Predictive strength of ready-mixed concrete: exemplified using data from the Norwegian market

Running head/short title: Predictive strength of ready-mixed concrete

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ABSTRACT

A hierarchical model for the variability of material properties in ready-mixed concrete is formulated. The model distinguishes between variation on the batch, recipe, plant, producer, durability class, strength class and regional standard level. By considering Bayesian inference and maximum likelihood estimators, the contributions from the different hierarchical levels to the variability can be estimated. The methodology is demonstrated by considering more than 14000 compressive strength recordings from Norwegian ready-mixed concrete plants. The results suggest that the compressive cube strength of lab-cured specimens can be represented by a log-normally distributed variable with mean $1.28 f_{\mathrm{ck,cube}}$ and coefficient of variation $V_{\mathrm{c,cube}} = 0.13$. Prior parameters for Bayesian updating are given for a range of strength and durability classes. The application of the results is demonstrated in two examples. Since the durability class gives a required maximum water-binder ratio, and the strength of the concrete is governed by the water-binder ratio, the durability class introduces a strength potential if the concrete is subject to strict durability requirements and low strength requirements. It is suggested that the designer should specify a strength class that utilizes this strength potential, and it is expected that a closer collaboration between the designer, contractor and producer will result in improved concrete specifications.

Keywords: Concrete compressive strength, hierarchical model for variability, Bayesian inference, informative prior distribution, maximum likelihood estimators, code calibration, structural reliability.

1. INTRODUCTION

Selecting the concrete type is an important decision in design of concrete structures. Following the Eurocodes [1-3] the concrete type is defined by requirements related to strength and durability. In Norway, this is implemented by assigning a strength and durability class, where the strength class is denoted by the letter B followed by the characteristic compressive strength of a lab-cured cylinder, see Tab. 1, and the durability class is denoted by the letter M or the letters MF followed by a number indicating the maximum effective water-binder ratio. The characteristic compressive strength is defined as the lower 5%-fractile of the strength. EN 206 gives the following durability classes for concrete in Norway: M90, M60, M45, MF45, M40 and MF40. In addition, The Norwegian Public Roads Administration introduces additional durability classes, e.g. SV30 and SV40, for infrastructure projects [4].

[TABLE 1]

Based on the strength and durability class, workability requirements and other requirements related to e.g. appearance or carbon footprint, the producer designs a recipe. For a given strength and durability class produced at a specific plant, the main differences between different recipes are related to the maximum aggregate size, the fractions of the different aggregate sizes, the cement type, the amount of supplementary cementitious materials and the amount of entrained air. Conformity control is performed based on the strength at 28 days. Cubes with sides 100 mm are used for conformity control in Norway due to their easier handling, preparation and testing compared to cylinders.

Mirza et al. [5] presents an extensive literature review, and suggest that the main sources for variation of the compressive strength are the variation in properties and proportion of the constituents of the concrete mix, the variations in mixing, transporting, placing and curing methods, the variations in testing procedures and variations due to concrete being in a structure rather than in control specimens. Several other sources report similar findings [6-15], also addressing topics like size and shape of control specimen, casting direction, workmanship and type of structural component and location within the component.

Rackwitz [16] suggests methods for predicting the strength of concrete using Bayesian inference, and estimate prior data based on a collection of data from Southern Germany. The prior data were later reworked and included in the *JCSS Probabilistic Model Code* [17].

The effect of compliance criteria are studied taking into account autocorrelation [18], different types of criteria [19] and the concept of concrete families [20,21]. Later, the effect of compliance control and strength estimation [22] on structural reliability are addressed [23-25]. Foster et al. [26] report from a study on a collection of strength recordings from Australia, and it can be shown that the 28-day compressive strength of lab-cured cylinders can be represented by a normally distributed variable with mean $\mu_{\mathrm{c}} = 1.21 f_{\mathrm{ck}}$ and a coefficient of variation of $V_{\mathrm{c}} = 0.12$. Correlation with other material parameters for concrete can be found elsewhere [27-29], and the relation between cylinder and cube strength is discussed in several contributions [30-35].

In Eurocode 2 [2,36], and similarly in fib *Model Code for Concrete Structures 2010* [37], the concrete strength is assumed represented by a log-normally distributed variable. The variability of the concrete strength is reflected in the partial factor

$$\gamma_{\mathrm{c}} = 1.15\exp\left(\alpha_{\mathrm{R}}\beta\sqrt{V_{\theta}^2 + V_{\mathrm{G}}^2 + V_{\mathrm{M}}^2} - 1.645V_{\mathrm{M}}\right) \cong 1.50, \quad (1)$$

where $\alpha_{\mathrm{R}} = 0.8$ is the sensitivity factor for resistance, $\beta = 3.8$ is the target reliability index for a 50-year reference period, $V_{\theta} = 0.05$ is the modelling uncertainty, $V_{\mathrm{G}} = 0.05$ is the geometrical uncertainty, $V_{\mathrm{M}} = 0.15$ is the material uncertainty [36] including the contributions discussed by Mirza et al. [5], and the factor 1.15 reflects the ratio of the lab-strength to the strength obtained in a structure. Eurocode 2 also suggests the relation $f_{\mathrm{cm}} = f_{\mathrm{ck}} + 8$ MPa between the mean and characteristic strength, assuming a standard deviation of approximately 5 MPa [38].

In the present work more than 14000 compressive strength recordings from Norwegian ready-mixed concrete plants were studied using a hierarchical model for the variability of material properties as suggested in the literature [16,17]. It is emphasized that the scope of the present work was to estimate the variation resulting from what the designer can control. The effects of the choices made by the contractor and the producer were thus not considered.

This work only provides statistical evidence for the lab-strength of cubes of ready-mixed concrete in Norway, such that the uncertain relation between the lab-strength of cubes and the strength obtained in a structure should be included if the present results are to be applied in e.g. a reliability assessment. For completeness, full details of the statistical analysis methods will be given along with a detailed summary of the results. This transparency is important for possible future extensions with additional data and to facilitate for correct application of the results.

2. HIERARCHICAL MODEL FOR THE VARIABILITY OF MATERIAL PROPERTIES IN CONCRETE

Fig. 1 shows how the hierarchical model for the variability of material properties in concrete was formulated in the present work. During concrete production, the producer controls for compliance using standardized test specimens. The variation between test specimens from one batch of concrete represents the *within-batch* variation. The variation *within* and *between* samples of observations on one level contributes to the variation *within* the next level, see Tab. 2. Hence, the variation within and between batches produced according to one *recipe* contribute to the *within-recipe* variation. Each batch is produced according to a given recipe, at a *concrete plant*, by a *concrete producer* in order to comply with a given *durability class* and *strength class*. The variation between plants and producers can be due to different availability and use of raw materials, but also due to cultural differences and the quality control regime at the respective plant. The concrete is produced within a region having a *supply controlled by a regional standard*, which is part of the *gross supply*. Since the designer specifies a strength and durability class, these levels are the entry points of information from the design process.

[FIGURE 1]

[TABLE 2]

3. METHODS FOR STATISTICAL ANALYSIS

3.1 Sample statistics for the hierarchical model

Assuming independent and interchangeable observations from a homogeneous population, unbiased estimators for the mean and variance of sample *i* with n_i observations are

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{i,j} \tag{2}$$

and

$$s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(y_{i,j} - \bar{y}_i\right)^2 \quad , \tag{3}$$

where $y_{i,j}$ is observation j in sample i. For example, i can refer to recipe i for obtaining a combination of strength and durability, and *j* can refer to a strength recording from batch *j* produced with that recipe. From this, one can derive the sample mean

$$\bar{y}_{\text{tot}} = \frac{1}{n_{\text{tot}}} \sum_{i=1}^{m} n_i \bar{y}_i \tag{4}$$

and variance

$$s_{\text{tot}}^2 = \frac{\sum_{i=1}^{m}([n_i - 1]s_i^2)}{n_{\text{tot}} - 1} + \frac{\sum_{i=1}^{m}(n_i \bar{y}_i^2) - n_{\text{tot}} \bar{y}_{\text{tot}}^2}{n_{\text{tot}} - 1} = s_{\text{tot,w}}^2 + s_{\text{tot,b}}^2 \tag{5}$$

of a group of m samples respectively, where $n_{\text{tot}} = \sum_{i=1}^{m} n_i$ is the total number of observations. Here, $\bar{y}_{\text{tot}}$, s_{tot}^2 and n_{tot} could include all the strength recordings for all recipes for a combination of strength and durability at a specific plant. For example the sample mean and variance for a combination of strength and durability class at a specific plant can thus be calculated directly by considering the sample mean, sample variance and number of observations for all the recipes obtaining the specified combination of strength and durability class at that plant, as indicated in Fig 1. Eq. (5) expresses the variance of the group of samples as the sum of the variance *within* and *between* the samples.

3.2 Bayesian inference

The derivations in this section are valid for normally distributed random variables, and are adapted from the literature [e.g. 16,39,40]. Following recommendations in the literature, the compressive cube strength of concrete, $f_{\text{c,cube}}$, is represented by a log-normally distributed variable [16,41], meaning that the natural logarithm of the cube strength is normally distributed. In the following, the variable y thus represents the natural logarithm of the cube strength, $y = \ln f_{\text{c,cube}}$.

Following *Bayes' theorem*, and assuming that y is normally distributed with mean μ and variance σ^2, the *joint posterior distribution* of the parameters μ and σ^2 given a set of n observations collected in the vector $\boldsymbol{y}$ is written as

$$f(\mu, \sigma^2 | \boldsymbol{y}) = \frac{f(\mu, \sigma^2) L(\boldsymbol{y} | \mu, \sigma^2)}{\iint_{-\infty}^{\infty} f(\mu, \sigma^2) L(\boldsymbol{y} | \mu, \sigma^2) \mathrm{d}\mu \mathrm{d}\sigma^2} \quad , \tag{6}$$

where $f(\mu, \sigma^2)$ is the prior distribution of the parameters and $L(\boldsymbol{y} | \mu, \sigma^2)$ is the likelihood of the observations. The likelihood is established by considering the distribution of y:

$$L(\boldsymbol{y} | \mu, \sigma^2) = \prod_{i=1}^{n} \mathrm{N}(y_i | \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{1}{2\sigma^2}[(n-1)s^2 + n(\bar{y} - \mu)^2]\right) \quad , \tag{7}$$

where

$$\mathrm{N}(y_i | \mu, \sigma^2) = \sqrt{\frac{1}{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{[y_i - \mu]^2}{\sigma^2}\right) \tag{8}$$

is the normal distribution. If there exists no prior information about y, the proportionality

$$f(\mu,\sigma^2) \propto \frac{1}{\sigma^2} \tag{9}$$

can be used as a non-informative prior distribution for μ and σ^2. By combining Eqs. (7) and (9) with Eq. (6), the joint posterior distribution of μ and σ^2 is

$$f(\mu,\sigma^2|\boldsymbol{y}) = \mathrm{N}\left(\mu\middle|\bar{y},{\sigma^2}/{n}\right)\mathrm{Inv}\text{-}\chi^2(\sigma^2|\nu s^2,\nu) \quad , \tag{10}$$

where

$$\mathrm{Inv}\text{-}\chi^2(\sigma^2|\nu s^2,\nu) = \frac{1}{\Gamma(\nu/2)}\left(\frac{\nu s^2}{2}\right)^{\nu/2}\left(\frac{1}{\sigma^2}\right)^{(\nu/2+1)}\exp\left(-\frac{\nu s^2}{2\sigma^2}\right) \tag{11}$$

is the *scaled inverse-χ^2 distribution* with scale νs^2 and ν degrees of freedom. $\Gamma(\cdot)$ is the *Gamma-function* and $\nu = n-1$, assuming that the sample variance and mean are estimated from the same sample. From Eq. (10), the marginal posterior distribution of each parameter is found by integrating over the other, e.g. $f(\sigma^2|\boldsymbol{y}) = \int_{-\infty}^{\infty} f(\mu,\sigma^2|\boldsymbol{y})\mathrm{d}\mu$. The posterior predictive distribution of y is found from the total probability theorem

$$f(y|\boldsymbol{y}) = \iint_{-\infty}^{\infty} \mathrm{N}(y|\mu,\sigma^2) f(\mu,\sigma^2|\boldsymbol{y})\mathrm{d}\mu\mathrm{d}\sigma^2 \quad , \tag{12}$$

where the integral is over all possible values of μ and σ^2. The posterior distributions for μ, σ^2 and y, and the corresponding expected values and variances, are summarized in Tab. 3. The posterior distribution of y is given in Eq. (13), which is a *t-distribution* with location $\bar{y}$, scale $s\sqrt{\frac{\nu+2}{\nu+1}}$ and ν degrees of freedom. Eq. (14) can be used to estimate values of y with a non-exceedance probability α, where $t_{\alpha,\nu}$ is the upper α-fractile of the t-distribution with ν degrees of freedom.

[TABLE 3]

$$f(y|\boldsymbol{y}) = t\left(y|\bar{y},s^2\frac{\nu+2}{\nu+1},\nu\right) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\sqrt{\frac{1}{\nu\pi}}\sqrt{\frac{\nu+1}{s^2(\nu+2)}}\left(1+\frac{\nu+1}{\nu(\nu+2)}\left[\frac{y-\bar{y}}{s}\right]^2\right)^{-\frac{\nu+1}{2}} \tag{13}$$

$$\tilde{y} = \bar{y} + t_{\alpha,\nu}s\sqrt{\frac{\nu+2}{\nu+1}} \tag{14}$$

If prior information about y exist, a *conjugate informative prior distribution* with prior parameters $\bar{y}'$, n', s'^2 and ν' on the same form as Eq. (10) can be written as

$$f(\mu,\sigma^2) = \mathrm{N}\left(\mu\middle|\bar{y}',{\sigma^2}/{n'}\right)\mathrm{Inv}\text{-}\chi^2\left(\sigma^2|\nu' s'^2,\nu'\right) \quad . \tag{15}$$

By combining Eqs. (7), (15) and (6) it can be shown that the joint posterior distribution of μ and σ^2 is given by Eq. (10) with *updated* parameters

$$n'' = n' + n \quad , \tag{16}$$

$$\bar{y}'' = \frac{1}{n''}(n\bar{y} + n'\bar{y}') \quad , \tag{17}$$

$$\nu'' = \nu' + \nu + 1 \quad , \tag{18}$$

and

$$\nu'' s''^2 = \nu s^2 + \nu' s'^2 + n\bar{y}^2 + n'\bar{y}'^2 - n''\bar{y}''^2 = \nu s^2 + \nu' s'^2 + \frac{nn'}{n + n'}(\bar{y} - \bar{y}')^2 \quad , \tag{19}$$

and the posterior distributions, expected values and variances of μ, σ^2 and y are given in Tab. 3, inserted for the updated parameters. Note that Eqs. (17) and (19) are parallel to Eqs. (4) and (5), and that in this case Bayesian updating involves inference on two samples of observations that are combined. The prior sample is often taken as a virtual sample where the sample size represents the information content in the sample.

If prior information exists only for the variance, the conjugate informative prior distribution would take the form

$$f(\mu, \sigma^2) = \text{Inv-}\chi^2\left(\sigma^2 | \nu' s'^2, \nu'\right) \quad , \tag{20}$$

and following the same derivation as above gives the updated parameters

$$n'' = n \quad , \tag{21}$$

$$\bar{y}'' = \bar{y} \quad , \tag{22}$$

$$\nu'' = \nu' + \nu \quad , \tag{23}$$

and

$$\nu'' s''^2 = \nu s^2 + \nu' s'^2 \quad . \tag{24}$$

Since the inference is based on the log-normally distributed random variable $y = \ln f_{\text{c,cube}}$, the results from the inference should be transformed in order to find the parameters of the distribution of $f_{\text{c,cube}}$. By applying a coordinate transformation such that Eq. (8) is expressed as a function of $f_{\text{c,cube}}$, and calculating the first two moments in a regular manner, it can be shown that the mean $\mu_{\text{c,cube}}$ and coefficient of variation $V_{\text{c,cube}} = \sigma_{\text{c,cube}}/\mu_{\text{c,cube}}$ can be calculated using

$$\mu_{\text{c,cube}} = \exp\left(\text{E}[y] + \frac{1}{2}\text{VAR}[y]\right) \approx \exp(\text{E}[y]) \tag{25}$$

and

$$V_{\text{c,cube}} = \sqrt{\exp(\text{VAR}[y]) - 1} \approx \sqrt{\text{VAR}[y]} \quad , \tag{26}$$

where the errors of approximation in Eqs. (25) and (26) are less than 2% for $V_{\mathrm{c,cube}} < 0.2$.

3.3 Estimate of parameters for an informative prior distribution

Rackwitz [16] suggests *maximum likelihood estimators* (MLE) for estimating parameters for an informative prior distribution. By considering Eqs. (11) and (8) the likelihoods

$$L(\boldsymbol{y}_{\mathrm{tot}}|s_{\mathrm{MLE}}^2,\nu_{\mathrm{MLE}}) = \prod_{i=1}^{m} \text{Inv-}\chi^2(s_i^2[\boldsymbol{y}_i]|\nu_{\mathrm{MLE}}s_{\mathrm{MLE}}^2,\nu_{\mathrm{MLE}}) \tag{27}$$

and

$$L(\boldsymbol{y}_{\mathrm{tot}}|\bar{y}_{\mathrm{MLE}},n_{\mathrm{MLE}}) = \prod_{i=1}^{m} \mathrm{N}(\bar{y}_i[\boldsymbol{y}_i]|\bar{y}_{\mathrm{MLE}},s_i^2[\boldsymbol{y}_i]/n_{\mathrm{MLE}}) \tag{28}$$

are established based on m samples of observations from a concrete type, where $\boldsymbol{y}_{\mathrm{tot}}$ represents the collection of all the m samples $\boldsymbol{y}_i$, and $\bar{y}_i[\boldsymbol{y}_i]$ and $s_i^2[\boldsymbol{y}_i]$ are the sample mean and variance of sample $\boldsymbol{y}_i$. By maximizing the natural logarithms of the likelihoods, the following MLE are found, with parameters in Eq. (33).

$$s_{\mathrm{MLE}}^2 = \frac{1}{A} \tag{29}$$

$$\nu_{\mathrm{MLE}} = \frac{1}{\ln A - B - \epsilon(\nu_{\mathrm{MLE}}^{-2})} \approx \frac{1}{\ln A - B} f\left(\frac{1}{\ln A - B}\right) \approx \frac{1}{\ln A - B} \tag{30}$$

$$\bar{y}_{\mathrm{MLE}} = \frac{C}{A} \tag{31}$$

$$n_{\mathrm{MLE}} = \frac{1}{D - \frac{C^2}{A}} \tag{32}$$

$$A = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{s_i^2} \quad , \quad B = \frac{1}{m}\sum_{i=1}^{m}\ln\frac{1}{s_i^2} \quad , \quad C = \frac{1}{m}\sum_{i=1}^{m}\frac{\bar{y}_i}{s_i^2} \quad , \quad D = \frac{1}{m}\sum_{i=1}^{m}\frac{\bar{y}_i^2}{s_i^2} \tag{33}$$

The error term in Eq. (30), $\epsilon(\nu_{\mathrm{MLE}}^{-2})$, is due to truncation after the second term of $\partial \ln \Gamma(\nu/2)/\partial\nu$, and can be compensated for by multiplying with the factor $f\left(\frac{1}{\ln A - B}\right)$ given in Tab. 4. ν_{MLE} and n_{MLE} are measures of the information content in the estimated values of s_{MLE}^2 and $\bar{y}_{\mathrm{MLE}}$, often denoted the *degree of belief*. ν_{MLE} and n_{MLE} attain large values if the sample variances and means are similar.

[TABLE 4]

If the concrete type is unknown, generalized prior parameters can be useful. Caspeele & Taerwe [42] suggest a method for obtaining approximated generalized prior parameters for the variance based on prior data from samples with unknown sample sizes m_j. In the present work, the sample sizes m_j are known, and generalized prior parameters $s^2_{\mathrm{MLE,gen}}$ to $n_{\mathrm{MLE,gen}}$ can be estimated by using the parameters A_{tot} to D_{tot} according to

$$m_{\mathrm{tot}} = \sum_{j=1}^{n} m_j \quad , \quad A_{\mathrm{tot}} = \frac{1}{m_{\mathrm{tot}}} \sum_{j=1}^{m} m_j A_j \quad , \ldots, \quad D_{\mathrm{tot}} = \frac{1}{m_{\mathrm{tot}}} \sum_{j=1}^{m} m_j D_j \tag{34}$$

where the subscript j refers to either strength class, durability class or combination of strength and durability class j. From this it is clear that $s^2_{\mathrm{MLE,gen}}$ and $\nu_{\mathrm{MLE,gen}}$ will be meaningful since the variances of the different classes are expected to be comparable. In contrast, $\bar{y}_{\mathrm{MLE},j}$ are not comparable due to the different target strengths. However, if $s^2_{\mathrm{MLE,gen}}$ is estimated with a reasonable degree of belief, $\delta_{\mathrm{MLE},j} = \bar{y}_{\mathrm{MLE,j}} - \ln f_{\mathrm{ck,cube},j}$ is expected to be comparable for different classes. δ should be interpreted as the natural logarithm of the ratio between the mean strength and the target cube strength, $f_{\mathrm{ck,cube}}$, from Tab. 1. Hence, $s^2_{\mathrm{MLE,gen}}$, $\nu_{\mathrm{MLE,gen}}$, $\delta_{\mathrm{MLE,gen}}$ and $n_{\mathrm{MLE,gen}}$ are estimated by replacing $\bar{y}_{\mathrm{MLE},j}$ with $\delta_{\mathrm{MLE},j}$ in the calculation of C_{tot} in Eq. (34), and inserting Eq. (34) in Eqs. (29) to (32). If $\delta_{\mathrm{MLE,gen}}$ is estimated with a reasonable degree of belief, its value can be used to estimate the location parameter of the prior distribution for a concrete with an arbitrary target strength.

3.4 Estimate of within-batch variation

Since the data acquired in the present work only included one strength measurement per batch, no direct inference could be made about the within-batch variation. However, it can be estimated. Assume that the natural logarithm of a strength recording from batch j, y_j, can be represented by a normally distributed variable expressed as $y = y_1 + y_2$ with $y_1 \sim \mathrm{N}(\mu_{\mathrm{y}}, \sigma^2/n)$ and $y_2 \sim \mathrm{N}(0, \sigma^2)$. Here, y_1 represents the uncertain mean of the batch and y_2 represents the random fluctuation due to an assumed known within-batch variance, σ^2. This is realistic in situations where more information is available about the variance compared to the mean. The variance of y is given by

$$\mathrm{VAR}[y] = \frac{\sigma^2}{n} + \sigma^2 = \sigma^2 \frac{n+1}{n} \quad , \tag{35}$$

with n being the number of batches that are considered. An estimate of $\mathrm{VAR}[y]$ is the sample variance s_i^2 of n_i observations for a given recipe i, and an estimator for the within-batch variation of recipe i thus becomes

$$\mathrm{s}^2_{\mathrm{wb},i} = s_i^2 \frac{n_i}{n_i + 1} \quad . \tag{36}$$

This indicates that the between-batch variance dominates the estimate if the number of observations is small, and that the within-batch variance dominates if the number increases. Based on Eq. (36), the MLE from Eqs. (29) and (30) can be used to estimate the within-batch variance.

4. RESULTS AND DISCUSSION

4.1 Compressive strength recordings from the Norwegian market

Three concrete producers provided more than 14000 compressive strength recordings covering most of the Norwegian supply in the period 2013-2017, shown in Tab. 5. The six strength classes B20-B55 were represented with 20 combinations of strength and durability classes. Only concretes with aggregates consisting of at least 50% coarse aggregates with size larger than 4 mm and a maximum aggregate size larger than 16 mm were considered.

[TABLE 5]

4.2 Bayesian inference with non-informative prior

The sample mean and variance for each recipe were calculated using Eqs. (2) and (3). Eqs. (4) and (5) were further used to calculate the sample mean and variance on higher levels of the hierarchy, and the equations in Tab. 3 were used to calculate the expected value and variance of $y = \ln f_{\mathrm{c,cube}}$ on the respective level.

For brevity, detailed results from the plant level are left out of this presentation. At plant level for producer A and C the durability class was governing for the strength prediction, e.g. B35M40, B35MF40 and B35SV40 were almost equally distributed as B45M40, B45MF40 and B45SV40, respectively. The within-recipe variation dominated, but the between-recipe variation was significant in most combinations of strength and durability classes. For the plants of producer C, a significant over-strength was observed in most instances of B35 concretes. The trend where the durability class was governing for the strength prediction was weaker for the plants of producer B, and the between-recipe variation was relatively large and dominated in eight combinations of strength and durability classes.

Tab. 6 shows the results on the producer level. The predicted characteristic strength should be compared to the target cube strength from Tab. 1. Due to statistical uncertainty, i.e. a small sample resulting in $\sqrt{\mathrm{VAR}[y]} \gg s$, a low value was predicted for the characteristic strength for B55M40 for producer A. For producer A, it can be seen that the between-plant variation was small and only dominant for B20M90. Due to large standard deviations, five strength predictions were lower than the target for producer B. For B25M90 and B30M60 this was due to within-plant variation, but for the higher strength classes the between-plant variation also influenced. Due to slightly low mean values, the predictions of B45MF40 and B45MF45 were lower than the target strengths for producer C. The contribution from between-plant variation was small in all cases. Since compliance control is based on samples where observations from different recipes are combined using the concrete family concept [20,21], the sometimes low predictions of the characteristic strength do not indicate that the producers deviate from the criteria in EN 206 [3].

Tab. 7 shows the results on the level of the combination of strength and durability class. Four predictions were lower than the target strength, either due to a large standard deviation due to within-producer variation, see e.g. B25M90, slightly low mean values, see B45MF40 and B45MF45, or due to statistical uncertainty, see B55M40. The contribution from between-producer variation was low in most cases, with the largest contribution for B35M40.

Finally, Tab. 8 shows the results from the level of the strength class. Only B25 did not reach the target strength. This was due to a slightly low mean strength and a large standard deviation with a significant contribution from between-durability class variation, where B25M60 was closer to a B30. B35 also got a significant contribution from between-durability class variation, as demonstrated in Fig. 2a, but the higher mean strengths of e.g. B35M40 resulted in a high mean strength for B35. The lower contribution from between-durability class variation for B45 is demonstrated in Fig. 2b.

[FIGURE 2]

In the bottom row of Tab. 8, the natural logarithm of the target strength is subtracted from the values of $\mathrm{E}[y]$, to get the variable δ as introduced in Sec. 3.3. It is interesting to note that $\mathrm{E}[\delta] = 0.25$ is approximately what could be estimated assuming $V_{\mathrm{c,cube}} = 0.15$, i.e. $\mathrm{E}[\delta] \approx 1.645 \cdot 0.15$. The results indicated that the cube strength could be represented by a log-normally distributed variable with mean

$$\mu_{\mathrm{c,cube}} \approx \exp\left(\ln f_{\mathrm{ck,cube}} + \mathrm{E}[\delta]\right) = 1.28 f_{\mathrm{ck,cube}} \tag{37}$$

and coefficient of variation

$$V_{\mathrm{c,cube}} \approx \sqrt{\mathrm{VAR}[y]} = 0.13 \quad . \tag{38}$$

The general coefficient of variation gets a dominant contribution from within-durability class variation, but also between-durability class variation, indicated by s_{w}^2/s^2 in Tab. 8, and between-strength class variation due to different $\mathrm{E}[\delta]$ for the different classes contribute.

[TABLE 6]

[TABLE 7]

[TABLE 8]

4.3 Parameters for an informative prior distribution

Informative prior parameters were estimated with the MLE on the strength and durability class level, based on groups of samples at the plant level. The results can be used in Eqs. (16) to (19), or Eqs. (21) to (24), assuming that the observations in the sample $\boldsymbol{y}$ originate from one producer and plant, and replacing $\bar{y}'$, s', n' and ν' with $\bar{y}_{\mathrm{MLE}}$, s_{MLE}, n_{MLE} and ν_{MLE}, respectively. Note that ν_{MLE} was estimated by solving Eq. (30) numerically.

Tab. 9 shows the results of the MLE per combination of strength and durability class. $\bar{y}_i$ and s_i^2 input to Eq. (33) were the sample mean and variance of a sample containing all the observations from the recipes obtaining a specified combination of strength and durability class at plant i. m could range from 2 to 17, excluding combinations represented at only one plant. Generally the results showed higher degrees of belief, in terms of n and ν, and lower standard deviations compared to the results by Rackwitz [16] shown in Tab. 10.

[TABLE 9]

[TABLE 10]

For B25M90, the variances at the plants of producer B dominated in Sec. 4.2, but were dominated by the lower variances at the plants of producer A in the MLE. This explains the low values for s_{MLE} and ν_{MLE}. The contrasting high value of n_{MLE} was recognized in the low contribution from between-plant and -producer variation shown in Tabs. 6 and 7. ν_{MLE} of B30M60 was associated with the small variation of the variances between the plants and producers. The low values for n_{MLE} for B35M40 and B35MF40 reflected that the durability class was governing for the strength prediction for producers A and C, but less governing for producer B. The low variation in the variance between plants and the relatively low contribution from between-plant and -producer variance for B45MF40 and B45SV40, resulted in high values for ν_{MLE} and n_{MLE} respectively. The low n_{MLE} for B45SV30 was reflected in the large contribution from between-plant variance for producer B.

Tab. 11 shows the MLE per durability class, where $\bar{y}_i$ and s_i^2 are the sample mean and variance of the group of samples with the same durability class at plant i, and the estimated variances would thus include a contribution from between-strength class variance. Interesting to note from the results were the high values for the degree of belief, both with respect to the mean and the standard deviation. These results were not surprising, since the durability class gives a required maximum water-binder ratio, and the strength of the concrete is governed by the water-binder ratio.

[TABLE 11]

Tab. 12 shows the MLE per strength class, where $\bar{y}_i$ and s_i^2 are the sample mean and variance of the group of samples with the same strength class at plant i, and the estimated standard deviations would thus include a contribution from between-durability class variance.

[TABLE 12]

Taking the values of the general MLE with the highest degree of belief, Tabs. 12 and 11 indicate that if the strength and durability class is unknown, and the producer and plant is known, the mean and coefficient of variation of the cube strength respectively, can be estimated with a reasonable degree of belief according to

$$\mu_{\mathrm{c,cube}} \approx \exp\left(\ln f_{\mathrm{ck,cube}} + \delta_{\mathrm{MLE}}\right) = 1.27 f_{\mathrm{ck,cube}} \tag{39}$$

and

$$V_{\mathrm{c,cube}} \approx s_{\mathrm{MLE}} = 0.09 \quad . \tag{40}$$

Since samples with low sample standard deviations tend to dominate, and the between-plant and -producer variance is not taken into account, the general MLE of the variance was smaller than the general estimate in Sec. 4.2.

4.4 Within-batch variation

For estimating the within-batch variation, the samples of observations for each recipe were treated separately, i.e. without combining samples from different recipes as in Sec. 4.3. The sample variance and the sample size of recipe i were input to Eq. (36) to obtain the estimator for the within-batch variation of recipe i, $s_{\mathrm{wb},i}^2$. Furthermore, the value of $s_{\mathrm{wb},i}^2$ for each recipe obtaining a specified strength class or combination of strength and durability class was input to Eqs. (33), (29) and (30). Tabs. 13 and 14 show the results per combination of strength and durability class and per strength class respectively. Note that Tab. 13 indicates for example that at the 17 plants considered in the present work, there were $m = 69$ recipes for obtaining a B45M40. The estimates of the standard deviation of the within-batch variation ranged from 0.03 for B45SV30 to 0.07 for B20M90, and from 0.05 for B25-B55 to 0.07 for B20. The general estimate of the within-batch variation was comparable to results from the literature [5,11,12].

[TABLE 13]

[TABLE 14]

4.5 General probability distribution

Based on the preceding sections, a general probability distribution was established, as summarized in Tab. 15. Considering Tabs. 8 and 12, the mean compressive cube strength was taken as

$$\mu_{\mathrm{c,cube}} \approx \exp\left(\ln f_{\mathrm{ck,cube}} + \mathrm{E}[\delta]\right) = f_{\mathrm{ck,cube}} \exp(0.25) \approx 1.28 f_{\mathrm{ck,cube}} \quad , \tag{41}$$

where $f_{\mathrm{ck,cube}}$ is the target cube strength from Tab. 1.

According to Tab. 8, the coefficient of variation of the cube strength at the highest level of the hierarchy can be taken as

$$V_{\mathrm{c,cube}} \approx \sqrt{\mathrm{VAR}[y]} = 0.13 \quad , \tag{42}$$

and assuming that Tabs. 11 and 14 represent the within-plant and within-batch variation respectively, the respective coefficients of variation can be taken as $V_{\mathrm{wp}} \approx s_{\mathrm{MLE,gen}} = 0.09$ and $V_{\mathrm{wb}} \approx s_{\mathrm{wb,MLE,gen}} = 0.05$. The total coefficient of variation can be given as

$$V_{\mathrm{c,cube}} = \sqrt{V_{\mathrm{wb}}^2 + V_{\mathrm{bbr}}^2 + V_{\mathrm{bpp}}^2} = 0.13 \quad , \tag{43}$$

where $V_{\mathrm{bbr}} = \sqrt{V_{\mathrm{wp}}^2 - V_{\mathrm{wb}}^2} \approx 0.08$ is the between-batch and -recipe variation and $V_{\mathrm{bpp}} = \sqrt{V_{\mathrm{c,cube}}^2 - V_{\mathrm{wp}}^2} \approx 0.09$ is the between-plant and -producer variation.

[TABLE 15]

5. APPLICATION EXAMPLES

5.1 General remarks

Two examples are introduced to demonstrate the application of the results: one existing and one new structure. Stewart [10] suggests that the compressive strength in a structure can be calculated based on the lab-strength of cylinders according to

$$f_{\mathrm{c}} = k_{\mathrm{cp}} k_{\mathrm{cr}} f_{\mathrm{c,lab}} \quad , \tag{44}$$

where k_{cp} and k_{cr} are factors considering the effects of compaction and curing respectively. It was assumed that the concrete was placed with fair performance of the compaction and exposed to fair curing conditions for at least seven days, resulting in mean values $\mu_{\mathrm{cp}} = 0.87$ and $\mu_{\mathrm{cr}} =$

1.00, and coefficients of variation $V_{\mathrm{cp}} = 0.06$ and $V_{\mathrm{cr}} = 0.05$. With a poor level of compaction and poor curing conditions, the mean values and the coefficients of variation of the factors decrease and increase, respectively. A nominal ratio between the cylinder and cube strength of 0.85 was used, and the coefficient of variation of the lab-strength of cylinders was assumed to be properly described by the one estimated for cubes, justified by evidence in the literature reviewed in the introduction. The compressive strength in the structure was thus represented by a log-normally distributed variable with mean

$$\mu_{\mathrm{c}} = 0.85 \cdot 0.87 \cdot 1.00 \cdot \mu_{\mathrm{c,cube}} \approx 0.74 \exp(\mathrm{E}[y]) \tag{45}$$

and coefficient of variation

$$V_{\mathrm{c}} = \sqrt{0.06^2 + 0.05^2 + V_{\mathrm{c,cube}}^2} \approx \sqrt{0.006 + \mathrm{VAR}[y]} \quad , \tag{46}$$

assuming that Eqs. (25) and (26) are valid approximations. It should be noted that the ratio between the cylinder and cube strength also has a significant coefficient of variation [30-35] which should be taken into account in Eq. (46) if a detailed strength prediction is necessary. However, for the present application examples, the nominal ratio was assumed sufficient.

5.2 Example 1: Existing structure

The sample of six cores drilled from an existing structure presented by Steenbergen & Vervuurt [43] was considered. The sample mean and standard deviation of the natural logarithm of the core strengths were $\bar{y} = 4.40$ and $s = 0.12$, and with six observations, $\nu = 5$. Assuming a non-informative prior, Eq. (14) can be used to estimate the lower 5%-fractile of the cylinder strength as

$$f_{\mathrm{c},0.05} = \exp\left(\bar{y} - t_{0.05,5}\, s\sqrt{\frac{7}{6}}\right) = 62.7\ \mathrm{MPa} \quad . \tag{47}$$

Assuming that the generalized prior data for the variance given in Tab. 12 is valid for the population of concrete from which the six cores originate, Eqs. (23) and (24) update the prior data for the variance, added the contributions from compaction and curing, according to

$$\nu'' = \nu_{\mathrm{MLE,gen}} + \nu = 9.1 \tag{48}$$

and

$$s'' = \sqrt{\frac{\nu_{\mathrm{MLE,gen}}\left(s_{\mathrm{MLE,gen}}^2 + V_{\mathrm{cp}}^2 + V_{\mathrm{cr}}^2\right) + \nu s^2}{\nu''}} = 0.12 \quad . \tag{49}$$

Eq. (49) is derived from Eq. (24), assuming that the prior value of the variance can be given by $s'^2 = s_{\mathrm{MLE,gen}}^2 + V_{\mathrm{cp}}^2 + V_{\mathrm{cr}}^2$. The estimated lower 5%-fractile of the cylinder strength becomes

$$f_{\mathrm{c},0.05} = \exp\left(\bar{y} - t_{0.05,9.1} s'' \sqrt{\frac{11.1}{10.1}}\right) = 64.8\ \mathrm{MPa} \quad . \tag{50}$$

The influence of the prior data on the updated standard deviation was small since the values of the sample standard deviation and the prior were similar. However, the prior data increased the *information content* in the posterior prediction, shown in Fig. 3, resulting in a 3% increase of the estimated lower 5%-fractile of the cylinder strength.

[FIGURE 3]

5.3 Example 2: New structure

A structure was assumed to be designed with a concrete B45M40. At an early stage in the design process, it is reasonable to consider the whole population of B45 when estimating the design compressive strength, i.e. the strength class level in the hierarchical model in Fig. 1. Considering Tab. 8 and the assumptions above, the mean compressive strength becomes

$$\mu_{\mathrm{c}} = 0.74 \exp(\mathrm{E}[y]) = 49.8\ \mathrm{MPa} \quad , \tag{51}$$

the total coefficient of variation becomes

$$V_{\mathrm{tot,c}} = \sqrt{V_{\theta}^2 + V_{\mathrm{G}}^2 + V_{\mathrm{c}}^2} = \sqrt{0.05^2 + 0.05^2 + (0.006 + \mathrm{VAR}[y])} = 0.15 \quad , \tag{52}$$

and the design compressive strength becomes

$$f_{\mathrm{c,des}} = \mu_{\mathrm{c}} \exp\left(-\alpha_{\mathrm{R}} \beta V_{\mathrm{tot,c}}\right) = 31.4\ \mathrm{MPa} \quad , \tag{53}$$

assuming that V_{θ}, V_{G} and $\alpha_{\mathrm{R}}\beta$, attain the values from Eq. (1). Comparing the design strength with the target cylinder strength from Tab. 1, f_{ck}, gives an effective partial material factor

$$\gamma_{\mathrm{c,eff.}} = \frac{f_{\mathrm{ck}}}{f_{\mathrm{c,des}}} = 1.44 \quad , \tag{54}$$

which could be compared with $\gamma_{\mathrm{c}} = 1.5$ from Eq. (1). By including more information by moving downwards in the hierarchical model, and considering Tabs. 7 and 6, the results in Tab. 16 are obtained. The different estimates of $\gamma_{\mathrm{c,eff.}}$ are results of considering different subpopulations of B45 and B45M40, having different mean values and coefficients of variation. If a sufficient amount of information is made available, it could be possible to move further downwards in the hierarchy, and possibly exclude both between-plant and between-recipe variation.

[TABLE 16]

6. CONCLUSION

By studying strength recordings from Norwegian ready-mixed concrete plants, the variability of the compressive cube strength has been quantified on different hierarchical levels. The highest studied level of the hierarchy was the strength class, which represents the entry point of information in the design process. During design of a new structure, the designer specifies a certain strength and durability class, and the fact that the producer, plant, recipe and batch is unknown is reflected in the coefficient of variation at the highest level of the hierarchy. The presented results are easily combined with additional data, e.g. from the European market, or from a supply controlled by a different regional standard.

It has been demonstrated how the level of knowledge of the designer influences the uncertainty that must be taken into account, and thus the estimated design compressive strength in the structure. With today's diversity in assessment methods both with regard to structural behaviour and uncertainty differentiation, and the strong focus on reassessments of existing structures, data on a form similar to what has been presented herein could be considered included in future design codes. This could stimulate to a safe use of advanced assessment methods, with an aim to reduce unnecessary conservatism and increase the competitiveness of concrete.

The scope of the present work was to estimate the variation resulting from what the designer can control. The results indicate that the designer should specify a strength class that utilizes the strength potential of the durability class, and avoid combinations like e.g. B35M40 and B25M60, where a resulting over-strength could introduce a safety margin, but also unintended variation in the population. A closer collaboration between the designer, contractor and producer is expected to result in improved concrete specifications. A natural continuation of this work could be to address the influence of the different constituents on the estimated variation, i.e. study the variation from a producer's point of view with the possible aim of reducing unintended variation, and obtaining a more homogenous population of concrete.

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Table 1: *Relation between target cylinder strength, f_{ck}, and cube strength, $f_{ck,cube}$, for the strength classes given in Eurocode 2, EN 206 and* fib *Model Code 2010.*

	B10	**B20**	**B25**	**B30**	**B35**	**B45**	**B55**	**B65**	**B75**	**B85**	**B95**
f_{ck} [MPa]	10	20	25	30	35	45	55	65	75	85	95
$f_{\mathrm{ck,cube}}$ [MPa]	12	25	30	37	45	55	67	80	90	100	110

Table 2: *Description of the levels of the hierarchical model. The right column indicates that if a sample of observations at one level in the hierarchy is considered, one can make inference about the between-variation on that level and a contribution to the within-variation on the next higher level.*

Hierarchical level	**Inference from a sample of observations at the respective level**
Standard test specimens from one batch	Within batch
Batch	Within recipe / between batch
Recipe	Within plant / between recipe
Concrete plant	Within producer / between plant
Concrete producer	Within durability class / between producer
Durability class	Within strength class / between durability class
Strength class	Within region / between strength class
Supply controlled by regional standard	Within the gross supply / between region
The gross supply	-

Table 3: *Marginal posterior distributions, $f(\cdot\,|\boldsymbol{y})$, expected values, $E[\cdot]$, and variances, $VAR[\cdot]$, for μ, σ^2 and y, starting from a non-informative prior distribution.*

Variable	$f(\cdot\,\vert\boldsymbol{y})$	$\mathrm{E}[\cdot]$	$\mathrm{VAR}[\cdot]$
μ	$t\left(\mu \middle\vert \bar{y}, s^2\frac{1}{\nu+1}, \nu\right)$	$\bar{y}$	$\frac{\nu}{(\nu-2)(\nu+1)}s^2$
σ^2	$\text{Inv-}\chi^2(\sigma^2\vert\nu s^2, \nu)$	$\frac{\nu}{\nu-2}s^2$	$\frac{2\nu^2}{(\nu-2)^2(\nu-4)}s^4$
y	$t\left(y \middle\vert \bar{y}, s^2\frac{\nu+2}{\nu+1}, \nu\right)$	$\mathrm{E}[\mu]$	$\mathrm{VAR}[\mu]+\mathrm{E}[\sigma^2]$

Table 4: *Correction factor for Eq. (30) found by including higher order terms of $\partial\, ln\, \Gamma(\nu/2)/\partial\nu$ in the numerical solution of Eq. (30).*

$\frac{1}{\ln A - B}$	0.5	1.0	1.5	2.0	3.0	5.0	10.0	20.0	40.0
$f\left(\frac{1}{\ln A - B}\right)$	2.42	1.44	1.20	1.14	1.10	1.06	1.03	1.02	1.01

Table 5: *Overview of the producers and plants that have contributed to the study.*

Producer	Plant	Observations
A	1	925
	2	751
	3	679
	4	419
	5	358
	6	258
B	1	612
	2	996
	3	543
	4	350
	5	479
	6	564
C	1	2065
	2	880
	3	841
	4	1698
	5	1789
Total		14207

Table 6: *Posterior inference per producer for different combinations of strength and durability classes, where $E[y]$ and $VAR[y]$ are the expected value and the variance of the posterior prediction of $y = ln\, f_{c,cube}$, s is the sample standard deviation, n is the sample size and $f_{c,cube,0.5}$ and $f_{c,cube,0.05}$ is the median and characteristic value of the posterior prediction in MPa. s_w^2/s^2 indicates the contribution from the within-plant variance to the total variance of the combination of strength and durability class at the respective producer.*

	$\mathrm{E}[y]$	$\sqrt{\mathrm{VAR}[y]}$	s	n	s_w^2/s^2	$f_{\mathrm{c,cube,0.5}}$	$f_{\mathrm{c,cube,0.05}}$
Producer A							
B20M90	3.52	0.13	0.13	33	0.40	33.7	26.8
B25M60	3.78	0.10	0.10	20	0.59	43.6	36.3
B25M90	3.59	0.10	0.09	33	0.90	36.4	30.8
B30M60	3.86	0.12	0.12	1076	0.81	47.4	39.2
B35M40	4.22	0.10	0.10	179	0.94	67.9	57.1
B35M45	4.05	0.12	0.12	409	0.95	57.5	47.5
B35M60	3.94	0.06	0.05	8	1.00	51.2	45.5
B35MF40	4.17	0.11	0.11	183	0.79	64.8	54.4
B35MF45	4.03	0.11	0.11	308	0.98	56.5	47.0
B35SV40	4.21	0.11	0.11	162	0.86	67.7	56.0
B45M40	4.21	0.10	0.10	235	0.89	67.3	56.7
B45MF40	4.20	0.10	0.10	212	0.97	66.7	56.6
B45SV30	4.19	0.10	0.10	68	1.00	65.9	55.5
B45SV40	4.24	0.11	0.11	458	0.77	69.4	58.2
B55M40	4.30	0.09	0.07	6	1.00	74.0	60.5
Producer B							
B20M90	3.55	0.12	0.11	16	0.69	34.9	28.0
B25M90	3.61	0.18	0.17	54	0.98	37.1	27.5
B30M60	3.81	0.12	0.12	956	0.78	45.0	36.8
B30MF45	3.98	0.11	0.07	5	1.00	53.6	41.8
B35M40	4.09	0.14	0.13	120	0.36	60.0	47.9
B35M45	4.03	0.13	0.13	466	0.58	56.3	45.5
B35MF40	4.09	0.19	0.18	59	0.34	59.5	43.3
B35MF45	4.01	0.12	0.12	276	0.66	55.2	45.4
B35SV30	4.10	0.08	0.04	4	1.00	60.2	48.4
B35SV40	4.14	0.11	0.11	62	0.47	62.7	51.7
B45M40	4.21	0.13	0.13	161	0.44	67.3	54.3
B45MF40	4.15	0.12	0.12	332	0.68	63.3	52.0
B45SV30	4.24	0.10	0.09	49	0.38	69.7	59.1
B45SV40	4.18	0.10	0.10	824	0.73	65.6	55.8
B55SV40	4.38	0.10	0.10	160	0.96	80.0	68.2
Producer C							
B20M90	3.59	0.11	0.11	131	0.72	36.3	30.1
B30M60	3.83	0.10	0.10	2363	0.91	45.9	38.8
B35M40	4.27	0.10	0.10	458	0.91	71.4	60.2
B35M45	4.13	0.10	0.10	1138	0.79	62.5	52.6
B35MF40	4.16	0.12	0.12	120	0.66	64.0	52.7
B35MF45	4.02	0.11	0.11	464	0.86	55.6	46.5
B35SV40	4.25	0.14	0.14	280	0.71	69.9	55.8
B45M40	4.30	0.10	0.10	343	0.91	73.4	61.7

B45M45	4.22	0.11	0.11	52	0.82	68.2	56.4
B45MF40	4.17	0.11	0.11	161	0.88	64.8	53.9
B45MF45	4.15	0.09	0.08	34	0.92	63.4	54.4
B45SV40	4.22	0.10	0.10	1729	0.88	68.4	57.7

Table 7: *Posterior inference per combination of strength and durability classes, where the variables are defined in Tab. 6.* s_w^2/s^2 *indicates the contribution from the within-producer variance to the total variance of the combination of strength and durability class.*

	$\mathrm{E}[y]$	$\sqrt{\mathrm{VAR}[y]}$	s	n	s_w^2/s^2	$f_{\mathrm{c,cube,0.5}}$	$f_{\mathrm{c,cube,0.05}}$
B20M90	3.57	0.12	0.12	180	0.94	35.7	29.3
B25M60	3.78	0.10	0.10	20	1.00	43.6	36.3
B25M90	3.61	0.15	0.15	87	1.00	36.8	28.7
B30M60	3.83	0.11	0.11	4395	0.97	46.1	38.3
B30MF45	3.98	0.11	0.07	5	1.00	53.6	41.8
B35M40	4.23	0.13	0.12	757	0.75	68.6	55.8
B35M45	4.09	0.12	0.12	2013	0.85	60.0	49.0
B35M60	3.94	0.06	0.05	8	1.00	51.2	45.5
B35MF40	4.15	0.13	0.13	362	0.94	63.6	51.5
B35MF45	4.02	0.11	0.11	1048	0.99	55.8	46.4
B35SV30	4.10	0.08	0.04	4	1.00	60.2	48.4
B35SV40	4.22	0.13	0.13	504	0.93	68.3	55.0
B45M40	4.25	0.12	0.12	739	0.87	70.1	57.7
B45M45	4.22	0.11	0.11	52	1.00	68.2	56.4
B45MF40	4.17	0.11	0.11	705	0.96	64.6	53.6
B45MF45	4.15	0.09	0.08	34	1.00	63.4	54.4
B45SV30	4.21	0.10	0.10	117	0.96	67.5	56.9
B45SV40	4.22	0.10	0.10	3011	0.96	67.7	57.1
B55M40	4.30	0.09	0.07	6	1.00	74.0	60.5
B55SV40	4.38	0.10	0.10	160	1.00	80.0	68.2

Table 8: *Posterior inference per strength class, where the variables are defined in Tab. 6.* s_w^2/s^2 *indicates the contribution from the within-durability class variance to the total variance of the strength class. The general posterior predictive inference is for the variable* $\delta = y - ln\, f_{ck,cube}$.

	$\mathrm{E}[y]$	$\sqrt{\mathrm{VAR}[y]}$	s	n	s_w^2/s^2	$f_{\mathrm{c,cube,0.5}}$	$f_{\mathrm{c,cube,0.05}}$
B20	3.57	0.12	0.12	180	1.00	35.7	29.3
B25	3.64	0.16	0.15	107	0.81	38.0	29.3
B30	3.83	0.11	0.11	4400	1.00	46.1	38.3
B35	4.12	0.14	0.14	4696	0.72	61.4	48.5
B45	4.21	0.11	0.11	4658	0.96	67.6	56.4
B55	4.38	0.10	0.10	166	0.98	79.8	68.0
General	0.25	0.13	-	-	-	-	-

Table 9: *MLE for different combinations of strength and durability class.*

	$\bar{y}_{\mathrm{MLE}}$	n_{MLE}	s_{MLE}	ν_{MLE}	m
B20M90	3.54	1.4	0.09	6.7	12
B25M60	3.77	1.8	0.07	6.5	2
B25M90	3.65	5.2	0.05	1.3	5
B30M60	3.83	3.8	0.10	21.4	17
B35M40	4.18	0.8	0.08	6.5	14
B35M45	4.06	1.6	0.09	7.2	17
B35MF40	4.14	0.8	0.09	7.0	14
B35MF45	4.01	2.8	0.09	8.7	15
B35SV40	4.21	1.0	0.08	4.7	12
B45M40	4.21	1.0	0.08	5.7	17
B45M45	4.19	5.7	0.07	2.9	4
B45MF40	4.17	2.5	0.09	11.6	15
B45MF45	4.14	8.1	0.09	33.8	2
B45SV30	4.24	0.9	0.06	11.0	5
B45SV40	4.21	3.4	0.08	9.5	16
B55SV40	4.36	9.8	0.06	3.8	3
General	0.26	0.6	0.08	5.0	170

Table 10: *Prior data for the cube strength of ready-mixed concrete as suggested by Rackwitz [16] assuming a log-normal distribution. $\bar{y}'$ and s' represent the prior knowledge about the mean and standard deviation, and n' and ν' are the degree of belief in $\bar{y}'$ and s' respectively.*

	$\bar{y}'$	n'	s'	ν'
C15	3.40	1.5	0.14	6.0
C25	3.65	1.5	0.12	6.0
C35	3.85	1.5	0.09	6.0
C45	3.98	1.5	0.07	6.0

Table 11: *MLE for different durability classes.*

	$\bar{y}_{\mathrm{MLE}}$	n_{MLE}	s_{MLE}	ν_{MLE}	m
M40	4.19	1.1	0.08	10.4	17
M45	4.06	1.6	0.09	7.4	17
M60	3.83	4.1	0.10	21.9	17
M90	3.57	2.4	0.10	6.8	13
MF40	4.17	2.2	0.09	10.7	16
MF45	4.01	2.8	0.09	8.3	15
SV30	4.24	0.8	0.06	11.2	5
SV40	4.21	3.2	0.09	8.0	16
General	-	-	0.09	7.7	116

Table 12: *MLE for different strength classes.*

	$\bar{y}_{\mathrm{MLE}}$	n_{MLE}	s_{MLE}	ν_{MLE}	m
B20	3.54	1.4	0.09	6.7	12
B25	3.67	1.2	0.06	1.4	7
B30	3.83	3.9	0.10	21.3	17
B35	4.07	2.2	0.11	22.8	17
B45	4.19	2.2	0.09	20.4	17
B55	4.35	4.9	0.06	5.0	4
General	0.24	1.1	0.09	4.1	74

Table 13: *MLE of the within-batch variation for different combinations of strength and durability class.*

	$s_{\mathrm{wb,MLE}}$	$v_{\mathrm{wb,MLE}}$	m
B20M90	0.07	3.8	26
B25M60	0.04	3.2	3
B25M90	0.05	1.4	16
B30M60	0.05	1.6	129
B35M40	0.05	1.9	59
B35M45	0.04	1.4	105
B35MF40	0.05	1.6	49
B35MF45	0.06	2.6	72
B35SV40	0.05	2.1	40
B45M40	0.05	2.0	69
B45M45	0.05	2.1	8
B45MF40	0.05	1.7	56
B45MF45	0.06	11.4	5
B45SV30	0.03	1.4	11
B45SV40	0.04	1.6	91
B55SV40	0.05	3.9	6
General	0.05	1.6	745

Table 14: *MLE of the within-batch variation for different strength classes.*

	$s_{\mathrm{wb,MLE}}$	$v_{\mathrm{wb,MLE}}$	m
B20	0.07	3.8	26
B25	0.05	1.5	19
B30	0.05	1.6	130
B35	0.05	1.6	327
B45	0.05	1.6	240
B55	0.05	4.2	7
General	0.05	1.6	749

Table 15: *Parameters for the general probability distribution for the compressive cube strength. $f_{ck,cube}$ is the target cube strength.*

Mean	$\mu_{\mathrm{c,cube}} = 1.28 f_{\mathrm{ck,cube}}$
Total variation	$V_{\mathrm{c,cube}} = \sqrt{V_{\mathrm{wb}}^2 + V_{\mathrm{bbr}}^2 + V_{\mathrm{bpp}}^2} = 0.13$
Within-batch variation Between-batch and -recipe variation Between-plant and -producer variation	$V_{\mathrm{wb}} = 0.05$ $V_{\mathrm{bbr}} = 0.08$ $V_{\mathrm{bpp}} = 0.09$

Table 16: *Results from Example 2, demonstrating the effect of including more information in the estimate of the design compressive strength.* μ_c *is the mean strength in the structure,* $V_{tot,c}$ *is the total coefficient of variation,* $f_{c,des}$ *is the design compressive strength and* $\gamma_{c,eff.}$ *is the effective partial material factor according to Eqs. (51) to (54).*

Knowledge	μ_{c} [MPa]	$V_{\mathrm{tot,c}}$ [-]	$f_{\mathrm{c,des}}$ [MPa]	$\gamma_{\mathrm{c,eff.}}$ [-]
B45	49.8	0.15	31.4	1.44
B45M40	51.8	0.16	31.9	1.41
B45M40, Prod. A	49.8	0.15	32.0	1.40
B45M40, Prod. B	49.8	0.17	30.0	1.50
B45M40, Prod. C	54.5	0.15	35.1	1.28

Figure 1: *Hierarchical model for the variability of material properties in concrete. The examples to the right in the figure indicates the application of the estimators in Sec. 3.1.*

a) B35, note that B35M60 and B35SV30 were left out of the figure due to the low numbers of observations.	*b) B45.*

Figure 2: *Posterior predictions according to Tabs. 7 and 8 and Eq. (14) for the combinations of strength and durability classes. The solid lines indicate the target cube strength and the squares and the triangles indicate the median and the lower 5%-fractile of the posterior predictive distributions respectively.*

Figure 3: *Posterior predictive distributions based on non-informative (dashed) and informative (solid) prior distributions for the cylinder strength in Example 1.*

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