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A method of road network vulnerability identification taking into account travelers' heterogeneous risk attitudes

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ABSTRACT: The analysis of road network vulnerability is a challenging and important research subject in the field of transportation reliability engineering because of the complex coupling relationships among travelers, vehicles, roads and environment. In view of the deficiency of existing researches on the description of travelers' risk averse and bounded rational behavior characteristics, both random utility theory and random regret theory are used to describe travelers' decision-making behaviors. Then a traffic assignment model expressed by variational inequality is constructed, in which travelers' risk aversion and bounded rationality as well as resultant heterogeneity of travelers' decision-making behaviors are explicitly taken into account. Subsequently, the vulnerability index of road network based on accessibility is defined, and a vulnerability identification model is built, then corresponding heuristic algorithm is also proposed. The example results show that the consequences of link closures could be misjudged and the vulnerability rankings could be misidentified, if ignoring the effects of the congestion levels of road network, travelers' perception errors and regret aversion degrees, as well as travelers' route choice decision criteria. Therefore, it is necessary to capture the travelers' behavior characteristics in the process of vulnerability analysis.

1 INTRODUCTION

1.1 *Background*

Robust road traffic networks have been regarded as one of the preconditions for a high quality of life. However, a traffic network can be vulnerable to various natural or man-made disasters. For example, adverse weather such as heavy snow and flooding could severely degrade the network performance. Although the occurrence probability of these major incidents is low, their consequences could be sufficiently large to generate a major problem that threats remedial actions. Therefore, it is vital to understand the potential vulnerability of traffic networks under such major incidents, so as to manage their risks.

A key issue in the vulnerability analysis is to identify the critical links of a network, where the failures of those links would have the most serious impacts on the whole network (Chen et al., 2012, Yin and Xu, 2010). After identifying the critical links, the network robustness can be enhanced through reinforcing these identified critical links or constructing new alternative parallel paths (Matisziw and Murray, 2009).

Modeling travelers' behavioral responses to link failures is another key issue involved in critical link identification (Chen et al., 2012). Subjected to link failures, the high degree of demand uncertainty and/or link capacity degradation will inevitably yield travel time variability, and consequently imposes additional disutility on travelers. Many empirical studies have revealed the significant influence of travel time uncertainty on travelers' route choice behavior (Liu et al., 2004, Shao et al., 2006, Wu and Nie, 2011). Travelers under travel time uncertainty tend to choose reliable shortest path, not only dependent on travel time saving, but also on reduction of travel time variability. This risk averse behaviors under travel time uncertainty have received considerable attention (Shao et al., 2006, Lo et al., 2006, Siu and Lo, 2008). However, most of the previous studies adopt Expected Utility Theory (EUT) and/or Random Utility Theory (RUT) to quantify travelers' perceptions of network uncertainty, and assume that the travelers are homogeneous. It is well known that both EUT and RUT are based on the assumption that travelers are absolutely rational when making route choice decisions. In reality, however, part of travelers' behaviors may be bounded rational, and can be influenced by his or her personality, psychological state, and environmental elements, etc. It goes without saying that the usefulness of EUT or RUT as a descriptive model of choice behaviour has been fiercely debated both inside and outside the transportation domain. It is particularly interesting to note that Regret Theory (RT) (Loomes and Sugden, 1982) and its extended Random Regret Theory (RRT) (Chorus, 2014), being widely considered one of the most prominent competitors of both EUT and RUT in the behavioral decision sciences, has been virtually ignored in the route choice domain. RT or/and RRT postulate that when choosing, people anticipate and try to avoid the situation where a non-chosen alternative outperforms the chosen one (which would cause post-decision regret). It is a pity that, to our best knowledge, travelers' risk aversion and bounded rationality as well as resultant travelers' heterogeneous behavior responses due to link closures have not yet been considered simultaneously in the studies of critical link identification.

In view of the above, this study proposed a method of road network vulnerability identification taking into account travelers' risk aversion, bounded rationality and heterogeneity simultaneously under travel time variations subjected to link failures. We assume that the total travel demand comprises of two parts, completely rational travelers and bounded rational travelers. A new vulnerability index based on accessibility is introduced to evaluate the consequences of a link closure with consideration of their effects.

1.2 *Outline*

The remainder of this paper is organized as follows. In Section 2, a stochastic mixed user equilibrium model is built. It follows with the definition of vulnerability index based on accessibility for evaluating the consequences of a link closure with consideration of travelers' risk aversion, bounded rationality and heterogeneity. In Section 4, numerical examples by means of the Nguyen and Dupuis network is provided to demonstrate the proposed model. Finally, the conclusions are given in Section 5, together with future research directions.

2 A STOCHASTIC MIXED USER EQUILIBRIUM MODEL

2.1 *Distributions of travel times under link closures*

Consider a road network represented by a strongly connected graph G = (N, A), where N and A are the sets of nodes and links respectively. Let W denote the set of Origin-Destination (OD), R_w represent the set of paths between OD pair w, $w \in W$.

Because of link failures, the high degree of demand uncertainty and/or link capacity degradation will inevitably lead to travel time uncertainty. Assume that a link travel time is a random variable. Let T_a represent the travel time on link a. Furthermore, assume that T_a follows the normal distribution with mean value t_a and variance $\rho_a t_a$, where ρ_a represents the variation coefficient of T_a. The mean travel time t_a can be described by the following BPR (Bureau of Public Roads) function:

$$t_a = t_a^0\left(1+\beta\left(\frac{v_a}{c_a}\right)^n\right), a \in A \tag{1}$$

where t_a^0 is the free-flow travel time on link a, v_a and c_a are the flow and the capacity on link a respectively, β and n are the constant parameters of BPR function.

Let the travel time on path k between the OD pair w be represented as T_k^w, which can be calculated according to the relationship of link and path, as follows:

$$T_k^w = \sum_{a\in A} T_a \delta_{a,k}^w, k \in R_w, w \in W \tag{2}$$

where $\delta_{a,k}^w$ is the link-path incidence variable, $\delta_{a,k}^w=1$ if link a is on path k, otherwise $\delta_{a,k}^w = 0$.

In order to simplify the problem, it is assumed that link travel times are independent of each other. Because a path is composed of several independent links, according to the Central Limit Theorem, a path travel time should obey the normal distribution approximately. According to equations (1) and (2), the mean and standard deviation of T_k^w can then be expressed as below:

$$t_k^w = \sum_{a\in A} \delta_{a,k}^w t_a^0\left(1+\beta\left(\frac{v_a}{c_a}\right)^n\right), k \in R_w, w \in W \tag{3}$$

$$\sigma_{T_k^w} = \sqrt{\sum_{a\in A} \delta_{a,k}^w \rho_a t_a}, k \in R_w, w \in W \tag{4}$$

where t_k^w and $\sigma_{T_k^w}$ are respectively mean and standard deviation of T_k^w.

The empirical researches show that, the travelers not only want to save travel time, but also hope to avoid the risk caused by the travel time uncertainty. Therefore, Lo et al. (2006) put forward the concept of Travel Time Budget (TTB), which is used to describe the route choice behavior of travelers avoiding travel risk. Let $\xi_k^w(\omega)$ be the TTB of route k between OD pair w, which can be expressed as follows:

$$\xi_k^w(\omega) = t_k^w + \Phi^{-1}(\omega)\sigma_{T_k^w}, k \in R_w, w \in W \tag{5}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of standard normal distribution, ω denotes the reliability parameter reflecting the probability that the actual trip time is within the TTB.

1.2 *Travel decision model for completely rational travelers based on RUT*

Considering the travelers' risk aversion in route choice decisions, it is assumed that the completely rational travelers use TTB as their route choice criterion. In addition, because of the travelers may not have perfect information on the travel time distributions, the travelers' perception errors on the travel times should be also taken into account. Therefore, according to RTU, the travel disutility of the completely rational travelers can be regarded as a random variable, which can be expressed as follows:

$$U_{w,k}^{\mathrm{CR}} = \xi_k^w(\omega) + \zeta_{w,k}^{\mathrm{CR}},\ k \in R_w, w \in W \tag{6}$$

where $U_{w,k}^{\mathrm{CR}}$ and $\zeta_{w,k}^{\mathrm{CR}}$ respectively represent the perceived travel disutility and perception error when the completely rational traveler' choose the route k between the OD pair w.

Furthermore, assuming that the perception error $\zeta_{w,k}^{\mathrm{CR}}$ are identically and independently Gumbel distributed random variables with mean zero, then the probability that a completely rational traveler chooses the route k between OD pair w can be described as follows:

$$p_{w,k}^{\mathrm{CR}} = \frac{\exp\left(-\theta^{\mathrm{CR}}\xi_k^w(\omega)\right)}{\sum_{r\in R_w}\exp\left(-\theta^{\mathrm{CR}}\xi_r^w(\omega)\right)},\ k \in R_w, w \in W \tag{7}$$

where $\theta^{\mathrm{CR}} > 0$ is the perception error parameter of the completely rational travelers which is used to measure the degrees of travelers' perception errors. It is noted that a higher θ^{CR} means smaller perception errors.

1.3 *Travel decision model for bounded rational travelers based on RRT*

In reality, due to the influences of information conditions, personality, preferences and other factors, not all travelers' route choice behaviors are completely rational, and some travelers show bounded rationality when choosing a route. In this paper, RRT is used to describe this phenomenon. In contrast with RUT, which postulates that a route's disutility is a function of its own performance only, RRT postulates that in addition, the performance difference with the competing route codetermines a route's disutility. In other words: RRT assumes that the traveler is regret averse in travel decision-making, that is, when the traveler make choice among all alternatives, the total value of regret that the current choice scheme compares with the other alternatives is considered, and the scheme with minimum value of regret is chosen.

Based on the above analysis, assume that the bounded rational travelers are risk averse and regret averse, who take perceived regret value as their route choice criterion. According to RRT, the travel disutility of the bounded rational travelers can be represented as below:

$$U_{w,k}^{\mathrm{BR}} = u_{w,k}^{\mathrm{BR}} + \zeta_{w,k}^{\mathrm{BR}},\ k \in R_w, w \in W \tag{8}$$

where $U_{w,k}^{\mathrm{BR}}$, $u_{w,k}^{\mathrm{BR}}$ and $\zeta_{w,k}^{\mathrm{BR}}$ respectively represent the perceived travel disutility (the perceived regret value), mean of the perceived travel disutility (mean of the perceived regret value) and perception error when the bounded rational traveler' choose the route k between the OD pair w.

Suppose that the bounded rationality travelers use the TTB as the absolute travel disutility, a specifically functional form of $u_{w,k}^{\mathrm{BR}}$ that satisfies RRT requirements can be stated as follows:

$$u_{w,k}^{\mathrm{BR}} = \sum_{m\neq k, m\in R_w}\exp\left(\gamma(\xi_k^w(\omega) - \xi_m^w(\omega))\right),\ k \in R_w, w \in W \tag{9}$$

where $\gamma > 0$ is a regret aversion parameter. When γ increases, regret becomes more and more important.

Similarly, assuming that the perception error $\zeta_{w,k}^{\mathrm{BR}}$ are identically and independently Gumbel distributed random variables with mean zero, then the probability that a bounded rational traveler chooses the route k between OD pair w can be described as follows:

$$p_{w,k}^{\mathrm{BR}} = \frac{\exp\left(-\theta^{\mathrm{BR}}u_{w,k}^{\mathrm{BR}}\right)}{\sum_{r\in R_w}\exp\left(-\theta^{\mathrm{BR}}u_{w,r}^{\mathrm{BR}}\right)},\ k \in R_w, w \in W \tag{10}$$

where $\theta^{\mathrm{BR}} > 0$ is the perception error parameter of the completely rational travelers which is used to measure the degrees of travelers' perception errors.

1.4 *Stochastic mixed user equilibrium model*

It is assumed that there are two kinds of travelers in the road network, completely rational travelers and bounded rational travelers, respectively. The completely rational travelers use the perceived travel time budget as their travel disutility and the bounded rational traveler use the perceived regret value as their travel disutility. In the process of routes selection, two kinds of travelers try to find the routes with the minimum travel disutility. The network is called to achieve the stochastic mixed user equilibrium state when each type of travelers can not decrease their travel disutility by unilaterally changing the routes.

According to the principle of stochastic user equilibrium (Sheffi, 1985; Huang, 1994), the network equilibrium condition can be expressed as follows:

$$f_{w,k}^{\mathrm{CR}} = q_w^{\mathrm{CR}} p_{w,k}^{\mathrm{CR}},\ k \in R_w, w \in W \tag{11}$$

$$f_{w,k}^{\mathrm{BR}} = q_w^{\mathrm{BR}} p_{w,k}^{\mathrm{BR}},\ k \in R_w, w \in W \tag{12}$$

$p_{w,k}^{\mathrm{CR}}$ and $p_{w,k}^{\mathrm{BR}}$ in formulas (11) and (12) are determined by formulas (7) and (10) respectively, and the following conditions of flow conservation constraint are required:

$$q_w = q_w^{\mathrm{CR}} + q_w^{\mathrm{BR}} = \sum_{k\in R_w} f_{w,k}^{\mathrm{CR}} + \sum_{k\in R_w} f_{w,k}^{\mathrm{BR}},\ w \in W \tag{13}$$

$$f_{w,k} = f_{w,k}^{\mathrm{CR}} + f_{w,k}^{\mathrm{BR}},\ k \in R_w, w \in W \tag{14}$$

$$f_{w,k}^{\mathrm{CR}} \geq 0, k \in R_w, w \in W \tag{15}$$

$$f_{w,k}^{\mathrm{BR}} \geq 0, k \in R_w, w \in W \tag{16}$$

$$v_a^{\mathrm{CR}} = \sum_{w\in W}\sum_{k\in R_w} f_{w,k}^{\mathrm{CR}} \delta_{a,k}^{w},\ a \in A \tag{17}$$

$$v_a^{\mathrm{BR}} = \sum_{w\in W}\sum_{k\in R_w} f_{w,k}^{\mathrm{BR}} \delta_{a,k}^{w},\ a \in A \tag{18}$$

$$v_a = v_a^{\mathrm{CR}} + v_a^{\mathrm{BR}},\ a \in A \tag{19}$$

where $q_w^{\mathrm{CR}}, q_w^{\mathrm{BR}}$ and q_w respectively denote the completely rational travelers demand, the bounded rational travelers demand and the total demand between OD pair w; $f_{w,k}^{\mathrm{CR}}$, $f_{w,k}^{\mathrm{BR}}$ and $f_{w,k}$ respectively represent the completely rational travelers flow, the bounded rational travelers flow and total flow on route k between OD pair w; v_a^{CR}, v_a^{BR} and v_a respectively represent the completely rational traveler flow, the bounded rational travelers flow and total flow on the link a.

The equilibrium conditions of (11) and (12) can be described by the following equivalent variational inequality (VI) model.

Find $f_{w,k}^{\mathrm{CR}*}, f_{w,k}^{\mathrm{BR}*} \in \Omega$, making it satisfy the condition:

$$\sum_{w\in W}\sum_{k\in R_w}\left(\xi_k^w(\omega) + \frac{1}{\theta^{\mathrm{CR}}}\ln\frac{f_{w,k}^{\mathrm{CR}*}}{q_w^{\mathrm{CR}}}\right)\left(f_{w,k}^{\mathrm{CR}} - f_{w,k}^{\mathrm{CR}*}\right) + \sum_{w\in W}\sum_{k\in R_w}\left(u_{w,k}^{\mathrm{BR}} + \frac{1}{\theta^{\mathrm{BR}}}\ln\frac{f_{w,k}^{\mathrm{BR}*}}{q_w^{\mathrm{BR}}}\right)\left(f_{w,k}^{\mathrm{BR}} - f_{w,k}^{\mathrm{BR}*}\right) \geq 0,$$
$$\forall f_{w,k}^{\mathrm{CR}}, f_{w,k}^{\mathrm{BR}} \in \Omega \tag{20}$$

where the superscript "*" is used to designate the solution of the VI problem; Ω is the feasible set for route flows satisfying the constraint condition formulated as formulas (13)–(16).

Let $\mathbf{f}^{\mathrm{CR}}$ and $\mathbf{f}^{\mathrm{BR}}$ denote the column vectors composed of $\left\{f_{w,k}^{\mathrm{CR}}, k \in R_w, w \in W\right\}$ and $\left\{f_{w,k}^{\mathrm{BR}}, k \in R_w, w \in W\right\}$ respectively, $\mathbf{v}^{\mathrm{CR}}(\mathbf{f})$ and $\mathbf{v}^{\mathrm{BR}}(\mathbf{f})$ denote the column vectors composed of $\left\{\xi_k^w(\omega) + \frac{1}{\theta^{\mathrm{CR}}}\ln\frac{f_{w,k}^{\mathrm{CR}}}{q_w^{\mathrm{CR}}}, k \in R_w, w \in W\right\}$ and $\left\{u_{w,k}^{\mathrm{BR}} + \frac{1}{\theta^{\mathrm{BR}}}\ln\frac{f_{w,k}^{\mathrm{BR}}}{q_w^{\mathrm{BR}}}, k \in R_w, w \in W\right\}$ respectively. Because $\mathbf{v}^{\mathrm{CR}}(\mathbf{f})$ and $\mathbf{v}^{\mathrm{BR}}(\mathbf{f})$ are continuous with respect to $\mathbf{f}^{\mathrm{CR}}$ and $\mathbf{f}^{\mathrm{BR}}$ respectively, and the feasible set Ω is a bounded closed convex set, there exists at least one solution of VI problem expressed by formula (20) according to the variational inequality theorem (Facchinei and Pang, 2003).

2 VULNERABILITY IDENTIFICATION MODEL OF ROAD NETWORK

The identification of key links and their criticality are important question in the road network vulnerability evaluation. The key links refers to the links that the failure will result a significant impact on the network vulnerability. In the literature, various vulnerability indices have been proposed to evaluate the consequences of link closures. For example, Jenelius et al. (2006) used the increase of the generalized cost, weighted by the demand, as a vulnerability measure to a link closure. Chen et al. (2007) proposed the utility-based accessibility index to take account of travelers' behavioral responses to the link closure. In this paper, the road network vulnerability is evaluated by the change of road network accessibility, and then identify the key links.

Accessibility can be defined as the convenience for travelers to arrive at a destination from a origin to a destination by the certain way in the certain period of time (Taylor et al., 2006, Taylor, 2008). The accessibility index of a single OD pair can be defined as follows:

$$AC_w = \frac{\sum_{w\in W}\left(q_w^{\mathrm{CR}} + q_w^{\mathrm{BR}}\right)}{\sum_{w\in W}\sum_{k\in R_w}\left(f_{w,k}^{\mathrm{CR}} + f_{w,k}^{\mathrm{BR}}\right)\xi_k^w(\omega)},\ w \in W \tag{21}$$

where AC_w expresses the accessibility between the OD pair w.

As shown in the formula (21), it can be seen that when the travel time of a unit OD travel demand is higher, the accessibility index of the OD pair is lower, which indicates that the traveling relative convenience is lower.

According to the formula (21), the accessibility index of a road network can be defined as follows:

$$AC(G)=\frac{\sum_{w\in W}\left(q_w^{\mathrm{CR}}+q_w^{\mathrm{BR}}\right)AC_w}{\sum_{w\in W}\left(q_w^{\mathrm{CR}}+q_w^{\mathrm{BR}}\right)} \tag{22}$$

where $AC(G)$ express the accessibility of road network G.

In this paper, the road network vulnerability is evaluated by the relative change of road network accessibility before and after link failures. The road network vulnerability index under the failure of link a is defined as follows:

$$VUL_a=\frac{AC_0(G)-AC_a(G)}{AC_0(G)},a\in A \tag{23}$$

where $AC_0(G)$ is the road network accessibility under normal condition; $AC_a(G)$ indicates the road network accessibility under the failure of road link a (removing road link a from road network). Obviously, it reflects the change of road network accessibility caused by the failure of link a.

The failure of single link is the lightest case of link failures in a road network. It is also the basis for studying the multi-link failures. For simplicity, this paper selects the situation of single link failure to identify the road network vulnerability.

Based on the above analysis, the specific vulnerability evaluation scheme is given below:

Step 1: Calculate road network accessibility $AC_0(G)$ under normal condition. Method of successive average (MSA) algorithm is used to calculate the equilibrium route flows and every route's TTB under normal conditions. Subsequently, the road network accessibility under normal conditions is calculated according to formula (21) and formula (22).

Step 2: Remove each link from the road network in turn, and the road network accessibility $AC_a(G)$ after the removal of link a is calculated according to the formulas (21) and (22).

Step 3: Calculate the road network vulnerability VUL_a. According to $AC_0(G)$ and $AC_a(G)$ obtained from Step 1 and Step 2, the road network vulnerability index VUL_a after the failure of link a can be calculated in turn by formula (23). If the removal of link a would result the road network is no longer connected directly, then the link a is immediately identified as the most critical section, making it $VUL_a=\infty$.

Step 4: Identify the critical links. Sort the VUL_a in descending order, and $rank_a$ express the order value of VUL_a (namely critical degree), and the key links are selected from the first N links with the minimum values of the $rank_a$, where N is the number of key links set in advance.

Where the steps of MSA algorithm are as following:

Step 1: Initialization. Set error parameter ε = 0.01 and iteration counter n = 1; initialize the route flows. The reasonable initial route flows of the completely rational travelers are set as $\mathbf{f}^{\mathrm{CR}(n)}=\left\{f_{w,k}^{\mathrm{CR}(n)},k\in R_w,w\in W\right\}$, and the initial route flows of the bounded rational travelers are set as $\mathbf{f}^{\mathrm{BR}(n)}=\left\{f_{w,k}^{\mathrm{BR}(n)},k\in R_w,w\in W\right\}$.

Step 2: Based on the current path flow $\mathbf{f}^{\mathrm{CR}(n)}$ and $\mathbf{f}^{\mathrm{BR}(n)}$, the vector of TTB $\xi^{(n)}=\left\{\xi_k^{w(n)},k\in R_w,w\in W\right\}$ and the vector of mean regret value $\mathbf{u}^{\mathrm{BR}(n)}=\left\{u_{w,k}^{\mathrm{BR}(n)},k\in R_w,w\in W\right\}$ are calculated respectively.

Step 3: Seek the iterative direction of route flows for the completely rational travelers $\mathbf{g}^{\mathrm{CR}(n)}$ and that of the bounded rational travelers $\mathbf{g}^{\mathrm{BR}(n)}$. Let $\mathbf{g}^{\mathrm{CR}(n)}=\mathbf{f}^{\mathrm{CR}(n)'}-\mathbf{f}^{\mathrm{CR}(n)}$, $\mathbf{g}^{\mathrm{BR}(n)}=\mathbf{f}^{\mathrm{BR}(n)'}-\mathbf{f}^{\mathrm{BR}(n)}$. where $\mathbf{f}^{\mathrm{CR}(n)'}=\left\{f_{w,k}^{\mathrm{CR}(n)'},k\in R_w,w\in W\right\}$, $f_{w,k}^{\mathrm{CR}(n)'}=q_w^{\mathrm{CR}}\exp\left(-\theta^{\mathrm{CR}}\xi_{w,k}^{(n)}(\omega)\right)\Big/\sum_{r\in R_w}\exp\left(-\theta^{\mathrm{CR}}\xi_{w,r}^{(n)}(\omega)\right),k\in R_w,$ $w\in W;\mathbf{f}^{\mathrm{BR}(n)'}=\left\{f_{w,k}^{\mathrm{BR}(n)'},k\in R_w,w\in W\right\}$, $f_{w,k}^{\mathrm{BR}(n)'}=q_w^{\mathrm{BR}}\frac{\exp\left(-\theta^{\mathrm{BR}}u_{w,k}^{(n)}(\omega)\right)}{\sum_{r\in R_w}\exp\left(-\theta^{\mathrm{BR}}u_{w,r}^{(n)}(\omega)\right)},k\in R_w,w\in W.$

Step 4: Update flow. Set $\mathbf{f}^{\mathrm{CR}(n+1)}=\mathbf{f}^{\mathrm{CR}(n)}+\frac{1}{n}\mathbf{g}^{\mathrm{CR}(n)}$ and $\mathbf{f}^{\mathrm{BR}(n+1)}=\mathbf{f}^{\mathrm{BR}(n)}+\frac{1}{n}\mathbf{g}^{\mathrm{BR}(n)}$.

Step 5: Check the convergence. If $\sum_{w\in W}\sum_{k\in R_w}\left|\frac{f_{w,k}^{\mathrm{CR}(n)}-f_{w,k}^{\mathrm{CR}(n-1)}}{f_{w,k}^{\mathrm{CR}(n)}}\right|+\sum_{w\in W}\sum_{k\in R_w}\left|\frac{f_{w,k}^{\mathrm{BR}(n)}-f_{w,k}^{\mathrm{BR}(n-1)}}{f_{w,k}^{\mathrm{BR}(n)}}\right|<\varepsilon$, then stop iteration, otherwise, set $n = n + 1$, turn to Step 2.

3 NUMERICAL STUDIES

In this section, the Nguyen and Dupuis network (Nguyen & Dupuis,1984) shown in Figure 1 is provided to demonstrate the proposed model, which consists of 13 nodes, 19 links, 25 routes, and 4 OD pairs. The free-flow travel time and design capacity for each link are shown in Table 1.

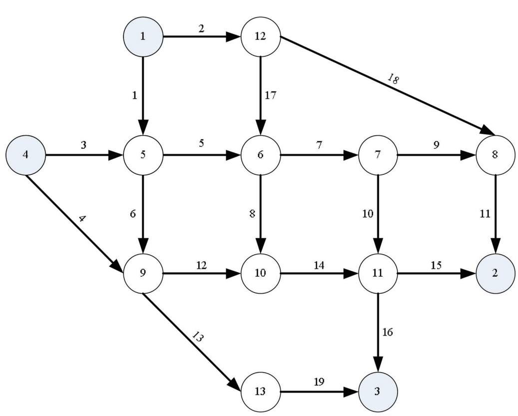

Figure 1. Nguyen and Dupuis network.

Table 1. Link characteristics.

Link	Free-flow travel time/min	Design capacity/pcu.h^{-1}
1	12	2500
2	12	2500
3	12	2500
4	24	2500
5	12	2500
6	12	2500
7	12	2500
8	12	2500
9	12	2500
10	12	2500
11	12	2500
12	12	2500
13	24	2500
14	12	2500
15	12	2500
16	12	2500
17	12	2500
18	36	1500
19	12	2500

For illustration purpose, the OD demand for each OD pair is described as follows:

$$q_w = \mu q_w^0 = \mu\left(q_w^{\mathrm{CR},0} + q_w^{\mathrm{BR},0}\right) = \mu\left(\eta q_w^0 + (1-\eta)q_w^0\right)\ w \in W \tag{24}$$

where q_w^0 represents the potentially maximum demand of OD pair w; μ is the multiplier of OD demand; η represents the proportion of completely rational travelers.

Other relevant parameters are set as: the parameters of BPR function in formula (1) are $\beta = 0.15$, $n = 4$; the variance coefficient $\rho_a = 0.5$, $a \in A$; the reliability parameter $\omega = 0.90$; the regret aversion parameter $\gamma = 0.15$; the perception error parameters $\theta^{\mathrm{CR}} = 1.0$ and $\theta^{\mathrm{BR}} = 0.2$ respectively; the maximum OD demands $q_{12}^0 = 5000$ (pcu·h^{-1}), $q_{13}^0 = 4000$ (pcu·h^{-1}), $q_{42}^0 = 4000$ (pcu·h^{-1}), and $q_{43}^0 = 3000$ (pcu·h^{-1}), respectively; the multiplier of OD demand $\mu = 0.5$; the proportion of completely rational travelers $\eta = 0.5$; the number of key links $N = 6$.

Table 2 shows the results of vulnerability evaluation for the 6 most critical links in the Nguyen and Dupuis network. It can be seen that the closures of these critical links decrease the network accessibility by 2% or more. The worst case is the closure of link 2, which leads to a 3.11% decrease in the network accessibility.

To test the effects of congestion level on network vulnerability, the consequences of link closures under various demand multipliers μ values are depicted in Table 3. From the table, it can be seen that as increases in demand level, the network vulnerability under link closures tends to go up. For example, when demand multiplier $\mu = 0.05$, the vulnerability index VUL_a is equal to 0.34% after the link 2 closure. However, when demand multiplier $\mu = 0.99$, the vulnerability index VUL_a rises to 5.45% under the link 2 closure. The results are not surprise because if the network is more congested, owing to closure, less spare capacity is available to absorb the rerouted traffic. In addition, From Table 3, it can be found that the vulnerability rankings may be varied owing to the congestion level. For instance, link 15 is ranked at $Rank_a = 5$ when $\mu = 0.05$. However, this link was ranked at $Rank_a = 6$ when $\mu = 0.49$, and $Rank_a = 7$ when $\mu = 0.99$.

In order to illustrate the effects of travelers' perception errors on network vulnerability in terms of different values of θ^{CR} and θ^{BR}, the results of link closures arising from various values of θ^{CR} and θ^{BR} are shown in Table 4. It should be pointed out

Table 2. The results of network vulnerability evaluation.

$Rank_a$	Link	VUL_a
1	2	3.11%
2	11	2.52%
3	14	2.44%
4	13	2.31%
5	19	2.31%
6	15	2.26%

Table 3. The effects of the congestion levels on network vulnerability.

	$\mu = 0.05$		$\mu = 0.49$		$\mu = 0.99$	
$Rank_a$	Link	VUL_a	Link	VUL_a	Link	VUL_a
1	2	0.34%	2	3.06%	2	5.45%
2	14	0.30%	11	2.48%	11	4.43%
3	7	0.29%	14	2.40%	14	4.26%
4	11	0.26%	13	2.27%	13	4.12%
5	15	0.26%	19	2.27%	19	4.12%
6	13	0.23%	15	2.22%	7	3.77%
7	19	0.23%	7	2.11%	15	3.76%
8	1	0.19%	1	1.76%	1	3.05%
9	3	0.17%	16	1.60%	16	2.83%
10	6	0.15%	3	1.57%	3	2.73%
11	4	0.15%	4	1.33%	4	2.43%
12	5	0.13%	5	1.26%	5	2.28%
13	16	0.12%	6	1.05%	6	1.83%
14	12	0.12%	18	0.92%	18	1.67%
15	17	0.11%	12	0.86%	12	1.50%
16	9	0.09%	17	0.80%	17	1.46%
17	18	0.09%	9	0.57%	9	1.02%
18	10	0.05%	10	0.43%	10	0.82%
19	8	–0.01%	8	0.27%	8	0.58%

that a higher θ^{CR} or θ^{BR} means smaller perception errors and vice versa. It can be observed from the table that travelers' perception errors can result in some impact on network vulnerability evaluation. Specifically, as change in values of θ^{CR} and θ^{BR}, the vulnerability rankings and the vulnerability index may be different accordingly. For example, link 13 is ranked at $Rank_a = 4$ when $\theta^{CR} = 5.0$ and $\theta^{BR} = 1.0$ whereas this link was ranked at $Rank_a = 7$ when $\theta^{CR} = 0.1$ and $\theta^{BR} = 0.02$.

Table 5 shows the impact of the regret aversion parameter on network vulnerability in terms of different values of γ. It can be found from Table 5 that the parameter γ can result in certain impact on network vulnerability evaluation. The vulnerability rankings may be varied due to the different γ values. For example, link 15 is ranked at $Rank_a = 4$ when $\gamma = 0.05$ whereas this link was ranked at $Rank_a = 6$ when $\gamma = 0.30$.

Table 6 depicts the impact of travelers' route choice decision criteria on network vulnerability according to different values of η. As shown in Table 6, with the variation on type structure of travelers, the vulnerability rankings and the vulnerability indices may change accordingly. For instance, when $\eta = 0.99$, which means that the completely rational travelers are dominant in the network, link 15 is ranked at $Rank_a = 6$ and the vulnerability index $VUL_a = 2.19\%$. However, this link was ranked at $Rank_a = 8$ and the vulnerability index $VUL_a = 2.06\%$ when $\eta = 0.01$.

The above analysis shows that ignoring the effects of the congestion levels of road network, travelers' perception errors and regret aversion

Table 4. The effects of the perception errors on network vulnerability.

	$\theta^{CR} = 5.0$, $\theta^{BR} = 1.0$		$\theta^{CR} = 1.0$, $\theta^{BR} = 0.2$		$\theta^{CR} = 0.1$, $\theta^{BR} = 0.02$	
$Rank_a$	Link	VUL_a	Link	VUL_a	Link	VUL_a
1	2	3.14%	2	3.11%	2	2.90%
2	11	2.55%	11	2.52%	14	2.55%
3	14	2.39%	14	2.44%	7	2.54%
4	13	2.33%	13	2.31%	11	2.28%
5	19	2.33%	19	2.31%	15	2.09%
6	15	2.19%	15	2.26%	19	2.02%
7	7	2.14%	7	2.15%	13	2.02%
8	1	1.74%	1	1.79%	1	1.63%
9	16	1.67%	16	1.63%	3	1.43%
10	3	1.56%	3	1.59%	6	1.33%
11	4	1.37%	4	1.35%	4	1.29%
12	5	1.26%	5	1.28%	5	1.10%
13	6	0.96%	6	1.07%	12	1.04%
14	18	0.94%	18	0.93%	16	1.03%
15	17	0.83%	12	0.88%	17	1.01%
16	12	0.81%	17	0.81%	9	0.82%
17	9	0.57%	9	0.58%	18	0.76%
18	10	0.46%	10	0.44%	10	0.40%
19	8	0.35%	8	0.28%	8	–0.08%

Table 5. The effects of the regret aversion parameter on network vulnerability.

	$\gamma = 0.05$		$\gamma = 0.15$		$\gamma = 0.30$	
$Rank_a$	Link	VUL_a	Link	VUL_a	Link	VUL_a
1	2	3.12%	2	3.11%	2	3.10%
2	11	2.52%	11	2.52%	11	2.51%
3	14	2.45%	14	2.44%	14	2.43%
4	15	2.31%	13	2.31%	13	2.34%
5	13	2.29%	19	2.31%	19	2.34%
6	19	2.29%	15	2.26%	15	2.20%
7	7	2.17%	7	2.15%	7	2.13%
8	1	1.83%	1	1.79%	1	1.74%
9	16	1.60%	16	1.63%	16	1.67%
10	3	1.58%	3	1.59%	3	1.62%
11	4	1.33%	4	1.35%	4	1.38%
12	5	1.30%	5	1.28%	5	1.27%
13	6	1.09%	6	1.07%	6	1.04%
14	18	0.94%	18	0.93%	18	0.93%
15	12	0.89%	12	0.88%	12	0.87%
16	17	0.82%	17	0.81%	17	0.80%
17	9	0.59%	9	0.58%	9	0.57%
18	10	0.45%	10	0.44%	10	0.44%
19	8	0.27%	8	0.28%	8	0.29%

Table 6. The effects of the travelers' route choice decision criteria on network vulnerability.

	$\eta = 0.01$		$\eta = 0.49$		$\eta = 0.99$	
$Rank_a$	Link	VUL_a	Link	VUL_a	Link	VUL_a
1	2	2.93%	2	3.11%	2	3.13%
2	14	2.72%	11	2.52%	11	2.53%
3	13	2.47%	14	2.45%	14	2.34%
4	19	2.47%	13	2.31%	19	2.32%
5	11	2.37%	19	2.31%	13	2.32%
6	7	2.27%	15	2.26%	15	2.19%
7	3	2.07%	7	2.15%	7	2.12%
8	15	2.06%	1	1.79%	1	1.72%
9	16	1.71%	16	1.63%	16	1.59%
10	1	1.63%	3	1.60%	3	1.45%
11	4	1.63%	4	1.35%	4	1.35%
12	6	1.45%	5	1.28%	5	1.23%
13	5	1.31%	6	1.08%	18	0.94%
14	12	1.13%	18	0.93%	6	0.90%
15	18	0.83%	12	0.88%	17	0.82%
16	17	0.79%	17	0.81%	12	0.79%
17	9	0.64%	9	0.58%	9	0.57%
18	10	0.43%	10	0.44%	10	0.44%
19	8	0.15%	8	0.28%	8	0.31%

degrees, as well as travelers' route choice decision criteria could misjudge the consequences of link closures and misidentify the most critical links.

4 CONCLUSIONS AND FUTURE RESEARCH

This study proposed a method of road network vulnerability identification taking into account travelers' risk aversion and bounded rationality simultaneously under travel time variations subjected to link failures. It is assumed that there are two kinds of travelers in the road network, completely rational travelers and bounded rational travelers, respectively. The completely rational travelers use the perceived travel time budget as their travel disutility while the bounded rational traveler use the perceived regret value as their travel disutility. According to the travelers' postulated route choice decision criteria, a mixed stochastic traffic assignment model formulated as variational inequality is constructed, a new vulnerability index of road network based on accessibility is defined, and a vulnerability identification model is built, and corresponding heuristic algorithm is also proposed. Numerical examples on the Nguyen and Dupuis network made apparent that the consequences of link closures could be misjudged and the vulnerability rankings could be misidentified, if ignoring the effects of the congestion levels of road network, travelers' perception errors and regret aversion degrees, as well as travelers' route choice decision criteria. Therefore, it is necessary to capture travelers' behavior characteristics for the vulnerability analysis.

The proposed vulnerability analysis only considers the scenarios of single link closure, and the consideration of multiple link closures is an important extension. Another valuable extension of this study is to take into account day-to-day adjustment processes for modeling travelers' behavioral responses to link closures.

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