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# PARAMETERS ESTIMATION OF NONLINEAR MANOEUVRING MODEL FOR MARINE SURFACE SHIP BASED ON PMM TESTS 

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## ABSTRACT

A nonlinear manoeuvring mathematical model of marine surface ship model is briefly introduced. In order to obtain the hydrodynamic coefficients, system identification is used to estimate the parameters using Planar Motion Mechanism (PMM) test data. Usually, the obtained parameters using measured data have a large uncertainty due to the illconditioned processes of identification. An optimal Truncated Singular Value Decomposition (TSVD) method is exploited to reduce the uncertainty of the estimated parameters. The optimal number of retaining singular values is calculated by using the Lcurve, which is a log-log plot of the norm of a regularized solution versus the norm of the corresponding residual norm. It is a graphical tool for displaying the trade-off between the estimated parameters norm and the corresponding residual error norm. The validation process is performed by comparison of the prediction values with the experimental data in the time domain.

## NOMENCLATURE

$v$
Velocity of rigid body, expressed in Body-fixed frame
$\boldsymbol{M}_{R B} \quad$ Rigid-body mass matrix
$\boldsymbol{C}_{R B} \quad$ Rigid-body Coriolis-centripetal matrix Hydrodynamic forces and moments
$X \quad$ Surge force
$Y \quad$ Sway force
$N \quad$ Yaw moment
$\boldsymbol{\alpha} \quad$ Hydrodynamic coefficients
$\mathbf{X} \quad$ Input matrix
y Measurement data
$\hat{y}\left(x, a^{*}\right) \quad$ The added mass in y direction
$\bar{y} \quad$ Mean value of measurement data
$\chi^{2}(a) \quad$ Chi-squared errors
$R^{2} \quad$ The goodness of fit criterion
$\boldsymbol{V}_{y} \quad$ Diagonal matrix of variances of y

| $\boldsymbol{V}_{a^{*}}$ | Error propagation matrix |
| :--- | :--- |
| $\boldsymbol{U}$ | Left-singular vectors |
| $\Sigma$ | Singular values |
| $\boldsymbol{V}$ | Right-singular vectors |

## INTRODUCTION

With the development of numerical computation and simulation power and techniques, the mathematical models are becoming more important in the whole process of marine ship design, and ship manoeuvring and operation. Many mathematical models of marine vessels have been proposed to meet application requirements, such as Abkowitz model [1], MMG [2], Nomoto model [3], vectorial model [4], and 4DOF nonlinear manoeuvring model developed using a Lagrangian approach [5]. These models have different features and are proposed considering the trade-off between the complexity and fidelity. Estimation of the hydrodynamic coefficients is a challenge and an interesting topic [6]. Captive model tests, carried out in a multi-purpose towing tank, is an effective method to measure the hydrodynamic forces and moments from which hydrodynamic coefficients in manoeuvring model can be identified.

System identification has been widely used for mathematical modelling and parameters estimation [7]. In [8], a neural network has been used to identify the nonlinear damping matrix for an underwater vehicle. The method of least squares (LS) is a standard approach for parameter estimation. In [9], the nonlinear viscous damping forces in the horizontal plane of a surface vessel at low speed was estimated using the least square method. In [10], least square method was used to estimate the hydrodynamic coefficients based on Planar Motion Mechanism (PMM) tests. The obtained mathematical model was then used to reproduce the manoeuvring test conducted in full-scale [11]. In [12, 13], the parameters of a nonlinear manoeuvring mathematical models were estimated using a least square method based on free-running model tests, and a classic genetic algorithm was used for minimizing a distance between the reference and recovered time histories.

A genetic algorithm is an intelligent method for solving both constrained and unconstrained optimization problems [14]. The parameters estimated by least square methods are usually largely affected by the noise of training data and it usually leads to non-consistent estimates [15]. In [16], a regularized least-square method was used to solve the hyperparameter estimation problem with large data sets and illconditioned computations. Truncated singular value decomposition (TSVD) [17] is also a good option to solve the ill-conditioned problem of the least square method [18]. The main assumption is to neglect its smallest singular values [19],
because the data corresponding to smaller singular values usually imposes more uncertainty in the process of estimating uncertain parameters.

Extended Kalman filter (EKF) has been used for parameter estimation of a ship motion model. In [20, 21], the extended Kalman filter was used to estimate the parameters of a modified Nomoto model for vessel steering. Fossen et al. [22] proposed an off-line parallel extended Kalman filter algorithm utilizing two measurement series in parallel to estimate the parameters of the dynamic positioning ship model. An adaptive wave filter coupled with a maximum likelihood parameter identification technique was proposed by Hassani et al. [23] and used for dynamic positioning control of marine vessels. Recently, the support vector regression (SVR) has been applied to estimate the hydrodynamic coefficients of a ship model [24]. In [25, 26], Least Squares Support Vector Machines has been applied to model the controller of the marine surface vehicle for path following scenarios based on manoeuvring test.

This paper adopts a 3-DOF nonlinear manoeuvring model for a surface ship from [5, 10] and by applying intelligent data processing techniques identifies the uncertain parameter of the manoeuvring model. The identification procedure uses a data set from a series of PMM tests, carried out by SINTEF Ocean [27] in their multi-purpose towing tank [28] using a scaled ship model of research vessel Gunnerus [29]. Various captive model tests recommended by ITTC [6] have been carried out, such as pure sway, pure yaw and mixed sway and yaw. The hydrodynamic coefficients of the nonlinear manoeuvring model were obtained using a least square method based on PMM test data. Optimal TSVD was used to reduce the uncertainty of the parameters due to the ill-conditioned processes of identification. The goal is to neglect the small singular values and the corresponding columns of the matrix, whose contribution to the solution vector can be dominated by random noise and round-off error in measurements.

Finally, the validation of the obtained nonlinear manoeuvring models is carried out by comparing the outputs of the model with the measured data for a portion of test data that was not used during the identification process. In order to quantify the fit to the data, the statistical metrics, the $\mathrm{R}^{2}$ goodness of fit criterion, was used to demonstrate the accuracy of the obtained model. R2 is the ratio of the variability in the data that is not explained by the model to the total variability in the data.

## NONLINEAR MANOEUVRING MATHEMATICAL MODEL

In this paper a 3-DOF manoeuvring model is considered, and the rigid-body kinetics in 3-DOF can be expressed as [4]:

$$
\begin{equation*}
\boldsymbol{M}_{R B} \dot{\boldsymbol{v}}+C_{R B}(\boldsymbol{v}) \boldsymbol{v}=\boldsymbol{\tau}_{R B} \tag{1}
\end{equation*}
$$

where $\boldsymbol{v}=[u, v, r]^{T}$ represents the velocities decomposed in a body-fixed reference and $\boldsymbol{\tau}_{R B}=[X, Y, N]^{T}$ is the generalized vector of external forces and moments expressed in the bodyfixed frame.

The hydrodynamic forces and moment can be derived by considering zero frequency added mass, Coriolis-centripetal forces, linear lift and drag, cross-flow drag. For a comprehensive survey of nonlinear manoeuvring model of ships, the reader is referred to [5]. As presented in [5, 10], the equations of the hydrodynamic forces and moment can be expressed as follow:

$$
\begin{align*}
& X=X_{\dot{u}}^{0} \dot{u}+Y_{\dot{v}}^{0} v r+\frac{1}{2}\left(N_{\dot{v}}^{0}+Y_{\dot{r}}^{0}\right) r r+X_{u u}^{L} u u+X_{u u u}^{L} u u u \\
& +X_{r v u}^{L} r v u+X_{v v}^{L} v v+X_{r v}^{L} r v+X_{u v v}^{L} u v v+X_{r r}^{L} r r  \tag{2}\\
& +X_{u r r}^{L} u r r+X_{u|v|}^{L} u|v| \\
& Y=Y_{\dot{v}}^{0} \dot{v}+Y_{\dot{r}}^{0} \dot{r}+X_{\dot{u}}^{0} u r+Y_{u v}^{L} u v+Y_{u r}^{L} u r+Y_{u u r}^{L} u u r \\
& +Y_{u u v}^{L} u u v+Y_{v v v}^{L} v v v+Y_{r r r}^{L} r r r+Y_{r r v}^{L} r r v+Y_{v v r}^{L} v v r  \tag{3}\\
& +Y_{v|r|}^{L} v|r|+Y_{v|v|}^{L} v|v|+Y_{r|v|}^{L} r|v|+Y_{r|r|}^{L} r|r| \\
& N=N_{\dot{v}}^{0} \dot{v}+N_{\dot{r}}^{0} \dot{r}+\left(Y_{\dot{v}}^{0}-X_{\dot{u}}^{0}\right) v u+\frac{1}{2}\left(N_{\dot{v}}^{0}+Y_{\dot{r}}^{0}\right) r u \\
& +N_{u v}^{L} u v+N_{u r}^{L} u r+N_{u u r}^{L} u u r+N_{u u v}^{L} u u v+N_{v v v}^{L} v v v  \tag{4}\\
& +N_{r r r}^{L} r r r+N_{r r v}^{L} r r v+N_{v v r}^{L} v v r+N_{v|r|}^{L} v|r| \\
& +N_{v|v|}^{L} v|v|+N_{r|v|}^{L} r|v|+N_{r|r|}^{L} r|r|
\end{align*}
$$

## PLANAR MOTION MECHANISM (PMM) TESTS

A series of captive model tests were carried out by SINTEF Ocean [27] during a research project [30] on the scaled ship model according to the recommended procedures by ITTC [6]. The captive model test is nowadays commonly used to provide data for identification and validation of mathematical models of ship manoeuvring motion. It can provide a reasonable estimation of the hydrodynamic coefficients, however, performing such tests is costly. In this section, a brief summary of different PMM tests is presented, such as pure surge, pure drift, pure sway, pure yaw and mixed sway and yaw, which were carried out in SINTEF Ocean's multi-purpose towing tank [28] using the scaled ship model, presented in Fig. 1. The motions in the surge, sway and yaw were controlled using a 6-DOF hexapod motion platform, which is mounted on the carriage. Each type of test emphasises different dynamic characteristics:

Pure Surge: A pure surge test tows the model forward with oscillations around a fixed velocity. It is usually sinusoidal oscillations. This test aims to achieve the full response of surge motion.

Pure Drift: A pure drift test tows the model forward with a fixed oblique angle. This test is usually used to isolate the static derivatives from yaw motion [10].

Pure Sway: A pure sway test is used to isolate the sway dynamics from the yaw motion. The ship will move forward with a constant velocity and with a sinusoidal oscillation in Sway. This test aims to achieve the full response of sway motion.

Pure Yaw: Similarly, in a pure yaw test, the model will move forward with a sinusoidal oscillation in yaw. The effect of sway can be neglected owing to the zero velocity in sway motion.

Mixed Sway and Yaw: This test was carried out using a ship model at a set of sway velocity and yaw rate. It is a generalization of pure yaw, except the model is held at a nonzero sway [5].


FIGURE 1: PLANAR MOTION MECHANISM TESTS IN TOWING TANK [ Courtesy of SINTEF Ocean [27]]

## OPTIMAL PARAMETER ESTIMATION METHODS AND UNCERTAINTY ANALYSIS

In this section, the optimal parameters estimation methods based on least square and truncated singular value decomposition (TSVD) will be presented. In order to estimate the hydrodynamic coefficients of the nonlinear manoeuvring model discussed in the previous section, the Equations 2 to 4 will be reordered in a vector format given by:

$$
\begin{equation*}
\boldsymbol{X} a=y \tag{5}
\end{equation*}
$$

where the matrix $X \in \mathbb{R}^{n * 38}$ contains the measured data, $a \in \mathbb{R}^{38^{* 1}}$ represents the uncertain parameters described in equation (6), and $y=[X, Y, N]^{T}$ is the matrix of the recorded forces and moments during the tests. In this study, there are 38 parameters to be estimated. Obviously, the linear equation is over-determined $n>m$.

$$
\begin{align*}
a= & {\left[X_{\dot{u}}^{0}, Y_{\dot{v}}^{0}, Y_{\dot{r}}^{0}, N_{\dot{v}}^{0}, N_{\dot{r}}^{0}, X_{u u}^{L}, X_{u u u}^{L}, X_{r v u}^{L}, X_{v v}^{L}, X_{r v}^{L}, X_{u v v}^{L}, X_{r r}^{L}\right.} \\
& X_{u r r}^{L}, X_{u|v|}^{L}, Y_{u v}^{L}, Y_{u r}^{L}, Y_{u u r}^{L}, Y_{u u v}^{L}, Y_{v v v}^{L}, Y_{r r r}^{L}, Y_{r r v}^{L}, Y_{v v r}^{L} \\
& Y_{v|r|}^{L}, Y_{v|v|}^{L}, Y_{r|v|}^{L}, Y_{r|r|}^{L}, N_{u v}^{L}, N_{u r}^{L}, N_{u u r}^{L}, N_{u u v}^{L}, N_{v v v}^{L}  \tag{6}\\
& \left.N_{r r r}^{L}, N_{r r v}^{L}, N_{v v r}^{L}, N_{v|r|}^{L}, N_{v|v|}^{L}, N_{r|v|}^{L}, N_{r|r|}^{L}\right]
\end{align*}
$$

Now, the problem of parameter estimations can be transferred to minimization of the difference between the estimated values $\hat{y}(x ; a)$ and the measured data $y$. In addition, several assumptions need to be made. The first assumption is that the sample of measurements $y_{i}$ are uncorrelated because every measurement is independent. Each measurement $y_{i}$ has a particular variance, $\sigma_{y}^{2}$ due to the environmental disturbance and sensors.

## Optimal parameter estimation using least square method

In order to find the optimal parameters, the residual error, $e=\hat{y}(x ; a)-y$ between the measured data $y$ and the estimated value $\hat{y}(x ; a)$ need to be minimized. Furthermore, the error needs to be dominated by the high-accuracy data (small-variance) and less affected by the low-accuracy data (large-variance). So the weighed sum of the squared residuals, also called 'chi-squared' is defined in terms of the vectors:

$$
\begin{equation*}
\chi^{2}(a)=\{\boldsymbol{X} a-y\}^{T} \boldsymbol{V}_{y}\{\boldsymbol{X} a-y\} \tag{7}
\end{equation*}
$$

where $\boldsymbol{V}_{y}$ is the diagonal matrix of variances of $y$. Usually, if the variances of $y$ is unknown in advance, the variance matrix, $\boldsymbol{V}_{y}$ can be assumed to be the identity matrix. The optimal parameters $a$ corresponds to the minimum value of the $\chi^{2}$ error function, which means the derivative of $\chi^{2}$ respect to the $a$ equals to zero.

$$
\begin{align*}
\left.\frac{\partial \chi^{2}}{d a}\right|_{a=a^{*}} & =0  \tag{8}\\
\boldsymbol{X}^{T} \boldsymbol{V}_{y}^{-1} \boldsymbol{X} a^{*}-\boldsymbol{X}^{T} \boldsymbol{V}_{y}^{-1} y & =0
\end{align*}
$$

Then the optimal values of the parameters $a^{*}$ can be obtained as

$$
\begin{equation*}
a^{*}=\left[\boldsymbol{X}^{T} \boldsymbol{V}_{y}^{-1} \boldsymbol{X}\right]^{-1} \boldsymbol{X}^{T} \boldsymbol{V}_{y}^{-1} y \tag{9}
\end{equation*}
$$

The $\chi^{2}$ error function can be minimized with respect to the parameters $a$. The estimated values, which have the best
agreement with the measured data, can be computed using $\hat{y}\left(x ; a^{*}\right)=X a^{*}$.

## The goodness of fit criterion

The $R^{2}$ goodness of fit criterion is used to measure the goodness of the fitness. It is defined as:

$$
\begin{equation*}
R^{2}=1-\frac{\sum\left[y_{i}-\hat{y}\left(x ; a^{*}\right)\right]^{2}}{\sum\left[y_{i}-\bar{y}\right]^{2}} \tag{10}
\end{equation*}
$$

where $\bar{y}$ is the mean value of the measured data. The $R^{2}$ is the ratio of the variability in the data that is not explained by the model to the total variability in the data. If $R^{2}$ equal to zero, it means that the model fails to explain the measurement variability. Otherwise, if $R^{2}$ equal to 1 , it means that all the variability of measured data can be fully explained by the model. If $R^{2}$ is negative, it means the model can explain the data worse than the mean value.

## Uncertainty analysis of the estimated parameters

The uncertainty analysis of the parameters is important for the mathematical modelling. Usually, poorly identified model (models with large parametric uncertainty) is very sensitive to the disturbance in the input data. Such model cannot reproduce the behaviour of the system with high accuracy. This is due to the fact that the parameters with large uncertainty will change dramatically with the errors in the measured data. In this study, the error propagation matrix or the covariance matrix is used to indicate how the random errors in $y$, as described by $\boldsymbol{V}_{y}$, propagate to the optimal parameter $a^{*}$. The error propagation matrix is given by

$$
\begin{equation*}
\boldsymbol{V}_{a^{t}}=\left[\frac{\partial a^{*}}{\partial y}\right] \boldsymbol{V}_{y}\left[\frac{\partial a^{*}}{\partial y}\right]^{T}=\left[\boldsymbol{X}^{T} \boldsymbol{V}_{y}^{-1} \boldsymbol{X}\right]^{-1} \tag{11}
\end{equation*}
$$

where the standard error of the parameters, $\sigma_{a^{*}}$, can get by calculation of the square-root of the diagonal of the error propagation matrix. Then the absolute error can be calculated easily.

## Optimal truncated singular value decomposition (TSVD)

The uncertainty analysis of the identified parameters in the model is of paramount importance to obtain a robust model. The uncertainty of parameters is affected by noise, and quantified by the error propagation matrix, $\boldsymbol{V}_{y}$. Large uncertainty or covariance of the parameters can be due to noise in data or an ill-conditioned model (or both). The matrix $\boldsymbol{X}$ can be rewritten as

$$
\begin{equation*}
\boldsymbol{X}=\sum_{i=1}^{n} u_{i} \sigma_{i} v_{i}^{T}=\boldsymbol{U} \Sigma \boldsymbol{V}^{T} \tag{12}
\end{equation*}
$$

where the matrix $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthonormal, $\boldsymbol{U}^{T} \boldsymbol{U}=I$ and $\boldsymbol{V}^{T} \boldsymbol{V}=I$. Furthermore, substitution of Eq. (12) into the optimal parameters estimation Eq. (5) give:

$$
\begin{equation*}
a=\sum_{i=1}^{n} \frac{v_{i} u_{i}^{T} y}{\sigma_{i}} \tag{13}
\end{equation*}
$$

As presented in Eq. (14), the smaller singular values can potentially dominate the solutions $a$. The smaller the singular value $\sigma_{i}$ is, the more uncertainty the estimated parameters have. The error in the measurement data, $y$, propagate to the estimated parameters, which can be expressed as follows

$$
\begin{equation*}
v_{i}^{T} \delta a=\frac{u_{i}^{T} \delta y}{\sigma_{i}} \tag{14}
\end{equation*}
$$



FIGURE 2: PLOT OF DISCRETE PICARD CONDITION.

So, in order to get a physically meaningful solution, the SVD coefficients $\left|u_{i}{ }^{T} \delta y\right|$ must decay faster than the $\sigma_{i}$, which is called the Discrete Picard Condition [31]. As presented in Fig.2, the solutions are dominated by the small singular values. So it is necessary to reduce the effect caused by the smaller singular values. In most cases, TSVD is an effective tool to reduce the uncertainty of the data set. The TSVD can be presented as

$$
\begin{equation*}
\boldsymbol{X}_{r}=\boldsymbol{U}_{r} \Sigma_{r} \boldsymbol{V}_{r}^{T} \tag{15}
\end{equation*}
$$

where the matrix $\Sigma_{r}$ is obtained by retaining the first $r$ singular values of $\Sigma$. Similarly, matrices $\boldsymbol{U}_{r}$ and $\boldsymbol{V}_{r}$ are found using the corresponding singular vectors. The resulting $\boldsymbol{X}_{r}$ represents the reduced data set where the data related to the omitted singular values are filtered. It should be noted that truncation of the original matrix will inevitably increase the bias error for the
parameters due to the loss of some information. But the uncertainty of the parameters (parameters drift) can be reduced significantly. As presented in the following section, the bias error for the parameters increases slightly, because the smaller singular values contribute little to the parameters. The optimal value of $r$ can be estimated using the L-curve. It is a log-log plot of the norm of a regularized solution versus the norm of the corresponding residual norm. It is a convenient graphical tool for displaying the trade-off between the size of a regularized solution and its fit to the given data, as the regularization parameter varies $[32,33]$ As presented in Figure 3 in our current data set the optimal $r$ equals to 29 .


FIGURE 3: OPTIMAL TSVD USING L-CURVE

## HYDRODYNAMIC COEFFICIENTS ESTIMATION AND VALIDATION

In this section, the parameter estimation based on the least square method (LS) and optimal truncated singular value decomposition (TSVD) are presented. Firstly, the PMM test data needs to be regrouped to be used as a training set in the identification process. The training set should contain enough information to excite the 3-DOF manoeuvring model (surge, sway and yaw motion). In this paper, the training set contains data collected from surge acceleration, pure drift, pure surge, pure sway and mixed sway and yaw tests. It is built by simply joining all the data in sequence. A small portion of the data was kept for validation. The same process is carried out to construct new data set for validation purpose. In order to assess the performance of the numerical model, the data for validation was not used for training.
In the first phase, the parameter estimation based on least square method have been carried out using training set. The prediction of forces and moments compared with training data is presented in Figure 4. From this figure, the curves fit well with each other, especially for sway force and yaw moments. A similar process was also carried out after treating the data set using the TSVD. The results are presented in Figure 5.

Furthermore, the obtained numerical model also can predict the system response successfully.


FIGURE 4: REGRESSED NUMERICAL MODEL OBTAINED BY LEAST SQUARE METHOD COMPARED WITH TRAINING SET.


FIGURE 5: REGRESSED NUMERICAL MODEL OBTAINED BY TSVD COMPARED WITH TRAINING SET.

The error propagation matrix of the estimated parameters are calculated using Eq.(11). The absolute errors of the estimated parameters based on least square method and TSVD are given in Table 2. From the table, the absolute errors of the parameters estimated using TSVD are reduced compared with the parameters obtained by the least square method. They are more stable and less affected by the error in the measured data. The absolute error of parameters estimated by TSVD is below $23 \%$, except the two parameters and $Y_{u u v}^{L}$, while for the least square method there are 8 parameters, whose absolute error is bigger
than $25 \%$. So TSVD is more robust and reduces the uncertainty for most parameters. Fig. 8 presents the same results graphically. The values of the absolute error of the parameters are presented in the Table 2.


FIGURE 6: VALIDATION OF NUMERICAL MODEL OBTAINED BY LEAST SQUARE METHOD COMPARED WITH TEST SET.


FIGURE 7: VALIDATION OF NUMERICAL MODEL OBTAINED BY TSVD COMPARED WITH TEST SET.

TABLE 1: THE $R^{2}$ GOODNESS OF FIT CRITERION FOR VALIDATION.

| Method | Surge | Sway | Yaw |
| :---: | :---: | :---: | :---: |
| LS | 0.6862 | 0.9981 | 0.9269 |
| TSVD | 0.6894 | 0.9981 | 0.9220 |

TABLE 2: THE ABSOLUTE ERROR (\%) OF THE ESTIMATED PARAMETERS USING LEAST SQUARE AND TSVD.

| NUM. | COEF. | TSVD | LS | NUM. | COEF. | TSVD | LS | NUM. | COEF. | TSVD | LS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $X_{u}^{0}$ | 5.55 | 5.17 | 15 | $Y_{u v}^{L}$ | 2.92 | 2.75 | 27 | $N_{u v}^{L}$ | 1.46 | 1.36 |
| $\mathbf{2}$ | $Y_{v}^{0}$ | 0.53 | 0.49 | 16 | $Y_{u r}^{L}$ | 2.28 | 8.45 | 28 | $N_{u r}^{L}$ | 0.26 | 1.00 |
| $\mathbf{3}$ | $Y_{r}^{0}$ | 15.18 | 13.94 | 17 | $Y_{u u r}^{L}$ | 21.72 | 20.15 | 29 | $N_{u u r}^{L}$ | 2.34 | 1.43 |
| $\mathbf{4}$ | $N_{v}^{0}$ | 0.38 | 0.35 | 18 | $Y_{u u v}^{L}$ | 86.59 | 62.54 | 30 | $N_{u u v}^{L}$ | 0.82 | 0.76 |
| $\mathbf{5}$ | $N_{r}^{0}$ | 0.11 | 0.10 | 19 | $Y_{v v v}^{L}$ | 4.44 | 4.26 | 31 | $N_{v v v}^{L}$ | 0.93 | 0.87 |
| $\mathbf{6}$ | $X_{u u}^{L}$ | 16.53 | 17.29 | 20 | $Y_{r r r}^{L}$ | 8.23 | 55.05 | 32 | $N_{r r r}^{L}$ | 2.32 | 4.98 |
| $\mathbf{7}$ | $X_{u u u}^{L}$ | 19.15 | 20.87 | 21 | $Y_{r v v}^{L}$ | 4.39 | 14.09 | 33 | $N_{r v v}^{L}$ | 1.11 | 10.17 |
| $\mathbf{8}$ | $X_{r v u}^{L}$ | 22.82 | 21.31 | 22 | $Y_{v v r}^{L}$ | 4.20 | 56.63 | 34 | $N_{v v r}^{L}$ | 0.26 | 1.59 |
| $\mathbf{9}$ | $X_{v v}^{L}$ | 19.25 | 17.56 | 23 | $Y_{v\|r\|}^{L}$ | 4.40 | 135.81 | 35 | $N_{v\|r\|}^{L}$ | 1.11 | 2.96 |
| $\mathbf{1 0}$ | $X_{r v}^{L}$ | 5.44 | 5.07 | 24 | $Y_{v\|v\|}^{L}$ | 7.05 | 6.35 | 36 | $N_{v\|v\|}^{L}$ | 1.29 | 1.22 |
| $\mathbf{1 1}$ | $X_{u v v}^{L}$ | 14.39 | 12.82 | 25 | $Y_{r\|v\|}^{L}$ | 3.85 | 19.21 | 37 | $N_{r\|v\|}^{L}$ | 0.24 | 2.05 |
| $\mathbf{1 2}$ | $X_{r r}^{L}$ | 17.05 | 26.86 | 26 | $Y_{r\|r\|}^{L}$ | 3.63 | 221.84 | 38 | $N_{r\|r\|}^{L}$ | 0.47 | 5.77 |
| $\mathbf{1 3}$ | $X_{u r r}^{L}$ | 15.54 | 65.10 |  |  |  |  |  |  |  |  |
| $\mathbf{1 4}$ | $X_{u\|v\|}^{L}$ | 200.01 | 106.93 |  |  |  |  |  |  |  |  |



FIGURE 8: THE UNCERTAINTY OF THE ESTIMATED PARAMETERS

The performance of both numerical models needs to be verified. The manoeuvring model is validated if the model can approximate the measured force and moments of the validation data set with high accuracy. The fit of the models obtained by least square and TSVD are presented in Figure 6 and 7. From the figures, both models work well and can successfully predict the test data. The $R^{2}$ goodness of fit criterion is summarized in Table 1. From this table, the two methods have almost equal accuracy in predicting the test data, However, figure 8 suggests that the TSVD can provide more stable parameters with less impact from the measurement noise.

## CONCLUSIONS

In this paper, a nonlinear manoeuvring mathematical model of a surface ship model in 3-DOF was revisited. The hydrodynamic coefficients were obtained using a least square method based on PMM test data. Usually, the obtained parameters using measured data have a large uncertainty and are largely affected by the noise in the measured data. Furthermore, the identification using least square method for such problems is usually ill-conditioned. In this study, by using an optimal truncated singular value decomposition (TSVD) approach, the size of data set was reduced. Identification of the uncertain parameters using the reduced data set resulted in better estimating the parameters with smaller covariance. The performance of the resulted nonlinear manoeuvring models was further tested against the portion of the data, which was not used in the identification process. The $R^{2}$ goodness of fit criterion was used to demonstrate the accuracy of the obtained models and TSVD algorithm proved to be able to provide more stable and accurate parameters based on the PMM data.

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