Towards Optimal Distributed Node Scheduling in a Multihop Wireless Network through Local Voting

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***Abstract*—In a multihop wireless network, it is crucial but challenging to schedule transmissions in an efficient and fair manner. In this paper, a novel distributed node scheduling algorithm, called *Local Voting*, is proposed. This algorithm tries to semi-equalize the load (defined as the ratio of the queue length over the number of allocated slots) through slot reallocation based on local information exchange. The algorithm stems from the finding that the shortest delivery time or delay is obtained when the load is semi-equalized throughout the network. In addition, we prove that, with Local Voting, the network system converges asymptotically towards the optimal scheduling. Moreover, through extensive simulations, the performance of Local Voting is further investigated in comparison with several representative scheduling algorithms from the literature. Simulation results show that the proposed algorithm achieves better performance than the other distributed algorithms in terms of average delay, maximum delay, and fairness. Despite being distributed, the performance of *Local Voting* is also found to be very close to a centralized algorithm that is deemed to have the optimal performance.**

***Index Terms*—Multihop wireless networks, Node scheduling algorithm, Wireless mesh networks, Load balancing.**

I. Introduction

MULTIHOP wireless networks are a paradigm in wireless connectivity which has been used successfully in a variety of network settings, including ad-hoc networks [1], wireless sensor networks [2], and wireless mesh networks [3]. In such networks, the wireless devices may communicate with each other in a peer-to-peer fashion and form a network, where intermediate wireless nodes may act as routers and forward traffic to other nodes in the network [4].

Due to their many practical advantages and their wide use, there have been a lot of studies on the performance of multihop wireless networks. For example, the connectivity of a multihop wireless network has been studied under various channel models in [4], [5]. Furthermore, their capacity has been studied analytically in [6]–[9]. In addition, the stability properties of scheduling policies for maximum throughput in multihop radio networks have been studied in [10], [11]. Also, a centralized scheduling algorithm that emphasizes on fairness has been proposed in [12]. In [13], the authors focused on the joint scheduling and routing problem with load balancing in multi-radio, multi-channel and multi-hop wireless mesh networks. They also designed a cross-layer algorithm by taking into account throughput increase with load balancing. Algorithms for joint power control, scheduling, and routing have been introduced in [14], [15]. In [16], the load balancing problem in a dense wireless multihop network is formulated where the authors presented a general framework for analyzing the traffic load resulting from a given set of paths and traffic demands.

Some more recent literature works include [17]–[25]. In [17], the authors present the state of the art in Time Division Multiple Access (TDMA) scheduling for wireless multihop network. Reference [18] proposes Genetic Algorithm for finding Collision Free Set (GACFS) which is a co-evolutionary genetic algorithm that solves the Broadcast Scheduling Problem (BSP) in order to optimize the slot assignment algorithm in WiMAX mesh networks. It is a centralized approach and does not take into consideration the traffic requirements or the load in the network. Another scheduling solution for wireless mesh networks based on a memetic algorithm that does not consider the traffic requirements is presented in [21]. An improved memetic algorithm is applied for energy-efficient sensor scheduling in [26]. Reference [20] investigates the mini-slot scheduling problem in TDMA based wireless mesh networks, and it proposes a decentralized algorithm for assigning mini-slots to nodes according to their traffic requirements. The authors in [19] propose a scheduling scheme for multicast communications where a conflict-free graph is created dynamically based on each transmission's destinations. Reference [22] presents a probabilistic topology transparent model for multicast and broadcast transmissions in mobile ad-hoc networks. The novelty of the scheme is that instead of guaranteeing that at least one conflict-free time slot is assigned to each node, it only tries to bring the probability of successful transmission above a threshold. The authors have further presented performance improvement for broadcasting in [27]. Another topology transparent scheduling algorithm is presented in [24]. The algorithm is not traffic dependent, and the achieved throughput is lower than the optimal mainly due to the requirement for a guaranteed slot for each node. Reference [23] proposes a distributed scheduling scheme for wireless sensor networks (WSNs). Finally, the *NP*-hardness of the minimum latency broadcast scheduling problem is proved in [25] under the Signal-to-Interference-plus-Noise-Ratio (SINR) model. Two distributed deterministic algorithms for global broadcasting based on the SINR model are presented in [28].

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Efficient traffic load balancing and channel access are essential to harness the dense and increasingly heterogeneous deployment of next generation 5G wireless infrastructure [29]. Channel access in 5G networks faces inherent challenges associated with the current cellular networks [30], e.g. fairness, adaptive rate control, resource reservation, real-time traffic support, scalability, throughput, and delay. For instance, being able to do frequency and time slot allocation enables more adaptive and sophisticated multi-domain interference management techniques [31], [32]. In [32], TDMA is used to mitigate the co-tier interference from time domain perspective in ultra-dense small cell networks. The modeling and the optimization of load balancing plays a crucial role in the resource allocation in the next generation cellular networks [33].

In this paper, we focus on the problem of *node scheduling* in multihop wireless networks. In the node scheduling problem, each transmission opportunity is assigned to a set of nodes in a such way which ensures that there will be no mutual interference among any transmitting nodes. More specifically, under node scheduling, two nodes can be assigned the same time slot (and transmit simultaneously) if they do not have any common neighbors. We introduce the *Local Voting* algorithm. The idea behind the algorithm was originated by the observation that the total delivery time in a network can be minimized, if the ratio of the queue length over the number of allocated slots is semi-equalized throughout the network. We call this ratio the *load* of each node. The proposed algorithm allows for neighboring nodes to exchange slots in a manner that eventually semi-equalizes the load in the network. The number of slots that are exchanged is determined by the relation between the load of each node and its neighbors, under the limitation that certain slot exchanges are not possible due to interference with other nodes. The preliminary results were presented in [34]. This paper presents new algorithm and an analysis of its performance, as well as new simulation results. The simulation results of the comparative study between *Local Voting* and other representative algorithms from the literature show that *Local Voting* achieves the shortest end-to-end delivery time and greatest fairness compared to other distributed algorithms for different network densities. We also show that its performance is very close to a centralized algorithm. The presented algorithm is a modification of the Local Voting protocol with non-vanishing to zero step-size which was suggested in [35]. It belongs to the more general class of stochastic approximation decentralized algorithms which have been studied early in [36], [37] with decreasing to zero step-size. However, changing the traffic parameters leads to an unsteady setting of the optimization problem. For similar cases the stochastic approximation with constant (or non-vanishing to zero step-size) is useful [38], [39].

The paper is organized as follows: Section II describes thoroughly the network model. Section III presents the proposed *Local Voting* algorithm where Section III-B presents an analysis of the performance of the algorithm in terms of achieving consensus. The simulation results in Section IV compare the performance of the proposed algorithm with other algorithms from the literature. Finally, Section V concludes the paper.

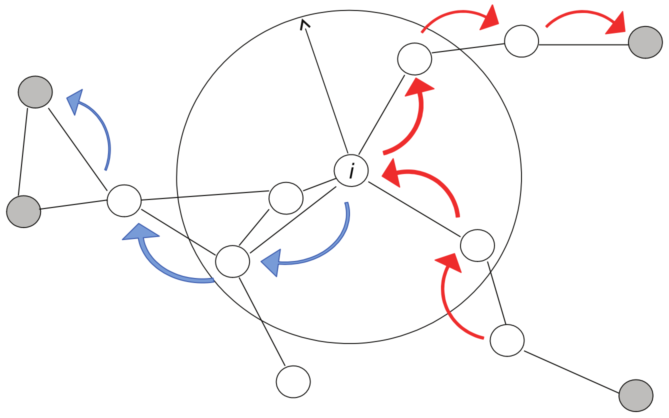


Fig. 1. A multihop wireless network where the communication range and the interference range of node i are denoted by the circle. The nodes with white background are two-hop neighbors of node i, and the nodes with gray background are outside the two-hop neighborhood of node i.

II. Network Model and Load Balancing

Consider a network that can be represented by a graph $\mathcal{G} = (N, E)$. N is the set of all wireless nodes that communicate over a shared wireless channel, i.e. $N = \{1, 2, \ldots, n\}$. E is the set of directional but symmetric edges which exist between two nodes if a broadcast from one node may cause interference on the other node. We use the terms edges and links interchangeably. Access on the channel is considered to follow a paradigm of time division multiple access. There is no spatial movement of the nodes.

The considered scheduling algorithm is a *node scheduling* algorithm, i.e. each slot is allocated to a node, instead of a communication link. We study a simple protocol interference model where two nodes are one-hop neighbors as long as their distance is less than the communication range. The interference range is considered to be equal to the communication range, and both values are considered constant throughout the network. A multihop network is presented in Fig. 1 where the nodes within the circle of node i are *one-hop neighbors* of node i, and the *one-hop neighborhood* of node i is denoted by $N_i^{(1)}$. We also define $N_i^{(2)}$ as a *two-hop neighborhood* of node i, i.e. the set of all the nodes that are neighbors to node i or that have a common neighbor with node i. Since the inclusion $N_i^{(1)} \subset N_i^{(2)}$ holds, the nodes with white background in Fig. 1 are *two-hop neighbors* of node i. The nodes presented with gray background are outside the two-hop neighborhood of node i. Note that the nodes within the circle of node i are also within the interference range of node i because the interference range and the communication range are equal. Two flows are depicted with red and blue arrows, respectively. According to the protocol interference model, two nodes can be assigned the same transmission slot, with no collision, as long as they do not have any common neighbors. Otherwise, a collision would happen, resulting in data loss. Node scheduling tries to guarantee that no such collision happens.

Each node contains a queue with packets to be transmitted, and the internal scheduling on the queue is first-come-first-serve. The maximum length of each queue is considered to be unbounded. Each node also has a set of slots that have been assigned to it, and neighboring nodes may exchange slots.

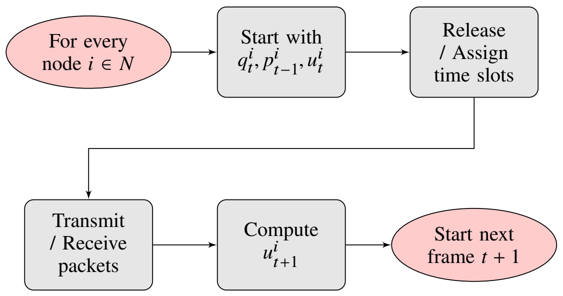

For every node $i \in N$
Start with q_t^i, p_{t-1}^i, u_t^i
Release / Assign time slots
Transmit / Receive packets
Compute u_{t+1}^i
Start next frame $t+1$

Fig. 2. Procedure of slot assignment and transmission of packets during frame t.

Time is divided into frames where each frame is denoted with t and $t = 0, 1, \ldots$. In addition, each frame t is divided into time slots. The number of time slots in each frame is considered to be fixed and equal to $|S|$ where all time slots have the same duration. The number of slots in a frame $|S|$ is considered to be large enough for every node to be able to obtain at least one slot in each frame, if needed. This value can be determined by the chromatic number of the graph, where there is an edge between any two-hop neighbors in the original graph $\mathcal{G}$. The Greedy Coloring Theorem provides an upper bound for this chromatic number which is equal to $\max_{i \in N} |N_i^{(2)}| + 1$ [40]. The duration of a time slot is sufficient to transmit a single packet.

The transmission schedule of the network is defined as,

$$X_t^{i,s} = \begin{cases} 1, & \text{if a slot } s \in S \text{ is assigned to a node } i \in N; \\ 0, & \text{otherwise;} \end{cases} \tag{1}$$

for $t \geq 0$, with $X_0^{i,s} = 0$ by convention.

The transmission schedule is *conflict-free*, if for any t,

$$X_t^{i,s} X_t^{j,s} = 0, \forall s \in S, i \in N, j \in N_i^{(2)}, i \neq j. \tag{2}$$

For each $i \in N$, let $\tilde{N}_t^i$ denote a set of such nodes j that node i can exchange slots with node j and the produced schedule remains conflict-free and E_t denote the corresponding subset of edges.

The objective of this work is to design a load balancing node scheduling strategy to schedule nodes' transmissions in such a way that the minimum maximal (min-max) nodal delay is achieved. We will study the following scheme of slot assignment and transmission of packets (see Fig. 2).

At the beginning of frame t, the state of each node i in the network is described by three characteristics:

- q_t^i is the queue length, counted as the number of slots needed to transmit all packets at node i at frame t;
- p_{t-1}^i is the number of slots assigned to node i at the previous frame $t-1$, i.e. $p_{t-1}^i = \sum_{s=1}^{|S|} X_{t-1}^{i,s}$;
- u_t^i is the number of time slots which are assigned ($u_t^i > 0$) or released ($u_t^i < 0$) by node i at the beginning of frame t (u_t^i is calculated by the scheduling policy).

For each node i, the slot assignment starts with releasing time slots according to the scheduling policy when $u_t^i < 0$, or otherwise with assigning slots to node i from free time slots or through redistribution of time slots with its neighbors. After that, the transmission of packets begins. During frame t new packets arrive. At the end of frame t, the scheduling policy calculates $\{u_{t+1}^i\}_{i \in N}$ locally based on the available data.

So, the dynamics of each node is described by

$$\begin{aligned} p_t^i &= p_{t-1}^i + n_t^i + u_t^i, \ i \in N, \ t = 0, 1, \ldots, \\ q_{t+1}^i &= \max\{0, q_t^i - p_t^i\} + z_t^i, \end{aligned} \tag{3}$$

where n_t^i is the number of free slots that are allocated to node i or the number of slots that are released due to an empty queue, and u_t^i is the number of time slots that node i gains or loses at frame t due to the adopted slot scheduling strategy. These are slots that are exchanged between neighboring nodes, while z_t^i is the number of slots needed to transmit new packets received by node i at frame t, either received as new packets from the upper layers or from a neighboring node. If $q_t^i = 0$, then no slot is allocated to the node i, i.e. we set $p_t^i = 0$.

For reader's convenience, we provide Table I with the key notations used in this paper.

A. *Load Balancing*

The ultimate objective of a scheduling algorithm in a multihop network is the packet flows to be delivered from the source to the destination in a short time. This can be measured by the end-to-end delay per packet, the end-to-end delivery time of a packet burst, the throughput of each flow, and the fairness in distributing the resources among the competing flows. In general, the problem of optimal scheduling in terms of approximating the optimal throughput in a multihop wireless network is *NP*-hard as it is proven in [41]. A specific challenge of having such a scheduling algorithm is that it needs to examine per flow information and use this information to schedule flows at every node which we believe is difficult to implement.

For this reason, we do not optimize the end-to-end delay for the whole wireless network, but instead we focus on optimizing the nodal (per-node) delay in each transmitter. The proposed *Local Voting* algorithm may be considered as a compromise, where we do node scheduling by using the slots without information about the individual flows. Since multihop end-to-end delay is the sum of nodal delays on the end-to-end path, we expect *Local Voting* to deliver also good multihop end-to-end delay performance. To validate this, the evaluation in Section 4 has been focused on multihop end-to-end delay, and the results indicate that *Local Voting* does give good or indeed better multihop end-to-end performance than various literature algorithms.

In the following we show that the nodal delay may be optimized (min-max), if the load of each node in the network is balanced. The load of node i at the beginning of frame t is defined as zero when $q_t^i = 0$, and otherwise it is defined as the ratio of the queue length q_t^i over the number of allocated slots p_t^i (note that slots are not assigned to nodes that have nothing to transmit in an optimal scheduling strategy, so we have $q_t^i = 0$ if $p_t^i = 0$), i.e.

$$x_i = \begin{cases} \left\lfloor \dfrac{q_i}{p_i} + 0.5 \right\rfloor, & \text{if } q_i > 0, \\ 0, & \text{if } q_i \text{ (and consequently } p_i) = 0. \end{cases}, \tag{4}$$

TABLE I
TABLE WITH NOTATIONS

$\mathcal{G} = (N, E)$	Graph of a network topology
i	Node
N	Set of nodes in the network
$\lvert N \rvert$	Number of nodes in the set N
E	Set of directional and symmetric edges between all two interfering nodes
S	Set of slots in a frame
$\lvert S \rvert$	Number of slots in a frame
s	Time slot
$X_t^{i,s}$	Transmission schedule for allocating slot s to node i at frame t
$N_i^{(1)}$	Set of one-hop neighbors of node i
$N_i^{(2)}$	Set of two-hop neighbors of node i
q_t^i	Queue length of node i at frame t
p_t^i	Number of slots assigned to node i at frame t
x_t^i	Load of node i at frame t
z_t^i	Number of required slots to transmit new packets received by node i at frame t
n_t^i	Number of free slots that are allocated to node i or released due to an empty queue at frame t
u_t^i	Number of slots that node i gains or releases at frame t
$\tilde{N}_t^i$	Set of neighbors that can exchange slots with node i at frame t
E_t	Set of edges between nodes that can exchange slots at frame t
A_t	Adjacency matrix corresponding to E_t
$a_t^{i,j}$	Weight of edge $(j, i) \in E_t$
$\mathcal{G}_{A_t}$	Graph defined by the adjacency matrix A_t
$E_{\max}$	Maximal set of communication links
$d^i(A)$	Weighted in-degree of node i (sum of i-th row of A)
$D(A)$	Diagonal matrix of weighted in-degree of A
$\mathcal{L}(A)$	Laplacian matrix of the graph $\mathcal{G}_A$
$\lambda_1, \ldots, \lambda_n$	Eigenvalues of the matrix $\mathcal{L}(A)$
E	Mathematical expectation
$\mathrm{E}_{\mathcal{F}_t}$	Conditional mathematical expectation with respect to the σ-algebra $\mathcal{F}_t$
A_{av}	Adjacency matrix of the averaged system
$a_{av}^{i,j}$	Mathematical expectation (average value) of $a_t^{i,j}$
$\lambda_2(A_{av})$	Second eigenvalue of the matrix B_{av} ordered by absolute magnitude
$[\cdot]$	Round function

where $[\cdot]$ is the round function (rounds a real number to the nearest integer). Using this definition we calculate the delay for each node i (in time slots) as $x_i \cdot |S|$.

Definition 1: Load balancing is the processes of equalizing the load between the nodes in the network by exchanging slots among them.

Definition 2: We define a conflict-free schedule as "nodally optimal" or just "optimal", if the maximum delay per node in the network is smaller or equal than the maximum per node delay for every other schedule (min-max).

Lemma 2.1: (Optimal schedules are maximal) An optimal schedule is a (or has an equivalent) maximal schedule in the sense that[1] $\nexists j \in N$ such that p_j can be increased without reducing p_k in at least one other node $k \in N$.

[1] Symbol $\nexists$ denotes the negation of existense $\exists$

Proof: Consider a schedule that is not maximal. That means there exists $j \in N$ such that p_j can be increased by one. For the new schedule, the delay for all the other nodes is unchanged (since we did not reduced slots for the other nodes). For node j, the new delay is $x_i' \cdot |S| = \left[\frac{q_i}{(p_i+1)} + 0.5\right] \cdot |S| \leq x_i \cdot |S|$. Thus, for every non-maximal schedule, there exists a maximal schedule that has smaller or equal maximum delay. ∎

Lemma 2.2: (Optimal schedules are balanced) Assume that node k is the most loaded node in the network, i.e $k = \mathrm{argmax}(x_i), i \in N$. For all optimal schedules, it holds $x_k \leq x_j/(1 - 1/p_j)$ for the load of the most loaded node k and the load of every other node j where $j \in \tilde{N}^k$.

Proof: Assume that an optimal schedule exists where for the most loaded node k, $x_k > x_j/(1 - 1/p_j)$ where $j \in \tilde{N}^k$. Since k is the most loaded node, the maximal delay for such a schedule is $x_k \cdot |S|$. Since node $j \in \tilde{N}^k$, it follows that a slot of node j can be reassigned to node k. After reassigning, the new load for node k is $[q_k/(p_k + 1) + 0.5]$, and the corresponding delay for node k is $\lceil q_k/(p_k + 1) \rceil \cdot |S| < x_k \cdot |S|$. In addition, node j loses a slot so the new delay for node j is $[q_j/(p_j - 1) + 0.5] \cdot |S| = [(q_j/p_j)/(1 - 1/p_j) + 0.5] \cdot |S| = [(q_j/p_j) + 0.5]/(1 - 1/p_j) \cdot |S| = x_j/(1 - 1/p_j) \cdot |S| < x_k \cdot |S|$. Thus, the new allocation has a maximal delay that is smaller than or equal to the maximal delay of the other allocation, so the allocation is not optimal. ∎

Based on the above reasoning, we design a load balancing strategy with two goals: 1) The produced schedule should be maximal, 2) The load in the schedule should be balanced in the sense of Lemma 2.2. For this reason, we define a slot exchange strategy that tries to equalize the load through load balancing, and in the next Section III-B we prove that the *Local Voting* algorithm converges to a such solution.

It should be noted that, in general, a schedule could be both maximal and balanced, but still not optimal. This is because there could exist a reallocation of the slots in the network that would produce a larger spectral efficiency. Optimizing the schedule in this sense would require finding a solution for the *NP*-complete broadcast scheduling problem. This is not easy, so for the purposes of this paper, we do not examine ways of escaping local optima and finding the global optimum. However, we can see from the simulation results that the performance of *Local Voting* is still better than the performance of other distributed algorithms that we compare with, and also we see that optimizing the maximal nodal delay also has a positive impact on the end-to-end delay.

Among all possible options for load balancing, the min-max nodal delay is achieved when all nonzero loads q_t^i/p_t^i are semi-equal. This comes as a result from the finding that the minimum expected nodal delay is achieved when the load in the network is equalized on nodes (Lemma 1 and Corollary 1 from [35]).

III. THE PROPOSED NODE SCHEDULING ALGORITHM: LOCAL VOTING

In the previous section we have shown that an optimal schedule has three properties: it is efficient, it is maximal, and it is balanced. These are the properties which guide us in the design of the *Local Voting* algorithm.

In order to be efficient, there should be no slots allocated to nodes that have an empty queue. For this reason, before the beginning of each frame, nodes with an empty queue release all time slots that they have reserved.

In order to be maximal, there should be no free time slot in the neighborhood of any node, if that node has a positive queue, and assigning the slot to the node would not cause a conflict with other nodes. In order to meet this objective, after the first step, free slots are allocated to the nodes that do not have an empty queue. Conflicts are resolved in a descending order of the load.

Finally, the third objective is to be balanced, which can be formulated with the following control goal: *to keep the ratio q_t^i/p_t^i semi-equal throughout the network (as much as possible) for the nodes i where the queue is not empty $q_t^i > 0$.* In other words, the number of slots assigned to each node should correspond to the amount of backlogged traffic. A consequent implication is that, in order to achieve this optimal strategy, we should be able to freely exchange slots among any two nodes in the network. However, in reality, it is not always possible due to the potential interference with other nodes in network. That is expressed through Eq. (2).

In the following, we propose a novel algorithm that adopts the local voting control strategy. For the proposed *Local Voting* algorithm, its semi-consensus properties with respect to the local balancing are proved in Section III-B.

A. The Proposed Algorithm: Local Voting

At the end of frame t, each node computes a scheduling policy. The u_{t+1}^i value is calculated as follows.

Each node uses the characteristics of its own state q_{t+1}^i, p_t^i and its neighbors' states q_{t+1}^j, p_t^j if $\tilde{N}_t^i \neq \emptyset$.

Let us for time frame t and for each node i, $i \in N : q_i^{t+1} > 0$, define semi-inverse load $\tilde{x}_t^i$: $\tilde{x}_t^i = p_i^t/q_i^{t+1}$, and consider the following modification in the already known *Local Voting (LV)* protocol [35]:

$$u_i^{t+1} = \left[\gamma \sum_{j \in \tilde{N}_t^i} a_t^{i,j} (\tilde{x}_t^j - \tilde{x}_t^i)\right] \tag{5}$$

where $\gamma > 0$ is a LV protocol step-size, and LV protocol matrix coefficients $a_t^{i,j}$:

$$a_t^{i,j} = \frac{q_{t+1}^j}{1 + \sum_{k \in \tilde{N}_t^i} \frac{q_{t+1}^k}{q_{t+1}^i}}.$$

Note, it is not so hard to see that

$$u_{t+1}^i = \left[\gamma \frac{\sum_{j \in \tilde{N}_t^i} q_{t+1}^i p_t^j - q_{t+1}^j p_t^i}{q_{t+1}^i + \sum_{j \in \tilde{N}_t^i} q_{t+1}^j}\right].$$

For all other case we define $u_i^{t+1} = 0$ and $\tilde{x}_t^i = p_i^t$. We set $a_t^{i,j} = 0$ for other pairs i, j and denote the matrix of the protocol as $A_t = [a_t^{i,j}]$. The elements $a_t^{i,j}$ in adjacency matrix A_t are $a_t^{i,j} > 0$ if node i can exchange slots with node j and the produced schedule remains conflict-free; and $a_t^{i,j} = 0$ otherwise.

When $\gamma = 1$, Eq. (5) has a form:

$$u_{t+1}^i = \left[q_{t+1}^i \times \frac{p_t^i + \sum_{j \in \tilde{N}_t^i} p_t^j}{q_{t+1}^i + \sum_{j \in \tilde{N}_t^i} q_{t+1}^j}\right] - p_t^i.$$

Example. Let's consider the network with $|S| = 50$, three nodes, all neighbors with each other (single hop), with the following initial queue lengths of $q_0^1 = 400$, $q_0^2 = 100$, $q_0^3 = 310$, and $p_0^1 = 20$, $p_0^2 = 20$, $p_0^3 = 10$. The initial values for the loads are the following: $x_0^1 = 20$, $x_0^2 = 5$, $x_0^3 = 32$.

The queue lengths at the end of frame $t = 0$ will be $q_1^1 = 400 - 20 = 380$, $q_1^2 = 100 - 20 = 80$, $q_1^3 = 310 - 10 = 300$. Using Eq. (5) we get

$$\begin{aligned} u_1^1 &= [380 \cdot (20 + 20 + 10)/(380 + 80 + 300)] - 20 &&= 5, \\ u_1^2 &= [80 \cdot (20 + 20 + 10)/(380 + 80 + 300)] - 20 &&= -15, \\ u_1^3 &= [300 \cdot (20 + 20 + 10)/(380 + 80 + 300)] - 10 &&= 10, \end{aligned}$$

and we have three semi-equal loads

$$x_1^1 = 380/25 = 15.2,\ x_1^2 = 80/5 = 16,\ x_1^3 = 300/20 \approx 15$$

Eventually, node i gains a slot in the following scenarios:

- Its queue length is positive and there exists an available time slot that is not allocated to one-hop or two-hop neighbors of node i;
- Its queue length is positive and there exists a neighbor $j \in \tilde{N}_t^i$ that has a value u_t^j lower than zero.

It is important to note that the quantities in protocol (5) are discrete-values, i.e. the state and other relevant quantities may only take a countable set of values. In that case, it makes sense to consider a quantised consensus problem [42], [43].

The proposed *Local Voting* algorithm consists of two functions: requesting and releasing free time slots, and load balancing.

For the first function (Fig. 3) nodes are examined sequentially at the beginning of each frame. If a node has an empty queue, then it releases all its time slots. If a node has a positive backlog (i.e. its queue is not empty), then it is given time slots. All time slots are examined sequentially, and the first available time slots that are found, which are not reserved by one-hop or two-hop neighbors for transmission, are allocated to the node. The message exchanges for requesting and releasing slots are considered equivalent to message exchanges in the DRAND algorithm [44]. If no available slot is found (all slots have been allocated to one-hop or two-hop neighbors of the examined node), then no new slot is allocated to the node. On the contrary, if the queue of the node is found to be empty and the node has allocated slots, then all slots are released.

The *load balancing* function (Fig. 4) is invoked in order to achieve the objective of keeping the load balanced. Every node $i \in N$ has a value u_t^i (from the scheduling policy calculated at the end of the previous frame) which determines how many slots the node should ideally gain or lose by the load balancing function. If a node has a positive u_t^i value, then it checks if any of its neighbors has a load lower that its own and may give a slot to it without causing a conflict. Note that this is not always the case, because the requesting node may not be able to obtain a slot if one of its other one-hop or two-hop

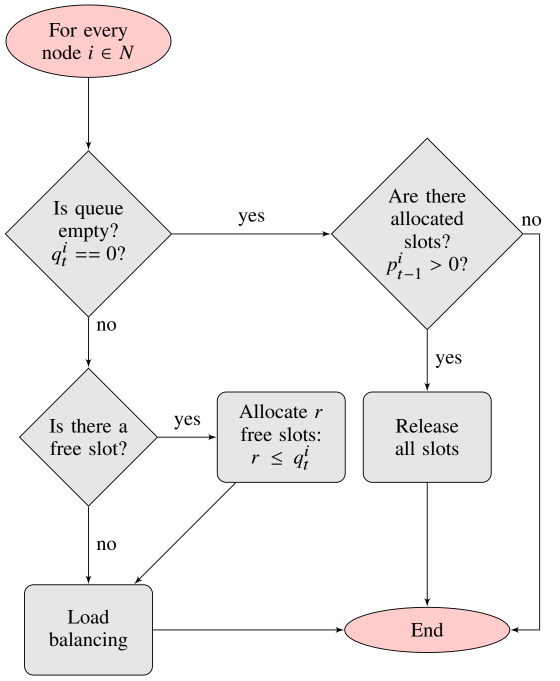

For every node $i \in N$
Is queue empty? $q_t^i == 0$?
yes
Are there allocated slots? $p_{t-1}^i > 0$?
no
no
yes
Is there a free slot?
yes
Allocate r free slots: $r \le q_t^i$
Release all slots
no
Load balancing
End

Fig. 3. Requesting and releasing time slots function for the *Local Voting* algorithm.

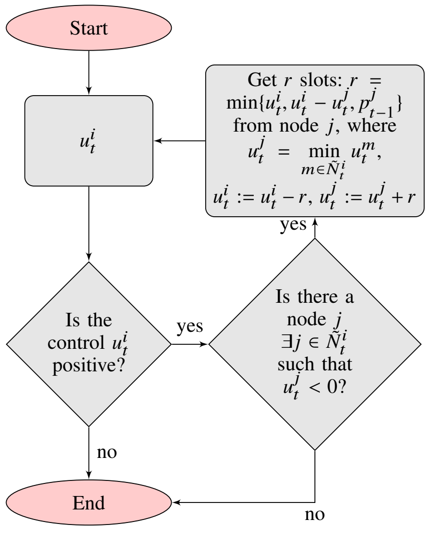

Start
u_t^i
Get r slots: $r = \min\{u_t^i, u_t^i - u_t^j, p_{t-1}^j\}$ from node j, where $u_t^j = \min_{m \in \tilde{N}_t^i} u_t^m$, $u_t^i := u_t^i - r$, $u_t^j := u_t^j + r$
yes
Is the control u_t^i positive?
yes
Is there a node j $\exists j \in \tilde{N}_t^i$ such that $u_t^j < 0$?
no
End
no

Fig. 4. Load balancing function for the *Local Voting* algorithm.

neighbors has also allocated the same slot. The neighbor with the smallest u_t^j value gives slots to node i. After the exchange u_t^i is reduced by $r = \min\{u_t^i, u_t^i - u_t^j, p_{t-1}^j\}$, and u_t^j is increased by r. This procedure is repeated until u_t^i is positive, or until none of the neighbors of node i can give any slots to node i without causing a conflict. In this way, in general, slots are removed from nodes with lower load and are offered to nodes with higher load, and eventually the load between nodes will reach a common value, i.e. semi-consensus will be achieved.

B. Consensus Properties of Local Voting

1) Notation: For the considered network, $N_i^{(1)}$ and $N_i^{(2)}$ do not change over time since there is no spatial movement of the nodes. However, the network changes over time due to the slot allocation which is dynamic. Taking this into consideration, we describe the structure of the dynamic network (network topology) using a sequence of directed graphs $\mathcal{G}_{A_t} = \{(N, E_t)\}_{t \ge 0}$, where $E_t \subseteq E$. In the considered case, E_t defines a subset which consists of links between the nodes that can exchange slots at time t. Note that these directed graphs $\mathcal{G}_{A_t}$ are not the same as the communication graph $\mathcal{G}$. Instead, they define to which of the other nodes a node can offer a slot. More specifically, if there is an edge from node i to node j in $\mathcal{G}_{A_t}$, it means that node i has a slot to offer to node j, and after the exchange the produced schedule will still remain conflict-free with respect to Eq. (2).

$A_t = [a_t^{i,j}]$ is the corresponding adjacency matrix. As defined earlier, $\tilde{N}_t^i = \left\{j : a_t^{i,j} > 0\right\}$ denotes the set of neighbors of node $i \in N$ at time t, i.e. the set of neighbors that can exchange slots with node i. Generally, $\tilde{N}_t^i \ne \emptyset$ if $\exists s \in S : X_t^{i,s} = 1$ and $\forall k \in N_i^{(2)} \cup i,\quad X_t^{i,s} X_t^{k,s} = 0$. Note that in contrast to $N_i^{(1)}$ and $N_i^{(2)}$, the set $\tilde{N}_t^i \subset N_i^{(1)}$ changes in time. Let $E_{\max} = \{(j,i) : \sup_{t \ge 0} a_t^{i,j} > 0\}$ stand for the maximal set of communication links (a set of edges that appear with non-zero probability in $\tilde{N}_t^i$). For any matrix A we define the weighted in-degree of node i as a sum of i-th row of the matrix A: $d^i(A) = \sum_{j=1}^n a^{i,j}$, and $D(A) = \text{diag}\{d^i(A)\}$ as the corresponding diagonal matrix. Let $\mathcal{L}(A) = D(A) - A$ denote the Laplacian matrix of the graph $\mathcal{G}_A$, and $\lambda_1, \dots, \lambda_n$ stand for the eigenvalues of the matrix $\mathcal{L}(A)$ ordered by increasing absolute magnitudes. The symbol $d_{\max}(A)$ accounts for a maximum in-degree of the graph $\mathcal{G}_A$.

2) Assumptions: Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space corresponding to the sample space, the collection of all events, and the probability measure, respectively, and $\{\mathcal{F}_t\}$ be a sequence of σ-algebras which are generated by $\{q_k^i, p_k^i\}_{i=1,\dots,n,k=1,\dots,t}$. The symbol E accounts for the mathematical expectation, $\mathrm{E}_{\mathcal{F}_t}$ is a conditional mathematical expectation with respect to the σ-algebra $\mathcal{F}_t$, and the following assumptions are satisfied:

A1. **a)** For all $i \in N, j \in N_{\max}^i$ an appearance of "variable" edges (j,i) in the graph $\mathcal{G}_{A_t}$ is an independent random event. $N_{\max}^i$ is defined by the topology $E_{\max}$.

Denote by $a_{av}^{i,j}$ the average value of $a_t^{i,j}$. Let A_{av} stand for the adjacency matrix of averaged values $a_{av}^{i,j}$.

b) For all $i \in N,\ t = 0, 1, \dots$, the number of slots z_t^i required to transmit new packets received by node i at frame t in Eq. (3) are random variables do not depend on $\mathcal{F}_t$.

Note that new packets refer to new incoming packets from new connections and new packets arrived from neighbors.

c) For all $i \in N,\ j \in \tilde{N}_t^i$ and $b_t^{i,j} = \frac{q_{t+1}^j}{q_{t+1}^i + \sum_{k \in \tilde{N}_t^i} q_{t+1}^k}$ there exist conditional average values $b_{av}^{i,j} = \mathrm{E}_{\mathcal{F}_{t-1}}(b_t^{i,j})$, which do not depend on t. Note that $b_t^{i,j} = a_t^{i,j}/q_{t+1}^i$ and $B_t = A_t Q_{t+1}^{-1}$

where $Q_{t+1} = \operatorname{diag}\{\max\{1, q^i_{t+1}\}\}$.

There exists a positive definite matrix $Q_{av} > 0$ such that $A_{av} = B_{av}Q_{av}$, and $\mathrm{E}\|Q^{-1}_{t+1} - Q^{-1}_{av}\|^2 \le \sigma^2_q$.

d) For matrices $B_t = [b^{i,j}_t]$ and $B_{av} = [b^{i,j}_{av}]$ there exists a matrix R such that

$$\mathrm{E}(\mathcal{L}(B_{av}) - \mathcal{L}(B_t))^{\mathrm{T}}(\mathcal{L}(B_{av}) - \mathcal{L}(B_t)) \le R,$$

and its maximum on the absolute magnitude eigenvalue: $\lambda_{\max}(R) < \infty$.

e) For all $i \in N$, $t = 0, 1, \ldots$, the errors of rounding in LV protocol (5)

$$w^i_t = \gamma \sum_{j\in \bar{N}^i_t} a^{i,j}_t(\tilde{x}^j_t - \tilde{x}^i_t) - [\gamma \sum_{j\in \bar{N}^i_t} a^{i,j}_t(\tilde{x}^j_t - \tilde{x}^i_t)] \tag{6}$$

are centered, independent, and they have a bounded variance $\mathrm{E}(w^i_t)^2 = \sigma^2_w$ and independent of $\mathcal{F}_t$.

f) For all $i \in N$, $t = 0, 1, \ldots$, the variables e^i_{t+1} are random, independent and identically distributed with mean values $\bar{e}^i$ and variance σ^2_e, and they do not depend on $\mathcal{F}_t$.

All variables z^i_t, e^i_{t+1}, w^i_t are mutually independent.

We assume that the following assumption for the average matrix of the network topology is satisfied:

A2: Graph $\mathcal{G}_{A_{av}}$ has a spanning tree, and for any edge $(j, i) \in E_{\max}$ it holds $a^{i,j}_{av} > 0$.

3) Mean Square ϵ-consensus: Consider the state vectors $\tilde{\mathbf{x}}_t \in \mathbb{R}^n$, $t = 0, 1, \ldots$, which consist of the elements $\tilde{x}^1_t, \tilde{x}^2_t, \ldots, \tilde{x}^n_t$. Note that if state values $\tilde{x}^i_t$, $i \in N$, are semi-equal then the inverse values q^i_{t+1}/p^i_t, $i \in N$ for $q^i_{t+1} > 0, p^i_t > 0$ are semi-equal.

The following theorem gives the conditions when the sequence $\{\mathbf{x}_t\}$ converges asymptotically in the mean squared sense to some bounded set around a trajectory $\bar{\mathbf{x}}_t$ of the corresponding averaged model

$$\bar{\mathbf{x}}_{t+1} = \bar{\mathbf{x}}_t - \gamma\mathcal{L}(B_{av})\bar{\mathbf{x}}_t + Q^{-1}_{av}\bar{\mathbf{e}}_{t+1},\ \bar{\mathbf{x}}_0 = 0(= \mathbf{x}_0). \tag{7}$$

If $\bar{\mathbf{e}}_t \equiv 0$ then $\bar{\mathbf{x}}_t \to \bar{\mathbf{x}}_\star$ as $t \to \infty$, and $\bar{\mathbf{x}}_\star$ is a left eigenvector of the matrix A_{av} corresponding to its zero eigenvalue. Note that if A_{av} is a symmetric matrix, then $\bar{\mathbf{x}}_\star$ is equal to $x_\star\mathbf{1}_n$ where $\mathbf{1}_n$ is n-vector of ones, i.e. we will get the asymptotical consensus for the state vectors $\{\bar{\mathbf{x}}_t\}$.

Theorem 1. If Assumptions **A1–A2** are satisfied and

$$0 < \gamma < \frac{1}{d_{\max}(B_{av})}, \tag{8}$$

then

$$\rho = (1 - \lambda_2(B_{av}))^2 < 1 \tag{9}$$

and the trajectory $\{\bar{\mathbf{x}}_t\}$ of the system (7) converges to the vector $\bar{\mathbf{x}}_\star$ which is a left eigenvector of the matrix $A_{\max}$ corresponding to its zero eigenvalues, and the following inequality holds:

$$\mathrm{E}\|\tilde{\mathbf{x}}_{t+1} - \bar{\mathbf{x}}_{t+1}\|^2 \le 2(\frac{\Delta}{1-\rho} + \rho^t\mathrm{E}\|\mathbf{p}_0\|^2 + \sigma^2_q\|Q_{av}\bar{\mathbf{x}}_{t+1}\|^2), \tag{10}$$

where

$$\Delta = n(\lambda_{\max}(R)|S| + \sigma^2_e + \sigma^2_w).$$

If $t \to \infty$, **then** the asymptotic mean square ε-consensus is achieved with

$$\varepsilon \le 2\frac{\Delta}{1-\rho} + 2\sigma^2_q\|Q_{av}\bar{\mathbf{x}}_{t+1}\|^2.$$

TABLE II
SIMULATION SETUP

Parameter	**Value**
Number of Nodes	100
Transmission/Interference range	10 units
Topology size	100 x 100 units
Frame length	10 time units
Number of concurrent connections	1 - 30
Number of packets per connection	100
Packet generation interval	Every 5 slots
Number of iterations	500

Proof is in the Appendix.

Theorem 1 shows that our protocol (5) provides an approximate consensus, i.e. gives an almost optimal behavior of the system.

IV. EVALUATION

We have performed a set of simulations in order to evaluate the performance of different scheduling algorithms. These simulations are carried out by using a custom–built, event-driven simulation tool developed in Java. The simulation setup is summarized in Table II.

Although several routing algorithms for load balancing in multihop networks exist, e.g. [45], in this paper we focus on the interaction of scheduling and load balancing algorithms. The routing in the network is considered to follow a simple shortest path routing algorithm.

A. The Simulation Tool

The source code that was developed for evaluating different scheduling algorithms has been made open source and is available.[2] The scripts for running the simulations and producing the results have also been made available.[3]

The simulation tool focuses on the evaluation of the scheduling algorithms. There are two types of scenarios that were evaluated. In the first class of scenarios, a variable number of connections is considered, each connection starts with a fixed number of packets. This represents the response to a sudden burst of traffic. Different load in the network is calibrated by changing the number of connections. The simulation is executed until all packets have reached their destinations. In the second class of scenarios, connections are added constantly, following a Poisson process. The load is calibrated by changing the connection arrival rate. This scenario is executed for a fixed time duration.

The measured metrics for each connection are:

- the delivery time, which is the time needed for all packets of a connection to reach their final destination;
- The delay, which is time from the moment each packet is generated until it has been received by its final destination;

[2]https://github.com/djvergad/local_voting_src
[3]https://github.com/djvergad/local_voting

- The throughput, which is the number of packets in the connection, divided over the time difference (in slots) between the start and the completion of the connection.

For each simulation we used the per connection metrics in order to take the average value between the connections per simulation, the maximum and minimum values for each connection, and the fairness, which was calculated using Jain's fairness index [46].

The simulation software is organized into four packages: the network package contains the implementation of the network elements and algorithms, the simulator package which contains the objects for implementing the discrete–event simulator, the application package which implements the network connections and the statistics gathering functionality, and the stability package which contains the different scenarios to be executed.

Some of the network functions that were implemented in the simulation tool include the following: a Connection object represents the application layer. For the purposes of this simulation, each connection has a random source and destination. It is initialized with a number of packets that are transmitted. For the first scenario (traffic bursts), each connection has 100 packets. For the steady state scenario, the number of packets are calculated based on an exponential distribution. The Node object represents each wireless station in the network. It contains an infinite FIFO queue that is common for all outgoing transmissions. It also has a routing table that is created using a shortest path algorithm. It contains a set of slot reservations, as well as X-Y coordinates. A Reservation object represents the slot reservation. It contains fields for the transmitting node, as well as the nodes that are blocked due to this reservation (all nodes in the two-hop neighborhood, except for the link-scheduling case). The Network object implements network functions, such as routing. The Scenario object contains the scenario to be executed, and defines the scheduler type, the transmission range, the number of time slots in each frame, the number of nodes in the network, and the size of the topology. Each Scheduler also has a different class which inherits from the TDMAScheduler class. The wireless channel is lossless (unless otherwise specified). Two nodes are one-hop neighbors if their distance is smaller than the transmission/interference range. All scheduling algorithms are conflict-free using the protocol interference model where two nodes are not scheduled to transmit as long as they are two-hop neighbors. We also consider a scenario with a link-scheduling algorithm where two transmissions are allowed to be concurrent, if each receiver receives at most one packet at a time.

B. Implemented Algorithms

In this subsection we briefly describe the operation of some algorithms for node scheduling from the literature. We have implemented these algorithms in our simulation platform, and compared their performance with the performance of *Local voting* algorithm.

A typical example of a distributed, traffic independent, topology dependent node scheduling algorithm is DRAND [44]. DRAND defines a communication protocol for obtaining a conflict-free schedule, using information from the two-hop neighborhood. The protocol assigns a single time slot to each node. The frame length is constant throughout the network, and it is determined by the maximum density of the nodes.

Another example of a distributed, traffic independent, topology dependent node scheduling algorithm is Lyui's algorithm [47], [48]. The algorithm first assigns a color to each node, using existing graph coloring techniques, with the limitation that two nodes are not assigned the same color if they are in the same two-hop neighborhood. Depending on the color that is assigned to a node, it is a candidate to transmit in any time slot for which $t \bmod p(c_k) = c_k \bmod p(c_k)$, where t is the time slot, c_k is the color assigned to node k, and $p(c_k)$ is the smallest power of 2 greater than or equal to c_k. Among these candidate nodes, in each two-hop neighborhood, the node with the largest color transmits. Therefore, in Lyui's algorithm, the nodes have more than one transmission opportunity in each frame, and there is no common frame length for the entire network. This makes slot assignment easier than in DRAND where the frame length must be known in advance. Lyui's algorithm also has better performance since the nodes can transmit more frequently, and the performance in sparse areas is not affected by larger node density elsewhere.

The *Load-Based Transmission Scheduling* (LoBaTS) [49] protocol is an example of a distributed, traffic dependent, topology dependent node scheduling scheme. It schedules the transmissions using Lyui's algorithm, but now instead of each node having a single color, additional colors can be assigned to nodes that experience high load. Each node maintains an estimate of the utilization of every node in its two-hop neighborhood. If the queue length exceeds a threshold, then the node tries to find an additional color that: a) is not assigned to any other node in the two-hop neighborhood, and b) does not cause the utilization of any other node in the neighborhood to exceed one. If such a color is found, then the node informs its neighbors about the new assignment, and it uses Lyui's algorithm to calculate the new transmission schedule.

A centralized, traffic dependent, topology dependent node scheduling algorithm was proposed in [50], called *Longest Queue First* (LQF) scheduling. According to this scheduling algorithm, nodes that have a packet to transmit are ordered according to their queue length in a descending order. The node with the longest queue is assigned to transmit in the current time slot. The remaining nodes are examined one by one, and any node that can transmit in the same time slot without causing a conflict is also assigned to transmit. The LQF policy is a simple heuristic for slot assignment, but it is not really practical, since it is centralized and the scheduler requires information about the queue lengths of all nodes in the network. Nevertheless, due to its simplicity and good performance, this algorithm has been often used for obtaining theoretical results and as a benchmark for comparing the performance of scheduling schemes. This algorithm is also known as the *Greedy Maximal Scheduling* algorithm, and its performance in terms of capacity has been analyzed in [51].

For the final scenario we used a link-scheduling variant of the LQF algorithm. In this version of the algorithm, again the nodes are examined in decreasing queue size. This time, however, whether the packet will conflict with other transmissions depends on the destination of the packet (since we have link scheduling). For this reason, we examine the packets from the start of the queue until we find the first packet that has a destination that doesn't cause a convict with the already scheduled transmissions in this slot. This packet is added to the slot, and the algorithm continues with the next node.

C. Delivery Time Scenario

In this experiment we investigate the delivery time of fixed sized messages, all initialized at the same time. The scenario has been repeated 500 times for each number of connections and for each of the algorithms. The total number of experiments is 500×30 connections $\times$ 5 protocols = 75000 experiments.

At the beginning of each simulation a varying number from 1 to 30 concurrent connections is generated with random sources and destination nodes. Each connection generates 1 packet every 5 time units until a total of 100 packets per connection is generated.

The results of the simulation are depicted in Fig. 5. For each number of concurrent connections and each algorithm, the above metrics are averaged over the 500 different simulation runs.

Fig. 5(a) depicts the average end-to-end delivery times among all the concurrent connections. The *LQF* and the *Local Voting* algorithms achieve the shortest delivery times, followed by *LoBaTS*. The *DRAND* and *Lyui* algorithms exhibit the worst performance, that is expected, since these two algorithms assign a fixed number of slots to each node without considering the traffic conditions. Fig. 5(b) presents the fairness in terms of the end-to-end delivery time among connections that is calculated using Jain's fairness index. The *LQF* and *Local Voting* algorithms clearly achieve superior fairness than other algorithms, regardless of the number of concurrent connections. This illustrates the significance of load balancing when considering fairness. The *LoBaTS* algorithm comes third (for most traffic loads) since it is also traffic dependent, while the *DRAND* follows it. *Lyui's* algorithm has the worst fairness, and this validates what is expected, since it assigns a different number of time slots according to the nodes' color, without considering the traffic conditions. The lack of fairness is noticeable for all algorithms except *LQF* and *Local Voting*, even when the number of connections is limited. As the number of connections increases, fairness deteriorates for all algorithms, but the difference in performance among the *Local Voting* and *LQF* algorithms and the remaining algorithms increases as the traffic load increases. It should be noted that even the *LQF* algorithm cannot achieve perfect fairness, and this is due to the different levels of congestion in various parts of the network. Namely, flows that encounter no (or only limited) congestion on their path have shorter delivery times than flows that encounter congestion, and this effect cannot be mitigated by scheduling policies alone.

Fig. 5(c) demonstrates the maximum end-to-end delivery time, which is the completion time of the connection that ends the latest. This is an important metric because it shows after how much time the system has delivered all packets to their destination, thus, it is related to the capacity of the network. The results confirm our expectations that the *LQF* algorithms achieves the best performance. However, the performance of the *Local Voting* algorithm is very close to optimal. This validates the results of Section II that load balancing can decrease the overall delivery time. The slight difference among these two algorithms can be explained by two facts: 1) the *Local Voting* algorithm is distributed, therefore, the delays in propagating the state affect its efficiency, and 2) slot exchange between two nodes is not always possible in real systems since allocations by other neighbors may cause a conflict, thus, it limits the amount of load balancing that is feasible. The *LoBaTS* algorithm exhibits worse performance than the first two algorithms, possibly because it assigns at least one slot to each node, even if the node does not have traffic. *DRAND* and *Lyui's* algorithms perform equally badly, i.e. several orders of magnitude behind the rest of the algorithms. This is expected since both algorithms do not adapt the scheduling to traffic requirements.

Fig. 5(d) depicts the end-to-end delivery time for the connections with the shortest delivery time. In general, the *Local Voting* algorithm has slightly better performance in terms of the minimum delay compared to the other algorithms.

D. The Effect of the Network Density

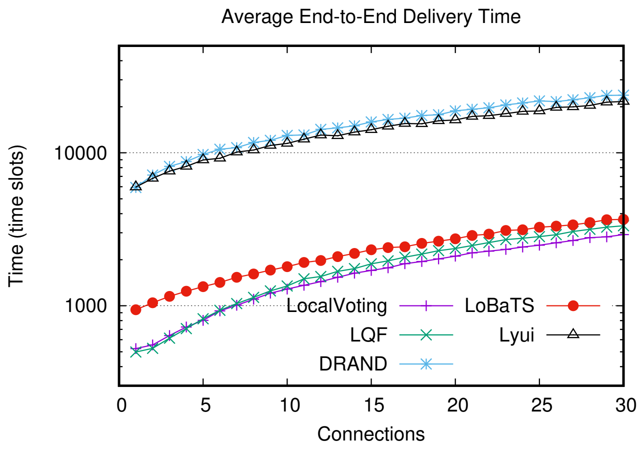
In this scenario we have repeated the experiments of section IV-C, but this time we have changed the size of the topology, while the number of nodes is kept constant. This allows us to investigate how the network density affects the performance of the algorithms.

We vary the size of the network from 10 units to 200 units, while the number of nodes is still equal to 100, and the transmission and the interference ranges are equal to 10 units. The results are depicted in Fig. 6 for 10 and 30 concurrent connections, respectively. In all cases the *Local Voting* and *LQF* algorithms have the best performance. Additionally, the performance of the proposed *Local Voting* algorithm is very close to the performance of the centralized *LQF* scheme in terms of maximum delivery time.

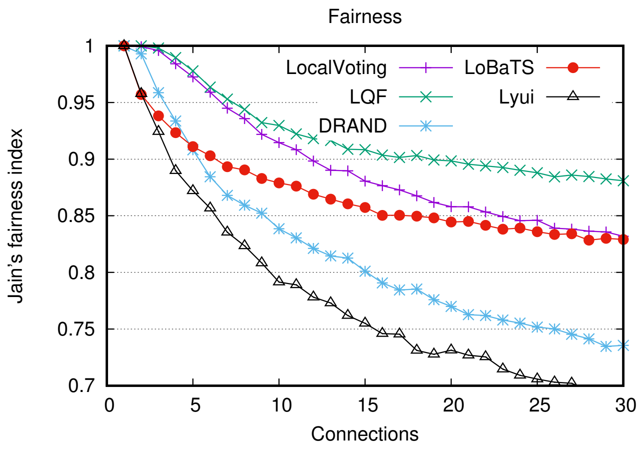
E. Steady State Scenario

In this subsection we evaluate the steady state performance of the load balancing algorithm. This scenario is set up on the same network as the previous one. However, instead of starting all connections at the beginning of the simulation, the connections start following a Poisson process where the arrival rate λ is in the range of $\left[10^{-4}, 10^{-1}\right]$ slots^{-1}, the duration of each connection is distributed exponentially with a parameter of $1/\mu = 10^{-3}$slots^{-1}, and the packet inter-arrival time within a connection is 1 packet every 5 time slots. The source and the destination of the connection are chosen randomly, following a uniform distribution. The duration of the simulation is 3×10^6 time slots. The packets that are received before 36666 slots have elapsed since the beginning of the simulation are ignored.

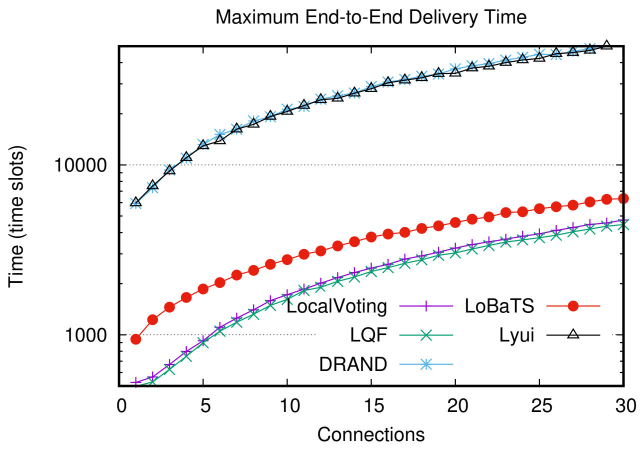
We measure the average end-to-end delivery time, the average end-to-end delay, the average throughput, and the fairness in terms of throughput. Fig. 7(a) presents the average end-to-end delivery time, from the transmission of the first packet to



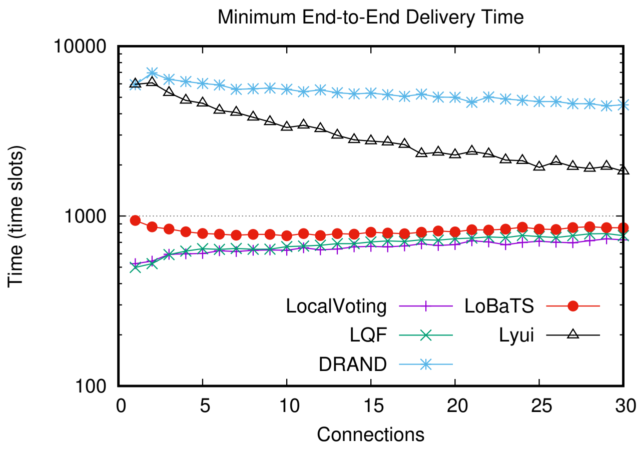
(a) The average end to end delay



(b) The fairness in end-to-end delay

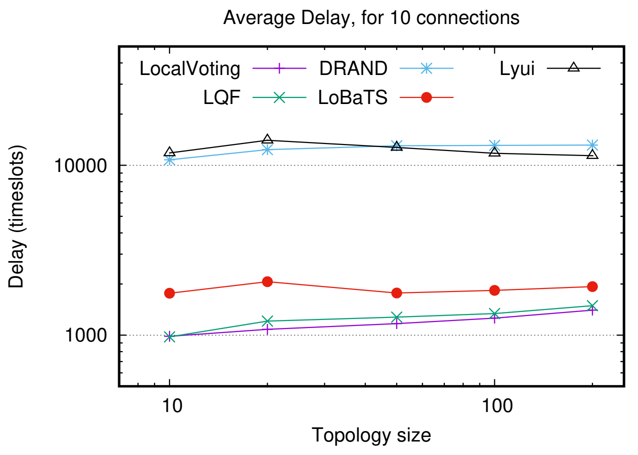


(c) The maximum end-to-end delay

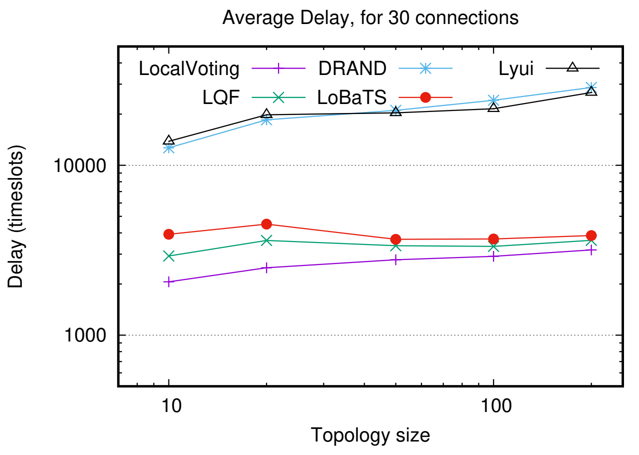


(d) The minimum end-to-end delay

Fig. 5. Simulation results of the different scheduling algorithms for different traffic load in the delivery time scenario.

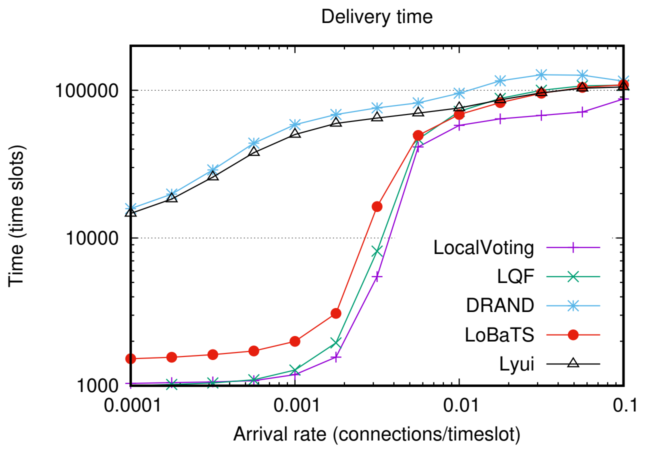


(a) The average delivery time (10 concurrent connections)

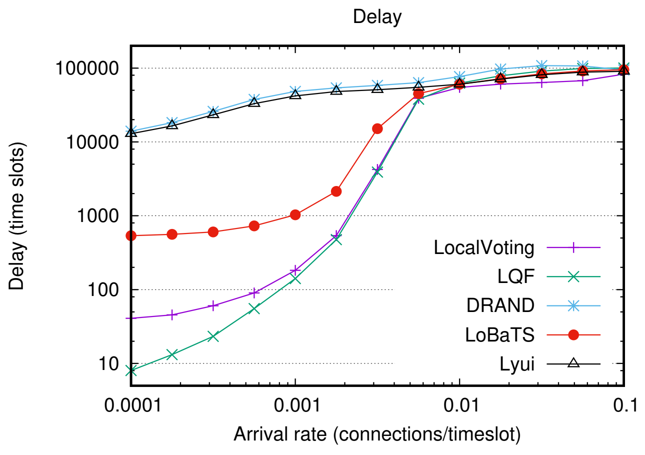


(b) The average delivery time (30 concurrent connections)

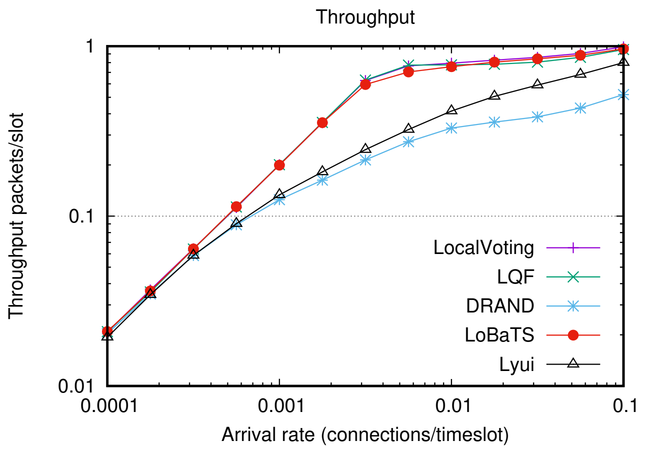
Fig. 6. Simulation results for varying network density.



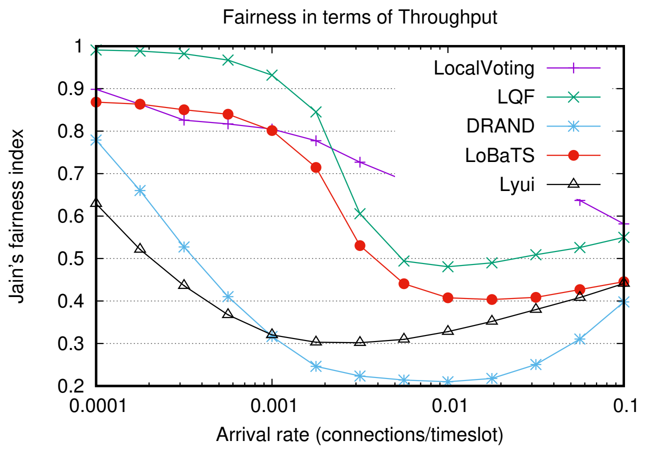
(a) The average delivery time



(b) The average end-to-end packet delay

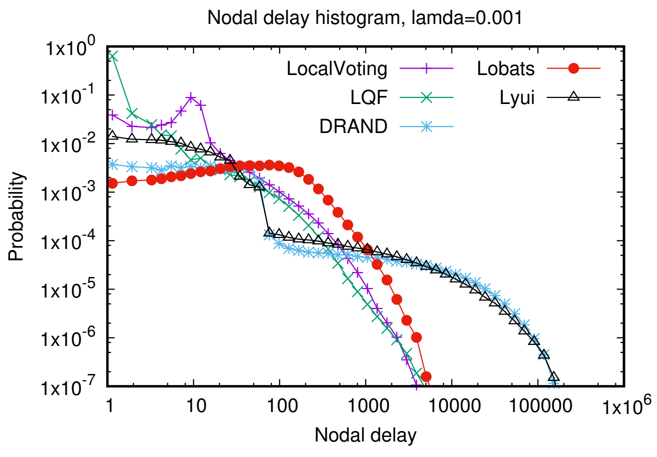


(c) The average throughput

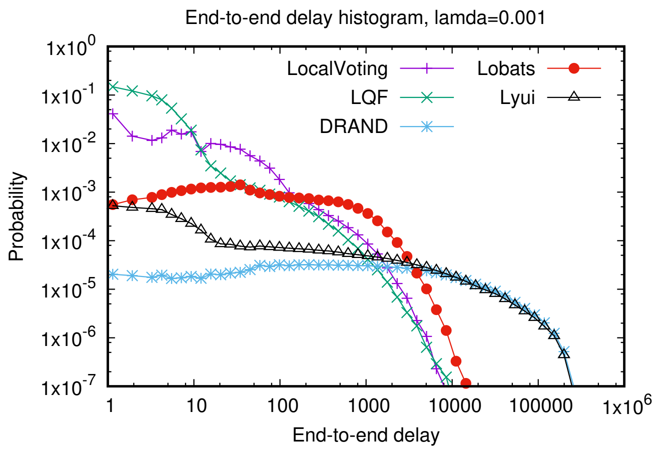


(d) The fairness in terms of throughput

Fig. 7. Simulation results for the steady state scenario.



(a) Nodal delay



(b) End-to-end delay

Fig. 8. The distribution of delay per packet (nodal and end-to-end) for the steady state scenario.

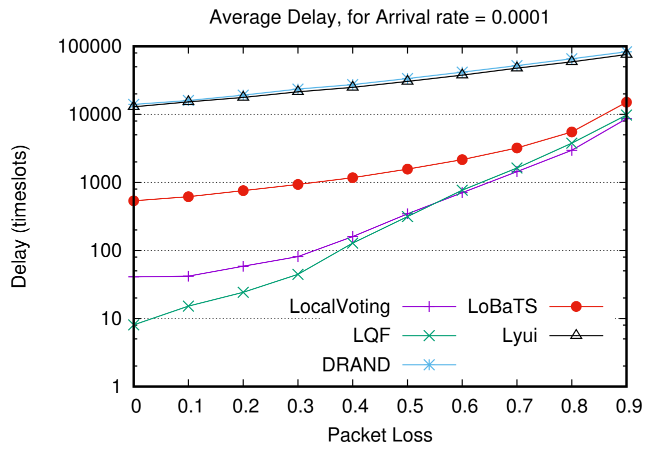


Fig. 9. Simulation results for different values of the packet loss, for an arrival rate of 10^{-4} connections/slot.

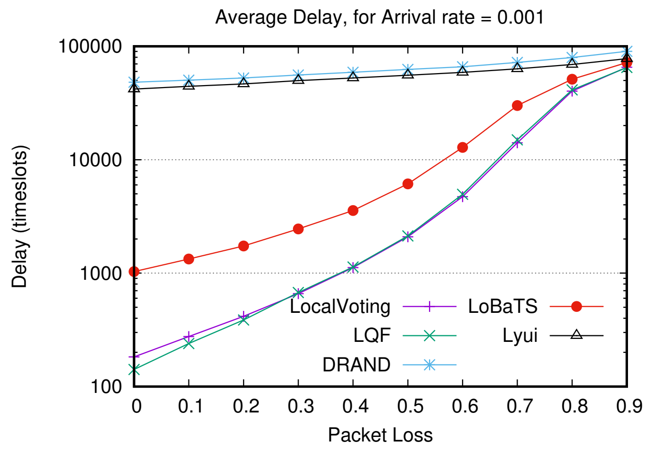


Fig. 10. Simulation results for different values of the packet loss for an arrival rate of 10^{-3} connections/slot.

the reception of the last packet of all connections. The *Local Voting* algorithm achieves the best performance that is very close to the *LQF* algorithm. The performance of the *LoBaTS* algorithm is a bit behind the first two algorithms, and the traffic independent algorithms achieve the worst performance. In Fig. 7(b) we can see the average end-to-end delay, from the moment a packet was generated until it was received by the final destination. For low arrival rates, the *LQF* algorithm has the smallest end-to-end delay, followed by the *Local Voting*, *LoBaTS*, *Lyui's* and *DRAND* algorithms. On the contrary, the average throughput for the *LQF*, *Local Voting*, and *LoBaTS* algorithms has a similar value, but *Lyui's* and *DRAND* achieve lower average throughput (Fig. 7(c)). Finally, in terms of fairness, the *Local Voting* algorithm is superior for medium arrival rates, but *LQF* has a superior performance for high and low arrival rates.

Fig. 8(a) shows the evolution of the delay per packet per node, for the different algorithms for an arrival rate of 10^{-3} new connections per time slot. The LQF algorithm has the higher percentage of packets with very low delay, and this is expected because there is no frame length, so packets are eligible to be transmitted at the next time slot. On the contrary, the *Local Voting* algorithm has a peak in the delay distribution that is close to the frame length of 10. The LoBaTS algorithm has higher delay, followed by DRAND and Lyui.

In Fig. 8(b) we plot the distribution of the end-to-end delay per packet. We can see that the ranking of the algorithms is similar to the per hop ranking. This result validates that optimizing per-node delay through load balancing has a positive effect on end to end delay in a multihop network.

F. The Effect of Packet Loss

In this scenario, we evaluate the performance of the scheduling algorithms when errors can occur during the transmission between nodes. We kept the same parameters as the previous scenario, but this time we considered a packet loss probability in a range from zero (i.e. no packet loss) to 0.9. We measure the average delivery time, the average end-to-end delay, and the average throughput for arrival rates of 10^{-4} and 10^{-3} connections per time slot.

Fig. 9 shows the results for an arrival rate equal to 10^{-4}, and Fig. 10 presents the results for an arrival rate equal to 10^{-3} connections per time slot. In both cases, when the packet loss increases, the end-to-end delay also increases. This is expected, because an increased packet loss causes the packets to be re-transmitted, thus, an additional delay is experienced. Similar results may be seen for the delivery time and the throughput, but are omitted due to space page limitation.

G. The Effect of the γ Value

In this scenario we investigate the effect of the γ value on the performance of the network. We execute the steady state scenario for the *Local Voting* algorithm, but this time, we set the γ parameter to different values, from 10^{-3} to 10^{3}. The results are depicted in Fig. 11. There are significant differences in terms of the end-to-end delay. For the network settings tested, we observed the best performance with in terms of delay for $\gamma = 1$.

H. Node Scheduling vs. Link Scheduling

All the algorithms studied in this paper are node-scheduling algorithms. This means that the destination of each transmission is not considered, so the interference model that is used under node-scheduling is more conservative than link-scheduling. On the other hand, node scheduling has a multiplexing advantage under intermittent load. Fig. 12 depicts the results of the first scenario, including a link-scheduling variant of the LQF algorithm.

V. Conclusion

The problem of scheduling is one of the big challenges in wireless networks. In this paper we studied the interaction of scheduling and load balancing. We showed that the problem of minimizing the overall delivery time through a multihop network can be modeled as a consensus problem, where the goal is to semi-equalize the fraction of the number of slots allocated to each node over the queue length of the node. We introduced the schedule exchange graph, that is a directed, time-varying graph, which represents whether a

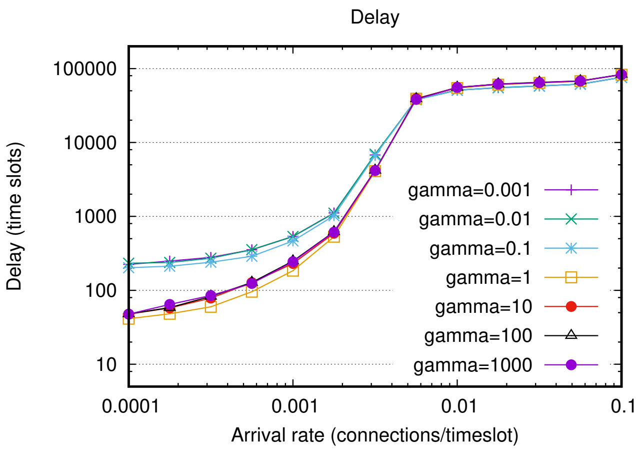


Fig. 11. Simulation results for local voting protocol, on the steady state scenario, with γ ranging from 10^{-3} to 10^{3}.

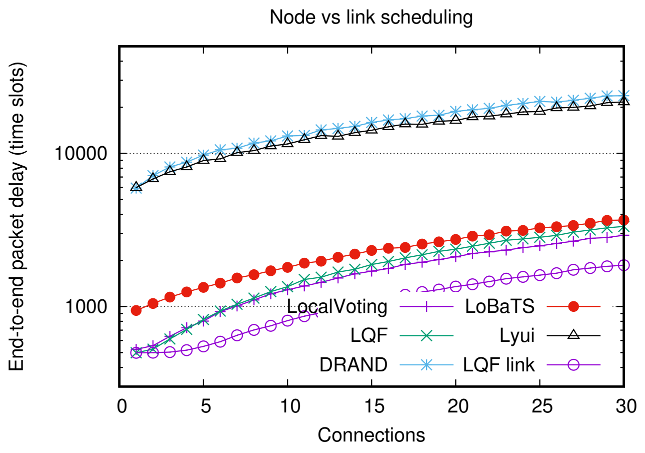


Fig. 12. Node vs. Link scheduling

node can give a slot to another node. The problem of wireless scheduling was modeled as a load balancing problem. Taking into consideration the dynamically changing network topology, we introduced *Local Voting* protocol (consensus protocol) to solve the scheduling/load balancing problem. Finally, we found the conditions that should be met in order for the *Local Voting* protocol to achieve approximate consensus, and therefore optimize the delivery time throughout the network.

We compared the performance of the *Local Voting* algorithm with other scheduling algorithms from the literature. Simulation results validated the theoretical analysis and showed that the delivery times are minimized with the use of the *Local Voting* algorithm. The proposed algorithm achieves better performance than the other known distributed algorithms from the literature in terms of the average delay, the maximum delay, and the fairness. Despite being distributed, the performance of the *Local Voting* algorithm is very close to the performance of the centralized *LQF* algorithm which is considered to have the best performance. To summarize, we showed the advantage of load balancing when performing scheduling in wireless multihop networks, proposed *Local Voting* algorithm for load balancing/scheduling, found theoretical conditions for convergence (reaching consensus), and demonstrated by simulations that the *Local Voting* algorithm shows good performance in comparison with other scheduling algorithms.

Appendix
Proof of Theorem 1

Proof: The result of this Theorem and its proof are different from corresponding parts in [35]. The difference is caused by the different ways of achieving consensus. While in [35], consensus is achieved through re-distributing the load or q_t^i, in this paper consensus is reached through re-distributing slots in a frame, i.e. p_t^i. The idea of the proof follows the paper [52].

By virtue the Eqs. (3) and (6), the dynamics p_t^i of the closed loop system with protocol (5) are as follows

$$p_{t+1}^i = p_t^i + e_{t+1}^i + [\gamma \sum_{\tilde{N}_t^i} a_t^{i,j}(\tilde{x}_t^j - \tilde{x}_t^i)] =$$

$$p_t^i + \gamma \sum_{\tilde{N}_t^i} a_t^{i,j}(\tilde{x}_t^j - \tilde{x}_t^i) + e_{t+1}^i + w_t^i. \qquad (11)$$

Denote by $\mathbf{p}_t \in \mathbb{R}^n$ a vector which consists of p_t^i, $\mathbf{e}_{t+1} \in \mathbb{R}^n$ a vector which consists of e_{t+1}^i, and by $\mathbf{w}_t \in \mathbb{R}^n$ a vector of the errors w_t^i, where $t = 0, 1, \ldots$.

Due to the view of the Laplacian matrix $\mathcal{L}(A_t)$ and definition of Q_{t+1}, we can rewrite Eq. (11) in a vector-matrix form as:

$$\mathbf{p}_{t+1} = \mathbf{p}_t - \gamma\mathcal{L}(A_t)Q_{t+1}^{-1}\mathbf{p}_t + \mathbf{e}_{t+1} + \mathbf{w}_t. \qquad (12)$$

We consider that $\bar{\mathbf{p}}_t = Q_{av}\bar{\mathbf{x}}_t$. If we multiply both sides of Eq. (7) by Q_{av}, we get that the sequence $\{\bar{\mathbf{p}}_t\}$ is a trajectory of the average system

$$\bar{\mathbf{p}}_{t+1} = \bar{\mathbf{p}}_t - \gamma\mathcal{L}(B_{av})\bar{\mathbf{p}}_t + \bar{\mathbf{e}}_{t+1}. \qquad (13)$$

The vector $\mathbf{1}_n$ is the right eigenvector of the Laplacian-type matrices $\tilde{\mathcal{L}}_t = \gamma\mathcal{L}(A_t)Q_{t+1}^{-1} = \gamma\mathcal{L}(B_t)$ and $\bar{\mathcal{L}}_B = \gamma\mathcal{L}(B_{av})$ corresponding to the zero eigenvalue: $\tilde{\mathcal{L}}_t\mathbf{1}_n\mathbf{1}_n = \bar{\mathcal{L}}_B\mathbf{1}_n = 0$. Sums of all elements in the rows of the matrices $\tilde{\mathcal{L}}_t$ or $\bar{\mathcal{L}}_B$ are equal to zero and, moreover, all the diagonal elements are positive and equal to the absolute value of the sum of all other elements in the row.

The next Lemma from [53] is useful.

Lemma [53]: Laplacian matrix $\mathcal{L}(B)$ of graph $\mathcal{G}_B$ has an algebraic multiplicity equal to one for its eigenvalue $\lambda_1 = 0$ if and only if graph $\mathcal{G}_B$ has a spanning tree.

Note that graph $\mathcal{G}_{B_{av}}$ has a spanning tree when conditions **A1.c** and **A2** hold.

Due to the definitions of the matrices $\bar{\mathcal{L}}_t$ and $\tilde{\mathcal{L}}_A$, we derive from (12),(13) for the difference $\mathbf{r}_{t+1} = \mathbf{p}_{t+1} - \bar{\mathbf{p}}_{t+1}$

$$\mathbf{r}_{t+1} = \mathbf{r}_t - \tilde{\mathcal{L}}_t\mathbf{p}_t + \bar{\mathcal{L}}_B\bar{\mathbf{p}}_t + \mathbf{e}_{t+1} - \bar{\mathbf{e}}_{t+1} + \mathbf{w}_t =$$

$$= (I - \bar{\mathcal{L}}_B)\mathbf{r}_t - (\bar{\mathcal{L}}_t - \tilde{\mathcal{L}}_B)\mathbf{p}_t + (\mathbf{e}_{t+1} - \bar{\mathbf{e}}_{t+1}) + \mathbf{w}_t,$$

where I is the identity matrix.

Consider the conditional mathematical expectation of the squared norm $\mathbf{r}_{t+1}$ according to σ-algebra $\mathcal{F}_t$. By virtue of Assumptions **A1.d–f** we derive

$$\mathrm{E}_{\mathcal{F}_t}\|\mathbf{r}_{t+1}\|^2 \leq \|(I - \tilde{\mathcal{L}}_B)\mathbf{r}_t\|^2 + \mathbf{p}_t^{\mathrm{T}}R\mathbf{p}_t + n\sigma_e^2 + n\sigma_w^2.$$

Further, by taking unconditional expectation we get: $\mathrm{E}\|\mathbf{r}_{t+1}\|^2 \le \rho \mathrm{E}\|\mathbf{r}_t\|^2 + \Delta$. By Lemma 1 from Chapter 2 of [54] it follows that

$$\mathrm{E}\|\mathbf{r}_{t+1}\|^2 \le \frac{\Delta}{1-\rho} + \rho^t \mathrm{E}\|\mathbf{p}_0\|^2. \tag{14}$$

Due to the definitions we have

$$\mathrm{E}\|\mathbf{x}_{t+1} - \bar{\mathbf{x}}_{t+1}\|^2 = \mathrm{E}\|Q_{t+1}^{-1}(\mathbf{p}_{t+1} - \bar{\mathbf{p}}_{t+1}) + (Q_{t+1}^{-1}Q_{av} - I)\bar{\mathbf{x}}_{t+1}\|^2 \le$$
$$2\mathrm{E}\|Q_{t+1}^{-1}\mathbf{r}_{t+1}\|^2 + 2\mathrm{E}\|(Q_{t+1}^{-1} - Q_{av}^{-1})Q_{av}\bar{\mathbf{x}}_{t+1}\|^2 \le$$
$$2(\frac{\Delta}{1-\rho} + \rho^t \mathrm{E}\|\mathbf{p}_0\|^2) + 2\sigma_q^2\|Q_{av}\bar{\mathbf{x}}_{t+1}\|^2.$$

The proof of the first part of Theorem 1 is completed.

The second conclusion about the asymptotic mean square ε-consensus follows from inequality (10) if $t \to \infty$. Since (9) is satisfied, then the third term of (10) exponentially tends to zero.

■

Acknowledgment

This work was supported by RFBR under Grants No.15-08-02640 and No.16-07-00890 and also by SPbSU (grant 6.38.230.2015). We would like to thank the anonymous reviewers for their very valuable comments.

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