Averaged strain energy density criterion for rupture assessment of cracked rubbers: A novel method for determination of critical SED

M.Heydari-Meybodi[a] M.R.Ayatollahi[a] F.Berto[b]

Abstract

In the present study, the application of averaged strain energy density (ASED) criterion has been extended to hyperelastic materials. Because of the material and geometry nonlinearities, commonly known for rubber-like materials, the use of conventional relations for determining the criterion parameters is no longer allowable. Therefore, by taking the advantage of a simple uniaxial state of stress field ahead of the crack tip in hyperelastic materials, a novel method has been proposed for determining the critical value of strain energy density. The sound agreement between the theoretical estimates based on the employed ASED criterion and the experimental data, taken from the literature, confirms the suitability of the proposed method.

- **Previous** article in issue
- **Next** article in issue

Keywords

Fracture mechanics

Failure assessment

Crack growth

Finite element analysis

Rubber

Nomenclature

a
: Crack length

C_{10}, C_{01}
: Material parameters of the Mooney hyperelastic material model

E
: Elastic modulus

$I_k (k=1,2,3)$
: The invariants of the right Cauchy–Green deformation tensor

$I_{jc} (j=1,2)$
: Critical value of the invariants of the right Cauchy–Green deformation tensor

K_{Ic}
: Fracture toughness

r
: Radial distance from the crack tip

$\bar{R}$
: Radius of control volume

R_c
: Critical radius of control volume

$\bar{W}$
: SED value averaged over control volume

W_c
: Critical value of SED

σ_t
: Ultimate tensile strength

ν
: Poisson's ratio

$\lambda_j (j=1,2,3)$
: Principal stretches

$\lambda_{jc} (j=1,2,3)$
: Critical principal stretches

λ_{ten}
: Rupture stretch obtained in a uniaxial tensile test

λ_U
: Loading direction stretch

μ_0, λ_m
: Material constants of the Arruda–Boyce material model

ASED
: Averaged strain energy density

CTR
: Crack tip radius

DEC
: Double edge crack sample

FEA
: Finite element analysis

SBR
: Styrene butadiene rubber

SED

Strain energy density

1. Introduction

Rubber-like materials are a group of engineering materials that are often known for unique mechanical behaviors such as high extensibility and a nearly full recoverable deformation in unloading. These characteristics are related to unique physics of rubber as it is made up of a large number of long entangled chains. One of the crucial conditions in engineering design is the case of structures containing a geometrical discontinuity like a defect or a crack. For this reason, it is important to present reliable criteria for fracture assessment of components weakened by cracks. Dealing with brittle or quasi-brittle materials, several fracture criteria have been presented in the past. These criteria can be divided into two main categories: (1) stress-based criteria, in which failure occurs when the stress at the atomic level exceeds the strength of material and (2) energy-based criteria that state the failure happens if the energy stored around the crack tip can overcome the surface energy of the material and create a free surface [1].

Among the various types of energy-based criteria (e.g. the Griffith energy balance [2], the energy release rate [3], [4], the strain energy density (SED) [5], [6], [7]; and J-integral [8], [9], [10]), the averaged strain energy density (ASED) criterion has been widely used in the recent years for the fracture assessment in a large number of materials. Dealing with both cracked and welded components, and ranging from static to fatigue tests, more than 2400 experimental data for different materials have been successfully analyzed by the ASED criterion [11]. From the sound agreement between the experiments and the results predictedby the ASED criterion, the high efficacy of this criterion in failure assessment of both brittle and ductile materials can be deduced [11], [12], [13].

According to the ASED criterion, the onset of fracture in a cracked component occurs when the mean value of SED over the control volume (i.e., circular sector) with the radius $R_c$reaches a critical value W_c[14]. In this regard, some expressions have been presented in the past for the determination of the values of R_c and W_c[15], [16], [17]. It is important to emphasize that these relations are valid only under linear elastic hypotheses even if some non-conventional extensions are possible at least as a preliminary fracture estimation tool [18].

On the other hand, to have an accurate tool for the fracture assessment of rubber-like materials, the employment of an appropriate non-linear theory of elasticity for these materials is a necessary step. Due to the nonlinearities involved, the problem would be more complicated in rubbers in comparison to the problems related to failure of brittle and quasi-brittle linear elastic materials. In other words, the previous relationships available for the two necessary inputs of the ASED criterion (i.e., R_c and W_c) cannot be directly employed in the case of hyperelastic materials [18].

Several fracture criteria can be found in literature for fracture assessment of hyperelastic materials (see for examples [18], [19], [20], [21], [22], [23]). One of the first criteria proposed for the study of crack growth in rubbers is the tearing energy criterion [19]. According to this criterion, the critical energy required for crack propagation (T_{cr}) is a material constant and nearly independent of the geometry and dimensions of the test-specimen. Moreover, in the work of Hamdi et al. [20], it has been shown that the stress-based criterion cannot be a good candidate to describe the crack growth in a hyperelastic material. In addition to the criteria mentioned above, the present authors have developed recently a new stretch-based criterion, namely the effective stretch (ES) criterion. This criterion, which considers the special physics and microstructures of rubbers, can be utilized for the materials with long entangled chains. The mechanical behavior of rubber-like materials, which are made up of long chains and networks with a high degree of flexibility, can be modeled via the statistical mechanics approaches, especially the well-known eight-chain model. Indeed, the ES criterion enjoys of a sound physical background which supports its accuracy. According to the ES criterion, when the maximum effective stretch at a critical distance from the crack tip reaches its critical value, rubber chains will rupture on the boundary of the damage zone and the crack will initiate from this point. Fracture loads of cracked-specimens under both mode-I and mixed-mode loading conditions have successfully been predicted by the ES criterion in the previous studies [24], [25]. Additionally, through examination of the maximum principal stretch fracture criteria for rubbers [26], it was revealed that the value of fracture controlling parameter depends significantly on the mesh size of the applied finite element model. Therefore, mesh dependency can be considered as an important disadvantage for this fracture criterion [24].

SED criterion is the next criterion used to predict the fracture behavior of rubber-like materials [20]. The results of Ref. [20] revealed that the critical value of the SED criterion may vary significantly with the crack orientation and therefore, the SED criterion could not be a good candidate for mixed-mode fracture assessment of soft materials. Recently, the ASED criterion has been utilized for fracture prediction of hyperelastic materials [18]. Berto used the ASED criterion for fracture assessment of rubbers subjected to mode-I and weakened by sharp V-notches (and cracks as a special case) using non-linear finite element analysis. However, because there were no simple equations for the critical radius (R_c) and the critical value of W_c for hyperelastic materials [18], he employed a simplified empirical approach for rough determination of these parameters very similar to that proposed in Ref. [27] for fracture assessment of brittle materials under pure compression loading. Indeed, since the critical load are characterized by the same value of W_c and R_c, independent of the length of the crack [27], Berto [18] determined the values of R_c and W_c by intersecting two different ASED curves plotted as a function of the critical radius for two different geometries. Therefore, no simple closed form expressions were provided by Berto [18] for the critical radius R_c and the critical energy W_c in hyperelastic materials.

The present study aims to extend the application of the ASED criterion to soft materials. Among the advantages of the ASED criterion, one can mention that the value of SED averaged in the control volume can be determined with high accuracy by using coarse meshes [28]. Another advantage of the ASED criterion is its simplicity and convenience in use which makes it possible to obtain the value of averaged SED by FEA conveniently. Moreover, the ASED criterion can be utilized for the fracture assessment in a wide range of materials as well as for both static and fatigue loading.

Considering the above mentioned points, it can be deduced that to facilitate the use of the ASED criterion in rubber-like materials, the presentation of a reliable method for determination of the critical value of SED is of paramount importance. Therefore, the main aim of the present study is to investigate a suitable relation for determination of the critical value W_c in hyperelastic materials. The key point in determination of W_c is the nearly uniaxial nature of the stress field in proximity of the crack tip in rubber-like materials subjected to pure mode-I loading. Afterwards, according to the obtained value of W_c, a procedure is elaborated to achieve the critical radius R_c. To validate the accuracy of the proposed procedure as well as the application of the ASED criterion in rubbers, two sets of experimental data available in the literature are used.

2. Analytical frame

In this section, first, the conventional ASED criterion used previously for linear elastic materials is reviewed. Then, taking into account some specific features of hyperelastic materials, the ASED criterion is extended and used for rupture assessment in these materials.

2.1. ASED criterion for linear elastic materials

Different from the SED criterion introduced by Sih [5], [6], [7], Lazzarin and Zambardi [14]suggested a criterion based on averaging the SED value over a control volume surrounding the crack (or notch) tip, called the ASED criterion. According to this criterion, brittle failureoccurs when the mean value of SED over a control volume (i.e., control area in two dimensional cases) reaches a critical value. Indeed, the

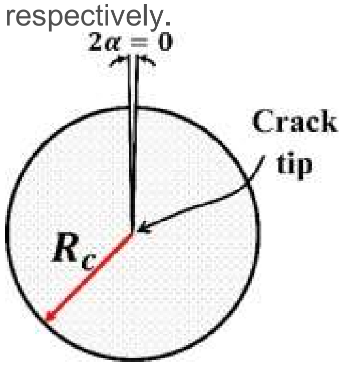
ASED criterion is mainly based on the precise definitions of two independent parameters which are assumed to be material properties: the radius of control volume, R_c, and the critical value of SED, W_c.
The critical value of SED for an ideally linear elastic material, under small strain and tensile stress conditions can be obtained as follows [16]:

$$(1)\quad W_c=\frac{\sigma_t^2}{2E}$$

where σ_t and E are the conventional ultimate tensile strength and the elastic modulus of the material, respectively. In addition, as stated in Ref. [29], when an unnotched specimen exhibits a non-linear behavior whereas the behavior of notched specimen remains linear, the stress σ_t should be substituted by the maximum normal stress existing at the notch edge at the moment of fracture initiation. Moreover, it is also recommended to use tensile specimenswith large semicircular notches to avoid any notch sensitivity effect [30].
On the other hand, the control volume (or control area in 2D cases) for a cracked body is defined as a circle with the radius of R_c centered at the crack tip (Fig. 1). This critical radiusdepends on the condition of loading (i.e., plane stress or plane strain) and can be obtained according to the following expressions [15], [31]:

$$(2)\quad R_c=\begin{cases}\frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 & \text{Plane Strain}\\ \frac{(5-3\nu)}{4\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 & \text{Plane Stress}\end{cases}$$

where K_{Ic} and ν are the fracture toughness and Poisson's ratio of the material, respectively.



1. Download high-res image (115KB)
2. Download full-size image

Fig. 1. Critical volume (area) around the crack tip.

2.2. Fundamental concepts for extending ASED criterion to hyperelastic materials

In this section, fundamental concepts for extension of the ASED criterion to rubber-like materials are explained.

2.2.1. The nearly uniaxial state of stress field near the crack tip

Although the crack-tip region is usually associated with a stress triaxiality in linear elastic materials and under small deformations [1], a different condition characterizes rubber-like materials. According to the previous experimental [32], numerical [25], [33], and theoretical [34], [35], [36], [33] investigations, the crack-tip stress field in rubber-like materials and under large strains is almost uniaxial. This important feature can be properly used for developing a novel method for determination of the critical SED value for these materials. It is useful to note that according to the analytical finite strain analyses, the type of singularity near the crack tip depends on the type hyperelastic material (incompressible/compressible) and on the assumed stress state (plane stress or plane strain). Detailed description of this issue is beyond the scope of the current paper, but the readers are encouraged to study, for example, Refs. [33], [34], [35], [36], [37], [38] to know more about the subject.

2.2.2. SED function in hyperelastic materials

In quasi-static loading conditions, a rubber-like material exhibits hyperelastic behavior [39]. A hyperelastic material is an ideally elastic material for which a SED function exists. Moreover, the stress-strain relationship for such materials can be then derived from the SED function [40], [41]. Therefore, the SED function plays an important role in characterization of hyperelastic materials.
Various forms of the SED function have been presented in the past for modeling the behavior of hyperelastic materials. These functions, often called hyperelastic material models, can be generally classified into two categories: phenomenological models and micro-mechanically based models. A good review of these models can be found in Refs. [42], [43].
On the other hand and from the point of view of continuum mechanics, the SED function, W, is a scalar function that for an isotropic hyperelastic material can be expressed as a function of the principal invariants of the right Cauchy–Green deformation tensor or principal stretches [44], as follows:

$$(3)\quad W=W(I_1,I_2,I_3)\quad \text{or}\quad W=W(\lambda_1,\lambda_2,\lambda_3)$$

where $I_k (k=1,2,3)$ and $\lambda_j (j=1,2,3)$ are the invariants of the right Cauchy–Green deformation tensor and principal stretches, respectively. In addition, since the rubber is incompressible, the third invariant of the right Cauchy–Green deformation tensor, I_3, is equal to one and thus, the SED function will be necessarily independent of I_3. As a result, the SED function can be described in terms of two other independent invariants of the right Cauchy–Green deformation tensor:

$$(4)\quad W=W(I_1,I_2)\quad \text{or}\quad W=W(\lambda_1,\lambda_2,\lambda_3)$$

It might be helpful to notice that I_1 and I_2 can be written in terms of the principal stretches as follows:

(5) $I_1=\lambda_1^2+\lambda_2^2+\lambda_3^2$ $I_2=\lambda_1^2\lambda_2^2+\lambda_3^2\lambda_2^2+\lambda_1^2\lambda_3^2 \rightarrow$ for an incompressible material $I_2=\lambda_1^{-2}+\lambda_2^{-2}+\lambda_3^{-2}$

2.2.3. Determination of the critical value of SED in rubber-like materials

According to Section 2.1, it can be concluded that the determination of the critical value of SED, W_c, is one of the main parameters towards employing the ASED criterion. Thus, to extend the use of the criterion to rubber-like materials, finding a method for determination of the value of W_c in these materials is of paramount importance.

One of the key aspects in characterization of rubber-like materials, as noted in Section 2.2.1, is the predominant uniaxial state of stress field next to the crack tip. Therefore, it is expected that similar condition exists between the rupture of a cracked rubber and its uniaxial tensile test. As a conclusion, in order to determine the critical SED value, W_c, in hyperelastic materials, it is suggested here to equate the critical value of SED in the uniaxial tensile test with the critical value for the cracked specimen subjected to mode I loading:

(6) $W_c^{tensile\,test}=W_c^{Mode\text{-}I\,fracture\,test}$

More details about the determination of the critical value of SED according to the experimental data will be discussed in the next section.

3. Implementation of ASED criterion in rubber-like materials

Similar to the ASED criterion used for brittle materials, the criterion can be applied to hyperelastic materials with the difference in using the conventional relations for the critical radius of R_c and the critical value of W_c (Eqs. (1), (2)). Indeed, due to restriction of Eqs. (1), (2) to linear elastic and small deformation hypotheses, they cannot be directly used for hyperelastic materials. Therefore, the ASED criterion for hyperelastic materials needs a non-linear analysis. Moreover, according to the ASED criterion, either in linear elastic or hyperelastic materials, when the mean value of SED over a control volume (i.e., ASED) reaches a critical value W_c, the failure occurs.

In order to determine the critical value of SED based on the experimental data and according to the method suggested in the previous section, the following procedure should be employed:

First, a uniaxial tensile test on the rubber should be carried out to obtain its rupture stretch, λ_{ten}, as well as its stress-strain behavior. Then, a curve fitting analysis needs to be performed on the provided uniaxial tensile test data to choose a hyperelastic material modelobeying well the experimental trend. It should be pointed out that the present method is applicable to both invariant-based hyperelastic material models (e.g., Arruda-Boyce, Mooney-Rivlin, and Polynomial) and stretch-based ones (e.g., Ogden).

On the other hand, in an incompressible hyperelastic material and under uniaxial tensile deformation mode, the principal stretches can be characterized in terms of the loading direction stretch, λ_U, as follows:

$$\lambda_1=\lambda_U;\quad \lambda_2=\lambda_3=\frac{1}{\sqrt{\lambda_U}} \tag{7}$$

Therefore, in the uniaxial tensile test, the first and second invariants of the right Cauchy–Green deformation tensor (i.e., I_1 and I_2) can be rewritten in terms of λ_U as follows:

$$I_1=\lambda_U^2+2\lambda_U^{-1};\quad I_2=2\lambda_U+\lambda_U^{-2} \tag{8}$$

Moreover, since in the critical (i.e., rupture) condition, $\lambda_U=\lambda_{ten}$, therefore, from Eqs. (7), (8) one can easily derive:

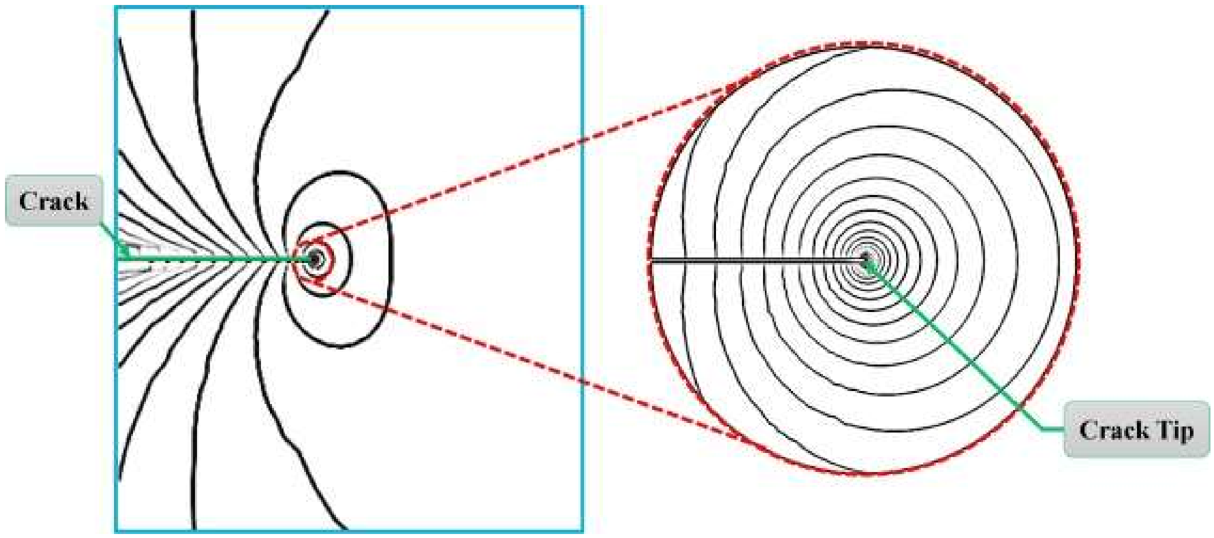
$$\lambda_{1c}=\lambda_{ten};\quad \lambda_{2c}=\lambda_{3c}=\frac{1}{\sqrt{\lambda_{ten}}} \qquad I_{1c}=\lambda_{ten}^2+2\lambda_{ten}^{-1};\quad I_{2c}=2\lambda_{ten}+\lambda_{ten}^{-2} \tag{9}$$

Finally, based on the selected hyperelastic material model, the critical value of SED can be obtained as follows:

$$W_c=W(I_{1c},I_{2c})\quad \text{or}\quad W_c=W(\lambda_{1c},\lambda_{2c},\lambda_{3c}) \tag{10}$$

It is useful to state that the above concept relies on the assumption of an incompressible hyperelastic material.

On the other hand, for determining the critical radius R_c, it is far from easy to develop a closed form expression for R_c in hyperelastic materials, as noted in Ref. [18]. In order to obtain the shape of control volume in a hyperelastic material, the practical pointrecommended in Ref. [16] is considered here. As it is suggested in Ref. [16], the shape of control volume in the ASED criterion can be determined based on the contour lines of SED in a notched component. Using this key point, Fig. 2 demonstrates the plot of SED for the hyperelastic material used in Ref. [22] in the initial (i.e., undeformed) configuration. It is clear from Fig. 2 that the isosurfaces of SED plot in the initial state in hyperelastic materials are of circular shape all centered at the crack tip.



1. Download high-res image (307KB)
2. Download full-size image

Fig. 2. The SED isosurface plot in a region near the crack tip for a sample hyperelastic material under mode-I loading.

Therefore, the shape of the control volume surrounding the crack tip in rubbers can be considered to be the same as that employed for brittle materials (as shown earlier in Fig. 1). Moreover, with the aim of obtaining the radius R_c for hyperelastic materials, it is suggested herein to intersect the graph of the SED values averaged over the control volume, $\bar{W}$, versus different radii of the control volume, $\bar{R}$. The point where $\bar{W}$ meets the obtained value of W_c gives the value of R_c.

4. Results and discussion

In the first part of this section, a brief description of the performed finite element analyses (FEA) is presented. Next, the almost uniaxial nature of the stress field near the crack tip in hyperelastic materials and under mode-I loading is numerically investigated. Finally, in order to evaluate the accuracy of the presented method for determination of the critical value of SED, W_c, as well as the prediction of the ASED criterion for hyperelastic materials, two sets of experimental data available in the open literature [45], [22] are used.

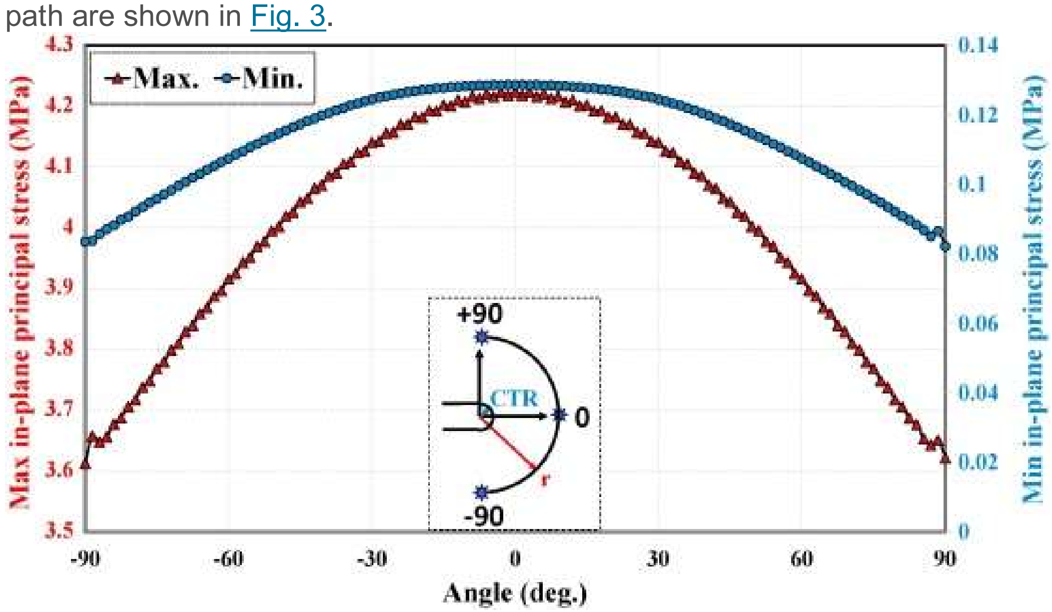
4.1. Numerical investigation of crack tip stress field in mode-I loading

The computation of SED value averaged over the control volume, $\bar{W}$, is the primary step towards employing the ASED criterion. In this regard, when the control volume next to the crack tip is defined, the value of $\bar{W}$ can be easily derived by using a commercial finite element code. Therefore, to quantify the values of $\bar{W}$, some FEA with 2D plane stress elements are performed. More details about the finite element modeling of large deformationbehavior in hyperelastic materials can be found in

Refs. [24], [25] according to which, the crack tip is usually modeled as a very narrow blunted notch in large deformation modeling. However, in the computation of the value of W^{-}, the region near the crack tip should be included in the control volume. Therefore, to be more precise, a very small crack tip radius (CTR) of 0.005 mm is considered for finite element modeling.

On the other hand, in order to investigate the stress field near the crack tip, the experimental study of Hocine et al. [22], as an example, has been considered. The double edge crack(DEC) specimen with the smallest crack length (i.e., a = 12 mm) has been investigated numerically with the reported rupture load of 30.8 N. More details about the experiment will be discussed in the next subsection.

In order to plot and then compare the values of maximum and minimum principal Cauchy stresses, the circumferential path considered in the initial (or undeformed) configuration and located at the radial distance of r = 0.1 mm, as a path near the crack tip, was selected. The graph of the principal stress values through the selected path are shown in Fig. 3.



1. Download high-res image (357KB)
2. Download full-size image

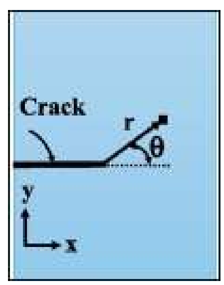
Fig. 3. Investigation of the nearly uniaxial state of stress field near the crack tip for the experimental study of Ref. [22] (the angle defined in the figure is evaluated in the undeformed configuration).

From Fig. 3, it can be clearly observed that the ratio of the maximum to minimum in-plane stress values is always larger than 30 for the explored rubber specimen. Therefore, it can be deduced that the state of the stress field near the crack tip is almost uniaxial tension. This result also confirms the key idea of the method presented in Section 2.2.3 for computing the critical value of SED, W_c.

Additionally, the principal directions for some elements located at different distances from the crack tip (r, θ) have been computed from FEA and the results are presented in Table 1. It can be concluded from Table 1 that the principal directions for the elements near the crack tip is about $90° \pm 10°$. This result also confirms the approximately uniaxial state of stress field near the crack tip in rubber-like materials.

Table 1. The principal direction for some sample elements at different locations relative to the crack tip.

r (mm)	θ (deg.)	Principal direction (deg.)	r (mm)	θ (deg.)	Principal direction (deg.)
0.05	+90	100	0.4	+90	103
	+45	96		+45	96
	0	90		0	90
	−45	84		−45	84
	−90	79		−90	76
0.1	+90	101	0.8	+90	103
	+45	96		+45	93
	0	90		0	90
	−45	84		−45	87
	−90	79		−90	78
0.2	+90	100			
	+45	96			
	0	90			
	−45	84			
	−90	78			

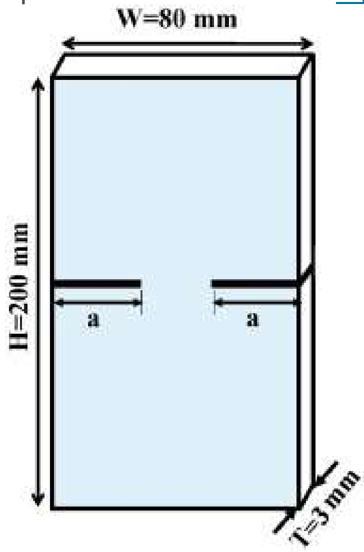


4.2. Experimental verification of the ASED criterion

In order to evaluate the accuracy of the ASED criterion in rubber-like materials, first, the experimental work of Hocine et al. [22] is used. They performed fracture tests using some DEC specimens made of a styrene butadiene rubber (SBR). To characterize the mechanical behavior of the SBR, they employed the Mooney hyperelastic material model, i.e.:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (11)$$

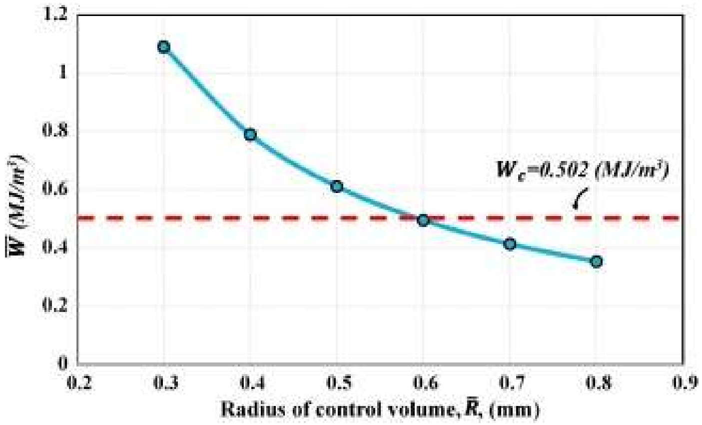
where $C_{10} = 0.0781$ MPa and $C_{01} = 0.0548$ MPa were obtained for the selected rubber. In addition, five specimens with different crack lengths (i.e., a = 12, 16, 20, 24 and 28 mm) were tested and then, their corresponding critical loads at the onset of fracture were reported in Ref. [22]. The geometry and dimensions of the DEC specimens are shown in Fig. 4.



1. Download high-res image (53KB)
2. Download full-size image

Fig. 4. The geometry and the dimensions of the DEC specimens used in Ref. [22].

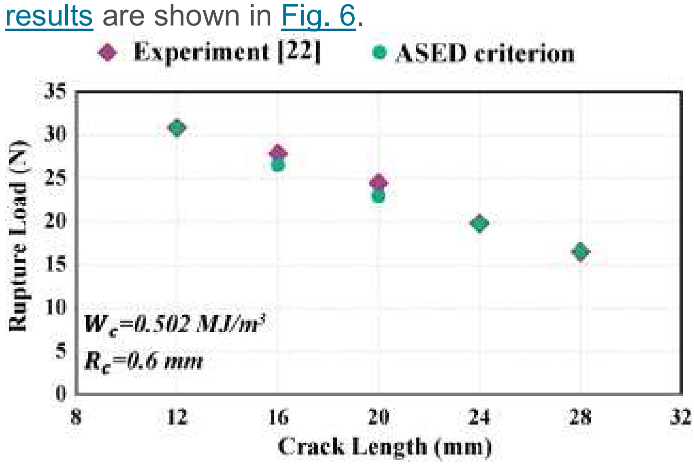
In order to determine the value of W_c in the ASED criterion, first, the uniaxial tensile testperformed in Ref. [22] was taken into consideration and then, the critical rupture stretch of uniaxial tensile test, λ_{ten}, was obtained as 2.64. Substitution of this value into Eq. (9) yields $I_{1c}=7.73$ and $I_{2c}=5.42$. Finally and according to Eqs. (10), (11), the critical value of SED in this case was achieved to be equal to $W_c = 0.502$ MJ/m^3. On the other hand and in order to determine the value of R_c, the specimen with a crack length of a = 12 mm was selected and then, a finite element simulation for the specimen with its fracture load of 30.8 N was carried out. During the simulation, different values of $\bar{R}$ were examined and their related values of $\bar{W}$ were obtained. Afterwards, the graph of $\bar{W}$ versus $\bar{R}$ was plotted and the point where $\bar{W}$ met W_c was determined (Fig. 5). Finally, according to this procedure, the critical distance of $R_c=0.6$ mm was obtained for the SBR used in Ref. [22].



1. Download high-res image (126KB)
2. Download full-size image

Fig. 5. Determination of the critical distance, R_c, for the experimental study of Ref. [22].

After determining the parameters involved in the ASED criterion (i.e., W_c and R_c) for the selected SBR, the criterion has been employed to assess the fracture load of the other specimens with different crack lengths. For this purpose, in the FEA performed separately for each specimen, the applied load was increased stepwise up to the point where the value of $\overline{W}$ reached the critical value of $W_c = 0.502$ MJ/m^3. Therefore, the corresponding load represented the critical load of the explored specimen based on the prediction of the ASED criterion. The theoretically obtained critical loads based on the ASED criterion in comparison with the experimental results are shown in Fig. 6.



1. Download high-res image (133KB)
2. Download full-size image

Fig. 6. Comparison of the predicted fracture loads using the ASED criterion with the corresponding experimental data of Ref. [22].

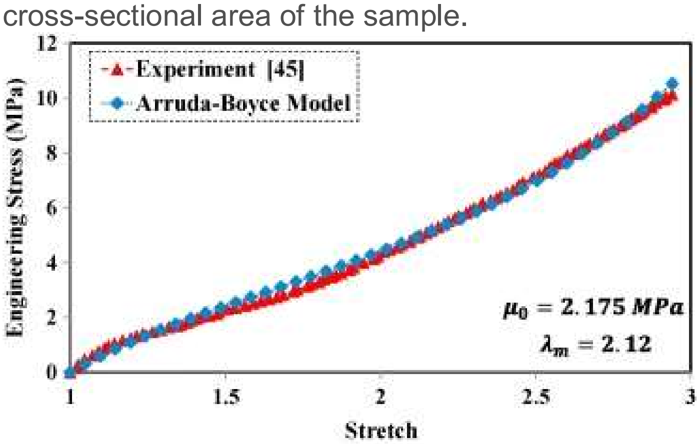
From the excellent agreement between the theoretical and experimental results shown in Fig. 6, one can deduce that the presented method for determination of the critical value of SED averaged over the control volume, W_c, provides reliable predictions.

For further verification of the ASED criterion in hyperelastic materials, the experimental study of Ref. [45] has also been considered. Pidaparti et al. performed some fracture tests on cracked specimens of different shapes under mode-I loading conditions. The tests were performed at room temperature and at a crosshead speed of 25.4 mm/min. The material used for the experiments was General Tire's SBR compound with the critical rupture stretch of $\lambda_{ten} = 2.93$ in uniaxial tensile test. The Mooney-Rivlin material model with two different material constants was suggested in Ref. [45] by the authors for characterization of the uniaxial tensile test data, but neither of those material constants could provide acceptable consistency between the model and the tensile test results. Therefore, to have a better accuracy in characterization of the test data presented in Ref. [45], the Arruda-Boyce hyperelastic material model [46], which had a good correlation with the experimental data, was employed in the present study. It might be helpful to notice here that the incompressible Arruda–Boyce material model has the following form [46]:

$$(12)\quad W=\frac{\mu_0}{f(\lambda_m)}\sum_{i=1}^{5}\alpha_i\frac{1}{\lambda_m^{2i-1}}\left(I_1^i-3^i\right)\quad \alpha_1=\frac{1}{2};\ \alpha_2=\frac{1}{20};\ \alpha_3=\frac{11}{1050};\ \alpha_4=\frac{19}{7000};\ \alpha_5=\frac{519}{673750};\ f(\lambda_m)=1+\frac{3}{5\lambda_m^2}+\frac{99}{175\lambda_m^4}+\frac{513}{875\lambda_m^6}+\frac{42039}{67375\lambda_m^8}$$

where I_1 is the first invariant of the right Cauchy-Green deformation tensor. Moreover, μ_0 and λ_m are the material constants that should be found through a least-squares-fit procedure.

After a curve fitting analysis with the provided uniaxial tensile test data given by Pidaparti et al. [45], the values of $\mu_0=2.175$ MPa and $\lambda_m=2.12$ were obtained for the Arruda-Boyce material model. As it can be observed from Fig. 7, a satisfactory agreement has been found to exist between the fitted Arruda-Boyce model and the experiments. It might be helpful to remind that "*Engineering stress*" presented in the vertical axis of Fig. 7 is derived by dividing the applied load directly by the original cross-sectional area of the sample.

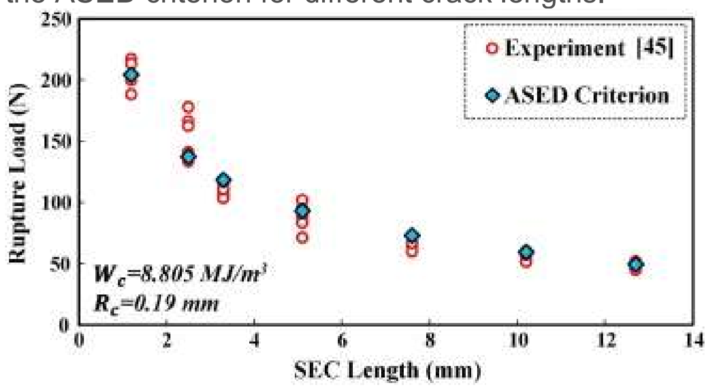


1. Download high-res image (142KB)
2. Download full-size image

Fig. 7. Comparison of the fitted Arruda-Boyce material model and the uniaxial tensile test data of Ref. [45].

With the aim to evaluate the predictions of the ASED criterion, the single edge crack (SEC) specimens with different crack lengths, which were experimentally investigated in Ref. [45], were selected. Then, to determine the value of W_c for the rubber selected in Ref. [45], the obtained critical rupture stretch of the uniaxial tensile test (i.e., $\lambda_{ten} = 2.93$) was substituted by the Arruda–Boyce material model relation (Eq. (12)) and therefore, the value of $W_c = 8.805$ MJ/m^3 was obtained. In the next step, to achieve the value of R_c, the specimen with the smallest crack length (i.e. a = 1.2 mm) was taken into account. Indeed, a finite element simulation was performed for this specimen, using the reported average fracture load of 204 N, and then, following the previously explained procedure, the critical radius was determined to be equal to R_c=0.19 mm for the selected SBR.

After determining the required material properties of the ASED criterion (i.e., W_c and R_c) for the target rubber, the criterion was utilized to predict the fracture loads of the other SEC specimens. The theoretical loads for the other crack lengths were obtained numerically by increasing the magnitude of the applied stress until the value of $\bar{W}$ reached the critical value of W_c=8.805MJ/m3. Fig. 8 shows a comparison between the experimental results and the values of the critical loads estimated using the ASED criterion for different crack lengths.



1. Download high-res image (117KB)
2. Download full-size image

Fig. 8. Experimental results versus the theoretical critical loads estimated using the ASED criterion for the SEC specimens of Ref. [45].

From Fig. 8, it can be seen that there is very sound agreement between the experimental fracture loads reported by Pidaparti et al. [45] and the predictions of the ASED criterion. In addition, the results reveal that the selection of the adopted shape for the control volume surrounding the crack tip as well as the obtained values of W_c and R_c are satisfactory.

5. Conclusions

In the present study, the averaged strain energy density (ASED) criterion was extended to investigate the failure loads in hyperelastic materials weakened by cracks and subjected to mode-I loading conditions. It is well-known that the ASED criterion is mainly based on the precise definitions of the radius of control volume, R_c, and the critical value of the strain energy density (SED), W_c. However, the available conventional relationships for these two parameters are restricted to linear elastic and small deformation hypotheses and they cannot be employed directly in materials with large deformation and non-linear behaviorslike rubbers. Therefore, for convenient application of the ASED criterion in hyperelastic materials, a novel method for determination of the critical value of SED in cracked rubber was presented in this paper. The basic idea of the proposed method for the computation of the value of W_c was the presence of a uniaxial stress state in the neighborhood of the crack tip in rubbers under mode-I loading. Moreover, after determining the value of W_c, a suitable procedure for determination of the value of R_c was presented as well. Finally, by employing non-linear finite element analyses, two sets of experimental data available in the open literature were utilized to evaluate the accuracy of the ASED criterion and the values of $W_c$and R_c computed by the proposed procedures. Very good agreement found between the experiments and the corresponding predictions based on the ASED criterion revealed that rubber-like materials can also be characterized by unique values of the critical SED and critical radius. Moreover, the results of present study confirmed that the selection of the adopted shape for the control volume surrounding the crack tip in hyperelastic materials was also satisfactory. Additionally, the nearly uniaxial nature of stress field close to the crack tip in mode-I loading conditions was shown numerically for rubbers.