Multiple Nash Equilibria in Electricity Markets with price-making Hydrothermal Producers

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Abstract

This paper proposes a novel approach incorporating scheduling decisions into a multi-nodal multi-period Cournot game. Through applying the Nikaido-Isoda function, market clearing is conducted without dual values being required. Maps of Nash equilibria are obtained through a branch-and-cut algorithm, based on tailored cutting rules. A case study of inertial response requirements shows that these maps and the resulting range of potential player profits can be used to analyze the impacts of policy decisions influenced by discontinuous variables. The case study also shows the financial impact on neighboring producers to the node with applied inertial response requirements.

Index Terms

Nikaido-Isoda, Nash Equilibrium, Energy Markets, Cournot Competition, Nonlinear Optimization, Competitive Game, Inertial Response, Generation Dispatch, Hydropower

I. Introduction

Equilibrium modeling in power systems represents an established method for analyzing player behavior and their reactions to system changes [1]. These methods have been traditionally based on systems of conventional means of power production participating in single-period games [1], [2]. Changes in system generation portfolios have, however, led to greater integration of fluctuating renewable electricity generation such as wind and solar power and to electricity storage facilities being added to the grid. These developments can result in traditional market models not being fit to adequately deal with arising problems.

To address this, several approaches have been proposed in the literature to deal with multi-period setups: Ref. [3] implements an equilibrium model on an assumed, already conducted hydro power scheduling, extending a single period Cournot model to a short term time frame. Ref. [4] introduces storage operators that behave as price-takers in a natural gas market with gas inventory holding being simplified through a fixed overall period capacity. Ref. [5] analyzes, based on [3], the implications of market power in a system with large shares of hydropower generation using a two stage model that clears a Cournot market and then (re)schedules units. Another analysis of market power in systems under hydrostorage is given by [6], which embeds hydropower decisions into a game played within a dynamic program and solved via interpolating the best response functions. Ref. [7] shows a leader-follower framework in a stochastic *Equilibrium Problem with Equilibrium Constraints*. It circumvents reliance on Karush Kuhn Tucker-conditions by using strong duality constraints in its bi-level problem setup of clearing the market and maximizing profits. Ref. [8] uses the Nikadio-Isoda function to establish an *active set* algorithm to clear a multi-period hydro-thermal market. The method presented below also relies on such a Nikadio-Isoda equilibrium framework.

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Scheduling of generation units $i \in I$ is incorporated in the strategic decision problem by considering binary variables. Such scheduling over a finite time frame T creates a finite number of possible iterations. Each iteration consists of a problem setup similar to the one presented in [9] and each shows none or a (potentially) unique Nash equilibrium (NE) [10] and therefore results in a finite number of potential equilibria. Multiple NE can, as explained in [11] and [7], vary greatly in appearance. In [11] polynomial algebra is used to define the equilibrium space and to establish the formulation presented. The approach presented below, however, makes use of a branch-and-cut algorithm, due to the finite number of potential 'equilibrium tuples', with analytically derived *optimality* and *feasibility cuts* making mapping of such tuples a possibility.
Integer problems in power systems have various applications. One is given by the question of inertial system frequency response and its interaction with individual plants [12]. The problem is defined by a fixed contribution of inertial response (frequently referred to as 'inertia') that is related to the on/off states of generators and thus is closely related to other market problems such as the market for e.g. spinning reserves. There are, however, no implemented remuneration schemes for inertial response that we are aware of. Thus and to give a practical example, a case study, based on spot prices, on inertia will be presented below.

II. NASH EQUILIBRIA

Assumed are players J that own generation units I participate in a game where they receive a payoff(/profit) function $\pi(x)$ that depends on a set of q_i (quantity) decisions $x = \{q_i, i \in I\}$.

The set of collective actions from the perspective of a player, a generation company j, can be described as $(y_j|\overline{x}_j) \equiv \{q_{i \in I_j}\}$ where $\overline{x}_j$ defines player j's assumptions of decisions on units not controlled by player j, denoted as $\overline{x}_j = \{\overline{q}_{j,i}, i \notin I_j\}$. $\overline{q}_{j,i}$ thus specifies a single players assumption on the output of a specific unit held by a competitor. Denoting optimal solutions with $*$ and using X as the set of viable decisions allows the NE to be defined (similarly to [13] and [8]) as the point y^* that fulfills:

$$\pi_j(y^*) = \max_{(y_j|\overline{x}_j^*) \in X} \pi_j(y_j|\overline{x}_j^*) \quad \forall j \tag{1}$$

The multivariate Nikadio-Isoda function can be defined as follows [14]:

$$\Psi(x, y) = \sum_j [\pi_j(y_j|\overline{x}_j) - \pi_j(x)] \tag{2}$$

As shown in [9], this function is able to yield the distance to a (potentially unique) *NE* for (weakly) concave profit functions.

III. A NON-COOPERATIVE, NON-CONVEX GAME

Adding additional dimensions to the game, such as a network of multiple nodes n or several time periods, expressed through t and so leading to q_i becoming $q_{i,t}$, does not necessarily change the ability of the function proposed above to find the NE distance. Ref. [13] shows this e.g. by solving the multi-nodal example presented in [2]. However, it should be mentioned that the complexity of the approach could increase due to the need for techniques to extend the solution to other time periods, as later periods could bear uncertainty.
Generation scheduling is strongly related to binary decisions, as the on/off states of

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NOMENCLATURE

Indices:

$i, i_2 \in I$	generation unit
i_{hydro}, i_{wind}	hydropower/windpower unit
$j \in J$	producer
n	network node(/area/country)
n_s, n_d	source, destination $[node]$
t	period $[h]$
$s \in S$	branching tuple

Variables:

$y^q_{i,n,t} \in \mathbb{R}^+$	quantity decision $[MWh]$
$y^b_{i,t} \in \{0,1\}$	scheduling decision
$d_{n,t} \in \mathbb{R}^+$	energy demand $[MWh]$
$\overline{q}_{j,i,n,t} \in \mathbb{R}^+$	quantity assumption $[MWh]$

Fixed Variables:

$q_{i,n,t} \in \mathbb{R}^+$	quantity provided $[MWh]$
$b_{i,t} \in \{0,1\}$	dispatch decision

Functions:

$\overline{p}_{j,n,t}$	price estimation $[€/MWh]$
$c_{i,t}$	generation cost [€]
$p^*_{n,t}$	market clearing price $[€/MWh]$
π_j	profit function of producer j [€]

Parameters:

q_i^{min}, q_i^{max}	generation capacity $[MW]$
$c^{var}_{i,t}$	variable cost portion $[€/MWh]$
c^{fix}_i	fixed cost portion [€]
$l^{n_s,n_d}_{i,n}$	line flow from n_s to n_d $[\%]$
$l^{max}_{n_{so},n_{de}}$	line capacity $[MW]$
$w_{i_{hydro}}$	inventory end value $[€/MWh]$
$r_{i_{hydro}}$	available reservoir quota $[MWh]$
$q^{cap}_{i,t} \in [q_i^{min}, q_i^{max}]$	available wind capacity $[MWh]$
H_i	inertial response constant
H^d_n	inertial response requirement

Sets:

I_j	generation units of producer j
I_n	generation units in node n
S^N	set of branching tuples in NE

units are factors that have to be considered in startup cost, ramp rate limits, reserve constraints, and up and down time limits [15]. Using algorithms such as those proposed in [16] or [17] allows the scheduling problem to be solved as a mixed integer cost minimization problem for the optimal dispatch of thermal plants.

Adding storage technologies such as hydropower to such a game allows players to strategically dispatch their resources. Providers with storage capabilities will actively aim to provide in high price (i.e. peak) hours and to withhold in low price (i.e. base) hours. Ref. [3] shows this concept for a Cournot game (a game with competition in quantity) by binding the time stages by their *marginal value of water*. This concept, which is often termed *water value* is used in both scheduling and in the optimization of bidding in hydro power dominated systems [18]. Problem formulations based on water values however often neglect the strategic impact of other players. To strengthen their position on the market, these players might actively aim to withhold production from peak periods in which other players aim to produce. In non-cooperative games, this means a player might act as a leader in some time stages and as a follower in others [19], so making the multi-period game more dynamic than single-period approaches.

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Adding (binary) integer variables to the problem setup leads to non-convex, non-continuous payoff-functions and so breaches the definition of convex games from [14]. Ref. [8] extends the concept of [9] by using an *active set method* to yield a combination of decision variables (which we will later refer to as *tuple*) that define a NE. A binary scheduling problem with I generation units, T time periods would, however, show a possible number of $2^{I\times T}$ tuples, and as shown in [19], multiple equally viable tuple equilibria - ranging from 0 to $I\times T$ (which is also discussed briefly in the appendix). Therefore, deriving a single equilibrium tuple might give an incorrect perspective on the existing array of equilibrium tuples. Such a misrepresentation could prove problematic, particularly in the consideration of ancillary services/markets for reserve energy, where the number of "running" (i.e. committed) units is of significant importance.
We thus propose an algorithm based on the *Nikaido-Isoda equilibrium algorithm* that incorporates branching and cutting based on analytical rules. The Nikaido-Isoda function, first proposed in [14] presents an auxiliary function that defines whether a given player's solutions yield a Nash Equilibrium. A step-wise algorithm as in [9] allows such an equilibrium to be derived for a system with shared constraints (e.g. a network). Based on this concept, the algorithm presented below is meant to bridge the economical approach of determining market power effects and the technical aspect of deriving explicit schedules for providing the commodity.

IV. Selective Cutting

This section will briefly introduce solving a Cournot game with binary variables using the *Nikaido-Isoda function*. This problem is not unique to power systems. A more general formulation will therefore be used and will be extended in the following sections to problems specific to electrical power systems. As discussed above, Cournot games find broad application in power systems, as they are suitable solutions for commodity market problems [1]. Other games such as Bertrand competition might also be applicable. This would, however, require additional analysis of the *cutting rules* presented below. Other modes of competition therefore have been excluded from this paper. We define the profit function of a single player as:

$$\begin{aligned}
&\pi_j(q_{i,n,t},b_{i,t}) = \sum_t \sum_{i\in I_j} \sum_n [\overline{p}_{j,n,t}(\sum_{i\in I_j} q_{i,n,t} \\
&\qquad + \sum_{i_2\notin I_j} \overline{q}_{j,i_2,n,t})q_{i,n,t} - c_{i,t}(q_{i,n,t},b_{i,t})] \\
&\text{where:} \\
&\overline{p}_{j,n,t}(\sum_{i\in I_j} q_{i,n,t} + \sum_{i_2\notin I_j} \overline{q}_{j,i_2,n,t}) = p^*_{n,t}(d_{n,t}) \forall j,n,t \\
&d_{n,t} = \sum_i q_{i,n,t} \qquad\qquad \forall n,t
\end{aligned} \tag{3}$$

This assumption of an existing market clearing quantity $d_{n,t}$ requires the underlying assumption of *information symmetry on price elasticity* among the competitors [1]. Thus, the expectations $\overline{p}_{j,n,t}$, $\overline{q}_{j,i_2,n,t}$ can be approximated as variables $p_{j,n,t}$, $q_{j,i_2,n,t}$ from the perspective of a player. To solve this problem, we establish the *Nikaido-Isoda function in the general form* as:

$$\begin{aligned}
&\Psi((q_{i,n,t},b_{i,t}),(y^q_{i,n,t},y^b_{i,t})) = \\
&\sum_j [\pi_j(y^q_{i,n,t},y^b_{i,t}) - \pi_j(q_{i,n,t},b_{i,t})]
\end{aligned} \tag{4}$$

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The total set of tuples $S = \{s^1, ..., s^{I \times T} | b^1 \neq ... \neq b^{I \times T}\}$ is established by all potential iterations of the binary variable $b_{i,t}$. By fixing $y^b_{i,t} := b_{i,t}$ it is possible to solve every tuple s for its equilibrium (assumed concave profit functions) through an iterative algorithm [9]:

Algorithm 1:

0) assume starting values for $(q_{i,n,t}, b_{i,t})$
1) solve for $\max\limits_{y^q_{i,n,t}} \Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t}))$
2) is $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = 0$?
   yes - **end**, $(y^q_{i,n,t}, y^b_{i,t})$ is the NE point (i.e. tuple is solved);
   no - $(q_{i,n,t}, b_{i,t}) := (y^q_{i,n,t}, y^b_{i,t})$, back to 1)

Repeating this algorithm can be compared to individual players applying stepwise (profit-)maximization, resulting in (supply-side) welfare maximization, whilst operating under shared constraints. As mentioned before, problems in the form of (4) are non-convex. As shown in [9], every individual tuple s therefore offers a (potentially unique) NE as long as the set of constraints added to the problem is convex and allows a feasible solution [10].

We define the set of viable NE $S^N \subseteq S$ as being the set of tuples for which no player has an incentive to dispatch another unit (i.e. increase $\sum\limits_{i \in I_j} b_{i,t}$). The system is computationally efficient solved by making use of two characteristics of the Nikadio-Isoda equilibrium algorithm presented in [8], [9]:

- solving one step of the convergence algorithm is, depending on the cost function, a problem of linear/quadratic nature and thus solved computationally quickly using available commercial software.
- the objective function $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t}))$ provides a quantitative statement about the improvement in each step of the algorithm (as the NE is defined by a value of 0, i.e. 'no improvement potential for any participant'). It is therefore possible to rank tuples by their rate of convergence (lower value of the Nikaido-Isoda function) and select the tuples s that are solved computationally more quickly than others.

Our proposed algorithm labels all tuples s as either:

- **pending** - there can be no definite statement made about the tuple as the Nikaido-Isoda function still returns a value above 0.
- **solved** - the NE of the tuple was found (Nikaido-Isoda function returns 0) and might be considered to be a Nash tuple.
- **sorted out** - the tuple will not be a Nash tuple, irrespective of the value of the Nikaido-Isoda function.

All tuples start as **pending**. Three transitions are possible:
1) **pending** $\Rightarrow$ **solved**, 2) **pending** $\Rightarrow$ **sorted out**, 3) **solved** $\Rightarrow$ **sorted out**.

S^N is the set of (Nash) tuples, for which no such transitions are possible anymore (all tuples are thus either solved or sorted out). The proposed **Nash tuple mapping algorithm** can be formulated as:

Algorithm 2:

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0) **sort out** tuples that do not fulfill (non-convex) constraints associated with discontinuous variables (the *preliminary cutting rules* are presented later in this paper).
1) conduct algorithm 1 on a number of (randomly) selected tuples s that are still **pending**
2) use already **solved** tuples to **sort out** other tuples (i.e. apply *dynamic cutting rules*)
3) are there any **pending** tuples left?
   no - proceed | yes - back to 1).
4) were any nodes **sorted out** in step 2)?
   yes - back to 2) | no - **end** (S^N shows the "map of Nash Equilibria")

V. Cutting Rule Design

Even though the ranking of tuples heavily depends on starting values and in the proposed framework tuples to be solved were selected randomly, generally applicable rules can formulated to sort out unfavorable tuples. For one, they may relate to the set of constraints and thus must be specifically tailored to the application. As such, they mostly depend on pre-selecting iterations of the integer variable to sort out tuples that do not fulfill given constraints. The *Inertial Response Requirement Rule* mentioned later in this paper is such a type. For the other, rules can be defined, that dynamically declare branches of tuples as infeasible or unfavorable, after a single tuple is declared as such. The *Marginal Cost Rule* presented later is an example of such a rule. These rules can, furthermore, draw dynamic conclusions based on already solved tuples. We formulate one such rule here (referred later to as the *Payoff-Function Cutting Rule*) based on two assumptions:

Assumption 1: players will not schedule units if that leads to a decrease in payoff
Assumption 2: adding additional units to the schedule will not increase any market clearing prices

Assumption 1 is a straight forward economical decision and is valid for players that aim to maximize their outcome. Assumption 2 is valid as long as units solely operate on the supply side. Purchases (for example pumped hydro storage) would result in negative supply effects and would increase $d_{n,t}$ and result in higher prices. The model presented here is therefore limited to competition on the supply side. Non-concave, decreasing market price functions are also a necessity. The assumptions presented here rely on the concept of dominance in games [20]. No economically rational player j would choose to commit generation units if the new equilibrium point would not dominate the previous.

To execute the proposed **Payoff-Function Cutting Rule**, two solved tuples denoted as s^* and s^{**} are required. In addition, several conditions must be fulfilled by the equilibrium solutions of the tuples , denoted as $\langle y^{q*}_{i,n,t}, y^{b*}_{i,t}\rangle$ and $\langle y^{q**}_{i,n,t}, y^{b**}_{i,t}\rangle$ respectively:

$$b^{**}_{i,t} \geq b^{*}_{i,t} \quad \forall i,t \tag{5a}$$

$$\pi_{\bar{j}}(y^{q*}_{i,n,t}, y^{b*}_{i,t}) > \pi_{j}(y^{q**}_{i,n,t}, y^{b**}_{i,t}) \quad \forall \bar{j} \tag{5b}$$

$$\sum_{i\in I_{\bar{j}}}\sum_{t} b^{**}_{i,t} > \sum_{i\in I_{\bar{j}}}\sum_{t} b^{*}_{i,t} \quad \forall \bar{j} \tag{5c}$$

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solved tuple s^*

j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	0	1
1	2	1	1	1
2	3	0	0	1
2	4	1	1	0

$\pi_1^* = 1000$, $\pi_2^* = 750$

solved tuple s^{**}

j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	1	1
1	2	1	1	1
2	3	0	1	1
2	4	1	1	0

$\pi_1^{**} = 500$, $\pi_2^{**} = 1500$

pending tuple s

j	i	$b_{i,1}$	$b_{i,2}$	$b_{i,3}$
1	1	0	1	1
1	2	1	1	1
2	3	1	1	1
2	4	1	1	0

$\pi_1 =???$, $\pi_2 =???$

As $\pi_1^* > \pi_1^{**}$ (i.e. π_1^* dominates), player 1 does not have an incentive to set $b_{1,2} := 1$. Thus, both the tuple s^{**} and its branch tuple s can be **sorted out**, independent of the profits of player 2.

Fig. 1: Numerical Cutting Example

$\bar{j}$ represents a specific player from the set of available players $\bar{j} \in J$. (5a) ensures, that tuple s^{**} is located on a branch of tuple s^*. (5b) holds where player $\bar{j}$ has a negative payoff effect from transitioning from tuple s^* to tuple s^{**}. Fulfilling requirement (5c) means that said player $\bar{j}$ made an active decision (committing an additional unit) that enabled this tree branch. According to assumption 1, no reasonable player $\bar{j}$ would choose such a decision. Thus, and according to assumption 2, the tree branch can be cut entirely: $\{s | b_{i,t} \geq b_{i,t}^{**}; \forall i, t\} :=$ **sorted out**

As one can see, this cutting method does not require the two tuples to be adjacent in the branching tree. The structure of the tree plays no role, as long as the stated conditions for s^* and s^{**} hold for the entire time frame. If the two assumptions hold, applicability to (Cournot) problems other than the case presented in this paper is given.

A numerical example of a cut is given by Figure 1. A practical application of the proposed algorithm with additional tailored cutting rules will now be presented.

VI. Multi Period Cournot Market Clearing

We developed an energy market clearing model based on problem (3) with affine cost functions:

$$c_{i,t}(q_{i,n,t}, b_{i,t}) = c_{i,t}^{var} \sum_n y_{i,n,t}^q + c_i^{fix} y_{i,t}^b \tag{6}$$

This allows the formulation of the *extended general form of the Nikaido-Isoda function for a single tuple s*:

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$$
\begin{aligned}
&\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = \\
&\sum_j \sum_t \sum_{i \in I_j} \sum_n \Big[\overline{p}_{j,n,t}\big(\sum_{i_2 \in I_j} y^q_{i_2,n,t} + \sum_{i_2 \notin I_j} q_{i_2,n,t}\big) y^q_{i,n,t} \\
&\qquad - c^{var}_{i,t} \sum_n y^q_{i,n,t} - c^{fix}_i y^b_{i,t}\Big] \\
&\qquad - \sum_{i \in I_j} \Big[\sum_n \overline{p}_{j,n,t}(d_{n,t}) q_{i,n,t} \\
&\qquad - c^{var}_i \sum_n q_{i,n,t} - c^{fix}_i b_{i,t}\Big]
\end{aligned} \tag{7}
$$

where:

$$d_{n,t} = \sum_i q_{i,n,t} \quad \forall n, t$$

$$y^b_{i,t} = b_{i,t} \quad \forall i, t$$

This function combines the objective functions of the players into a single optimization problem that allows conjoint optimization under consideration of previous optimization results entered in the form of previous tuple solutions $\langle q_{i,n,t}, b_{i,t} \rangle$. Using this function as the objective function (8) in an optimization problem and applying the shared constraints allows the distance to the Nash equilibrium for a specific tuple (i.e. a tuple with similar schedules $y^b_{i,t} = b_{i,t} \quad \forall i, t$) to be found.

The generation units show minimum and maximum output restrictions based on whether the unit is running or not. The constraint (9) therefore has to be added to the model. Line constraints connecting the different network nodes were also implemented, one in the positive and one in the negative direction: (10), (11). The concept proposed here extends the formulation proposed in [2], [13] by allowing the exclusion of specific generation units from participating in competing in certain market nodes n or using certain transfer lines $l^{n_s,n_d}_{i,n}$. A transmission system operator and arbitrageurs as independent players (as e.g. displayed in [2]) were excluded from the model for two reasons: 1.) the Nikaido-Isoda function would require additional complexity for such heterogeneous players to be incorporated, so increasing notational complexity unnecessarily; 2.) as shown in [9], the stepwise algorithm is capable of dealing with such shared constraints and can thus be used to assign line capacities shared by players. This comes as a result of the Nikaido-Isoda function allowing solving all players problems bundled within the single objective function (7) compared to other methods from literature such as derivation of the Karush-Kuhn-Tucker conditions. There will, however, be no direct result for wheeling fees, which can limit the applicability of the model in certain markets such as those found in the USA (which would require heterogeneous players). Furthermore, higher granularity of the problem (solving small scale problems within limited areas) would require additional technical specifications and thus additional (shared) constraints, both omitted in the here presented model. Assumptions such as demand curve elasticity can be considered valid assumptions for large scale problems. The case study was therefore chosen to represent an excerpt of cross-country trading within the European electricity market.

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$$\max_{y^q_{i,n,t}, y^b_{i,t}} \Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) \quad (8)$$

$$\text{s.t. } q_i^{min} y^b_{i,t} \leq \sum_n y^q_{i,n,t} \leq q_i^{max} y^b_{i,t} \forall i, t \quad (9)$$

$$\sum_i \sum_n (l^{n_s,n_d}_{i,n} - l^{n_d,n_s}_{i,n}) y^q_{i,n,t} \geq -l^{max}_{n_s,n_d} \forall t, n_s, n_d \quad (10)$$

$$\sum_i \sum_n (l^{n_s,n_d}_{i,n} - l^{n_d,n_s}_{i,n}) y^q_{i,n,t} \leq l^{max}_{n_s,n_d} \forall t, n_s, n_d \quad (11)$$

Solving the maximization problem (8) iteratively, as described above, would result in a NE point (i.e. $\Psi((q_{i,n,t}, b_{i,t}), (y^q_{i,n,t}, y^b_{i,t})) = 0$). As can be seen, this point fulfills the price clearing condition of (3): $\overline{p}_{j,n,t}(\sum_{i \in I_j} q_{i,n,t} + \sum_{i_2 \neq i} \overline{q}_{j,i_2,n,t}) = \overline{p}_{j,n,t}(d_{n,t}) = p^*_{n,t}(d_{n,t})$

Different constraints and parameter specifications must be added depending on plant type. It should be noted that this paper shows a limitation similar to the literature sources - the equilibrium considers only a deterministic representation. Uncertainty can affect a number of parameters including market prices, hydrological inflow, available wind power capacity, and fuel prices. Omitting stochastic representation, which was considered necessary to deal with model complexity - limits the model to shorter time frames that impose less uncertainty. To give an example, wind power is simulated through stochastic parameters in unit commitment models, see e.g. [21], [22]. Such an approach would, however, require additional techniques (i.e. *sampling*, *decomposition*) and therefore exceed the limits of this paper. It was therefore decided to instead use preselected wind capacity scenarios (i.e. *point forecasts* as presented in [23]).

A. Hydropower Plant

Hydropower plants show low cost profiles for production. Models therefore usually exclude the generation cost [18]. The opportunity cost of storing water is instead taken into consideration, defining the decision to generate or store in a single time period [24]. Due to applied formulation of the reservoir function, the approach presented in this paper manages the transition between time periods without[1] calculating the dual values of inventory that are commonly referred to as water value. It still, however, requires a finite set of time periods $t = 1, ..., T$ and an assumption of end values of variables, which are traditionally the end levels of reservoirs. This paper instead applies assumptions of the end value of stored hydrological inventory to demonstrate a different approach. The variable cost of the hydro units were therefore assigned the opportunity cost of stored water: $c^{var}_{i_{hydro},t} := w_{i_{hydro}} \forall t$.

The possibility of holding inventory effectively enables arbitrage over time stages. To incorporate this, a concept similar to [3], [4], [8] was implemented. Thus, a predetermined maximum allowance of available hydropower inventory for the total time frame being given as a parameter. This indirectly represents the hydrological inflow by approximating the state transition caused by reservoir storage as a capacity constraint over the total time frame. To realize this, additional constraints for each of the hydropower units are required:

$$\sum_n \sum_t y^q_{i_{hydro},n,t} \leq r_{i_{hydro}} \quad \forall i_{hydro} \quad (12)$$

[1]with the exception of an initial, fixed assumption of the value after the observed time frame

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In contrast to [3], [4], [8], the approach presented here is realized through an inequality constraint. This is made possible by assuming an end period water value instead of reservoir storage, whereas the storage level is now subject to player decision. In the case study presented here, the water values were considered to be the (assumed) spot price of electricity rounded down to 100s, in the next period after the analyzed time frame (i.e. $T + 1$) within the location node of the respective plant.

Both of these changes to traditional hydropower equilibrium models, i.e. no requirement for dual values and alleviation of the inventory constraints, led to gains in computational efficiency that support the performance of the Nash tuple mapping algorithm. Similar to [5], spillage of hydrological inventory is not considered.

B. Wind Power Plant

Wind, unlike water, which can be physically stored, is a fluctuating resource, that cannot be transferred from time stage to time stage. Availability depends on external factors which the players have no control of (there is no market for the 'procurement' of wind). Wind curtailment can therefore be considered to be a parameter and requires additional constraints for the generation units of 'wind power' type:

$$\sum_{n} y^{q}_{i_{wind},n,t} \leq q^{cap}_{i,t} \quad \forall i_{wind}, t \tag{13}$$

C. Thermal Power Plant

An introduction of CO_2 caps or the ability to store coal or fossil fuels would add a constraint similar to (12) into the mix. However, such a constraint was omitted, for the sake of simplicity. It was considered sufficient for the case study to have higher variable and fixed cost factors than the renewable generation forms, which implicitly forces the players to minimize up-times and therefore CO_2 emissions. The here presented case also is focused on short term modeling. Thermal restrictions such as minimum and maximum downtime were therefore neglected. Constraints for the *contribution to nodal inertial response* were instead chosen to demonstrate a modern application which the algorithm shown here offers. Nonetheless, we propose future extensions to the model in the form of a more sophisticated representation of intertemporality in players' dispatch decisions. I.e. startup and stopping cost, maximum and minimum runtimes, etc.

D. Inertial Response Requirements

As debated in [12], evolving power systems shifting their production portfolio to higher shares of renewable generation, increases the demand for additional security services. One such service would be providing kinetic energy, or inertial response capabilities. The *inertia constant* (in the literature commonly denoted as H) was used to implement this characteristic in our presented market competition model and to rate the individual impact of generation units and formulate inertia "demand" constraints. Defining inertial response contribution as a parameter H_i supplied at an equal level as long as the unit is running (i.e. $y^{b}_{i,t} = 1$) and summation of those contributions to define the nodal/system inertia was considered to be an appropriate approximation [24]. This is given to create a realistic example to showcase the capabilities of the designed framework and is not necessarily aimed at providing a statement about quantitative impacts of inertial response that can be considered without further analysis.

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We assumed the fictional scenario in which nodes can be assigned minimum inertia requirements, relating to the model in the form of a nodal demand constraint:

$$\sum_{i \in I_n} y^b_{i,t} H_i \geq H^d_n \quad \forall t, n \tag{14}$$

E. Additional Cutting Rules

As mentioned in section V, analytically derived cutting rules are an integral component of the algorithm presented. In literature, cuts are commonly categorized into *feasibility* and *optimality* cuts. We also use the definitions *preliminary* and *dynamic* cuts. Preliminary refers to cuts that can be conducted before any tuple equilibria are obtained (i.e. relate to step 0 in the tuple sorting algorithm in III). Dynamic cuts require one or more already solved s^*(i.e. relate to step 2 in the Nash tuple mapping algorithm in III).

1) Feasibility Cuts: Some combinations of the binary variables $b_{i,t}$ cause infeasibility and thus yield no possible market equilibrium. Certain tuples therefore can and must be sorted out before calculating the tuple NE $\Psi((q_{i,n,t}, b_{i,t}), (y^{q*}_{i,n,t}, y^{b*}_{i,t})) = 0$.

- Inertial Response Requirement Cut (preliminary): Too few committed units in a certain node n in period t will result in a breach of (14). Tuples leading to such a situation can be sorted out before solving them. This can be formulated as:

$$\sum_{i \in I_n} b_{i,t} H_i < H^d_n \quad \text{for any } t, n \tag{15}$$

Tuples s that fulfill rule (15) must therefore be sorted out.

- Minimum Hydropower Output Cut (preliminary): Certain tuples can, due to their minimum outputs over the total time frame being higher than available reservoir volume, similarly show a constellation of binary values that breach constraint (12). The cutting rule reads:

$$\sum_t q^{min}_{i_{hydro}} b_{i_{hydro},t} > r_{i_{hydro}} \quad \text{for any } i_{hydro} \tag{16}$$

The affected s that breach (16) have to be (as for the other cuts) sorted out.

2) Optimality Cuts: One of the core aspects of the Nikaido-Isoda function is that some tuples converge faster than others. Therefore, (dynamic) optimality cuts can be conducted to stepwise decrease the amount of unsolved tuples.

- Payoff-Function Rule (dynamic): As explained above, two tuples are required to be in state **solved** for their NE whereas s^{**} has to be located on a branch of s^*.

- Marginal Cost Rule (dynamic): Players in Cournot competition are able to influence prices by varying their bidding quantity. It is therefore possible for prices to end up at a level where no production quantity could compensate for the involved cost. A tree branch, where the *Marginal Cost* of a unit i exceeds the *Market Clearing Price* is therefore not economically viable for the player controlling that unit. This means choosing the maximum price of all nodes n in a single period t as a benchmark clearing price:

$$\begin{gathered} \text{for a tuple } s^* \text{ cut all tuples } s \text{ where:} \\ b_{i,t} \geq b^*_{i,t} \ \forall i, t \\ \sum_{i,t} [b_{i,t} | p^{MC}_{i,t} > \max_n p^*_{n,t}] > 0 \\ p^{MC}_{i,t} = b_{i,t} c^{var}_{i,t} \end{gathered} \tag{17}$$

This shows that cuts might overlap. A tuple affected by the Marginal Cost Rule would show $q_{i,n,t} = 0$ for i and t where $p^{MC}_{i,t} > \max_n p^*_{n,t}$. Otherwise, the generator i

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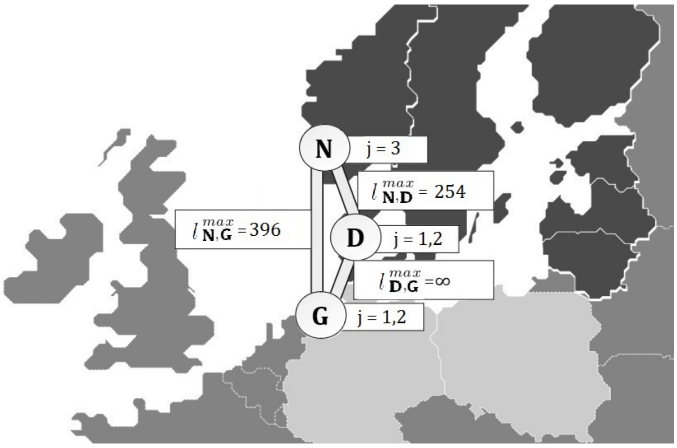

Fig. 2: Case Study Setup

would produce at a loss. However, as the cost portion c_i^{fix} must be paid by unit i due to $b_{i,t} = 1$, then the payoff-function will inherently yield a lower result $\pi_j(q_{i,n,t}, b_{i,t})$ for the owner of $i \in j$. Therefore, such a case would also be cut through the payoff-function rule presented above.

The following section introduces a case study to demonstrate a practical application of the framework presented here.

VII. Case Study

As illustrated by Figure 2, the case study is designed to relate to an excerpt of the European power system, nodes representing the countries of **N**orway, **D**enmark and **G**ermany. As discussed above, a representation of areas or countries in which little regard is paid to wheeling fees can be considered fitting for the model in the here proposed form. Further granularity would require adequate adjustment (i.e. the introduction of further agents). The test case resembles part of a week in late fall with medium to high available wind capacity (especially in the North Sea) and low to medium available hydropower capacities. The parameters can be found in the appendix. The importance of this case study is highlighted by the lack of literature on market power in hydrothermal competition and market power in the European system. Hydrothermal competition is based on the technical constraints related to the state variables, whereas analysis of market power is based on legislature aiming to hinder exercise of such (but not strategic bidding). We argue, particularly in the light of the introduction of new products such as commercialization of inertial response, that a careful analysis of market robustness to such actions should be incorporated in the design process.

Different types of generation units[2] (Table II) meet in a 3-bus network to conduct trade under the assumption of similar information on market clearing price elasticity (Table III). The power line flows $l_{i,n}^{n_s,n_d}$ found in Table IV were assumed to be similar to the P*ower Transfer Distribution Factors* (PTDF) presented in [2]. A single convergence criteria was added to the model:

$$\Psi((q_{i,n,t}, b_{i,t}), (y_{i,n,t}^{q*}, y_{i,t}^{b*})) \leq 0.00001 \equiv 0 \tag{18}$$

[2]note that the plants are assumed to be continuously running or idle for a whole day

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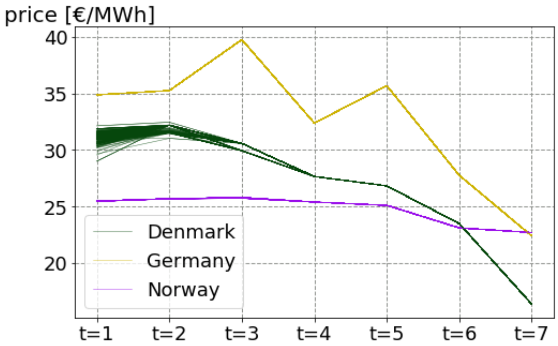
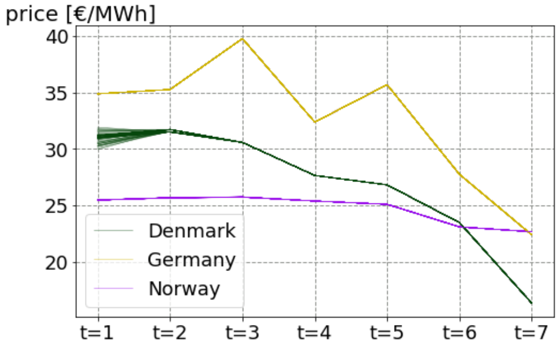


(a) case #1: no requirements for inertial response



(b) case #2: requirements for inertial response in node $n = D$

Fig. 3: Price curves for each Nash tuple $s \in S^N$

No further tolerances were added on the constraints, as they are not required due to tuple problems being represented by quadratic optimization problems (an advantage of the Nikaido-Isoda method compared to more traditional methods such as using Karush-Kuhn-Tucker conditions) that can be solved by most commercial solvers.

The case study aims to analyze the impacts of applying minimum requirements for inertial response in the wind power dominated node $n = D$. Therefore the case study can be adjusted to increase computational efficiency by analyzing generation schedules. Several plants show no fixed cost. No negative effects of the on/off states of the generation units on the profit functions can therefore be expected. These units can therefore be assumed to run continuously, i.e. $\bar{b}_{i,t} := 1 \forall i = \{1, 2, 5, 6, 8, 9, 10\}, t$. For hydropower plants, this is only possible as their minimum generation is assumed to be 0. Schedules would have to be included for minimum output capacities > 0 and the cut presented in (16) would have to be applied. However, as this is not the case, these tuples were removed, reducing the number of total tuples from $2^{10\times7}$ to $2^{3\times7}$.
The model does not consider the possibility of shared inertial response within the whole system but instead focuses on modeling nodal inertia demands. Thus, and as the scheduling of the thermal plant $i = 7$ does not affect the inertial response in node $n = D$, it was assumed to be predetermined as $\bar{b}_7 := [1, 1, 1, 1, 0, 0, 0]$. This led to a reduction in tuples from $2^{3\times7}$ to $2^{2\times7} = 16384$. This remaining set of tuples was solved twice:

#1: no requirements for inertial response:
$H_N^d = H_D^d = H_G^d = 0$

#2: requirements for inertial response in Denmark:
$H_N^d = H_G^d = 0, H_D^d = 1$

The Nash tuple mapping algorithm required solving 629 (randomly selected) tuples for case #1 and 385 tuples for case #2 until the mapping algorithm converged. Processing times on an Intel i7-5600 core @2.6 GHz were below 1 second for an iteration, with an average of 15 iterations until a single tuple converged. The resulting set of Nash tuples S^N contained 390 elements in case #1 and 128 elements in case #2. The model does not show a large range of infeasible states. Most cuts were therefore conducted dynamically.

Figure 3 shows a reduction in the ranges of price scenarios from case #1 to #2. Scheduling decisions however seem to mostly affect the node of the two plants with

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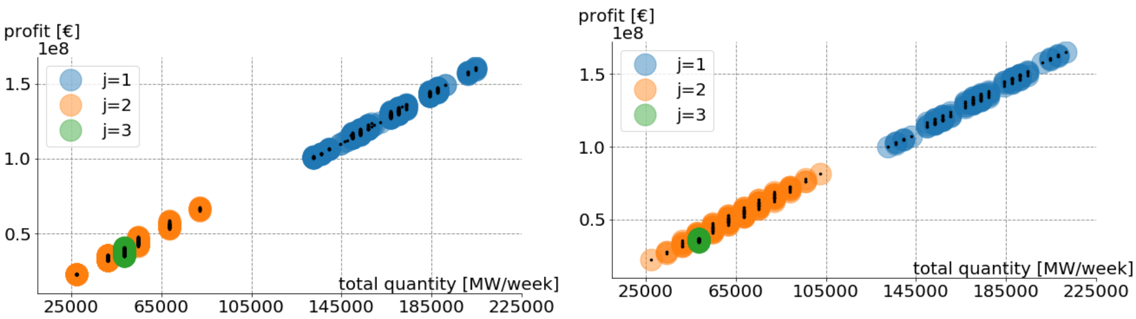
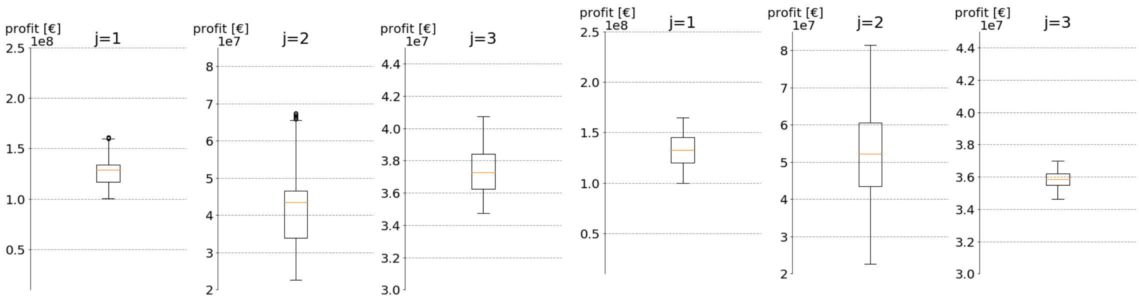


(a) case #1: no requirements for inertial response

(b) case #2: requirements for inertial response in node $n = D$

Fig. 4: Map of Nash tuples $s \in S^N$



(a) case #1: no requirements for inertial response

(b) case #2: requirements for inertial response in node $n = D$

Fig. 5: Firm profits for each Nash tuple $s \in S^N$

variable schedules. The map of NE is displayed in Figure 4. The figure demonstrates the importance of showing different equilibrium tuples. Every chosen schedule yields strongly different outputs and player profits, whereas each tuple is an equilibrium and thus represents a potential outcome. The figure shows that the range of equilibrium results increases slightly for player $j = 1$, strongly for $j = 2$ and decreases slightly for $j = 3$. This change in the range of profit scenarios is also displayed in Figure 5. Player $j = 1$ profits marginally and $j = 2$ strongly from apparent effects of "forced cooperation". This change in profit stems from that the respective owner would choose to not schedule in order to result in an alternate optimum (i.e. it would be profitable to shut them down, thus they would be sorted out by rule (5a) to (5c)).

Scheduling these unprofitable units however occupies transfer line capacities and thus reduces the possibility of the hydropower player $j = 3$ accessing other market nodes, resulting in lower profits across all scenarios. The effect of additional line congestion can be observed in the increase in average capacity in line $N \to D$, as displayed in result Table I. It shows that increasing exports and decreasing local production leads to lower impact of binary variables on the price 'spread'. This is shown by the wide gap of prices in $t = 1$ in Figure 3 and the low spread in $t = 7$. The case study demonstrates that influencing unit commitment decisions (as ancillary services such as primary reserves or the inertial response requirements discussed here do) has an impact on otherwise unaffected generators in the system - here represented by hydropower producer $j = 3$. This negative effect, i.e. a profit decrease, comes as a result of the market share that is shifted to generators that would choose to not schedule in the optimum, but who

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TABLE I: Transfer Results (averaged over all tuples)

line utilization	**case #1**	**case #2**
line $D \rightarrow G$	278.3 MW	330.8 MW
line $G \rightarrow N$	-194.4 MW	-191.8 MW
line $N \rightarrow D$	60.6 MW	113.3 MW

t =	day 1	day 2	day 3	day 4	day 5	day 6	day 7
Local generation $[MWh]$							
case #1	1281	1449	1274	549	81	243	0
case #2	1283	1620	1469	549	81	243	0
Exported generation $[MWh]$							
case #1	911	848	1048	633	823	716	1177
case #2	912	916	1055	709	907	759	1392

are, through system constraints, forced to participate in certain periods. This distorts competition by removing market share from more competitive players such as the time stage abitrageurs (hydro power producers) and assigning them to less competitive forms of generation such as thermal producers. This effect comes from the inertia requirements making certain equilibria from case #1 infeasible, thus effecting the tree and enabling branches that support less efficient equilibria. Averaged over the tuples, case #1 results in a generator welfare of $144M€$ whereas case #2 shows $224.6M€$, a welfare increase that would have to come at the expense of the demand side, i.e. consumers.

VIII. Conclusions

The proposed framework and case study in this paper presents a number of contributions:

The main contribution is the consideration of strategic scheduling decisions in a model with price-making generators and multiple interconnected time stages, a novelty in the literature [25]. Furthermore, the resulting mapping of a finite pool of Nash equilibrium tuples demonstrates a new view on discontinuous problems in energy systems, that have traditionally been occupied with converging towards single solution tuples (e.g. [9]) whilst disregarding potential other equilibria. This allows the discontinuous decisions of a player to relate to its market impact and vice versa, so determining the impact players have on each others' scheduling. In addition, the proposed cutting techniques and adjustments to other models proposed in the literature allows for a more computational efficient approach to model hydro-thermal(-renewable) systems. Finally, the proposed case study itself constitutes a novelty. It shows that introducing minimum requirements for committed units in single nodes has an effect on the profits of other participants in the system. The reason for this is found in transmission capacities being used by the newly committed units, occupying transmission lines that could be otherwise used by different actors to conduct nodal price arbitrage, resulting in a worse outcome for those arbitrageurs. This result and the framework proposed in the paper might aid future discussions of system design options, for example the analyzed requirements for inertial response. One limitation of the paper is demonstrated by the case study. The requirement of in-depth problem analysis does not allow for the plug-and-play of the solution framework. Tailored cuts and predetermining the generation units that are valid in active scheduling decisions requires a case-by-case analysis. As mentioned above, the problem in its current form might be applicable for similar large area applications such as the analysis of spinning reserves. However, and as for most equilibrium problems,

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real world uncertainty and resulting forecast volatility influence the outcome and thus provide an important starting point for future research.

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TABLE II: Generation Unit Parameters (prescheduled units marked with a *)

i	j	type	q_i^{min}	q_i^{max}	H_i	c_i^{fix}	$c_{i,t}^{var}$ / $w_{i_{hydro}}$	$r_{i_{hydro}}$/ $q_{i,t}^{cap}$
1*	1	wind	0	370	0	0	0,...,0	349, 337, 152, 128, 195, 256, 349
2*	1	wind	0	166	0	0	0,...,0	66, 91, 136, 78, 46, 62, 71
3	1	coal/fuel oil	145	715	2.8	4654	29.83, 30.29, 29.67, 29.67, 29.67, 29.67, 28.75	-
4	2	hard coal	285	570	3	3690	20.33, 37.25, 20.92, 43.0, 28.29, 23.5, 23.5	-
5*	1	wind	0	800	0	0	0,...,0	501, 661, 766, 325, 56, 200, 689
6*	2	wind	0	138	0	0	0,...,0	72, 58, 34, 19, 25, 43, 68
7*	2	coal/fuel oil	151	757	2.8	4921	26.67, 26.42, 26.04, 26.25, 25.38, 24.42, 23.29	-
8*	3	hydro	0	376	3.5	0	21	25265
9*	3	hydro	0	342	3.5	0	21	20110
10*	3	hydro	0	118	3.5	0	21	3092

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Appendix

Note on Existence of Nash Equilibria

In the proposed framework, a stepwise Nikaido-Isoda convergence algorithm is applied to find a Nash Equilibrium for a tuple s that is defined by a fixed set of binary variables. This transforms the original Cournot Game of players optimizing Mixed Integer Problems into a number of individual continuous Cournot Games that are solved via a branch-and-cut algorithm. The existence of Nash Equilibria is represented by one of two forms: 0 **Nash tuples** - this situation can only occur due to infeasibility. The preliminary cuts proposed in this model will **sort out** all infeasible states, leaving no tuples that are able to transition towards the **solved** state. An infeasible problem also means that no Nash Equilibrium could be found by the Nikaido-Isoda optimization problem, thus leaving no result for the continuous problem that could represent a tuple. ≥ 1 **Nash tuples** - where the problem is feasible and due to the convexity of the continuous problem, each tuple can yield (at least one) Nash equilibrium (even if the solution is that every player produces at minimum/maximum levels) [9]. Multiple Nash equilibria might exist within one tuple (see e.g. [6]). However the Nikaido Isoda function is able to determine the optimal profits for a single equilibrium tuple which subsequently allows the comparison of tuple equilibria. It can be therefore stated that a Nash tuple represents a definite solution for the integer variables but can include a continuum of solutions for the continuous variables that yield similar player profits.

Data Sets

Table II lists plant types, specifications and related generation firms (i.e. players). Note that daily values were obtained through a factor of 24 on the parameters denoted in hours $[h]$ as the plant is assumed to consistently run/stand idle for a whole day. The selected data set is based on real world data from NordPool and from selected power plant data (slightly distorted to ensure anonymity). The fuel mixes are a hydro power generator in Norway, a representative Danish offshore wind/thermal mix and a representative slice of German generation in form of a large thermal plant, an onshore wind farm and an offshore wind farm. The remaining generation in the countries are expressed indirectly by the elasticity of the price curves given in Table III. These are based on the spot market volume obtained through NordPool. Table III shows the market price curves during the observed week. Table IV lists the locations and the PTDF associated with the plants adapted from the three-node case in [2]. It should be noted for $l_{i,n}^{n_s,n_d}$ that the superscript represents a lineflow $n_s \rightarrow n_d$ and the subscript represents the source unit i and the target node n.

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TABLE III: Market Price Parameters

$p^*_{n,t} =$	$n = N$	$n = D$	$n = G$
$t = 1$	$25.5 - 6\text{E}{-6}d_{n,t}$	$33.04 - 10\text{E}{-5}d_{n,t}$	$34.92 - 10\text{E}{-6}d_{n,t}$
$t = 2$	$25.7 - 6\text{E}{-6}d_{n,t}$	$33.63 - 7\text{E}{-5}d_{n,t}$	$35.29 - 10\text{E}{-6}d_{n,t}$
$t = 3$	$25.8 - 5\text{E}{-6}d_{n,t}$	$30.58 - 7\text{E}{-5}d_{n,t}$	$39.79 - 8\text{E}{-5}d_{n,t}$
$t = 4$	$25.4 - 5\text{E}{-6}d_{n,t}$	$27.67 - 7\text{E}{-5}d_{n,t}$	$32.42 - 10\text{E}{-6}d_{n,t}$
$t = 5$	$25.1 - 5\text{E}{-6}d_{n,t}$	$26.83 - 8\text{E}{-5}d_{n,t}$	$35.71 - 10\text{E}{-6}d_{n,t}$
$t = 6$	$23.1 - 8\text{E}{-6}d_{n,t}$	$23.5 - 6\text{E}{-5}d_{n,t}$	$27.79 - 10\text{E}{-6}d_{n,t}$
$t = 7$	$22.9 - 7\text{E}{-6}d_{n,t}$	$16.38 - 5\text{E}{-5}d_{n,t}$	$22.42 - 10\text{E}{-6}d_{n,t}$

TABLE IV: Plant locations and connections

$i = 1, 2, 3, 4$	location: $n = D$	$l^{D,G}_{i,G} = 67\%$, $l^{D,N}_{i,G} = l^{N,G}_{i,G} = 33\%$ $l^{D,N}_{i,N} = 67\%$,$l^{D,G}_{i,N} = l^{G,N}_{i,N} = 33\%$
$i = 5, 6, 7$	location: $n = G$	$l^{G,D}_{i,D} = 67\%$, $l^{G,N}_{i,D} = l^{N,D}_{i,D} = 33\%$ $l^{G,N}_{i,N} = 67\%$,$l^{G,D}_{i,N} = l^{D,N}_{i,N} = 33\%$
$i = 8, 9, 10$	location: $n = N$	$l^{N,D}_{i,D} = 67\%$, $l^{N,G}_{i,D} = l^{G,D}_{i,D} = 33\%$ $l^{N,G}_{i,G} = 67\%$,$l^{N,D}_{i,G} = l^{D,G}_{i,G} = 33\%$

Sensitivity of Water Values

Expectations of water values also impact the range of equilibrium tuples of non-hydro players. A low expectation of future prices and resulting low water values for the hydropower players leads to a higher range of potential schedules for the thermal players. A high price expectation furthermore leads to a reduction in potential tuples. This is a result of the additional flexibility of a hydropower producer having to shift production to another time stage if profitable.

This is shown by the result of setting water values to 16.67€ per hour (or 400€ per day) which increases the total number of equilibrium tuples to 412. Another extreme can be given by water values of 25€ per hour, which results in only a single equilibrium tuple. This indicates that flexibility in storage creates flexibility in the schedules in a system, even though the units do not belong to the same players.

Note on Performance

With increasing problem complexity, specifically additional actively scheduled units and extended time periods, decreasing performance can be expected. However, additional tests indicate that resource-efficient scaling is possible and the algorithm allows for more complex problem settings than the one presented. To provide an example, thermal unit $i = 7$ was considered with flexible schedule. The result was 15 Nash equilibrium tuples, that required solving 999 tuples (~ 15000 seconds) for a problem with a total of 2097152 tuples.

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