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Evaluation of four-point transient potential drop on conductive plates

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We have derived an analytical solution for the transient potential drop due to a step function excitation of a four-point probe on a conducting plate. Similar expressions have already been developed based on a previous analysis for a conducting half-space. The purpose of this article, however, is to extend the theory to measurements on conductors of arbitrary thickness and thereby broaden the practical applicability of the technique. The results are useful for non-destructive measurements of the conductivity, permeability and wall thickness of metals. Further applications of the technique include monitoring material loss due to corrosion and measurement of factors that affect the electromagnetic properties of materials such as mechanical stress. *Published by AIP Publishing.*
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Potential drop (PD) methods are widely established for materials characterization,[1] non-destructive measurement of cracks,[2] and detection of corrosion.[3] Measurements are commonly made using an arrangement of four pin electrodes in contact with a conducting material. An electrical current is injected via two terminals, and the resulting potential drop is measured as the differential voltage between two voltage electrodes. Probes can be either portable, where electrical contact is achieved using spring-loaded pins, as shown in Fig. 1, or permanently attached to the specimen, for example, by spot welding. Although we are mainly motivated by measurements on macroscopic specimens and structures, the technique also finds applications in semiconductor characterization using special probes of sizes approaching the nanoscale.[4]

Alternating current potential drop (ACPD) measurements are commonly made at frequencies where the electromagnetic skin depth is small, meaning that the current flows in a near-surface layer which results in high sensitivity to surface-breaking cracks. For this reason, ACPD is often used as an alternative to the traditional direct current potential drop (DCPD) method. Furthermore, multi-frequency potential drop measurements have the advantage of providing information at different depths in the inspected material. For example, four-point probe measurements were used by Saguy and Rittel[5] for crack identification in specimens containing internal cracks. Bowler and Huang[12] demonstrated the use of multi-frequency ACPD for improved conductivity and permeability measurements in homogeneous plates, in particular, for ferromagnetic materials, for which conventional eddy current conductivity probes are inaccurate. In addition, Bowler *et al.*[6] showed that the technique is suitable for model-based measurement of the depth of case hardening in steel, circumventing the need of calibration.

When pulsed current is used as the excitation source, the resulting potential drop is a transient signal carrying information on the variation of electromagnetic properties with depth. In practice, the technique combines the capability of ACPD and DCPD and requires simpler instrumentation compared to multi-frequency ACPD since the depth-dependence is contained in a single transient signal that can be simply captured using a digital oscilloscope. In an earlier investigation, the transient potential drop method was used for the evaluation of stress in ferromagnetic steel[7] based on the principles of the magneto-mechanical effect.[8]

Recently, as a step in the theoretical analysis of the transient potential drop, analytical expressions were derived for the time varying response of a four point probe on a homogeneous metal half-space.[9] The half-space corresponds to an idealized geometry that extends infinitely in the thickness direction, as well as in the directions parallel to the surface. In practical applications, the thickness of the material can sometimes be ignored, for example, when measurements are made on plates that are thick compared to the electrode separation, and half-space results can be used.[10] In this article, we take into account the thickness of materials by finding analytical expressions for the transient potential drop of a four-point probe for the case of a step current being injected

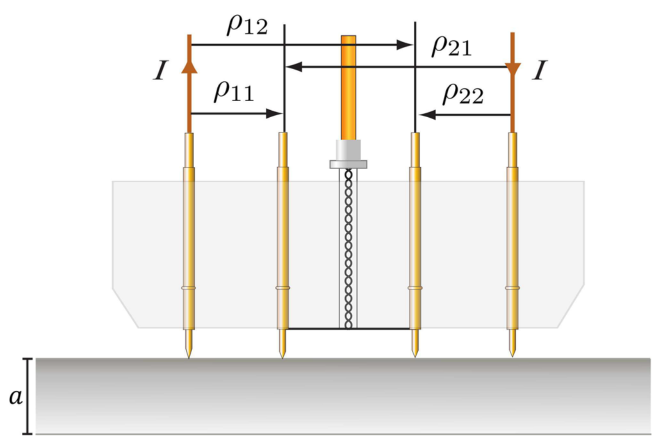

FIG. 1. A four-point probe consisting of spring-loaded pins in contact with a plate of thickness a.

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into a metal plate. First, the limiting case of a plate that is thin compared to the probe length is considered. Then, the solution is found for a plate of arbitrary thickness.

The time-dependent potential drop of a four-point probe on a planar surface can be written as

$$V(t) = \frac{I_0}{2\pi\sigma}[F(\rho_{22},t) - F(\rho_{12},t) - F(\rho_{21},t) + F(\rho_{11},t)], \quad (1)$$

where σ is the electrical conductivity of the material and I_0 is the magnitude of the injected current. For the alternating current potential drop, frequency replaces time as the dependent variable.[11] The function $F(\rho, t)$ represents the surface potential at a radial distance ρ from a single current source and ρ_{ij} represents the distance between electrode contact points on the surface, as indicated in Fig. 2. The structure of this expression reflects the approach used to derive the surface potential drop where the potential due to a single current injection wire is first found and then the complete four-point solution is found by the superposition of two current sources having opposite polarity. An elementary example is the response due to direct current injected into a conducting half-space which can be found by using $F(\rho) = 1/\rho$, where ρ is the standard radial variable in a cylindrical polar coordinate system centered at the injection point.

Finding the response $V(t)$ due to a time-dependent source requires that we solve Maxwell's equations for a time-dependent field. A common approach is to seek a solution using the Laplace transform

$$v(s) = \int_0^\infty e^{-st} V(t)\mathrm{d}t, \quad (2)$$

where the voltage is expressed in terms of the complex frequency parameter s. The time-domain solution of interest is then found by first solving for the characteristic function $f(\rho, s)$ related to the surface field represented by a transverse magnetic potential for a single point injection and then finding the time dependence by using the Bromwich integral or lookup tables.

We begin by considering an approximate solution that is valid when the plate is thin compared to the probe electrode spacing. This is in contrast to the existing half-space solution that represents the limit in which the electrode separation is small compared to the thickness of the specimen.

In the thin-plate regime, the response to a time-harmonic current source can be expressed by the function[14]

$$f_{\mathrm{tp}}(\rho,\omega) = -\imath k \coth(\imath ka) \ln\rho, \quad (3)$$

whose real and imaginary parts determine the amplitude and phase of the response to alternating current. Here, a is the thickness of the plate and $k^2 = \imath\omega\mu\sigma$ where ω is the angular frequency of the source and μ is the magnetic permeability of the material. Since f_{tp} depends on ρ only through the factor $\ln\rho$, the potential drop, in the form of a time-harmonic version of Eq. (1), can be written simply as

$$v_{\mathrm{tp}}(-\imath\omega) = v_0 \imath ka \coth(\imath ka). \quad (4)$$

In the limit of low frequency ($\omega \to 0$) of the injected current, $\coth(\imath ka) \to 1/\imath ka$ and the potential drop reduces to v_0 which is equivalent to the DCPD result and given by

$$v_0 = \frac{I_0}{2\pi\sigma a} \ln\left(\frac{\rho_{12}\rho_{21}}{\rho_{11}\rho_{22}}\right). \quad (5)$$

Inversion to the time-domain is done by interpreting the frequency response as the Laplace transform of the time-domain impulse response of the system consisting of the probe in contact with the material. Using the correspondence $s \to -\imath\omega$ in the expression for v_{tp} and multiplying the result by the Laplace transform of the injected current, $I(s)$, give the following expression for the probe response:

$$v_{\mathrm{tp}}(s) = I(s) v_0 \kappa\sqrt{s} \coth(\kappa\sqrt{s}), \quad (6)$$

where $\kappa^2 = \mu\sigma a^2$. In the following, $I(s)$ will be expressed on a normalized form by assuming that I_0 is contained in v_0. In the case of an ideal step current excitation, $I(s) = 1/s$ which gives

$$v_{\mathrm{tp}}(s) = v_0 \frac{\kappa}{\sqrt{s}} \coth\left(\kappa\sqrt{s}\right). \quad (7)$$

Although this expression cannot be inverted directly, it can be expressed in a series form by first writing $\coth(\kappa\sqrt{s})$ as

$$\coth\left(\kappa\sqrt{s}\right) = \left(1 + e^{-2\kappa\sqrt{s}}\right) \frac{1}{1 - e^{-2\kappa\sqrt{s}}}, \quad (8)$$

and expanding the fraction on the right hand side

$$\frac{1}{1 - e^{-2\kappa\sqrt{s}}} = \sum_{n=0}^{\infty} e^{-2n\kappa\sqrt{s}}. \quad (9)$$

The following series expression is then obtained for the transform of the step response:

$$v_{\mathrm{tp}}(s) = v_0 \frac{\kappa}{\sqrt{s}} \left(1 + 2\sum_{n=1}^{\infty} e^{-2n\kappa\sqrt{s}}\right). \quad (10)$$

The time-domain response can now be obtained by using the transform pair[13]

$$\frac{1}{\sqrt{s}} e^{-c\sqrt{s}} \leftrightarrow \frac{1}{\sqrt{\pi t}} e^{-c^2/4t}, \quad (11)$$

where the constant $c \geq 0$ and doing a term-by-term inversion of the expression for $v_{\mathrm{tp}}(\rho, s)$ in Eq. (10), which gives the result

$$V_{\mathrm{tp}}(t) = v_0 \sqrt{\frac{\kappa^2}{\pi t}} \left(1 + 2\sum_{n=1}^{\infty} e^{-n^2\kappa^2/t}\right). \quad (12)$$

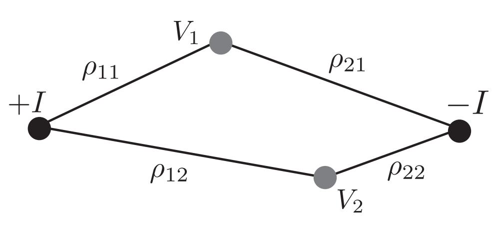

FIG. 2. Definition of four-point probe distances on a planar surface.

Fig. 3 shows the first few terms of the normalized potential drop. A relatively few number of terms are required in calculations since the transient decays rapidly towards the steady state (DC) value and we only require that the late part of the decay has converged sufficiently towards this value.

Note that in the thin-plate approximation the decay of the transient is determined only by the material properties and the plate thickness through the parameter $\kappa^2 = \mu\sigma a^2$ and is independent of the separation between probes. The probe parameters appear only in the factor v_0, meaning that their influence on the signal is due to the non-uniform spatial current distribution at the surface of the conductor.

When $t \ll \kappa^2$, the exponential terms in the expression for $V_{\rm tp}$ disappear and the voltage decays according to

$$V_{\rm tp}(t) \approx v_0\sqrt{\frac{\kappa^2}{\pi t}}, \tag{13}$$

which is independent of the plate thickness since a^2 appearing in κ^2 is canceled by the $1/a$-dependence of v_0. Physically, this represents a short-time regime that corresponds to the diffusion of the fields in the conductor, while the penetration depth of the fields is small compared to the plate thickness. As t approaches and exceeds the value of the parameter κ^2, the exponential terms, representing multiple field reflections from the bottom and top surfaces of the plate, are no longer negligible and combine to form the characteristic decay towards the steady value.

It can be noted that the thin-plate approximation is equivalent to a far-field approximation to the fields[11] since the imposed requirement is that the distance between current injection and voltage probe is large compared to the plate thickness. The far-field approximation, which is valid in regions a few skin depths away from the current injection, holds in the thin-plate approximation since the penetration depth is effectively limited by the plate thickness. Consequently, the thin-plate formula is valid provided that the potential is measured at a distance a few times the plate thickness away from the current injection points. A distance of more than twice the thickness is usually sufficient for practical purposes.[10]

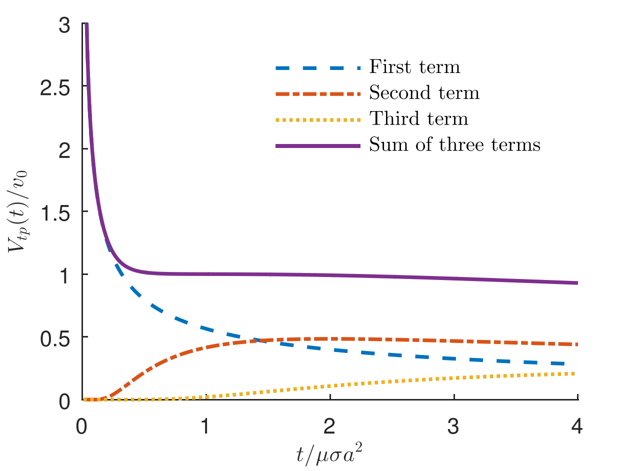

FIG. 3. Plot of the first three terms appearing in the summation formula for the transient thin-plate approximation. Also shown is the sum of these terms, forming the transient step response.

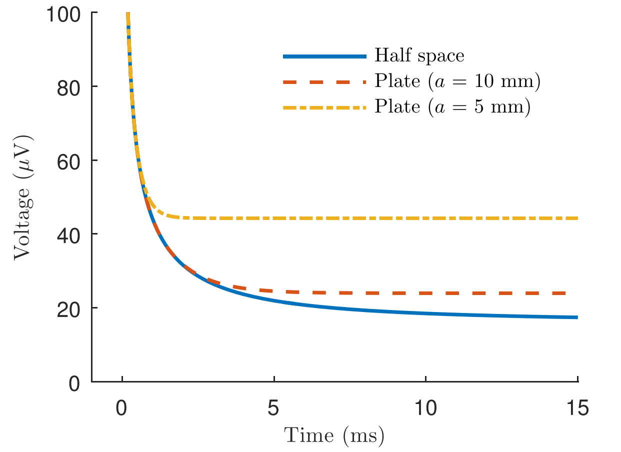

FIG. 4. The transient step responses for different values of the plate thickness together with the half space solution. In the calculations, a co-linear equidistant probe is assumed with $\rho_{11} = \rho_{22} = 10$ mm, $\rho_{12} = \rho_{21} = 20$ mm and $I_0 = 1$ A and material properties $\mu_r = 100$ and $\sigma = 4$ MS/m.

In order to get a transient response due to a plate whose thickness cannot be considered small compared to the probe spacing, the following solution can be used for the frequency domain response, expressed by the thin-plate approximation and additional image summation terms that take into account both the finite plate thickness and the electrode separation:[14]

$$f_{\rm p}(\rho,\omega) = -\imath k\coth(\imath ka)\ln\rho + \sum_n\left\{\frac{\exp(\imath k a_n)}{a_n} + \imath k e^{2\imath kna}E_1[-\imath k(a_n - 2na)]\right\}, \tag{14}$$

where $a_n^2 = \rho^2 + (2na)^2$. The first term in this expression is simply the thin-plate approximation treated above.

Proceeding as in the case for the thin-plate approximation, the time-domain response due to an ideal step current is sought by replacing every occurrence of $\imath k$ with $-\tilde{\kappa}\sqrt{s}$, where $\tilde{\kappa} = \mu\sigma$, and multiplying by $I(s) = 1/s$. The first term inside the summation can be inverted directly using the transform pair[13]

$$\frac{1}{s}e^{-c\sqrt{s}} \leftrightarrow \operatorname{erfc}\left(\frac{c}{\sqrt{4t}}\right).$$

For the third and final terms, here denoted $f_3(s)$, apply the following definition of the exponential integral function[13]

$$E_1(x) = \int_1^\infty \frac{1}{u}e^{-ux}\mathrm{d}u,$$

which, after multiplying by $1/s$ and replacing $\imath k$ with $\tilde{\kappa}\sqrt{s}$, gives

$$f_3(s) = -\frac{\tilde{\kappa}}{\sqrt{s}}\int_1^\infty \frac{1}{u}\exp\left\{-\tilde{\kappa}\sqrt{s}[2na + u(a_n - 2na)]\right\}\mathrm{d}u, \tag{15}$$

where the factor $e^{2\imath kna}$ appearing in Eq. (14) has been absorbed into the exponential term in the exponential integral function. Since the integration is with respect to u, the integrand expression can be inverted using Eq. (11) to get

$$F_3(t) = -\frac{\tilde{\kappa}^2}{\sqrt{\pi t}}\int_1^{\infty}\frac{1}{u}\exp\left\{-\frac{\tilde{\kappa}}{4t}[2na+u(a_n-2na)]^2\right\}\mathrm{d}u. \quad (16)$$

Finally, the step response of a four-point probe (Fig. 4) can now be obtained from Eq. (1) with

$$F_{\mathrm{p}}(\rho,t)=\sum_{n=-\infty}^{\infty}-\sqrt{\frac{\tilde{\kappa}}{\pi t}}\exp\left[-\tilde{\kappa}(na)^2/t\right]\ln\rho+\frac{1}{a_n}\mathrm{erfc}\left(\sqrt{\frac{\tilde{\kappa}a_n^2}{4t}}\right) -\sqrt{\frac{\kappa}{\pi t}}\int_1^{\infty}\frac{1}{u}\exp\left\{\frac{\kappa}{4t}[2na+u(a_n-2na)]^2\right\}\mathrm{d}u. \quad (17)$$

From these results, we can deduce that in the intermediate regime where the probe size is similar to the material thickness the decay of the transient response is controlled by two timescales. One is related to the plate thickness through the parameter $\kappa^2 = \mu\sigma a^2$, which is the same as that found in the thin-plate approximation. The second is related to the probe distance, through $\kappa_\rho^2 = \mu\sigma\rho^2$, which is the characteristic timescale appearing in the half-space solution.

To conclude, a current pulse excitation of a four point probe in general gives rise to a transient potential drop that contains information on the variation of electromagnetic properties with depth in a material. The particular problem we have considered is that of predicting transient potential drop signals due to a step current excitation on homogeneous conducting plates, while considering the effects of finite plate thickness. The technique and its analysis have potential for use in materials characterization and in monitoring material degradation due to corrosion for example.

Future work includes the analysis of transient signals for measurements made on materials where the conductivity and permeability vary with depth and where the material is anisotropic due to stress.

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