

Tangential strain-based criteria for mixed-mode I/II fracture toughness of cement concrete

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Abstract

Experimental and theoretical works are performed on the mixed-mode I/II brittle fracture of cement concrete tested by edge cracked semicircular bend specimens. Theoretical background of the traditional fracture criteria including strain energy density, maximum tangential stress, and maximum tangential strain (MTSN) are introduced. The ability of each fracture criterion in prediction of the fracture test data is investigated. The comparison between the evaluations by the traditional criteria and the experimental data shows that none of them are capable of successfully estimating the fracture resistance of cement concrete. An enhanced version of the MTSN criterion is then employed to predict the test data. It is demonstrated that the extended MTSN criterion can successfully predict the test data in a higher accuracy than traditional criteria.

1 INTRODUCTION

One of the most common concerns for engineers in designing structures is to evaluate the resistance of the materials against crack creation and propagation. Cracks are seen in any engineering structure and are usually due to the combination of mechanical, thermal, and chemical loads. Due to such inevitable discontinuities, the brittle materials are always exposed to sudden fracture, which may lead to catastrophic failure in structures made of brittle materials. Therefore, the study of fracture behaviour in brittle materials is of great importance. Fracture toughness is considered as a common index for evaluation of the mechanical response of brittle materials against crack propagation and is usually determined by running fracture experiments utilizing different fracture test specimens.

Concrete is a brittle composite material containing dispersed aggregates with different size, bonded together with cement paste, and is one of the most frequently used materials for construction of the structures and infrastructures. The relatively low tensile strength leads hardened concrete to contain cracks at both early ages and in-service conditions. Due to external loading, cracks are created in concrete depending on the loading configuration and are extended in-plane (I and II) or out of plane (III). However, in reality, a combination of all 3 modes occurs for a crack existing in an in-service

structure. Several research papers have been published by many researchers so far investigating mixed-mode brittle fracture behaviour of engineering components made of cement concrete.[1-10](#) However, it has always been a major concern to find a criterion that can properly evaluate the fracture behaviour of cement concrete under mixed-mode loading conditions.

Generally speaking, a variety of fracture criteria are available in the literature to assess brittle fracture characteristics of materials. Three categories of criteria are used: energy-based,[11-15](#) stress-based,[16-20](#) and strain-based[21-27](#) criteria. Each classification employs a different concept to describe the fracture mechanism and has intrinsic advantages and disadvantages. For instance, the stress-based fracture criteria are described with a simpler set of equations than the energy-based criteria, and the role of each stress field parameter is clearly represented in estimating the onset of fracture. On the other hand, the energy-based criteria utilize the concept of energy dissipation due to the crack extension and are formulated based on more realistic platforms.

A literature review unfolds this fact that in some materials and specimen geometries, the fracture propagation can be explained by a strain-based fracture criterion in a more precise way than by applying the energy and stress-based criteria.[22-28](#) Among them, a research conducted by Wu[26](#) manifests that fracture behaviour of concrete should be assessed by a strain-based fracture theory. As was pointed out by Wu,[26](#) a scalar-valued function of strain tensor determines the onset of fracture. Based on the research performed by Wu,[26](#)Chang[27](#) suggested a fracture criterion based on the maximum hoop strain considering only singular terms of the crack tip strain field. However, it has recently been proven that the nonsingular terms of the crack tip stress and strain fields significantly influence the fracture behaviour of brittle materials. For interface cracks (cracks existing at the interface of 2 dissimilar materials), Mirsayar[17](#) and Mirsayar and Park[19](#) have recently indicated that the first nonsingular crack tip stress term, called T-stress, remarkably influences the onset of fracture and the crack extension angle.

The traditional maximum tangential strain (MTSN) criterion proposed by Chang[27](#) has recently been modified by Mirsayar,[22](#) incorporating the effect of the first nonsingular strain term, called T-strain. It has been shown in Mirsayar[22](#) that the T-strain makes a significant impact on the fracture initiation behaviour. However, the extended MTSN (EMTSN) criterion proposed by Mirsayar[22](#) is the first model considering the role of the first nonsingular strain term in mixed-mode I/II brittle fracture of materials. The EMTSN criterion has successfully been applied to poly(methyl methacrylate),[22](#) polycrystalline

graphite,[28](#) and soda lime glass,[29](#) but it has not been applied to the cement concrete components so far.

This paper examines the fracture resistance of concrete focusing on the strain-based criteria, using edge cracked semicircular bend (SCB) specimens. The SCB specimen has been widely used for mixed-mode fracture analysis of different engineering materials such as poly(methyl methacrylate)[30](#), rock,[31](#), [32](#) concrete,[4](#) and asphalt concrete.[33](#), [34](#) The mixed-mode I/II stress intensity factors as well as the first nonsingular tangential strain term around the crack tip are calculated by using the numerical simulation for the SCB specimens. The traditional strain energy density (SED), maximum tangential stress (MTS), MTSN, and EMTSN criteria are introduced and applied to the test data. The accuracy of each model is investigated and discussed by comparing its results to the experimental data.

2 BRIEF REVIEW OF BRITTLE FRACTURE CRITERIA

2.1 Crack tip stress field

According to Williams,[35](#) the crack tip elastic stress field equations under mixed-mode I/II loading can be found as follows:

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} f_{rr,1}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{rr,2}(\theta) + T \cdot \cos^2\theta + HOT \quad (1)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} f_{\theta\theta,1}(\theta) - \frac{K_{II}}{\sqrt{2\pi r}} f_{\theta\theta,2}(\theta) + T \cdot \sin^2\theta + HOT \quad (2)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} f_{r\theta,1}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{r\theta,2}(\theta) - T \cdot \sin\theta \cos\theta + HOT \quad (3)$$

where r and θ are polar coordinates with origin at the crack tip and HOTA are the higher order terms. In Equations [1](#) to [3](#), the terms corresponding to K_I and K_{II} (modes I and II stress intensity factors) are called singular terms because they go to infinity as r approaches crack tip. The parameter T is the coefficient of the first nonsingular stress (T-stress) or strain (T-strain) term. The functions $f_{ij,k}$ are known functions of θ .[22](#), [35](#) The stress field can be extended in infinite nonsingular higher order terms. However, the effects of higher order terms are negligible in the vicinity of the crack tip.

2.2 Strain energy density criterion

Sih¹³ proposed a mixed-mode fracture theory based on the SED concept. According to the SED criterion, the conditions associated with the crack extension occur when the strain energy-density factor (ψ) attains its minimum value as follows:

$$\frac{\partial \psi}{\partial \theta} = 0 \quad \text{at } \theta = \theta_0 \quad (4)$$

$$\frac{\partial^2 \psi}{\partial \theta^2} > 0 \quad \text{at } -\pi < \theta_0 < \pi \quad (5)$$

According to this theory, the crack extension occurs when $\psi = \psi_c$ at $\theta = \theta_0$, where ψ_c is the critical value of ψ and θ_0 is the direction of the crack extension. Considering only singular terms and mixed-mode I/II conditions, it can be shown that ψ can be related to K_I and K_{II} as follows¹³:

$$\psi = \frac{1}{16G} \{ (1 + \cos\theta)(k - \cos\theta) \} K_I^2 + 2 \sin\theta [2 \cos\theta - (k-1)] K_I K_{II} + \{ (k+1)(1 - \cos\theta) + (1 + \cos\theta)(3 \cos\theta) \} K_{II}^2 \quad (6)$$

where $G = E/[2(1 + \nu)]$ is the shear modulus, ν is the Poisson ratio (E is the Young modulus), and k equals $3 - 4\nu$ for plane strain and $(3 - \nu)/(1 + \nu)$ for plane stress conditions.

2.3 Maximum tangential stress criterion

The MTS criterion proposed for brittle materials by Erdogan and Sih¹⁶ postulates that crack propagation initiates from the crack tip in the direction of the maximum hoop stress ($\sigma_{\theta\theta}$) when this stress component reaches the ultimate tensile strength of the material (σ_c).

Considering only singular stress terms of Equation [2](#), the crack initiation angle (θ_0) and the fracture conditions can be calculated through Equations [7](#) and [8](#), according to MTS criterion:

$$\frac{d}{d\theta} \{ K_I f_{\theta\theta,1}(\theta) - K_{II} f_{\theta\theta,2}(\theta) \} = 0 \rightarrow \theta_0 \quad (7)$$

$$\frac{K_I}{K_{IC}} f_{\theta\theta,1}(\theta_0) - \frac{K_{II}}{K_{IC}} f_{\theta\theta,2}(\theta_0) = 1 \quad (8)$$

where $K_{IC} = \sigma_c \sqrt{2\pi r_c}$ is the well-known mode I fracture toughness and parameter r_c is known as the critical distance determining the size of the crack tip process zone.^{36, 37}

2.4 Traditional maximum tangential strain criterion

As a strain-based criterion, the MTSN predicts the crack extension angle (θ_0) and the fracture conditions based on the crack tip tangential strain component ($\varepsilon_{\theta\theta}$):

$$\begin{cases} \left. \frac{\partial \varepsilon_{\theta\theta}}{\partial \theta} \right|_{r=r_c; \theta=\theta_0} = 0 \\ \left. \frac{\partial^2 \varepsilon_{\theta\theta}}{\partial \theta^2} \right|_{r=r_c; \theta=\theta_0} < 0 \end{cases} \quad (9)$$

$$\varepsilon_{\theta\theta}(r_c, \theta_0) = \varepsilon_T = \frac{\sigma_c}{E} \quad (10)$$

where ε_T is the ultimate tensile strain of the material. The tangential component of the crack tip elastic strain field is expressed as follows:

$$\varepsilon_{\theta\theta} = \frac{1}{E} [\sigma_{\theta\theta} - \nu \times \sigma_{rr}] = \frac{K_I}{E\sqrt{2\pi r}} [f_{\theta\theta,1}(\theta) - \nu \times f_{rr,1}(\theta)] + \frac{K_{II}}{E\sqrt{2\pi r}} [f_{\theta\theta,2}(\theta) - \nu \times f_{rr,2}(\theta)] + \frac{T}{E} [\sin^2(\theta) - \nu \cos^2(\theta)] \quad (11)$$

Considering only the singular terms and substituting Equation [11](#) into Equations [9](#) and [10](#), the traditional MTSN criterion, proposed by Chang, [27](#) is presented as follows:

$$\frac{K_I}{E\sqrt{2\pi r_c}} \times \frac{\partial}{\partial \theta} [f_{\theta\theta,1}(\theta_0) - \nu \times f_{rr,1}(\theta_0)] + \frac{K_{II}}{E\sqrt{2\pi r_c}} \times \frac{\partial}{\partial \theta} [f_{\theta\theta,2}(\theta_0) - \nu \times f_{rr,2}(\theta_0)] = 0 \rightarrow \theta_0 \quad (12)$$

$$\frac{K_I}{E\sqrt{2\pi r_c}} \times [f_{\theta\theta,1}(\theta_0) - \nu \times f_{rr,1}(\theta_0)] + \frac{K_{II}}{E\sqrt{2\pi r_c}} \times [f_{\theta\theta,2}(\theta_0) - \nu \times f_{rr,2}(\theta_0)] = \varepsilon_T = \frac{\sigma_c}{E} \quad (13)$$

2.5 Extended maximum tangential strain criterion

In the EMTSN, the effect of the T-strain term is considered along with the singular strain terms. Mirsayar [22](#) theoretically showed that the T-strain term, which was neglected in the traditional MTSN criterion, plays a remarkable role in the prediction of the mixed-mode crack extension angle and the onset of fracture. Based on the EMTSN, the angle and onset of initiation can be predicted by Equations [14](#) and [15](#) as follows:

$$\frac{K_I}{E\sqrt{2\pi r_c}} \times \frac{\partial}{\partial \theta_0} [f_{\theta\theta,1}(\theta_0) - \nu \times f_{rr,1}(\theta_0)] + \frac{K_{II}}{E\sqrt{2\pi r_c}} \frac{\partial}{\partial \theta_0} [f_{\theta\theta,2}(\theta_0) - \nu \times f_{rr,2}(\theta_0)] + \frac{T}{E} \cdot (1 + \nu) \cdot \sin(2\theta_0) = 0 \rightarrow \theta_0 \quad (14)$$

$$\frac{K_I}{E\sqrt{2\pi r_c}} \times [f_{\theta\theta,1}(\theta_0) - \nu \times f_{rr,1}(\theta_0)] + \frac{K_{II}}{E\sqrt{2\pi r_c}} \frac{\partial}{\partial \theta_0} [f_{\theta\theta,2}(\theta_0) - \nu \times f_{rr,2}(\theta_0)] + \frac{T}{E} [\sin^2(\theta_0) - \nu \cos^2(\theta_0)] = \varepsilon_T = \frac{\sigma_c}{E} \quad (15)$$

3 EXPERIMENTAL STUDY

3.1 Materials

The concrete mixture was made of ordinary Portland cement (ASTM Type II), maximum size coarse aggregates of 19 mm, and a fineness modulus of 3.4. The water absorption and specific gravity were obtained for coarse aggregates as 0.56% and 2.59 and for fine aggregates as 1.75% and 2.47, respectively. To control the workability of the mixture, carboxylic 110 M (BASF) was added as a superplasticizer. The trial and error-based mixing procedure was performed as follows: After mixing the binder and fine aggregate for 1 minute, half of the mixing water and superplasticizer were mixed for 2 minutes. The remaining water together with the coarse aggregate were added and mixed for 5 minutes. Details on mixture proportion of the concrete are listed in Table 1.

Table 1. Mixture proportion of the concrete types

Concrete	Cement (kg/ m ³)	Water/Cement Ratio	Coarse Aggregate (kg/ m ³)	Fine Aggregate (kg/ m ³)	Super Plasticizer (kg/ m ³)
Mixture	350	0.46	862	972	1.5

The Poisson ratio of 0.2 is assumed for concrete materials, and the Young modulus and the tensile strength of the cement concrete were obtained as 30.12 GPa and 4.78 MPa, respectively.

3.2 Test specimen

The cracked SCB specimen is chosen for conducting the mixed-mode fracture experiments on cement concrete. As is illustrated in Figure 1, the SCB specimen is a semidisc of radius R and thickness t , containing an inclined edge crack of length a , under 3-point loading configuration. The broad mixed-mode loading configuration from pure I to II can be achieved by changing the crack inclination angle α .

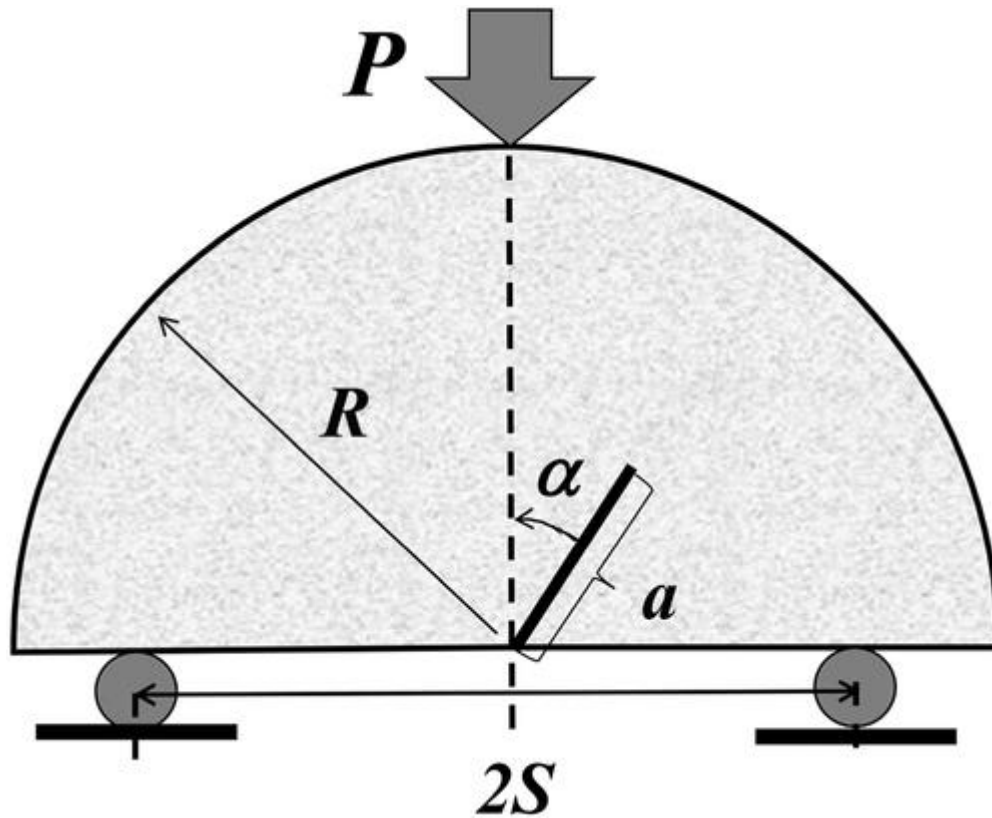


Figure 1

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General configuration of the semicircular bend (SCB) specimen

For this specimen, the stress intensity factors and T-stress can be determined in normalized forms by the following general equations:

$$K_i = \frac{P}{2Rt} \sqrt{\pi a} Y_i(a/R, S/R, \alpha) \quad i = I, II \quad (16)$$

$$T = \frac{P}{2Rt} T^*(a/R, S/R, \alpha) \quad (17)$$

where T is the normalized T and Y_I and Y_{II} are the geometry factors corresponding to each fracture mode.

As is stated in Equations [16](#) and [17](#), these normalized parameters are functions of the crack length (a/R), normalized location of loading supports (S/R), and the crack angle α in the SCB specimen. For the current experiments, these normalized parameters are chosen as $a/R = 0.3$ and $S/R = 0.43$. The corresponding geometry factors and the normalized T parameter are determined by the finite element simulation at different crack inclination angle (α), as is demonstrated in [Figure 2](#).

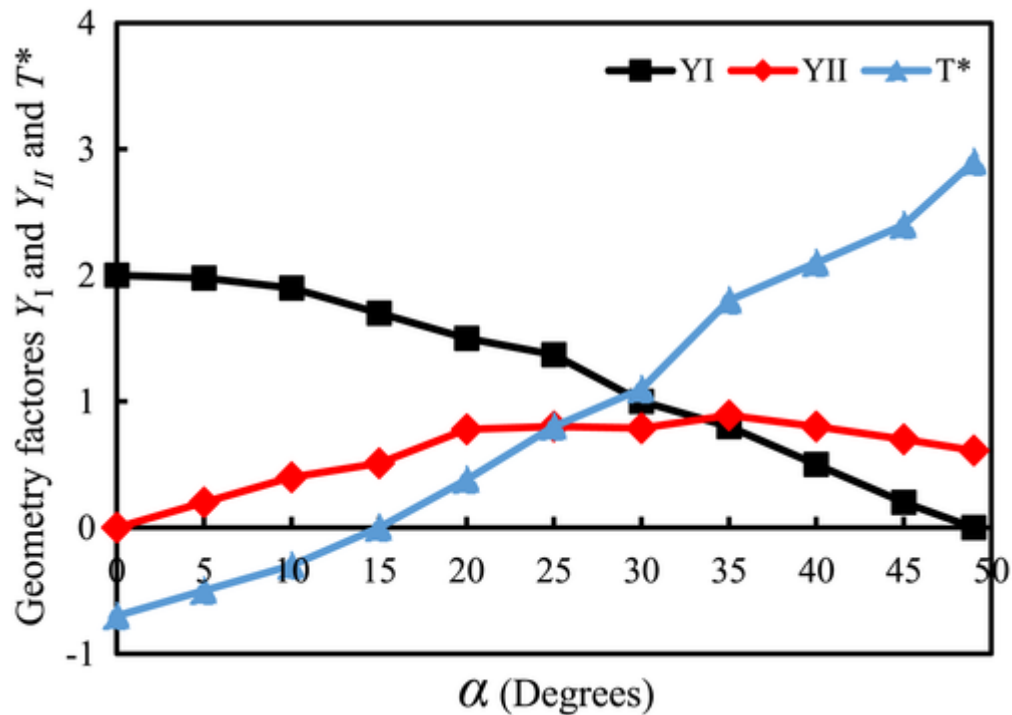


Figure 2

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Geometry factors Y_I and Y_{II} and T^* for various crack inclination angles in the semicircular bend (SCB) specimen ($a/R = 0.3$, $S/R = 0.43$) [Colour figure can be viewed at wileyonlinelibrary.com]

A typical mesh pattern generated for the simulation of the SCB specimens is depicted in Figure 3. Around the crack tip, a very fine mesh is used due to the high stress (strain) concentration to avoid computational errors. The SCB specimens are meshed by 8-node quadrilateral elements, except for the elements adhered to the crack tip, which is 6-node quarter-point singular element. The coefficient of the crack tip field, shown in Figure 2, is obtained numerically by utilizing J-integral approach adapted to the field equations.

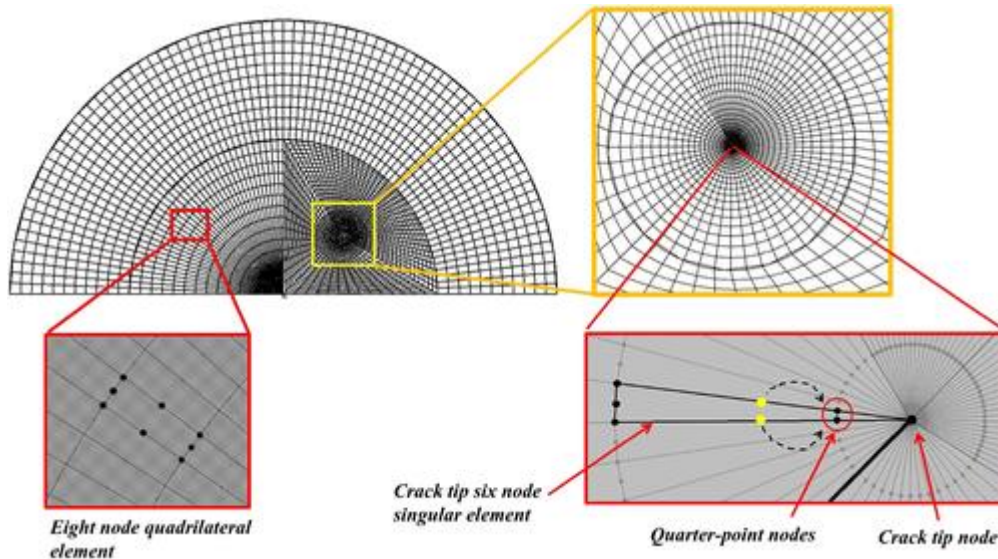


Figure 3
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Detailed of finite element mesh pattern used for the simulation of the semicircular bend (SCB) specimen ($\alpha = 45^\circ$) [Colour figure can be viewed at wileyonlinelibrary.com]

In the test program, fracture tests were conducted at crack inclination angles of $\alpha = 0^\circ$, 15° , 30° , 40° , 45° , and 49° to cover from pure mode I ($\alpha = 0^\circ$) to pure mode II ($\alpha = 49^\circ$). The geometrical parameters a , R , and t were kept constant as 15, 50, and 30 mm, respectively. The preparation process for the cracked SCB specimens is illustrated in Figure 4.



Figure 4
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Preparation process for the cracked semicircular bend (SCB) specimens [Colour figure can be viewed at wileyonlinelibrary.com]

3.3 Fracture test data

Table 2 lists the fracture test data for cracked SCB specimens under different mixed-mode conditions. In Table 2, the parameter $M^{\#}$, called mode mixity, is a conventional dimensionless parameter that represents the contribution of each fracture mode under

mixed-mode conditions. This parameter ranges from unity (at pure mode I) to 0 (at pure mode II) and is given as follows:

$$M^c = \frac{2}{\pi} \tan^{-1} \left(\frac{K_I}{K_{II}} \right) \quad (18)$$

The corresponding stress intensity factors and parameter T can be determined by substituting the fracture loads and the geometry factors (Y_I and Y_{II}) depicted in Figure 2 and Table 2 into Equations 16 and 17. Table 3 lists the critical stress intensity factors (K_{Ic} and K_{IIc}) obtained for different values of the mode mixity. In this table, the parameter K_{eff} , called effective stress intensity factor, represents the effective value of the mixed-mode I/II fracture toughness and is defined as follows:

$$K_{eff} = \sqrt{K_{Ic}^2 + K_{IIc}^2} \quad (19)$$

4 RESULTS AND DISCUSSION

This part deals with the evaluations of the fracture toughness by using 4 different fracture criteria: SED, MTS, MTSN, and EMTSN. By employing the calculated values of the crack tip parameters Y_I , Y_{II} , and T , the fracture curves are plotted by using different fracture criteria. The critical distance of $r_c = 2.3$ mm is selected for the concrete materials that correlate well with the previously reported values.¹ Figure 5 shows the test data along with the theoretical predications based on the conventional fracture criteria (SED, MTS, and MTSN) for the mixed-mode fracture toughness of cement concrete tested by SCB specimens. Among the traditional criteria, it is observed that the MTSN criterion provides closer estimates of the test data than other criteria. However, it is seen that all traditional fracture criteria overestimate the test data. Remarkable deviations are observed between the theoretical predictions by conventional fracture criteria and the reported experimental data, more near pure mode II. This behaviour can be addressed when tracking the variation of T-term against crack inclination angle, shown in Figure 2. It is seen from Figure 2 that for α greater than 15° , the magnitude of normalized T-term increases as α increases (approaching pure mode II). This relationship may bring this fact to the mind that the discrepancy between the prediction results obtained by traditional criteria and the test data can be resolved reasonably if the T-term is taken into account. This hypothesis is proven when employing EMTSN criterion that takes into account the T-strain together with the singular strain terms.

Table 2. Mixed-mode fracture loads for cracked semicircular bend (SCB) specimens made of cement concrete

Crack Angle, α	M^c , Mixity Parameter	Load P_{cr} (kN)	Average Fracture Load P_{cr} (kN)
0°	1	3.35	3.45
		4.23	
		2.77	
15°	0.81	3.14	3.75
		3.66	
		4.45	
30°	0.57	3.30	4.12
		4.38	
		4.60	
40°	0.35	3.30	4.34
		4.82	
		4.90	
45°	0.18	3.69	4.87
		5.01	
		5.91	
49°	0	4.68	5.35

Crack Angle, α	M^c , Mixity Parameter	Load P_c (kN)	Average Fracture Load P_c (kN)
		5.25	
		6.12	

Table 3. Mixed-mode fracture resistance of cement concrete

Crack Angle, α	M^c , Mixity Parameter	K_{Iff} (MPa. \sqrt{m})	K_{IIff} (MPa. \sqrt{m})	K_{eff} (MPa. \sqrt{m})
0°	1	0.558	0	0.558
		0.704		0.704
		0.461		0.461
15°	0.81	0.444	0.133	0.463
		0.518	0.155	0.540
		0.630	0.189	0.657
30°	0.57	0.274	0.217	0.349
		0.364	0.288	0.464
		0.389	0.308	0.496
40°	0.35	0.137	0.219	0.258
		0.200	0.321	0.378
		0.204	0.326	0.384
45°	0.18	0.061	0.245	0.252

Crack Angle, α	M^c , Mixity Parameter	K_{Iff} (MPa. \sqrt{m})	K_{IIff} (MPa. \sqrt{m})	K_{eff} (MPa. \sqrt{m})
49°	0	0.083	0.292	0.303
		0.098	0.344	0.357
		0	0.237	0.237
			0.267	0.267
			0.311	0.311

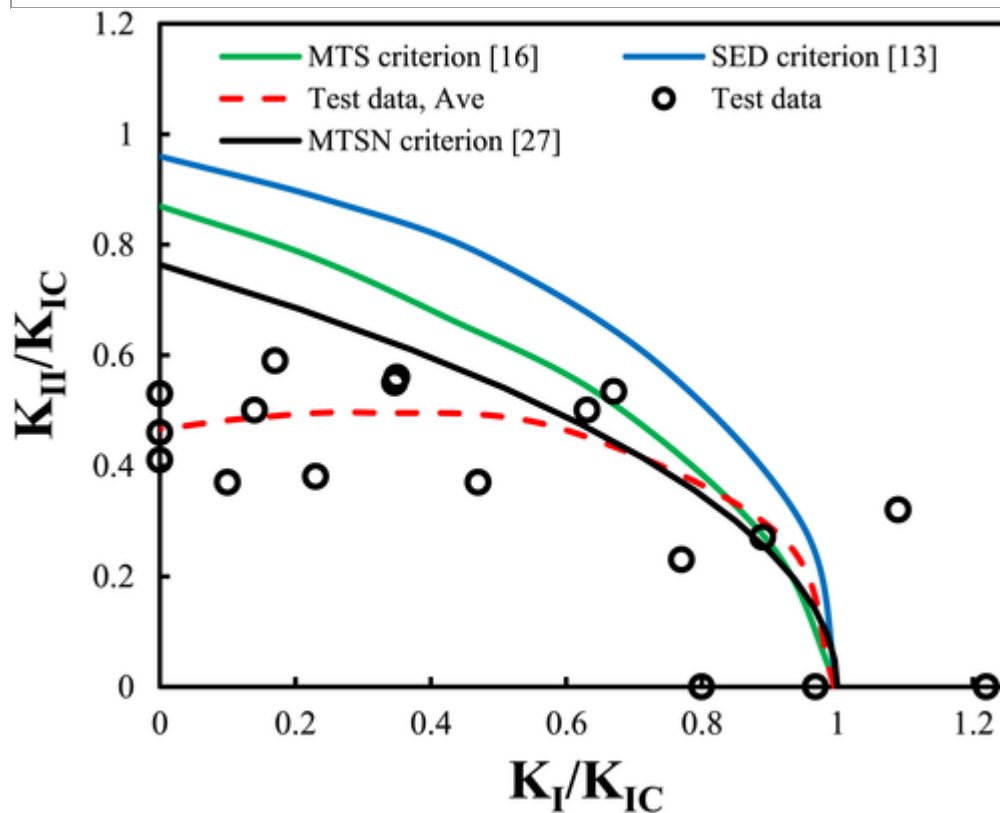


Figure 5

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Mixed-mode fracture loci predicted by different traditional criteria (strain energy density [SED], maximum tangential stress [MTS], and maximum tangential strain [MTSN]) as well as the mixed-mode fracture test data for cement concrete tested by semicircular bend (SCB) specimens [Colour figure can be viewed at wileyonlinelibrary.com]

A comparison between the traditional MTSN and the EMTSN criterion is presented in Figure 6. To explicitly demonstrate the effects of the T-strain on the mixed-mode crack propagation conditions, Mirsayar²² suggested to plot mixed-mode fracture test data in the $K_{I\text{f}}/K_{IC}^* - K_{II\text{f}}/K_{IC}^*$ diagram. The parameter K_{IC}^* , called generalized fracture toughness, is expressed in Equation 20 and considers the effect of T-term and the Poisson ratio (see Mirsayar²² for more details):

$$K_{IC}^* = \frac{\sigma_c \sqrt{2\pi r_c}}{1 - \nu \left(1 + \frac{T \sqrt{2\pi r_c}}{K_{\text{eff}}} \right)} \quad (20)$$

The effect of first nonsingular strain term is not taken into account in the conventional MTSN criterion, while it is covered in EMTSN criterion. According to Figure 6, it is observed that the EMTSN criterion provides remarkably better predictions for the experimentally obtained fracture toughness of cement concrete than the MTSN criterion. Based on the results demonstrated in Figures 5 and 6, it can be concluded that the first nonsingular strain term significantly improves the predictions obtained from strain-based criteria and, hence, it is necessary to be considered in the fracture prediction model. It is seen from Figure 5 that the maximum $K_{I\text{f}}$ value does not occur at pure mode II, as predicted by the traditional criteria, which can be explained by the positive T-strain value at pure mode II. In other words, the SCB specimen is very much affected by the positive T at pure mode II, which results in lower mode II fracture toughness. This fact is not considered by traditional fracture criteria shown in Figure 5 but is taken into account by EMTSN criterion in Figure 6.

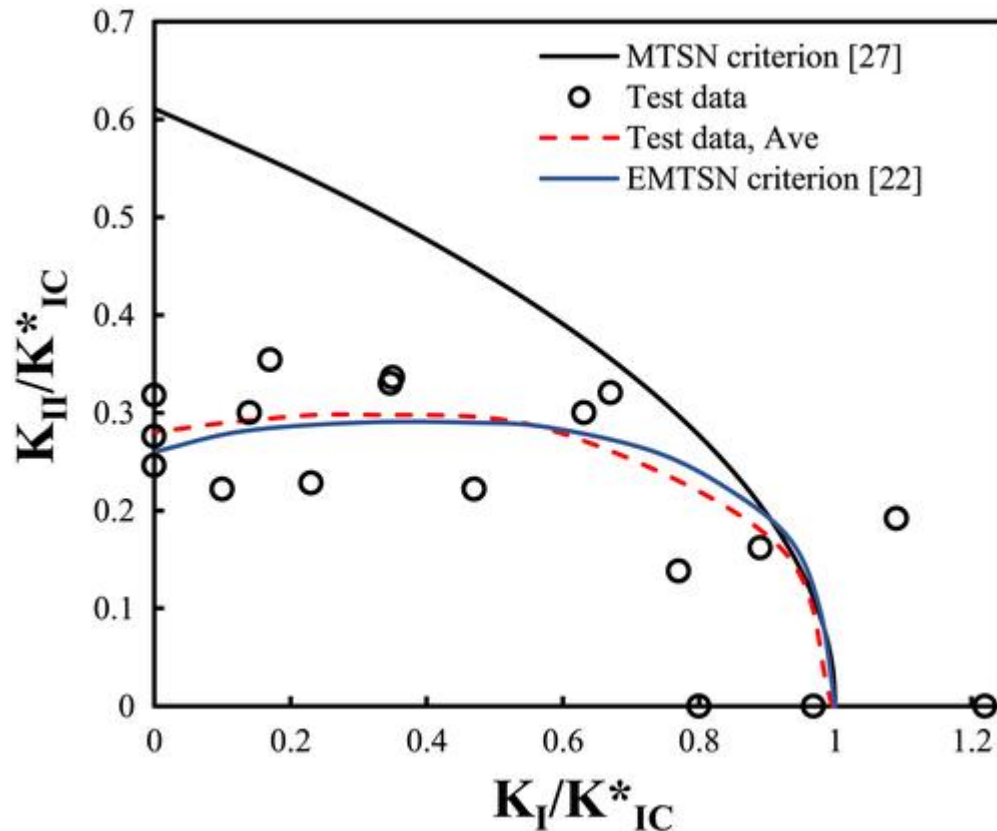


Figure 6

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Mixed-mode fracture loci predicted by maximum tangential strain (MTSN) and the extended MTSN (EMTSN) criteria, compared with the mixed-mode fracture test data for cement concrete tested by semicircular bend (SCB) specimens [Colour figure can be viewed at wileyonlinelibrary.com]

It is worth mentioning that the role of the first nonsingular term of the Williams series expansion in the brittle fracture of the engineering materials has widely been studied in many stress-based and energy-based criteria. However, in many cases, considering only the first nonsingular term in the fracture criterion does not lead to the more precise predictions (see, for example, GMTS criterion by Smith et al³⁸). It is frequently observed that one needs to add more nonsingular terms to obtain accurate results when using stress-based fracture criteria (see, for example, Aliha et al,³⁹ Wang et al,⁴⁰ and Saghafi et al⁴¹). One of the reasons of not obtaining a proper prediction results could be neglecting the effect of material properties (ie, Poisson ratio) in the fracture criterion. For example, the MTS criterion¹⁶ and its extended version³⁸ are independent of the material properties. Indeed, the role of material properties on the brittle fracture behaviour is also considered in energy-based criteria (eg, SED criterion¹³). However, adding the T-term to the energy-based criteria leads to more complex formulations than

strain-based and stress-based criteria. Therefore, the strain-based criteria can be considered as a good choice because they take into account the effect of material properties and are represented in a relatively simple formulation.

5 CONCLUSIONS

Mixed-mode fracture behaviour of cement concrete is explored experimentally and theoretically by using SCB specimens. The mixed-mode fracture toughness is evaluated by using the SED, MTS, and MTSN and EMTSN criteria, and the results obtained are compared with the experimental data. The comparison shows that the traditional fracture criteria (SED, MTS, and MTSN), which only take into account the singular terms of the crack tip field equations, are not able to predict the test results properly. The test data are then evaluated by using the EMTSN criterion, which takes into account the effect of T-strain as well as the singular strain terms. It is found the EMTSN criterion produces more accurate predictions of the test data than traditional fracture criteria. It is also concluded that the strain-based criteria can be considered as proper fracture models because of their simplicity as well as taking into account the effect of material properties.