MPC-based Collision Avoidance Strategy for Existing Marine Vessel Guidance Systems*

I. B. Hagen, D. K. M. Kufoalor, E. F. Brekke, T. A. Johansen

Abstract— **This paper presents a viable approach for incorporating collision avoidance strategies into existing guidance and control systems on marine vessels. We propose a method that facilitates the use of simulation-based Model Predictive Control (MPC) for collision avoidance (COLAV) on marine vessels. Any COLAV strategy to be applied in real traffic must adhere to the international regulations for preventing collisions at sea (COLREGS). The proposed MPC COLAV method does not rely on an accurate model of the guidance system to achieve vessel behaviors that are compliant with the COLREGS. Rather, it depends on transitional costs in the MPC objective for collision avoidance maneuvers that are being executed by the marine vessel. Hence, it is straightforward to implement the MPC COLAV on different vessels without specific knowledge of the vessel's guidance strategy. Moreover, it offers the possibility to switch between different (possibly application specific) guidance strategies on the same vessel while running the same MPC COLAV algorithm. We present results from full scale experiments that show the viability of our method in different collision avoidance scenarios.**

I. INTRODUCTION

The existing matured technology platforms on marine vessels form an essential part of the emerging autonomous surface vehicles (ASVs). Such platforms include mission planning systems, guidance, navigation and control systems, which have several advanced capabilities such as path and trajectory tracking, dynamic weather routing, dynamic positioning (see e.g. [1]).

Important aspects of ASVs that are still at early development stages are automatic obstacle tracking and collision avoidance (COLAV). The COLAV aspect requires the capability to make safe and reliable decisions in hazardous situations, and its success may depend immensely on how well the collision avoidance strategy incorporates the relevant components and functionalities mentioned above.

Much research has been done in the field of COLAV and a number of different approaches for solving this type of problems have emerged. Methods especially relevant for comparison are in this case velocity obstacle (VO) [2] and dynamic window (DW) [3], [4]. Other strategies include set-based methods [5], potential fields [6] and inevitable collision states (ICS) [7]. With many methods there is a limitation to the extent to which the dynamics of the ASV and the effect of other essential components can be incorporated into COLAV algorithms. The use of Model Predictive Control (MPC) allows the possibility to explicitly include models of relevant components that influences the ASV's dynamics [8]. Within this framework it is also possible to include models of the obstacles' motion, the evolution of the dynamic environment, and different operational constraints. This introduces a design flexibility (and possibly performance gains) superior to other approaches explored in the collision avoidance literature.

A considerable amount of literature has been published on the use of MPC for collision avoidance within a range of fields: ground vehicles [9]–[11], aircrafts [12] and underwater vehicles [13]. Recently it has also been employed in the case of marine crafts [8], [14], [15]. MPC is a general and powerful method that can compute optimal trajectories and employ nonlinear vehicle models. Environmental forces are easily included, and risk, hazard and operational constraints along with mission objectives can be formalized in the cost function. However, computational complexity and convergence issues is a challenge for real time implementation. To evade these issues, one approach is to reduce the search space to a finite number of control behaviors. Optimization can then be reduced to evaluating the cost associated with each behavior and comparing these [10].

Although accurate vessel models can be used in predicting the effect of the autopilot, steering and propulsion systems within the MPC framework, it may neither be feasible nor convenient to replicate the numerous capabilities of existing advanced guidance systems in the COLAV algorithm. Moreover, discrepancies between the predicted and actual maneuvering commands generated by the guidance system may lead to an undesired behavior of the ASV. In an attempt to avoid these issues, this work investigates the option of excluding the underlying decision methods of the guidance system from the prediction model of the simulation-based MPC scheme proposed in [8]. We therefore look at the collision avoidance system as an extension to the guidance system where the decisions of the latter are used as desired setpoints to the MPC COLAV method.

In addition to this, we propose and discuss the use of transitional costs as part of the MPC objective for collision avoidance maneuvers that are in progress. The discussion is supported by results from a simulation study [15], where comparisons with the Velocity Obstacles (VO) method provide further insight into the performance and capabilities of our approach. To conclude the work and to verify the viability of our approach full scale experiments were conducted, and results from four key scenarios are presented.

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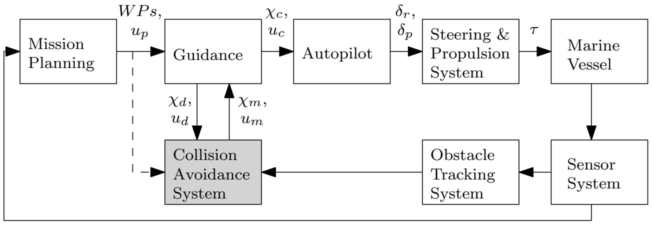


FIG. 1. Information flow for guidance and motion control with collision avoidance (proposed architecture and parameterization).

II. COLLISION AVOIDANCE SYSTEM ARCHITECTURE

The architectural components of the proposed collision avoidance system are shown in Fig. 1. The architecture focuses on information flow between the collision avoidance system and the other components. We consider a mission planning system that generates a path in terms of a desired forward speed (u_p) and a set of waypoints (WPs) for the ASV to visit. These are the inputs to the guidance system that provides the necessary course (χ_c) and speed (u_c) commands to the autopilot in order to reach the waypoints and desired speed. The autopilot determines the steering and propulsion control commands (δ_r and δ_p, respectively). The result of the steering and propulsion system are forces and moments (τ) that determine the vessel's motion.

Due to disturbances and obstacles that may be detected along the vessel's planned path, re-planning and updates to motion control may be necessary. Such updates depend on information available from the sensor system and the capabilities of the obstacle tracking system. This work focuses on the collision avoidance system as an extension to the guidance system, and we propose the use of the guidance decisions (χ_d, u_d) as desired reference to the collision avoidance system. The task of the collision avoidance system is therefore to determine the amount of modification (χ_m, u_m) required in order to ensure compliance with COLREGS and thereby avoid collision.

III. MPC COLLISION AVOIDANCE STRATEGY

The MPC COLAV scheme presented in this paper is based on the simulation-based control behavior selection approach of [8]. The MPC is designed according to the architecture proposed in Section II. Note that the COLAV has been separated from the guidance module. This implies that the simple internal simulation model of the MPC does not include the known guidance behavior as was assumed in [8].

The main objective of the MPC is to compute modifications to the desired course (χ_d) and speed (u_d) that lead to a COLREGS-compliant ASV trajectory (cf. Fig. 2). In this work, an obstacle's future motion is predicted as a straight-line trajectory, and we focus on a hazard minimization criterion (i.e. a cost function) that considers dynamic obstacles and COLREGS compliance. Including static obstacles is straightforward [8].

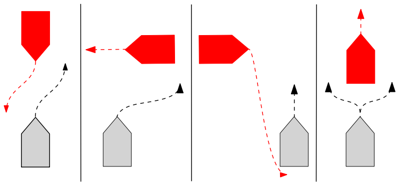


FIG. 2. Main COLREGS scenarios and correct vessel behavior. The ASV is marked in gray and obstacle vessel in red. From left: head-on, crossing from right, crossing from left, overtaking. Furthermore, any action taken to avoid collision must be significant enough to be readily apparent to other vessels (cf. COLREGS, Rule 8). For a comprehensive guide to steering and sailing rules, see [16].

A scenario in the MPC is defined by the current state of the ASV, the trajectories of obstacles, and a control behavior candidate [8]. The set of control behaviors are chosen so that the resulting maneuvers are easily observable from other vessels (cf. COLREGS). The following set of alternative control behaviors are evaluated and assumed to be fixed on the prediction horizon:

- Course offset in degrees (χ_m):
  -90, -75, -60, -45, -30, -15, 0, 15, 30, 45, 60, 75, 90.
- Speed factor (u_m): 1, 0.5, 0
  i.e. 'keep speed', 'slow down', 'stop'

The modifications are in turn applied to the desired decisions (χ_d, u_d) from the guidance system to obtain a course and speed command (i.e. $\chi_c = \chi_d + \chi_m$, and $u_c = u_d \cdot u_m$). Therefore, choosing $\chi_m = 0$ and $u_m = 1$ simply recovers the desired course χ_d and speed u_d. This parametrization leads to a total of $13 \cdot 3 = 39$ possible scenarios to be simulated and evaluated. Trajectories for the obstacles must also be predicted. The computational complexity thus depend on the number of scenarios, the number of obstacles and the chosen prediction horizon. The internal model and cost function are described next.

A. *Internal simulation model*

A model of the ASV is necessary to generate the trajectories to be evaluated by the cost function. The limited computational resources of the target platform in our experiments require a much simpler model than the 3-degrees of freedom model used in [8]. In the experiments the ASV is only expected to perform long-range, deliberate maneuvers, this along with its relatively fast dynamics, makes the time the ASV needs to change its course/speed negligible. We therefore argue that a sufficiently accurate trajectory can be achieved using only the kinematic equation

$$\dot{\eta} = R(\chi)\upsilon, \tag{1}$$

where $\eta = (x, y, \chi)$ denotes the position and course in the earth-fixed frame, $\upsilon = (\upsilon_x, \upsilon_y, r)$ denotes the velocities in surge, sway, and yaw, decomposed in the body-fixed frame, and $R(\chi)$ is the rotation matrix from body-fixed to earth-fixed frame. The prediction of the ASV's trajectory is made by inserting the desired values from scenario k into the equation (1), ie. $\upsilon = (\upsilon_x = u_d \cdot u_m^k, \upsilon_y = 0, r = 0)$ and $R(\chi = \chi_d + \chi_m^k)$. This model implies an instant turn and it also assumes no drift due to wind and ocean current.

This is clearly a very simplified model but its applicability for our experiments is confirmed by [15], where both the kinematic equation (1) and the full 3-DOF model were tested, producing only minor differences in the simulation results.

B. Cost function components

The cost function specifies the hazard evaluation criterion used in the collision avoidance strategy. We adopt the main components proposed in [8]. Specifically,

- a cost associated with collision with an obstacle,
- a cost for violating COLREGS,
- and a cost for the choice of maneuvering effort.

In addition, we introduce a new cost component:

- a COLREGS-transitional cost,

which penalizes control behaviors that abort a COLREGS-compliant maneuver. The new cost makes it possible to use decisions from a guidance strategy as reference to the MPC COLAV scheme, without including the same guidance strategy in the MPC's internal model (cf. Fig. 1).

With the guidance strategy included in the MPC COLAV, as in [8], a cost penalizing the change of control behavior is sufficient to deter the abortion of COLREGS-compliant maneuvers, provided that an adequate prediction horizon has been chosen. Not including the guidance strategy in the MPC COLAV results in a chattering behavior appearing in overtaking and crossing scenarios, as can be seen in the simulations of [15].

The problem arises because the modification to the guidance decision is made under the assumption that the desired guidance decision is constant on the prediction horizon.

Using a LOS guidance strategy as an example, i.e. $\chi_d = \chi_{LOS}$. When a COLAV maneuver is initiated by a modification to the course command, the ASV will deviate from the desired path. At the next run of the MPC, χ_{LOS} points back towards the desired path. Setting $\chi_c = \chi_{LOS}$ ($\chi_m = 0$) will cause the ASV to cross the desired path and pass the obstacle on the side opposite to what was initially predicted. If this new path is collision free and COLREGS-compliant, this scenario has the lowest cost and will be chosen. This process repeats itself until another crossing would lead to a violation of the requirement of keeping well clear (cf. COLREGS, Rule 16).

Note that the complex decision process outlined above may not be straightforward to address using a simple implementation of hysteresis (see e.g. [2]) that is merely dependent on the rate at which COLAV decisions switch. The transitional cost systematically addresses this issue by penalizing control behaviors that will cause the ASV to pass an obstacle on a different side than what is predicted with the current control behavior. Furthermore, by ensuring that the cost of collision with an obstacle dominates the corresponding transitional cost, a change in decision that is necessary due to a high cost of collision will still be allowed.

C. Cost function details

The MPC COLAV objective is to evaluate the scenarios $k \in \{1, 2, \ldots, N_s\}$ for each obstacle vessel $i \in \{1, 2, \ldots, N_o\}$ at time t_0 and select the control behavior that minimizes the cost $\mathcal{H}^k(t_0)$. Specifically,

$$k^*(t_0) = \arg\min_k \mathcal{H}^k(t_0), \tag{2}$$

where

$$\mathcal{H}^k(t_0) = \max_i \max_{t \in \mathcal{D}(t_0)} \left(\mathcal{C}_i^k(t)\mathcal{R}_i^k(t) + \kappa_i \mathcal{M}_i^k(t) + \lambda_i \mathcal{T}_i^k(t) \right) + f(u_m^k, \chi_m^k)$$

The terms of the above cost function will now be defined. For the following, the definitions are as proposed in [8]:

- the cost associated with collision with obstacle i at time t in scenario k, i.e. $\mathcal{C}_i^k(t)$, and the corresponding collision risk factor, $\mathcal{R}_i^k(t)$,
- the cost for violating COLREGS, $\kappa_i \mathcal{M}_i^k(t)$, were κ_i is a tuning parameter.
- and the cost of maneuvering effort associated with scenario k, i.e. $f(u_m^k, \chi_m^k)$.

Each scenario is evaluated at discrete sample times along the horizon T using the discretization interval T_s, i.e. $\mathcal{D}(t_0) = \{t_0, t_0 + T_s, \ldots, t_0 + T\}$. The costs at a given time t are calculated based on the position, speed and course of the ASV and the obstacles at time t, obtained from the simulations of their respective trajectories (cf. Section III-A).

The COLREGS-transitional cost $\lambda_i \mathcal{T}_i^k(t)$ is formulated using the binary indicator $\mathcal{T}_i^k \in \{0,\ 1\}$ and weight λ_i, which is a tuning parameter. The indicator value is specified using

$$\mathcal{T}_i^k(t) = O_i^k(t) \vee Q_i^k(t) \vee X_i^k(t),$$

where the binary indicators $O_i^k(t) = 1$, $Q_i^k(t) = 1$ and $X_i^k(t) = 1$ indicate the type of situation at time t, (the ASV is overtaking a vessel, the ASV is being overtaken and a crossing situation, respectively) and that the control behavior of scenario k will at time t cause the vessels to pass each other on the side opposite to what is predicted with the current control behavior. The following paragraphs define the indicator for each situation type.

Overtaking:

If the ASV is currently overtaking obstacle i, a control behavior in scenario k at a future time t is associated with a transitional cost if the predicted location of obstacle i at time t is not on the same side of the ASV as observed at the current time t_0. That is, for $t \in \{t_0 + T_s, \ldots, t_0 + T\}$,

$$O_i^k(t) = \begin{cases} O_i(t_0) \wedge S_i^k(t) & \text{if } \neg S_i(t_0) \\ O_i(t_0) \wedge \neg S_i^k(t) & \text{if } S_i(t_0) \end{cases}$$

where $S_i(t_0) = 1$ indicates that obstacle i is currently on the ASV's starboard side, whereas $S_i^k(t) = 1$ indicates that obstacle i appears on the ASV's starboard side at the future time t in scenario k. The ASV is currently overtaking obstacle i, i.e. $O_i(t_0) = 1$, if the obstacle is considered *close, ahead*, and traveling at a *lower speed*. If the obstacle's speed $|\vec{v}_i(t_0)|$ is not close to zero, the following condition must also hold:

$$\vec{v}(t_0) \cdot \vec{v}_i(t_0) > \cos(\phi_{ot}) |\vec{v}(t_0)| |\vec{v}_i(t_0)|,$$

FIG. 3. The Polar Circle 845 Sport vessel Telemetron.

where ϕ_{ot} is a suitable angle according to COLREGS, $\vec{v}(t_0)$ is the current velocity of the ASV, and $\vec{v}_i(t_0)$ is the current velocity of obstacle i. In the situation where the ASV is being overtaken by the obstacle, the binary indicators defined above are appropriately adapted from the perspective of the obstacle.

Crossing:

If obstacle i is currently crossing the path of the ASV from starboard side, a COLREGS-compliant maneuver to starboard should result in the obstacle appearing on port side when the crossing situation is over. Therefore, an alternative control behavior in scenario k at a future time t is associated with a transitional cost if the obstacle is on starboard side at time t and the control behavior suggests a change in maneuver to port side. That is, for $t = t_0 + T_s, \dots, t_0 + T$,

$$X_i^k(t) = X_i(t_0) \wedge S_i(t_0) \wedge S_i^k(t) \wedge \text{turn to port}.$$

The ASV is said to be currently in a crossing situation with obstacle i if the obstacle is *ahead* and

$$\vec{v}(t_0) \cdot \vec{v}_i(t_0) < \cos(\phi_{cr}) |\vec{v}(t_0)| |\vec{v}_i(t_0)| \wedge \neg O_i(t_0) \wedge \neg Q_i(t_0),$$

where ϕ_{cr} is a suitable angle according to COLREGS.

IV. EXPERIMENTS

A. Test setup and objectives

Experiments were performed in the Trondheimsfjord to test the performance of the proposed MPC COLAV scheme in realistic situations where deliberate COLREGS-compliant maneuvers are expected, more than 1 nautical mile away from a dynamic obstacle.

The ASV used is Maritime Robotics' Polar Circle 845 Sport vessel called Telemetron (Fig. 3). Telemetron is a relatively small Rigid Bouyancy Boat (RBB) with a V-shaped hull, making it both stable and highly maneuverable. We used the Trondheim Port Authority's Munkholmen II tugboat as the obstacle vessel. Some technical specifications of the vessels are provided in Table I.

TABLE I. Vessel specifications

Parameter	**Telemetron** (ASV)	**Munkholmen II** (obstacle)
Length [m]	8.0	14.0
Width [m]	3.0	6.0
Weight [kg]	$\sim$ 2000	–
Power [hp]	225	520
Max. speed [kn]	$\sim$ 34	$\sim$ 10

The MPC COLAV scheme was implemented in C++ and installed on the embedded computer of the Telemetron vessel. In addition, the interface between the COLAV system and the existing systems was implemented according to the proposed architecture shown in Fig. 1. We used the Automatic Identification System (AIS) as the sensor for tracking the motion of the obstacle vessel. The accuracy of AIS depends on the GPS system of the target vessel and is not a tool for precision navigation. It is however sufficient for our purposes.

Both the guidance system and the MPC COLAV extension installed on the ASV were run at a rate of 0.5Hz. However, since the AIS data received is updated at least once in 10 s, a linear prediction is used until new information about the obstacle is received. Moreover, the predicted position of the obstacle vessel is considered *close* to that of the ASV when it is 1000 m away, the safety distance used in computing the collision risk factor $\mathcal{R}_i^k$ (defined in [8]) was 200 m, and the prediction horizon T was set to 400 s, with $T_s = 5$ s discretization interval.

The experiments were performed in weather conditions that introduced significant disturbances into the dynamics of the ASV. Although no measurements of the weather condition were available during the experiments, updated weather forecast close to the time of the experiments reflect the conditions experienced: wind speeds up to 15 m/s, wave height of about 1 m, and up to 0.5 m/s currents.

B. Results

The results from different collision avoidance scenarios are shown in Fig. 5–8. The figures show snap shots of the trajectories and the main variables that describe the behavior of the ASV and the obstacle vessel. An aerial photo taken during the experiments can be seen in Fig. 4. In Fig. 5 the ASV is the give-way vessel, and it performs a COLREGS-compliant maneuver in order to avoid collision. A clear deviation from the desired course from the guidance system can be seen in Fig. 5b. The next results represent cases where both the ASV and the obstacle vessel are expected to perform COLREGS-compliant maneuvers in order to avoid collision. We examine the ASV's behavior in the case where the obstacle vessel 'stays on' (Fig. 6) and in cases where the obstacle vessel either performs a COLREGS-compliant maneuver (Fig. 7) or makes the situation worse through a more dangerous maneuver (Fig. 8).

The snap shots in Fig. 8a reveal an important property of the MPC COLAV method. That is, the capability to abort a COLREGS-compliant maneuver when a drastic change in situation is detected. Moreover, the COLAV scheme does not prevent the ASV from making necessary reactive maneuvers to its port side when in a close range situation as observed in the second snap shot of Fig. 8a and the corresponding course modification in Fig. 8b (between samples 220 and

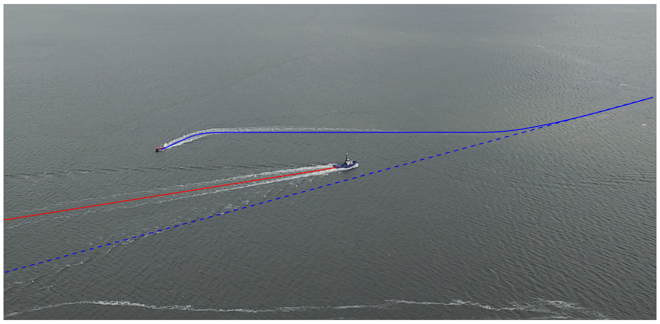
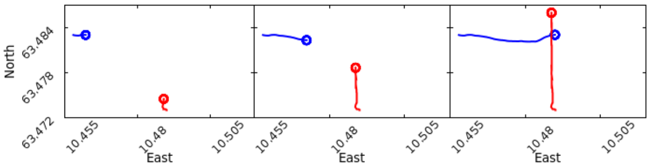
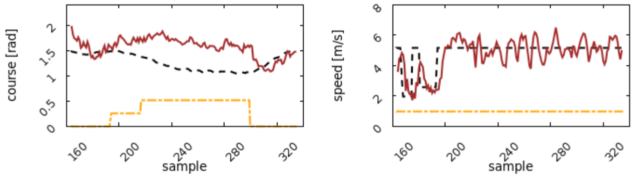


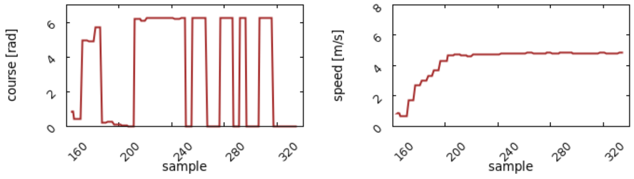
FIG. 4. Head-on situation: Planned path of ASV (– – –) and actual trajectories of ASV (——) and obstacle vessel (——). This image is a snapshot from the video accompanying the article.



(a) Trajectories of the ASV (——) and the obstacle vessel (——)



(b) Desired value from guidance (– – –), COLAV modification (–·–), and measured value (——)



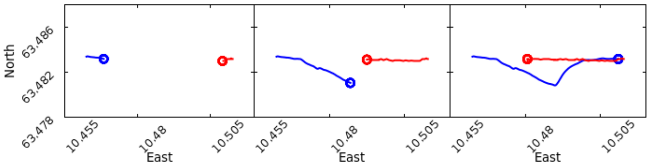
(c) Obstacle course and speed values from AIS

FIG. 5. Obstacle vessel crossing from starboard.

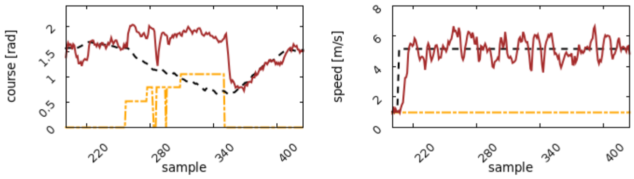
280). Although the control behavior parameterization and tuning prioritize course modification, the speed is reduced in critical situations (cf. Fig. 8b).

Although the environmental disturbances are not explicitly accounted for in the COLAV implementation, the results confirm the viability of the MPC COLAV method. An important observation is that an acceptable level of robustness to disturbances is achieved due to the choice of parameterization of alternative control behaviors (χ_m, u_m) and the cost function components (see Section III) that ensure that the control behavior remains unchanged unless an alternative behavior provides a significant reduction in the collision hazard.

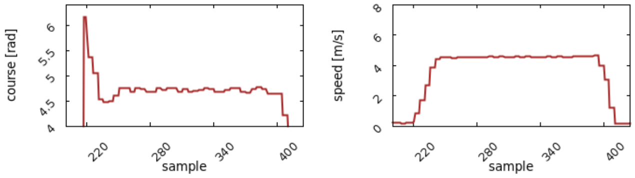
Considering the weight and design of the ASV, the weather conditions also introduce significant uncertainty into the guidance system. Although the obstacle vessel is much heavier, it is less maneuverable at low speeds and therefore



(a) Trajectories of the ASV (——) and the obstacle vessel (——)

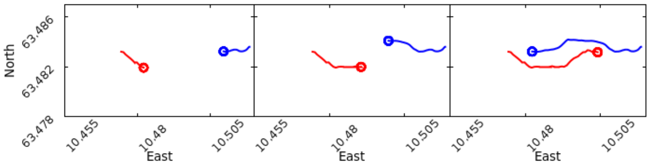


(b) Desired value from guidance (– – –), COLAV modification (–·–), and measured value (——)

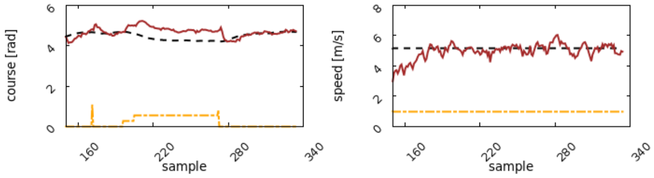


(c) Obstacle course and speed values from AIS

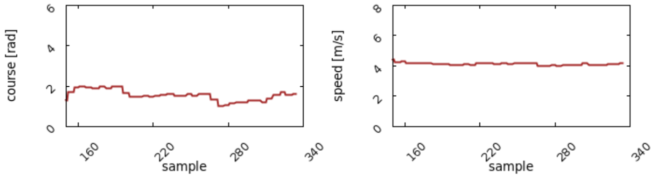
FIG. 6. Obstacle vessel approaching head-on.The ASV performs a COLREGS compliant avoidance maneuver, before returning to its planned path.



(a) Trajectories of the ASV (——) and the obstacle vessel (——)

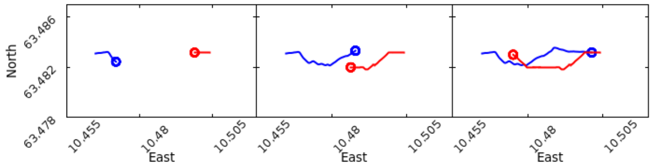


(b) Desired value from guidance (– – –), COLAV modification (–·–), and measured value (——)

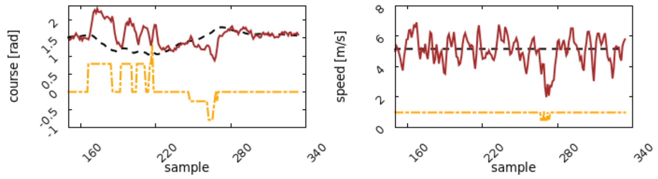


(c) Obstacle course and speed values from AIS

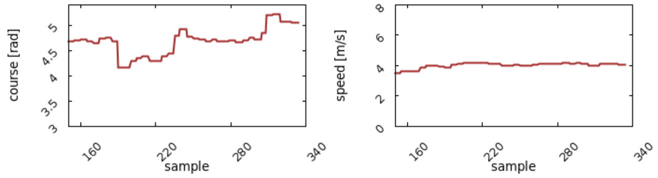
FIG. 7. Obstacle vessel approaching head-on and turns to starboard. In this situation both vessels act according to COLREGS.



(a) Trajectories of the ASV (——) and the obstacle vessel (——)



(b) Desired value from guidance (– – –), COLAV modification (–·–·), and measured value (——)



(c) Obstacle course and speed values from AIS

FIG. 8. Obstacle vessel approaching head-on and turns to port, contrary to what is advised by COLREGS. The ASV then performs a necessary reactive maneuver to port.

not always successful in keeping a steady course. Consequently, the experiments provide results for scenarios where the COLAV system's capability of predicting the obstacle's future trajectory is uncertain. The results suggest that a more careful tuning of the cost function weights is needed to avoid spikes in the COLAV modifications, see Fig. 6b, 7b and 8b.

V. CONCLUSIONS

This paper has presented a collision avoidance system capable of avoiding dynamic obstacles in a COLREGS-compliant manner while following a predefined path. The suggested COLAV system does not include a model of the ASV's guidance system and can easily be implemented in already existing guidance and control architectures, without the need for further knowledge of the guidance system's behavior. This also makes it possible to switch between different guidance strategies while the COLAV is running. The model of the ASV dynamics used in the experiments was a generic kinematic model.

A transitional cost was proposed to increase the incentive to continue an already started COLREGS maneuver and alleviate the oscillating behavior displayed in overtaking and crossing situations. The tests also showed that, when drastic changes in the situation are detected, it is also capable of aborting the maneuver and perform a reactive maneuver not normally sanctioned by COLREGS to avoid collision.

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