Hydro-thermal Power Market Equilibrium with Price-Making Hydropower Producers

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Abstract

This paper formulates an electricity market dominated by price-making hydro-thermal generation. Generation companies optimize their unit commitment, scheduling and bidding decisions simultaneously as a Mixed Integer Programming problem and participate in a market under quantity competition, giving rise to a discontinuous Nash-Cournot game. Both hydropower and thermal units are considered as price-makers. The market equilibrium under uncertainty is computed via time stage decomposition and nesting of a Continuous Nash game into the original Discontinuous Nash game that can be solved via a search algorithm. To highlight applicability of the proposed framework, a case study on the Scandinavian power market is designed and suggests positive welfare effects of large scale storage, whereas the implications on scheduling of conventional units are subsequently discussed. Reformulation allows computationally efficient scaling of the problem and possible extensions to allow large scale applications are discussed.

Keywords: Hydropower, Hydro-Thermal, Cournot game, Nash equilibrium, discrete game, electricity market

1. Introduction

1.1. Background

Larger integration of renewable resources increases the challenges on liberal electricity markets. Such means of generation are, compared to conventional forms of generation, characterized by their low cost curves and uncertain capacity profiles. Higher shares of renewable generation could thus lead to increased supply side volatility as well as increased gaps between peak and base prices. Those effects will be eventually carried financially by the end consumer and, in interconnected systems, might spread to otherwise unaffected nodes or areas[1]. Applying flexible means of production mitigates this issue by applying the principle of 'peak skimming'[2], where a producer strategically schedules generation for the periods showing the highest market prices. Such flexible generation can come in form of conventional plants or energy storage, whereas hydropower plants provide the most

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prevalent large-scale application for latter. Despite their negligibly small cost curves, hydropower units with large enough storage capacities [1] compete with conventional generation for peak loads rather than for base loads with other means of renewable generation[3]. This paper addressed the issue of an electricity market dominated by hydro-thermal generation as price makers. Differing from the existing works on this topic, in this paper the hydropower producers simultaneously decide their unit commitment and scheduling strategies under uncertainty.

Table 1: Model Feature Comparison

Ref	Hydropower	Thermal power	Uncertainty	Multiple periods	Multiple players	Price-makers	Non-convex players
[1]	✓	✓			✓		
[3]	✓	✓	✓	✓	✓	✓	
[4]		✓			✓	✓	
[5]		✓		✓	✓	✓	
[6]	✓	✓		✓	✓	✓	
[7]	✓		✓	✓	✓	✓	
[8]	✓	✓		✓			✓
[9]	✓	✓	✓	✓	✓		
[10]	✓			✓	✓	✓	
[11]	✓	✓		✓	✓	✓	
[12]	✓	✓		✓	✓	✓	✓
[13]	✓		✓	✓	✓	✓	
[14]	✓	✓		✓	✓	✓	
[15]	✓	✓				✓	✓
[16]	✓			✓			✓
[17]	✓	✓	✓	✓	✓		✓
[18]		✓			✓	✓	
[19]					✓	✓	
[20]	✓	✓	✓	✓	✓		✓
[*]	✓	✓	✓	✓	✓	✓	✓

[*] refers to the model proposed in this paper

1.2. Related Works

In the literature, there are a multitude of examples given for analyzing the strategic aspects of conventional means of generation[21]. In Ref.[4] nodal prices were

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[1]In relation to their generation capacities, as a reservoir with large storage capacity and smaller output capacity has higher flexibility regarding the time stages it chooses to feed into the system.

derived through modeling transmission system operators and market operators as players participating in a *Cournot competition*. Ref.[22] shows electricity market applications of modeling market clearing through *supply function equilibria*, subsequently deriving *Nash equilibria* on the base of the cost functions of market participants. Ref.[23] extends the concept of Stackelberg games to multi-leader games and solves it through an *Equilibrium Problem with Equilibrium Constraints*(EPEC) formulation. In the model presented in Ref.[5], demand sided players are presented as strategic entities in a pool market.

Not considering technical specifications such as nonlinear efficiency curves, hydropower shows two prominent characteristics that differentiate it from conventional generation and make the above presented methodologies hard or impossible to apply: negligibly small generation cost functions and uncertain, period-transferable capacity in form of hydrological inventory. As a result, contrary to their conventional counterparts, bidding models for hydropower units generally consider price-taker approaches[24], leading to models for hydrothermal competition usually strictly separating between exercise of market power by conventional plants and efficient unit commitment by hydrological plants. Game-theoretical applications that focus on market clearing are found in Ref.[6, 7], whereas applications that focus on unit commitment are given in Ref.[8, 9].

There is indication that anticipation of price-making storage operators can impact market outcomes[10]. A few examples of literature analyze this topic: Ref.[11] describes a Cournot market clearing based on a Nash equilibium convergence algorithm through the Nikaido-Isoda function that has an active set method applied to stepwise converge to an equilibrium with optimal storage. Ref.[12] extends this concept to a deterministic multi-nodal Mixed-Integer market clearing problem and finds the optimal unit schedules via branch-and-cut. In Ref.[13] two different approaches are considered to model Nash equilibria in hydro-thermal systems. As the system is hydro-dominated focus is put on modeling uncertainty. Ref.[14] implements hydrological storage through a capacity constraint connecting time stages and thus otherwise individual models into a single market clearing model under Cournot competition. Ref.[15] focuses on the scheduling decisions of the thermal plants, ignoring inventory transfers through hydropower reservoirs and solving a series of deterministic Mixed Integer Problems to converge towards a balance in supply and demand to represent a cleared market. Despite the listed approaches, no literature is found on a problem as shown in Fig.1: hydropower producers participating in markets with changing access (on/off) for marginal (thermal) units, i.e. a game with a time-dynamic set of participants. The reason herefore, as e.g. listed in Ref.[25] is the difficulty of dealing with the duality gap created by such integer decisions. The model presented in this paper however aims to connect exercise of market power in hydrothermal systems with optimal scheduling and unit commitment, which has historically been focus of cost minimization[26]. This competitive market is formulated as a Discrete game[27] and can be solved by commercial solvers for its (potentially multiple) Nash equilibria, raising computational efficiency through reformulation[28].

Figure 1: Multi Period Hydro-Thermal Game
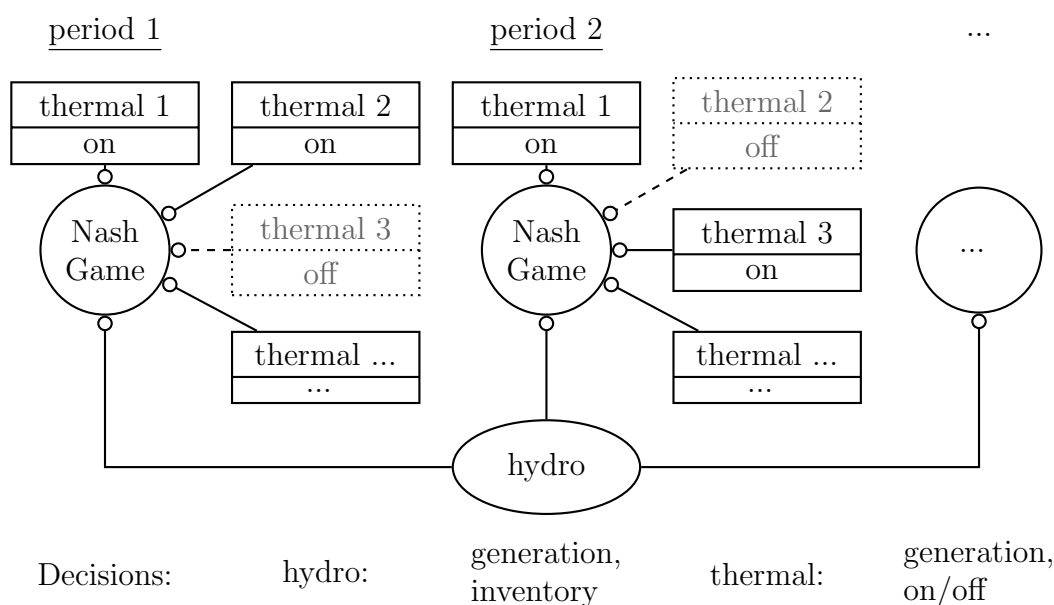
1.3. Contributions

In the here presented paper the hydropower companies optimize their unit commitment and bidding strategies simultaneously under the consideration of uncertainty. The salient features of this paper are summarized as follows.

* **Combination of scheduling and price-maker bidding:** the presented model provides a novel tool for both generation companies and system operators to analyze the impact of storage capacities onto the network, thus incorporating the change of market power over time.

* **Model compactness:** the nested problem of a continuous game under uncertainty shows strong computational efficiency and thus has the potential to itself be used as an analytic tool (without solving the scheduling problem).

* **Practical applicability:** future applications of the presented model have a wide range. For example this could include analysis of capacity mechanisms or the impact of maintenance and involuntary down-times, refinement of hydropower-bidding through ability to model price impact, and analysis of the interactions of strategic scheduling and strategic storage. The here presented base model might be extended by additional dimensions (e.g. more nodes, market types) and constraints (e.g. reserve provision) in a similar manner to traditional equilibrium models[21] to enable it to cope with real-world problems.

A direct comparison to models from literature can be found in Tab.1.

1.4. Organization

The rest of this paper is organized as follows. In Section 2, the hydro-thermal model under consideration of uncertainty, periodic inflow and binary unit commitment decisions is formulated. Section 3 specifies the solution techniques used to

yield what is later defined as 'Nash tuples'. Case studies are presented in Section 4 with discussions on welfare effects. Section 5 concludes the paper.

Nomenclature

J	players	
I_j	generation units of player j	
I_j^{Th}	thermal units of player j	
I_j^{Hy}	hydrological units of player j	
T	time periods	
Ξ	scenarios	
N	predefined schedules	
Φ	equilibrium tuples	
Indexes		
j	player	$j \in J$
i	generation unit	$i \in I_j$
t	time period	[h]
s	inflow source period	[h]
t_2	inflow sink period	[h]
Variables		
$q_{i,t}$	generation level	[MWh]
$b_{i,t}$	scheduling variable	[binary]
$q_{i,t}^s$	production from source period s used in period t	[MWh]
$q'_{i,t}$	generation decisions of other players $i \notin I_j$	[MWh]
n_i	selected schedule	$n_i \in \mathbb{R}^+$
Parameters		
ξ	stochastic parameter	$\xi \in \Xi$
$\underline{q_i}, \bar{q}_i$	generation capacities	[MW]
R_i	reservoir capacities	[MW]
P^ξ	scenario probability	[%]
k	convergence parameter	$k \in \mathbb{R}^+$
$c_{\text{fix}}, c_{\text{var}}^a, c_{\text{var}}^b$	cost function parameters	
Functions		
$\Pi_{i,t}$	profit function	[€]
Q_i^s	inflow in hour s	[MWh/h]
$p_{j,t}$	price expectation of player j	[$\frac{€}{\text{MWh}}$]
$c_{i,t}$	cost function of unit i	[€]
d_t	demand function	[MWh]
$b_{i,t}$	scheduling function	[binary]
Dual variables		
$\underline{\delta}_{i,t}, \bar{\delta}_{i,t}$	generation capacities	
σ_i	reservoir inflow	
$\gamma_{i,t}$	split representation	
$\psi_{i,t}$	reservoir capacity	
$\mu_{i,t}$	non-negativity	
$\omega_{i,t}^{p,\xi}, \omega_i^{Q,\xi}$	stochastic - deterministic gap	

2. Model

2.1. Hydro-Thermal Generation

We assume a single area with a single pool market and competition in quantity. All supply sided participants, further referred to as generation companies or players, aim to solve a Mixed Integer profit maximization problem in the form of:

$$\max_{q_{i,t}, q_{i,t}^s, b_{i,t}} \Pi_j = \sum_{i \in I_j} \sum_{t \in T} \Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) \tag{1a}$$

$$\text{s.t.} \quad \underline{q}_i b_{i,t} \leq q_{i,t} \leq \bar{q}_i b_{i,t} \qquad \forall i \in I_j, t \in T \tag{1b}$$

$$\sum_{t=s}^{\max(T)} q_{i,t}^s \leq Q_i^s(\xi) \qquad \forall i \in I_j^{Hy}, s \in T \tag{1c}$$

$$\sum_{s=1}^{t} q_{i,t}^s = q_{i,t} \qquad \forall i \in I_j^{Hy}, t \in T \tag{1d}$$

$$\sum_{s=1}^{t-1} \sum_{t_2=t}^{\max(T)} q_{i,t_2}^s \leq \bar{R}_i \qquad \forall i \in I_j^{Hy}, (t > 1) \in T \tag{1e}$$

$$q_{i,t} \in \mathbb{R}, q_{i,t}^s \in \mathbb{R}^+, b_{i,t} \in [0, 1]$$

Objective function Eq.(1a) incorporates all generation units owned by the player. The profit function of a single generation unit in a single time period, as shown below in Eq.(2), depends on a stochastic parameter representing uncertainty, as well as the chosen levels of generation and the scheduling variables. Each player might hold both thermal and hydropower units. The generation capacities given in Eq.(1b) depend on the unit schedules that define if the units are able to supply between their given minimum and maximum generation limits in a certain period. This depends on if the respective unit is running (i.e. $b_{i,t} = 1$) or shut down ($b_{i,t} = 0$). The inflow consistency constraint Eq.(1c) ensures that the hydropower units only use their given inflows, whereas s indicates the source period in which the inflow arrives at the reservoir. Decision variable $q_{i,t}^s$ represents how much of inflow from a source period is used for generation in period t. Subsequently, Eq.(1d) ensures that the total generation of those units matches this split representation. Physical capacities of reservoirs are considered in Eq.(1e): transfers from a source period $s < t$ into a sink period $t_2 \geq t$ count to the total inventory in period t which cannot exceed the upper limit of the reservoir. The reason why it is conducted over $(t > 1) \in T$ periods is that for a number of $\max(T)$ periods there are a number of $\max(T) - 1$ inventory transfers between periods. For the sake of simplicity and similar to Ref.[14], no (mandatory) end reservoir levels are assumed. Starting reservoir levels are determined by the inflow in period 1.

The profit functions of the players are defined as:

$$\Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) = p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) q_{i,t} - c_{i,t}(q_{i,t}, b_{i,t}) \tag{2}$$

In Cournot competition, players calculate their individual profits based on assumptions representing decisions of other market participants, which are subsequently in this paper marked by $'$. As the model is aimed for short term applications, cost

functions are assumed to be independent of uncertainty, thus limiting the profit impact of the stochastic parameter ξ on the components[2] of the price functions $p_{j,t}$. As hydropower cost components are generally considered to be negligible in short to medium term applications, the respective cost functions $c_{i,t}$ can be omitted, simplifying $\Pi_{i,t}(\xi, q_{i,t}, b_{i,t}) \rightarrow \Pi_{i,t}(\xi, q_{i,t}) \forall i \in I_j^{Hy}, t \in T$. In addition, this removal of the binary variables from the profit function also allows for a simplification of Eq.(1b) - since, for hydropower units, there are now no (negative) profit effects caused by the binary variables, they can be considered fixed as $b_{i,t} = 1 \forall i \in I_j^{Hy}, t \in T$. Furthermore, depending on the form of price and cost curves of the thermal units, the problem of a single player can take the form of a Mixed Integer Linear Program or a Mixed Integer Quadratic Program.

2.2. Market Clearing

A system operator would aim to clear the market by calculating a periodical demand, in the here presented framework with uniform pricing:

$$d_t = \sum_{i \in I_j} q_{i,t} + \sum_{i \notin I_j} q'_{i,t} \quad \forall j, t \tag{3}$$

To allow derivation of a definite market price, symmetric information on price curves has to be assumed:

$$p_{j_1,t}(\cdot) = p_{j_2,t}(\cdot) \quad \forall j_1 \in J, (j_2 \neq j_1) \in J, t \in T \tag{4}$$

As liberalized pool markets generally provide historical data publicly, such an assumption can be considered valid in practical applications. Thus, Eq.(2) can be simplified by:

$$p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) \rightarrow p_t(\xi, d_t) \forall j \in J, t \in T \tag{5}$$

2.3. Model Limitations

As shown in Ref.[16], real-world hydropower short term unit commitment includes a large range of complicating factors such as interconnected inflows and technical characteristics (e.g. head and tail effects). In addition, due to increasing uncertainty through intermittent generation such as wind, unit commitment problems might require advanced techniques to cope with their tasks[17]. As mentioned below, in this paper however, uncertainty will be dealt with on an approximated level - through addition of slack variables and minimizing their weighted distance to a deterministic solution. Thus, the here presented problem gives a support tool to show the interaction between unit commitment and equilibrium problems and cannot replace optimal scheduling of units.

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[2]In case of linear demand curves both slope and intercept would be subject to uncertainty.

Figure 2: Solution Algorithm Flowchart (Discrete Game)
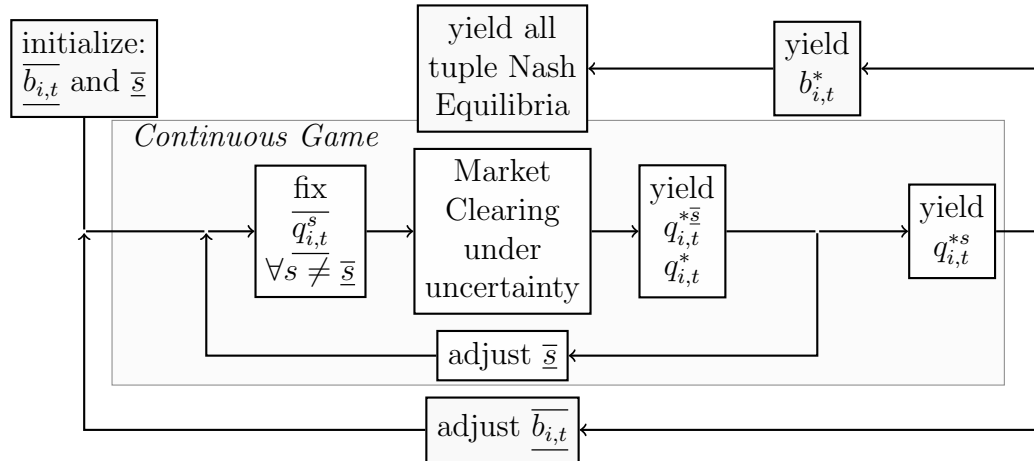
3. Solution Approaches

Even though the previously listed assumptions and simplifications support efficient solving of the model, several complications have still to be dealt with. Mainly, the previously mentioned duality gap caused by the binary scheduling variables that eliminates the possibility of straight-forward application of some of the conventional techniques shown in the introduction to this paper. Furthermore, the 'curse of dimensionality' related to problem size - i.e. amounts of scenarios, time stages, agents - has to be approached to allow for practical applications of the proposed framework. Thus, we propose a triple layer approach to derive multiple Nash Equilibria for the player decisions as shown in Eq.(1).

Regarding annotation: below we will mark fixed variables/parameters as $\bar{\cdot}$ and optimal solutions of variables with $*$. The structure of the solution framework is shown in Fig.2.

3.1. Market Clearing under Uncertainty (Continuous Game part I)

The core problem is represented by the Karush Kuhn Tucker conditions of Eq.(1), with adjustments to cope with stochasticity and with initial presets for certain decision variables. Namely, the schedules - i.e. the binary decision variables - are given fixed values $\overline{b_{i,t}}$, and a specific inflow source period $\bar{s}$ is chosen, leading to all quantity decisions that are not related to this inflow period being considered as fixed parameters instead of variables, i.e. $q_{i,t}^s := \overline{q_{i,t}^s} \forall (s \neq \bar{s}) \in T$ with starting values of $\overline{q_{i,t}^s} = 0$.
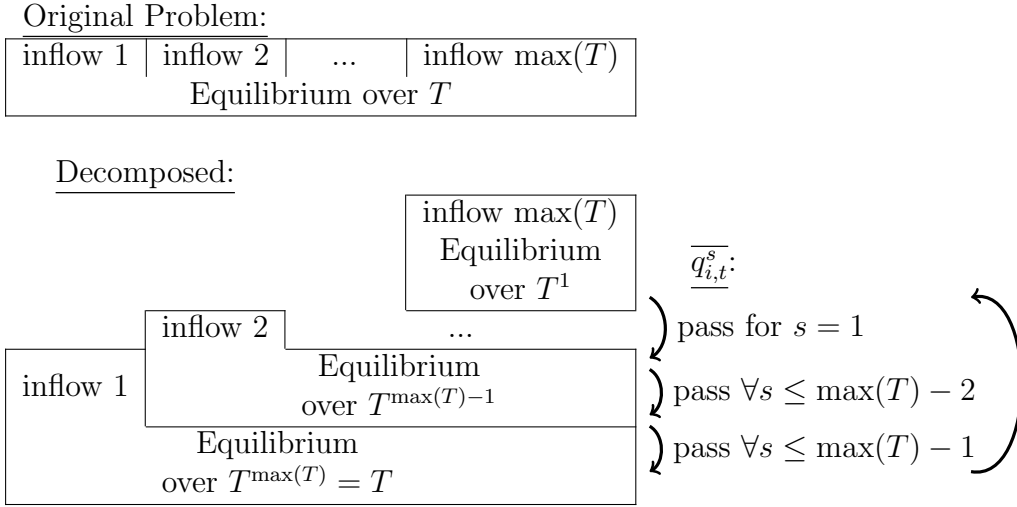
As presented in Fig.3 this decomposes the model in a number of smaller equilibria.

As with fixed schedules for thermal plants, price arbitrage between periods can only happen through hydropower units, considering a specific inflow period $\bar{s}$ affects only the following periods. Thus a new period set can be defined:

$$T^{\bar{s}} = \{t | t \in T, t \geq \bar{s}\} \tag{6}$$

Figure 3: Continuous Game - Decomposition

Original Problem:

inflow 1	inflow 2	...	inflow max(T)
Equilibrium over T			

Decomposed:

This allows simplification and reformulation of Eq.(1):

$$\max_{q_{i,t},q_{i,t}^{\overline{s}}} \quad \Pi_j = \sum_{i \in I_j} \sum_{t \in T} \Pi_{i,t}(\xi, q_{i,t}, \underline{\overline{b_{i,t}}})$$

$$\text{s.t.} \quad \underline{q_i}\overline{\underline{b_{i,t}}} \leq q_{i,t} \leq \bar{q}_i\overline{\underline{b_{i,t}}} \qquad \begin{matrix} \forall i \in I_j, \\ t \in T^{\overline{s}} \end{matrix} \qquad (\underline{\delta}_{i,t}, \bar{\delta}_{i,t})$$

$$\sum_{t \in T^{\overline{s}}} q_{i,t}^{\overline{s}} \leq Q_i^{\overline{s}}(\xi) \qquad \forall i \in I_j^{Hy} \qquad (\sigma_i)$$

$$\sum_{s=1}^{t} \begin{cases} q_{i,t}^{\overline{s}}, & \text{if } s = \overline{s} \\ \underline{q_{i,t}^s}, & \text{if } s \neq \overline{s} \end{cases} = q_{i,t} \qquad \begin{matrix} \forall i \in I_j^{Hy}, \\ t \in T^{\overline{s}} \end{matrix} \qquad (\gamma_{i,t}) \qquad (7)$$

$$\sum_{s=1}^{t-1} \sum_{t_2=t}^{\max(T)} \begin{cases} q_{i,t_2}^{\overline{s}}, & \text{if } s = \overline{s} \\ \underline{q_{i,t_2}^s}, & \text{if } s \neq \overline{s} \end{cases} \leq \bar{R}_i \qquad \begin{matrix} \forall i \in I_j^{Hy}, \\ (t > \underline{s}) \in T^{\overline{s}} \end{matrix} \qquad (\psi_{i,t})$$

$$q_{i,t}^{\overline{s}} \geq 0 \qquad \begin{matrix} \forall i \in I_j^{Hy}, \\ t \in T^{\overline{s}} \end{matrix} \qquad (\mu_{i,t})$$

For a single scenario, taking the Karush Kuhn Tucker conditions of this problem allows derivation of an equilibrium solution. To clear the problem under several scenarios $\xi \in \Xi$ with individual scenario probabilities we apply a similar approach as found in Ref.[18], extending the model by two dual variables $\omega_{i,t}^{p,\xi}$ and $\omega_i^{Q,\xi}$ that represent the shadow prices of a scenario deviating from a selected deterministic solution, therefore minimizing the residuals between a deterministic solution and all scenarios. Discussion on this approach is provided in Appendix A.

Subsequently, the Karush Kuhn Tucker conditions are formulated and provided in Eq.(B.1) which can be found in Appendix B. The Lagrangians are shown in Eq.(B.1a) to (B.1c), the feasibility and complementarity conditions are found in Eq.(B.1d) to (B.1h). Eq.(B.1j) and (B.1k) respectively present the price and inflow consistency constraints that minimize the residuals between the different scenarios whose probability adjusted shadow prices can be found to affect Eq.(B.1a), Eq.(B.1b), and Eq.(B.1f).

3.2. Backwards Pass (Continuous Game part II)

As shown in Fig.2 and Fig.3, readjustment of the fixed inflow source period $\bar{\underline{s}}$ is required to accommodate for all inflow periods. By nesting the single period problem into a backwards pass algorithm, the multi-period problem can be solved. After choosing a value for the convergence parameter this algorithm can be implemented in the following manner:

Algorithm 1.

    0. *initialize* $\bar{\underline{s}} := \max(T)$, *run* $:= 1$, $\Pi_j^0 := -\infty \ \forall j \in J$

    1. *solve KKT conditions*

    2. *update:* $\overline{q_{i,t}^s} := q_{i,t}^{*\bar{\underline{s}}} \ \forall j \in J, i \in I_j^{hy}, t \in T, s = \bar{\underline{s}}$
       and $\Pi_j^{run} := \Pi_j^* \ \forall j \in J$

    3. *if* $\bar{\underline{s}} > 1$: $\bar{\underline{s}} := \bar{\underline{s}} - 1$ *and back to 1.*

    4. *if* $\sum\limits_{j \in J} |\Pi_j^{run-1} - \Pi_j^{run}| > k$: *run* $:= run + 1$ *and back to 1.*

    5. **converged:** $q_{i,t}^{*s} = \overline{q_{i,t}^s} \ \forall j \in J, i \in I_j^{hy}, t \in T, s \in T$

As mentioned in Ref.[13], such an equilibrium will only show one potential schedule for the hydropower units. Even with a convergence parameter of $k = 0$(as in the latter presented case study) and therefore requiring a global optimum, other schedules might result in similar profits making the optimum non-unique. Discussion on this matter can be found in Appendix C. However, as the derivation of the discontinuous Nash equilibrium only uses the information on objective values of the player problems (which are constant over all equilibria) and not the specific generation decisions (which might diverge), validity of the results still holds.

3.3. Discontinuous Nash Equilibrium (Discrete Game)

As displayed in Fig.2 the Backwards Pass algorithm determines optimal generation in the equilibrium, considering that unit commitment is predefined. Such fixed ramping schedules as utilized e.g. in Ref.[19, 20] which assume that binary schedules of the units are pre-established and treated as input parameters to the problem. However, as the here presented model aims to give the thermal plants the possibility to react to hydropower decisions (i.e. withdraw from time stages with peaks lowered through storage arbitrage). Fixing the ramping schedules and thus the binary variables $b_{i,t}$ eliminates those actions by the thermal players, thus weakening their model strategies in relation to their options in reality and thus distorting the model results and subsequently displaying a skewed representation of the equilibrium.

On the contrary to fixed schedules, all potential iterations of the binary variables would amount to a number of $2^{\sum\limits_{j \in J} |I_j^{th}| \times \max(T)}$. Each of those iterations would in turn represent a 'Nash tuple', that is an equilibrium solution derived by the Continuous game as shown above.

Thus, fixing schedules to a single outcome might not represent the reality adequately, whilst keeping all iterations in the game introduces the problems traditionally related to Mixed Integer Programming: increasing complexity and the possibility of ending up in local maximums.

To adequately address this issue, we apply an approach that can be considered a middle course between those mentioned. By using a reformulation similar to Ref.[28], we are able to reduce computational complexity whilst still keeping the core strategies of the players intact: instead of solving for the binary decision variable, we replace $b_{i,t} \to b_{i,t}(n_i)$. The function $b_{i,t}(n_i)$ represents preselected schedules indexed by the decision variable $n_i \in N_i$. This reduces complexity to a new number of iterations: $iter = 1 \leq 2^{\sum\limits_{j \in J} \sum\limits_{i \in I_j^{th}} |N_i|} \leq 2^{\sum\limits_{j \in J} |I_j^{th}| \times \max(T)}$. Even though this reformulation of the scheduling decisions results in a computationally less demanding problem setup, it still is of $\mathcal{NP}$-hard nature[29]. Modern techniques usually tend to work with various branch-and-cut approaches to derive solutions for such discontinuous problems[30]. However, comparing and evaluating different outcomes for Nash equilibria in their "validity" itself poses a problem that is not straightforward[27]. Furthermore, Nash equilibria are not necessarily globally optimal for the players, as demonstrated by famous examples such as the prisoners' dilemma. As a result, instead of applying an approach using bounds and risking jeopardizing potential viable equilibria, pre-selection of an adequately sized number of predefined schedules to enable brute-forcing all equilibrium tuples was conducted. This is made possible as every tuple (if feasible) can be solved for an equilibrium that defines profits for each player, allowing to determine dominant strategies (i.e. scheduling decisions). By defining a player j's assumption on other generation units' schedules as $n'_{i_2} \ \forall i_2 \in I_{j_2 \neq j}$ a *Nash tuple equilibrium* can be defined as:

$$\Phi = \langle q_{i,t}^*, q_{i,t}^{*s}, b_{i,t}(n_i^*) \rangle$$

where

$$\Pi_j^* = \max_{n_i} \sum_{i \in I_j} \sum_{t \in T} \Pi_{i,t}(\xi, q_{i,t}^*/q_{i_2,t}'^*, b_{i,t}(n_i)/b_{i_2,t}(n'_{i_2})) \quad \forall j \in J \tag{8}$$

As each iteration can potentially represent an equilibrium, the number of tuple equilibria $|\Phi|$ will range within $0 \leq |\Phi| \leq iter$.

To derive the equilibria a search algorithm, e.g. in the following form, can be used:

Algorithm 2.

0. *initialize* $\phi(n_i) := \{0 | \mathbb{N}^{\prod\limits_{i \in I_j} N_i}, j \in J\}$
1. *for* $j \in J$:
2.   *initialize* $N'_j = \{n_{i_2} | n_{i_2} \in N_{i_2}, i_2 \in I_{j_2 \neq j}\}$
3.   *while* $N'_j \neq \{\varnothing\}$:
4.   *choose any* $n'_{i_2} \in N'_j$
5.   *solve* Π_j^*
6.   *increment* $\phi(n_i/n'_{i_2}) := \phi(n_i/n'_{i_2}) + 1$
7.   *remove* n'_{i_2} *from* N'_j
8. *for all* n_i *where* $\phi(n_i) = \max(J)$: $n_i = n_i^*$

This algorithm builds on the requirement of a computationally feasible set of available schedules. Adding more sophistication in terms of a larger set of unit

Figure 4: Obtaining Scenarios through Lattice Separation
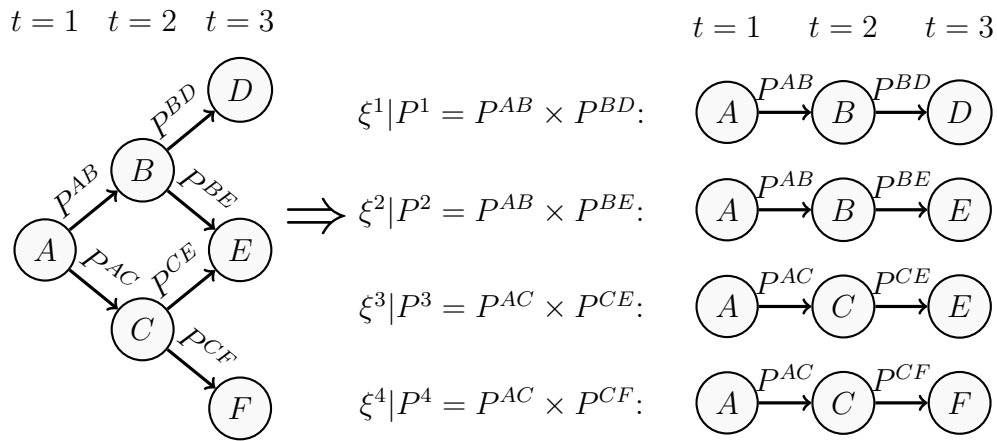
schedules would require application of additional techniques, i.e. branch-and-bound or branch-and-cut as demonstrated in e.g. Ref.[31]. However, as the here presented model aims to focus on short term time frames, it is reasonable to assume that a range of potential unit commitment schedules is already established. As mentioned before, literature traditionally assumes a single such schedule, whereas we relax this by providing a set of potential schedules to our players to choose from.

4. Case Study

To validate the proposed model and methods, a test case representing the Scandinavian power system was designed to represent a late spring scenario in Southern Sweden. The aim of the case study is to provide a showcase of the capabilities of the demonstrated model framework.

Test System. Three oligopolistic players - respectively holding a hydropower, thermal and mixed generation portfolio, were considered competing over 7 periods. Out of the five thermal plants three heterogeneous plant types (Gas/Coal/Oil) were introduced, whereas units of a specific type were modeled in homogeneous manner in regards to generation capacities, up-/down-time limits and cost curves. The thermal units were given quadratic cost functions in the form of:

$$c_{i,t}(n_i, q_{i,t}) = c_{\text{fix}} b_{i,t}(n_i) + c_{\text{var}}^a q_{i,t} + c_{\text{var}}^b (q_{i,t})^2 \tag{9}$$

For the sake of simplicity in demonstration, ramp-up and -down cost were replaced with fixed rates for up-time.

The five hydropower units were modeled heterogeneous in regards to generation capacities, reservoir sizes, degree of regulation (relation of generation to reservoir size, where generally a low value can be used to represent a run-of-river unit and a large value a long term storage unit), inflow (base level, variability and trend). This can be observed in Fig.5 which shows the scenarios for latter[3]. The total

---

[3]The size of the outer rings and strength of the connection lines display the likelihood of the scenario.

Figure 5: Inflow Scenarios for the Hydropower Units[3]

starting period inventory:
  unit $i = 6$: high
  unit $i = 7, 8, 9$: medium
  unit $i = 10$: low
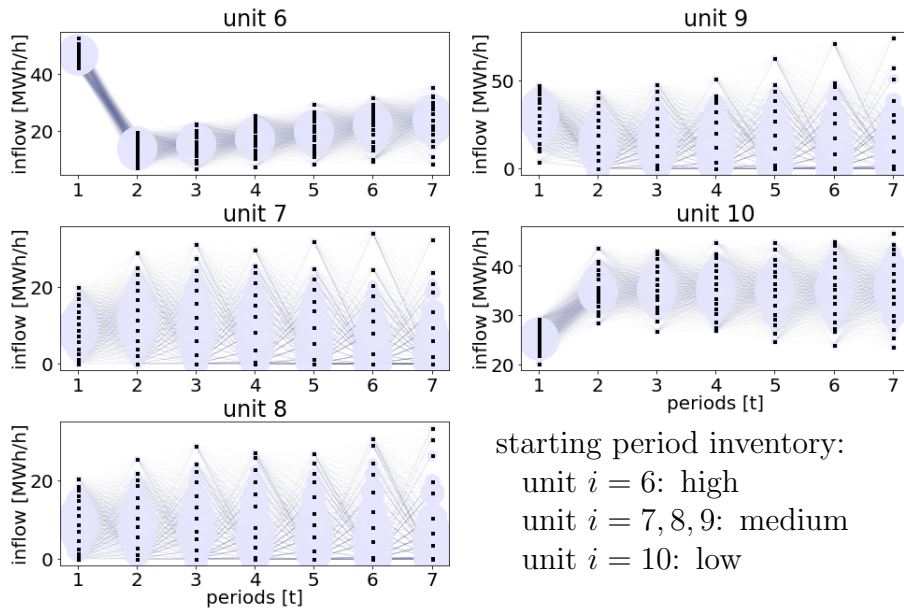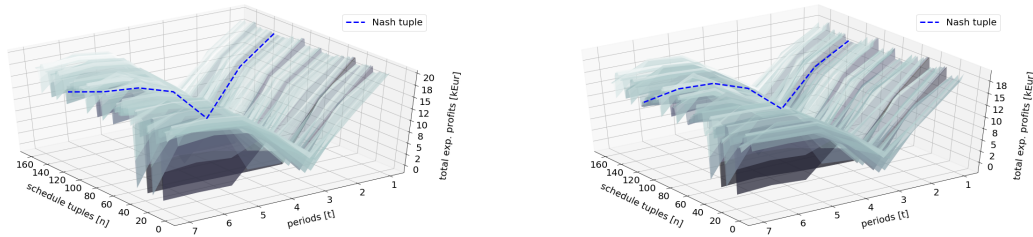
Figure 6: Nash Tuples

(a) Nash Tuple Hydro-Thermal Case   (b) Nash Tuple Thermal-dominated Case

generation capacities for the thermal and hydropower plants were $1300MW$ and $280MW$ respectively.

Solvers. The model was coded in Python, using the multi solver interface Pyomo[32]. This interface provides nonlinear transformations for complementarity problems with continuous variables[33] based on a constrained optimization technique from Ref.[34], enabling the usage of freeware tools such as the nonlinear interior-point solver IPOPT[35]. Performance on an Intel i7-5600U was an average of 80.5 seconds to yield a tuple solution in each case, i.e. to solve the Continuous Game, for both cases running in parallel.

Base Case (Hydro-thermal competition). In the case study, the chosen form of representing uncertainty was using Markov processes transformed into samples. Techniques to obtain such scenario lattices that adequately represent the distribution exist plenty in literature, i.e. Ref.[36]. Therefore, we will assume that an adequate discrete representation of price and inflow distributions are given and can be transformed into scenarios as Fig.4 shows for a three period example. Historical price data was obtained from the public database of Scandinavian market operator Nordpool,

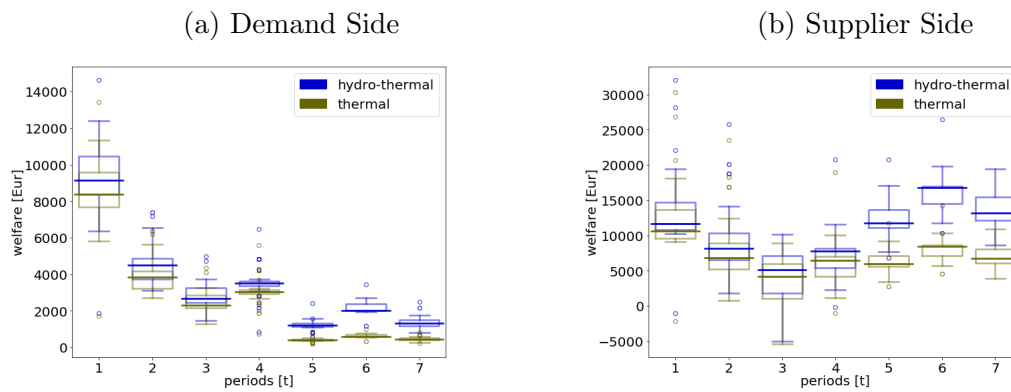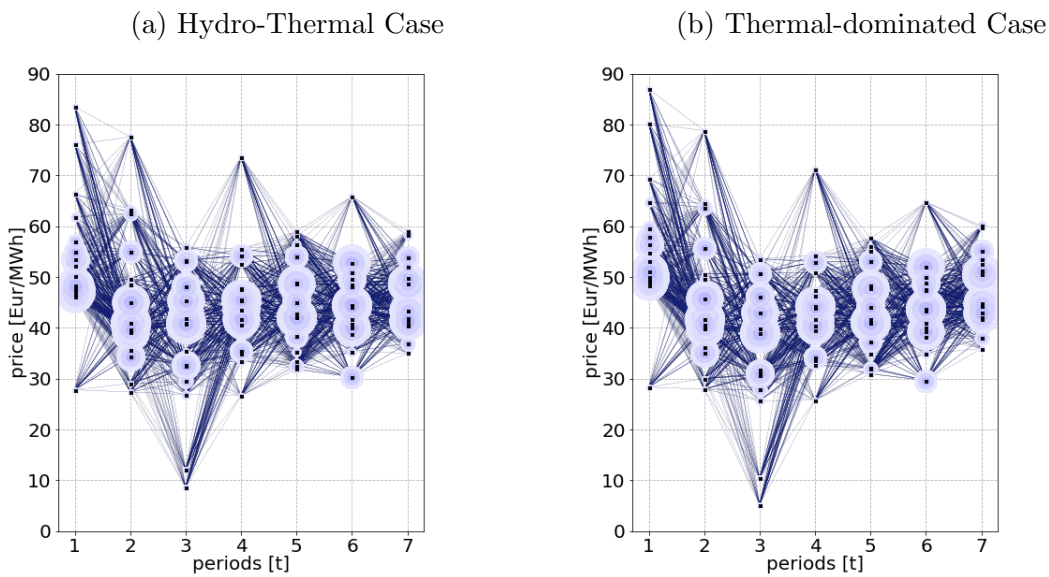13 of 26

Figure 7: Welfare Distribution

(a) Demand Side

(b) Supplier Side

Figure 8: Price Scenarios for Nash Tuples[3]

(a) Hydro-Thermal Case

(b) Thermal-dominated Case

whereas the elasticity of the linear price curves was extrapolated based on volume data[4]. 100 uniformly distributed initial scenarios were obtained and reduced to 15 normally distributed scenarios per time period.

162 schedule iterations(/tuples) of the thermal units were realized, whereas the per-period profits of every iteration as well as the tuple representing the (single) Nash equilibrium in schedules can be found in Fig.6a.

Thermal-dominated competition. For the second case analyzed, the hydropower player holding four out of five units (and thus holding a quasi-monopoly on storage, apart from unit $i = 6$) was removed, leading to a thermal dominated game between two players with a total capacity of $1350MW$ resulting in a different Nash equilibrium tuple found in Fig.6b.

Fig.7 shows that removing the hydropower player has a higher impact on the supply side welfare than the demand side, whereas this effect is amplified in periods

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[4]The elasticity was adjusted accordingly for the thermal-dominated case presented below.

Figure 9: Expected Utilizations of Maximum Unit Capacity

(a) Hydro-Thermal Case (b) Thermal-dominated Case
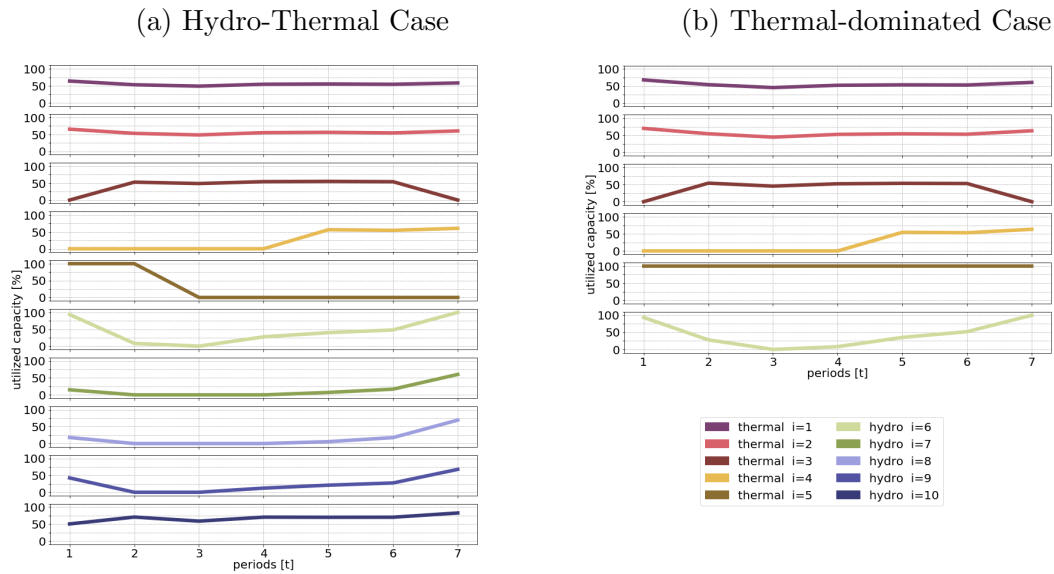
with lower variance in prices. This suggests that peak skimming has not only strong positive effects on customer welfare but also benefits the generators, even though the introduction of additional storage shows dampening effects over the whole spectrum of price scenarios, as Fig.8 illustrates. Paradoxically, as seen in Fig.7b this drop of prices through addition of storage creates positive welfare effects on the supplier side. The reason herein lies in that the hydropower producer holds close to $\frac{3}{4}$ of the supplier profits by being able to generate at infinitesimally low cost. Further, this positive impact on the supplier side seems to be of greater extend than on the demand side, indicating that suppliers profit more from the cost savings of introducing storage technology. This can partially be explained by low elasticities of the price curves, as characteristic for electricity markets[21], but also is a result of the here presented model being able to accurately capture costs related to schedules.

Especially in period 3, another negative effect for the players holding thermal plants can be observed: due to plants aiming to schedule for upcoming high price periods, certain periods shows welfare potentially taking negative values, i.e. generators supplying under loss. In systems with storage inflow from the market (e.g. pumped hydro storage), capacity could be taken and transferred to later stages. In systems like in the here analyzed cases, no such possibility for moving capacity directly exists. Thus the thermal producers are given only two options: reduce the outputs with current schedule or change schedule. Either way, the thermal producers are forced away from their optimal point and punished for inflexibility. Thus, in systems with a large share of storage facilities with natural inflow (traditional hydropower plants) inflexible thermal units might see adverse effects of inflexibility amplified, whereas in systems with market inflow (e.g. pumped hydro storage), such effects might be dampened.

This change in schedule can be observed in Fig.9, which shows the adjustments in the output quantities of player $j = 1$ holding units $i = 1, 2$ and player $j = 2$ holding units $i = 3, 4, 5, 6$. All units apart from $i = 5$ react with slight adjustments of their

Table 3: Fixed Cost $c_{\text{fix}}b_{i,t}(n_i)$ [€]

case	unit	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hydro-Thermal	$i=1$	500	500	500	500	500	500	500
	$i=2$	2000	2000	2000	2000	2000	2000	2000
	$i=3$	0	500	500	500	500	500	0
	$i=4$	0	0	0	0	2000	2000	2000
	$i=5$	850	850	**0**	**0**	**0**	**0**	**0**
	$\sum_{i=1}^{5}$	3350	3850	**3000**	**3000**	**5000**	**5000**	**4500**
Thermal	$i=1$	500	500	500	500	500	500	500
	$i=2$	2000	2000	2000	2000	2000	2000	2000
	$i=3$	0	500	500	500	500	500	0
	$i=4$	0	0	0	0	2000	2000	2000
	$i=5$	850	850	**850**	**850**	**850**	**850**	**850**
	$\sum_{i=1}^{5}$	3350	3850	**3850**	**3850**	**5850**	**5850**	**5350**

Table 4: System Marginal Cost of Capacity

case		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hy.-Th.	marginal unit	$i=3$	$i=4$	$i=5$	$i=5$	$i=5$	$i=5$	$i=3$
	mc_t^{cap} [€/MW]	6.67	14.29	14.17	14.17	14.17	14.17	6.67
Th.	marginal unit	$i=3$	$i=4$	$i=4$	$i=4$	-	-	$i=3$
	mc_t^{cap} [€/MW]	6.67	14.29	14.29	14.29	∞	∞	6.67

∞ relates to no available capacity

output levels, withholding a minor amount of output in the thermal-dominated case and thus causing the price increase mentioned above. Reducing the amount of players leading to resuming players withholding quantity is an expected characteristic of Cournot models. However, in our presented model this effect is reduced by unit $i=5$ switching the schedule in the thermal case in order to produce on maximum generation level. This leads to the conclusion that giving players the option to enter or leave a Cournot game (in our model through scheduling units for the respective time periods) dampens the effects of exercising market power. The impact on the total fixed cost in the system can be found in Tab.3.

This gives the possibility to formulate the system marginal cost of additional capacity by finding the minimum cost for adding an additional MW to the system:

$$mc_t^{\text{cap}} = \min_i \frac{\partial c_{i,t}(q_{i,t},b_{i,t})}{\partial b_{i,t}} / \underline{q}_i \cdot (1 - b_{i,t}(n_i)) + \infty \cdot b_{i,t}(n_i) \tag{10}$$

The results of this for both cases can be found in Tab.4 which show an increase in cost for the case where more thermal capacity is procured.

Table 5: Marginal Cost of Energy[€/MWh]

case	unit	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
Hydro-Thermal	$i=1$	41.95	36.48	34.42	37.24	37.6	37.15	39.14
	$i=2$	37.95	33.61	32.00	34.37	34.72	34.09	36.14
	$i=3$	10	36.48	34.42	37.24	37.6	37.15	10
	$i=4$	15	15	15	15	34.72	34.09	36.14
	$i=5$	∞	∞	23	23	23	23	23
	system	37.95	33.61	32.00	34.37	34.72	34.09	36.14
Thermal	$i=1$	43.99	37.09	32.78	36.17	36.82	36.57	40.36
	$i=2$	39.62	34.10	30.63	33.48	34.06	33.61	37.17
	$i=3$	10	37.09	32.78	36.17	36.82	36.57	10
	$i=4$	15	15	15	15	34.06	33.61	37.17
	$i=5$	∞	∞	∞	∞	∞	∞	∞
	system	39.62	34.10	30.63	33.48	34.06	33.61	37.17

∞ relates to no available capacity

Further, the marginal costs of energy can be derived from Eq.(9):

$$p_{j,t}(\xi, \sum_{i_2 \in I_j} q_{i_2,t} + \sum_{i_2 \notin I_j} q'_{i_2,t}) \rightarrow p_t(\xi, d_t) \forall j \in J, t \in T \tag{11}$$

The results, presented in Fig.5 show that the cost effects of additional storage can not necessarily be found in reduction of cost, as the average marginal cost over all time periods is with 34.70€/MWh in fact slightly higher in the hydro-thermal case than the 34.67€/MWh in the thermal-dominated case. In addition, a flattening effect of storage can be observed. In traditional economic models that do not consider start-up and shut-down decisions, the system marginal costs would be given by $i=4,5$ in the hydrothermal case and by $i=4$ in the thermal case. In the here presented model however, those units are not participating in the market, as they are either not started in the first period (unit $i=4$) or actively decide to withdraw from the market (unit $i=5$ in the hydrothermal case). Thus, our model is able to capture both the indirect impact of changing capacity costs on energy prices as well as on marginal cost of energy. This plays a role regarding the modeling of otherwise homogeneous units. Units $i=1,3$ as well as units $i=2,4$ are assumed to be of similar types regarding cost curves and capacities. However, with different initial states (units $i=1,2$ running from the start of the time frame and units $i=3,4$ being off) and different proposed schedules, the unit commitment and scheduling decisions differ vastly in the Nash equilibrium tuples of both cases, as shown in Fig.9.

These scheduling tuples being Nash equilibria can also be supported by the results of the individual players. Fig.10 shows that player $j=2$ has an economic incentive to switch the schedule on unit $i=5$, with the chosen optimal schedules (taken from the Nash equilibrium tuple) showing a more beneficial outcome for a *risk-neutral* player.

The case studies indicate, that in a system relying on an energy only market, the increasing fixed cost from changes in schedule as presented in Fig.3 and thus the cost of optimal generation capacity are carried by both suppliers and generators.

Figure 10: Profit Distributions of Player $j = 2$ in Thermal-dominated Case (probability distributions indicated in blue)

(a) Optimal Schedules

(b) Schedules from Hydro-Thermal Case
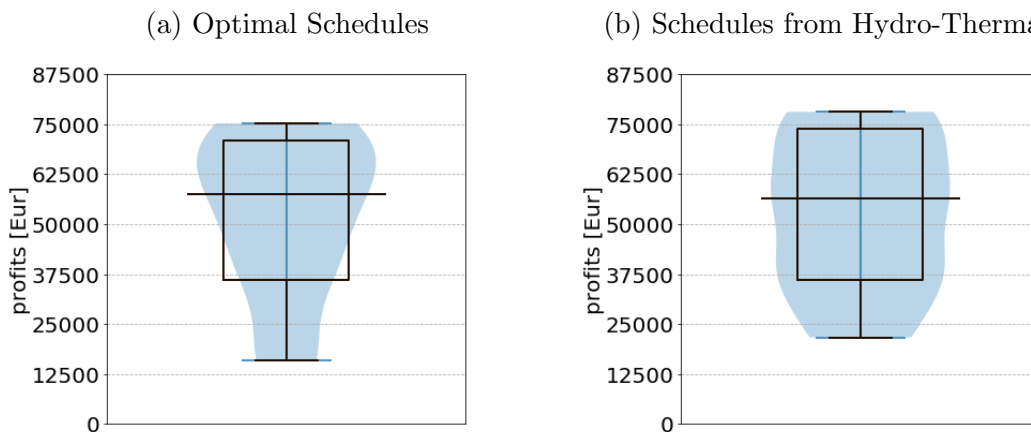
Even in the here presented monopolistic cases, both suppliers and generators share the loss of welfare after losing flexibility (by removing the storage provider from the system).

However, adding additional capacities would require a payment in height of the marginal values presented in Tab.4. The derived marginal costs in the here presented case studies however lie significantly below the observed prices of regulating power in the analyzed area of Nordpool (commonly close to equal to energy prices), indicating high potential profit margins for participants in this market type. However, technical simplifications such as assuming a fixed cost rate instead of start-ups and shutdowns might play a role and thus we advise further research on this topic.

Model Discussion. The proposed approach presents a novelty in literature: solving an equilibrium model with two groups of agents that either actively choose to participate depending on the period or are able to distribute inventory over the time frame. This gives the possibility to quantify the impact of scheduling on market equilibria and thus allow to derive marginal cost of capacity, even for an energy-only market as the example of the case study demonstrates.

Initial tests hint that scalability of the model for larger applications in dimensions of time, player size and scenarios is of satisfying performance, especially considering the original problem is $\mathcal{NP}$-hard. The strictly disconnected nature of the Continuous and the Discrete Game also allows for partial adjustments of the model. Examples for former would be additional nodes and related transfer flows, more than a single energy markets or other uncertain parameters as the ones already considered. Examples for latter would be addition of capacity mechanisms or consideration of unit maintenance. As an extension of the model, we propose further discussion and research on replacing the Continuous Game by a Non-Convex Game and the resulting conditions to keep the Nash tuple equilibria yielded by the Discrete Game valid.

5. Conclusion

The presented base model shows a novel problem setup: players optimizing unit commitment and bidding simultaneously (in form of a Mixed Integer Lin-

ear/Quadratic Problem) whilst competing against each other in a multi-period Cournot market under uncertainty. A nested equilibrium approach is proposed, first finding the Nash equilibria within a decomposed Continuous Game and subsequently comparing the resulting discrete decision tuples to yield what we refer to as Nash tuple equilibria.

Both the proposed Continuous and Discrete Game are complete novelties in literature and allow for a wide range of applications and adaptions. One such application is presented in form of a case study on the Scandinavian power market, where the quantitative influence of removing hydropower storage capacity is analyzed. Without having the hydropower generator in the system, sufficient capacities to conduct 'peak-skimming' are missing, leading to additional thermal unit start-ups. The welfare effects of those are observed and indicate that apart from the demand side also the supplier side is influenced negatively by a lack of storage. In addition, the effects on marginal cost on both continuous and discontinuous variables is calculated, which allows for derivation of a system marginal cost of capacity, even in the presented energy-only market. This allows for a variety of practical applications. In the here presented case study this is shown by being able to analyze the welfare losses of withdrawing flexible (storage) capacity. The results indicate that a loss of flexibility is shared by both suppliers and generators. However, increasing flexibility through additional dispatch over the point of optimal capacity has to be carried by the supply side (as e.g. through capacity or reserve payments to generators). In addition, through the possibility of deriving 'marginal cost of capacity' our results indicate that the approximate 1:1 relation of reserve and spot/intraday prices in Nordpool might be an overestimate in favor of reserve providers.

Acknowledgment

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Appendix A. Market Equilibria under Uncertainty

In order to elaborate on the method used to deal with uncertainty, we will make use of an adjusted one-period example of the previously presented problem. In this example players maximize profits under uncertain output capacities:

$$
\begin{aligned}
\max_{q_i} \quad & \sum_{i \in I_j} p(\xi, \sum_{j \in J} \sum_{i_2 \in I_j} q_{i_2}) q_i - c_i(q_i) \\
\text{s.t.} \quad & 0 \leq q_i \leq \bar{q}_i(\xi) \qquad (\underline{\delta}_i, \bar{\delta}_i)
\end{aligned}
\tag{A.1}
$$

The reader might notice that this resembles the bidding problem of a renewable energy producer such as wind or solar closer than the hydro-thermal example of this paper. This however is intended as the displayed example provides a problem under two different equations - an objective function and an inequality constraint - both under uncertainty. It also has to be noted that the market clearing condition under

symmetric information is already included in the price function by using the other players actual bids q_i instead of using an assumption q_i'.

Assuming the players have symmetric information on the outcome of the uncertain parameter, thus fixing ξ as $\bar{\bar{\xi}}$ allows to formulate the KKT conditions as:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial q_i} = &-\frac{\partial p(\bar{\bar{\xi}}, \sum\limits_{j \in J} \sum\limits_{i_2 \in I_j} q_{i_2})}{\partial q_i} q_i \\
&-p(\bar{\bar{\xi}}, \sum\limits_{j \in J} \sum\limits_{i_2 \in I_j} q_{i_2}) \qquad \forall j \in J, i \in I_j \\
&+\frac{\partial c_i(q_i)}{\partial q_i} - \underline{\delta}_i + \bar{\delta}_i = 0 \\
&0 \leq \underline{\delta}_i \perp -q_i \leq 0 \qquad \forall j \in J, i \in I_j \\
&0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\bar{\bar{\xi}}) \leq 0 \qquad \forall j \in J, i \in I_j
\end{aligned}
\tag{A.2}
$$

Using uncertainty in form of scenarios $\xi \in \Xi$ however might result in complications, especially related to the objective function (which might be caused to not be able to clear). To deal with this issue, Ref.[18] propose introduction of a slack variable into the complementarity conditions. By denoting this variables as $\omega \in \mathbb{R}^i$ and assuming two scenarios ξ_1 and ξ_2 the uncertain generation maximum capacity constraint from the previous example can be reformulated as:

$$
\begin{aligned}
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi_1) + \omega_i \leq 0 \quad &\forall j \in J, i \in I_j \\
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi_2) - \omega_i \leq 0 \quad &\forall j \in J, i \in I_j
\end{aligned}
\tag{A.3}
$$

This shows that the slack variable finds a single solution q_i that minimizes the residuals between the two different scenarios. Reformulation for an open number of scenarios leads to a similar formulation as in Ref.[18]:

$$
\begin{aligned}
0 \leq \bar{\delta}_i \perp q_i - \bar{q}_i(\xi) + \omega_i^\xi \leq 0 \quad &\forall j \in J, i \in I_j, \xi \in \Xi \\
\sum\limits_{\xi \in \Xi} \omega_i^\xi = 0 \quad &\forall j \in J, i \in I_j
\end{aligned}
\tag{A.4}
$$

Below, we extend this formulation by two characteristics.

Appendix A.1. Consideration of Probability Distributions

The original formulation does not explicitly consider probability distributions. Arguably, this could lead to distortions as outlier scenarios would be considered with similar priority as more likely outcomes. In theory, this can be circumvented by adding latter scenarios with a higher rate (i.e. scenario 2 is x times as likely as scenario 1, so add 1 scenario 1 and x scenario 2). In practical applications however, this would lead to an increase in model complexity, as every single additional scenario represents an additional complementarity constraint for each constraint affected by the uncertainty. Thus in this paper we decided to use the probabilities as weighting parameters to replace ω_i^ξ with $\frac{\omega_i^\xi}{P^\xi}$. Thus, the higher the likelihood of an outcome, the lesser the impact of the slack variable on diverging from that scenario. As a result, the solution will be closer to the scenario with higher likelihood and deviate more from the scenario with lower likelihood. It has to be noted, that the proposed formulation does not consider any risk preferences, making all participants *risk-neutral* players.

Appendix A.2. Extension to Lagrangians

The Lagrangians can be relaxed in similar manner to the complimentarity constraints, yielding single solutions for the decision variables that allow clearing the market for objective functions under uncertainty. For the previously defined example this would result in the following KKT conditions:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial q_i} = & -\frac{\partial p(\xi, \sum\limits_{j \in J} \sum\limits_{i_2 \in I_j} q_{i_2})}{\partial q_i} q_i \\
& -p(\xi, \sum\limits_{j \in J} \sum\limits_{i_2 \in I_j} q_{i_2}) \\
& +\frac{\partial c_i(q_i)}{\partial q_i} - \underline{\delta}_i + \bar{\delta}_i + \frac{\omega_i^{p,\xi}}{P^\xi} = 0 \\
& 0 \le \underline{\delta}_i \perp -q_i \le 0 \\
& 0 \le \bar{\delta}_i \perp q_i - \bar{q}_i(\xi) + \frac{\omega_i^{\bar{q},\xi}}{P^\xi} \le 0 \\
& \sum_{\xi \in \Xi} \omega_i^{p,\xi} = 0 \\
& \sum_{\xi \in \Xi} \omega_i^{\bar{q},\xi} = 0
\end{aligned}
\qquad
\begin{aligned}
& \forall j \in J, i \in I_j, \\
& \quad \xi \in \Xi \\
\\
& \forall j \in J, i \in I_j \\
& \forall j \in J, i \in I_j, \\
& \quad \xi \in \Xi \\
& \forall j \in J, i \in I_j \\
& \forall j \in J, i \in I_j
\end{aligned}
\qquad (A.5)
$$

Again, this yields a single solution for the quantity decision of every generation unit and thus allows clearing the market similar to traditional (in this case: Cournot) clearing procedures.

Appendix B. Karush Kuhn Tucker(KKT)-Conditions

The KKT-conditions in extend form are:

$$\frac{\partial \mathscr{L}}{\partial q_{i,t}} = -\frac{\partial p_t(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2,t})}{\partial q_{i,t}} q_{i,t} - p_t(\xi, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2,t}) \quad \begin{aligned}&\forall j \in J, i \in I_j^{Th},\\& t \in T^{\overline{s}}, \xi \in \Xi\end{aligned} \qquad \text{(B.1a)}$$
$$+\frac{\partial c_{i,t}(q_{i,t}, \overline{b_{i,t}})}{\partial q_{i,t}} - \underline{\delta}_{i,t} + \bar{\delta}_{i,t} + \frac{\omega_{i,t}^{p,\xi}}{P^\xi} = 0$$

$$\frac{\partial \mathscr{L}}{\partial q_{i,t}} = -\frac{\partial p_t(\xi, \sum\limits_{j\in J}\sum\limits_{i_2\in I_j} q_{i_2,t})}{\partial q_{i,t}} q_{i,t} - p_t(\xi, \sum_{j\in J}\sum_{i_2\in I_j} q_{i_2,t}) \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& t \in T^{\overline{s}}, \xi \in \Xi\end{aligned} \qquad \text{(B.1b)}$$
$$-\underline{\delta}_{i,t} + \bar{\delta}_{i,t} - \gamma_{i,t} + \frac{\omega_{i,t}^{p,\xi}}{P^\xi} = 0$$

$$\frac{\partial \mathscr{L}}{\partial q_{i,t}^{\overline{s}}} = \sigma_i + \gamma_{i,t} + \sum_{t_2=\underline{s}+1}^{t} \psi_{i,t_2} - \mu_{i,t} = 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1c)}$$

$$0 \le \underline{\delta}_{i,t} \perp \underline{q}_i \overline{b_{i,t}} - q_{i,t} \le 0 \quad \begin{aligned}&\forall j \in J, i \in I_j,\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1d)}$$

$$0 \le \bar{\delta}_{i,t} \perp q_{i,t} - \bar{q}_i \overline{b_{i,t}} \le 0 \quad \begin{aligned}&\forall j \in J, i \in I_j,\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1e)}$$

$$0 \le \sigma_i \perp \sum_{t\in T^{\overline{s}}} q_{i,t}^{\overline{s}} - Q_i^{\overline{s}}(\xi) + \frac{\omega_i^{Q,\xi}}{P^\xi} \le 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& \xi \in \Xi\end{aligned} \qquad \text{(B.1f)}$$

$$\sum_{s=1}^{t} \begin{cases} q_{i,t}^{\overline{s}}, & \text{if } s = \overline{s} \\ \underline{q}_{i,t}^{s}, & \text{if } s \ne \overline{s} \end{cases} - q_{i,t} = 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1g)}$$

$$0 \le \psi_{i,t} \perp \sum_{s=1}^{t-1}\sum_{t_2=t}^{\max(T)} \begin{cases} q_{i,t_2}^{\overline{s}}, & \text{if } s = \overline{s} \\ \underline{q}_{i,t_2}^{s}, & \text{if } s \ne \overline{s} \end{cases} - \bar{R}_i \le 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& (t > \overline{s}) \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1h)}$$

$$0 \le \mu_{i,t} \perp -q_{i,t}^{\overline{s}} \le 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1i)}$$

$$\sum_{\xi\in\Xi} \omega_{i,t}^{p,\xi} = 0 \quad \begin{aligned}&\forall j \in J, i \in I_j,\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1j)}$$

$$\sum_{\xi\in\Xi} \omega_i^{Q,\xi} = 0 \quad \begin{aligned}&\forall j \in J, i \in I_j^{Hy},\\& t \in T^{\overline{s}}\end{aligned} \qquad \text{(B.1k)}$$

Appendix C. Multiple Solutions for Hydropower Commitment

Assumed be a game in two periods $t = 1, 2$ yields a player j holding two hydropower units $i = 1, 2$ an optimal profit of Π_j^* for clearing prices p_1^* and p_2^*. As mentioned above, hydropower units are assumed to operate cost-neutral, thus the optimal profits cannot be decreased by changing commitment decisions as long as $p_t(q_{1,t} + q_{2,t} + \sum_{i_2\notin I_j} q'_{i_2,t}) = p_t(d_t^*) = p_t^*$ holds for both time periods and additional constraints such as reservoir and generation capacities are fulfilled. Assumed there is only a single deterministic inflow in period 1, denoted as Q_i and no end reservoir values are required (thus, the full inflow will be used in the two time periods), the

previous condition can be reformulated as:

$$\begin{array}{ll} q_{1,t} + q_{2,t} = d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} & \forall t = 1,2 \\ Q^i \geq q_{i,1} + q_{i,2} \geq 0 & \forall i = 1,2 \end{array} \tag{C.1}$$

Assuming constant quantities provided by other players, player j can choose i as either 1 or 2 and freely select any quantities $q_{i,t}$ as long as they fulfill:

$$\begin{array}{ll} 0 \leq q_{i,t} \leq d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} & \forall t = 1,2 \\ q_{ii,t} = d_t^* - \sum_{i_2 \notin I_j} q'_{i_2,t} - q_{i,t} & \forall ii \neq i, t = 1,2 \\ Q^i \geq q_{i,1} + q_{i,2} \geq 0 & \forall t = 1,2 \\ Q^{ii} \geq q_{ii,1} + q_{ii,2} \geq 0 & \forall ii \neq i, t = 1,2 \end{array} \tag{C.2}$$

There is a range of potential commitment solutions that fulfill these conditions. They differ in reservoir held over the time stage as well as the periodical utilization of the generation units but yield the same (i.e. the optimal) profits for the player and end up in similar end reservoir values (here = 0).