Localization of an Acoustic Fish-Tag using the Time-of-Arrival Measurements: Preliminary results using eXogenous Kalman Filter

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***Abstract*— This paper addresses the source localization problem of an acoustic fish-tag using the Time-of-Arrival measurement of an acoustic signal, transmitted by the fish-tag. The Time-of-Arrival measurements denote the pseudo-range information between the acoustic receiver and the fish-tag, except that the Time-of-Transmission of the acoustic signal is unknown. Starting with the pseudo-range measurement equation, a globally valid quasi-linear time-varying measurement model is presented that is independent of the Time-of-Transmission of the acoustic signal. Using this measurement model, an Uniformly Globally Asymptotically Stable (UGAS), three stage estimation strategy (eXogenous Kalman Filter) is designed to estimate the position of an acoustic fish-tag and evaluated against a benchmark Extended Kalman Filter based estimator. The efficacy of the developed estimation method is demonstrated experimentally, in presence of intermittent observations using an array of receivers mounted on three Unmanned Surface Vessels (USVs).**

I. Introduction

A. Motivation

Acoustic fish-tags have been used traditionally to monitor the migratory pattern of a certain species of fishes and aquatic animals in studies related to marine biology [1], [2]. These fish-tags are basically acoustic transmitters that transmit an acoustic signal periodically, containing information such as ID, temperature, depth, conductivity, etc., depending on the type of transmitter. The transmitted acoustic signals are then picked up by an array of acoustic receivers that are fixed in a region of interest. The collected data are later used to reconstruct and analyze the migratory patterns of the aquatic animals. Recently, with the advent of enabling technologies such as low cost Unmanned Surface Vessels (USVs), efforts have been made to use acoustic receivers mounted on the USVs [3], [4], [5] that track and follow the fish-tag thereby allowing operations in a larger area when compared to the traditional moored receiver systems. The acoustic communication means are prone to measurement losses and delays. Hence, robust estimation of the position of the acoustic fish-tag in the presence of intermittent observations naturally assumes importance for the successful operation of such an mobile aquatic telemetry system.

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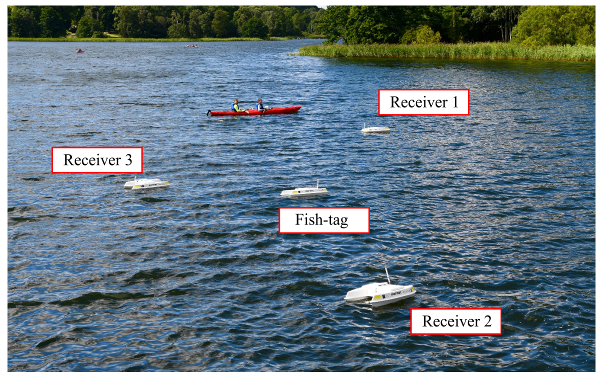


Fig. 1: Mobile acquatic telemetry system

The methods to solve the source localization problem in the literature can be roughly classified into approaches that aim at design of optimal action strategies to achieve improved localization [6], [7] and approaches that improve localization accuracy through the design of efficient estimation strategies [8], [9], [10], [11]. This paper addresses the latter problem and presents a practical solution to the source localization problem of an acoustic fish-tag using only the Time-of-Arrival (ToA) of the acoustic signal, as measured by the mobile acoustic receivers (shown in Figure 1) using an eXogenous Kalman Filter (XKF) [12] based estimation strategy. The ToA measurement usually denotes a pseudo-range measurement between a receiver and the transmitter. The term *pseudo* refers to the fact that the measured range is not the actual range and is inclusive of measurement errors due to clock synchronization, delay, uncertain speed of sound, etc. Since the fish-tag is submerged in the water, the ToT of the acoustic signal is in general not known. Therefore, the source localization problem can be posed as a position estimation problem using pseudo-range measurements, that is known to be a highly nonlinear estimation problem further

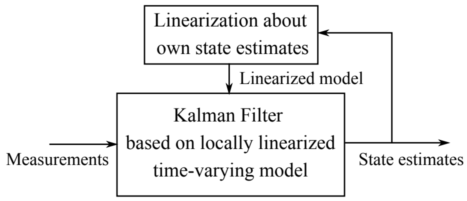


Fig. 2: Extended Kalman Filter architecture

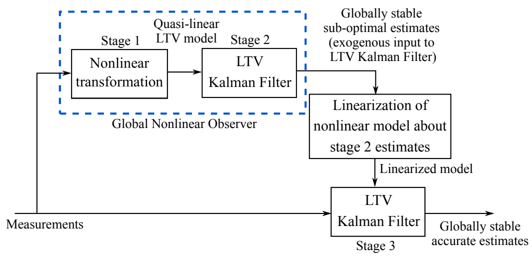


Fig. 3: eXogenous Kalman Filter architecture

complicated by the unavailability of the ToT information.

B. Related Work

Range based positioning systems have been widely investigated and finds applications in search and rescue, mobile positioning in wireless networks, aquatic telemetry, navigation of marine vehicles, warehouses etc. A comprehensive review of range based positioning algorithms and methods can be found in [13], [8] and the references therein. A class of methods use the first order Taylor series approximation of the nonlinear measurement equations, and transform the position estimation problem into a nonlinear least squares problem or used in an Extended Kalman Filter (EKF) formulation [13], [14]. While these methods have been successfully used in practice, they suffer from the lack of convergence and stability guarantees. Another class of methods [13], [10] derive a quasi-linear time-varying models using a globally valid nonlinear algebraic transformations of the pseudo-range equations. In a noise free case, these equations are solved algebraically to obtain the position estimates of the source. However, in the presence of measurement noise, these methods usually yield sub-optimal position estimates. In order to recover performance, weighted least squares methods have been used in [11], [10].

In the context of acoustic localization, [15] presents a method to localize an Autonomous Underwater Vehicle (AUV) using range and depth measurements to a set of stationary beacons using Minimum Energy estimation method where Linear Time-Varying (LTV) models were obtained by augmenting the state vector with auxiliary variables that replace the quadratic relations arising in range measurement equations. References [6], [4] presents a robotic telemetry system for localizing a radio-tagged fish using coarse bearing measurements. The approach however, is to select the sensing locations that minimize localization uncertainty and employs an EKF for estimation. Particle filter based approaches to estimate and track a leopard shark using single AUV [5] and multiple-AUVs [16], [9] have been reported. The Particle Filters ability to handle ambiguous sensor measurements is

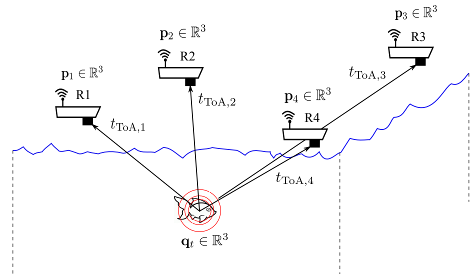


Fig. 4: Source localization scenario with receivers R1 to R4

leveraged to deal with a stereo hydrophone and receiver system that cannot determine the sign of the relative angle to a detected fish tag. A three stage estimation strategy recently introduced in [8] was used in [17] to localize an AUV using pseudo-range and pseudo-range-rate measurements in presence of unknown wave speed and in [18] using a single transponder. The three stage filter proposed in [8], provides convergence and stability guarantees for the source localization problem using pseudo-range measurements. A globally valid nonlinear algebraic transformation [13], [19] is applied in stage one to obtain a set of quasi-linear time-varying equations. These equations are then used as the measurement model in the second stage *Linear Time-Varying* Kalman Filter (LTV-KF) based estimator to obtain sub-optimal position estimates. Additionally, the clock bias was estimated to account for the synchronization error between the receiver and transmitter clocks. Further, a third stage *linearized* Kalman Filter was designed using the locally linearized models of the nonlinear pseudo-range measurement equations in order to improve the accuracy of the estimates. The linearization was performed about the estimates of the stage two LTV-KF, thereby deviating from the EKF formulations where the linearization is performed about the filter's own estimates. Figure 2 and 3 illustrate the difference between the EKF and the three stage estimation strategy. The three stage filter can be seen as a variant of a more generic eXogenous Kalman Filter (XKF) [12] framework, where the output of stage two filter form the exogenous input to the third stage Kalman Filter and hence referred to as XKF in the remainder of the paper.

C. Contributions

The main contributions of this paper are the design and experimental validation of an Uniformly Globally Asymptotically Stable (UGAS) XKF based estimator to localize the fish-tag using mobile acoustic receivers. A globally valid, algebraic nonlinear transformation [10] is applied to the nonlinear pseudo-range measurement equations to obtain a quasi-linear time-varying model that is independent of ToT of the acoustic signal, thereby deviating from the method presented in [8]. The designed XKF is validated against a benchmark EKF through experiments using acoustic receivers mounted on the USVs. Furthermore, experiment results illustrate the efficacy of the proposed approach in the presence of intermittent observations.

The remainder of the paper is organized as follows.

Section II describes the measurement models necessary for the design of the estimator. The globally valid, algebraic nonlinear transformation used in order to obtain the quasi-linear time varying model is discussed in Section III. The design of the XKF based estimator is presented in Section IV. The performance of the XKF is validated against a benchmark EKF experimentally in Section V, followed by conclusions in Section VI.

II. Measurement Model

Let $\mathbf{q}_t \in \mathbb{R}^3$ denote the location of the acoustic fish-tag, expressed in a fixed frame of reference, that needs to be estimated. Consider $i = \{1, 2, \cdots, m\}$ acoustic receivers with known positions $\mathbf{p}_i \in \mathbb{R}^3$ as shown in Figure 4. The data received by the acoustic receiver i consist of the Time-of-Arrival (ToA) of the acoustic signal transmitted by the fish-tag, denoted by $t_{\mathrm{ToA,i}}$. It is assumed that the clocks on the acoustic receivers are synchronized. Such an assumption is satisfied in practice by synchronizing the time at the receivers with the GPS clocks. The pseudo-range measurement available with each receiver i is given as

$$r_i = c(t_{\mathrm{ToA,i}} - t_{\mathrm{ToT}}) + c\tau_c + e_i \tag{1}$$

where c is speed of sound in water [m/s] that is assumed to be known, t_{ToT} is the Time-of-Transmission of the acoustic signal, the term $c\tau_c$ denotes the error in range measurement due to clock bias, τ_c is clock synchronization error between transmitter and receiver (constant for every pseudo range measurement since receivers are synchronized) and $e_i \sim \mathcal{N}(0, \sigma_{r_i}^2)$ is the Zero-Mean Gaussian Noise (ZMGN) acting on the pseudo-range measurement. In general, the ToT of the acoustic signal may not be known. If known, it could be possible that the transmitter and receiver clocks are not synchronized. Hence, the uncertainties in the range measurement can be combined into an unknown variable $\beta = -ct_{\mathrm{ToT}} + c\tau_c$ that could be estimated [8] or eliminated as discussed in the next section. The pseudo-range measurement is then given as

$$r_i = ct_{\mathrm{ToA,i}} + \beta + e_i \tag{2}$$

In order to design an estimator for the localization of the fish-tag, a mathematical model for the sensing system is developed. Since the pseudo-range measurement (2) essentially captures the geometric distance between the fish-tag and the receiver combined with the uncertainty due to clock synchronization errors, it follows that the pseudo-range measurement can be described as

$$r_i = \|\mathbf{q}_t - \mathbf{p}_i\| + \beta + e_i \tag{3}$$

The pseudo-range measurement equation (3) forms the basis for the design of estimators for localization of the fish-tag in the remainder of the paper.

III. Quasi-linear Time-varying Model

The pseudo-range measurement equations of (3) are nonlinear in nature. However, it is shown in [13], [10] that a globally valid quasi-linear time-varying model can be obtained starting from (3). Two possible transformations exist to obtain a quasi-linear time-varying model. First, *Squared-range* difference of the pseudo-range equations with respect to the measurement obtained from a reference receiver leads to elimination of the quadratic nonlinear term from the resulting set of equations. This approach leads to four unknowns - $[\mathbf{q}_t^T \;\; \beta]^T$ that needs to be estimated. This model forms the basis for estimator design presented in [8]. Such an approach is suitable when the ToT of the acoustic signal is known and β represents the slowly-varying clock bias signal. However, when the ToT is unknown as considered in this paper, this approach can be sensitive to the initialization of the parameter β. Therefore, a quasi-linear model is obtained by taking squared *range-difference* of the pseudo-range equations, that is independent of the parameter β and hence the unknown ToT, leading to three unknowns - $\mathbf{q}_t$ that needs to be estimated.

Given m acoustic receivers, let the m^{th} receiver be the reference receiver without loss of generality. The square of the pseudo-range measurement equation (3) can be written for all $i \in \{1, 2, \cdots, m-1\}$ as

$$r_i^2 = (r_i - r_m + r_m)^2 = r_{im}^2 + 2r_{im}r_m + r_m^2 \tag{4}$$

where $r_{im} = r_i - r_m$. From (3) with $d_i = \|\mathbf{q}_t - \mathbf{p}_i\|$, $r_i^2 = d_i^2 + \beta^2 + 2d_i\beta$ and $r_m^2 = d_m^2 + \beta^2 + 2d_m\beta$, where the gaussian noise term e_i is excluded for the sake of brevity. Substituting for r_i^2 and r_m^2 in (4), we have

$$r_{im}^2 + 2r_{im}r_m = d_i^2 - d_m^2 + 2(d_i - d_m)\beta \tag{5}$$

Since the geometric distance between the i^{th} receiver and the fish-tag is unknown, (5) needs to be rewritten in terms of the available measurements r_i. Through simple algebraic manipulation, the term $2r_{im}r_m$ on the left hand side of (5) can be written as

$$\begin{aligned} 2r_{im}r_m &= 2r_{im}(d_m + \beta) \\ &= 2r_{im}d_m + 2(d_i - d_m)\beta \end{aligned} \tag{6}$$

and the term $d_i^2 - d_m^2$ on the right hand side of (5) satisfies

$$\begin{aligned} d_i^2 - d_m^2 &= \|\mathbf{q}_t\|^2 + \|\mathbf{p}_i\|^2 - 2\mathbf{p}_i^T\mathbf{q}_t \\ &\quad - \left[\|\mathbf{q}_t\|^2 + \|\mathbf{p}_m\|^2 - 2\mathbf{p}_m^T\mathbf{q}_t\right] \\ &= -2(\mathbf{p}_i - \mathbf{p}_m)^T\mathbf{q}_t + \|\mathbf{p}_i\|^2 - \|\mathbf{p}_m\|^2 \end{aligned} \tag{7}$$

Using (6) and (7) in (5)

$$r_{im}^2 - \|\mathbf{p}_i\|^2 + \|\mathbf{p}_m\|^2 = -2(\mathbf{p}_i - \mathbf{p}_m)^T\mathbf{q}_t - 2r_{im}d_m \tag{8}$$

Stacking the instances of (8) for all $i \in \{1, 2, \cdots, m-1\}$ into vectors and matrices results in the globally valid quasi-linear time varying model

$$\mathbf{y} = 2C_{yq}\mathbf{q}_t - 2d_m\boldsymbol{\eta} \tag{9}$$

where the vectors $\mathbf{y} \in \mathbb{R}^{m-1}$, $\boldsymbol{\eta} \in \mathbb{R}^{m-1}$ and matrix $C_{yq} \in \mathbb{R}^{(m-1\times 3)}$ are defined as

$$\mathbf{y} := \begin{bmatrix} r_{1m}^2 - \|\mathbf{p}_1\|^2 + \|\mathbf{p}_m\|^2 \\ \vdots \\ r_{(m-1)m}^2 - \|\mathbf{p}_{m-1}\|^2 + \|\mathbf{p}_m\|^2 \end{bmatrix}$$

$$C_{yq} \coloneqq \begin{bmatrix} -(\mathbf{p}_1 - \mathbf{p}_m)^T \\ \vdots \\ -(\mathbf{p}_{m-1} - \mathbf{p}_m)^T \end{bmatrix} \quad \boldsymbol{\eta} \coloneqq \begin{bmatrix} r_{1m} \\ \vdots \\ r_{(m-1)m} \end{bmatrix}$$

The following lemma presents the algebraic solution to the quasi-linear time-varying measurement model (9) for the estimate of the position of the fish-tag $\mathbf{q}_t$.

Lemma 1. *The two candidate algebraic solutions* $\mathbf{q}_t$ *and* d_m *given by*

$$\mathbf{q}_t = \bar{\mathbf{y}} + d_m\bar{\boldsymbol{\eta}}, \ \ \bar{\mathbf{y}} = \frac{1}{2}C_{yq}^{+}\mathbf{y}, \ \ \bar{\boldsymbol{\eta}} = C_{yq}^{+}\boldsymbol{\eta} \tag{10}$$

$$d_m = \begin{cases} \frac{-b\pm\sqrt{b^2-4ac}}{2a} & \text{if } a \neq 0 \\ -\frac{c}{b} & \text{otherwise} \end{cases} \tag{11}$$

where $a = (1-\bar{\boldsymbol{\eta}}^T\bar{\boldsymbol{\eta}})$, $b = 2(\mathbf{p}_m^T\bar{\boldsymbol{\eta}} - \bar{\mathbf{y}}^T\bar{\boldsymbol{\eta}})$, $c = -(\mathbf{p}_m - \bar{\mathbf{y}})^2$ *and* $C_{yq}^{+} = (C_{yq}^TC_{yq})^{-1}C_{yq}^T$ *is the Moore-Penrose pseudo-inverse, under the assumption* $\operatorname{rank}(C_{yq}) = 3$*, solves the quasi-linear measurement model* (9)*. Furthermore, at least one of the solutions is the true position of the source in the absence of measurement noise, provided the condition*

$$\boldsymbol{\epsilon} = \left(C_{yq}C_{yq}^{+} - I_{m-1}\right)\left(\frac{1}{2}\mathbf{y} + d_m\boldsymbol{\eta}\right) = 0 \tag{12}$$

is satisfied.

Proof. See Lemma 1 of [8] for outline. □

The above mentioned results are stated considering the absence of measurement noise. In practice, application of Lemma 1 to obtain the source position in presence of the measurement noise leads to sub-optimal estimates. Therefore, a Linear Time-Varying Kalman Filter (LTV-KF) based on the quasi-linear time varying model (9) is designed as stage 2 estimator (illustrated in Figure 3) with a new measurement vector $\mathbf{z} = \frac{1}{2}\mathbf{y} + d_m\boldsymbol{\eta}$ to filter out the measurement noise. Consequently, the algebraic position estimate (10) is not used. However, (11) is used to compute the estimated geometric distance between the reference receiver and the fish-tag in order to construct the measurement vector $\mathbf{z}$ for the stage 2 LTV-KF of the XKF framework.

Remark 1. Equation (11) suggest that there are two possible solutions to the estimate of the geometric distance between the reference receiver and the fish-tag, except in cases where $a = 0$ or equivalently $\bar{\boldsymbol{\eta}}^T\bar{\boldsymbol{\eta}} = 1$. This leads to ambiguity in the estimate of d_m as the correct estimate needs to be chosen to construct the measurement vector $\mathbf{z}$ for the stage 2 filter. The ambiguity can be resolved by selecting d_m that satisfies (12). When $m = 4$ and $\operatorname{rank}(C_{yq}) = 3$, (12) is always satisfied since $C_{yq}C_{yq}^{+} = I_{m-1}$. In such cases, the physical interpretation of d_m can be useful to resolve ambiguity.

IV. eXogenous Kalman Filter

This section describes the design of an XKF that consists of three filtering sub-systems connected in a cascade. The result of the previous section is used as a first stage preprocessing of the measurements leading to a LTV model (9) and new measurement vector $\mathbf{z}$. A LTV-KF based on the quasi-linear time-varying model and measurements obtained from stage one forms the stage two of the XKF that produces sub-optimal estimates since the prior information about nonlinear relationship is lost due to application of algebraic nonlinear transformation. Therefore, a third stage *Linearized* KF, that takes the data from stage two LTV-KF as an exogenous input in order to obtain locally linearized model of the original nonlinear measurement equations is designed to recover performance. Assuming lack of any information regarding the motion of the fish-tag, a random walk model is used to describe its motion, i.e.,

$$\dot{\mathbf{q}}_t = \mathbf{u}_q \tag{13}$$

where $\mathbf{u}_q \sim \mathcal{N}(0, \Sigma_u^2)$ and $\Sigma_u^2 \in \mathbb{R}^{3\times3}$. The discretized motion model is therefore given as

$$\mathbf{q}_{t,k} = A\mathbf{q}_{t,k-1} + D\mathbf{u}_{q,k-1} \tag{14}$$

where $A = I_3$, $D = \Delta t I_3$ with Δt being the sampling time, for the random walk model and k is the discrete time index.

Remark 2. Other motion models that would more accurately represent the fish motion could be used. Further, the type of motion model used would depend on the type of the fish that is being tracked. For example, in [9], a hybrid Brownian and Levy Flight motion model is used that is better able to account for multiple hypothesis of the motion of a leopard shark.

A. Stage 1: Algebraic Nonlinear Transformation

The globally valid algebraic nonlinear transformation presented in Section III is applied to obtain a quasi-linear time varying measurement model

$$\mathbf{z}_k = C_{yq,k}\mathbf{q}_{t,k} \tag{15}$$

that is equivalent to (9) with new measurement vector $\mathbf{z}_k = \frac{1}{2}\mathbf{y}_k + d_{m,k}\boldsymbol{\eta}_k$. Note that the construction of measurement vector $\mathbf{z}_k$ requires knowledge of $d_{m,k}$. Hence results of Lemma 1 are applied to estimate $d_{m,k}$ and it is assumed that the ambiguities are resolved through the application of (12) or Remark 1.

B. Stage 2: LTV Kalman Filter

Based on the discrete time motion and measurement model (14) and (15) respectively, a Linear-Time Varying Kalman Filter in predictor-corrector form is developed to estimate the position of the fish $\bar{\mathbf{q}}_{t,k} \sim \mathcal{N}(\mathbf{q}_{t,k}, \bar{P}_k)$, where $\bar{P}_k$ is the covariance associated with the estimates. The filter equations are given as

$$\bar{\mathbf{q}}_{t,k}^{-} = A\bar{\mathbf{q}}_{t,k-1} \tag{16a}$$
$$\bar{P}_k^{-} = A\bar{P}_{k-1}A^T + D\bar{Q}_kD^T \tag{16b}$$
$$\bar{K}_k = \bar{P}_k^{-}C_{yq,k}^T(C_{yq,k}\bar{P}_k^{-}C_{yq,k}^T + \bar{R}_k)^{-1} \tag{16c}$$
$$\bar{\mathbf{q}}_{t,k} = \bar{\mathbf{q}}_{t,k}^{-} + \bar{K}_k(\mathbf{z}_k - C_{yq,k}\bar{\mathbf{q}}_{t,k}^{-}) \tag{16d}$$
$$\bar{P}_k = (I - \bar{K}_kC_{yq,k})\bar{P}_k^{-} \tag{16e}$$

where $\bar{Q}_k = \Sigma_u^2$ and $\bar{R}_k$ is computed using the covariance terms, $\operatorname{Cov}(z_i, z_i) = 0.5(\sigma_{r_i}^2 + \sigma_{r_m}^2)^2 + (r_{im} + d_m)^2(\sigma_{r_i}^2 + \sigma_{r_m}^2)$ and $\operatorname{Cov}(z_i, z_j) = 0.5\sigma_{r_m}^4 +$

$\left[r_{im}r_{jm} + d_m(r_{im} + r_{jm}) + d_m^2\right] \sigma_{r_m}^2$ (see Appendix of [8] for outline of derivation). The LTV-KF is Uniformly Globally Asymptotically Stable (UGAS) following the arguments in Proposition 1 and 2 of [8].

Remark 3 (Observability conditions). Efficient operation of the LTV-KF requires that the acoustic receivers are positioned such that there exists $\sigma^* > 0$ such that $C_{yq,k}^T C_{yq,k} \geq \sigma^* I_3$ holds, at all times k and $m \geq 4$.

C. Stage 3: Linearized Kalman Filter

The globally stable, sub-optimal estimates obtained in the stage two LTV-KF are used as auxiliary estimates that provide a linearization point for a third stage *linearized* Kalman Filter, where the measurement model is based on the local linear approximation of the original nonlinear measurement equation given by

$$\mathbf{r}(\mathbf{q}_t) := \begin{bmatrix} \|\mathbf{q}_t - \mathbf{p}_1\| - \|\mathbf{q}_t - \mathbf{p}_m\| \\ \vdots \\ \|\mathbf{q}_t - \mathbf{p}_{m-1}\| - \|\mathbf{q}_t - \mathbf{p}_m\| \end{bmatrix} \tag{17}$$

Note that the pseudo-range-difference equations are considered that eliminates the term β from the nonlinear measurement model. Furthermore, the linearization is performed using the position estimate $\bar{\mathbf{q}}_{t,k}$ from the second stage LTV-KF, thereby digressing from the Extended Kalman Filter architecture. The local linearization of the nonlinear model (17) satisfies

$$\mathbf{r}_k \approx \hat{\mathbf{r}}_k + C_k\left(\mathbf{q}_t - \bar{\mathbf{q}}_{t,k}\right) \tag{18}$$

where $\mathbf{r}_k := \begin{bmatrix} r_{1m,k} & \cdots & r_{(m-1)m,k} \end{bmatrix}^T$, $\hat{\mathbf{r}}_k = \mathbf{r}(\bar{\mathbf{q}}_{t,k})$ and C_k has the form

$$C_k := \left.\frac{\partial \mathbf{r}(\mathbf{q}_t)}{\partial \mathbf{q}_t}\right|_{\mathbf{q}_t = \bar{\mathbf{q}}_{t,k}} = \begin{bmatrix} \frac{(\mathbf{p}_m - \bar{\mathbf{q}}_{t,k})^T}{d_m} - \frac{(\mathbf{p}_1 - \bar{\mathbf{q}}_{t,k})^T}{d_1} \\ \vdots \\ \frac{(\mathbf{p}_m - \bar{\mathbf{q}}_{t,k})^T}{d_m} - \frac{(\mathbf{p}_{m-1} - \bar{\mathbf{q}}_{t,k})^T}{d_{m-1}} \end{bmatrix} \tag{19}$$

Note that the matrix C_k is the not the same as $C_{yq,k}$ used in the second stage LTV-KF. The equations for the third stage, *Linearized* KF in predictor-corrector form, with estimates of the position of the fish-tag $\hat{\mathbf{q}}_{t,k} \sim \mathcal{N}(\mathbf{q}_{t,k}, \hat{P}_k)$ are given by

$$\hat{\mathbf{q}}_{t,k}^- = A\hat{\mathbf{q}}_{t,k-1} \tag{20a}$$

$$\hat{P}_k^- = A\hat{P}_{k-1}A^T + D\hat{Q}_k D^T \tag{20b}$$

$$\hat{K}_k = \hat{P}_k^- C_k^T (C_k \hat{P}_k^- C_k^T + \hat{R}_k)^{-1} \tag{20c}$$

$$\hat{\mathbf{q}}_{t,k} = \hat{\mathbf{q}}_{t,k}^- + \hat{K}_k\left[\mathbf{r}_k - \hat{\mathbf{r}}_k - C_k\left(\hat{\mathbf{q}}_{t,k}^- - \bar{\mathbf{q}}_{t,k}\right)\right] \tag{20d}$$

$$\hat{P}_k = (I - \hat{K}_k C_k)\hat{P}_k^- \tag{20e}$$

Since the motion model remains unchanged, $\hat{Q}_k = \Sigma_u^2$ is the same as in the second stage, while $\hat{R}_k$ is computed using the covariance terms, $\mathrm{Cov}(r_{im}, r_{im}) = \left(\sigma_{r_i}^2 + \sigma_{r_m}^2\right)$ and $\mathrm{Cov}(r_{im}, r_{jm}) = \sigma_{r_m}^2$. The UGAS property of the filter is retained as the linearization is performed using stage two LTV-KF estimates that are UGAS. The UGAS of the XKF then follows from the cascade of UGAS subsystems [12].

(a) (b)

Fig. 5: (a) Representative image of the Themla Biotel fish-tag of 18 mm diameter and 104 mm length with a mass of 43 gram. (b) Thelma Biotel TBR-700 acoustic receivers of 75 mm diameter and 230 mm length with a mass of 1140 gram.

D. Practical Issues

As mentioned before, the ToA of the acoustic signal is a function of the geometric range between the receiver and the fish-tag. Therefore, the measurements are obtained asynchronously. The effects such as multi-path, error due to density/temperature variations in the water or even the loss of acoustic signal is possible in a challenging underwater environment. Since the ToT is unknown, it is often not possible to differentiate between a correct and an erroneous measurement. Therefore, additional care must be taken while taking the difference between two measurements to construct the measurement vector $\mathbf{z}_k$ and $\mathbf{r}_k$ for the stage two and stage three filter design respectively. Since the operational range of the fish-tag is known, it is possible to reject measurements when the difference between ToA of two receivers exceed a certain value i.e, $r_i - r_m > c\tau_{\max}$, where $\tau_{\max}$ is maximum allowable time difference between two ToA measurements. Since it is also possible that one or more measurements are lost, the filter must be able to handle such intermittent observations. In order to deal with such situations, the stage one pre-processing and stage two LTV-KF are executed only when sufficient measurements (at least three in order to estimate 3D position of the fish-tag) are available to compute the algebraic estimate d_m using (11) of Lemma 1, that is also required to compute the measurement vector $\mathbf{z}_k$ for stage two LTV-KF. This prevents inaccurate d_m estimate to be used in the stage two LTV-KF that may lead to inaccurate position estimates, thereby severely affecting the performance of the third stage Linearized KF. The third stage linearized KF however can incorporate any number of measurements when they are available, since the locally linearized models are obtained using stable, stage two LTV-KF estimates.

V. Experiment Results

A. Experiment setup

The proposed source localization method was tested on an experimental testbed that consist of four Maribot Duckling[1] USVs (Figure 1), three TBR-700 Thelma Biotel acoustic receivers (Figure 5b) mounted on three USVs and a Thelma Biotel acoustic fish-tag (Figure 5a) attached to the fourth USV. The fish-tag, designed for long duration missions has

[1]The Maribot Duckling USVs are developed at the Maritime Robotics Laboratory, Royal Institute of Technology, Stockholm, Sweden.

a range of approximately 800 [m] and nominally transmits an acoustic signal once every 8 seconds. Depth measurement is obtained from the fish-tag, which enables the use of XKF with only three receivers. The depth measurement equation given by

$$r_z = [0 \;\; 0 \;\; 1]^T \mathbf{q}_t + e_z \tag{21}$$

where $e_z \sim \mathcal{N}(0, \sigma_z^2)$, is used to augment the matrices $C_{yq,k}$ and C_k in LTV-KF and Linearized KF respectively. The receivers on-board the USVs provides the ToA (in UNIX time since epoch, synchronized with GPS clocks), and uses GPS to record the receiver position. The receiver data is transmitted from three Ducklings over wireless communication link to the fourth Duckling that caries the fish-tag and also performs the position estimation. The GPS data of the USV carrying the fish-tag is used as the ground truth for performance comparison. The position estimation algorithms are implemented using C++ in DUNE and the entire system is monitored using Neptus through a mission center. DUNE and Neptus are a part of LSTS toolchains [20] that provide an integrated framework for guidance, navigation, control, and monitoring tasks for marine and aerial vehicles. In order to satisfy the observability conditions outlined in Remark 3, a distance based formation controller [21] for target-tracking is executed on three Ducklings carrying the acoustic receivers.

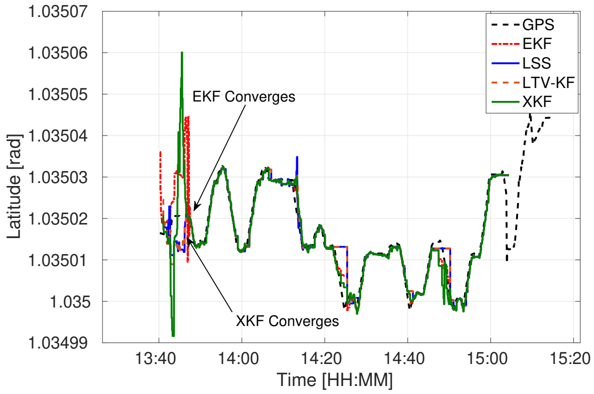
B. Results

The performance of the XKF is validated through experimental data gathered over field tests conducted for a duration of approximately 90 minutes. An EKF based estimator is also designed using the nonlinear measurement model (17) and forms a benchmark estimator for performance comparison. Since the fish-tag data nominally arrives every 8 seconds, both XKF and EKF were executed at a low sampling frequency of 1 Hz with identical initial conditions and covariance matrices. The process noise covariance matrix was chosen as $\Sigma_u^2 = \mathrm{diag}(1, 1, 0.1)$ and the measurement noise covariance of $\sigma_{r_i}^2 = 0.9$ for all $i \in \{1, \cdots, m\}$ and $\sigma_z^2 = 0.2$ was used. The initial condition was set to $\mathbf{q}_{t,0} = [100, 100, 7]^T$ in [m] with respect to a fixed reference point (lat-lon coordinates) and the initial covariance matrix P_0 was set as $\mathrm{diag}(100, 100, 10)$. Note that the position estimation is performed in a local coordinate frame expressed with respect to the fixed reference frame. The estimated position of the fish-tag output by the EKF and the XKF were then transformed into global lat-lon coordinates. The speed of sound in water was set as $c = 1485$ [m/s].

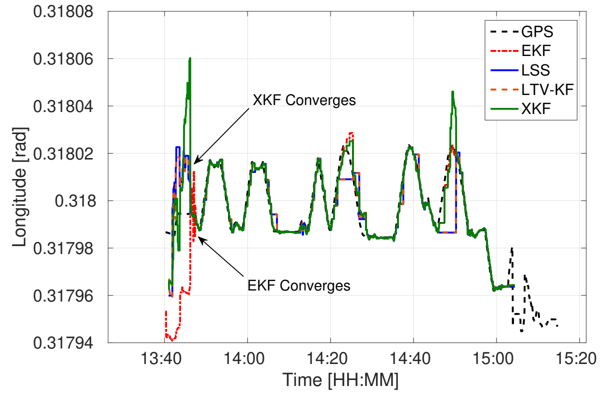
Estimator Performance – Figure 6a and 6b shows the plot of estimated latitude and longitude of the fish-tag over time. The 2D latitude-longitude plot is shown in Figure 6c. The plots illustrate the known GPS data of the USV that carries the fish-tag - considered as ground truth, solutions of the EKF, stage one Least Squares Solution (LSS), stage two LTV-KF and the stage three Linearized KF (also the output of XKF). The estimated depth is shown in Figure 6d. From the plots, it can be observed that both the EKF and XKF are able to estimate the position of the fish-tag, however, the performance of the XKF is superior compared to the EKF on the following accounts. From Figure 6a and 6b it can be noticed that XKF achieves faster convergence when compared to the EKF. This is due to the UGAS three stage estimation strategy adopted by the XKF. Given three measurements and a valid computation of d_m with no ambiguity (Lemma 1), and its subsequent use in the stage-two LTV-KF (to construct the measurement vector $\mathbf{z}_k$), results in a sub-optimal, correct solution to the position estimation problem. The estimated position of stage two is used as a linearization point by the stage three linearized KF to further enhance the accuracy, thereby providing faster convergence to the correct solution. This is also reflected in the estimates of the depth, where XKF estimates are more accurate than the EKF. Note that the fish-tag was immersed at a constant depth of 2 [m] from the USV. The initial estimation errors with the XKF are due to that fact that the application of Lemma 1 would lead to an invalid computation of d_m possibly due to the geometric configuration of the receivers and/or erroneous ToA measurements at the initialization phase of the experiments. As mentioned before, convergence is achieved once a valid d_m is obtained. Figures 6e and 6f show the norm of innovation vector and the trace of covariance matrix respectively. It can be noticed that the uncertainty associated with the EKF (red peak in Figure 6f) is higher until it converges to the true position of the fish-tag while the uncertainty associated with XKF is lower. The initial peak in the norm of the innovation vector for the XKF is due to unavailability of a valid d_m during the initialization phase of the experiments. It is clear from the plots, that the XKF exhibits better performance when compared to the EKF in the terms of faster convergence and UGAS property, which are the most important advantages.

Initial Conditions – It is known that the convergence with EKF cannot be guaranteed and is true especially when sufficiently accurate initialization of the estimator states cannot be made. Therefore, the initial position of the fish-tag was arbitrarily changed to $\mathbf{q}_{t,0} = [1000, 1000, 70]^T$ [m] and the performance of the estimators were assessed using a part of measurements starting from approximately 14:43 [HH:MM], where the receivers were in a favorable position that satisfies the observability condition (Remark 3). Figures 7a and 7b show the estimated latitude and longitude of the fish-tag and the superior performance of the XKF is evident. While the EKF tracks the true position only after 5-7 minutes at 14:50 [HH:MM], XKF exhibits a fast convergence to the true position within seconds even with an arbitrary initialization of the estimator states.

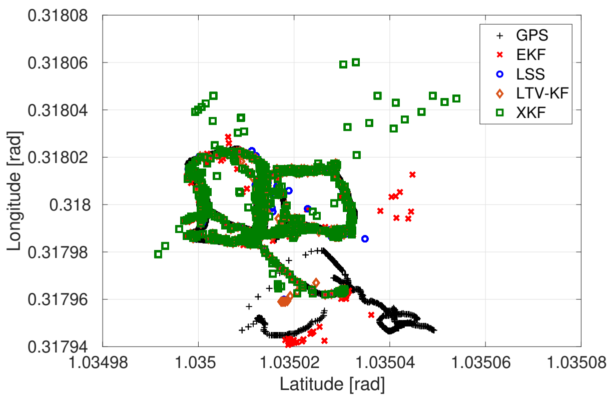
Intermittent Observations – It can be noted that the XKF based estimator is able to handle the case of intermittent observations. Specifically, at around 14:10, 14:20 and 14:50 [HH:MM] of the Figure 6a and 6b, it can be noticed that the stage one Least Squares Solution (LSS) and stage two LTV-KF solution remains constant as sufficient measurements (3 in this case) are not available to apply the result of Lemma 1. The third stage Linearized KF however, uses the measurements when available in order to estimate the



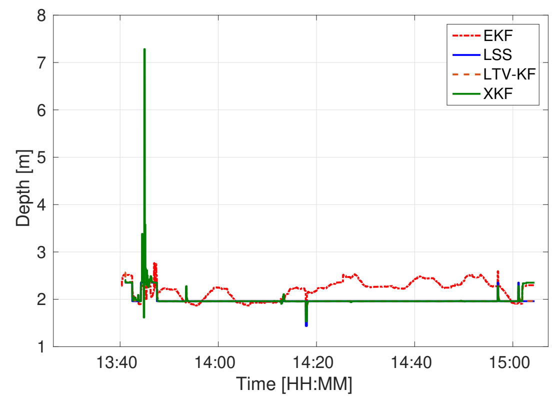
(a) Estimated Latitude of the fish-tag



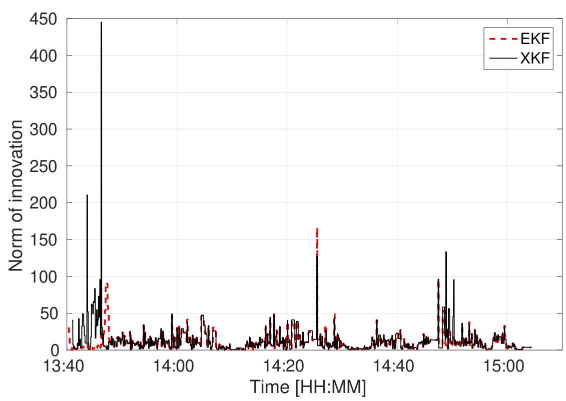
(b) Estimated Longitude of the fish-tag



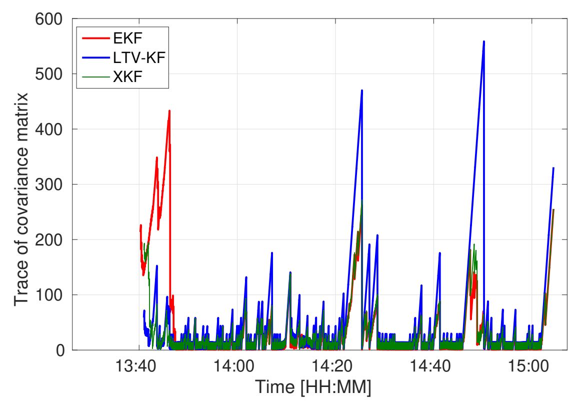
(c) 2D estimated lat-lon of the fish-tag



(d) Estimated depth



(e) Norm of the EKF and XKF innovation



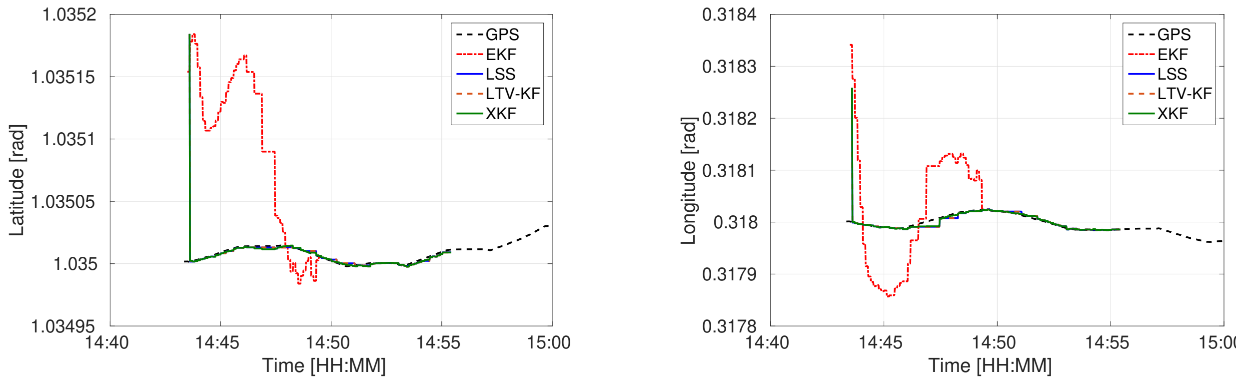
(f) Trace of the covariance matrix P_k for EKF and XKF

Fig. 6: Results for estimation of the fish-tag using XKF and EKF with $m = 3$ receivers and depth measurement.

position of the fish-tag. Another important observation where the XKF deviates from the ground truth is at 14:50 of Figure 6b. This is the instance when sufficient measurements are not available for a long duration that prevents measurement update of stage two LTV-KF. Consequently, the stage three linearized KF relies on the linearized model that remains constant until the LTV-KF is updated and leads to error in the estimates of the position. This indicates that a formal analysis of allowable duration for which the intermittent observations can be tolerated and its implication on the performance of the XKF is needed. The effect of intermittent observations on the variance of the estimated states can be seen in the Figure 6f where the uncertainty associated with the LTV-KF, Linearized KF are higher (blue peaks). The performance is recovered once the new measurements are available.

VI. Conclusion

This paper addressed the problem of design of an XKF based estimator that is UGAS, for localization of an acoustic fish-tag using ToA measurements. The proposed filter design was validated experimentally using three receivers mounted on USVs and a depth measurement obtained from the fish-



(a) Estimated Latitude of the fish-tag

(b) Estimated Longitude of the fish-tag

Fig. 7: Position estimates of the fish-tag for an arbitrary initialization of the estimator states

tag attached to a fourth USV. The performance of the XKF was compared against a benchmark EKF based estimator. It was observed that the performance of XKF is superior when compared to the EKF. The performance of the XKF was further validated with poor initialization of the estimator states and it was found that XKF achieves significantly faster convergence to the true position while EKF outputs incorrect position estimates. Additionally, it was shown that the designed XKF perform adequately in the presence of intermittent observations. However, a formal analysis of the performance of the XKF in the presence of intermittent observations is left as a future work.

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