

1 **Parallel Channels' Fracturing Mechanism during Ice**
2 **Management Operations. Part II: Experiment**

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1 **Abstract**

2 During ice management operations, cutting parallel channels with narrow spacing in the ice using icebreakers can
3 effectively reduce the size of ice floes reaching the protected vessel/structure. A narrow channel spacing width
4 generates smaller ice floes. However, a spacing that is too narrow can lead to excessive or even impractical ice
5 management operations. Therefore, it is beneficial to establish a theoretical model that correlates the channel
6 spacing width with the frequency of ice fracturing events and the reduction of managed ice floe sizes. This is
7 achieved in the current study with two sequential papers, i.e. Papers I and II. In Paper I, a theoretical model
8 involving an ‘edge crack model’ was formulated to predict the following conditions for an ice management
9 operation: 1) the maximum ice floe with size L_{MCD} that can be produced; 2) the maximum channel spacing width
10 h_{max} beyond which long cracks will not develop between the channels; and 3) the required force to initiate long
11 cracks between parallel channels. In this paper (Paper II), we describe two dedicated ‘parallel channel tests’
12 conducted separately on September 26th and 29th in 2015 during an expedition to the Arctic Ocean (around 82° N
13 and 16° E) with the icebreakers Oden and Frej. The tests had ‘well-controlled’ channel spacing in each test run.
14 Several different channel spacing values were tested with the Oden and the consequent fracturing information was
15 documented by camera images. Image processing enabled us to extract information, such as maximum floe sizes
16 and floe size distributions, given different channel spacing widths. In addition, the ship’s data, such as ship velocity
17 and propulsion history, enabled us to validate the theoretical model’s capability to predict the onset of long cracks
18 between two parallel channels. Despite uncertainties (e.g., non-uniform ice thickness, fracture properties of sea
19 ice, etc.) involved in the tests, favourable comparisons between the experimental results and the theoretical
20 predictions were achieved. Both the theoretical model and experimental results help clarify the parallel channel
21 fracturing mechanism.

22

23

24 **Keywords:**

25 Ice management operations; Parallel channels; Ship–ice interactions; Image Processing; Multi-body Dynamics;

26

27 **1 Introduction**

28 One of the primary goals for icebreakers during an ice management operation is to break the incoming ice features
29 with maximal efficiency. In this regard, fracturing sea ice plays an important role. In principle, one may increase
30 the number of icebreakers or employ more powerful and versatile ones to increase the overall fracturing efficiency.
31 Tactically, one must consider the given icebreaking resources to design an optimal icebreaking pattern that
32 incorporates the varying environmental factors, e.g., ice drifting speed, directional variation and ice properties,
33 such that 1) the ice floe size reduction is under the design threshold; and 2) the protected asset is within the created
34 channel. Practically, for effective and uniform downstream floe size reduction, the deployed icebreakers are
35 expected to follow a systematic track pattern and tackle all ice floes that are encountered, no matter the size
36 (Hamilton et al., 2011a). Depending on ice drifting speeds, directional variations, and ice properties, the optimal
37 shape of the icebreaker's track pattern varies. Despite this variety, most of the track patterns create successive
38 'parallel' channels in the ice field. In between two successive parallel channels, long cracks usually form and
39 effectively lead to floe size reductions.

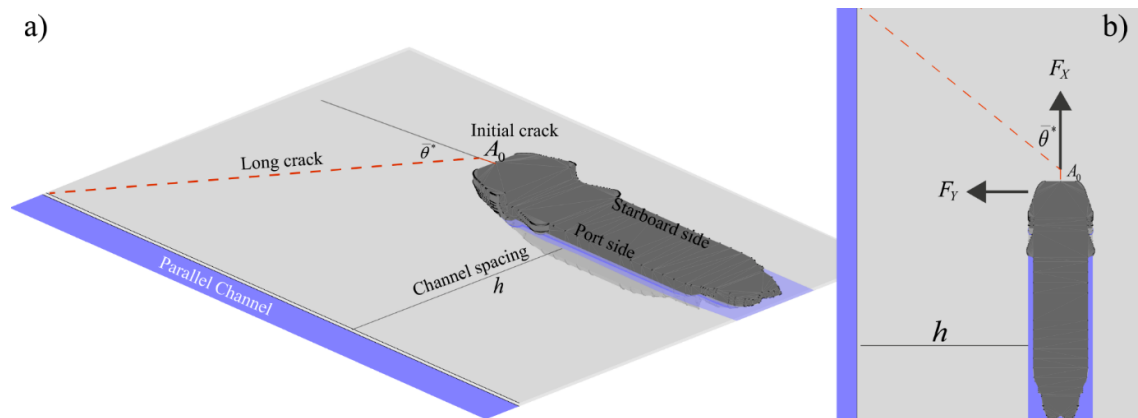
40 This parallel channel fracturing phenomenon is important to consider while planning the deployment of a fleet of
41 icebreakers for an ice management operation. It directly influences the produced ice floe sizes downstream of the
42 icebreakers. For example, Hamilton et al. (2011a, 2011b) developed a numerical simulator to quantify the
43 effectiveness of an ice management operation (e.g., downstream floe size distribution). Within the simulator, the
44 outgoing floe size was assumed to be equivalent to the spacing between two successive parallel channels (i.e.,
45 produced floe size L : Channel spacing $h \approx 1:1$). Similar floe size distributions were also observed in other ice
46 management trials (Farid et al., 2014, Lubbad et al., 2012, Lubbad et al., 2013). However, to the authors'
47 knowledge, until now, there are no existing theoretical formulations with well-controlled tests in relation to the
48 parallel channel fracturing mechanisms that are reported in the literature.

49 Therefore, in a series of two papers, we strive to offer both a theoretical explanation (Paper I) and experimental
50 validation (Paper II) of the observed parallel channel fracturing mechanism. In Paper I, a theoretical model
51 concerning an edge crack's propagation was proposed and formulated to predict 1) the maximum channel spacing
52 h_{\max} beyond which long cracks cease to develop; and 2) the maximum ice floe size L_{MCD} that can be generated
53 between two parallel channels. In this paper (i.e., Paper II), we report two dedicated field tests during the Oden
54 Arctic Technology Research Cruise in 2015 (OATRC2015) in the Arctic Ocean (Lubbad et al., 2016). With the
55 icebreaker Oden, the first parallel channel test was carried out on September 26th from 13:00 to 14:00 (unless

56 otherwise stated, UTC time is used in this paper). Valuable information from helicopter images was collected
57 during this first test. The second parallel channel test was conducted on September 29th from 08:00 to 11:30, which
58 showed well-controlled channel spacing. During the second test, cracks propagating between two successive
59 parallel channels were documented by a dedicated on-board camera. This paper focuses on the test description,
60 data interpretation, comparison with proposed theoretical models, and fracture mechanism discussion.

61 2 Background

62 Before we proceed to the detailed test descriptions, it is beneficial to provide a brief description of the theoretical
63 development carried out in Paper I to reveal the important parameters that are to be measured for validation
64 purposes. To explain the mechanism of long crack formation, Paper I presented a theoretical model regarding the
65 ship–ice interaction with the presence of a neighbouring free channel. This model is re-sketched in Fig. 1, with
66 Oden as an example. Fig. 1a illustrates the major parameters utilised in the proposed interaction model. Fig. 1b
67 illustrates the force components F_X and F_Y that are applied on the ice sheet through contact with the ship's bow.
68 The influence from both force components are considered in the proposed theoretical model.



69

70 Fig. 1. Interaction model proposed for the parallel channels' fracture mechanism, with Oden as an example, a) perspective view with major
71 geometric definitions; b) top view with the force components applied to the ice sheet.

72

73 Following the interaction model in Fig. 1, two groups of equations were developed. The first group of equations
74 is re-written in Eq. (1), in which the relationship between the channel spacing h and the contact forces F_X and
75 F_Y are established by an 'edge crack model' (detailed information can be found in Paper I).

$$\begin{aligned}
 h &= \left(\frac{F_X}{tK_{IC}}\right)^2 [f_1^2\left(\frac{A_0}{h}, \beta_{YX}\right) + f_2^2\left(\frac{A_0}{h}, \beta_{YX}\right)] \\
 \beta_{YX} &= \frac{F_Y}{F_X} \\
 f_1\left(\frac{A_0}{h}, \beta_{YX}\right) &= \sqrt{\frac{4\pi}{\pi^2 - 4} \left(\frac{A_0}{h}\right)^{-1/2} \left(\beta_{YX} - \frac{1}{\pi}\right) + \frac{2.0284(A_0/h) + 2.9890}{(A_0/h)^{-1.3569} + 1.0499} \beta_{YX} + \frac{1}{2} \frac{0.1856 - 0.2174(A_0/h)^{0.1635}}{(A_0/h)^{-6.2861} + 0.4092(A_0/h)^{0.1635}} } \\
 f_2\left(\frac{A_0}{h}, \beta_{YX}\right) &= \frac{1.8134(A_0/h) + 0.5498}{(A_0/h)^{-1.4702} + 1.2041} \beta_{YX} + \frac{1}{2} \frac{0.4051 - 1.3238(A_0/h)^{0.4339}}{(A_0/h)^{-0.9953} + 1.0099(A_0/h)^{0.4339}}
 \end{aligned} \tag{1}$$

76 In Eq. (1),

t is the ice thickness [m];

A_0 is the initial crack length, which can be approximated by $A_0 = 2\ell$;

$\ell = \sqrt[4]{D/k}$ is the characteristic length expressed by the flexural rigidity $D = Et^3/[12(1-\nu^2)]$ and fluid foundation stiffness $k = \rho_w g$ (with more details presented in Paper I);

K_{IC} is the fracture toughness of sea ice [$\text{kPa}\sqrt{\text{m}}$];

$f_i\left(\frac{A_0}{h}, \beta_{YX}\right)$ are functions dependent on the ratio of A_0/h and the interaction force ratio β_{YX} ; they were fitted from numerical simulations together with theoretical asymptotic analysis.

78 Eq. (1) quantifies the minimum force F_X that is required to propagate the existing crack that may eventually lead
79 to a long crack between the parallel channels with spacing h . Therefore, in the designed tests, it is important to
80 measure/control the channel spacing, document the occurrences/absence of long cracks, and provide an account
81 of the available contact force F_X .

82 The second group of equations characterises the parallel channels' fracturing path in a simplified fashion with an
83 easy-to-implement numerical recipe presented in Paper I (Table 1), to identify an intermediate variable L'_{\max} ,
84 which is derived further to the variable L_{\max} and maximum floe size L_{MCD} according to Eq. (2), as follows:

$$\begin{aligned}
 L_{\max} &= f_3(P/Q) \cdot L'_{\max} \\
 f_3(P/Q) &= 0.0334P/Q + 1.1813 \\
 L_{MCD} &= \sqrt{4(L_{\max}h)/\pi}
 \end{aligned} \tag{2}$$

86 in which, L'_{\max} is calculated with the numerical recipe in Table 1 of Paper I, taking into account the idealised crack
87 tip's Stress Intensity Factors (SIFs), which further determines the crack's kink angle; $f_3(P/Q)$ is function
88 obtained by fitting the numerical results to scale L'_{\max} up; L_{MCD} is derived following the conventional definition

89 of a Mean Calliper Diameter (MCD) of an ice floe. It represents the diameter of a disk which has the same area of
90 a geometry of irregular shape.

91 A further exploitation of Eq. (2) leads to the formulation of the maximum floe size ratio between two parallel
92 channels, as shown in Eq. (3).

$$93 \quad \text{Maximum floe ratio} = L_{MCD} / h = \sqrt{4(L_{\max} / h) / \pi} \quad (3)$$

94 To verify Eqs. (2) and (3), the paths of the developed long cracks and floe size ratio between the parallel channels
95 should be documented in the tests. This is achieved via the different camera systems that were employed in the
96 tests.

97 **3 Test Description**

98 Bearing in mind the relationships among the important parameters stated above, we carried out ‘parallel channel
99 tests’ during OATRC2015. Two icebreakers (Oden and Frej) were employed during the research cruise in the
100 Arctic Ocean (around 82° N and 16° E). Both icebreakers are heavily instrumented for various research activities
101 (Lubbad et al., 2016). Performing the ice management trials using a range of supporting hardware and software
102 were among the prioritised activities (Shafrova et al., 2016, Hamilton et al., 2016, Matskevitch et al., 2016).
103 Because most ice management operations involve the creation of parallel channels, we were privileged with an
104 abundant amount of data for the evaluation of their fracturing mechanisms. In particular, among all these ice
105 management operations, two dedicated ‘parallel channel tests’ were carried out by the icebreaker Oden with strictly
106 controlled channel spacing h and a continuous logging of ship data (i.e., ship’s position, inertia, propulsion and
107 wind data).

108 **3.1 Channel Spacing and Ship Data**

109 During the test, the channel spacing h was measured by a laser rangefinder with an accuracy to meters; and was
110 maintained by the captain to his best capability. Even though the spacing was prescribed as a constant value while
111 planning the tests, it is rather challenging to maintain an accurate and consistent channel spacing due to practical
112 limits imposed by varying ice drifting speeds and directions and inhomogeneous ice features. During the tests, the
113 actual channel spacing values were logged constantly. For different test runs, the channel spacing was varied, and
114 the fracturing events were documented by a series of cameras to be described. These cameras yield information
115 such as crack paths (i.e., by the on-board camera) and floe size distributions (i.e., by the helicopter camera).

116 At the same time, ship data such as ship's position, inertia, propulsion and wind data were also continuously stored
117 with conventional instruments. These data are utilised to back calculate the global ice force encountered by the
118 icebreaker during the tests. Detailed instrumentation information along with the adopted method are presented in
119 Section 4.2.

120 3.2 On-board Camera

121 Among all the installed camera systems, a specific 'parallel channel camera' at the port side of Oden (see Fig. 2)
122 was employed to visually document the fracturing events during the tests. The camera is Pan-Tilt-Zoom (PTZ)
123 enabled, meaning that we can remotely control its pan, tilt angle and in/out zoom. A consistent recording frequency
124 of 1 fps was used throughout the duration of the test. Note here that we are not studying the crack propagation
125 process in great detail; it is only the crack occurrence frequency, propagation length and directions that are of
126 interest. Therefore, a higher image rate is not needed to track the details of each crack.

127 After a careful selection of camera parameters and considerations regarding installation locations, we managed to
128 remotely adjust the camera before each test to capture both the neighbouring channel (from 30~300 m apart) and
129 a part of the ship's bow as a reference (see the coverage in Fig. 3). Additionally, important parameters, such as
130 camera viewing angle and tilt angle, are available as outputs for each image. This enables us to quantify each
131 individual crack's propagation distance and direction towards the neighbouring parallel channels. The schematic
132 set-up of the on-board camera's coverage and a sample image with a channel spacing of approximately 80 m is
133 illustrated in Fig. 3.

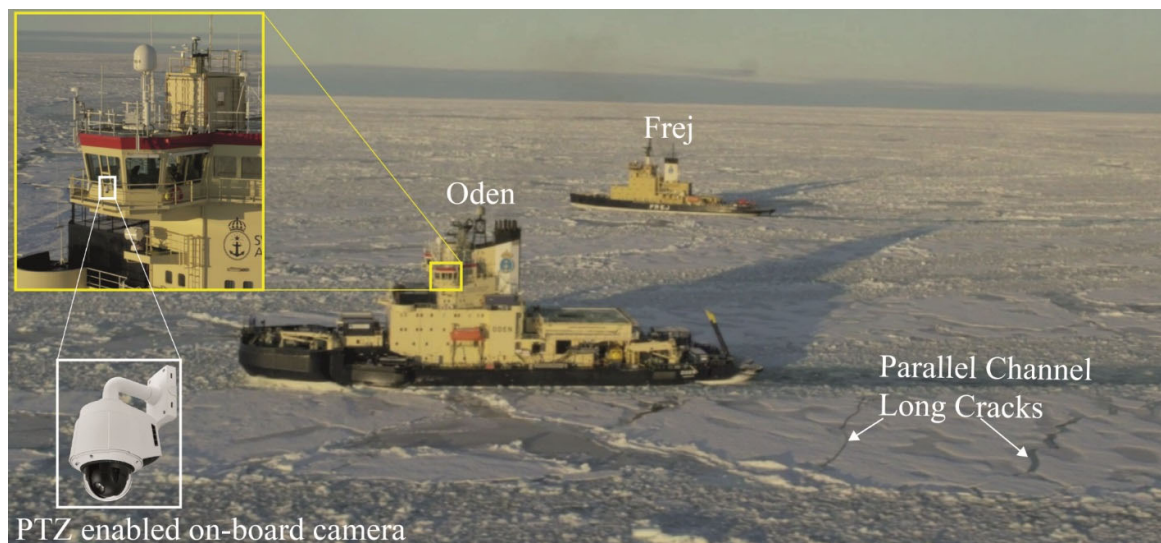
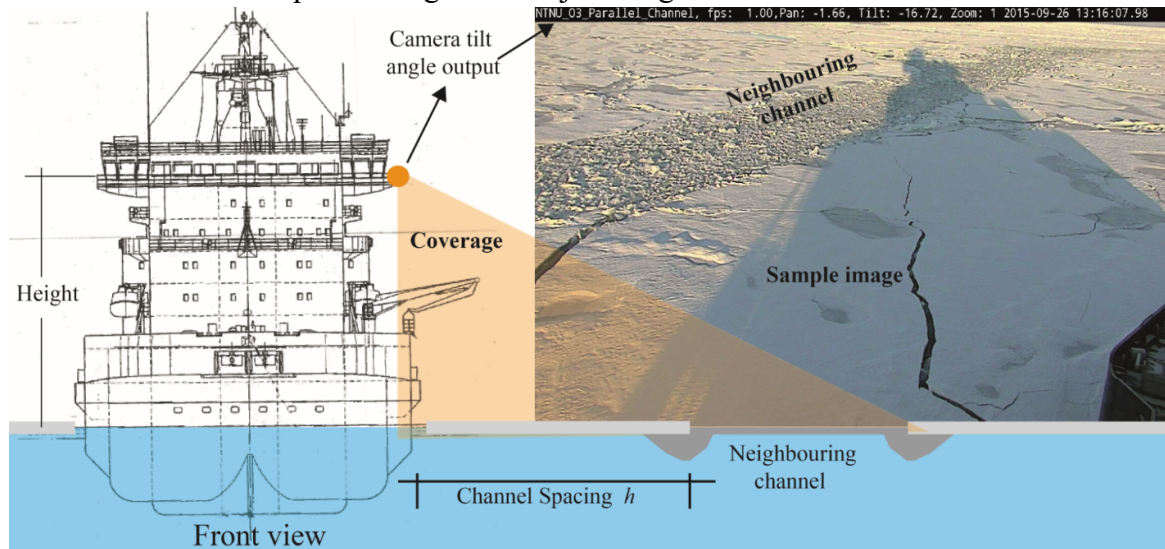


Fig. 2. On-board camera installation location and the overall test environment.



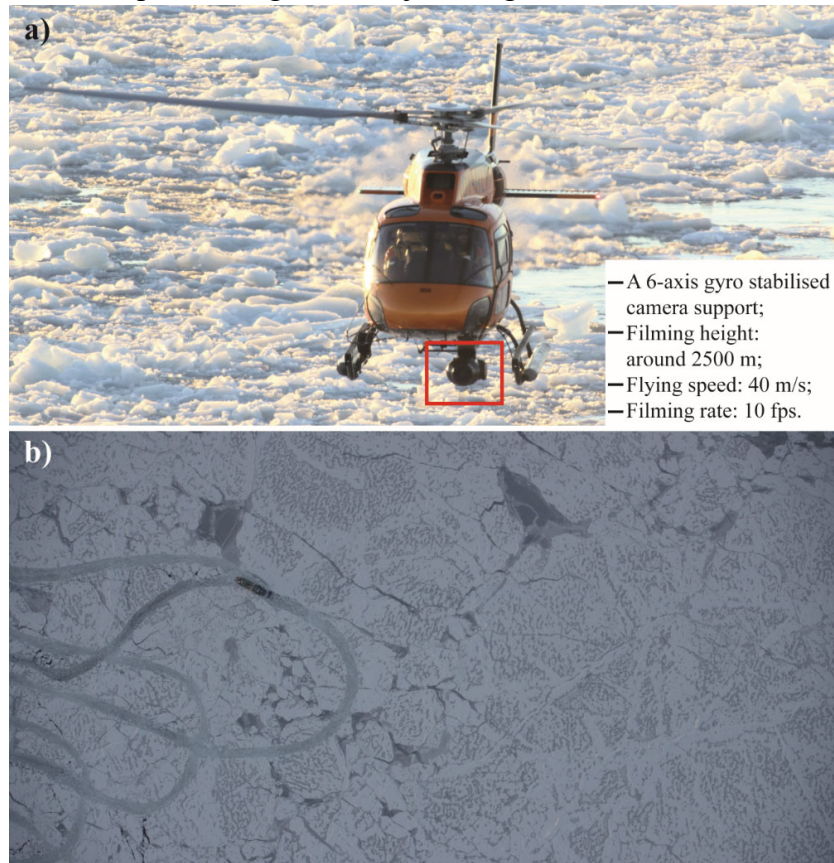
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Fig. 3. Schematic illustration of the camera coverage and a sample image.

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140 3.3 Helicopter Camera Images

141 During the expedition, an AS-335NP helicopter based on Oden was employed to capture images over the ice field
142 with a camera system before and after each ice management operation. The camera system consists of a camera
143 support (i.e., ShotOver F1, see Fig. 4a) and a Red Dragon camera with a Fujinon 25–300 mm lens. The images
144 taken by the helicopter camera are available for one of the parallel channel tests. The parallel channel tests can
145 also benefit from these images for evaluating the actual test track on the ice and floe size distributions in between
146 two channels. The camera's installation location and filming conditions are illustrated in Fig. 4a, and a sample
147 image is displayed in Fig. 4b. All the images are enriched with real-time information, such as latitude, longitude,
148 and the camera's filming parameters (i.e., Pan, Tilt and Roll angles), which are useful for later image stitching.



149

150

Fig. 4. Helicopter camera and a sample image.

151 As opposed to the on-board camera, which dynamically records the cracking events, the helicopter images, having
152 been stitched together, can statically illustrate the overview of the test results with reasonably fine details (i.e.,
153 with discernible crack paths).

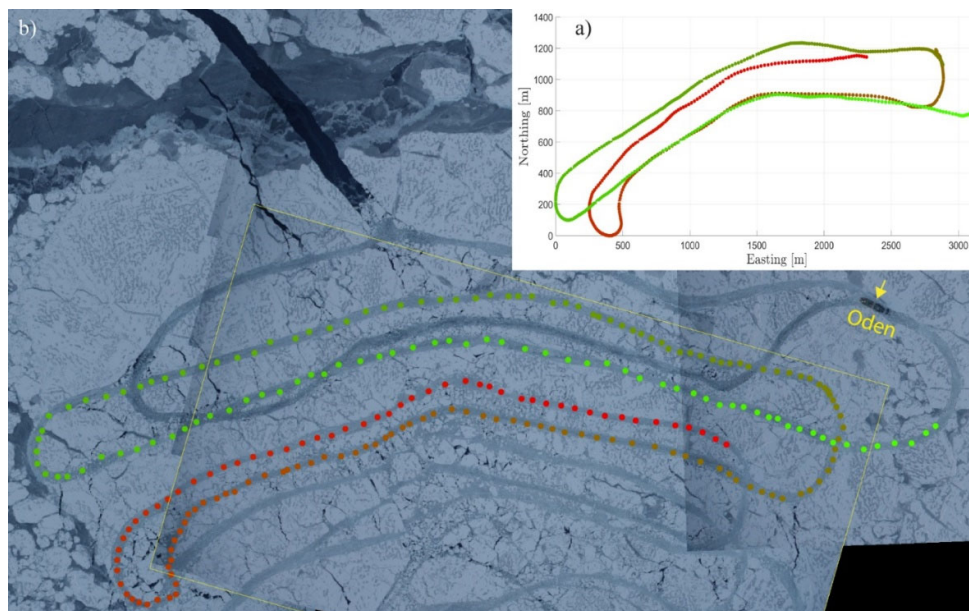
154 3.4 Parallel Channel Tests

155 We carried out three parallel channel tests during the cruise using the icebreaker Oden. These tests were usually
156 conducted at the end of an ice management trial. The first, Test #0, lasted for only 30 minutes and was considered
157 an initial trial. We only report the last two tests (i.e., Tests #1 and #2) in this paper.

158 Regarding the ice condition, a relatively more uniform ice sheet extending over a large area (or the so-called level
159 ice) would have been ideal to validate the proposed theoretical model. However, in both of our tests, it was mainly
160 first-year ice with melt ponds on discontinuous ice floes which are frozen together. The ice thickness is not uniform,
161 but varies between 0.6 m to 1.5 m in the tested region. Pre-existing but refrozen or newly opened leads/cracks are
162 also present. These discontinuous features would undoubtedly influence the occurrence and directional preference
163 (at least locally) of the parallel channel crack. Though clouded with the presence of such inhomogeneity in the ice

164 field, we are still able to identify the free boundary influences from the neighbouring channel on the fracturing
165 events.

166 Test #1 was performed from 13:00 to 14:00 on September 26th, 2016. The track of Oden, with reference to the
167 ground, is plotted in Fig. 5a. The track starts with a red colour and gradually changes to green along the plotted
168 course. The same track, but with reference to the ice, is also highlighted in the stitched helicopter images taken
169 immediately after the completion of Test #1 (see Fig. 5b). Compared to common ice management operations (see
170 the tracks without colour in Fig. 5), the dedicated parallel channel tests have a relatively stricter channel spacing
171 requirement, and the new channel is created as parallel as practically possible to the neighbouring ones. The
172 purpose of this is to reproduce, as close as possible, the boundary conditions of the theoretical model proposed in
173 Paper I.



174
175 Fig. 5. Test #1: a) track over ground; b) track over ice (the track starts with the red colour and gradually changes to green along the course).

176 With accumulated experience, Test #2 was performed from 08:00 to 11:30 on September 29th, 2016. The initially
177 planned track is plotted in Fig. 6a, with the intended parallel channel fracturing events exposed to Oden's portside,
178 where the on-board camera is installed. Unfortunately, no immediate helicopter flights were arranged after the test,
179 and therefore, no useful helicopter images are available to study the floe size distribution of Test #2. During the
180 test, a northward ice drifting speed of approximately 0.18 m/s to 0.2 m/s was logged at 09:38. After some trial
181 plots, by assuming a uniform ice drifting speed of 0.15 m/s northward, we managed to produce Fig. 6b as an
182 approximate ship track over the ice. This approximated track corresponds well with the satellite image (in its
183 original form in Fig. 7) covering the area where Test #2 was performed.

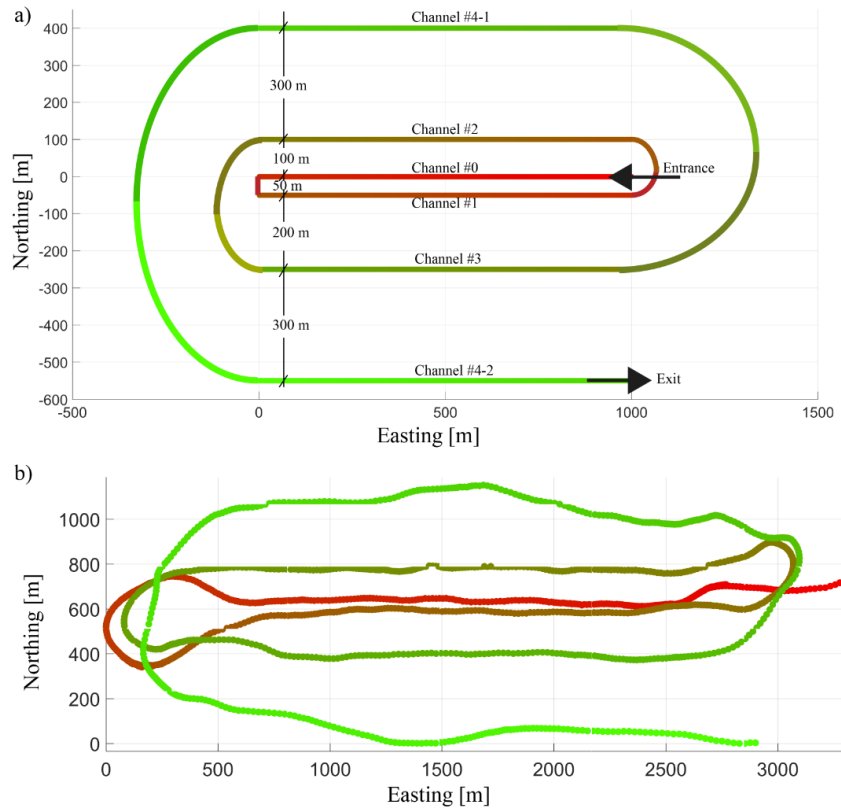
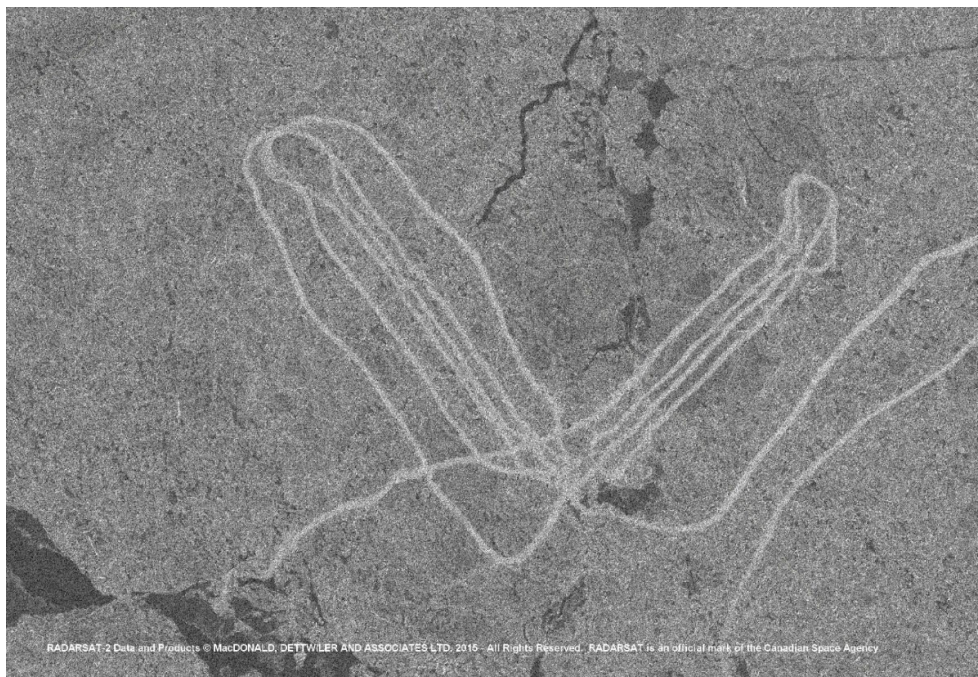


Fig. 6. Test #2: a) planned track; b) estimated ship track over ice, assuming a uniform northward ice drifting speed of 0.15 m/s.

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Fig. 7. Actual ship track over ice of Test #2 (the left one) with reasonable resemblance to Fig. 6b.

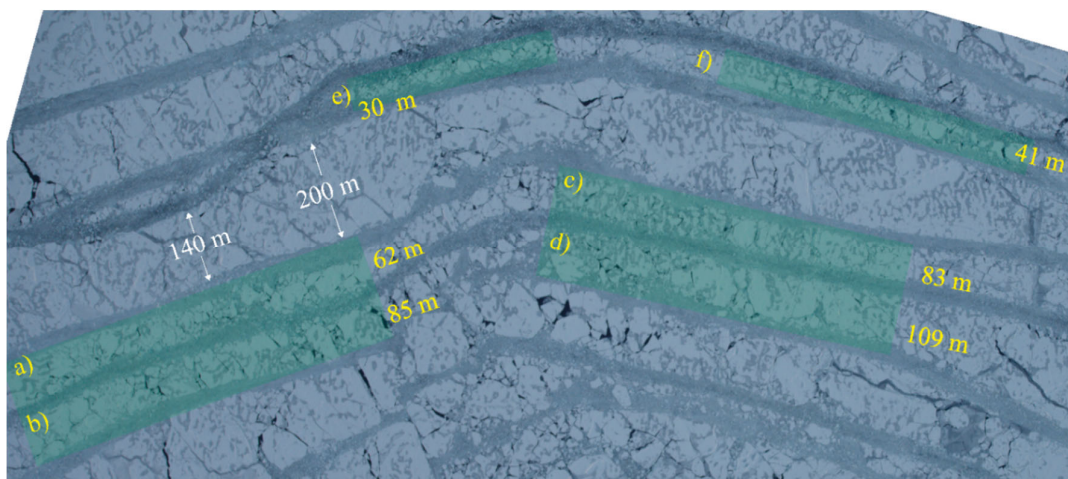
188 **4 Methods**

189 Most of the collected data are in the form of images. Image analysis algorithms have been developed to extract
190 values from the images, such as the floe size (L , defined in Eq. (4)), floe ratio (i.e., L/h) and crack paths (Lu et
191 al., 2016). This paper extends the analysed results further to examine their statistical properties. Specifically, we
192 extracted the floe size distributions from the helicopter images in Test #1. This yields information regarding the
193 maximum floe size L_{MCD} over channel spacing h , which is used to validate the theoretical prediction in Eq. (3).
194 In Test #2, aside from the on-board camera dynamically recording the occurrence of long crack formations, the
195 ship's position, inertia, propulsion, and wind data were continuously recorded. These data enable us to calculate
196 the ice force acting on the ship and can further be converted into comparable ice fracturing force components
197 against the theoretical predictions in Eq. (1).

198 **4.1 Image Analysis**

199 **4.1.1 Helicopter images from Test #1**

200 The helicopter was sent off immediately (approximately 30 min) after Test #1 to take an overall photograph of the
201 ice management effects in the region. Information on parallel channel Test #1 can be retrieved from the relevant
202 helicopter images (highlighted in the yellow box in Fig. 5). We choose Fig. 8 as the base image for our analysis
203 because it covers the major part of Test #1. With the average channel spacing information labelled in the figure,
204 we can immediately see that most of the parallel channels' fracturing events take place with spacings below 100
205 m. More fracturing events are found with narrower channel spacing, e.g., 30 m. As the spacing distance reaches
206 approximately 140-200 m, few fully through ice-floe fractures can be identified.

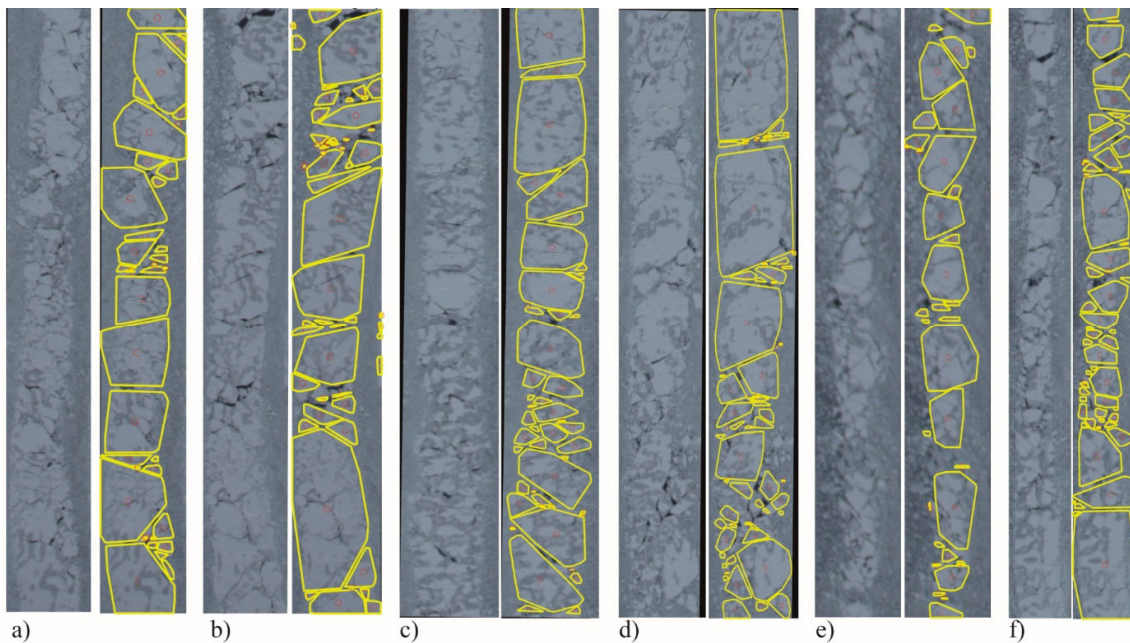


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Fig. 8. Chosen regions for analysis.

209 We chose six regions (shaded in green in Fig. 8) with different parallel channel spacing h values to study its
210 influence on the fracturing events. The floe size distribution within each region is extracted for evaluation using
211 the image processing technique described by Lu et al. (2016). The technique involves the following procedures.
212 We first transform the original image into binary images with different grey scales to identify the border of ice
213 floes and visually discernible cracks. The identified ice borders usually cover only a part of the perimeter of the
214 ice floe. Then the visually discernible cracks were propagated to merge with the identified partial ice borders until
215 a convex shaped ice floe is identified/extracted. The extracted ice floes, highlighted with yellow polygons, are
216 illustrated in Fig. 9 for the selected regions.



217

218 Fig. 9. Ice floe extraction based on propagating seeding cracks: a)–f) represent the different regions in Fig. 7.

219 To put the visual results into perspective, the extracted ice floes in Fig. 9 are patched back into the original ice
220 field in Fig. 11, which shows the potential of the previously developed algorithm to quantify the floe size
221 distribution after an ice management operation (Lu et al., 2016).

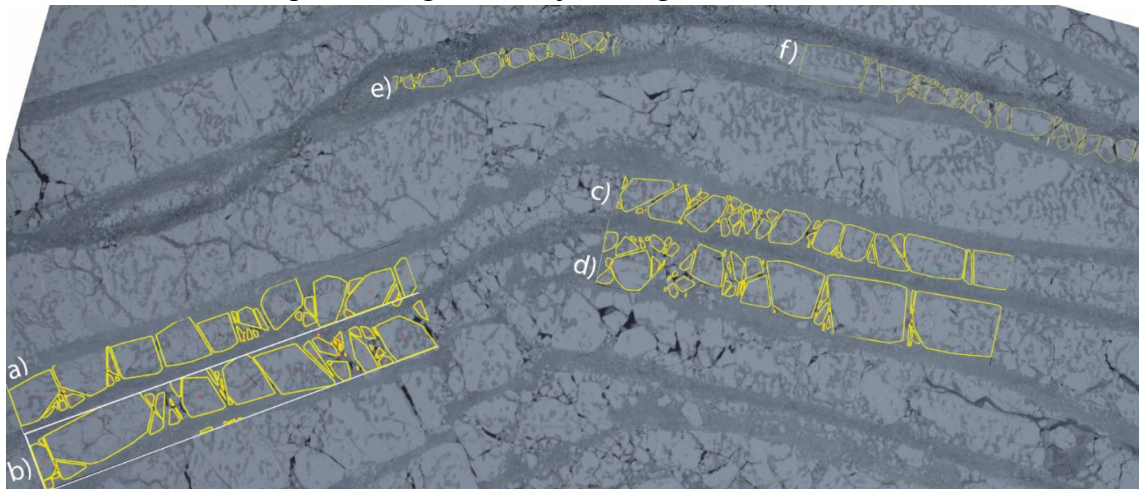


Fig. 10. Ice floe segmentation in the selected ice field.

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224 Despite the limited test samples presented in Figs. 8 and 9, statistical analyses of the floe size distributions are
225 conducted. Emphasis is placed on the ratio of the maximum floe size L_{MCD} over its channel spacing h , which can
226 theoretically be predicted by Eqs. (2) and (3). While quantifying the general floe size L (and L_{MCD}) from Fig. 9,
227 the concept of the Mean Calliper Diameter (MCD) is used as in Eq. (4) to characterise the irregular floe size, as
228 follows:

$$L_i = \sqrt{4A_i / \pi} \quad i = 1, 2, \dots, N_{total} \quad (4)$$

$$L_{MCD} = \max(L_i)$$

229

230 A_i in Eq. (4) is the area of the individual ice floe and N_{total} is the total number of ice floes.

231 For the floe size distributions, different distributions in Eqs. (5) to (7) have been proposed in the ice literature (e.g.,
232 see the review by Lu et al. (2008)). The formulations are presented with parameters C_0 , D , γ , L_0 , and L_r , which
233 need to be fitted given the measured floe size distributions. Particularly, L_0 and L_r correspond to the mean and
234 maximum floe size, respectively (Lu et al., 2008). In this paper, all three types of distribution functions are utilised
235 to fit the floe sizes in Fig. 9 with a nonlinear least square fitting method.

236

$$\text{Power-Law distribution: } \frac{N(>L)}{N_{total}} = C_0 L^{-D} \quad (5)$$

237

$$\text{Upper Truncated Power-Law distribution: } \frac{N(>L)}{N_{total}} = C_0 (L^{-D} - L_r^{-D}) \quad (6)$$

238 Weibull distribution: $\frac{N(>L)}{N_{total}} = \exp[-(\frac{L}{L_0})^\gamma]$ (7)

239

240 **4.1.2 On-board camera images from Test #2**

241 During the test, the on-board camera captured all the long cracks emanating from the portside bow area of Oden.

242 With the acquired images, it was later found that automatically tracking the occurrence of fracturing events

243 appeared to be rather challenging due to the presence of significant visual noise. Therefore, the on-board camera

244 images were processed manually in a rather straightforward manner. We reviewed all the recorded images and

245 identified all the occurrences of long cracks. In each of the events, we highlighted the far side parallel channel and

246 the corresponding visible crack. Afterwards, these images were rectified following a procedure described and

247 applied in other relevant Arctic marine operations (Lu and Li, 2010, Zhang et al., 2012a, Zhang et al., 2012b).

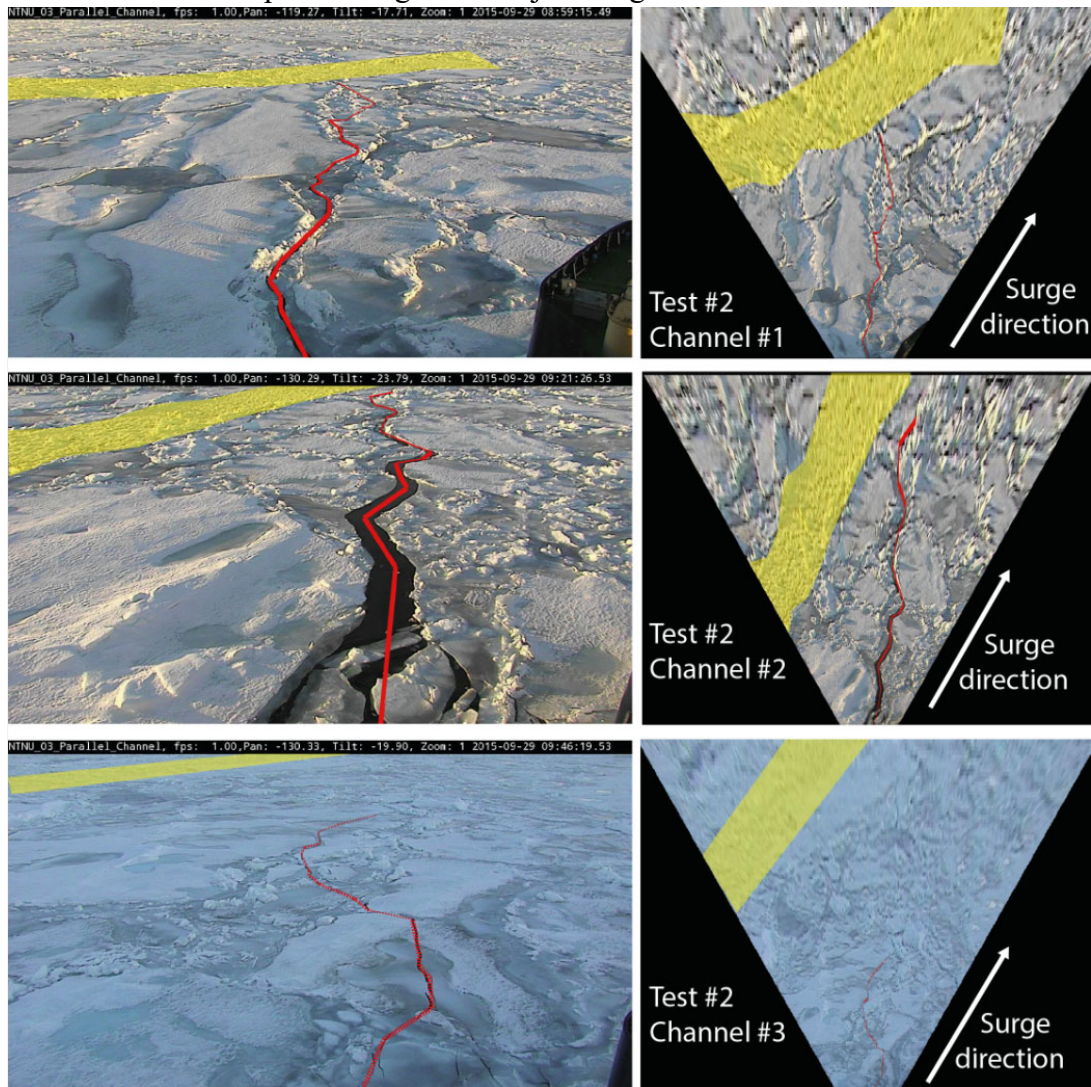


Fig. 11. Fracturing events captured by the on-board camera with different channel spacing, left: original highlighted images; right: rectified images.

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252 Fig. 11 shows three sampled fracturing events with different channel spacing widths. Rows #1 to #3, respectively,
253 correspond to tests in Channels #1 to #3 in Fig. 6. The original images are in the left column and the rectified
254 images are in the right column. The images show that the tortuous crack propagation path is influenced locally by
255 the weak zones within the ice field. However, the overall propagation direction of these long cracks is largely
256 dictated by the neighbouring parallel channel when the spacing is less than approximately 200 m, i.e., they tend to
257 propagate towards free boundaries. On the other hand, the long cracks, labelled by 'dashed lines' in Channel #3
258 (Row #3 in Fig. 11), are events in which the cracks did not manage to propagate fully through to the far side
259 channel. In this scenario, the channel spacing of approximately 200 m is too far for the long crack to develop fully.

260 These rectified images are utilised to document the frequency of long cracks occurring, which is useful in
261 supporting the floe size distribution analysis in Test #1.

262

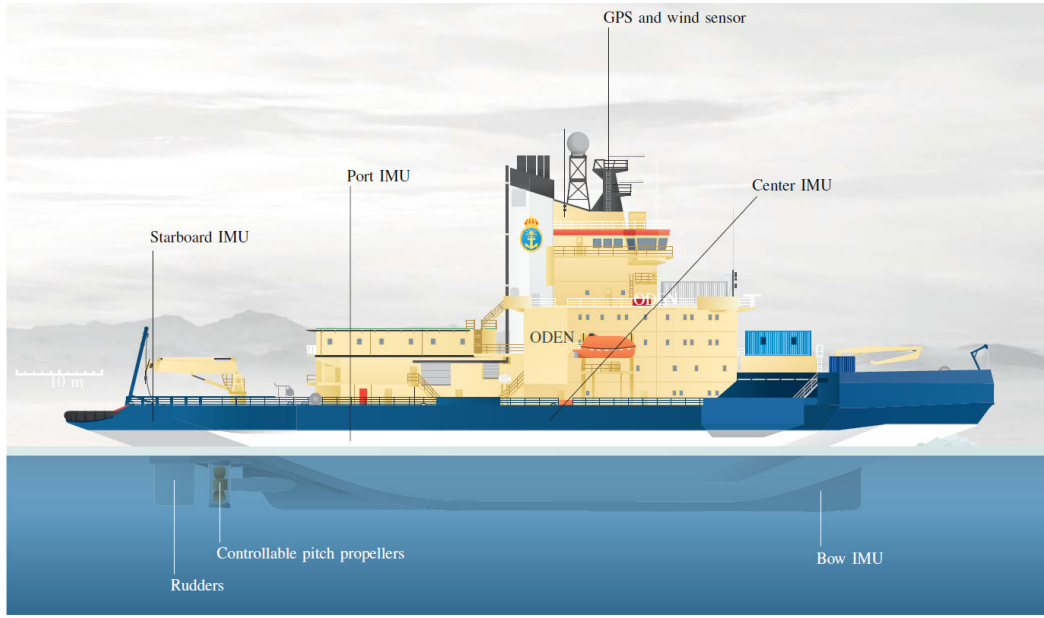
263 **4.2 Ice Force Estimation based on measurements**

264 The direct measurement of the global ice force R_{ice} on an icebreaker is usually challenging. In this paper, we adopt
265 an alternative approach to derive the ice force history R_{ice} during Test #2 by a purposely developed method
266 (Kjerstad et al., 2018). The method is largely based on Eq. (8), in which the ship's propulsion force R_{prop} ,
267 hydrodynamic resistance R_{hydro} , and wind resistance R_{wind} are calculated with established methods and the
268 conventional measurements on board. In particular, a state estimator algorithm was formulated to estimate the 6
269 Degrees of Freedom (DoFs) accelerations \dot{V} of the ship to calculate the term $M\dot{V}$ (Kjerstad et al., 2018).
270 Thereafter, the ice force history R_{ice} is derived.

$$271 \quad M\dot{V} = R_{prop} + R_{hydro} + R_{wind} + R_{ice} \quad (8)$$

272 Fig. 12 illustrates the relevant sensors collecting the useful information for Eq. (8) and their respective installation
273 locations. Four Inertia Measurement Units (IMUs) were installed on board Oden. The local linear accelerations
274 and rotational velocities at the installation location were measured. Combining information from all these non-
275 coplanar IMUs, we can retrieve rather accurate accelerations \dot{V} of the ship. For the term R_{prop} , Oden is equipped
276 with two Controlled Pitch (CP) propellers with a diameter of 4.8 m. One is installed on the port-side and the other
277 on the starboard-side. The propellers are propelled by 4 medium speed, 8-cylinder Sulzer diesel engines. The total
278 propulsion is 18 MW (Johansson and Liljeström, 1989b). During Test #2, the ship propulsion P_s in [kW] is
279 measured from the two propellers in the port and starboard sides of Oden. Such measurements can be utilised to
280 calculate R_{prop} following the established method (Kjerstad et al., 2018).

281 To validate the crack driving force F_X and/or F_Y in Eq. (1), R_{ice} is calculated according to Eq. (8) with other
282 known terms. However, R_{ice} is further composed of many other terms involving different ice-structure physical
283 interaction processes, e.g., ice crushing at the contact area, ice fracturing, and submerging and sliding of ice blocks
284 along the ship body. The component of interest to our validation is the ice fracturing component R_f , and we
285 propose a method to quantify it. This is achieved by utilising a modified Lindqvist formulation (Lindqvist, 1989)
286 for ice resistance in the current test condition.



287

288

Fig. 12. Relevant sensors and their respective installation locations (originally from Kjerstad et al. (2018)).

289

290 **4.2.1 From ice force R_{ice} to its ice fracturing component R_f**

291 First, for the ship-level ice interaction process (e.g., see Section 2.1 in Paper I), the level ice resistance R_{ice}^{Level} is
292 formulated in Eq. (9) by Lindqvist (1989). R_{ice}^{Level} consists of the following components: the ice crushing
293 component R_c ; the ice bending component R_b ; a component R_s , which involves the submerging and sliding of
294 broken ice blocks during the ship transit; and its speed dependency component $(9.4v / \sqrt{gL_{ship}})R_s$.

295

$$R_{ice}^{Level} = (R_c + R_b)(1 + 1.4 \frac{v}{\sqrt{gt}}) + R_s(1 + 9.4 \frac{v}{\sqrt{gL_{ship}}}) \quad (9)$$

296

Specially, the submergence term R_s is re-written in Eq. (10), as follows:

297

$$\begin{aligned} R_s &= -1 \cdot (\rho_w - \rho_{ice})gt(A_{s1} + \mu_{i-s}A_{s2} + \mu_{i-s}A_{s3}) \\ A_{s1} &= BT \frac{B+T}{B+2T} \\ A_{s2} &= B(0.7L_{ship} - T / \tan \phi - 0.25B / \tan \alpha) \\ A_{s3} &= BT \cos \psi \cos \phi \sqrt{1 / \sin^2 \phi + 1 / \tan^2 \alpha} \end{aligned} \quad (10)$$

298

in which,

ρ_w and ρ_{ice} are the density of sea water and sea ice, respectively, in [kg/m^3];

-1 denotes that R_s acts in the opposite 'x' axis in Fig. 1b;

L_{ship} , B , and T are length, breadth and draft of the ship, in [m];

μ_{i-s} is the ice-structure friction coefficient. In the current paper, it is chosen as $\mu_{i-s} = 0.1$, considering Oden's low friction paint (Johansson and Liljestrom, 1989a) and the recommendation by Lindqvist (1989);

A_{s1} is related to the potential energy generated by submerging ice floes;

A_{s2} is the ice-covered area of the ship's flat bottom, in [m^2];

A_{s3} is the projected area (to the water plane) of the bow area, in [m^2];

ϕ , α , and ψ are the stem angle, waterline entrance angle, and a definition of $\tan \psi = \tan \phi / \sin \alpha$.

Considering Oden's bow form, in the current paper's calculation, we utilised $\phi = 15^\circ$ and $\alpha = 90^\circ$.

299 Eq. (9) is semi-empirical formula based on several assumptions and field data fitting. Its major contributor is the
300 submergence term in Eq. (10). One of the important assumptions of R_s is that 70% of the ship's bottom area is
301 covered with broken ice blocks. Despite Eqs. (9) and (10)'s typical use as engineering tools for the estimation of
302 level-ice resistance as pointed out by Lindqvist (1989), the formula (or a part of the formula, especially the
303 submergence component in Eq. (10)) is widely used for the calculation of level ice resistance in support of or for
304 validation of more advanced model developments (e.g., (Aksnes, 2010, Lubbad and Løset, 2011, Su et al., 2010,
305 Valanto, 2001).

306 Next, for the ice resistance R_{ice} during the ship-ice interactions with the presence of a nearby free channel, we
307 argue that similar interaction processes are taking place, with some exceptions, regarding the weight of different
308 force components (a detailed interaction process is presented in Paper I). In this paper, the ice resistance R_{ice}
309 during a parallel channel test is assumed to be composed of the following: 1) a component R_f involving the
310 fracture of ice dominated by forming long cracks propagating towards the nearby channel; and 2) a new
311 submergence term, $R'_s = R_s(\xi B)$, with reduced submergence area and coverage of broken ice floes beneath the
312 icebreaker. R_{ice} is expressed in Eq. (11), as follows:

313
$$R_{ice} = R_f + R'_s \left(1 + 9.4 \frac{v}{\sqrt{gL_{ship}}}\right) \quad (11)$$

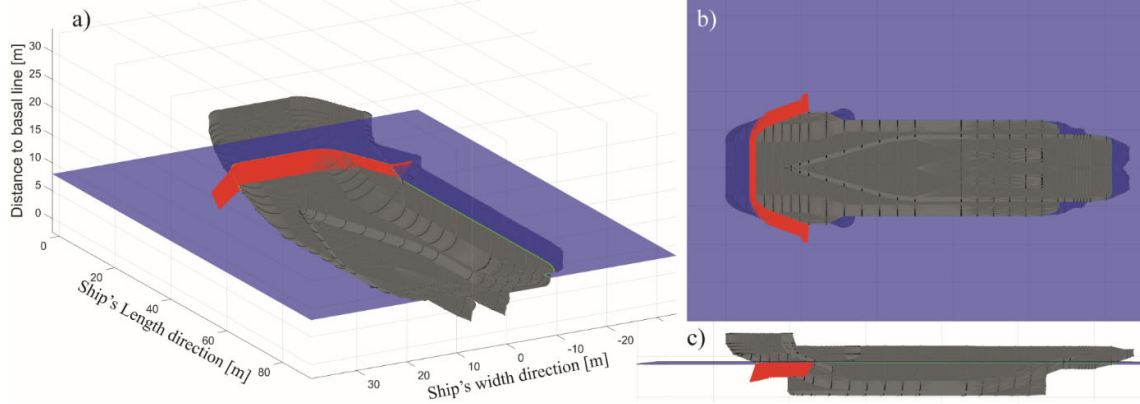
314 in which the calculation of R'_s is the same as in Eq. (10) except that the ship's breadth B is reduced into ξB ,
315 with $0 \leq \xi \leq 1$ denoting a reduced coverage of broken ice blocks under the ship. This is a reasonable assumption
316 considering the fact that a majority of the ice was fractured and pushed sideways due to the presence of the
317 neighbouring parallel channel. This differs from the scenario of ship-level ice interactions in which dominant
318 bending failure modes push broken ice blocks downward, forming a larger coverage area beneath the ship (e.g.,
319 70%, as assumed in Eq. (10)). In the current experiment, however, we have no means to directly measure either
320 the actual broken ice block coverage or the submergence term. Therefore, the coefficient ξ is kept open as a
321 degree of uncertainty in the forthcoming analysis.

322 Based on the above derivations, we thus established the relationship between the component R_f , pertaining to sea
323 ice fracturing during parallel channel tests, and the measurements of the ship's inertia, propulsion, hydrodynamic
324 and wind resistance, which was further converted to the net trust R_{ice} , based on Eq. (8). An explicit form of this
325 is written in Eq. (12).

326
$$R_f = MV - R_{prop} - R_{hydro} - R_{wind} - R'_s \left(1 + 9.4 \frac{v}{\sqrt{gL_{ship}}}\right) \quad (12)$$

327 **4.2.2 Relationship between F_X and F_Y**

328 With Fig. 1b, we see that $|R_f| = |F_X|$ and both act at the ship bow with equal magnitude but in opposite directions.
329 However, in Eq. (1), which is to be validated, the relationship between F_X and F_Y (i.e., β_{YX}) also needs to be
330 identified. Therefore, the relationship between F_Y and F_X (or R_f) should be established. This is achieved by
331 considering Oden's bow geometry. We assumed that the normal directions around Oden's bow region at the water
332 line are evenly distributed. This is illustrated in Fig. 13, with red arrows signifying the normal directions. Fig. 13b
333 shows that it is mainly the bow region closer to the reamer of Oden that contributes the force component in the F_Y
334 direction; whereas it is primarily the stem region that contributes to forces in the F_X direction. Nevertheless, all
335 the normal vectors in the ship's bow region are projected to the F_X and F_Y directions. The relationship in Eq. (13)
336 is thus established.



337

338

Fig. 13. Vectors (in red arrows) normal to ship bow, a) perspective view; b) view from the bottom; c) side view.

339

340

$$\begin{aligned} F_Y &= \beta_{YX} F_X = -\beta_{YX} R_f \\ \beta_{YX} &= 0.6945 \end{aligned} \quad (13)$$

341 5 Results

342 5.1 Floe Size Distributions in Test #1

343 One of the important measurements during the test is the channel spacing h . From Fig. 8, we extracted the channel

344 spacing h values for Test #1. Within the selected regions, from west to east, the varying channel spacing h and

345 its average h_{mean} are plotted in Fig. 14. The averaged channel spacing h_{mean} shall be utilised to normalise the floe

346 size L in the following results presentations. Without further differentiation, h_{mean} is from here on written as h

347 for convenience and also for comparison purposes with results from Test #2.

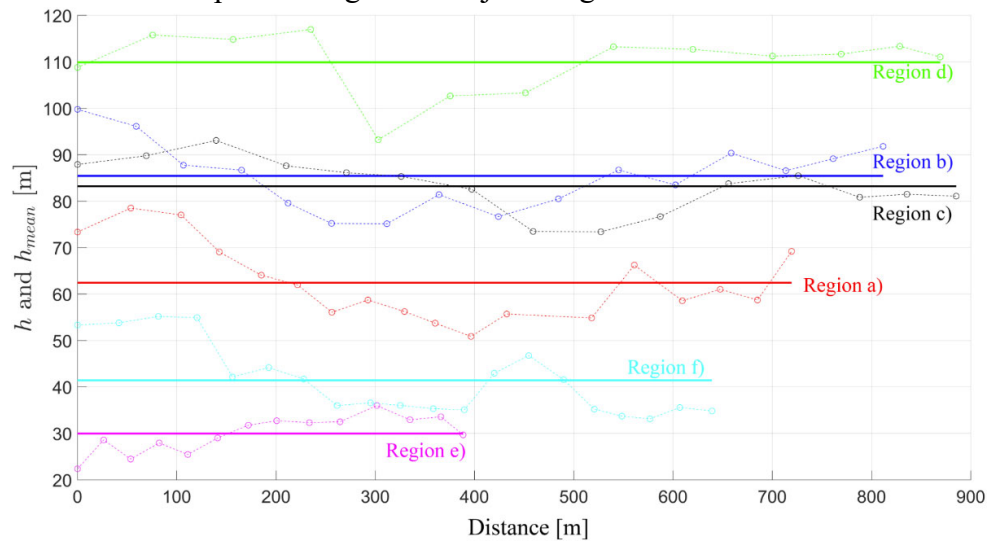
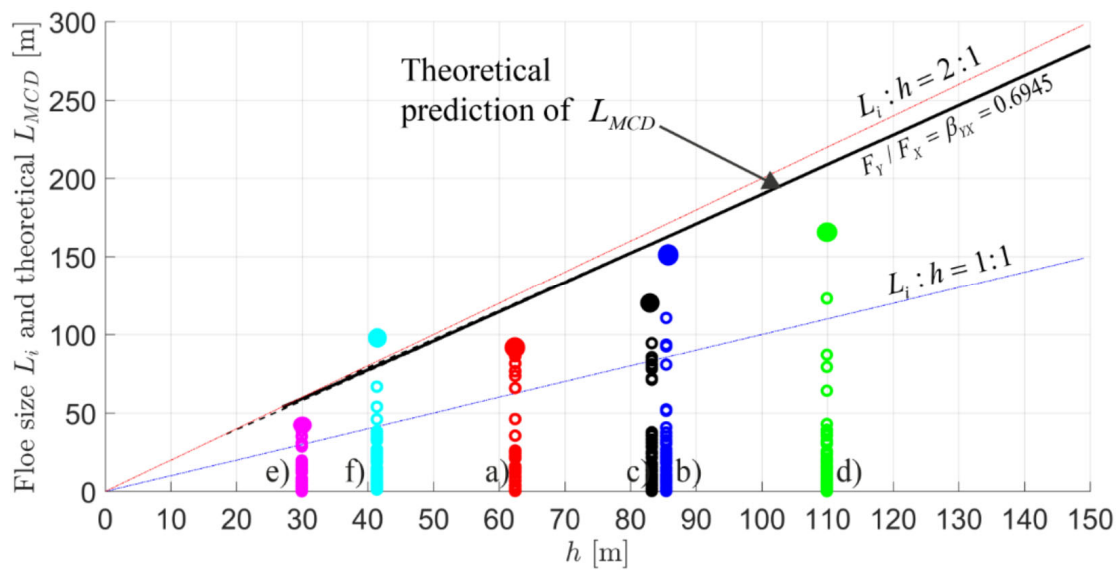


Fig. 14. Channel spacing variations for the selected regions in Test #1.

348
349

350 First, all the managed floe sizes L_i , i.e., the MCD defined in Eq. (4), are collected and plotted in Fig. 15, with
351 different channel spacing h values. In addition, the theoretical prediction of L_{MCD} using Eqs. (2) and (3) are also
352 plotted with a varying ice thickness of $t = 0.6, 1.0, \text{ and } 1.5 \text{ m}$. The theoretical predictions show little variation with
353 the ice thickness and overlap with each other in Fig. 15. Attention is primarily given to the measured maximum
354 floe size, plotted as enlarged solid markers in Fig. 15. Furthermore, in Fig. 15, we also plot the often assumed floe
355 size versus channel spacing ratio, i.e., $L_{MCD} : h = 1$, according to Hamilton et al. (2011a, 2011b), and the proposed
356 $L_{MCD} : h = 2$ as an upper limit by Lu et al. (2016).



357

358 Fig. 15. Floe sizes L_i collected in Test #1 versus theoretical predictions of maximum floe size L_{MCD} given different channel spacing h
359 values.

360 In a managed ice field, in addition to the maximum floe size L_{MCD} , the overall floe size distribution is also
361 important for ice resistance considerations. Therefore, the cumulative distribution $N(<L)/N_{total}$ of all the
362 measured floe sizes L_i is illustrated in Fig. 16. As mentioned in Section 4.1.1, three different distribution functions
363 were utilised to fit the measured data. An exemplified fitting for the data obtained in Channel a) by three different
364 distribution functions is demonstrated in Fig. 17. One can see that the Weibull distribution gives a better overall
365 fitting compared to the other two fitting methods. Similar data fitting results for other channels were also obtained.

366

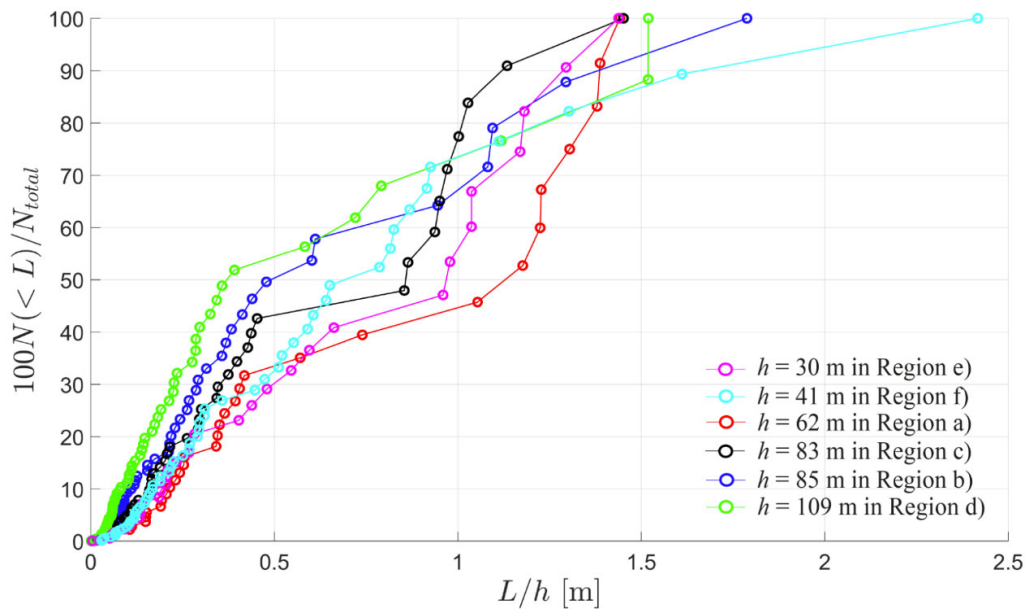
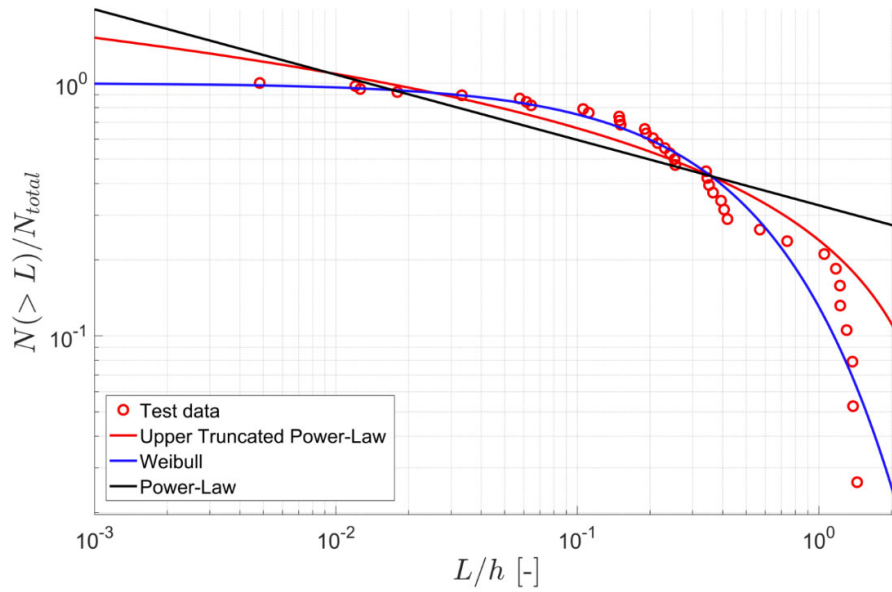


Fig. 16. The cumulative distribution of the managed ice floe size in different channels.



369

370

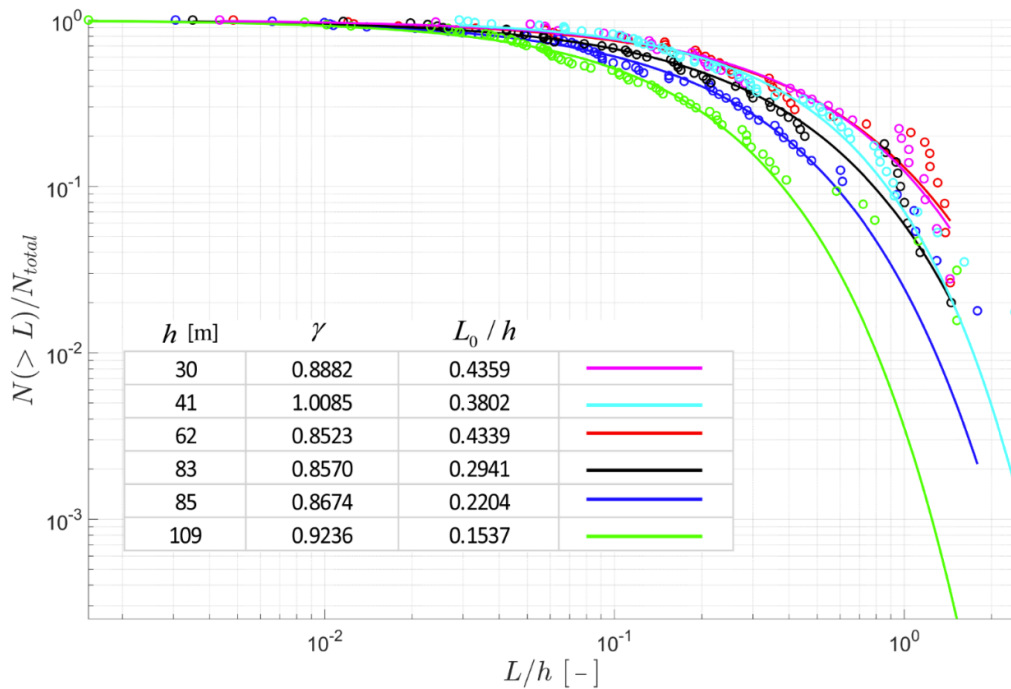
Fig. 17. Data fitting by different distribution functions for Channel a).

371

372 Given the relatively better Weibull fitting function, all the test data in the different channels are fitted accordingly

373 in Fig. 18, along with the necessary fitting parameters γ (the shape parameter) and L_0 / L (indicating the average

374 floe size) (Lu et al., 2008).



375

376

Fig. 18. Floe size distribution in Test #1 fitted by the Weibull distribution function.

377 Practically, it is convenient to give an account of the floe size distribution with reference to the channel spacing
378 h . This is summarised in Table 1.

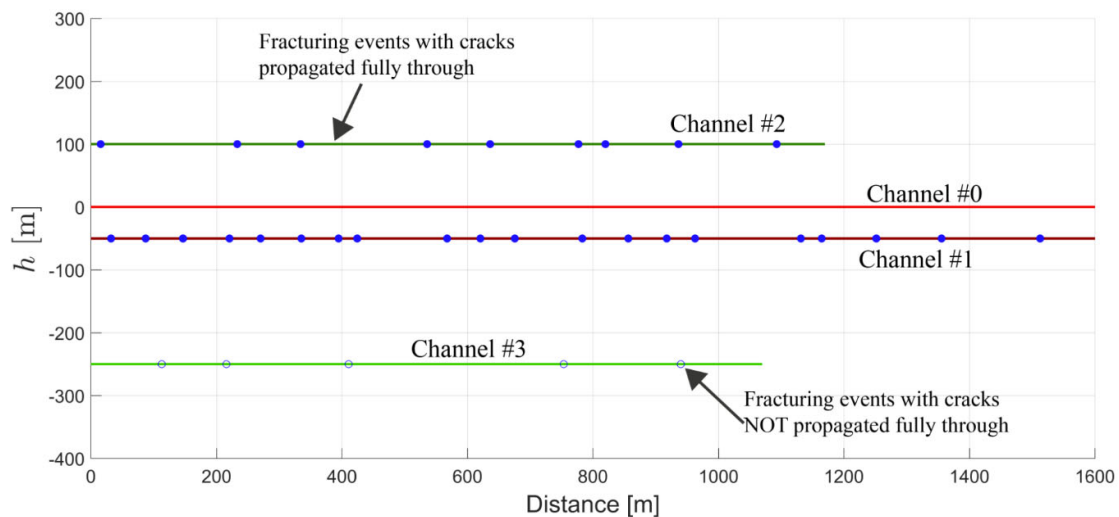
379 Table 1 Channel Spacing and its influence on the floe size.

Region	h	L_{MCD}	$N(< h) / N_{total}$		$N(< 1.5h) / N_{total}$		$N(< 2h) / N_{total}$	
	[m]	[m]	Test	Weibull	Test	Weibull	Test	Weibull
e)	30	41	65%	87.65%	100%	95.01%	100%	97.91%
f)	41	100	80%	92.95%	90%	98.15%	95%	99.52%
a)	62	90	46%	86.96%	100%	94.38%	100%	97.47%
c)	83	120	77%	94.12%	100%	98.17%	100%	99.39%
b)	85	157	70%	97.56%	90%	99.49%	100%	99.89%
d)	109	163	80%	99.64%	90%	99.97%	100%	100.0%

380

381 5.2 Long Crack Formation Events in Test #2

382 Based on the visual analysis introduced in Section 4.1.2, the fracturing events in Test #2 were gathered and
383 illustrated along different channels in Fig. 19. Similarly, the channel spacing h are presented in its average values.
384 The fracture events involving the development of matured long cracks, which travelled fully through the ice
385 reaching the nearby channel (e.g., events in the first two rows in Fig. 11), are labelled with solid markers. On the
386 other hand, events with long cracks that did not manage to propagate fully through the ice are plotted with hollow
387 markers.



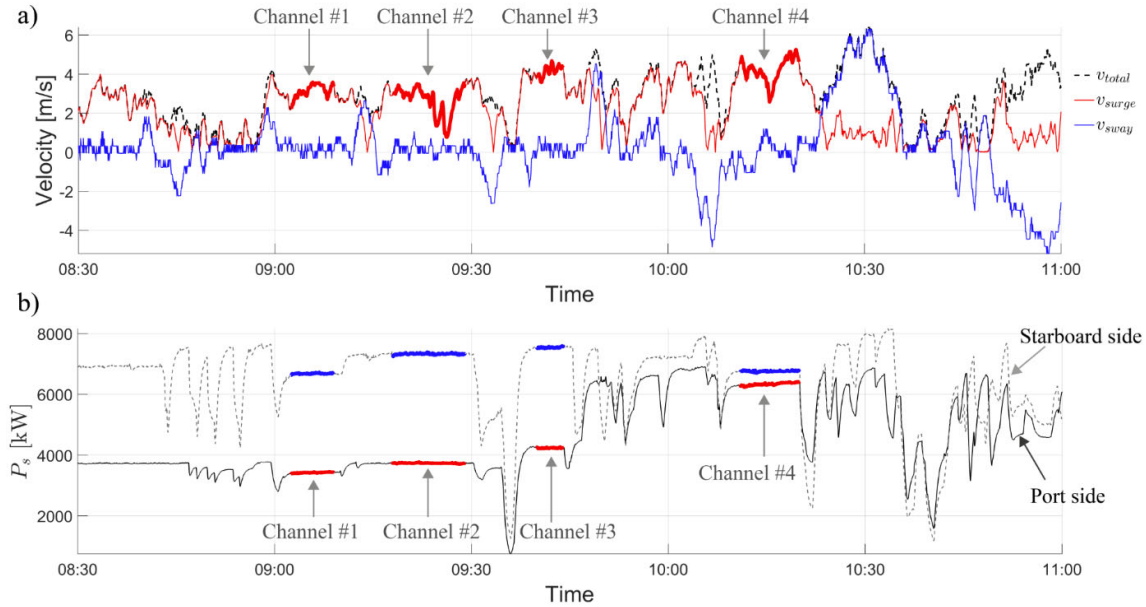
388

389 Fig. 19. Frequency of parallel channel fracturing events in Test #2 (the overall tracks are plotted in Fig. 6).

390

391 **5.3 Long Crack Propagation versus Channel Spacing in Test #2**

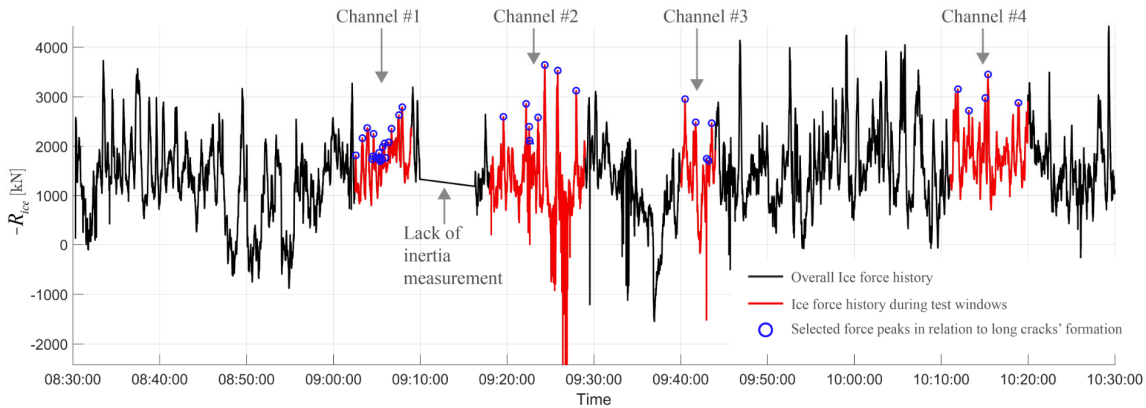
392 For Test #2 on September 29th, 2016, the measured propulsion power P_s (for both the port- and starboard-sides)
393 and velocities (calculated based on the ship's GPS data) are plotted in Fig. 20.



394

395 Fig. 20. Ship propulsion history and velocity data in Test #2 in different channels.

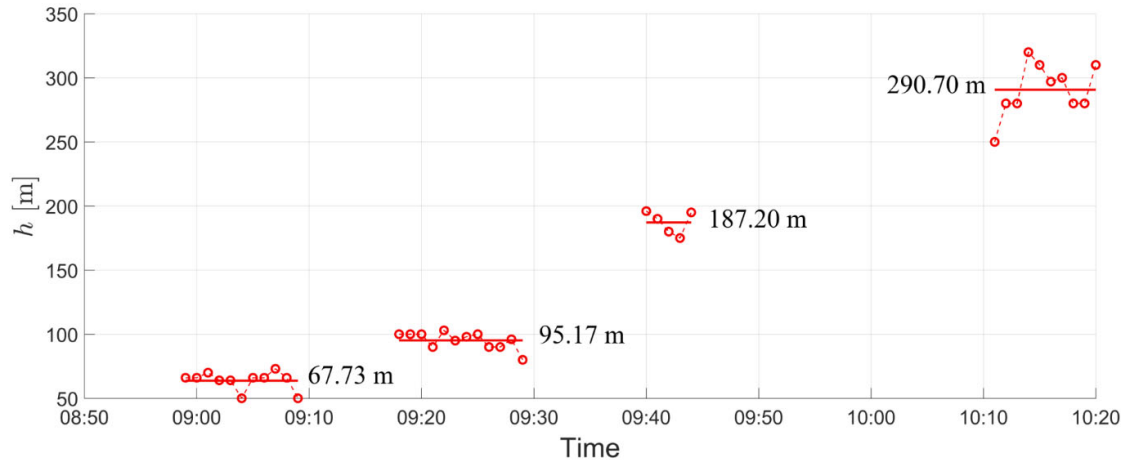
396 With the method developed by Kjerstad et al. (2018), based on Eq. (8) with relevant measurements, the estimated
397 ice force history R_{ice} is presented in Fig. 21. The results within each parallel channels' test window are highlighted.
398 Within each test window, the registered peak forces are expected to be correlated to the occurrence of long cracks
399 observed in Fig. 19. These force peaks are averaged within each channel and are utilised to validate the theoretical
400 prediction of F_X based on Eq. (1).



401

402 Fig. 21. Estimated ice force history based on relevant measurements in Eq. (8).

403 Within Eq. (1), the channel spacing h is an important input. Based on experiences from Test #1, we logged in the
404 measurements of h in a much more frequent and ordered manner (i.e., one record per minute). The measured
405 channel spacing during the test is presented in Fig. 22, along with their average values.



406

407

Fig. 22. Measured channel spacing with 1 min^{-1} frequency during Test #2.

408 In addition to h , two more input parameters (i.e., ice thickness t and fracture toughness of K_{IC}) are needed in
409 Eq. (1). During the tests, varying ice thicknesses (0.6 m to 1.5 m) are encountered (Lu et al., 2016). Because the
410 force peaks from Fig. 21 are utilised for comparison purpose, $t = 1.5 \text{ m}$ is utilised herein to complete the
411 calculation of F_X . For K_{IC} , because the actual fracture toughness of sea ice was not measured onsite, the
412 calculations of F_X are based on two K_{IC} values, which can be found in the literature; i.e., $K_{IC} = 115 \text{ kPa}\sqrt{\text{m}}$ is
413 based on laboratory measurements (see, e.g., (Schulson and Duval, 2009)) and $K_{IC} = 250 \text{ kPa}\sqrt{\text{m}}$ is based on
414 field measurements (Dempsey et al., 1999). The comparison between the theoretically predicted F_X based on the
415 proposed edge crack model and the calculated ice fracturing component R_f based on the measurements is
416 presented in Fig. 23.

417

418

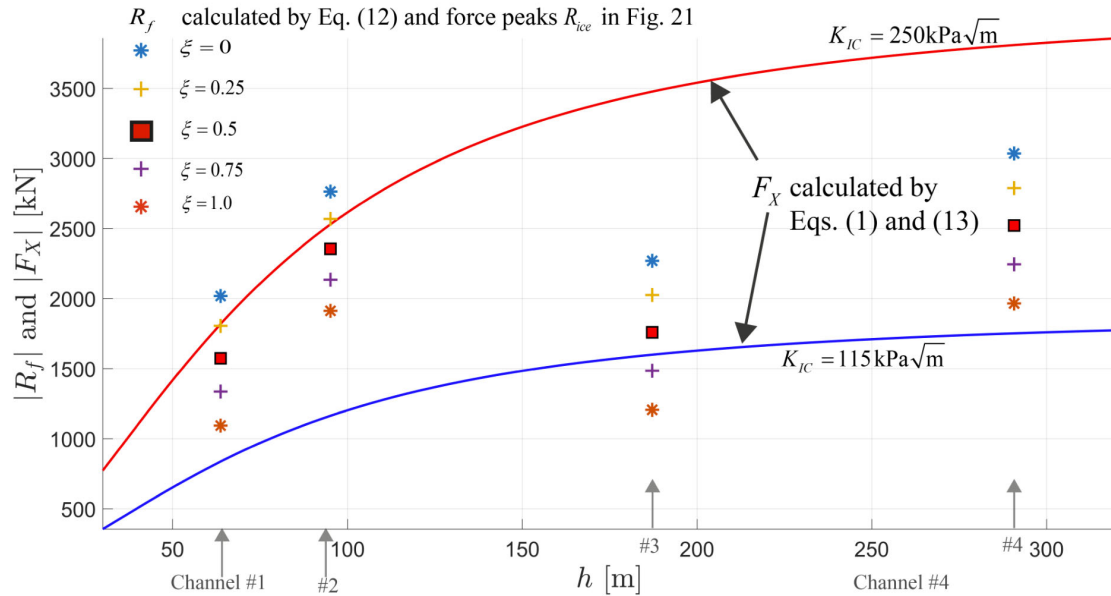
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421

422

Fig. 23 presents two comparable force components, R_f and F_X , for the different channel spacing values. R_f is
calculated based on Eq. (12) in Test #2 and is presented with discrete markers for Channels #1 – 4. During the
calculation of R_f , the formulation of R_f' involves a reduction factor ξ , which was undetermined. In Fig. 23,
different scenarios of ξ values are included. This means that for a measured channel spacing h , corresponding to
the measured discrete value in Fig. 22, there are five markers, from the top to the bottom, sequentially represented
as $\xi = 0, 0.25, 0.5, 0.75$ and 1.0 .



423

424

Fig. 23. Comparison between R_f , based on measured data, and F_X , based on theoretical prediction, for Test #2.

425 6 Discussion

426 6.1 Managed Ice Floe Size L_i and L_{MCD}

427 For Test #1, both Fig. 15 and Fig. 16 show that a significant number of small ice floes were generated in all regions
428 regardless of the channel spacing. However, the largest ice floe, i.e., L_{MCD} , is generally increasing with channel
429 spacing h . This is in line with the theoretical prediction plotted in Fig. 15. The enlarged solid markers in most of
430 the regions are in close proximity to the theoretical predictions. The theoretical results appear to be only slightly
431 influenced by the thickness variations from $t = 0.6 \text{ m}$ to 1.5 m . The theoretical predictions draw an upper bound
432 of the maximum possible ice floe size L_{MCD} , which is supported by the current measured data in all regions (except
433 for a slight underestimation in Region f).

434 In engineering practices, it is common to characterise the generated floe size with reference to the channel spacing
435 h . Fig. 15 illustrates two common floe size approximations, i.e., $L : h = 1$ according to Hamilton et al. (2011a,
436 2011b) and the proposed $L : h = 2$ as an upper limit by Lu et al. (2016). One can see that for the approximation of
437 L_{MCD} , the $L : h = 2$ relationship appears to be a better estimation compared to its counterpart, $L : h = 1$. On the
438 other hand, the theoretical model-based formulation, i.e., Eqs. (2) and (3), takes into account the ratio of F_X and
439 F_Y , and it can be conveniently implemented following a simple numerical recipe presented in Table 1 of Paper I.
440 It is expected to yield more accurate results for more general engineering applications.

441 For Test #2, the manually determined fracturing events in Fig. 19 also corroborate the above discussions. The
442 fracturing events in Fig. 19 indicate the maximum possible floe sizes L_{MCD} produced between two parallel
443 channels. We can see that most of the fracturing events occur with $1h$ or $2h$ intervals. This indicates that the
444 produced floe sizes have possible maximum size L_{MCD} in the range of approximately $2h$.

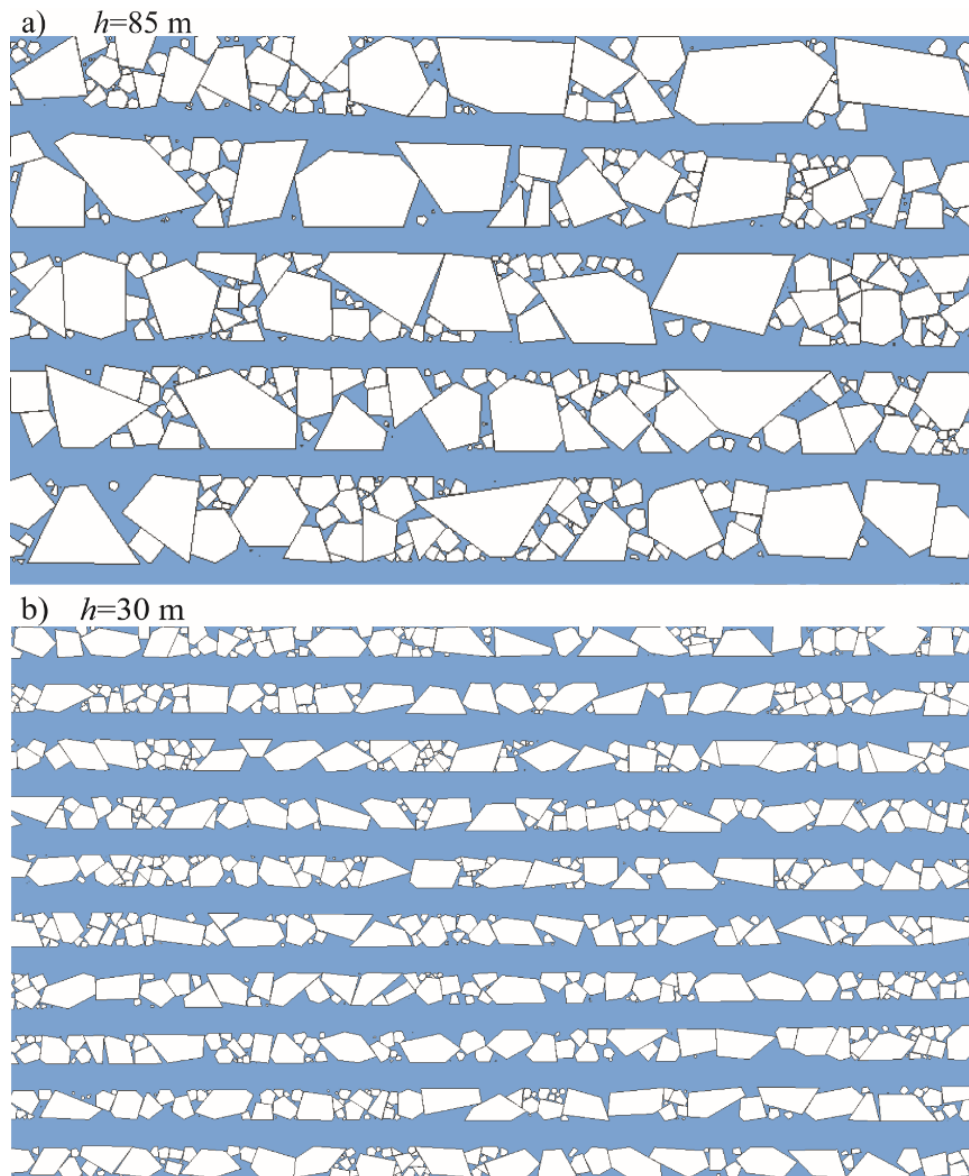
445 6.2 Managed Ice Floe Size Distribution

446 In addition to the maximum floe size L_{MCD} , which sets an upper bound on the managed ice floe size, it is equally
447 important to consider the floe size distributions. Fig. 16 illustrates that most of the ice floes produced are of smaller
448 sizes than h . Among all three distribution functions, Fig. 18 shows that the Weibull distribution gives a better
449 overall fit to the floe sizes extracted from Fig. 10. This is in accordance with the proposition of Lu et al. (2008),
450 who argue that the Weibull distribution often gives a better fit. One disadvantage of the Weibull distribution
451 function, however, is that large ice floes are often underestimated. This shortcoming can be overcome by the Upper
452 Truncated Power-law distribution function, which often yields relatively conservative theoretical values in the
453 large floe size region. Therefore, depending on one's interest, either to focus on the overall floe size distribution
454 or to be conservative when accounting for large ice floes, different distribution functions can be chosen.

455 For Weibull distributions, the fitted parameters for different channels are presented in Fig. 18. For different channel
456 spacing h values, the shape parameter γ varies little compared to the scaling parameter L_0 . The general trend of
457 L_0/h is decreasing with increasing channel spacing values. Recalling the physical meaning of L_0 which
458 represents the average floe size, this indicates that as the channel spacing increases, more, relatively smaller, ice
459 floes are created. With the theoretical distribution functions and parameters at hand, the corresponding managed
460 ice field can be numerically generated according to a similar method described by Yulmetov et al. (2014, 2016).
461 The method first generates sufficient convex shaped ice floes of different size following the prescribed distribution.
462 Then all the numerically generated ice floes were 'thrown' into a domain within which a 'multi-body dynamics'
463 bases algorithm (Coutinho, 2013) is employed to resolve the overlap between ice floes. Fig. 24 demonstrates two
464 managed ice fields with floe size distributions following the Weibull distribution and parameters in Fig. 18. To
465 highlight the managed ice floes, brash ice within the channels is not included in Fig. 24. In practice, small rigid
466 bodies of various shapes can be generated within the channel, reproducing the effect of brash ice (Konno et al.,
467 2013, Konno et al., 2011, Konno, 2009). The managed ice floe size distributions demonstrated in Fig. 24 resemble
468 the corresponding scenarios in the field in Fig. 10 b) and e). Fig. 24 also visually demonstrates that with a larger

469 channel spacing, larger managed ice floes are produced, and a larger amount of comparatively smaller ice floes
470 are also present.

471 In practice, for the readers' convenience, the cumulative distribution function's values for different channels are
472 summarised in Table 1. The table demonstrates that nearly all the produced ice floe sizes are smaller than $2h$;
473 more than 90% of the ice floe sizes are smaller than $1.5h$, and a majority (46% to 80%) of the ice floe sizes are
474 within $1h$. With the Weibull distribution, these corresponding values are relatively higher, signifying the Weibull
475 distribution's underestimation of large ice floes.



476

477 Fig. 24. Numerically generated managed ice field with different parallel channel spacing: a) channel spacing is 85 m and b) is 30 m (Note
478 that the brash ice within the channels are not illustrated)

479 6.3 Long Crack Propagation

480 Previous discussions primarily focus on the characterisation of managed ice fields (e.g., L_i , L_{MCD} and distribution)
481 with a pre-condition that long cracks are already formed between parallel channels. On the other hand, the
482 formation of long cracks is influenced by many factors, among which, the channel spacing h plays an important
483 role. Eq. (1) formulates the minimum requirements for the propagation of a long crack. The formula was tested
484 with the calculated R_{ice} based on field measurements in Test #2.

485 First, the propulsion power history during Test #2 is plotted in Fig. 20. As the channel spacing h increases, the
486 power requirement on the port side (the side that is in close proximity to the neighbouring channel) also increases.
487 Qualitatively, this is in line with Eq. (1), that under certain limits, a larger channel spacing leads to higher ice
488 resistance and *vice versa*. In particular, for the case in Channel #4, the power delivery on the port- and starboard-
489 sides becomes equivalent. This indicates that as the channel spacing increases to approximately 300 m, the parallel
490 channels' effect becomes negligible. For more quantitative comparisons, the propulsion data is converted into the
491 ice force R_{ice} and further into the resistance component involving ice fracture R_f with the established methods.

492 This conversion process involves several simplifications and uncertainties, among which the largest variance lies
493 in the reduced ice sliding component $R_s^i = \xi R_s$. To account for this uncertainty, the different values of $\xi = 0, 0.25,$
494 $0.5, 0.75$ and 1.0 are considered in the calculations, with the results plotted in Fig. 23. In addition, the theoretical
495 predictions based on Eqs. (1) and (13) are plotted with the available fracture toughness values of the sea ice within
496 the literature. Despite the scatters in Fig. 23's comparisons, we can generally see that a higher chance of favourable
497 comparison agreement exists for the tests in Channels #1 and #2 with the results calculated by the fracture
498 toughness of $K_{IC} = 250 \text{ kPa}\sqrt{\text{m}}$. This is in line with the visual observations presented in Fig. 19, in which matured
499 long cracks are frequently formed within Channels #1 and #2 as opposed to Channels #3 and #4. This means that
500 the primary ice failure pattern in the tests within Channels #1 and #2 are dominated by the formation of long cracks,
501 which can be predicted by the edge crack model involving Eqs. (1) and (13), and a fracture toughness of
502 $K_{IC} = 250 \text{ kPa}\sqrt{\text{m}}$.. We further consider the fact that with a narrower channel spacing h , the ice sliding
503 component would become less significant, i.e., a smaller ξ value. In this regard, we see that a majority of R_f ,
504 calculated by a reduced ξ for Channels #1 and #2, resides well within the theoretical prediction ranges. The
505 current quantitative comparisons made in Fig. 23, though clouded with uncertainties, show Eq. (1)'s promising
506 potential for determining a long crack's propagation in the presence of a neighbouring parallel channel.

507 **6.4 Parallel Channels' Fracture Mechanisms**

508 There are generally two ice fracturing modes during ice structure interactions, namely, the in-plane (Dempsey et
509 al., 1993, Bhat, 1988, Bhat et al., 1991) and out-of-plane failure modes (Lu, 2014). The out-of-plane fracture's
510 crack extension, in the form of radial cracking, is usually rather limited in length. The radial crack is bounded by
511 a length of 2ℓ (Lu et al., 2015b, Sodhi, 1996), beyond which, local bending failure with the formation of a
512 circumferential crack at a distance slightly larger than 1.83ℓ (Nevel, 1972, Lu et al., 2015c) shall take place. ℓ is
513 the characteristic length defined in Eq. (1). For typical ice material floating on water, ℓ can be approximated by
514 $\ell = 13.5t^{3/4}$ (Gold, 1971). For parallel channel spacing $h \leq 2\ell \approx 30\text{ m}$, much more frequent fracture events are
515 observed in Channel e) in Test #1, Fig. 8. Practically, the channel spacing is much larger than 2ℓ ; thus, most of
516 the observed long crack formations are essentially under the category of the in-plane failure mode, which enables
517 a crack to propagate for a much longer distance (Lu et al., 2015a, Bhat, 1988, Bhat et al., 1991). For ships creating
518 parallel channels in an ice field, because of the nearby free boundaries, circumferential cracks can potentially cease
519 to take place; instead, the formed radial crack, with a maximum length of $A_0 = 2\ell$ can be further propagated,
520 leading to the formation of long cracks. The initial location and orientation of these long cracks are difficult to
521 characterise deterministically. In a limiting sense, in these two associated papers, we choose to analyse and validate
522 one type of long crack, i.e., Type #1 in Figure 4 of Paper I. Based on the chosen edge crack model, a new developed
523 formulation in Paper I enables us to determine this type of long crack's propagation and path. Consequently, the
524 formulation yields information on the maximum channel spacing h_{\max} and maximum floe size L_{MCD} . The
525 experimental results described in this paper corroborate the theoretical prediction hitherto. However, the other
526 failure types in Figure 4 of Paper I should not be forgotten. The joint occurrence of all these types of in-plane
527 fracturing processes leads to the observed floe size distribution presented in this paper. The combined effect of all
528 these fracturing processes is considered as the complete mechanism behind the observed parallel channel fracturing
529 mechanism.

530 **7 Conclusions**

531 In the associated Paper I, a theoretical model was proposed to uncover the fracture mechanism for the observed
532 long crack formations occurring between two parallel channels during ice management operations. As a
533 continuation to the theoretical development, we reported in this paper (i.e., Paper II) two parallel channel tests
534 performed by the icebreaker Oden during the Oden Arctic Technology Research Cruise 2015 (OATRC2015).

535 During the tests, we varied the channel spacing for different test runs. Within each test run, the channel spacing
536 was maintained to the captain's best ability. In the tests, the fracturing information were collected in the form of
537 images taken from a helicopter camera (for Test #1) and the ship's on-board camera system (for Test #2). The
538 helicopter camera images were processed to extract the managed ice floes in Test #1 with different channel spacing
539 h . This yields information such as the maximum ice floe size L_{MCD} and floe size distributions. In addition, major
540 fracturing events were also documented by the on-board cameras in Test #2. The documented fracturing events
541 indicate the maximum possible ice floes L_{MCD} . All these test results satisfactorily support the theoretical prediction
542 in terms of L_{MCD} . Aside from the validation made upon the theoretical model, some general conclusions regarding
543 L_{MCD} are drawn as follows:

- 544 • The size of the largest managed ice floe, L_{MCD} , within two parallel channels generally increases with the
545 channel spacing h .
- 546 • The size L_{MCD} can be well predicted by Eqs. (2) and (3) with reasonable precision. However, for
547 straightforward engineering applications, L_{MCD} can be approximated by twice the channel spacing, i.e.,
548 $L_{MCD} = 2h$.

549 In addition to the maximum managed ice floe size, we also studied the overall floe size distributions given different
550 channel spacings h . Conclusions in this regard are drawn as follows:

- 551 • Among all three tested distribution functions, the Weibull distribution yields a comparatively better
552 fitting to the overall floe size distributions, whereas certain underestimation exists for large ice floes;
- 553 • The Upper Truncated Power-law distribution shows a better fit to large ice floes, whereas the overall
554 fitting is not as satisfactory as the Weibull distribution;
- 555 • A great majority of the managed ice floe sizes are smaller than the L_{MCD} values extracted in the
556 experiment or predicted by theories;
- 557 • For Test #1, all (100%) of the managed ice floe sizes are 2 times smaller than the spacing (i.e., $< 2h$);
558 90% of them are smaller than 1.5 times the spacing (i.e., $< 1.5h$); and a majority of them (around 46%
559 to 80% depending on the channel spacing) are smaller than the spacing (i.e., $< h$). The fitted Weibull
560 distribution functions give relatively higher percentage values due to its underestimation regarding the
561 existence of large ice floes;

562 • The fitted distribution function, e.g., Weibull distribution function, can be utilised to numerically
563 generate a managed ice field (exemplified in Fig. 24). This uncovered the numerical potential in
564 simulating the ice force in different representative managed ice fields.

565 Moreover, we analysed the ship's inertia, propulsion history, hydrodynamic and wind force on the icebreaker Oden
566 in Test #2. The force component pertaining to ice fractures was derived. The conversion process involves several
567 simplifications and uncertainties. However, the calculated results based on the measurement agree favourably with
568 the theoretical predictions (i.e., Eq. (1)), signifying its theoretical validity. Based on the comparison and discussion,
569 the following conclusions are drawn:

- 570 • For ice fracturing scenarios dominated by the formation of long cracks between parallel channels, Eq. (1)
571 gives reasonable predictions for the crack propagation;
- 572 • Conversely, Eq. (1) can be utilised to quantify the maximum channel spacing h_{\max} , beyond which long
573 cracks of Type #1 cease to take place;

574 In these two associated papers, a theoretical model was proposed and validated both numerically and
575 experimentally. The theoretical model is based on an 'edge crack model' and is a highly idealised fracture scenario
576 of long cracks of Type #1 (see Paper I, Figure 4). The theoretical model, in a limiting sense, predicts the maximum
577 managed ice floe size L_{MCD} , maximum channel spacing h_{\max} and required force combination (F_X and F_Y) for
578 long crack propagation. However, the varying managed ice floe sizes produced between two parallel channels are
579 the joint effect of several types of fracturing processes. These different types of fracturing processes (see Paper I,
580 Figure 4) share the similarities in the sense that 1) they are under the category of in-plane splitting failure; and 2)
581 they are influenced by a nearby free boundary. The joint effect is considered as the observed parallel channel
582 fracturing mechanism.

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