Maritime Fleet Deployment with Speed Optimization and Voyage Separation Requirements

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Abstract. A shipping company operates a heterogeneous fleet of ships to service a given number of voyages on a number of trade routes over the planning horizon. Each ship has a predefined speed range within which it can sail. Fuel consumption, and hence fuel cost, significantly depends on the chosen speed. Furthermore, the shipping company makes Contracts of Affreightments with the shippers stating that the voyages on each trade route should be fairly evenly spread. This leads to the maritime fleet deployment problem with speed optimization and voyage separation requirements. We propose two formulations for this problem, i.e. one arc flow and one path flow model. The non-linear relationship for fuel consumption as a function of ship speed is linearized by choosing discrete speed points and linear combinations of these. Computational results show that the path flow model performs better than the arc flow model and that incorporating speed decisions in the fleet deployment gives better solutions and more planning flexibility.

Keywords: Maritime fleet deployment, Speed optimization, Voyage separation requirements

1 Introduction

Maritime transportation is the main distribution network for international trade and has a key role in today's globalized world. According to the International Maritime Organization (IMO), 90% of all transported goods across borders worldwide is transported by the shipping industry, corresponding to approximately 10 billion tons in 2015 [9]. Even though the global demand has steadily been increasing over decades, there has been a tendency of overcapacity in the fleet since the financial crisis around 2009 [9]. In 2015, the shipping industry, with the exception of tankers, suffered from historic low levels of freight rates and weak earnings. As a result, the margins are pushed down. For an industry that has high investment and operational costs, the quest for profitable operations is of higher importance than ever. One of the main targets in order to achieve this is to utilize the fleet capacity at all times and reduce ballast sailing (i.e. sailing without payload) to a minimum. Proper planning of maritime routes and schedules is therefore important.

In this paper, we extend the problem studied by Norstad et al. [5]. They considered a real maritime fleet deployment problem with voyage separation constraints for a shipping company operating in the open hatch dry bulk segment. The voyage separation constraints arise from contracts with the shippers which require that the trade routes are serviced regularly and that consecutive voyages along each trade are sufficiently separated in time. The main task in this problem is to assign available ships to the voyages on the different trade routes, such as to utilize the fleet in an optimal manner. Two models, an a priori path generation method and an arc flow method, were presented in [5], where the path flow model performed best. Vilhelmsen et al. [10] developed a Branch-and-Price procedure for the problem studied in [5]. They used a dynamic programming algorithm and a modified time window branching scheme, and found solutions that were at least as good as those by Norstad et al. [5] in shorter time.

Within other transportation modes, several examples of voyage separation requirements and time dependencies between routes can be found, though in a different context than ours. In vehicle routing, Reinhardt et al. [8] consider a dial-a-ride problem for airport passengers with complicating synchronization constraints. Dohn et al. [4] also consider synchronization and precedence constraints in two compact formulations of the vehicle routing problem with time windows. Dantzig–Wolfe decompositions of these formulations are presented and four different master problem formulations are proposed.

Most of the models found in the maritime transportation literature assume fixed and known speeds for the ships, either as implicit input or explicit input [7]. This is also the case in [5] and [10]. However, in reality fuel consumption, and hence sailing costs, is strongly dependent on speed. Therefore, incorporating speed in ship routing and scheduling can yield significant improvements in profits for the shipping company [6]. In addition, fuel consumption influences the emissions of Greenhouse Gas (GHG). Many papers assume that daily fuel consumption is a cubic function of ship speed. Andersson et al. [2] use a linear combination of predefined discrete speed alternatives and interpolation in order to provide the desired fuel consumption as a piecewise linear function of speed. It should be noted that this problem differs from most other problems where speed optimization have been incorporated, including [6] and [2], in that we cannot optimize speed locally in each route due to the voyage separation requirements, resulting in inter-dependency among the ship routes.

Based on these findings, we extend the models [5] and [10] by integrating speed decisions along the different sailing legs, and we denote it the Maritime Fleet Deployment problem with Speed Optimization and Voyage Separation requirements (MFDSOVS). Our main contributions are to propose two models for the MFDSOVS, i.e. an arc flow and a path flow model, both which are extended based on [5] by integrating speed decisions. A number of realistic instances based on data from a shipping company are used to test the performance of the models and the effect of incorporating speed optimization on the solution quality, and it is shown that the path flow model performs better than the arc flow model.

2 Problem Description

We will now give a description of the MFDSOVS. Section 2.1 describes the fleet deployment part of the problem, which basically consists of assigning voyages to ships in the fleet (and implicitly ship routes). Section 2.2 describes the speed optimization part of the problem. Section 2.3 describes the voyage separation requirements before we end the section by summarizing the MFDSOVS.

2.1 Fleet Deployment

The fleet deployment problem can be described as a tactical planning problem of assigning ships from a heterogeneous fleet to voyages on different trade routes efficiently in terms of costs and service. A trade route is a predefined, typically intercontinental, sailing route from an origin region (including one or more ports) to a destination region (including one or more ports). Figure 1 shows intercon-

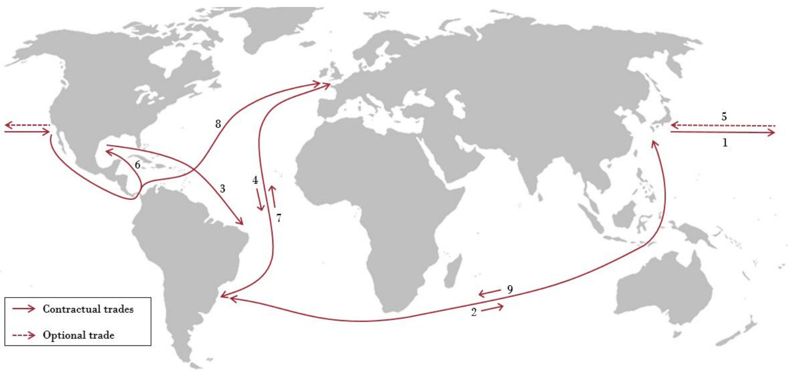

Fig. 1. Contractual and optional trade routes

tinental trade routes. A voyage is a sailing along a trade route. The number of voyages to be serviced along each trade may vary according to some frequency requirements. The trades can be separated into two types; contractual (mandatory) and optional trades. The shipping company seeks to maximize its profit by servicing voyages on optional trades while satisfying all contractual voyages on the contractual trades. If the company's own fleet is not capable to carry all contractual voyages, additional spot ships are chartered to serve contractual voyages. It is assumed that the ballast sailing costs associated with chartering a spot ship is included in the charter costs. The ships usually serve several voyages in a sequence within the planning horizon. To start the next voyage, a ship might have to sail in ballast from the end of its previous voyage to re-position itself.

Each voyage has a predefined time window within which the voyage must start, instead of a fixed start-up time as is common in container shipping [3], which provide some flexibility for the shipping company.

2.2 Fuel Consumption and Speed

The operational costs of a fleet depend heavily on fuel consumption, which is also an environmental concern. Thus, optimizing sailing speeds along the ships' routes should be integrated with the fleet deployment. Fuel consumption is typically a cubic (quadratic) and convex function of speed per time (distance) unit.

Speed optimization means to adjust the sailing speed to seek higher profits. Increasing the speed increases the total available fleet capacity, which can in some cases be cheaper than chartering in spot ships to service contractual voyages. Increased fleet capacity also enables the possibility for the company to service optional voyages, which leads to additional revenue. On the other hand, higher sailing speeds incurs higher sailing costs. Therefore, it is not straight forward to find the optimal sailing speeds along all sailing legs that a ship performs during the planning horizon. In addition, a ship's fuel consumption also depends on the load onboard the ship as illustrated in Figure 2.

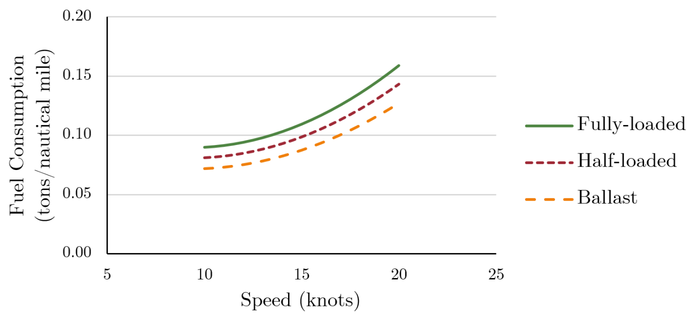

Fig. 2. Fuel consumption curves for ballast, half-loaded, and fully-loaded sailing [11].

2.3 Voyage Separation Requirements

Shippers enter Contracts of Affreightment (CoAs) with the shipping company. The most important part of the CoAs is where the cargo is heading, the amount transported, at what time and the freight rate. A commonly used term in CoAs, regarding the frequency and timing of voyages on a trade route, is "fairly evenly spread". This means that consecutive voyages on the same trade should be sufficiently spread in time. This introduces voyage separation requirements to the MFDSOVS. Norstad et al. [5] shows an example of the spread of voyages on a trade with or without voyage separation requirements as in Figure 3, which clearly shows that without voyage separation constraints some of the consecutive voyages start very close in time to each other, which would possibly be in conflict with the "fairly evenly spread" terms that are stated in the CoAs.

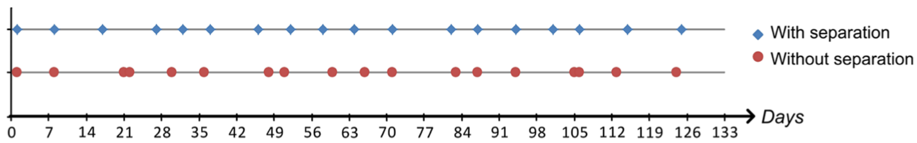

Fig. 3. Starting days for voyages on a trade with or without voyage separation [5].

2.4 Problem Summary

The objective in the MFDSOVS problem is to maximize profit, i.e. total freight income minus the sum of operation costs of ships in fleet and the charter costs for spot ships. The decisions to be made are: 1) the ship routes (i.e. which ship should perform which voyages and in what sequence), 2) the ships' sailing speeds for each sailing leg along their routes, 3) the start time for each voyage, 4) which optional voyages to sail, and 5) which voyages should be serviced by spot ships.

The decisions must comply with 1) that all contractual voyages are serviced within their given time windows, either by a ship from the company's fleet or by a spot ship, and 2) that all consecutive voyages along each trade route are fairly evenly spread.

3 Mathematical Formulations

In this section, two mathematical formulations, one arc flow, and one path flow model, for the MFDSOVS problem are given. Both are based on the ones from [5], though extended with speed optimization.

3.1 Arc Flow Model

Notation Let $\mathcal{V}$ be the set of heterogeneous ships in the fleet of the shipping company, indexed by v. The ships have individual starting positions and maintenance schedules, and should therefore be treated individually, as treating them as a group could lead to infeasible solutions. We use the same approach as Andersson et al. [2] for handling the non-linear relationship between speed and fuel consumption/cost and sailing times, where we choose a number of discrete speed alternatives from the non-linear function (see Figure 2) and allocate weights to these speed points. The set $\mathcal{S}$ is an ordered set containing all available discrete speed points, from minimum to maximum speed, indexed by s.

The set $\mathcal{R}$ denotes the set of trade routes operated by the company, indexed by r. $\mathcal{R}_v$ is a subset of $\mathcal{R}$ for which trade routes ship v can carry out. Let the set $\mathcal{I}_r$={1,2,3, ..., n_r} be the set of voyages on trade route r, where n_r is the number of voyages on trade route r that has to be serviced over the planning horizon. The set of voyages is indexed by i.

The given problem can be formulated on a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ denotes nodes, and $\mathcal{A}$ represents the set of arcs. The set $\mathcal{N}$ consists of four different kinds of nodes: Origin nodes, destination nodes, voyage nodes and maintenance nodes. The set $\mathcal{N}_v \subseteq \mathcal{N}$ consists of the nodes that ship v can visit. For each ship v, its origin node *o(v)* in set $\mathcal{N}$ represents the initial position and its destination node *d(v)* in set $\mathcal{N}$ corresponds to an artificial destination which does not exist physically. Each voyage i on trade route r is given by a voyage node (r,i). There are two types of voyage nodes (contracted voyage nodes and optional voyage nodes), which consists of two disjoint subsets of $\mathcal{N}$. The set $\mathcal{N}^C$ represents the contracted voyages that the shipping company must service, while the set $\mathcal{N}^O$ represents the optional voyages. The set $\mathcal{N}_v^M$ is the set of maintenance nodes for ship v, indexed by (r,i) like voyage nodes. For each ship v without any maintenance requirements during the planning period, the set $\mathcal{N}_v^M$ will be empty. If ship v is due for maintenance, it is assumed to visit exactly one maintenance node during the planning period. The set $\mathcal{A}$ includes all arcs. The set $\mathcal{A}_v \subseteq \mathcal{A}$ consists of the arcs that can be traversed by ship v. The arc $((r,i),(q,j))$ corresponds to sailing ballast directly from the end of voyage or maintenance node (r,i) to the start of voyage or maintenance node (q,j). The arcs sailing directly from the origin node of ship v to voyage or maintenance node (r,i), $((o(v)),(r,i))$, and the arcs travelling directly from the voyage or maintenance node (r,i) to the destination node of ship v, $((r,i),(d(v)))$, are also included in $\mathcal{A}$. The set $\mathcal{A}_v$ consists of the arcs $(o(v),d(v))$ such that the ship v sails directly from its starting node *o(v)* to the ending node *d(v)*, i.e. the ship is idle over the planning horizon.

Let T^B_{vriqjs} be the time ship v takes to sail ballast from the last unloading port of voyage (r,i) to the first loading port of voyage (q,j), or in other words sailing the arc $((r,i),(q,j))$, with speed alternative s. The corresponding cost is C^B_{vriqjs}. The time it takes to sail ballast from the starting position to start position of voyage (r,i) with speed alternative s is $T^B_{vo(v)ris}$, and the corresponding cost is $C_{vo(v)ris}$. The time it takes to sail voyage (r,i) with speed alternative s is denoted by T_{vris}, which corresponds to sailing time between all ports on a trade route plus the operation time at all port calls. The corresponding cost is C_{vris}, which mainly consists of fuel costs. The estimated freight income minus the port costs, for sailing voyage (r,i) is R_{ri}. C^S_{ri} is the cost of chartering a ship from the spot market to service voyage (r,i). Each voyage has to start at its first port within a given time window, $[E_{ri}, L_{ri}]$. The parameter E_{ri} is the earliest time for starting voyage i on trade r, while L_{ri} is the latest time for starting the voyage. Let $E_{o(v)}$ be the earliest time ship v can depart from its initial starting position. B_r represents the minimum accepted time interval between two consecutive voyages on trade r.

Let variable x_{vriqj} be a binary variable, which is 1 if ship v sails directly from node (r,i) to node (q,j), otherwise 0. The binary flow variable $x_{vo(v)ri}$ is 1 if ship v travels directly from it initial position to node (r,i), otherwise 0. Let variable $x_{vrid(v)}$ equal 1 if (r,i) is the last node ship v services, and 0 otherwise. Similarly, variable $x_{o(v)d(v)}$ is 1 if ship v is idle, and 0 otherwise. Let variable u^S_{ri} be 1 if voyage i on trade r is carried out by a spot ship, and 0 otherwise. The start time of voyage i on trade r is defined by the variable t_{ri}. Let variable w_{vris} be the weight of speed alternative s for ship v sailing voyage (r,i). Let variable w^B_{vriqjs} be the weight of speed alternative s for ship v sailing ballast from the last unloading port of voyage (r,i) to the first loading port of voyage (q,j). Let variable $w^B_{vo(v)ris}$ be the weight of speed alternative s for ship v sailing ballast from its initial position $o(v)$ to the first loading port of the voyage (r,i). The weights of the speed alternatives should sum up to 1 if an arc is serviced by that ship, and 0 otherwise. The maritime fleet deployment problem with speed optimization and voyage separation requirements can be formulated as follows:

Objective function The objective function (1) maximizes the total profit by summing the profits from servicing the voyages by ships in the fleet (the estimated freight income minus the voyage costs minus ballast sailing costs) and the spot ships (the estimated freight income minus the voyage costs).

$$\begin{aligned} \max \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{i\in\mathcal{I}_r}\Bigg[\sum_{s\in\mathcal{S}}(R_{ri}-C_{vris})w_{vris} - \sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q}\sum_{s\in\mathcal{S}} C^B_{vriqjs}w^B_{vriqjs} \\ - \sum_{s\in\mathcal{S}} C^B_{vo(v)ris}w^B_{vo(v)ris}\Bigg] + \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}(R_{ri}-C^S_{ri})u^S_{ri} \end{aligned} \quad (1)$$

Service constraints Constraints (2) represent that each contracted voyage should be serviced exactly once by either a ship in the fleet or a spot ship. Constraints (3) state that each optional voyage can be serviced at most once by a ship in the fleet. Constraints (4) ensure that all required maintenance for ships in the fleet are performed.

$$\sum_{v\in\mathcal{V}_r}\Bigg[\sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q} x_{vriqj} + x_{vrid(v)}\Bigg] + u^S_{ri} = 1, \quad (r,i)\in\mathcal{N}^C \quad (2)$$

$$\sum_{v\in\mathcal{V}_r}\Bigg[\sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q} x_{vriqj} + x_{vrid(v)}\Bigg] \le 1, \quad (r,i)\in\mathcal{N}^O \quad (3)$$

$$\sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q} x_{vriqj} + x_{vrid(v)} = 1, \quad v\in\mathcal{V}, (r,i)\in\mathcal{N}^M_v \quad (4)$$

Network flow constraints Constraints (5)-(7) ensure network flow for each ship. Constraints (5) state that a ship must either be idle or leave its starting position to a node $(r,\, i)$, while constraints (7) state that a ship must either be idle or arrive at its ending position from a node (r, i). Constraints (6) ensure that each voyage starts in an origin node, that every visited voyage or maintenance node is also exited, and that each voyage ends up in a destination node. Constraints (8)-(10) describe the relation between the flow variables and the speed weighting variables for initial ballast sailing, ballast sailing and voyage sailing, respectively.

$$x_{vo(v)d(v)} + \sum_{r\in\mathcal{R}_v}\sum_{i\in\mathcal{I}_r} x_{vo(v)ri} = 1, \quad v\in\mathcal{V} \tag{5}$$

$$x_{vrid(v)} + \sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q} x_{vriqj} - \sum_{q\in\mathcal{R}_v}\sum_{j\in I_q} x_{vqjri} - x_{o(v)ri} = 0, \quad v\in\mathcal{V}, r\in\mathcal{R}_v, i\in\mathcal{I}_r \tag{6}$$

$$x_{vo(v)d(v)} + \sum_{r\in\mathcal{R}_v}\sum_{i\in\mathcal{I}_r} x_{vrid(v)} = 1, \quad v\in\mathcal{V} \tag{7}$$

$$x_{vo(v)ri} - \sum_{s\in\mathcal{S}} w^B_{vo(v)ris} = 0, \quad v\in\mathcal{V}, r\in\mathcal{R}_v, i\in\mathcal{I}_r \tag{8}$$

$$x_{vriqj} - \sum_{s\in\mathcal{S}} w^B_{vriqjs} = 0, \quad v\in\mathcal{V}, ((r,i),(q,j))\in\mathcal{A}_v \tag{9}$$

$$x_{vrid(v)} + \sum_{q\in\mathcal{R}_v}\sum_{j\in\mathcal{I}_q} x_{vriqj} - \sum_{s\in\mathcal{S}} w_{vris} = 0, \quad v\in\mathcal{V}, r\in\mathcal{R}_v, i\in\mathcal{I}_r \tag{10}$$

Time constraints Constraints (11) state that time spent sailing ballast from the initial position of ship v to its first voyage (r,i) does not exceed the latest start time of voyage (r,i). Constraints (12) ensure the time spent on voyage (r,i) and ballast sailing to the start of voyage (q,j) does not exceed the latest start time of voyage (q,j). Constraints (13) secure that time window for each voyage is not violated. Constraints (11) and (12) have been linearized by applying the big-M method.

$$E_{o(v)} + \sum_{s\in\mathcal{S}} T^B_{vo(v)ris} w^B_{vo(v)ris} - t_{ri} - E_{o(v)}(1 - x_{vo(v)ri}) \le 0, \quad v\in\mathcal{V}, r\in\mathcal{R}_v, i\in\mathcal{I}_r \tag{11}$$

$$t_{ri} + \sum_{s\in\mathcal{S}} (T_{vris} w_{vris} + T^B_{vriqjs} w^B_{vriqjs}) - t_{qj} - L_{ri}(1 - x_{vriqj}) \le 0, \quad v\in\mathcal{V}, ((r,i),(q,j))\in\mathcal{A}_v \tag{12}$$

$$E_{ri} \le t_{ri} \le L_{ri}, \quad r\in\mathcal{R}, i\in\mathcal{I}_r \tag{13}$$

Voyage Separation constraints Constraints (14) take care of the minimum accepted time between two consecutive voyages on a trade route.

$$t_{r,i+1} - t_{ri} \ge B_r, \quad r\in\mathcal{R}, i\in\mathcal{I}_r\backslash\{n_r\} \tag{14}$$

Binary and Non-negativity Constraints.

$$x_{vo(v)d(v)} \in \{0,1\}, \quad v \in \mathcal{V} \tag{15}$$

$$x_{vo(v)ri}, x_{vrid(v)} \in \{0,1\}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{16}$$

$$x_{vriqj} \in \{0,1\}, \quad v \in \mathcal{V}, ((r,i),(q,j)) \in \mathcal{A}_v \tag{17}$$

$$w^B_{vo(v)ris}, w_{vris} \in [0,1], \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r, s \in \mathcal{S} \tag{18}$$

$$w^B_{vriqjs} \in [0,1], \quad v \in \mathcal{V}, ((r,i),(q,j)) \in \mathcal{A}_v, s \in \mathcal{S} \tag{19}$$

$$t_{ri} \geq 0, \quad r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{20}$$

$$u^S_{ri}, \in \{0,1\} \quad r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{21}$$

3.2 Path Flow Model

Notation Some of the notation presented for the arc flow model is still valid for the path flow model. Only new notation for the path flow model is presented in this section. $\mathcal{P}_v$ represents the set of all feasible paths for ship v. $\mathcal{P}_{vriqj}$ is a subset of $\mathcal{P}_v$ including all paths where ship v travels directly from voyage i on trade route r to voyage j on trade route q. $\mathcal{P}_{vri}$ is a subset of $\mathcal{P}_v$, which contains all paths where ship v services voyage i on trade route r. Another subset of $\mathcal{P}_v$, $\mathcal{P}_{vo(v)ri}$, which contains all paths where ship v sails directly from its initial position to voyage i on trade route r as its first voyage.

Let E_{vpri} be a the earliest service start time for ship v at voyage i on trade route r for a path p.

Let variable z_{vp} be a binary variable, which equals 1 if ship v sails path p, and 0 otherwise. Let t_{vri} be a variable that sets the start time of voyage i on trade route r with ship v. Variable t^S_{ri} applies when a spot ship starts sailing voyage i on trade route r. A path flow model describing the fleet deployment problem with speed optimization and voyage separation constraints can be described as follows.

Objective function The objective function (22) aims to maximize profit by finding the optional speed on the paths.

$$\begin{aligned} \max \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} \sum_{i \in \mathcal{I}_r} \Bigg[& \sum_{s \in \mathcal{S}} (R_{ri} - C_{vris}) w_{vris} - \sum_{q \in \mathcal{R}_v} \sum_{j \in \mathcal{I}_r} \sum_{s \in \mathcal{S}} C^B_{vriqjs} w^B_{vriqjs} \\ & - \sum_{s \in \mathcal{S}} C^B_{vo(v)ris} w^B_{vo(v)ris} \Bigg] + \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} (R_{ri} - C^S_{ri}) u^S_{ri} \end{aligned} \tag{22}$$

Service constraints Constraints (23) ensure that all contractual voyages are carried out exactly once, either by a ship within the fleet or by a spot ship, where constraints (24) ensure that the optional voyages may be carried out at most once by a ship within the fleet. All ships have to be assigned to exactly one path, as in constraints (25).

$$\sum_{v \in \mathcal{V}_r} \sum_{p \in \mathcal{P}_{vri}} z_{vp} + u^S_{ri} = 1, \quad (r,i) \in \mathcal{N}^C \tag{23}$$

$$\sum_{v \in \mathcal{V}_r} \sum_{p \in \mathcal{P}_{vri}} z_{vp} \leq 1, \quad (r,i) \in \mathcal{N}^O \tag{24}$$

$$\sum_{p \in \mathcal{P}_v} z_{vp} = 1, \quad v \in \mathcal{V} \tag{25}$$

Network flow constraints Constraints (26)-(28) ensure that the speed weighting variables for each ship on a path can take non-zero values only when the ship sails that path.

$$\sum_{s \in \mathcal{S}} w_{vris} = \sum_{p \in \mathcal{P}_{vri}} z_{vp}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{26}$$

$$\sum_{s \in \mathcal{S}} w^B_{vo(v)ris} = \sum_{p \in \mathcal{P}_{vo(v)ri}} z_{vp}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{27}$$

$$\sum_{s \in \mathcal{S}} w^B_{vriqjs} = \sum_{p \in \mathcal{P}_{vriqj}} z_{vp}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r, q \in \mathcal{R}_v, j \in \mathcal{I}_q \tag{28}$$

Time constraints Constraints (29) say that the start time for a voyage has to be within the time window. The same goes for the start time for a voyage by spot ships as in constraints (30). Constraints (31) ensure that a ship can not start a voyage before it has sailed ballast from its origin position to the start point of the voyage. Likewise, constraints (32) ensure that a ship can not start a voyage before it has completed the previous voyage and sailed ballast to the start of the next voyage.

$$\sum_{p \in \mathcal{P}_{vri}} E_{vpri} z_{vp} \leq t_{vri} \leq \sum_{p \in \mathcal{P}_{vri}} L_{ri} z_{vp}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{29}$$

$$E_{ri} u^S_{ri} \leq t^S_{ri} \leq L_{ri} u^S_{ri}, \quad r \in \mathcal{R}, i \in \mathcal{I}_r \tag{30}$$

$$\sum_{s \in \mathcal{S}} \big(T^B_{vo(v)ris} + E_{o(v)}\big) w^B_{vo(v)ris} \leq t_{vri}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{31}$$

$$\begin{aligned} & t_{vri} + \sum_{s \in \mathcal{S}} \big(T_{vris} w_{vris} + T^B_{vriqjs} w^B_{vriqjs} + (L_{ri} + T_{vri,1}) w^B_{vriqjs}\big) \\ & - L_{ri} - T_{vri,1} - t_{vqj} \leq 0, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r, q \in \mathcal{R}_v, j \in \mathcal{I}_q \end{aligned} \tag{32}$$

Voyage Separation constraints Constraints (33) show the voyage separation constraints, which ensures a minimum time spread between two consecutive voyages on the same trade route.

$$B_r + \sum_{v \in \mathcal{V}} t_{vri} + t^S_{ri} - \sum_{v \in \mathcal{V}} t_{vr,i+1} - t^S_{r,i+1} \leq 0, \quad r \in \mathcal{R}, i \in \mathcal{I}_r \backslash \{n_r\} \tag{33}$$

Binary and Non-negativity Constraints.

$$z_{vp} \in \{0,1\}, \quad v \in \mathcal{V}, p \in \mathcal{P}_v \tag{34}$$

$$u^S_{ri} \in \{0,1\}, \quad r \in \mathcal{R}, i \in \mathcal{I}_r \tag{35}$$

$$w_{vris}, w^B_{vo(v)ris} \in [0,1], \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r, s \in \mathcal{S} \tag{36}$$

$$w^B_{vriqjs} \in [0,1], \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r, q \in \mathcal{R}_v, j \in \mathcal{I}_q, s \in \mathcal{S} \tag{37}$$

$$t^S_{ri} \geq 0, \quad r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{38}$$

$$t_{vri} \geq 0, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, i \in \mathcal{I}_r \tag{39}$$

4 Computational Study

The mathematical models presented in Section 3 have been implemented in Mosel and solved using Xpress 31.01.09. All computational tests are performed on a HP Elitedesk computer with Intel Core i7-7700 CPU (4 3.60 GHz) and 32 GB RAM running on Windows 10.

4.1 Test Instances

The instances are based on data from the case shipping company as in [5] and shown in Table 1. The instances are divided into four sets, with three (six), five (ten), seven (14) and nine (18) trades (ships), respectively. All four sets have also been divided into planning horizons of 60, 90 and 120 days, totaling 12 instances. For example, the nine trade routes instances shown in Figure 1 correspond to the largest set, set 4 in Table 1. Instance sets 1-3 are reduced versions of instance set 4 where some trade routes and ships have been removed. The fifth column shows the number of voyages that should be serviced for each instance. The numbers represent the contractual voyages out of the total number of voyages (including optional voyages). The optional voyages are organized as one trade that consists of optional voyages only. All feasible paths for each instance are generated using Matlab code as input to the path flow model in Section 3.2, and are shown in the last column in Table 1. The fuel cost is set to 388 USD/ton, which is the global average for the first quarter of 2018 for the 20 largest ports in the world [1].

Table 1. Summary of test instances

Set	Instance	Ships	Trades	Voyages	Planning days	Paths
	1	6	3	11/11	60	159
1	2	6	3	15/15	90	299
	3	6	3	20/20	120	985
	4	10	5	13/15	60	364
2	5	10	5	18/21	90	823
	6	10	5	24/28	120	3277
	7	14	7	24/26	60	1886
3	8	14	7	34/37	90	8711
	9	14	7	46/50	120	69776
	10	18	9	30/32	60	3073
4	11	18	9	44/47	90	16199
	12	18	9	59/63	120	138292

4.2 Comparison of the Arc Flow and Path Flow Models

The 12 test instances in Table 1 have been solved by both the arc flow and the path flow models using three speed points (i.e. the minimum, maximum and the middle speed points). The results of these comparisons are shown in Table 2. The columns Time report the computational times in seconds. Here, we have allocated a maximum running time of one hour (i.e. 3600 seconds). The columns Obj val. show the objective values found by the two models. The columns Gap show the gap in percentage between the best integer solution and the best bound found after the time limit. The columns LP Rel. show the LP relaxation.

From the results in Table 2 we see that for the smallest problem instances (1, 2 and 4) there are no significant differences in performance between these two models. They both find the optimal solutions to these instances in little computational time. For the larger problem instances, however, there is a tendency that the path flow model is faster than the arc flow model (14.05% improvement in time). For large test instances 8-12, both the arc flow and the path flow models are not able to prove optimality within the time limit with an average of 17.25% and 3.46% gaps, respectively.

Comparing the average performance of the two models, we see that that the path flow models gives 11.5% improvement in solution quality with reduced solution times compared to the arc flow model. The LP relaxation achieves a 4.1% improvement for the path flow compared to the arc flow model. The conclusion from this comparison is that using the path flow model gives better (or equally good) solutions for all large instances due to smaller average gaps, better LP relaxation, and less computational times. Therefore, only the path flow model is used in Section 4.3 for analyzing the speed optimization in detail.

Table 2. Comparison of arc flow and path flow models using three speed points.

	Arc Flow Model				Path Flow Model			
Instance	Time	Obj val.	Gap	LP Rel.	Time	Obj. val.	Gap	LP Rel.
1	0.6	13,837	0.00%	14,742	0.1	13,837	0.00%	13,949
2	1.7	17,350	0.00%	19,874	1.2	17,350	0.00%	17,965
3	80.6	22,223	0.00%	26,123	10.6	22,223	0.00%	23,308
4	1.3	17,456	0.00%	18,035	0.3	17,456	0.00%	17,555
5	1857.1	22,949	0.00%	24,485	13.0	22,949	0.00%	23,845
6	3600.0	28,141	11.77%	32,018	3600.0	28,795	3.99%	31,090
7	3600.0	24,995	4.72%	26,186	1704.3	25,339	0.00%	25,835
8	3600.0	31,579	18.62%	37,752	3600.0	33,934	4.56%	35,967
9	3600.0	40,751	23.10%	50,587	3600.0	42,227	12.62%	47,621
10	3600.0	29,435	5.20%	31,610	3600.0	30,288	0.14%	30,621
11	3600.0	34,419	28.61%	44,466	3600.0	40,755	6.43%	43,465
12	3600.0	27,510	114.99%	59,340	3600.0	51,161	13.76%	58,223
Average	2261.8	25,887	17.25%	32,101	1944.1	28,860	3.46%	30,787

4.3 Comparison of using different speed points for linearization

All test instances in Table 1 with planning horizon of 120 days have been solved using the path flow model with one, two, and three speed points, respectively. When solving with one speed point (without speed optimization), we have used the maximum speed, as this was shown to give better solutions compared to planning with only the medium (or service) speed of the ships [6]. The results are shown in Table 3. For a fair comparison, it should be noted that the column Profit show the best solutions found by the model after a posteriori speed optimization (using 10 points), and will therefore slightly deviate from Obj. value in Table 2 for three speed points. The columns Gap show the gap between the best integer solutions and the best bounds found after the 3600 seconds time limit. The columns #Spot show the number of voyages performed by spot ships in the problem instance, while the columns Time report the computational time in seconds.

Table 3. Comparison of different number of speed points.

	1 speed point (max)				2 speed points (min/max)				3 speed points (min/avg/max)			
Instance	Profit	Gap	# Spot	Time	Profit	Gap	# Spot	Time	Profit	Gap	# Spot	Time
3	20,175	0.00%	3	0.1	22,246	0.00%	3	11.1	23,308	0.00%	3	10.6
6	27,666	0.00%	1	0.2	28,555	5.56%	3	3600.0	28,852	3.99%	2	3600.0
9	41,207	0.00%	0	211.4	42,367	13.61%	7	3600.0	42,359	12.62%	6	3600.0
12	51,173	0.00%	0	994.3	50,781	16.16%	1	3600.0	51,553	13.76%	3	3600.0
Average	35,055	0.00%	1	301.5	35,987	8.83%	4	2702.8	36,518	7.59%	4	2702.7

Comparing the average solutions from the different number of speed points, we can see that using three speed points gives the best solution quality, though with longer solution times. A larger number of speed points might provide better results with longer solution times. As a compromise, three speed points are used in our study. The one speed point instances are all solved to optimality within the maximum time limit. The average number of voyages performed by spot ships with two and three speed points are higher than that with one speed point, especially for the medium and large instances (instances 9 and 12), which implies that integrating speed optimization in a fleet deployment problem not only achieves better profits, but also gives much more planning flexibility for shipping companies.

4.4 Path Reduction Heuristics

It was shown in the previous sections that the gaps are large for the largest instances with long planning horizons. We have therefore tested three simple path reduction rules on instances 9 and 12. In the first, we remove all paths/routes that have higher percentage ballast sailing than a threshold level. In the next two, which will be used during the path generation, we remove paths with any ballast seiling leg and consecutive waiting time (assuming maximum speed) above specified threshold levels. We show results for the following four combinations of applying these rules in Table 4 (Max percentage ballast sailing - Max length ballast sailing in nautical miles - Max consecutive waiting days): A) 45% - 10.500 - 20, B) 35% - 10.500 - 20, C) 40% - 10.500 - 10, and D) 30% - 10.000 - 10.

Table 4. Effect of heuristic combinations

	Instance 9			Instance 12			Average	
Comb.	Paths	Obj. val.	Impr.	Paths	Obj. val.	Impr.	Obj. val.	Impr.
None	69776	42,227'	-	138392	51,161'	-	46,694'	-
A	19603	42,716'	1.2%	42338	51,841'	1.3%	47,279'	1.3%
B	8127	43,075'	2.0%	20431	52,098'	1.8%	47,587'	1.9%
C	7542	43,208'	2.3%	18294	52,305'	2.2%	47,757'	2.3%
D	2365	42,113'	-0.3%	6956	52,740'	3.1%	47,427'	1.6%

Table 4 shows both the number of paths and the solution improvement compared to the results without any path reduction rules. We see that the number of paths are significantly reduced and that we are able to obtain improved solutions to both instances 9 and 12, except for combination D on instance 9, where we obviously lose at least one of the optimal paths when using the path reduction heuristic.

5 Concluding Remarks

We have extended a previously studied problem [5, 10] by incorporating speed optimization. This gives the Maritime Fleet Deployment problem with Speed Optimization and Voyage Separation requirements (MFDSOVS). Two formulations for this problem, one arc flow formulation and one path flow formulation, are proposed in this paper. The non-linear relationship for fuel consumption as a function of ship speed is linearized by choosing discrete speed points and linear combinations of these. Computational results show that the path flow model is faster and generate better results than the arc flow model. Furthermore, we show that speed in the fleet deployment results in not only better profits, but also gives much more planning flexibility for shipping companies by having more voyages taken by spot ships. Finally, a priori path reduction heuristics are tested to solve the large instances more efficiently.

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