

ASSESSMENT OF METHODS FOR SHORT-TERM EXTREME VALUE ANALYSIS OF RISER COLLISION

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ABSTRACT

Risers are commonly arranged as clusters with relatively small spacing due to economic necessity. As a consequence, collision between risers becomes an essential problem. This study presents a comprehensive assessment of various methods for riser collision probability analysis. A pair of tandem arrangement risers subjected to combined current and wave loads is modelled. Three hours short-term simulation is performed in order to obtain the time history samples for the collision probability analysis. The wake effect due to the presence of the upstream riser is considered. The shortest distance between risers is calculated at each time step. Four methods for estimation of the extreme value distribution, e.g. Gumbel probability paper method, Weibull based method, average conditional exceedance rate method and moment based Hermit method, are presented, and the results obtained from different methods are compared and discussed.

INTRODUCTION

As the offshore industry moves to deeper water, risers are commonly arranged as clusters with small spacing due to limited size of the platform as well as the cost considerations. The dynamic response of riser clusters, induced by the waves, currents, and platform motions, becomes significantly important. The complication of the response is increased by the arrangement of the risers. When the risers are in tandem arrangement and close enough, the local fluid kinematics in terms of amplitude, frequency and phase around the downstream riser is signif-

icantly modified, compared to that for the upstream riser. Additionally, the differences in excitation force on neighbouring risers will cause large relative distance, leading to possibility of riser collision, especially when they are subjected to a severe sea state.

There are two different design strategies for riser collision assessment according to DNV-RP-201 [1]. One is called 'No Collision Allowed', which means that riser collision is not allowed under normal, extreme or survival conditions. The problem is then reduced to determine the probability of the relative distance between risers over a given threshold value. Another one is 'Collision Allowed', indicated that infrequent collision may be allowed in some extreme conditions. Hence, assessment of structural interaction will be required. For the present study, the former design strategy will be considered.

Duggal and Niedzwecki [2] performed experiments for top tension risers subjected to random waves in order to estimate the riser collision probability. They considered the relative distance between two nodes located at the same water depth as a random process, so that the collision probability problem was equivalent to a crossing process for a threshold value. The probabilistic model is developed by adapting first-passage time formulations, and is extended to account for a non-Gaussian collision process by applying the Hermite transformation technique. He and Low [3] provided an approach for estimation of the probability of collision between two flexible risers, and the relative distance is calculated as the shortest distance between two lines. Fu et al. [4] developed an approach for estimation of collision probability between two flexible risers by accounting for the uncertainties of

the important parameters, e.g., shielding effects, floater motion and current velocity.

The purpose of this paper is to estimate the collision probability of a pair of flexible risers in tandem arrangement which are subjected to combined current and wave loads. The approach developed in this study is based on the short-term time domain simulations. Particular attention is given to the flexible riser in the wave configuration. Several 3-hour short-term simulations are performed to obtain the time history samples, and the relative distance is treated as a random process for the collision probability analysis. The performances of extreme value analysis methods, such as Gumbel probability paper, Weibull based method, average conditional exceedance rate method and moment-based Hermite method are evaluated. The importance of the threshold value is discussed for better modelling the tail of the distribution and the estimating of the extreme value.

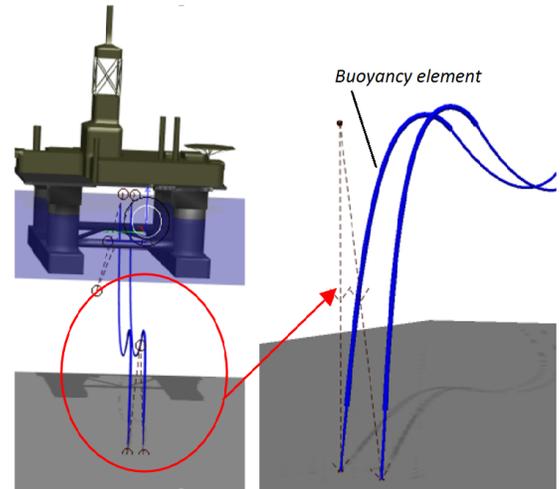


FIGURE 1. RISER CONFIGURATION.

TIME DOMAIN SIMULATIONS

Time domain simulations are necessary for calculating the nonlinear riser response under combined current and wave loads. The nonlinearity is increased when the shielding effects are taken into account. The shielding effects is considered by combining the finite element software Riflex [5] along with the Blevins wake model [6]. The Riflex is specially designed to handle static and dynamic analyses of slender marine structures. The Blevins weak model expresses both drag force and lift force on the downstream riser as a function of the relative distance between two risers.

Structural and Environmental Modelling

The flexible riser used in present study is the wave riser. A wave riser has the addition of buoyancy modules along a part of the riser length in order to form a 'wave' shape, so that some of the axial tensile forces acting on the riser can be relieved, as shown in Fig. 1. The total length of the riser is $l = 160$ with diameter $D = 0.25$ m. The length of the buoyancy module is $l_b = 50$ m with diameter $D_b = 0.63$ m, and along the riser at water depth $h = 60 \sim 90$ m. The main riser properties are summarized in Table 1. Two identical risers arranged in tandem are modelled in Riflex. The top ends are rigidly connected with a semi-platform. The bottom ends are fixed at the seabed. The gap between ends is $L_0 = 10$ m. The platform is to be modelled as a rigid body with six-DOF, and the motion of the body is specified through the Response Amplitude Operators (RAOs). For simplicity, only the first-order wave loads are considered in the dynamic analyses.

For short-term analyses it is assumed that the most critical response occurs during a design sea state corresponding to a given return period, i.e., 100 years. The JONSWAP spectrum is selected using a γ factor of 3.3, and a significant wave height

TABLE 1. RISER AND BUOYANCY ELEMENTS PROPERTIES.

	Unit	Riser	Buoyancy elements
Outside diameter	[m]	0.25	0.63
Inside diameter	[m]	0.05	0.05
Mass coefficient	[kg/m]	100	100
EI	[kNm ²]	104	104
Content density	[kg/m ³]	1000	1000
Total length	[m]	110	50

$H_s = 14$ m and a spectral peak period $T_p = 18$ s. The current velocity V_c is set to be 1.0 m/s at the sea surface and to decrease linearly to 0.8 m/s at the seabed. Only collinear wave-current interaction is considered.

Hydrodynamic Loads

The hydrodynamic forces are calculated based on two-dimensional strip theory. The wave-induced excitation forces (Froude-Kriloff and diffraction forces) are computed by a long wavelength approximation which involves added mass and potential damping of the actual cross section together with the wave kinematics. The viscous loads are computed using the drag term in a modified Morisons equation, taking into account the relative motion between riser and fluid flow. In addition, shielding effects generated by the upstream cylinder has to be accounted for. In the present study, the downstream riser is placed at the wake center-line so that the lift force caused by the asymmetry flow can be neglected. The drag force is reduced due to the wake shielding from the upstream riser. In this study, the Blevins wake

model is used, by which the reduction of the local flow velocity is transformed to the reduction of the drag coefficient. The formulation is given in Eqs. 1.

$$C_D(x) = C_{D0} \left\{ 1 - k_1 \left(\sqrt{\frac{C_{D0} D_u}{x}} \right) \exp \left(\frac{-k_2 y^2}{C_{D0} D_u x} \right) \right\}^2 \quad (1)$$

where x is the position of the downstream cylinder with respect to the upstream cylinder; C_D is the downstream cylinder drag coefficient based on local flow velocity; C_{D0} is the reference drag coefficient based on undisturbed flow velocity; D_u is the diameter of the upstream cylinder; Parameters $k_1 = 1$ and $k_2 = 4.5$ are constants, and determined by fitting curve to the experimental data at $x/D = 3, 5, 9$ and 20.3 using the least-squares method [7]. However, more data is required in order to validate this model.

The upstream riser is considered as a single, isolated cylinder. However, the drag force on the downstream riser depends on the relative distance with respect to the upstream riser. Due to the current profile and the riser boundary condition, the drag force varies along the riser. Therefore, an iteration process searching for the static equilibrium position and reduced drag force is necessary. This is achieved by combining the wake model in Eq. 1 and static analysis in Riflex. The equilibrium static position of the risers and the associated drag coefficient is used further in the dynamic analysis.

Definition of Random Process

In Riflex, a riser is modelled as line represented by a series of line segments, according to their property and geometry. To estimate the probability of collision between the discrete lines, the distance between each pair of the line segments should be found. As mentioned previously, the risers are initially arranged in tandem with relative distance $L_0 = 10$ m. The initial position can be calculated by using the catenary equilibrium calculations. When the current loads are applied, the final static position can be found by an iteration process as described. The shortest distance between the risers at the static position is denoted as L_s . For the dynamic analysis, the shortest distance is then defined as the minimum distance between risers at each time step, i.e. $L_d(t)$. Figure 2 illustrates the definitions of the different distances.

The probability of the riser collision, actually, is an extreme minimum value problem. However, it is convenient to transform the extreme minimum value problem to the non-dimensional extreme maximum value problem by writing the process as:

$$X(t) = -L_d(t)/D \quad (2)$$

In this case, the process $X(t) < 0$, and the risers clash when $X(t) = -1$. Figure 3 compares the time history of X and L_d ,

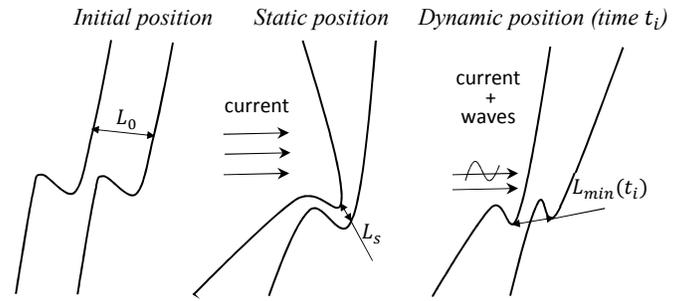


FIGURE 2. DEFINITION OF THE RELATIVE DISTANCE, L_0 , L_s and L_d .

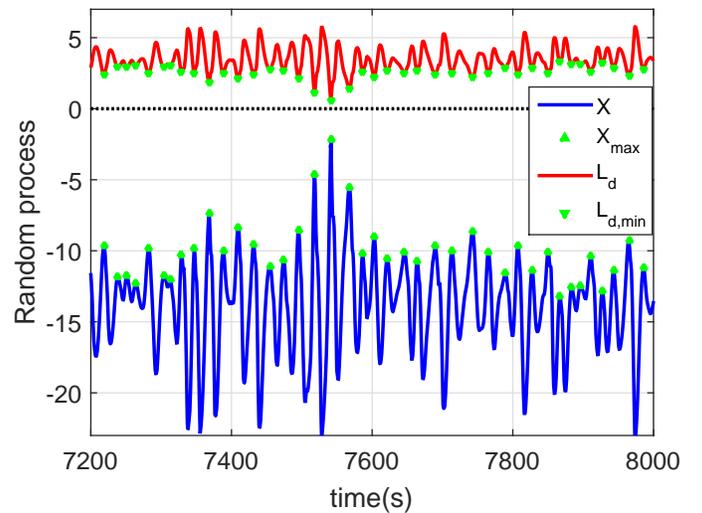


FIGURE 3. TIME SERIES OF THE PROCESS X and L_d .

with the concerning maximum and minimum values, i.e. X_m and $L_{d,min}$.

EXTREME VALUE ANALYSIS

This section introduces four existing methods evaluating the extreme value distribution of a stochastic process during the time duration T .

For classical extreme value theory, it is assumed that the sequence of maxima X_{mi} is independent and identically distributed with common distribution function $F_{X_m}(x)$. The extreme value of a finite number is then $X_e = \max\{X_{m1}, X_{m2}, \dots, X_{mm}\}$. The distribution of X_e can be derived as:

$$F_{X_e}(x) = \text{Prob}\{X_e \leq x\} = [F_{X_m}(x)]^n \quad (3)$$

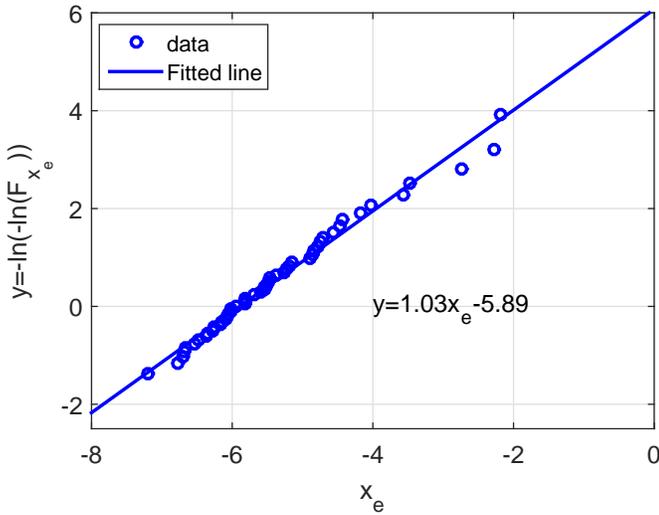


FIGURE 4. GUMBEL PROBABILITY PAPER.

This equation will normally converge towards one of three possible asymptotic extreme value distributions as $n \rightarrow \infty$. In the present study, the Gumbel distribution function will be adopted due to the behaviour of the upper tail of the distribution of the maxima.

Gumbel Probability Method

The Gumbel probability paper method is a simple and efficient method to determine the distribution parameters. The cumulative distribution function is given by:

$$F_{X_e}(x) = \exp\{-\exp(-\alpha(x-u))\} \quad (4)$$

where α and μ are the scale and location parameters, respectively. By taking the logarithm of both left and right hand side of this equation twice, the following equation is obtained:

$$-\ln[-\ln(F_{X_e}(x))] = \alpha(x-u) \quad (5)$$

Further, by introducing $y = -\ln[-\ln(F_{X_e}(x))]$ a linear function $y = \alpha(x-u)$ is obtained, which implies that in a x-y axis system, the cumulative distribution becomes a straight line. The parameters α and μ can be estimated by the least-square fitting of the samples to the straight line.

The fitted straight line and the extreme samples identified from the 50 three-hour simulations with different random seeds for generating time series of wave are plotted in Fig. 4. The shape and location parameters are $\alpha = 1.03$ and $\mu = -5.72$, respectively.

The Gumbel method requires a large set of simulated time series samples in order to estimate the distribution with acceptable accuracy. Only the largest maxima will be used for the estimation. In order to use more information from the time series samples, some other estimations of the extreme values based on the time series sample or based on the individual maxima will be introduced in the following sections.

Weibull Based Method

The Weibull based method for extreme value estimation is based on the assumption that, if the local maxima follows a three parameters Weibull distribution, the extreme response will follow a Gumbel extreme value distribution. The Gumbel parameters α and μ can be expressed in term of the Weibull parameters as following [8]:

$$\begin{aligned} \mu + \frac{\gamma}{\alpha} &= a + b \left\{ (\ln n)^{\frac{1}{c}} + \frac{0.57722}{a} (\ln n)^{\frac{1-c}{c}} \right\} \\ \alpha &= \frac{c}{b} (\ln n)^{\frac{1-c}{c}} \end{aligned} \quad (6)$$

where γ is the Euler's constant; n is the number of maxima for a given time duration; u , β and λ are the location, scale and shape parameters of the Weibull distribution function, respectively. The three parameters Weibull function is given as:

$$F_{X_{max}}(x) = 1 - \exp\left\{-\left(\frac{x-u}{\beta}\right)^{\lambda}\right\} \quad (7)$$

The constants u , β and λ can be determined by applying moment estimator according to Farnes and Moan [9] based on experience. The detail of the moment estimator can found in Appendix A. It is also recommended that only the global maxima, i.e. the largest maxima between zero up-crossings should be used for calculation of the sample statistical moments in order to obtain optimal results.

Since the aim of the fitting is to obtain a Weibull model to be used for estimation of extremes, it is more important that the fitting procedure gives a good fit to the upper tail of the sample maxima distributions. Therefore, a threshold is necessary in order to avoid including small maxima. Choosing of the threshold is empirical, and in this paper, the following values $E(x) + \eta$ are used as the threshold levels, where $E(x)$ the expected value; $\eta = [0, 0.5\sigma, \sigma, 1.2\sigma, 1.4\sigma]$ where σ is the standard deviation.

The global maxima over a threshold $\eta = \sigma$ identified from the time history sample and its 3-parameter Weibull density function are presented in Fig. 5. The effect of the thresholds on the fitting result is shown in Fig. 6. The Largest maxima identified from each simulations are plotted in the same figure. It appears

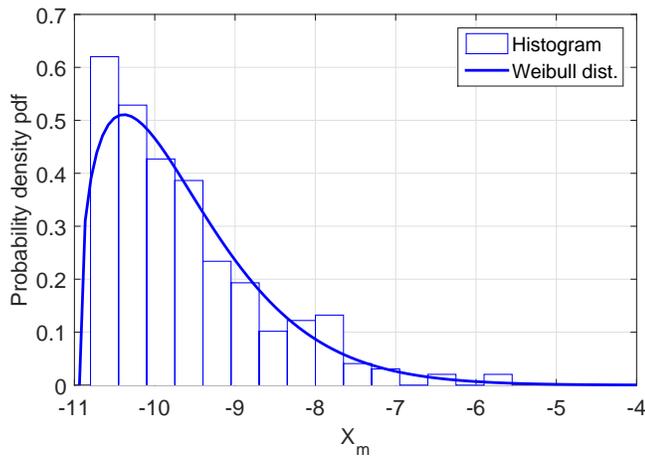
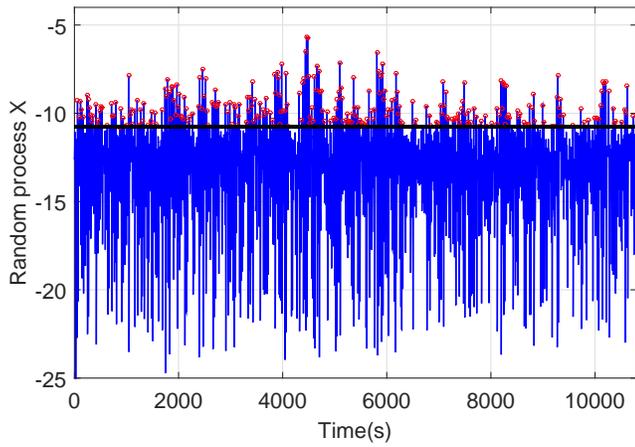


FIGURE 5. GLOBAL MAXIMA WITH THRESHOLD $\eta = \sigma$ FOR A THREE-HOUR SIMULATION. TOP: TIME HISTORY AND GLOBAL MAXIMA OVER THRESHOLD; BOTTOM: PROBABILITY DENSITY FUNCTION

that the choice of threshold has a significant effect on the statistics and the shape of the distribution. The lower threshold value, i.e. $\eta = 0$ and $\eta = 0.5\sigma$, preserves the greatest number of maxima from the time series. However, the distribution is heavily weighted to lower values of the maxima and the fitted distribution does not agree well for the upper tail data. As the threshold value increases, the weight of the upper tail data becomes important, but the amount of the data is reduced. Therefore, it is found that $\eta = \sigma$ gives a good agreement with the extreme data for the present study, and will be adopted in the following calculation.

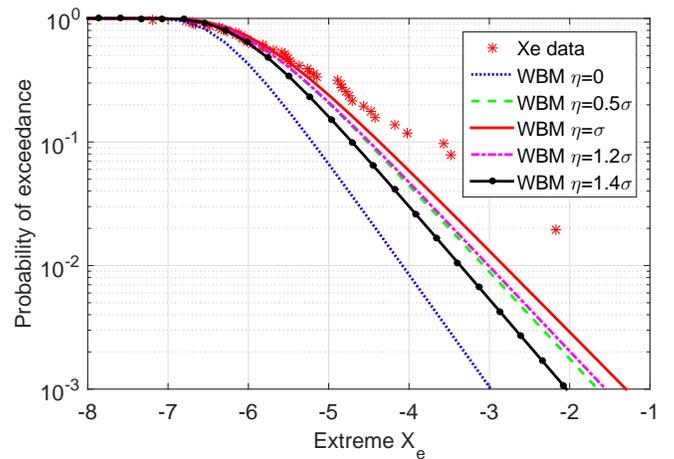
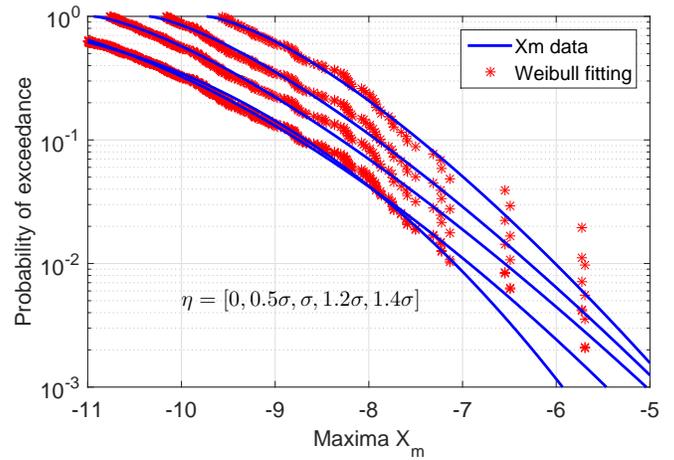


FIGURE 6. MAXIMA AND EXTREME VALUES DISTRIBUTION FOR DIFFERENT THRESHOLDS. TOP: WEIBULL DISTRIBUTION OF MAXIMA; BOTTOM: GUMBEL DISTRIBUTION OF EXTREMES

Average Conditional Exceedance Rate

Unlike the above mentioned methods based on the parametric distribution functions, the average conditional exceedance rate (ACER) method estimates the exact extreme value distribution by constructing a sequence of non-parametric distribution functions, i.e. the ACER functions. The principle and development of the ACER functions are given in Refs. [10] and [11].

With the time series of the individual maxima, the extreme

value can be expressed as:

$$F_{x_e}(x) \approx P_k(x) \approx \exp\{-(N-k+1)\hat{\varepsilon}(x)\} \quad (8)$$

where k is the order of the ACER function; P_k is the approximation of the extreme value distribution based on the k -th order ACER function; $\hat{\varepsilon}(x)$ is the empirical ACER function of order k , which can be determined by applying the existed time series. As the order k increases, the accuracy of Eq. 8 improves, but the amount of data for calculating $\hat{\varepsilon}(x)$ reduces.

In order to predict the extreme value distribution in the tail region, an extrapolation scheme is applied. Specifically, in the upper tail region (e.g. $x \geq x_0$), the ACER functions behaves similarly to $\exp\{-a(x-b)^c\}$, where $a > 0$, $b \leq x_0$ and $c > 0$ are suitable constants.

The empirical ACER function is assumed to be in the form of:

$$\hat{\varepsilon}_k(x) = q_k \exp\{-a_k(x-b_k)^{c_k}\}; \quad x \leq x_0 \quad (9)$$

where a_k , b_k , c_k and q_k are suitable constants which dependent on the order k . These parameters can be found by an optimized fitting on the log scale. It should be noted that Eq. 9 is applicable at the upper tail region, i.e., $x \geq x_0$, where $x \geq x_0$ is an appropriately chosen tail marker. By comparing the empirical $\hat{\varepsilon}_k(x)$ with different value of k , an appropriate value of k is selected to capture the dependence structure of the time series.

These parameters can be determined by the following mean-square-error function:

$$F(q_k, a_k, b_k, c_k) = \sum_{i=1}^N \rho_j |\ln \hat{\varepsilon}_k(x_i) - \ln q_k + a_k(x_i - b_k)^{c_k}|^2 \quad (10)$$

where x_i , $i = 1, \dots, n$ are levels at which the ACER functions have been empirically estimated. The weight factor ρ_j is given by the relationship $\rho_j = (\ln CI^+(x_i) - \ln CI^-(x_i))^{-2}$, where CI represents the 95% confidence interval, which can be approximately expressed as:

$$CI^\pm(x_i) = \hat{\varepsilon}_k(x_i) \left\{ \frac{1.96}{\sqrt{(N-k+1)\hat{\varepsilon}_k(x_i)}} \right\} \quad (11)$$

Therefore, it is seen in Eqs. 10 and 11, that the weight factor ρ_j decreases as the level x_i increases which implies that the extrapolation scheme puts more emphasis on the more reliable data points. Moreover, it should be noted that there is a level x_i beyond which the weight factor ρ_j is no longer defined since the

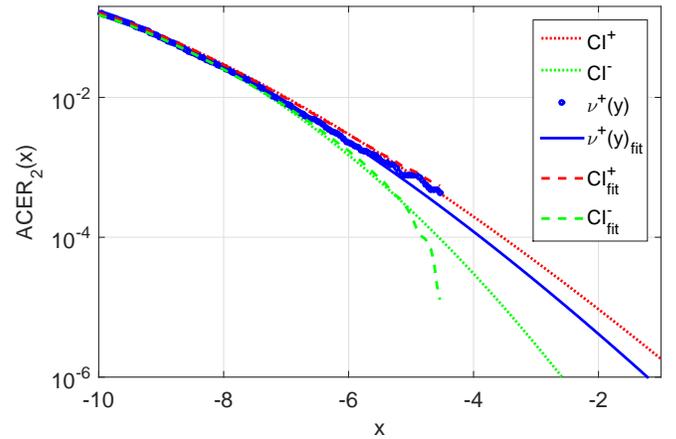
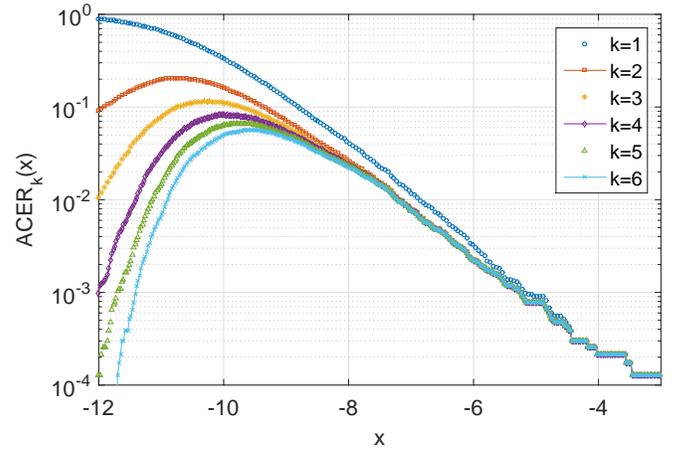


FIGURE 7. ESTIMATIONS OF ACER FUNCTION FOR $k = 1 \dots 6$

CI^- estimated by Eq. 11 would be negative as the levels exceed x_i .

The empirical ACER functions, $\hat{\varepsilon}_k(x)$ for different orders of k and the corresponding estimated ACER function with $k = 2$ and estimated confidence interval are plotted in Fig. 7. From the top figure it appears that, for the lower range of the individual maxima, there is a noticeable variation of the empirical ACER functions for different orders of k , which implies significant effect of dependence between the data points. Nevertheless, these functions coalesce in the tail region as $k \geq 2$, which means that $\hat{\varepsilon}(x)$ can be used for the extrapolation purpose. This is advantageous since in the cases with $k \leq 2$, the second order empirical ACER function is the one most accurately estimated because more data

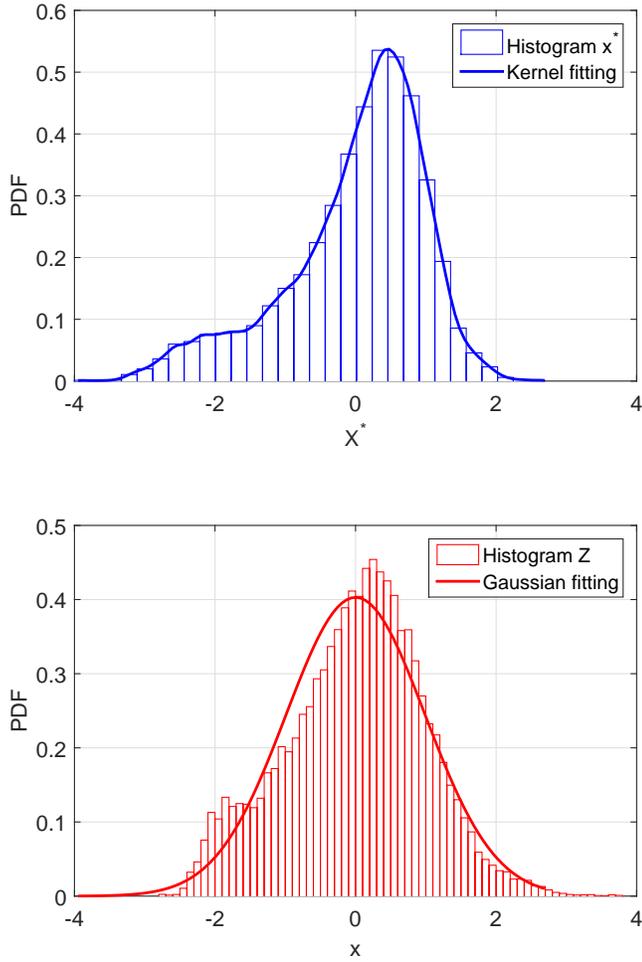


FIGURE 8. TRANSLATION OF PROBABILITY DENSITY BY THE HERMITE POLYNOMIALS FOR A THREE HOURS SIMULATION. TOP: NON-GAUSSIAN PROCESS; BOTTOM: GAUSSIAN PROCESS.

are available for its estimation. The bottom figure presents the $\hat{\epsilon}_2(x)$ and the 95% confidence interval obtained from the time series as well as the estimated curve in the upper tail region and the corresponding estimated confidence interval provided by the extrapolation scheme.

Moment Based Hermite Method

Moment-based Hermite Method provides a transformation between the non-Gaussian process and Gaussian process through a memoryless monotonic translations, as given in Eq.

12, so that the cumulative distribution function of the extreme value can be estimated based on the traditional Gaussian process:

$$F_X(x^*) = \Phi(z) \quad (12)$$

$$x^* = g(z) = F_X^{-1}[\Phi(z)]$$

where $x^* = (x - m_x)/\sigma$ is the non-Gaussian processes where m_x and σ are the mean and standard deviation of X ; Z is the standard Gaussian process; F_X and $\Phi(z)$ are CDFs of $X^*(t)$ and $Z(t)$; $g(\cdot)$ is the translation function; F_X^{-1} is inverse function of F_X^* . The extreme value distribution is then can be determined by application of the extreme value theory of the Gaussian process, which can be expressed as:

$$F_{X_e}(x) \approx \exp\{-v^+(x)T\} \approx \exp\{-v_0 T \exp\{[g^{-1}(x^*)]^2/2\}\} \quad (13)$$

where $v^+(x)$ is the up-crossing rate at a threshold x ; v_0 is the zero up-crossing rate.

Winterstein [12] demonstrated that the translation function can be expressed as the Hermite polynomials:

$$x^* = g(z) = \kappa\left\{z + \sum_{n=3}^{\infty} h_n H_{e_n}\right\} \quad (14)$$

$$\approx \kappa\{Z + \bar{h}_3(Z^2 - 1) + \bar{h}_4(Z^3 - 3Z) + \dots\}$$

where h_n and κ are the shape and scale factors of the model, respectively; $H_{e_n}(z)$ is the n -th Hermite polynomial function. For $n = 4$ moments, the parameters can be expressed as:

$$h_4 = \frac{\sqrt{1 + 1.5(\alpha_4 - 3)} - 1}{18} \quad (15)$$

$$h_3 = \frac{\alpha_3}{4 + 2\sqrt{1 + 1.5(\alpha_4 - 3)}}$$

$$\kappa = 1/\sqrt{1 - 2h_3^2 + 6h_4^2}$$

where α_3 and α_4 are the skewness and kurtosis of the process, respectively. The inverse translation function is then given as:

$$z = g^{-1}(x^*)$$

$$= \left\{ \sqrt{\xi^2(x^*) + c + \xi(x^*)} \right\}^{\frac{1}{3}} - \left\{ \sqrt{\xi^2(x^*) + c - \xi(x^*)} \right\}^{\frac{1}{3}} - a \quad (16)$$

where $\xi(x^*) = 1.5b(a + x^*/\kappa) - a^3$; $a = h_3/3h_4$; $b = 1/3h_4$; $c = (b - 1 - a^2)^3$. It should be noted that the above Hermite model is

only applicable for softening non-Gaussian processes, i.e., $\alpha_4 \leq 3$.

Figure 8 presents the PDFs of the initial process X^* with $E(X^*) = 0$ and $\sigma(X^*) = 1$ as well as the translated standard Gaussian process Z . The distribution of the initial process is fitted by the kernel sampling density function. However, this will not be elaborated further here; see reference [13] for more details. The figure it is found that the standard Gaussian distribution translated by the Hermite polynomials agrees well with the upper tail values. However, it is not a good translation function for the lower tail values.

COMPARISON OF THE RESULTS

The results obtained from the above methods are based on one specific 3-hour simulation. In practice, due to the statistical uncertainties which are inherent in the random response process, repeated simulations are required in order to obtain a reliable estimation of the extreme response. Therefore, the so-called average expected value is introduced in order to consider the uncertainties. More specifically, for the Weibull based method, a reasonable estimation is that the extreme response follows the Gumbel distribution with the average expected value and the average standard deviation from each sample, given in Eq. 17:

$$E[x_e] = \frac{1}{N} \sum_{i=1}^N (E_i[x_e]) \quad (17)$$

$$STD[x_e] = \left\{ \frac{1}{N-1} \sum_{i=1}^N (E_i[x_e] - E[x_e])^2 \right\}^{\frac{1}{2}}$$

Here N is the number of the time histories, and $E_i[x_e]$ is the expected value of the simulation number i .

For the ACER method and moment based Hermite method, a suitable estimation is to use the mean up-crossing rate for the samples of the total time history, which is defined as:

$$\hat{v}_0^+ = \frac{1}{N \cdot T} \sum_{i=1}^N n_i^+ \quad (18)$$

where n_i is the counted zero up-crossing number for simulation number i .

The estimated extreme value distribution in terms of exceedance probability for the different methods is shown in Fig. 9. The vertical dash line denotes the criterion of the collision. Physically, the collision will occur when $X \geq -1$. However, due to the hydrodynamic interference, it is reasonable to believe that the collision will take place when the riser clearance is smaller than $2D$ where the drag force becomes negative [14]. The collision probabilities obtained by using both definitions of collision are

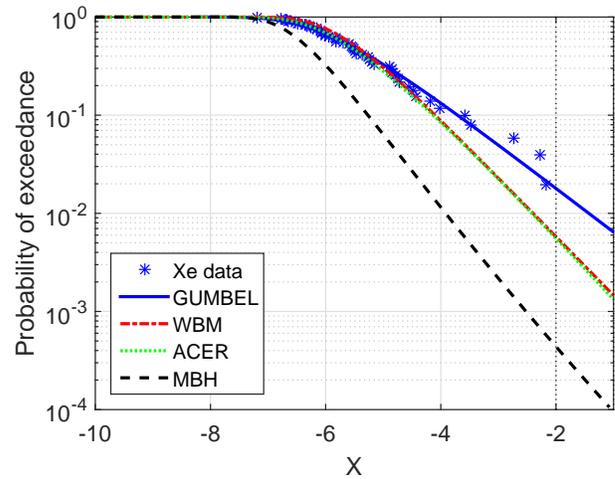


FIGURE 9. COMPARISON OF PROBABILITIES OF EXCEEDANCE OF EXTREME VALUE FOR DIFFERENT METHODS

TABLE 2. COLLISION PROBABILITY

Method	Probability of exceedance	
	D	2D
GUMBEL	6.40×10^{-3}	1.78×10^{-2}
WBM	1.52×10^{-3}	6.46×10^{-3}
ACER	1.35×10^{-3}	5.71×10^{-3}
MBH	1.06×10^{-4}	3.92×10^{-4}

summarized in Table 2. It is observed that the moment based Hermite method is not a good estimation for the present extreme distribution, as it fails to translate the highly skewed distribution to a standard Gaussian distribution. The Weibull based method and ACER method with the threshold value $E(X) + \sigma$ cover most of the extreme samples from each time histories, and give satisfactory estimation of the collision probability. The Gumbel probability paper method, by contrast, give the most critical result because all the largest maxima from each simulation are equally weighted.

CONCLUSIONS

This paper evaluated the performance of different methods for the short-term extreme value analysis for the riser collision. A pair of tandem arrangement risers with steep-wave configuration, which are subjected to combined current and wave loads, are modelled. The Blevins wake model is used to calculate the reduced drag force caused by the wake effect. The minimum relative distance between the risers at each time step is com-

puted. The random process is obtained by changing the sign of the minimum distance in order to deal with the maxima extreme value problem. The performance of the Gumbel probability paper method, Weibull based method, ACER method and moment based Hermite method are evaluated. Firstly, it appears that the Gumbel probability paper method requires a large number of simulations to achieve acceptable results. Secondly, the Weibull based method is more practical when the data is limited. However, the selection of an appropriate threshold has a significant effect on the estimation. Moreover, the results obtained by using the ACER and Weibull based methods are quite similar, giving satisfactory results. Lastly, the moment based Hermite method does not give a good estimation, as it fails to translate the highly skewed distribution to a standard Gaussian distribution.

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APPENDIX A

The moment of x are given by:

$$m_x^n = \int_0^\infty x^n f_X dx = b^n \Gamma\left(1 + \frac{n}{c}\right) \quad (19)$$

The moment of the sample are given by:

$$\hat{x}^n = \frac{1}{n} \sum_{i=1}^n \hat{x}_i^n \quad (20)$$

where n is the number of the max. Then, the expected value, variance and skewness can be found as following:

Expected value:

$$\frac{1}{n} \sum_{i=1}^n \hat{x} = a + b \Gamma\left(1 + \frac{1}{c}\right) \quad (21)$$

Variance:

$$\frac{1}{n} \sum_{i=1}^n (\hat{x} - a)^2 = b^2 \left\{ \Gamma\left(1 + \frac{2}{c}\right) - \Gamma^2\left(1 + \frac{1}{c}\right) \right\} \quad (22)$$

Skewness:

$$\begin{aligned} & \left(\frac{\hat{x}^3}{\hat{x}^2} \right)^{3/2} \\ &= \frac{\Gamma(1 + 3/c) - 3\Gamma(1 + 1/c) \cdot \Gamma(1 + 2/c) + 2T^3(1 + 1/c)}{\{\Gamma(1 + 2/c) - \Gamma^2(1 + 1/c)\}^{3/2}} \end{aligned} \quad (23)$$

where $\Gamma(\cdot)$ is the gamma function. The distribution parameters a , b and c can be obtained by solving the Eqs. 21 to 23.