

T-stress effects on fatigue crack growth – Theory and experiment

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Abstract

The effects of specimen geometry on the fatigue crack growth rate and the number of elapsed cycles have been investigated experimentally and theoretically for Al 7075-T6 using four different cracked specimens with positive and negative T-stresses. All the specimens have been tested under high cycle fatigue loading. An energy-based method, namely the generalized strain energy density (GSED) criterion has been proposed to estimate the fatigue crack growth rate and the number of elapsed cycles by considering the T-stress effects. It is shown that the proposed criterion can provide significantly improved predictions for the fatigue crack growth behavior of the pre-cracked specimens.

1. Introduction

Fatigue failure often occurs by the initiation and propagation of cracks. There are different approaches to predict fatigue crack growth rate and fatigue life in the components containing a crack. One of the most well-known methods is the Paris–Erdogan law in the high cycle fatigue region [1], in which the correlation between the fatigue crack growth rate da/dN and the cyclic stress intensity factor (SIF) range ΔKI under pure mode I loading is and the fatigue parameters C and m are assumed to be material constants independent of the specimen geometry.

Two reasons have been suggested by researchers to show the inadequacy of the Paris–Erdogan law in estimating the fatigue crack growth behavior of components particularly under complex loading conditions [2]. First, the fatigue loading is properly defined when the fatigue crack growth expression, da/dN , contains both the stress amplitude, $\Delta\sigma$, and the mean stress level, σ_m , as two loading parameters while the Paris law involves only $\Delta\sigma$. Second, the equation is restricted to pure mode I crack growth where the crack is assumed to propagate always along its initial direction. However, under complex and mixed mode cyclic loading, cracks do not propagate normal to the applied load and deviate from their initial lines significantly. This has prompted researchers to suggest some improved fatigue crack growth theories to overcome the shortcomings of the Paris–Erdogan model. One of these models is based on the strain energy density (SED) range near the crack tip as proposed by Sih and Barthelemy [2] who used only the singular terms of crack tip stresses to calculate the strain energy density. However, by considering the higher order terms of Williams' series expansion, in particular the T-stress, one can derive more accurate formulation for the strain energy density around the crack tip.

The effects of T-stress (or the second term) in the Williams series expansion on the fracture and fatigue behavior of cracked specimens have been studied by several researchers. Modified energy-based and stress-based models were proposed to study the influence of T-stress on pure mode-I, pure mode-II and mixed mode brittle fracture [3], [4] under static loading. Ayatollahi et al. explored

the role of T-stress in linear elastic fracture mechanic (LEFM) through a large number of experimental, numerical and analytical studies [6], [7], [8]. They showed that under static loading, modified criteria are able to provide significantly better estimates for the onset of mixed mode fracture in comparison with the conventional criteria which ignore the T-stress effects. Under cyclic loading, the effects of specimen geometry on the fatigue crack extension have been investigated both theoretically and experimentally by using stress or energy-based approaches [9], [10], [11], [12], [13]. Tong [14] showed that depending on the value of T-stress, fatigue cracks under mode I cyclic loading sometimes deviate from their initial plane. The importance of T-stress in the crack growth path prediction and crack curving has also been discussed in detail by Cotterell and Rice in [5] and more recently by Ayatollahi et al. [6], [15] for mode-I static loading.

In this paper, a generalized form of minimum strain energy density criterion is developed to consider the effect of T-stress on fatigue crack growth rate and the number of elapsed cycles in Al 7075-T6. For this purpose, a number of test samples with both positive and negative T-stresses are selected for conducting the tests under pure mode-I cyclic loading. The test specimens are the compact tension (CT), the double cantilever beam (DCB) and the Brazilian disc (BD) specimens. It is shown that for the DCB specimen with positive T-stress, the crack path kinks from the original crack line in a non-coplanar way, but for the CT and BD specimens, the crack extension takes place along the original crack. Moreover, it is shown that very good agreement exists between the experimentally obtained number of elapsed cycles and theoretical results predicted using the generalized SED criterion which considers the effect of T-stress in calculating the strain energy density around the crack tip.

2. Experiments

In this section, the material and specimens used for fatigue crack growth experiments are described. Aluminium alloys are known as favorite materials for conducting fatigue crack growth experiments. Among various Al alloys, the Al 7075 (with different heat treatments such as T6, T651) is used extensively for airplane and automobile applications owing to its high strength, good fracture toughness and fatigue strength, appropriate ductility, low density and high thermal conductivity. Due to strict safety measures in airplane structures and vehicles, different mechanical properties of aluminum have been studied in the past by numerous researchers experimentally and theoretically.

Four specimens of different shapes were selected to show the effect of specimen geometry on fatigue crack growth, including: the compact tension (CT) specimen (width of $W = 30$ mm, height of $h = 32$ mm), the double cantilever beam (DCB1) specimen (width of $W = 60$ mm, height of $h = 32$ mm), the DCB2 specimen (width of $W = 90$ mm, height of $h = 32$ mm) and the Brazilian disc (BD) specimen (radius of $R = 35$ mm). Fig. 1 shows the specimens which were all produced from an Al 7075-T6 sheet of thickness $t = 12.5$ mm. As described later, the specimens provide a wide range of T-stress. Considering the relatively large value of thickness of the specimens in comparison with other dimensions, all the specimens are assumed to be under plane strain condition in the computational investigations.

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Fig. 1. Four types of specimens used for fatigue crack growth experiments.

For creating the cracks, first a high-precision 2D CNC water jet cutting machine was employed to create a notch with an initial depth slightly less than $a/R = 0.45$ in the BD specimen and $a/W = 0.3$ in the CT and DCB specimens. Then, a fatigue pre-crack of length about 1.4 mm was created in notched samples by pre-cycling to make the final crack length of each specimen equal to $a/R = 0.45$ in the BD specimen and $a/W = 0.3$ in the CT and DCB specimens.

The tensile test was also carried out based on ASTM E8[16] and the elastic modulus E , the yield strength σ_y and the ultimate strength σ_u were determined as $E = 71.7$ GPa, $\sigma_y = 415$ MPa, $\sigma_u = 488$ MPa. Based on an earlier study [17], the Poisson's ratio was taken to be 0.33. The engineering stress-strain curve for the tested Al 7075-T6 is shown in Fig. 2.

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Fig. 2. The engineering stress-strain curve for the tested Al 7075-T6.

Three samples for each specimen geometry were tested by a servo hydraulic fatigue testing machine at room temperature with loading frequency of 7 Hz (420 cycles/min) under load control cyclic tension conditions. The maximum applied loads in CT, DCB1, DCB2 and BD specimens were 2.40 kN, 2.20 kN, 2 kN and 30 kN, respectively. The applied cyclic form was sinusoidal and the minimum/maximum load ratio, R , was 0.1. The fatigue experiments were conducted on the CT, DCB1 and DCB2 specimens with pin-loading fixture where the external load was applied via two pin holes designed on each sample. Two flat disks were utilized throughout the fatigue experiments in BD specimens to apply the compressive loads. The crack length data at different intervals data were recorded during each test by using pictures taken by a digital camera (Canon EOS 600D with an EF 100 mm f/2.8 Macro Lens). The rate of crack growth was determined by calculating the slope of the straight line connecting two adjacent data points on the a versus N curve. This method, known as the secant method, has been recommended in ASTM E647[18]. Fig. 3 shows sample specimens before and after the fatigue experiments.

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Fig. 3. The specimens before (a) and after (b) the high cycle fatigue experiments.

It is seen from Fig. 3b that the path of crack growth is significantly curved in the cases of DCB1 and DCB2 specimens. Therefore, mixed mode fatigue crack growth models are required for predicting

the crack path and crack length during the fatigue crack extension when a considerable number of fatigue cycles is associated with the curved part of crack path. In the next section, a generalized form of minimum strain energy density criterion is employed to predict the experimental results.

3. Computational fatigue crack growth model

In this section, an energy-based criterion is proposed by considering the effect of T-stress in addition to the singular term for fatigue life estimation of cracked components. The proposed criterion is then used in the forthcoming section for predicting the paths of crack growth and the fatigue lives of cracked Al 7075-T6 samples.

In the case of plane elasticity, strain energy density factor S can be expressed for unit thickness in terms of the stress components as [19]:

(1)

where dW/dV is the minimum strain energy density function and μ is defined as $3-4\nu$ for plane strain problems and $\mu = 2$ for plane stress ones. G is the modulus of rigidity which is equal to $E/2(1 + \nu)$ and E and ν are the modulus of elasticity and the Poisson's ratio, respectively. Each component of stresses in the polar co-ordinate system (r, θ) and can be expressed as an infinite series expansion [20] as:

(2)

where (r, θ) are the polar coordinates with the origin located at the crack tip. K_I and K_{II} are the mode-I and mode-II stress intensity factors and the first non-singular term is called the T-stress which is independent of distance r from the crack tip. The higher order non-singular terms $O(r^{1/2})$ are often negligible very close to the crack tip. By replacing the crack tip stresses $(\sigma_r, \sigma_\theta, \tau_{r\theta})$ from Eq. (2) into Eq. (1) and simplifying the obtained equation, one may write

(3)

in which

(4)

For midrange fatigue crack growth rate, Sih and Barthelemy [2] suggested a relation between da/dN and ΔW based on the experimental data which can be described by a power function as:

(5)

where A and n are two material constants. ΔW is the cyclic strain energy density range under the maximum and minimum loads during one cycle defined as:

(6)

where W_{max} and W_{min} are the maximum and the minimum values of the strain energy density factor corresponding to the maximum load and minimum load during one cycle in the direction θ , respectively.

(7)

The angle θ indicates the angle between the initial direction and the direction of new crack growth increment. According to the minimum strain energy density criterion proposed by Sih [19], [21], the direction of fracture initiation (θ) coincides with the direction of minimum strain energy density factor S along a critical distance r_c from the crack tip. This can be written mathematically as

(8)

By replacing the minimum strain energy density factor S from Eq. (3) into Eq. (8), the direction of fracture initiation θ is found from

(9)

where

(10)

By substituting the strain energy density factor from Eq. (3) into Eq. (6) and simplifying the obtained equation, one may write at the critical distance r_c and along θ in the following form:

(11)

where a_1 to a_6 are all functions of θ . Considering Eqs. (5), (11), it is seen that similar to the well-known Paris model [1], the fatigue crack growth model proposed in this paper is also dependent on the maximum and minimum stress intensity factors. However, Eq. (11) is a function of the maximum and minimum T-stresses as well. Eq. (11) may be re-written in terms of the cyclic stress intensity factor range ΔK , the T-stress range ΔT , the mean stress intensity factor K_m and the mean T-stress, (Eq. (12)). These parameters are defined in Eq. (13).

(12)

(13)

To normalize the effect of T-stress, a dimensionless parameter called the biaxiality ratio B is used that is independent of the level of load [22]. The critical distance from the crack tip r_c is also presented by the dimensionless parameter α , as

(14)

By using B and α , Eq. (12) can be rewritten in terms of $B\alpha$ as

(15)

The terms involving $B\alpha$ in Eq. (15) represent the contribution of T-stress in the value of strain energy density around the crack tip. For pure mode-I loading where θ is equal to zero and fracture initiation, Eq. (15) can be simplified as:

(16)

If the terms containing the T-stress in Eqs. (15), (16) vanish, the cyclic strain energy density range (ΔW) under mixed mode I/II and pure mode I loading can be written as [2]

(17)

(18)

The superscript zero refers to the case of $T = 0$. In the generalized SED (GSED) criterion, the value of cyclic strain energy density range in Eq. (5) is calculated by using Eq. (15) for mixed mode I/II loading and Eq. (16) for pure mode I loading. The generalized SED criterion considers the influence of T-stress on the fatigue crack growth rate while in the conventional SED criterion the value of ΔW is determined by using Eqs. (17), (18) where the effect of T-stress is ignored.

4. Results and discussion

The generalized strain energy density criterion described in Section 3 is employed in this section to predict the fatigue crack growth rate and fatigue life of Al-7075 for the four specimens used in the experiments. The theoretical results are then compared with the experimental data.

In this study, a numerical code was developed and linked to finite element (FE) software ABAQUS to predict the values of SIFs and T-stress required for the CT, DCB1, DCB2 and BD specimens in every incremental length of crack growth under mode I loading. An interaction integral method built in ABAQUS was used for obtaining directly the crack tip parameters (SIFs and T-stress) [23]. The path of crack growth was simulated by an iterative procedure in which a constant pre-specified incremental length of crack growth Δa was considered in every step of computation. An incremental crack length of $\Delta a = 0.5$ mm was considered in every computation step and the direction of crack extension in each step was determined from Eqs. (9), (10). The geometry of finite element model was redefined and remeshed in every step after extending the crack length by an increment Δa . The crack length at the next step was considered as $a + \Delta a$ and by repeating these steps, the whole crack path was predicted by finite element simulations. Fig. 4 shows the variations of the biaxiality ratio B determined in each length of crack in the CT, DCB1, DCB2 and BD specimens.

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Fig. 4. Variations of biaxiality ratio (B) with the crack length for the CT, DCB1, DCB2 and BD specimens.

Under pure mode I loading, a dimensionless parameter called Λ , is introduced here to show the contribution of the T-stress in the value of strain energy density by dividing the cyclic strain energy

density range without T-stress term (Eq. (18)) by the cyclic strain energy density range which includes the T-stress term, (Eq. (16)) and is defined as

(19)

According to Eq. (19), the dimensionless parameter Λ is a function of two parameters α and B . Fig. 5 shows the variations of the parameter Λ versus fatigue crack length obtained from Eq. (19) for two different values of the critical distance r_c ($r_c = 0.1$ mm and $r_c = 1$ mm) in the DCB1, DCB2 and BD specimens. It can be seen from Fig. 5a that for $r_c = 0.1$ mm, although the biaxiality ratio B in these specimens are significant, the variations of the parameter Λ is small. However, when $r_c = 1$ mm, the variations of the parameter Λ are considerable. It can be deduced that in addition to the T-stress, the critical distance r_c has an important effect on the value of the parameter Λ . Therefore, the effect of T-stress on the value of strain energy density around the crack tip is more significant in materials having larger radii of r_c .

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Fig. 5. Variations of the parameter Λ versus fatigue crack length for two different values of critical distance $r_c = 0.1$ mm (a) and $r_c = 1$ mm (b) for DCB1, DCB2 and BD specimens.

As suggested in [24], the value of r_c can be obtained by using the following relationship in terms of the yield strength σ_{ys} and the mode I fracture toughness K_{Ic} of the material.

(20)

By replacing the values of mode I fracture toughness ($K_{Ic} = 18.75$ MPa $m^{0.5}$) and yield strength ($\sigma_{ys} = 415$ MPa) obtained from our experiments into Eq. (20), the calculated value of r_c for the tested Al 7075-T6 is found to be 1.3 mm.

In the present investigation, first some high cycle fatigue experiments were performed using the CT specimen, in which the dimensionless term B is negligible, to determine the value of the constants A and n in Eq. (5). Then, Eqs. (5), (16) were used to predict the number of elapsed cycles and the fatigue crack growth rate of the DCB1, DCB2 and BD specimens in high cycle fatigue region. For the DCB1 and DCB2 specimens which had large positive values of T-stress the path of crack growth deviated (see Fig. 2). This is in agreement with the results reported by Cotterell and Rice [5] where they have shown that the T-stress is of an essential role in crack curving in the DCB specimens. For those cases, $K_{II} \neq 0$ and the number of elapsed cycles and the fatigue crack growth rate were determined from Eqs. (5), (15). The crack curving and the crack trajectory can be estimated theoretically by using a modified form of the strain energy density criterion as elaborate in [6]. However, in the DCB1 specimens, when the crack growth kinked from the original crack line, the final fatigue failure occurred at a low number of cycles shortly of the deviation. Thus, the part of crack growth which deviated was not considered in the DCB1 specimens.

4.1. Material fatigue constants

As described above, the material fatigue constants A and n were calculated by using the high cycle fatigue experiments on the CT specimen. Fig.6 shows the fatigue crack propagation rate data, da/dN , of the CT specimen made of Al 7075-T6 alloy versus the cyclic strain energy density range, ΔS_{min} , in a logarithmic scale. The value of the cyclic strain energy density range was determined by using Eq. (16) for any desired fatigue crack length. This Figure shows that there is a linear relation between the fatigue crack propagation rate and the cyclic strain energy density range. The material fatigue constants were determined by using a linear regression analysis. The calculated material fatigue constants together with the vertical intercept are presented in Table 1.

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Fig. 6. Fatigue crack growth rate data for the CT specimens made of Al 7075-T6 alloy.

Table 1. Material fatigue constants in Eq. (5) obtained for the CT specimen made of Al 7075-T6 alloy [da/dN] = m/cycle; [Δ] = MPa m.

Sample	Y_intercept	A	n (Line slope)
CT	1.84	69.18	1.58

4.2. Fatigue crack growth rate data in DCB1, DCB2 and BD specimens

The generalized SED and the conventional SED criteria described in Section 3 was used to predict the numbers of elapsed cycles, N , and the fatigue crack propagation rate da/dN , in samples containing a mode I crack. The theoretical results are compared in this section with the experimental data. The numbers of elapsed cycles were determined from the equation in each incremental crack growth by using the material fatigue constants (A and n) presented in Table 1. The crack extension in the DCB1, BD specimens and a part of crack growth path in the DCB2 specimen took place along the original crack in high cycle fatigue region and the cyclic strain energy density range, ΔS , was calculated from the GSED criterion by substituting the numerical values of K_I and T-stress into Eq. (16). When the crack growth kinks from the original crack line in the DCB2 specimens, first the crack growth direction of each step (θ) has to be determined by substituting the numerical values of K_I , K_{II} and T-stress into Eq. (9). Afterwards, the cyclic strain energy density range was determined using the mixed mode relations (Eq. (15)). To predict the number of elapsed cycles by using the conventional SED criterion, Eqs. (18), (19) were used for the mixed mode and for pure mode I loading, respectively. Fig. 7, Fig. 8, Fig. 9 display the experimental data and the theoretical predications based on the GSED and the conventional SED criteria in terms of the number of elapsed cycles for the DCB1, DCB2 and BD specimens. It can be seen from Fig. 7, Fig. 8, Fig. 9 that the GSED criterion which takes into account the T-stress in addition to the stress intensity factors could provide good estimates for the elapsed cycles obtained from the high cycle fatigue experiments on the DCB1, DCB2 and BD specimens made of Al 7075-T6 alloys whereas the conventional SED criterion fails to estimate the elapsed cycles.

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Fig. 7. Theoretical predictions of the generalized SED and conventional SED criteria for the number of elapsed cycles of Al 7075-T6 alloys obtained from the DCB1 specimen.

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Fig. 8. Theoretical predictions of the generalized SED and conventional SED criteria for the number of elapsed cycles of Al 7075-T6 alloys obtained from the DCB2 specimen.

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Fig. 9. Theoretical predictions of the generalized SED and conventional SED criteria for the number of elapsed cycles of Al 7075-T6 alloys obtained from the BD specimen.

Fig. 10, Fig. 11 show the fatigue crack propagation rate data versus the cyclic strain energy density range predicted by using the conventional SED and the generalized SED criteria based on Eq. (5), respectively. It is seen in Fig. 10 that the conventional SED criterion that takes into account only the singular terms related to the stresses and strains in the William's series expansion fails to predict the fatigue crack propagation rate data obtained from fatigue tests on the DCB1, DCB2 and BD specimens. As shown earlier in Fig. 4, the absolute values of the biaxiality ratio B (and hence the T -stress) in the DCB1, DCB2 and BD specimens are significant. The contribution of the T -stress in the value of the cyclic strain energy density range around the crack tip could be considered when the fatigue crack growth rate is predicted using the GSED criterion (Eqs. (15), (16)). It is seen from Fig. 11 that the GSED criterion provides significantly improved estimates for the fatigue crack growth rate in the DCB1, DCB2 and BD specimens and for a wide range of fatigue crack lengths.

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Fig. 10. Predictions of conventional SED criterion for the fatigue crack growth behavior of CT, DCB1, DCB2 and BD specimens.

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Fig. 11. Predictions of GSED criterion for the fatigue crack growth behavior of CT, DCB1, DCB2 and BD specimens.

Fatigue crack growth models based on the strain energy density criterion postulate that an increment of progressive damage occurs under fatigue loading when a critical amount of energy is accumulated within an element after a finite number of loading cycle. The crack is then assumed to grow by a value of Δa . The crack growth rate is written in terms of the range of strain energy density factor as $\frac{da}{dN}$. One can expect that by using a more accurate expression for elastic stresses near the crack tip, the value of SED factor range and hence the crack growth rate become more accurate. The conventional strain energy density criterion makes use of only the singular terms of stresses around the crack tip to calculate ΔW . However, it was suggested by the generalized strain energy density criterion that in addition to the crack tip singular stresses, the T-stress is also considered when calculating ΔW . Therefore, as shown in this paper the generalized SED criterion could provide significantly better predictions for the experimental results obtained from fatigue crack growth tests performed on specimens of different geometry and loading conditions. Finally, it is worth mentioning that Rashidi Moghaddam et al. [13] recently developed a modified average strain energy density (ASED) criterion for fatigue life estimation of cracked component. By taking into account the effect of T-stress, the modified ASED criterion utilizes the deformation energy averaged over a given finite-size volume surrounding the crack tip (as a highly stressed region). However, the modified ASED criterion was not able to predict the crack extension trajectory and also the crack curving which sometimes take place during the fatigue crack growth processes. Therefore, the generalized SED criterion was proposed in the present paper in order to estimate the path of fatigue crack growth, for example in the DCB specimens which experienced crack curving in the experiments. There are also other models based on strain energy density criteria, among them the local strain energy density based on the definition of a material-dependent control volume. The criterion is able to automatically take into account three-dimensional effects [25], [26], [27], [28]. It has been recently used for the fracture and fatigue assessment of different materials [29], [30]. Future work will be carried out by the same authors considering this advanced local criterion. Moreover, by performing larger numbers of experiments on each test specimen, statistical analyses of the experimental results can be performed as future work. The proposed fatigue crack growth model could also be further validated by performing similar experiments on other engineering materials and also on other specimen shapes which can provide wider range of T-stresses.

5. Conclusions

The effect of T-stress on the fatigue crack growth rate and the number of elapsed cycles was studied theoretically and experimentally by using the CT, DCB1, DCB2 and BD specimens made of Al 7075-T6 alloys under mode-I loading. The stress intensity factors and the T-stress were calculated for these specimens in each incremental length of the crack growth using a large number of FE analyses. The numerical results showed that the absolute values of T-stress in the DCB1, DCB2 and BD specimens are significant but its value is not considerable in the CT specimen.

The conventional SED criterion could not predict the experimental results obtained from high cycle fatigue tests performed on the four different tested specimens. Then, a generalized SED criterion was suggested which took into account both the stress intensity factors and the non-singular T-stress in calculating the value of the averaged strain energy density around the crack tip. Improved predictions were provided for the fatigue crack growth experimental results obtained from Al 7075-

T6 specimens with both positive and negative T-stresses when the generalized SED criterion was employed. It was shown that the effect of T-stress on the value of averaged strain energy density around the crack tip and hence on the fatigue life of cracked components is more significant in materials having larger critical distances.

References