $g$
$\bar{E}_b^0$	[MWh]	stored electricity in storage device $b$
$\bar{E}_b$	[MWh]	maximum electricity that can be stored in storage device $b$
$\underline{E}_b$	[MWh]	minimum electricity that can be stored in storage device $b$
$\eta_b^{\text{out}}$	[-]	discharge loss coefficient of storage device $b$ ( $0 \leq \eta_b^{\text{out}} \leq 1$ )
$\eta_b^{\text{in}}$	[-]	charge loss coefficient of storage device $b$ ( $\eta_b^{\text{in}} \geq 1$ )
$\bar{\theta}_i$	[-]	minimum phase angle of node $i$
$\underline{\theta}_i$	[-]	maximum phase angle of node $i$
$\bar{V}_i$	[kV]	minimum voltage magnitude of node $i$
$\underline{V}_i$	[kV]	maximum voltage magnitude of node $i$
$C_{i,t}$	[MW]	active load of node $i$ at time $t$
$D_{i,t}$	[Mvar]	reactive load of node $i$ at time $t$
$G_i$	[S]	shunt conductance of node $i$
$B_i$	[S]	shunt susceptance of node $i$
$\bar{T}A_{i,j}$	[MW]	maximum active power on line $(i, j)$
$\underline{T}A_{i,j}$	[MW]	minimum active power on line $(i, j)$
$\bar{T}R_{i,j}$	[Mvar]	maximum reactive power on line $(i, j)$
$\underline{T}R_{i,j}$	[Mvar]	minimum reactive power on line $(i, j)$
$\bar{T}I_{i,j}$	[A]	maximum current on line $(i, j)$
$\underline{T}I_{i,j}$	[A]	minimum current on line $(i, j)$
$\delta_{i,j}$	[-]	loss angle of series impedance on line $(i, j)$
$X_{i,j}$	[S]	transversal conductance of node $i$
$Y_{i,j}$	[S]	transversal susceptance of node $i$
$Z_{i,j}$	[Ω]	series impedance of line $(i, j)$
$Vn_{i,j}$	[kV]	rated voltage of transformer primary winding $i$
$Vn_{j,i}$	[kV]	rated voltage of transformer secondary winding $j$
$u_{i,j}$	[-]	maximum increase relative to rated voltage for transformers with tap changer installed on the primary winding $i$
$d_{i,j}$	[-]	minimum increase relative to rated voltage for transformers with tap changer installed on the primary winding $i$
$u_{j,i}$	[-]	maximum increase relative to rated voltage for transformers with tap changer installed on the secondary winding $j$
$d_{j,i}$	[-]	minimum increase relative to rated voltage for transformers with tap changer installed on the secondary winding $j$

### Decision Variables

$\Delta P_{g,t}^+$	[MW]	increase of active power output for generator $g$ at time $t$
$\Delta P_{g,t}^-$	[MW]	decrease of active power output for generator $g$ at time $t$
$\Delta Q_{g,t}^+$	[Mvar]	increase of reactive power output for generator $g$ at time $t$
$\Delta Q_{g,t}^-$	[Mvar]	decrease of reactive power output for generator $g$ at time $t$
$P_{b,t}^{\text{out}}$	[MW]	active power discharge of storage device $b$ at time $t$
$P_{b,t}^{\text{in}}$	[MW]	active power charge of storage device $b$ at time $t$
$Q_{b,t}^{\text{out}}$	[Mvar]	reactive power injection of storage device $b$ at time $t$
$Q_{b,t}^{\text{in}}$	[Mvar]	reactive power absorption of storage device $b$ at time $t$
$P_{g,t}$	[MW]	active power output of generator $g$ at time $t$
$Q_{g,t}$	[Mvar]	reactive power output of generator $g$ at time $t$
$P_{b,t}$	[MW]	active power exchange between network and storage device $b$ at time $t$
$Q_{b,t}$	[Mvar]	reactive power exchange between network and storage device $b$ at time $t$
$V_{i,t}$	[kV]	voltage of node $i$ at time $t$
$\theta_{i,t}$	[ $-$ ]	phase angle of node $i$ at time $t$
$TI_{i,j,t}$	[A]	current transit on line $(i, j)$ at time $t$
$TA_{i,j,t}$	[MW]	active power flow on line $(i, j)$ at time $t$
$TR_{i,j,t}$	[Mvar]	reactive power flow on line $(i, j)$ at time $t$
$TC_{i,j,t}$	[kV]	voltage at time $t$ of the primary winding for transformer with OLTC $(i, j)$ installed in the primary winding
$TC_{j,i,t}$	[kV]	voltage at time $t$ of the secondary winding for transformer with OLTC $(i, j)$ installed in the secondary winding

The optimal values of the decision variables are determined by solving the following NLP problem.

$$\begin{aligned} \min \quad & \sum_{t \in \mathcal{T}} \left[ \sum_{g \in \mathcal{G}} \left( c_{g,t}^{\Delta P^+} \Delta P_{g,t}^+ + c_{g,t}^{\Delta P^-} \Delta P_{g,t}^- + c_{g,t}^{\Delta Q^+} \Delta Q_{g,t}^+ + c_{g,t}^{\Delta Q^-} \Delta Q_{g,t}^- \right) \right. \\ & \left. + \sum_{b \in \mathcal{B}} \left( c_{b,t}^{\text{out},P} P_{b,t}^{\text{out}} + c_{b,t}^{\text{in},P} P_{b,t}^{\text{in}} + c_{b,t}^{\text{out},Q} Q_{b,t}^{\text{out}} + c_{b,t}^{\text{in},Q} Q_{b,t}^{\text{in}} \right) \right] \end{aligned} \quad (16)$$

subject to

$$P_{g,t} = P_{g,t}^0 + \Delta P_{g,t}^+ - \Delta P_{g,t}^- \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (17)$$

$$Q_{g,t} = Q_{g,t}^0 + \Delta Q_{g,t}^+ - \Delta Q_{g,t}^- \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (18)$$

$$\sum_{g \in \mathcal{G}_i} P_{g,t} = C_{i,t} + V_{i,t}^2 G_i - \sum_{(i,j) \in \mathcal{L}} TA_{i,j,t} \quad i \in \mathcal{N}, t \in \mathcal{T} \quad (19)$$

$$\sum_{g \in \mathcal{G}_i} Q_{g,t} = D_{i,t} + V_{i,t}^2 B_i - \sum_{(i,j) \in \mathcal{L}} TR_{i,j,t} \quad i \in \mathcal{N}, t \in \mathcal{T} \quad (20)$$

$$\eta_b^{\text{out}} P_{b,t}^{\text{out}} - \eta_b^{\text{in}} P_{b,t}^{\text{in}} = TA_{i(b),j(b),t} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (21)$$

$$Q_{b,t} = TA_{i(b),j(b),t} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (22)$$

$$P_{b,t} = P_{b,t}^{\text{out}} - P_{b,t}^{\text{in}} \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (23)$$

$$E_b \leq E_b^0 - \sum_{\tau \leq t} P_{b,\tau} \leq \bar{E}_b \quad b \in \mathcal{B}, t \in \mathcal{T} \quad (24)$$

$$TA_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}Vn_{i,j}Vn_{j,i}} \sin(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\sin \delta_{i,j}}{Z_{ij}Vn_{i,j}^2} + \frac{X_{i,j}}{Vn_{i,j}^2} \right) \quad (i,j) \in \mathcal{L} \setminus (\mathcal{L}_1^{TC} \cup \mathcal{L}_2^{TC}), t \in \mathcal{T} \quad (25)$$

$$TR_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}Vn_{i,j}Vn_{j,i}} \cos(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\cos \delta_{i,j}}{Z_{ij}Vn_{i,j}^2} - \frac{Y_{ij}}{2Vn_{i,j}^2} \right) \quad (i,j) \in \mathcal{L} \setminus (\mathcal{L}_1^{TC} \cup \mathcal{L}_2^{TC}), t \in \mathcal{T} \quad (26)$$

$$TA_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}TC_{i,j,t}Vn_{j,i}} \sin(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\sin \delta_{i,j}}{Z_{ij}TC_{i,j,t}^2} + \frac{X_{i,j}}{TC_{i,j,t}^2} \right) \quad (i,j) \in \mathcal{L}_1^{TC}, t \in \mathcal{T} \quad (27)$$

$$TR_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}TC_{i,j,t}Vn_{j,i}} \cos(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\cos \delta_{i,j}}{Z_{ij}TC_{i,j,t}^2} - \frac{Y_{ij}}{2TC_{i,j,t}^2} \right) \quad (i,j) \in \mathcal{L}_1^{TC}, t \in \mathcal{T} \quad (28)$$

$$TA_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}Vn_{i,j}TC_{j,i,t}} \sin(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\sin \delta_{i,j}}{Z_{ij}Vn_{i,j}^2} + \frac{X_{i,j}}{Vn_{i,j}^2} \right) \quad (i,j) \in \mathcal{L}_2^{TC}, t \in \mathcal{T} \quad (29)$$

$$TR_{i,j,t} = \frac{V_{i,t}V_{j,t}}{Z_{ij}Vn_{i,j}TC_{j,i,t}} \cos(\theta_{i,t} - \theta_{j,t} - \delta_{i,j}) + V_{i,t}^2 \left( \frac{\cos \delta_{i,j}}{Z_{ij}Vn_{i,j}^2} - \frac{Y_{ij}}{2Vn_{i,j}^2} \right) \quad (i,j) \in \mathcal{L}_2^{TC}, t \in \mathcal{T} \quad (30)$$

$$TI_{i,j,t} = \frac{1}{\sqrt{3}V_{i,t}} \sqrt{TA_{i,j,t}^2 + TR_{i,j,t}^2} \quad (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad (31)$$

$$\underline{\theta}_i \leq \theta_{i,t} \leq \bar{\theta}_i \quad i \in \mathcal{N}, t \in \mathcal{T} \quad (32)$$

$$\underline{V}_i \leq V_{i,t} \leq \bar{V}_i \quad i \in \mathcal{N}, t \in \mathcal{T} \quad (33)$$

$$\underline{P}_g \leq P_{g,t} \leq \bar{P}_g \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (34)$$

$$\underline{Q}_g \leq Q_{g,t} \leq \bar{Q}_g \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (35)$$

$$(1 - d_{i,j})Vn_{i,j} \leq TC_{i,j,t} \leq (1 + u_{i,j})Vn_{i,j} \quad (i,j) \in \mathcal{L}_1^{TC}, t \in \mathcal{T} \quad (36)$$

$$(1 - d_{j,i})Vn_{j,i} \leq TC_{j,i,t} \leq (1 + u_{j,i})Vn_{j,i} \quad (i,j) \in \mathcal{L}_2^{TC}, t \in \mathcal{T} \quad (37)$$

$$\underline{TA}_{i,j} \leq TA_{i,j,t} \leq \bar{TA}_{i,j} \quad (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad (38)$$

$$\underline{TR}_{i,j} \leq TR_{i,j,t} \leq \bar{TR}_{i,j} \quad (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad (39)$$

$$\underline{TI}_{i,j} \leq TI_{i,j,t} \leq \bar{TI}_{i,j} \quad (i,j) \in \mathcal{L}, t \in \mathcal{T} \quad (40)$$

$$\Delta P_{g,t}^+, \Delta P_{g,t}^-, \Delta Q_{g,t}^+, \Delta Q_{g,t}^-, P_{b,t}^{\text{out}}, P_{b,t}^{\text{in}} \geq 0 \quad g \in \mathcal{G}, b \in \mathcal{B}, t \in \mathcal{T} \quad (41)$$

The optimization model used in Step 2 is based on a classical Optimal Power Flow as in [8, 9], suitably modified to account for power redispatch problems. Given the resources selected by the MILP model in Step 1, the model in Step 2 determines

- active and reactive power productions of controllable generators,
- modules and phases of voltages of all nodes,
- current flows in lines,
- rated voltage of tap-changer transformers,

so as to minimize redispatch costs, while satisfying the technical constraints

- load-flow equations, i.e. balance of active and reactive power at every node,
- equations of transit of active and reactive power in all lines,
- equations of current transits, for ensuring security,
- generators capability curves, that define the feasible values of active and reactive power production for each generator,
- bound constraints on power productions, current transits, voltage modules and phases.

Equations (17) and (18) establish a relation between power output and power variations. Equations (19) and (20) represent the nodal balance constraints. Equations (21) define the active power exchanges between the storage devices and the network, where charge and discharge losses are taken into account, while equations (22) are related to reactive power exchanges between storage devices and the network. Equations (23) define the net power discharge of storage devices, while constraints (24) define the bounds on the energy level at time  $t$ . Equations (25)-(30) define the power flow through lines and transformers. Equations (31) define the current transit in lines and transformers. Box constraints (32)-(40) are the upper and lower bounds on phase angles and nodal voltages of nodes, on active and reactive power outputs of generators, on voltages of tap changers, on active and reactive power transits and current transits on lines and transformers.

An application of the introduced framework may be found in [4]. The proposed procedure can be used a simulation tool for the DSO to optimize the configuration of MV network, e.g. determine effective positions of storage units. It can also be used as a simulation tool for regulators to analyze the impact of costs associated to the usage of controllable resources.

#### **4 Fast Estimation of Equivalent Capability for Active Distribution Networks**

Another relevant application of OPF tools in the operation of medium voltage grids is related to possible future market interactions between distribution and transmission grids [23]. Indeed, the contribution of both distributed renewable energy sources and flexible loads (demand side management) on the share of electricity production is going to be more relevant [7] in a near future and the stochastic and non-programmable nature of many renewable resources, along with the decrease of the conventional generation, will impact on the stability of the power system. In order to maintain an adequate balancing reserve, distributed resources are likely to be allowed to participate in the ancillary service market (e.g. voltage management of the transmission network). However, the contribution of the distribution network needs to be considered together with the related network constraints. Different coordination schemes, with different potentialities depending on the market evolution, have been recently proposed to foster the participation of distribution resources to

the balancing market [10]: in these schemes the interface between transmission and distribution network, the so called *Point of Common Coupling* (PCC), plays a fundamental role since the operation of the HV network requires detailed information on the actual flexibility of each transmission node. Fast and efficient methods need to be developed for computing the equivalent P-Q capability as seen from the HV node, since the actual power provision from resources on the MV grid is affected by the constraints on the MV grid operation; moreover, the active power can change in real time depending on the availability of the primary source. In [23] different methods for estimating the active distribution networks capabilities have been investigated. The proposed approach for computing the equivalent capability of the distribution network is based on the OPF tool used in Step 2 of the procedure presented in the previous section, with the additional consideration of circular and triangular capabilities of generators, represented by the following constraints

$$\underline{\varphi}_g P_{g,t} \leq Q_{g,t} \leq \overline{\varphi}_g P_{g,t} \quad g \in \mathcal{G}^T, t \in \mathcal{T} \quad (42)$$

$$\sqrt{P_{g,t}^2 + Q_{g,t}^2} \leq |\overline{S}_g| \quad g \in \mathcal{G}^C, t \in \mathcal{T} \quad (43)$$

where  $\mathcal{G}^T$  is the set of generators subject to triangular capability,  $\mathcal{G}^C$  is the set of generators subject to circular capability,  $\underline{\varphi}_g$  and  $\overline{\varphi}_g$  are the minimum value and the maximum value, respectively, of the ratio of reactive power output to active power output for generator  $g$  and  $\overline{S}_g$  is the maximum apparent power of generator  $g$ . The resulting model allows taking into account resources capabilities, local controllers and inter-temporal constraints [20].

When network constraints are not taken into account the aggregated flexibility can be easily calculated by algebraically summing all the capabilities of the available resources: in the test case considered in [23] and reported in Figure 2 this aggregated flexibility is similar to a trapezium (green curve in Figure 3) with rounded edges, due to the circular capability of the storage device. However, this theoretical capability cannot be entirely ensured because of the network operational limits (i.e. voltage and current constraints). These constraints bound the capability of the network when high share of both active and reactive power have to be exchanged (red curve in Figure 3).

The inner frontier, in red colour, determines the area corresponding to the set of activations and/or modulations of the connected flexible resources that do not determine network congestions. The constrained capability could be potentially computed by defining a grid of (P-Q) values and, for each of the defined couples, solving an OPF problem to check if the active and reactive power can be exchanged with the HV grid. In case the OPF returns an infeasibility then the point lies outside of the capability area. Albeit defining a quite accurate method of inspection, it is time consuming to run this methodology with an acceptable resolution. This aspect is extremely important, especially for real time markets (such as the balancing one) in which the prompt estimation of the ancillary services provision is fundamental. Moreover, since the working point of operations of distribution grids is subject to continuous variations, the reconstruction of the aggregated capability has to be fre-

resource (g)	capability shape	$ \bar{S} _g$ [MVA]	$Q_g^0$ [Mvar]	$\underline{Q}_g$ [Mvar]	$\bar{Q}_g$ [Mvar]	$P_g^0$ [MW]	$\underline{P}_g$ [MW]	$\bar{P}_g$ [MW]	$c_g^{P+}$ [€/MW]	$c_g^{P-}$ [€/MW]
Storage	circular	1.1	0	-1	1	0	-1	1	50	0
DG 4	circular	2	0	-1	1	1	1	2	100	80
DG 1	triangular (pf = 0.9)	2	0	-0.5	0.5	1	0	1	120	100
DG 2	triangular (pf = 0.9)	1	0	-0.5	0.5	0.5	0	0.5	120	100
DG 3	triangular (pf = 0.9)	2	0	-0.5	0.5	0	0	1	120	100
Load	constant (pf = 0.9)	2	-0.4	-0.4	0	-0.8	-0.8	0	220	—

Fig. 2: Example of distributed resources of a medium voltage network

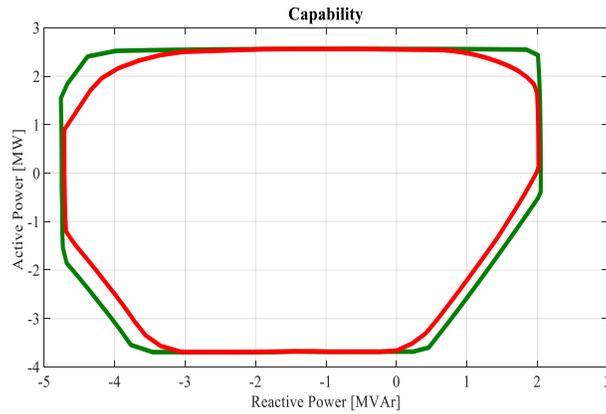


Fig. 3: Unconstrained capability (green) vs network constrained capability (red)

quently reprocessed. A faster procedure can be adopted using a conventional OPF tool. The basic principle of calculation can be summarized with the following steps:

1. a dummy unit is added to the network model in correspondence of the PCC;
2. a positive cost is assigned to the dummy unit when its power exchange is nonzero;
3. zero cost is assigned to distribution flexible resources;
4. a power exchange ( $P_{PCC}$ ,  $Q_{PCC}$ ) at the PCC is imposed, which has to be outside the capability area determined without considering the network constraints ;
5. a starting operational point ( $P_0$ ,  $Q_0$ ) is defined for the network
6. the OPF is performed.

The general principle behind this method is that the dummy unit will exchange the minimum amount of active and reactive power ( $P_{FU}$  and  $Q_{FU}$  respectively),

fulfilling the constraints. Therefore, the OPF returns a situation in which the distribution resources will try to exchange the maximum power to minimize the dummy unit contribution (which has a cost) and to maintain the network in its safe operation area. The dummy unit is modeled in order to exchange power with a fixed power factor ( $\tan(\phi_{FU})$ ) and with a cost proportional to  $|P_{FU}|$ . Thanks to this, the OPF solution lies on an arbitrarily selected straight line, for which the slope and the position on the (P-Q) plane depend on the imposed power exchange ( $P_{PCC}, Q_{PCC}$ ) and on  $\tan(\phi_{FU})$ . The capability curve is scanned by polling a series of OPFs for which different PCC power exchanges and FU power factors have been imposed as shown in Figure 4. According to the working principle describe above, each OPF converges on a point that corresponds to the intersection between the distribution network capability curve and the selected straight line.

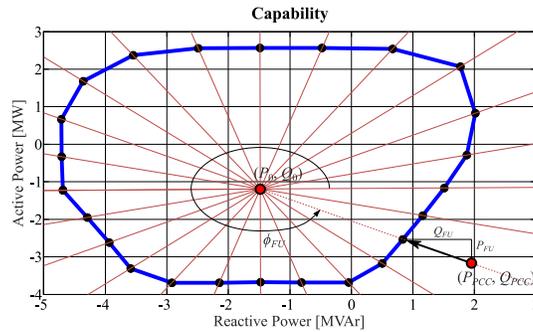


Fig. 4: Radial reconstruction of equivalent capability (fixed power factor method)

Finally, the proposed methods can be adapted in order to determine an approximation of the costs associated to the ((P-Q)) points belonging to the capability area. This can be obtained by reprocessing the estimation methods and activating a limited set of resources which is then gradually increased (the first process is executed for the cheapest flexible device, the next one includes also the second cheapest units, etc.). With this procedure a series of concentric capability areas can be extracted (with the desired cost resolution). Figure 5 reports the obtained results.

The frequent recalibration of the capability area, due to changes in the network conditions, calls for the usage of an efficient solution method for the OPF tool. Primal Dual Interior Point methods, with the Mehrotra Predictor-Corrector modification, provide a suitable approach for the solution of this kind of problem [14, 18, 19, 17].

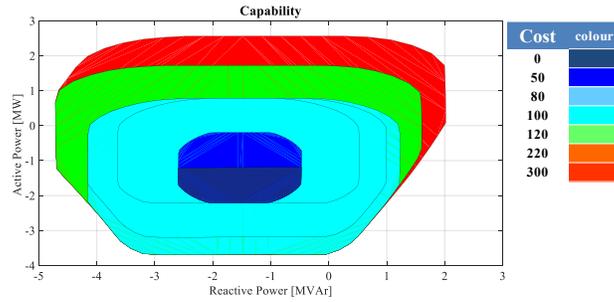


Fig. 5: Maximum capability for increasing number of resources according to the marginal cost, computed with the proportional cost method.

## 5 Conclusions

Two optimal power flow based tools have been presented which are expected to be useful for planning and operation of medium voltage networks in the near future. The first application allows for an efficient use of the network resources and provides assistance in interfacing the medium voltage grid with the high voltage grid. The nonconvexity of the AC OPF models and the mixed integer nature of selecting the adequate resources require defining special procedures to compute a solution for grid operation. The case studies suggest that the usage of these tools for distribution networks are beneficial for voltage control and for relieving line congestions, thus providing power quality and network security as well as ensuring a continuous provision to the customers. We have also shown that OPF tools can be efficiently used for providing a market interface with the high voltage grid and allow the distributed resources to participate in the provision of ancillary services to the transmission network. Namely the OPF tool can be used to build the equivalent capability region for the provision of active and reactive power from the distribution network to the transmission network and can be used, to some extent, to define bids to be supplied to the transmission network. Nevertheless, these operations require to be frequently reprocessed as the network conditions shift over time. This requires the Distribution System Operator to use efficient algorithms and software for the management of the Distribution Network.

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