Evolution of quasi-history of the Planck Blackbody radiation equation in a Physics Textbook

J.R. Persson*

 $NTNU, Norwegian\ University\ of\ Science\ and\ Technology$ $N\text{-}7491\ Trondheim$

Norway

(Dated: March 26, 2018)

Abstract

In physics textbooks, the emphasis is normally placed on the physics. In order to give the subject a deeper dimension, the inclusion of history and persons involved in the development has become increasingly used. However, the actual historical development does not always follow a linear progression, which is the way the physics is presented in textbooks. In order to study how the inclusion of "history" has influenced the presentation, a study of the development of a well-known history in different editions of a physics textbook is done. The gradual development of a quasi-history is observed and possible explanations for this are investigated.

I. INTRODUCTION

The primary aim, in physics education, is that the student should gain an understanding of the principles of physics and how to apply them to different problems. A secondary aim is to allow the students to appreciate the scientific approach and significance of it in the evolution of science and society, i.e. learning about science. This has been described by Hodson¹ as being three aspects of science education: learning science, doing science and learning about science. One common approach for the second aim has been to include historical material in physics textbooks. The question one has to ask; what is the purpose of this material? Has it been included to give physics a "human dimension", so to say make it easier to digest or to show how physics has developed by the efforts made by specific persons? The former emotional argument may serve as a motivation for students, but at the risk of creating an idol, a genius they can never match. By presenting the scientists of the past in a hagiographic way, students will not be able to identify themselves or their own problems in understanding the subject, something that can be de-motivating. The latter argument seeks the basis of scientific progress, how it can be done and why it is done as it is. It offers a lesson on the mistakes and problems that formed the science of today. This can be called a humanistic or historical argument, with the same aims as historical research to explaining why. Students can, in this case, identify their own problems in understanding with the ones the originators had. One should note that this goes beyond just giving short biographies of scientists.

The quantity of the historical material included in different textbooks is quite diverse, from textbooks with a more historical approach to others without any historical material at all. The quality, in historical accuracy, of the material included in the textbooks is also diverse. There have been done a number of studies on the historical content of science textbooks. ^{2,3} Examples of deviations in the historical narration in the textbooks are the development of the Planck blackbody radiation equation, Bohr's atom model and the acceptance of Einstein's theory of photoelectric effect. In this paper, I focus on the development of the historical material, i.e. the presentation of the Blackbody radiation equation, in a specific textbook (Sears & Zemansky's University Physics) over a number of editions from the first edition in 1949 to the 13th edition in 2012. This textbook is especially suited as the different editions spans over 6 decades. The aim is to see in which edition the historical

material is included and how it is included. How well it describes the history and how the presentation changes between different editions. Will the presentation focus on the historical development or the "logical" physics development?

The event of interest is the introduction of the Blackbody radiation formula or Planck's radiation formula. This is well known out of a historical perspective, but also a case where a quasi-historical seems to be quite common in textbooks as noted by Brush.⁴ The aim of the article is not to discuss the use of the history of science in teaching, but rather to show that the textbooks might be unsuited for this purpose. For the use of the history of science in teaching, I refer to the book edited by Mathews.⁵

II. STUDY OF HISTORY IN PHYSICS TEXTBOOKS

Studies of the historical content in science textbooks have been performed by a number of authors over the years, for example, Leite² and Niaz et al.³ The studies can in principle be differentiated into two different types. A domain general type, based on a number of dimensions derived from the history and nature of science, the work of Leite² is an example of this type of studies. A domain specific type, based on a given topic where details are important. An example of this type of study is Niaz et al.³ on the photoelectric effect. There is an overlap between these types of studies, but it is important to be aware of the difference as the aims of the studies are slightly different and they cannot be directly compared. General studies are best used for investigating the inclusion of historical material, that is the quantity but also quality based on the dimensions used in the analysis, while the specific is better in studying how the history is presented in detail, that is more focused on quality. In this article, I take a specific view, but in a historical textbook context, that is, I am interested in how the presentation has changed over time. This will place this work in a domain of specific development, which differs from most other articles in this subject.

A. History of modern physics in textbooks.

The history of modern physics in textbooks has been the subject of a number of studies. For example, Thomas Kuhn⁶ argued, from studies of the history, that Ehrenfest and Einstein were the first to recognize that the blackbody law could not be derived without restricting the

energy to multiples of $h\nu$. That is that Planck had just introduced a mathematical approximation based on classical physics and that Ehrenfest and Einstein where the first to propose a "full" quantum hypothesis. Brush⁴ studied 28 general physics textbooks and found that only six supported Kuhn's analysis. A similar study performed by Niaz and Fernandez⁷ showed that only one out of 55 general chemistry textbooks supported Kuhn's analysis. Brush⁴ also analyzed 26 general physics textbooks in connection with the relationship between the Michelson-Morley experiment and Einstein's theory of special relativity. Nine textbooks gave the Michelson-Morley experiment an important role in motivating Einstein 's theory of special relativity. Niaz and Fernandez⁸ investigated how well the development of atomic structure was presented out of a history and philosophy of science perspective in general physics textbooks published in the USA between 1970 and 2001. Out of 39 textbooks, only one could be considered as satisfactory when it comes to historical accuracy. The photoelectric effect and how it is presented has been investigated by Niaz et al.³ in 103 physics textbooks published in the USA between 1953 and 2007. Niaz et al.³ considered six criteria based on the historical development; 1. Lenard's trigger hypothesis to explain the photoelectric effect; 2. Einstein's quantum hypothesis to explain the photoelectric effect; Lack of acceptance of Einstein's quantum hypothesis in the scientific community; 3. Millikan's experimental determination of the Einstein photoelectric Equation and Planck's constant, h; 4. Millikan's presuppositions about the nature of light; 5. The historical record presented and its interpretation within a history and philosophy perspective. They found that criterion 2 where presented satisfactory or better in 48% of the books. Criterion 4 was satisfactorily presented in 16% all other criteria in less than 5%. Most criteria were not mentioned at all in the books, from 69% (criterion 4) to 95% (criterion 5) of the books. It was also possible to do a temporal analysis, where the scores improved significantly in the beginning of the 1960s, something they thought might be due to the massive science education reform in the USA after the launch of the Sputnik. However, later reforms have not shown a similar effect.

III. PLANCK'S RADIATION FORMULA

There exist excellent accounts of the development of quantum theory in general^{9–12} as well as detailed studies of Planck's radiation formula^{13–15}, why we will not go into detail on

the history but try to give a general outline.

The development of spectroscopy made it possible to study spectra from different elements but also from macroscopic objects. In 1859, Gustav Robert Kirchhoff argued that black-body radiation was of a fundamental nature¹⁶, thus initiating an interest in studies of black-body radiation. The spectral distribution was investigated by several physicists, both experimentally and theoretically. In 1896 Wilhelm¹⁷ found a radiation law that was in agreement with precise measurements performed at the Physikalisch-Technische Reichsanstalt in Berlin.

According to Wien the spectral density is described as:

$$u(f,T) = \alpha f^3 e^{\frac{-\beta f}{T}}$$

where α and β are constants to be determined empirically, f and T are frequency and temperature, respectively. However, this lacked a theoretical foundation, something that Max Planck could not accept. Planck formulated a "principle of elementary disorder", which he used to define the entropy of an ideal oscillator. Using this it was possible for Planck, early in 1899, to find an expression from which Wien's law followed.

However, measurements by Lummer and Pringsheim in November 1899¹⁸, showed a deviation from Wien's law at low frequencies. Planck had to revise his calculations, using a new expression for the entropy of a single oscillator. The new distribution law was presented at a meeting of the German Physical Society on 19 October 1900. The spectral density was now given as:

$$u\left(f,T\right) = \frac{\alpha f^{3}}{e^{\frac{\beta f}{T}} - 1}$$

where α and β are constants to be determined empirically, f and T are frequency and temperature, respectively. The problem was, as Planck realized, that his new expression was no more than an inspired guess. Planck had not used energy quantization or Boltzmann's probabilistic interpretation of entropy.

In what he himself describes as "an act of desperation", he turned to Boltzmann's probabilistic notion of entropy. Even if he adopted Boltzmann's view, he did not convert to the probabilistic notion of entropy. He remained convinced that the law of entropy was absolute, not probabilistic, and therefore reinterpreted Boltzmann's theory in his own non-

probabilistic way. Using the "Boltzmann equation"

$$S = k \log W$$

which relates the entropy, S, to the molecular disorder, W, Planck had to be able to count the number of ways a given amount of energy can be distributed in a set of oscillators. It was in doing this Planck introduced what he called "energy elements", that is the total energy of the black-body oscillators, E, divided into finite portions of energy, ε , via a process known as "quantization". The energy of the finite energy elements was given by a constant, h, multiplied by a frequency f.

$$\varepsilon = hf$$

Using this it was easy for Planck to follow the procedure that Boltzmann used in deriving Maxwell's distribution of velocities of molecules in a gas and derive the spectral density:

$$u(f,T) = \frac{8\pi}{c^3} \frac{hf^3}{\left(e^{\frac{hf}{kT}} - 1\right)}$$

This was presented to the German Physical Society on 14 December 1900¹⁹, followed by four papers in 1901.

Lord Rayleigh published a paper in June 1900^{20} where he presented an improved version of Wien's radiation law. Using Maxwell-Boltzmann's equipartition theorem, he obtained a different radiation law:

$$u\left(\lambda,T\right) = c_1 \frac{T}{\lambda^4} e^{\frac{-c_2}{\lambda T}}$$

This law was noted and tested experimentally but got very little attention since Planck had produced a formula that was a better fit to the experimental results. In 1905 Rayleigh²¹ came up with a refined radiation law:

$$u\left(\lambda,T\right) = \frac{64\pi kT}{\lambda^4}$$

An error in the calculations was corrected by Jeans^{22,23} and the new radiation law was therefore named Rayleigh-Jeans law:

$$u\left(\lambda,T\right) = \frac{8\pi kT}{\lambda^4}$$

The result is an energy density that increases as the frequency gets higher, becoming "catastrophic" in the ultraviolet region. This made Paul Ehrenfest coin the name "ultraviolet catastrophe" in 1911, thus becoming a matter of discussion quite late in the development of quantum theory.

Einstein was the first to fully adopt the quantization principle in his derivation of Planck's radiation law in 1906.²⁴ Something that caused an increased interest among physicists, leading to more discussions.

The historical time scale is quite clear: Planck's Radiation law origins from 1900, Rayleigh-Jeans law from 1905 and the term "ultraviolet catastrophe" from 1911.

From a physical and physics educational point of view, the order of the radiation laws will be quite different from a historical view. As Rayleigh-Jeans law is based on classical physics, i.e. no quantization, it should be placed before Planck's law, by presenting the problems with classical physics. Also, the use of the Maxwell-Boltzmann's equipartition theorem makes it natural to derive it from the Maxwell-Boltzmann distribution. It is notable that one of the first textbooks on "modern physics" choose to present the subject following the physics from classical to modern physics, with Rayleigh-Jeans before Planck. The reason possibly being to give a full description using classical physics before moving into "modern" physics. But this will not follow the historical and conceptual development at the end of the nineteenth century and will create a history with incorrect reasons why the quantum physics were developed.

IV. SEARS & ZEMANSKY'S UNIVERSITY PHYSICS

The textbooks chosen for this study, Sears & Zemansky's University Physics with the first edition published in 1949²⁶ has the advantage of being used for over six decades. In addition, it is widely used around the world today, even if the first editions were mainly used in the USA. At many universities this textbook is the first (and sometimes the only) physics textbook for many students, making a rather large impact on them. As this textbook has a long history will it be a good candidate when studying the development of textbooks? Since the first edition in 1949²⁶, a number of editions have been published. The responsible au-

thors have changed over the years. The first four editions, 1949^{26} , 1955^{27} , 1963^{28} and 1970^{29} were written by Sears and Zemansky. The fifth edition, from 1976^{30} , was co-authored by Young. This edition was also the last co-authored with Sears who died suddenly during the final preparing stages of the manuscript. Six years later, in 1982^{31} , the sixth edition was published, this being the last with Zemansky who died in 1981 after the new manuscript had been finished. The seventh and eight editions were authored by Young, 1987^{32} and 1992^{33} , respectively. The ninth edition included Freedman as co-author, something that has been the case for the last editions up to date, ninth edition in 1996^{34} , tenth to thirteenth, 2000^{35} , 2004^{36} , 2008^{37} , and 2012^{38} , respectively.

It should also be mentioned that a number of contributing authors has been used over the years.

Having a textbook that has evolved from the start over 60 years ago, gives a unique opportunity to probe the development of physics education in the view of the authors. Both in the presentation and the layout, but also in the examples and derivations used. In this case, we are more interested in the development of how a specific historical development is presented and how it is changing. An article describing physics texts of the last 150 years has been written by Holbrow³⁹. Clearly, University Physics was not written in a vacuum, there existed other textbooks before. Sears had written a book before³⁹ mainly aimed at his students at MIT but used throughout the USA, as described by Templin⁴⁰, in a way the template of University Physics. Sears then a Professor at MIT joined forces with Zemansky, a professor at the City College of New York, to create a new introductory physics textbook.³⁹ The first edition followed a basic outline, with a conventional selection and sequence. Mechanics, heat, sound, electricity and magnetism, and optics. Modern Physics were added after the first editions, with atomic physics as the first modern subject to be included. It is clearly stated in the preface of the first editions (up to the seventh) that:

The emphasis is on physical principles; historical background and practical applications have been given a place of secondary importance.²⁸ (Preface)

The eighth edition³³ is a comprehensive revision aimed at the changes in the background and needs of the students as well as a change in the philosophy of introductory physics courses as stated in the preface.³³ (Preface) One effect of this being an attempt to make physics more human, in part by including the history of physics. One should note that the

third edition²⁸ was an attempt to include modern physics in an introductory textbook with for example an innovative approach to the theory of special relativity³⁹. However, this was removed from the fourth edition.²⁹

A. Blackbody radiation in University Physics

In the first and second editions, the Blackbody radiation is described from a practical or experimental view, without mentioning Planck's quantization or giving the blackbody radiation formula. This is hardly surprising as the book was developed in an engineering milieu. Blackbody radiation and Planck's quantization is mentioned in the third edition of University Physics, in the thermodynamics section (Chapter 17-7 Planck's law.²⁸) The description differs from earlier editions, with a discussion on the origin of Planck's law, as the earlier editions just described Blackbody radiation from a radiation point of view without mentioning Planck's quantization. The presentation in the third edition does not emphasize the history, but on a more experimentally observed approach:

Max Planck, in 1900, developed an empirical equation that satisfactory represented the observed energy distribution in the spectrum of a Blackbody. After unsuccessful attempts to justify his equation by theoretical reasoning based on the laws of classical physics, Planck concluded that these laws did not apply to energy transformations on an atomic scale. Instead, he postulated that a radiating body consisted of an enormous number of elementary oscillators, some vibrating at one frequency and some at another, with all frequencies from zero to infinity being represented.²⁸ (p 380)

The engineering approach is clearly visible as the result is important, not the derivation. The basic ideas of Planck are given and the formula presented. It is also interesting to note that Wien's displacement law and Stefan's law are given in the following chapter (17-8) further confirming the practical approach, without much historical content.

The discussion in the fourth, fifth and sixth editions, is very similar to the discussion in the first two editions, making the third edition special in the case of presenting modern physics.

It is with the seventh edition (1987) that Planck makes his re-entrance in a chapter on continuous spectra (Chapter 41-7³²). The presentation now takes a starting point in the empirical Stefan-Boltzmann Law and the Wien displacement law and describes how Planck in 1900 used the principle of equipartition of energy together with a quantization of the energy that is emitted or absorbed. The presentation does not emphasize the history but states:

Finally, in 1900 Max Planck succeeded in deriving a function, now called the Planck radiation law, that agreed with experimentally obtained powerdistribution curves. To do this he added to the classical equipartition theorem the additional assumption that a harmonic oscillator with frequency f can gain or lose energy only in discrete steps of magnitude hf, where h is the same constant that now bears his name.

Ironically, Planck himself originally regarded this quantum hypothesis as a calculational trick rather than a fundamental principle. But as we have seen, evidence for the quantum aspects of light accumulated, and by 1920 there was no longer any doubt about the validity of the concept. Indeed, the concept of discrete energy levels of microscopic systems really originated with Planck, not Bohr, and we have departed from the historical order of things by discussing atomic spectra before continuous spectra.³² (p995)

It is notable that there is no discussion on the development of physics, it is again the result that is important. The basic ideas seem to be the same as in the third edition but are treated more in detail. The description differs from the historical development but cannot be considered as quasi-history.

B. Changes in the eighth edition

The eighth edition³³ shows some major changes, the title now includes modern physics, and the material in this section has changed a lot and new material has been added. In the case of continuous spectra (Chapter 40-8), the discussion about the physics behind has been modernized, the history has also been extended with the derivation of Rayleigh-Jeans law from 1905, presented as a precursor to Planck's Law from 1900!

During the last decade of the nineteenth century, many attempts were made to derive these empirical results from basic principles. In one attempt, Rayleigh considered light enclosed in a rectangular box with perfectly reflecting sides. Such a box has a series of possible normal modes for the light waves, analogous to normal modes for a string held at both ends (Section 20-3). It seemed reasonable to assume that the distribution of energy among the various modes was given by the equipartition principle (Section 16-4), which had been successfully used in the analysis of heat capacities. A small hole in the box would behave as an ideal Blackbody radiator.

Including both the electric- and magnetic-field energies, Rayleigh assumed that the total energy of each normal mode was equal to kT. Then by computing the number of normal modes corresponding to a wavelength interval $d\lambda$, Rayleigh could predict the distribution of wavelengths in the radiation within the box. Finally, he could compute the intensity distribution $I(\lambda)$ of the radiation emerging from a small hole in the box. His result was quite simple:

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4} \qquad (40-30)$$

At large wavelengths, this formula agrees quite well with the experimental results shown in Fig. 40- 27, but there is serious disagreement at small λ . The experimental curve falls to zero at small λ ; Rayleigh's curve approaches infinity as $1/\lambda^4$, a result called in Rayleigh's time the "ultraviolet catastrophe." Even worse, the integral of Eq. (40- 30) over all λ is infinite, indicating an infinitely large *total* radiated intensity. Clearly, something is wrong.

Finally, in 1900, Planck succeeded in deriving a function, now called the **Planck** radiation law, that agreed very well with experimental intensity distribution curves. To do this, he made what seemed at the time to be a crazy assumption; he assumed that in Rayleigh's box a normal mode with frequency f could gain or lose energy only in discrete steps with magnitude hf, where h is the same constant that now bears Planck's name.³³ (p1129-1130)

The presentations follow the "logical" transition from classical to modern physics as presenting a classical (Rayleigh) result before the "modern" (Planck) result. One should also note the rather sharp wording (crazy assumption) in connection with Planck's assumption.

We also find a change in the description of the development of physics, which is now a part of the discussion. This might indicate a desire to discuss how the results were obtained. But we can also notice that the description of events does not follow the actual history, in which Rayleigh plays a minor part.

Planck was not comfortable with this quantum hypothesis; he regarded it as a calculational trick rather than a fundamental principle. In a letter to a friend he called it "an act of desperation" into which he was forced because "a theoretical explanation had to be found at any cost, whatever the price." But five years later, Einstein extended this concept to explain the photo-electric effect (Section 40-2), and other evidence quickly mounted. By 1915 there was no longer any doubt about the validity of the quantum concept and the existence of photons. By discussing atomic spectra before discussing continuous spectra, we have departed from the historical order of things. The credit for inventing the concept of quantization goes to Planck, even though he didn't believe it at first.³³ (p1130)

Note that the year for general acceptance of the concept of quantization and the existence of photons has changed from 1920 to 1915. Here we also find that photons are discussed, something that is not the case in the earlier editions where the quantum aspects are discussed. The authors also raise the question of who was the first to introduce the quantum concept in a deeper sense, without discussing it further.

It is clear that the style of the book has changed in such a way that it is not sufficient to just present the results but the development on how the results were found is becoming more important. Still, the emphasis is on the practical aspect of physics and problem solving.

The ninth³⁴, tenth³⁵ and eleventh³⁶ editions are almost the same as the eight, with minor changes in the presentation.

C. Changes in the twelfth edition

In the twelfth edition³⁷ the change is not in the presentation but in the fact that Rayleigh's derivation is now given as a separate subsection with a special heading, **Rayleigh and the** "Ultraviolet Catastrophe", in section 38.8. As before placed before Planck's derivation

which is also given a separate subsection with a special heading, **Planck and the Quantum Hypothesis**. With the presentation and now subsection, the dawning quasi-history from the eighth edition is enhanced, leading further away from the actual development.

D. The thirteenth edition

In the preface where news to this edition is presented we find the following quote:

The **core modern physics chapters** (Chapters 38-41) are revised extensively to provide a more idea centered, less historical approach to the material.

Despite this, the presentation in the thirteenth edition³⁸ is the same when it comes to history. But now an analogy is introduced to explain the difference between line spectra and continuous spectra. In Planck's derivation, the discussion is extended and coupled more towards thermodynamics and Boltzmann distributions, as to give the idea behind.

V. DISCUSSION

It is clear that the presentation style changed from a clear result-centred approach to a much broader approach as from the eighth edition (1992), where the idea to make physics more interesting by introducing a "human dimension" by incorporating the history of physics. However, the basic ideas of "University Physics" are still the same as in the first editions:

The emphasis is on physical principles; historical background and practical applications have been given a place of secondary importance.²⁸ (Preface)

The inclusion of historical material is quite late compared with other textbooks as can be seen in the study of Niaz et al.³ The aim of textbooks is to present a logical presentation of the scientific facts and development of physics, but also to provide a historical framework where the scientific facts fit easily. Thus creating a development of physics that will make sense and is easily remembered out from a physical point of view. In this process, the historical events have to adapt to the physics, in such way that students can follow a "logical" development. However, history is seldom "logical", so it is easy to rewrite history to fit the physics. In doing so one misses the chance to discuss the foundations of physics and why

these might seem to be counterintuitive. It is important to show that understanding and finding the correct theory is seldom a straightforward process. If the students have trouble to understand a theory it might be comforting to know that the persons developing the theory also had problems in understanding it and why they had these problems. Some of the problems they had are identical to what we today call misconceptions. A discussion on these will help students in their understanding of the basic physical concepts.

The textbook studied, does not follow the historical development but rather a "logical" development in the physical conceptual sense, something that tends to enhance a quasi-historical myth, when placing a result-centered presentation in a false historical context. The intentions and the pedagogical motivation can be considered as solid out of a result-centered approach, but unfortunately, will this give a false picture of the history and nature of physics. It is possible that earlier textbooks such as²⁵ have served as an inspiration to the narrative in University Physics and other modern textbooks, but as the context and personal "knowledge" of the historical development have changed, the result is unfortunate.

The most important question to ask is if we want students to learn just physics (results and principles) or in addition understand the processes that led to the development of physics. These views are complementary as an understanding of how physics came to develop also gives insights, helping the personal learning, as well as an understanding of how problems were solved and will be solved in the future. This raises the question of how much and what type of historical material is included. Should all material presented have a historical approach or should one limit the historical material to certain case studies? It is important to ask why it should be included and can it help students understanding and learning. If only the results (products) of physics is important, very little or no historical material might be included. However, if the ideas and processes are important should be included, but then as close to the actual development as possible without any quasi-history.

As has been studied in this work, the development of the historical narrative in University Physics follows an evolution from a result-centred presentation to a more general presentation, where the development is also getting important but where the logical structure is kept intact at the expense of a correct historical description. This is clearly the case with blackbody radiation, but studies of other important concepts should also be performed. Modern physics might suffer most from this, why a study of other concepts, such as Photoelectric effect and development of atomic models should be done.

* Electronic address: jonas.persson@ntnu.no

- ¹ Hodson, D. (1992) Redefining and reorientating Practical Work in School Science. School Science Review 73 (264), 65-78.
- ² Leite, L. (2002). History of science in science education: Development and validation of a checklist for analyzing the historical content of science textbooks. Science & Education, 11, 333-359.
- Niaz, M., Klassen, S., McMillan, B., & Metz, D. (2010). A historical reconstruction of the photoelectric effect and its implications for general physics textbooks. Science Education, 94, 903931.
- 4 Brush, S.G. (2000) Thomas Kuhn as a historian of science. Science & Education, 9(1-2), 39-58
- ⁵ Matthews, M. (Ed.) (2014). International handbook of research in history, philosophy and science teaching. Dordrecht: Springer Verlag.
- ⁶ Kuhn, T.S. (1978) Black-body theory and the quantum discontinuity: 1894-1912. New York: Oxford University Press.
- Niaz, M & Fernandez, R. (2008) Understanding quantum numbers in general chemical textbooks. International Journal of Science Education, 30(7), 869-901.
- ⁸ Rodriguez M.A & Niaz, M, (2004) A reconstruction of the atom and its implications for general physics textbooks: A history and philosophy of science perspective. Journal of Science Education and Technology, 13, 409-424.
- ⁹ Mehra, J., & Rechenberg, H. (1982). The Historical Development of Quantum Theory (Vol.1).
 New York: Springer-Verlag.
- $^{10}\,$ Jammer, M. (1966). Conceptual Development of Quantum Mechanics . New York: McGraw-Hill.
- ¹¹ Herman, A., (1971) The Genesis of Quantum Theory (1899-1913). Cambridge. Mass.:MIT Press.
- ¹² Kragh, H. (1999). Quantum Generations. Princeton, Mass.: Princeton University Press.
- ¹³ Klein, M. J. (1962). Max Planck and the Beginning of the Quantum Theory. Archive for History of Exact Sciences, 1(5), 459-479.
- ¹⁴ Klein, M. J. (1963). Einstein's First Paper on Quanta. The Natural Philosopher, 2, 59-86.
- ¹⁵ Kragh, H. (2000). Max Planck: the reluctant revolutionary. Physics World December 2000.

- ¹⁶ Kirchhoff, G., (1860). Ueber das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht. Annalen der Physik, 185(2), 275–301.
- Wien, W. (1896). Ueber die Energievertheilung im Emissionsspectrum eines schwarzen Körpers.
 Annalen der Physik 58, 662-669
- ¹⁸ Lummer, O., & Pringsheim, E. (1899). Die Vertheilung der Energie im Spectrum des Körpers und des blanken Platins. Verhandlungen des Deutschen Physikalischen Gesellschaft, 1, 215-235.
- ¹⁹ Planck,M., (1900). Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum. Verhandl. Dtsch. phys. Ges., 2, 237-245
- Rayleigh, L. (1900). Remarks upon the Laws of Complete Radiation. Philosophical Magazine, 49(5), 539-540.
- $^{21}\,$ Rayleigh, L. (1905). The Dynamical Theory of Gases and of Radiation. Nature, 72, 54.
- ²² Jeans, J., (1905). The partition of energy between matter and ether. Phil. Mag. 10,91-98.
- ²³ Jeans, J., (1905). The law of radiation. Proc.Roy.Soc. A, 76,545-552
- ²⁴ Einstein, A. (1906). Zur Theorie der Lichterzeugung und Lichtabsorption. Annalen der Physik, 20(4), 199-207.
- ²⁵ Richtmyer, F. K. (1928) Introduction to modern physics (1 ed.). New York: McGraw-Hill.
- ²⁶ Sears, F.W., & Zemansky, M.W., (1949). University Physics (1 ed.). Cambridge Mass.: Addison Wesley.
- ²⁷ Sears, F.W., & Zemansky, M.W., (1955). University Physics (2 ed.). Reading Mass.: Addison Wesley.
- ²⁸ Sears, F.W., & Zemansky, M.W., (1963). University Physics (3 ed.). Reading Mass.: Addison Wesley.
- ²⁹ Sears, F.W., & Zemansky, M.W., (1970). University Physics (4 ed.). Reading Mass.: Addison Wesley.
- ³⁰ Sears, F.W., Zemansky, M.W., & Young, H. D. (1976). University Physics (5 ed.). Reading Mass.: Addison Wesley.
- Sears, F.W., Zemansky, M.W., & Young, H. D. (1982). University Physics (6 ed.). Reading Mass.: Addison Wesley.
- ³² Young, H. D. (1987). University Physics (7 ed.). Reading Mass.: Addison Wesley.
- Young, H. D. (1992). University Physics, Extended version with modern physics (8 ed.). Reading Mass.: Addison Wesley.

- Young, H. D., & Freedman, R. A. (1996). University Physics (9 ed.). San Fransisco Boston New York: Addison Wesley.
- Young, H. D., & Freedman, R. A. (2000). University Physics (10 ed.). San Fransisco Boston New York: Addison Wesley.
- Young, H. D., & Freedman, R. A. (2004). University Physics (11 ed.). San Fransisco Boston New York: Pearson Addison Wesley.
- Young, H. D., & Freedman, R. A. (2008). University Physics (12 ed.). San Fransisco Boston New York: Pearson Addison Wesley.
- Young, H. D., & Freedman, R. A. (2012). University Physics (13 ed.). San Fransisco Boston New York: Pearson Addison Wesley.
- ³⁹ Holbrow, C.H., (1999) Archaeology of a Bookstack: Some Major Introductory Physics Texts of the Last 150 Years. Physics Today 52(3)
- ⁴⁰ Templin, J.D., (1996) The three volume Sears 50 years later. Am. J. Phys. 64, 369-373