Bundled Generation and Transmission Planning under Demand and Wind Generation Uncertainty based on a Combination of Robust and Stochastic Optimization

Arash Baharvandi, Jamshid Aghaei, *Senior Member, IEEE*, Taher Niknam,
Miadreza Shafie-khah, *Senior Member, IEEE*, and João P. S. Catalão, *Senior Member, IEEE*

***Abstract*—Bundled generation and transmission expansion planning (BGTEP) is aimed to solve problems related to ascendant demand of power systems. In this paper, a BGTEP model is considered and the optimal planning for a long-term period is obtained such that the cost of installation and operation would be minimized. Also, due to the recent orientation towards renewable energy sources, the influence of wind farms is involved in the methodology. An important aspect of load and wind power is their uncertain nature and the characteristic of being unforeseen. This matter would be under consideration by a bounded and symmetric uncertainty optimization approach. In fact, the combination of two uncertainty methods, i.e., robust and stochastic optimization approaches are utilized and formulated in this paper. Moreover, to cope with this uncertainty, Weibull Distribution (WD) is considered as wind distribution and load distribution is counted by a Normal Distribution (ND). A unique approximation approach for WD to be considered as ND is presented. In addition, the linear formulation is obtained by alternative constraints, to reduce the level of complexity of formulation drastically. Accordingly, a Mixed Integer Linear Programming (MILP) formulation is proposed to solve the BGTEP problem. The modified 6-bus and IEEE 24-bus RTS test systems are used to prove the applicability of the proposed method.**

***Index Terms*—Bundled Generation and Transmission Expansion Planning (BGTEP), Bounded Symmetric Optimization (BSO), Mixed Integer Linear Programming (MILP), Robust Optimization (RO), Normal Distribution (ND), Weibull Distribution (WD)**

NOMENCLATURE

A. Indices and Sets

ϕ_{ln}	Set of candidate lines.
ϕ_{in}	Set of candidate units.
ϕ_t	Set of demand intervals.
ϕ_i	Set of candidate and existing units.
ϕ_l	Set of candidate and existing lines.
φ_n^i	Set of units placed at bus n.
φ_n^j	Set of loads placed at bus n.
γ_k	Set of components of disabled due to contingency k.
ζ	Set of all candidate and existing components covering units and lines.
$s(l)$	Sending bus of line l.
$r(l)$	Receiving bus of line l.

B. Constants

IC_l	Investment cost of candidate line l (\$).
IC_i	Investment cost of candidate unit i (\$).
du_t	Duration of demand interval t (hour).
OM_i	Operation and maintenance cost of unit i (\$/MWh).
$p_i^{\max}$	Maximum output power of unit i (MW).
L_{jt}^f	Value of forecast load j at demand interval t (MW).
pw_{jt}^f	Value of forecast wind power at bus n related to load j in the demand interval t (MW).
X_l	Reactance of line l.
$f_l^{\max}$	Maximum power flow of line l (MW).
R	Forced outage rate.

C. Variables

b_l	Binary variable that for constructed line l is equal to 1 and 0 otherwise
b_i	Binary variable that for constructed unit i is equal to 1 and 0 otherwise
p_{it}	Output power of unit i in the demand interval t (MW).
$\theta_{n,t}$	Voltage angle of bus n under demand interval t.
f_{lt}	Flow power of line l in the demand interval t (MW).

Arash Baharvandi and Jamshid Aghaei and Taher Niknam are with the Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz, Iran (e-mails: aghaei@sutech.ac.ir; Niknam@sutech.ac.ir; ar.baharvandi@sutech.ac.ir).
Miadreza Shafie-khah is with C-MAST, University of Beira Interior, Covilhã 6201-001, Portugal (miadreza@gmail.com).
João P. S. Catalão is with INESC TEC and the Faculty of Engineering of the University of Porto, Porto 4200-465, Portugal, also with C-MAST, University of Beira Interior, Covilhã 6201-001, Portugal, and also with INESC-ID, Instituto Superior Técnico, University of Lisbon, Lisbon 1049-001, Portugal (e-mail: catalao@ubi.pt).

τ_{jtk} Load shedding of load j at demand interval t under contingency k.

Ω_0 Availability probability of component without contingency.

Ω_k Probability of contingency k.

EENS_{jtk} Expected energy not supplied at load j in the demand interval t under contingency k.

I. Introduction

A. Aims and Background

EXTENSION of the entire network has been introduced as an important issue in recent years. Considering the new components which can be installed in order to supply the extra loads culminates in bundled generation and transmission expansion planning (BGTEP) [1], [2]. In this problem, it should be decided when to invest new capacity and which kind of generation or transmission is needed. Moreover, in the BGTEP problem, the optimum location of newly constructed components should be assessed [3]. It is a foregone conclusion that, as time passes, the number of devices which ought to be supplied increases.

The final goal of BGTEP is to have a secure reliability level for the forecasted electricity demand. In this situation, the generation and transmission constraints should be satisfied. In addition, the amount of emissions corresponding to greenhouse gases is increasing progressively, so renewable energies are being increasingly used to provide a more friendly climate. Renewable Energy Sources (RES) such as wind power and solar cells are clean sources. Nevertheless, their associated generation has inherent uncertainty [4], [5].

B. Literature Review

To cope with the above-mentioned problem, there are many types of research in the area of BGTEP. In [6], superconducting fault current limiters are implemented to decrease the current faults in a model of combined generation and transmission network expansion planning. There are several methodologies for solving a multi-objective BGTEP model. In addition to the cost, the reliability is another objective function for [7], and this reference solves a multi-objective probabilistic expansion model. There is another multi-objective transmission expansion planning to cover the uncertain investment budget and uncertain demand in [8].

Note that there are many published types of research working on a separate generation or transmission expansion planning. Indeed, Generation Expansion Planning (GEP) is considered as the main objective in some works in the area [9]–[11].

In [9], the application of stochastic MILP is considered in multi-stages (periods), and the uncertainty of hydrological resources are analyzed. Reference [10] solve a GEP problem while the effect of different units such as nuclear, renewable energy and different fossil fuel-fired units, is considered. A co-optimizing methodology in the form of charging or discharging of electric vehicles is proposed in [11]. Recently, the investors have unprecedented challenges on Transmission Expansion Planning (TEP), so the authors solve this problem with several different points of view [12]–[14]. In [12], TEP problem is solved by implementing a multi-objective framework considering cost and risk as two contradictory goals. The authors in [13] propose a method utilizing the power transfer distribution factors by which some important transmission lines would be observable, and afterward, they try to create a reliable system by this observability. A short-circuit level constrained TEP problem is analyzed in [14]. This reference uses an MILP approach with considering the transmission investment cost as a master problem and three different sub problems.

Moreover, the inherent uncertainties such as load uncertainty and unforeseen wind power have been challenged by the authors in many published types of research. The robust optimization (RO) method is a prevalent approach to consider the forecasting uncertainty of load or renewable sources [15]–[17]. In [15], a scenario-based RO is implemented to cope with load and wind uncertainty for TEP problem. Similarly, a single and two-stage robust optimization is used in [16] to solve the uncertainty in GEP problem. Likewise, the authors in [17] present robust optimization and do not utilize the traditional probabilistic model used in stochastic approaches. In fact, in the RO methodology, there is no need for probability distribution functions of uncertain parameters.

C. Contribution

In this paper, the problem of expansion planning regarding the lines and generators is formulated while the wind turbine is considered as a renewable source. In the presence of network uncertainties, the complexity of expansion planning problem increases. Here, uncertainties are related to a variety of wind velocity culminating in varied wind power and also the diversity of demand. RO and stochastic programming (SP) are two different methods which are implementing to cope with system's uncertainty nature. However, the proposed method is based on a combination of RO and SP. Additionally, wind power is estimated by Weibull Probability Distribution Function (WPDF). However, since the combination of RO and SP necessitates the uncertain variable to have a Normal Probability Distribution Function (NPDF), in the proposed approach, there is a new approximation methodology to consider WPDF as an NPDF with minimum error. In fact, the suggested methodology approximates Weibull distribution pertaining to the wind turbine to the normal distribution with the specified mean and standard deviation which is the closest distribution to WPD, and there is not any other NPDF which has an error less than the error calculated in the proposed approximation. Therefore, by this approximation and considering wind power by NPDF which is the closest distribution to WPD, the bounded and symmetric approach can be $\forall l, \forall t$ integrated while the effect of uncertain wind power is covered.

Although, in [15]–[17], the RO strategy has been presented for TEP and GEP problems, while the symmetric aspect of uncertainty related to the stochastic problem has not been addressed in these works.

To the best authors' knowledge, finding the best approximation of Weibull to Normal distribution is not considered in previous researches.

II. Mathematical Formulation

In this section, the BGTEP problem is formulated, and an objective function containing operation and investment cost is minimized. In addition to, the expected energy not supplied (EENS) as the probabilistic reliability criteria are aggravated to the objective function. Meanwhile, technical and operational constraints should be satisfied.

A. Problem Formulation without Wind

The investment cost for constructed units and new lines, as well as the operation cost of existing units, should be minimized as follows [18], [19]:

$$Cost = \sum_{l\in\phi_{ln}} b_l IC_l + \sum_{i\in\phi_{in}} b_i IC_i + \sum_{t\in\phi_t}\sum_{i\in\phi_i} p_{it} OM_i du_t \qquad (1)$$

The first two terms are related to the investment cost (C_{inv}), and the third term relates to operation cost (C_{gen}). Constraints are given below [20], [21]:

$$b_l = 1 \qquad \forall l \in \{\phi_l - \phi_{ln}\} \qquad (2)$$

$$b_i = 1 \qquad \forall i \in \{\phi_i - \phi_{in}\} \qquad (3)$$

$$b_l \in \{0,1\} \qquad \forall l \in \phi_{ln} \qquad (4)$$

$$b_i \in \{0,1\} \qquad \forall i \in \phi_{in} \qquad (5)$$

$$0 \le p_{it} \le b_i p_i^{\max} \qquad \forall i, \forall t \qquad (6)$$

$$\sum_{i\in\varphi_n^i} p_{it} - \sum_{l:s(l)=n} f_{lt} + \sum_{l:r(l)=n} f_{lt} = 0 \qquad \forall n \notin \varphi_n^j, \forall t \qquad (7)$$

$$\sum_{i\in\varphi_n^i} p_{it} - \sum_{l:s(l)=n} f_{lt} + \sum_{l:r(l)=n} f_{lt} \ge \sum_{j\in\varphi_n^j} L_{jt}^f \qquad \forall n \in \varphi_n^j, \forall t \qquad (8)$$

$$\sum_{i\in\varphi_n^i} p_{it} - \sum_{l:s(l)=n} f_{lt} + \sum_{l:r(l)=n} f_{lt} \le \sum_{j\in\varphi_n^j} L_{jt}^f + \delta \sum_{j\in\varphi_n^j} L_{jt}^f - \varepsilon\lambda \sum_{j\in\varphi_n^j} L_{jt}^f \qquad \forall n \in \varphi_n^j, \forall t \qquad (9)$$

$$f_{lt} = \frac{b_l}{X_l}(\theta_{s(l),t} - \theta_{r(l),t}) \qquad \forall l, \forall t \qquad (10)$$

$$f_{lt} \le f_l^{\max} \qquad \forall l, \forall t \qquad (11)$$

$$f_{lt} \ge -f_l^{\max} \qquad \forall l, \forall t \qquad (12)$$

Constraints (2) and (3) pertain to binary variables of existing components. Constraints (4) and (5) describe the situation of new components, if b_l=1 and b_i=1, new components have been added to the system and otherwise b_l=0 and b_i=0. Constraint (6) states the power limitations of the generators. Constraint (7) states the balance equation for buses without load. Constraint (8) states the balance equation for buses with the load that is relaxed [20], [21]. In fact, balance equation represents that difference of generation power and sending power at each bus is equal to consumption in that bus. Constraint (9) is the same as the balance equation with considering the uncertain demand j at period t where ε denotes uncertainty level. δ denotes infeasibility tolerance, and the relationship between λ and κ (reliability level) is given as follows:

$$\kappa = 1 - F(\lambda) \qquad (13)$$

$$\kappa = 1 - \Pr\{\Psi \le \lambda\} \qquad (14)$$

$$\Psi = -\xi \qquad (15)$$

$$\kappa = 1 - \int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}) dx \qquad (16)$$

In the proposed method was $\delta \ge \varepsilon\lambda$ and λ between -1 and 1. Constraint (10) represents the power flow of lines. Constraints (11) and (12) are the limitations of power flow. In view of the contingency, EENS is added and will have the following:

$$0 \le p_{itk} \le b_i p_i^{\max} \qquad \forall i, \forall t, \forall k \qquad (17)$$

$$\sum_{j\in\varphi_n^j} \tau_{jtk} \ge \sum_{j\in\varphi_n^j} L_{jt}^f - \sum_{\substack{i\in\varphi_n^i\\ i\notin\gamma_k}} p_{itk} + \sum_{\substack{l:s(l)=n\\ l\notin\gamma_k}} f_{ltk} - \sum_{\substack{l:r(l)=n\\ l\notin\gamma_k}} f_{ltk} \qquad \forall n \notin \varphi_n^j, \forall t, \forall k \qquad (18)$$

$$\sum_{j\in\varphi_n^j} \tau_{jtk} \le \sum_{j\in\varphi_n^j} L_{jt}^f + \delta \sum_{j\in\varphi_n^j} L_{jt}^f - \varepsilon\lambda \sum_{j\in\varphi_n^j} L_{jt}^f - \sum_{\substack{i\in\varphi_n^i\\ i\notin\gamma_k}} p_{itk} + \sum_{\substack{l:s(l)=n\\ l\notin\gamma_k}} f_{ltk} - \sum_{\substack{l:r(l)=n\\ l\notin\gamma_k}} f_{ltk} \qquad \forall n \notin \varphi_n^j, \forall t, \forall k \qquad (19)$$

$$f_{ltk} = \frac{b_l}{X_l}(\theta_{s(l)tk} - \theta_{r(l)tk}) \qquad \forall l, \forall t, \forall k \qquad (20)$$

$$f_{ltk} \le f_l^{\max} \qquad \forall l, \forall t, \forall k \qquad (21)$$

$$f_{ltk} \ge -f_l^{\max} \qquad \forall l, \forall t, \forall k \qquad (22)$$

Constraint (17) is unit power limitations. τ_{jtk} is the load lost pertaining to load j at bus n at period t under contingency k, it was seen in constraint (18) that is relaxed. In other words, the difference between the forecasting load with generation power and receiving power at each bus represents the load lost. τ_{jtk} is defined as load shedding. Constraint (19) is the same as the previous constraint with considering the uncertainty of demand. Constraints (20)-(22) are the power flow and limitations of the line under contingency.

A binary variable is defined that describes the state of total components as following:

$$\boldsymbol{\omega}_{1\times K} = [\boldsymbol{\omega_e}\ \boldsymbol{\omega_n}] \qquad (23)$$

ω_e is related to the existing components and is one. ω_n denotes the state of new components and is one or zero. ω_n at Built-in components is 1 and otherwise ω_n=0. Access or lack of access to components makes it a binomial probability distribution is defined as the Bernoulli distribution [22]:

$$\Omega_0 = \prod_{\alpha\in\zeta} (1 - \omega_\alpha R_\alpha) \qquad (24)$$

$$\Omega_k = \omega_k R_k \prod_{\substack{\omega \neq k \\ \omega \in \zeta}} (1-\omega_w R_w)$$
$$= \omega_k R_k (1-R_k)^{-1} \prod_{\omega \in \zeta} (1-\omega_w R_w) \qquad (25)$$
$$= \omega_k R_k (1-R_k)^{-1} \Omega_0 \qquad \forall k$$

R_α is defined as the forced outage rate (FOR) of component α. The Eq. (24) states the probability of availability of components in non-contingent status. The Eq. (25) denotes the probability of contingency k.

The EENS at load j, in period t under contingency k, is denoted:

$$\text{EENS}_{jtk} = \Omega_k \tau_{jtk} du_t \qquad \forall j, \forall t, \forall k \qquad (26)$$

The cost of EENS is defined:

$$Cost_{EENS} = \text{VOLL} \times \sum_{k} \sum_{t \in \phi_t} \sum_{j \in \phi_d} \Omega_k \tau_{jtk} du_t \qquad (27)$$

where the value of lost load (VOLL) is found in [23]. Also, with considering the contingency, the objective function is denoted:

$$\min \left\{ \begin{array}{l} C_{inv} + \Omega_0 C_{gen} + \sum_k \Omega_k \left(\sum_{t \in \phi_t} \sum_{\substack{i \in \phi_i \\ i \notin \gamma_k}} p_{itk} OM_i du_t \right) + \\ Cost_{EENS} \end{array} \right\} \qquad (28)$$

B. Problem Formulation with Wind

Adding wind power to the system will affect the constraints (8), (9), (18), (19). This impact is given as follows:

$$\sum_{i \in \varphi_n^i} p_{it} - \sum_{l:s(l)=n} f_{lt} + \sum_{l:r(l)=n} f_{lt} \geq P_{ND}^f \qquad \forall n \in \varphi_n^j, \forall t \qquad (29)$$

$$\begin{array}{l} \sum_{i \in \varphi_n^i} p_{it} - \sum_{l:s(l)=n} f_{lt} + \sum_{l:r(l)=n} f_{lt} \leq P_{ND}^f \\ + \delta P_{ND}^f - \varepsilon \lambda P_{ND}^f \end{array} \qquad \forall n \in \varphi_n^j, \forall t \qquad (30)$$

$$\begin{array}{l} \sum_{j \in \varphi_n^j} \tau_{jtk} \geq P_{ND}^f \\ - \sum_{\substack{i \in \varphi_n^i \\ i \notin \gamma_k}} p_{itk} + \sum_{\substack{l:s(l)=n \\ l \notin \gamma_k}} f_{ltk} - \sum_{\substack{l:r(l)=n \\ l \notin \gamma_k}} f_{ltk} \end{array} \qquad \forall n \in \varphi_n^j, \forall t, \forall k \qquad (31)$$

$$\begin{array}{l} \sum_{j \in \varphi_n^j} \tau_{jtk} \leq P_{ND}^f \\ + \delta P_{ND}^f - \varepsilon \lambda P_{ND}^f \\ - \sum_{\substack{i \in \varphi_n^i \\ i \notin \gamma_k}} p_{itk} + \sum_{\substack{l:s(l)=n \\ l \notin \gamma_k}} f_{ltk} - \sum_{\substack{l:r(l)=n \\ l \notin \gamma_k}} f_{ltk} \end{array} \qquad \forall n \in \varphi_n^j, \forall t, \forall k \qquad (32)$$

$$P_{ND}^f = \sum_{j \in \varphi_n^j} (L_{jt}^f - pw_{jt}^f) \qquad \forall n \in \varphi_n^j, \forall t \qquad (33)$$

Constraint (29) is similar to constraint (8) that is relaxed and constraint (30) is same as constraint (29) with considering the uncertainty of demand and wind power. In fact, with the wind generation to the network, these two constraints replace the constraints (8) and (9). Constraint (31) is the amount of load shedding that impact of wind power in P^f_{ND} appears and constraint (32) is with considering the uncertainty of demand and wind power. These two constraints also replace the constraints (18) and (19). The difference of forecasting load and wind power is defined as the forecasting net demand power (P^f_{ND}) at constraint (33).

III. Approximation of WPDF to NPDF

In order to use the mentioned approach, the uncertain parameter should be described by a normal distribution [20], [21]. In the previous section, the difference between load and wind power was defined as the net demand power. The load is defined by a normal distribution, but the distribution of wind power isn't. Using empirical observations of wind power in wind farms can be considered as a normal distribution [24]. According to the empirical wind data and a curve fitting, it is observed that the single-Weibull, bi-Weibull or tri-Weibull distributions are good approximations for the available wind data. Using Akaike information criterion (AIC) and Bayesian information criterion (BIC), it can be determined which of these three distributions are much more suitable for the available wind data [25].

Data of the Weibull distribution mixture is found in [25]. So should the wind power distribution, approximated by a normal distribution and then benefitted from the approach of a combination of RO and SP. In this section, the distribution of wind power in three states (one, two and three Weibull) is approximated by the normal distribution. The WPDF is expressed as:

$$f(x) = \frac{\hbar}{\rho} \left(\frac{x}{\rho}\right)^{\hbar - 1} \exp^{\left(-\left(\frac{x}{\rho}\right)^{\hbar}\right)} \qquad (34)$$

where ρ and $\hbar$ are the Weibull scale parameter and shape parameter.

The Weibull distribution mixture of wind power is given following:

$$f(pw) = \sum_{N=1}^{c} \Gamma_N f(pw \mid \varpi_N) \qquad (35)$$

where Γ_N is the weight of each term, N is the number of terms, $f(pw|\varpi_N)$ is the Weibull distribution function, and ϖ is included ρ and $\hbar$. However, the WPDF should be approximated by an NPDF that its error is less of any other NPDF. This approximation is obtained by the following error:

$$Error = \left| \begin{array}{l} \left(\sum_{z=1}^{c} \Gamma_z f(pw \mid \varpi_N) \right) \\ - \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-0.5 \left(\frac{pw - \mu}{\sigma}\right)^2 \right) \end{array} \right| \qquad (36)$$

The second term of error is NPDF where σ and μ are the standard deviation and expected value, respectively. Error acquires the difference of WPDF and specified NPDF. It also determines the maximum difference. This procedure is done for different NPDFs and every time the maximum difference is determined. Between of maximum differences, minimum difference is attained. The selected minimum difference is related to a specified NPDF. It is the best approximation for WPDF of wind power.

A. Approximation of Single Weibull Distribution

If c=1, then wind power distribution is single Weibull that

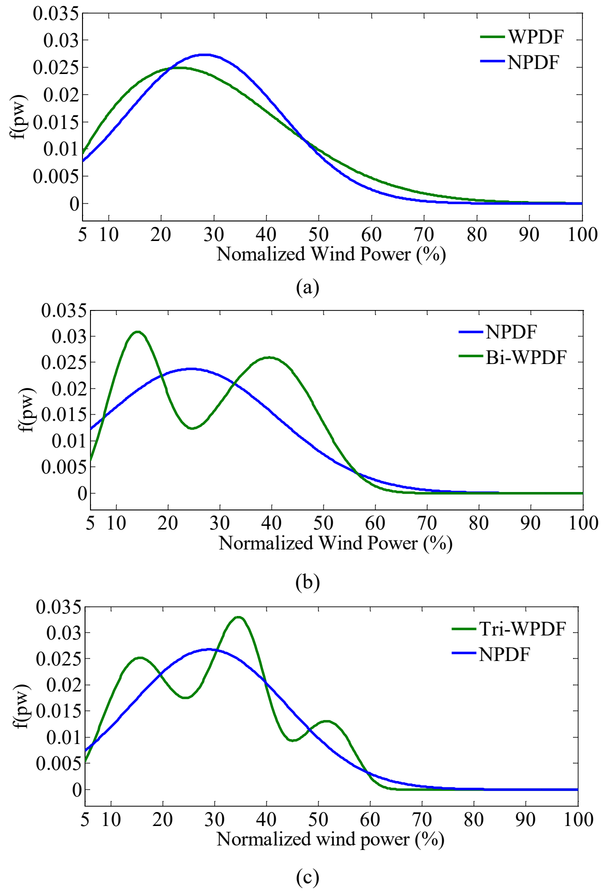

Fig. 1. Approximation WPDF to NPDF. (a) Single WPDF to NPDF. (b) bi-WPDF to NPDF. (c) tri-WPDF to NPDF

the parameters include: ρ=33.86, $\hbar$=1.95. The approximation of single Weibull to a specified NPDF is shown in Fig. 1(a). The NPDF derived has σ=14.6 and μ=28.2. In this case, maximum error and percent of forecast value error are 0.0042 and 6%, respectively.

B. Approximation of Bi-Weibull Distribution

If c=2, then wind power distribution is bi-Weibull that the parameters include: Γ_1=0.63, Γ_2=0.37, ρ_1=41.73, ρ_2=15.54, $\hbar_1$=4.55, $\hbar_2$=3.18. The approximation of bi-Weibull to a specified NPDF is shown in Fig. 1(b). The resulting NPDF has σ=16.8 and μ=24.4. In this case, maximum error and percent of forecast value error are 0.0114 and 16.3%, respectively.

C. Approximation of Tri-Weibull Distribution

If c=3, then the wind power distribution is tri-Weibull that the parameters include: Γ_1=0.44, Γ_2=0.37, Γ_3=0.19, ρ_1=35.22, ρ_2=17.47, ρ_3=52.19, $\hbar_1$=6.86, $\hbar_2$=2.95, $\hbar_3$=9.73. The approximation of tri-Weibull to a specified NPDF is shown in Fig. 1(c). The resulting NPDF has σ=14.9 and μ=28.9.

In this case, maximum error and percent of forecast value error are 0.0083 and 2.6%, respectively. By comparing the three approximations, it is shown that the approximation for single Weibull and tri-Weibull is better than bi-Weibull. Maximum error that is the maximum difference between the normal distribution and the Weibull distribution, but forecast value error is obtained according to the mean definition of probability distribution function.

IV. Linearization of Formulations

In the above formulation, there are several nonlinear terms. These terms are because of production of continues variable and binary variables, e.g., (10), (20), (27) and (28).

Let r_i be the product of a bounded free variable y and set of binary variables m. The nonlinear terms in section II have the following form:

$$r_i = m_i \prod_{\substack{j=1 \\ j \neq i}}^{E} (1 - A_j m_j) y \qquad \forall i \in I = (1, 2, ..., E) \tag{37}$$

where A_j is a parameter such as forced outage rate, and E denotes the number of binary variables. Consequently, we can write:

$$r_i = m_i h \qquad \forall i \in I \tag{38}$$

$$h = \prod_{\substack{j=1 \\ j \neq i}}^{E} (1 - A_j m_j) y \tag{39}$$

The term in (38) is expanded as follows:

$$h = y \times (1 - A_1 m_1) \times (1 - A_2 m_2) \times ... \times (1 - A_E m_E) \tag{40}$$

Next, we assume the following expressions:

$$W_j = W_{j-1} \times (1 - A_j m_j) \qquad \forall j \neq i \tag{41}$$

$$h = W_E \tag{42}$$

The first row in (41), W_1, is nonlinear since y is a variable and m_1 is a binary variable. Production of these two variables makes W_1 nonlinear. The equivalent linear form of W_1 can be given by:

$$y - m_1 Z \le W_1 \le y + m_1 Z \tag{43}$$

$$y(1 - A_1) - (1 - m_1) Z \le W_1 \le y(1 - A_1) + (1 - m_1) Z \tag{44}$$

where Z is a positive large enough constant, it should be greater than y. If m_1 equals zero, W_1 should be equal to y from (43) while the bounds in (44) are inactive. Otherwise, when m_1 equals one, W_1 from (44), should be equal to $y(1–A_1)$. Therefore, W_1 is converted to equivalent linear inequalities by (43)–(44). Then, we derive the linear form of W_j ($j \in I$–{1} and $i \in I$) which is formulated as:

$$W_j \le W_{j-1} + m_j Z \tag{45}$$

$$W_j \ge W_{j-1} - m_j Z \tag{46}$$

$$W_j \le W_{j-1}(1 - A_j) + (1 - m_j) Z \tag{47}$$

$$W_j \ge W_{j-1}(1 - A_j) - (1 - m_j) Z \tag{48}$$

According to (43)-(44), W_1 is either y or y $(1–A_1)$ wherein both cases, $0 \le W_1 \le y$. likewise, W_2 is either W_1 or W_1 $(1–A_1)$. If m_j = 1, then W_j would be equal to $W_{j-1}(1–A_1)$ based on the inequalities (47) and (48) while inequalities (45) and (46) are inactive. If m_j = 0, the inequalities (45) and (46) state that W_j=W_{j-1} while the inequalities (47) and (48) do not bind. It is obvious that each W_j is a linear function of W_{j-1} and the expression associates to W_1 which is linear. Therefore, by (42)-(48), we can extract h as a linear function of binary variables m_i and continues variable y. Now, r_i expressed in (38) can be linearized by the equivalent following linear inequalities:

$$-m_i Z \le r_i \le m_i Z \qquad \forall i \in I \qquad (49)$$

$$W_E - (1-m_i)Z \le r_i \le W_E + (1-m_i)Z \qquad \forall i \in I \qquad (50)$$

If x_i equals zero, r_i must be equal to zero from (49), while the bounds of (50) are inactive. Otherwise, if x_i equals one, r_i must be equal to W_E or h from (50), while the bounds of (49) are inactive.

Finally, it should be noted that to linearize the equations such as (10) and (20) which have just one binary variable multiplying by a continuous variable, the procedure in (43) and (44) can be applied. So, we have a mixed integer linear programming (MILP) problem.

V. Case Studies

In this section, the proposed methodology is applied to BGTEP problem of IEEE 6-bus and IEEE 24-bus reliability test systems (RTS). All case studies are considered using CPLEX solver within GAMS [26] on a personal computer with Core i7 processor and 16 GB RAM.

A. 6-Bus Test System

The data for all components is founded in [27]. The planning horizon in this paper is one year. It is distributed to five sectors and for each sector a determined load factor. Multiplying the load factor of a sector at the annual peak load is defined as the load for each sector.

Table I shows the load factor of each sector. The weight of the load is the ratio of the existing load at bus n to the total load. The weight of loads of buses 3, 4, 5 is 0.4, 0.3 and 0.3, respectively. As well as, the value of VOLL is 1000$/MWh.

In this section, the case studies are considered under two modes. In the first case, it is assumed that renewable energy sources (RES) in the system does not exist. In the latter case, it is assumed that wind turbines as renewable energy sources can be added to the network.

1) Case A: Probabilistic BGTEP Model without the Effect of RES: In this case, the RES will be discarded, and the only uncertain parameter is the load. Accordingly, the problem formulation without wind is applied. The simulation results of BGTEP are shown in Table II. As can be seen, with increasing annual peak load, the number of candidate components added to the network increases and thus it increases the value of the objective function. The simulation results represent that the BGTEP model would be infeasible for annual peak load equal to 80MW. This means that in the presence of new and existing components, the system is not able to meet the demand of the network. The advantage of BGTEP than to GEP is less constructed units. In other words, due to using new lines, BGTEP supplies the demand with more less generating units. As well as, it supplies the higher demand. Because limitation of lines is refused that GEP supplies the more load. Because of the investment cost of the line is lowered the new units and lines have no operation cost, we conclude that BGTEP is effective economically.

2) Case B: Probabilistic BGTEP Model with the Effect of RES: In this case, the effect of RES is considered, and the problem formulation with wind is used. In this case, the problem of wind power is its uncertainty. So, uncertainty is related to the load and wind power that can be expressed in the form of net demand power. For the sake, according to the proposed method, wind power should be described by a normal distribution. It was discussed in detail in section III. Two wind turbines can be placed at buses 3, 4 that the capacity of each of them is 2.21 MW. The data of wind power in this paper is founded in [28]. The simulation results of BGTEP is shown in Table III. In the presence of RES, the value of objective function decreases. As part of the demand is met by wind turbines and the output power of generating units is reduced. The simulation results represent that the BGTEP model would be infeasible for annual peak load equal to 85MW while this amount was 80 MW in the previous case. To clarify this issue, it can be a wind turbine placed on a special bus to be considered. Part of the load is supplied by wind power, as a result of reduced power flow of lines leading to the bus bar. The situation is similar to increasing of capacity lines, and more load is supplied. The simulation results are shown that the value of the objective function for 60MW in case B is 12.786(10^6\$) while in case A is 14.342 (10^6\$). So, the effect of wind is evident. The objective function versus reliability level is shown in Fig. 2 by the variation δ and ε for annual peak load 50 MW. As can be seen, with increased reliability level, the value of objective function increases. In fact, to have a higher reliability level, cost more should be spent.

TABLE I: DATA OF LOAD FACTOR FOR IEEE 6-BUS TEST SYSTEM

Time sector duration (h)	1510	2800	2720	1120	610
Load factor	0.5	0.65	0.8	0.9	1

TABLE II: RESULTS OF BGTEP WITHOUT RES

Annual peak load (MW)	New lines	New units	Number of new components	Cost (10^6\$)
30	-	A4,A5,B4,B8	4	6.656
35	-	A5,B2,B3,B4,B8	5	7.604
40	-	A4,A5,B2,B3,B4, B7,B8	7	8.887
45	T2,T3	A1,A5,B1,B2, B3,B4,B7,B8	10	10.19
50	T2,T3	A1,A4,A5,B1,B2, B3,B4,B7,B8	13	11.550
55	T2,T3,T6,T7	A1,A4,A5,B2,B3, B4,B5,B7,B8	13	12.968
60	T2,T3,T6,T7	A1,A4,A5,B1,B2, B3,B4,B5,B7,B8	14	14.342
65	T2,T3,T6,T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B7, B8	15	15.870
70	T2,T3,T4,T6, T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B6, B7,B8	17	17.533
75	T2,T3,T4,T6, T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B6, B7,B8	17	19.207
80	Infeasible	Infeasible	Infeasible	Infeasible

TABLE III: RESULTS OF BGTEP WITH RES

Annual peak load (MW)	New lines	New units	Number of new components	Cost (10^6\$)
30	-	A5,B3,B4,B8	4	5.230
35	-	A5,B2,B3,B4,B8	5	6.416
40	-	A5,B2,B3,B4,B7, B8	6	7.587
45	-	A4,A5,B2,B3,B4, B7,B8	7	8.687
50	T2,T3	A1,A4,A5,B2,B3, B4,B7,B8	10	9.987
55	T2,T3	A1,A4,A5,B1,B2, B3,B4,B7,B8	11	11.370
60	T2,T3,T6,T7	A1,A4,A5,B2,B3, B4,B5,B7,B8	13	12.786
65	T2,T3,T6,T7	A1,A4,A5,B1,B2, B3,B4,B5,B7,B8	14	14.140
70	T2,T3,T6,T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B7, B8	15	15.706
75	T2,T3,T4,T6 ,T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B6, B7,B8	17	17.377
80	T2,T3,T4,T6 ,T7	A1,A3,A4,A5,B1, B2,B3,B4,B5,B6, B7,B8	17	19.055
85	Infeasible	Infeasible	Infeasible	Infeasible

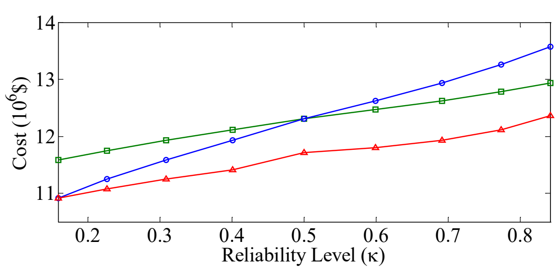

Fig. 2. Cost versus Reliability Level under different uncertainty level and infeasibility tolerance

TABLE IV: NEW LINES FOR IEEE 24-BUS

New lines	(1-5),(3-9),(3-24),(4-9),(6-10),(7-8),(9-12) (10-12),(11-13),(12-13)

TABLE V: RESULTS OF BGTEP WITHOUT RES FOR IEEE 24-BUS RTS

Annual peak load (MW)	New lines	New units	Number of new components	Cost (10^6\$)
4000	-	G2,G5,G6,G12,G14,G26	6	3.563
5000	T5	G1,G2,G5,G6,G12,G13, G14,G15,G17,G19,G22, G23,G26	14	9.073
6000	T5,T6	G1,G2,G3,G4,G5,G6,G7, G8,G9,G10,G11,G12, G13,G14,G21,G22,G23, G24,G25,G26	22	17.719

TABLE VI: RESULTS OF BGTEP WITH RES FOR IEEE 24-BUS RTS

Annual peak load (MW)	New lines	New units	Number of new components	Cost (10^6\$)
4000	-	G5,G26	2	1.581
5000	-	G1,G2,G5,G6,G12,G13, G14,G23,G26	9	6.554
6000	T6	G1,G2,G5,G6,G7,G8, G10,G11,G12,G13,G14, G20,G22,G23,G24,G26	17	14.104

TABLE VII: RUN TIME OF SIMULATIONS

Case studies	6 bus without RES	6 bus with RES	24 bus without RES	24 bus with RES
Run time	1’:20’’	1’:28’’	5’’:03’’’	4’’:37’’’

Also, the variation of δ and ε affect the value of objective function. In Fig. 2 with different δ and ε, the effect of reliability level on cost is observed. It should be noted that the value of ε, δ, λ is 0.05 at all the results of the simulation.

B. IEEE 24-Bus RTS

In this case study, existing generating units and lines are 32 and 38, respectively. The relevant data are available in [29], [30]. Table I is used for this test system, and the value of VOLL is 1000\$/MWh. Due to the limitations in water resources, hydro units are not considered as candidate generators. The new lines are represented in Table IV, and relevant data is found in [31]. Also, the capacity of wind turbines used is 66.916 MW and placed at buses 1, 6, 9, 13, 16, 20.

This system test is simulated for the both of case *A* and *B*. The simulation results are shown in Table V and Table VI. The effect of wind farms can be observed in the value of objective function. For instance, the value of the objective function is 9.073 (10^6\$) for 5000MW in case *A*, but this value is 6.554 (10^6\$) in case *B*. So, in the presence of wind farms, in addition to reducing greenhouse gas emissions, operation costs will also be degraded. Table VII represents the run time of simulations.

VI. Conclusions

In this work, a new approach has been proposed to address the bundled generation and transmission planning under uncertainty based on a combination of robust and stochastic optimization strategies, which when applied to MILP problems produce “robust” solutions in the sense of being immune against wind generation and demand uncertainties. A unique feature of the proposed approach is that it can address many uncertain parameters in the BGTEP problem. Indeed, the approach can be applied to address the BGTEP problem with different uncertain resources. It should be noted that since the combination of RO and SP necessitates the uncertain variable to have a normal PDF, in the proposed approach, a new approximation methodology is proposed to consider Weibull PDF as a normal PDF with minimum error. To validate the formulation, the variation of reliability levels under different uncertainty levels and infeasibility tolerance have been studied in the test network. Also, the linearization of formulation provided less complexity in the simulation results. Besides, the effect of contingency and reliability on the BGTEP problem has been considered. The computational results show that this approach provides an effective way to address planning problems under uncertainty, producing reliable schedules and generating helpful insights on the tradeoffs between conflicting objectives. Accordingly, due to the efficient and easy to handle formulation, the approach is capable of solving real-world problems with a large number of uncertain parameters.

APPENDIX

In this section, the proposed method is proved. The problem based on combination RO and SP is expressed as follows [21]:

$$\min/\max \quad q^T x + j^T y \tag{51}$$

$$Gx + Dy \le e \tag{52}$$

$$\sum_m g_{lm} x_m + \sum_i d_{li} y_i + \varepsilon\lambda \sqrt{\sum_{m\in M_l} g_{lm}{}^2 x_m{}^2 + \sum_{i\in I_l} d_{li}{}^2 y_i + e_l^2} \le e_l + \delta \max[1, |e_l|] \quad \forall l \tag{53}$$

$$\underline{x} \le x \le \overline{x} \tag{54}$$

$$y_i = 0,1 \qquad \forall i \tag{55}$$

$$g_{lm}^{true} = (1 + \varepsilon\xi_{lm}) g_{lm} \tag{56}$$

$$d_{li}^{true} = (1 + \varepsilon\xi_{li}) d_{li} \tag{57}$$

$$e_l^{true} = (1 + \varepsilon\xi_l) e_l \tag{58}$$

Also, constraints (13), (14), (16) are considered. In the above MILP problem, G and D are uncertain parameters. As well as, x and y are variables. M_l and I_l are the set of indices regarding uncertain parameters. Constraints (56)-(58) denote the relation between true value and nominal value. In order to prove this problem, two conditions must be established:

(i) the problem is feasible for the nominal value;

(ii) $\Pr\left\{ \sum_m g_{lm}^{true} x_m + \sum_i d_{li}^{true} y_i > e_l^{true} + \delta \max[1, |e_l|] \right\} \le \kappa$

where $\lambda = F_n^{-1}(1-\kappa)$.

Proof of condition (ii):

$$\Pr\left\{ \sum_m g_{lm}^{true} x_m + \sum_i d_{li}^{true} y_i > e_l^{true} + \delta \max[1, |e_l|] \right\}$$

$$= \Pr\left\{ \begin{array}{l} \sum_m g_{lm} x_m + \varepsilon \sum_{m\in M_l} \xi_{lm} |g_{lm}| x_m + \sum_i d_{li} y_i + \varepsilon \sum_{i\in I_l} \xi_{li} |d_{li}| y_i > \\ e_l + \varepsilon\xi_l |e_l| + \delta \max[1, |e_l|] \end{array} \right\}$$

$$\le \Pr\left\{ \begin{array}{l} (\sum_{m\in M_l} \xi_{lm} |g_{lm}| x_m + \sum_{i\in I_l} \xi_{li} |d_{li}| y_i - \xi_l |e_l|) \\ / \sqrt{\sum_{m\in M_l} g_{lm}{}^2 x_m{}^2 + \sum_{i\in I_l} d_{li}{}^2 y_i + e_l^2} > \lambda \end{array} \right\}$$

$$= 1 - \Pr\left\{ \begin{array}{l} (\sum_{m\in M_l} \xi_{lm} |g_{lm}| x_m + \sum_{i\in I_l} \xi_{li} |d_{li}| y_i - \xi_l |e_l|) \\ / \sqrt{\sum_{m\in M_l} g_{lm}{}^2 x_m{}^2 + \sum_{i\in I_l} d_{li}{}^2 y_i + e_l^2} \le \lambda \end{array} \right\}$$

$$= 1 - F_n(\lambda) = 1 - (1-\kappa) = \kappa$$

where $\dfrac{(\sum_{m\in M_l} \xi_{lm} |g_{lm}| x_m + \sum_{i\in I_l} \xi_{li} |d_{li}| y_i - \xi_l |e_l|)}{\sqrt{\sum_{m\in M_l} g_{lm}{}^2 x_m{}^2 + \sum_{i\in I_l} d_{li}{}^2 y_i + e_l^2}}$ is a random variable with standardized normal distribution.

As seen in the appendix and references [20], [21], the wind distribution approximation is not applied in the formulation but is used to prove the proposed method.

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