Statistical Modelling of Financial Risk

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Preface

The business of finance becomes constantly more complex, requiring more advanced statistical tools. Moreover, due to new international regulations, it is more important than ever for financial institutions to understand and measure their risk. The topic of this thesis is to develop new statistical tools for several specific financial applications. The main focus is modelling of risk.

The thesis consists of an introduction followed by six papers. Each paper can be read independently of the others. However, the thesis can be divided into two main parts. First a theoretical part, consisting of papers I-III, and then a more application-oriented part consisting of papers IV-VI. Risk management confronts us with heavy-tailed risks, rapid changes and complex interdependencies, which force us to go beyond standard statistical models to develop more sophisticated methodology. Part I treats two themes that are especially important; non-normal distributions (papers I and II) and dependency modelling (paper III). These two statistical themes are very important for a huge set of applications, and in the second part of the thesis, two specific practical problems from finance are treated. In papers IV and V we present a model for measuring total risk for a financial institution, while paper VI develops a joint model for a portfolio of electricity forward products.

To summarise, this thesis is a collection of the following six publications:

Paper I K. Aas and I. Hobæk Haff, "The Generalised Hyperbolic Skew Students t-distribution", *Journal of Financial Econometrics*, Vol 4, pp 275-309, 2006.

Paper II K. Aas, I. Hobæk Haff and X. K. Dimakos, "Risk Estimation using the Multivariate Normal Inverse Gaussian Distribution", *Journal of Risk*, Vol 8, pp. 39-60 Winter 2005/2006.

Paper III K. Aas, C. Czado, A. Frigessi and H. Bakken, "Pair-copula constructions of multiple dependence", *Insurance: Mathematics and Economics*, 2007.

Paper IV X. K. Dimakos and K. Aas, "Integrated risk modeling" , *Statistical modeling*, Vol. 4, pp. 265-277, 2004.

Paper V K. Aas, X. K. Dimakos and A. Øksendal, "Risk Capital Aggregation", *Risk Management*, Vol. 9, pp. 82-107, 2007.

Paper VI K. Aas and K. Kåresen, "The Matrix", *Energy Power Risk Management*, Vol. 9, pp. 50-55, 2004.

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Further, I am grateful to Håvard Rue, who introduced me to the concept of vines, and also helped Ingrid Hobæk Haff and me writing the skew-t paper. Moreover, he is the one who convinced me to finally put together a PhD thesis after all these years.

On a more personal level I want to thank my parents for endless devotion and support, and my sisters with families for always being there for me. Finally, my friends deserve a big thank you for indirectly contributing to this thesis by filling my life with happiness and pleasure.

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Contents

natural disasters and criminal acts). In addition, credit and insurance risk are the major risks for banks and insurers, respectively, while commodity price risk and production risk are the most important for oil and electricity companies.

Risk management confront us with heavy-tailed risks, rapid changes and complex interdependencies, which force us to go beyond simplifying assumptions in standard statistical models to develop more sophisticated methodology. Two themes are especially important, non-normal distributions and dependency modelling. Very few risk factors are normally distributed. The distributions of financial returns, oil and energy prices, operational losses and insurance claims all have heavier tails. Moreover, some of them are skewed, with one of the tails heavier than the other. In this thesis we produce new models to study heavy-tailed and skewed phenomena.

Appropriate modelling of dependencies is very important. Examples are pricing credit derivative products referencing a portfolio of underlying assets, understanding the relationships among different lines of business for an insurer, and maximizing the profit of a power-generating plant (need a joint model for the prices of power and the fuel, usually gas). The dependence structure is non-linear, meaning that novel methods for capturing dependency must be produced. One class of alternatives is copula-based approaches. In this thesis we develop new promising ways to construct multivariate distributions from smaller components. Such constructions are particularly useful when looking to extreme behaviour.

This thesis can be divided into two main parts. First a theoretical part, consisting of papers I-III, and then a more application-oriented part consisting of papers IV-VI. Part I treats the two statistical themes described above.

These themes are very important for a huge set of applications. In the second part of the thesis, two specific practical problems from finance are treated; modelling of the total risk for a financial institution, and developing a joint model for a portfolio of electricity forward products. In what follows, we briefly describe the different concepts dealt with in the different papers.

Part I: Non-normal distributions and dependency structures

Skewed and heavy-tailed distributions

It is a well-known fact that returns from financial market variables such as exchange rates, equity prices, and interest rates, measured over short time intervals, i.e. daily or weekly are characterised by non-normality. The empirical distribution of such returns is also more peaked and has heavier tails than the normal distribution. This implies that very large changes in returns occur with a higher frequency than under normality. In addition it is often skewed with a heavier left tail, indicating that big losses are more frequent than big gains of the same magnitude.

The generalised hyperbolic (GH) class of distributions is a promising alternative for such returns. This class was introduced by Barndorff-Nielsen (1977) in connection with a study of grains of sand. The GH distributions possess a number of attractive properties, e.g. they are closed under conditioning, marginalisation and affine transformations. They can be both symmetric and skew, and their tails are heavier than those of the normal distribution. While several specific subclasses of the GH distribution have been applied

Introduction

With the advent of new financial instruments and new risk structures, the business of finance becomes more complex, asking for advanced statistical tools. Further, under pressure from shareholders and with the international changing regulatory environment, it is more important than ever for banks and insurance companies to understand and evaluate their risk exposures.

In banking, the Basel II international capital framework creates heavy demands for statistical methods. The continuing evolution of financial products characterised by increasing complexity, such as collateralised debt obligations and credit derivatives, underlines the need to improve knowledge and practice in the area of risk. Similar demands arise from the new reporting standards IFRS in accountancy, which will require mathematically sound valuation of derivatives, executive share options and intangibles. In insurance, Solvency II introduces a need for better methods of quantifying risk beyond traditional actuarial models.

The topic of this thesis is to develop new statistical tools for several specific financial applications. The main focus is modelling of risk. Most companies are exposed to market risk (e.g. due to fluctuations in interest rates and currencies) and operational risk (the risk of loss resulting from inadequate or ineffective internal processes and systems, and from external events such as in various situations, the distribution itself is very seldom used in practical applications. This is probably due to the fact that it is not particularly analytically tractable, and that it is very challenging to estimate its parameters, especially the parameter that determines the subclass. Even for very large sample sizes, it may be hard to make a distinction between different values of the subclass parameter because of the flatness of the GH likelihood function in this parameter. See for instance Prause (1999).

The purpose of paper I is to argue for a special case of the GH family that we denote as the GH skew Student's t-distribution. This distribution has the important property that one tail has polynomial, and the other exponential behaviour. Further, it is the only subclass of the GH family of distributions having this property. Although the GH skew Student's t-distribution has been previously proposed in the literature, it is not well known, and specifically, its special tail behaviour has not been addressed. Paper I presents empirical evidence of exponential/polynomial tail behaviour in skew financial data, and demonstrates the superiority of the GH skew Student's t-distribution with respect to data fit, compared with some of its competitors. Through Value-at-Risk (VaR) and expected shortfall calculations we show why the exponential/polynomial tail behaviour is important in practice.

In addition to non-normality, a stylised fact of financial returns is volatility clustering. Volatility clustering means that small changes in the price tend to be followed by small changes, and large changes by large ones. The success of the GARCH class of models (Bollerslev 1986) at capturing volatility clustering in financial markets is extensively documented. Recent surveys are given in Ghysels et al. (1996) and Shepard (1996). On the other hand, it is well recognized that GARCH models, coupled with the assumption of conditionally normally distributed errors, are unable to fully account for the tails of the distributions of daily returns. In paper I we show that the GH skew Student's t-distribution also performs very well as the conditional distribution of a GARCH(1,1)-model.

Appropriate modelling of time-varying dependencies is very important for quantifying financial risk, such as the risk associated with a portfolio of financial assets. Most of the papers analysing financial returns have focused on the univariate case. The few that are concerned with their multivariate extensions are mainly based on the multivariate normal assumption. GARCH models have been extended to the multivariate case, see Bauwens et al. (2006) for a survey, and the idea of paper II is to use the multivariate normal inverse Gaussian (MNIG) distribution as the conditional distribution for a multivariate GARCH model.

Like the GH skew Student's t-distribution, the MNIG distribution is a subclass of the GH distribution. The MNIG distribution possesses a number of attractive theoretical properties that are not shared by the GH distribution in general, among others its analytical tractability. Moreover, maximum likelihood estimation of its parameters is quite straightforward using the EM-algorithm (Dempster et al. 1977). In paper II we show that these features make the MNIG-GARCH model very useful for practical applications, such as forecasting VaR of a portfolio of equities.

Dependency structures

Understanding and quantifying dependence is the core of all modelling efforts in financial econometrics. The linear correlation coefficient, which is by far the most used measure to test dependence in the financial community (and also elsewhere), is not a measure of general, but only of *linear* dependence.

If asset returns are well represented by an elliptical distribution, such as the multivariate Gaussian or the multivariate Student's t, their dependence structure is linear. Hence, the linear correlation coefficient is a meaningful measure of dependence. Outside the world of elliptical distributions, however, the use of the linear correlation coefficient as a measure of dependence may induce misleading conclusions. In financial markets, there is often a non-linear dependence between returns. Thus alternative methods for capturing co-dependency should be considered, such as copula-based ones. Copulas are used to combine marginal distributions into multivariate distributions.

The concept of copulas was introduced by Sklar (1959), and has for a long time been recognized as a powerful tool for modelling dependence between random variables. The use of copula theory in financial applications is a relatively new (introduced by Embrechts et al. (1999)) and fast-growing field. A systematic development of the theory of copulas, particularly bivariate ones, with many examples is found in Nelsen (1999). A recent reference on copula methods in finance is Cherubini et al. (2004). From a practical point of view, the advantage of the copula-based approach is that appropriate marginal distributions for the components of a multivariate system can be selected freely, and then linked through a suitable copula. Hence, the dependence structure may be modelled independently of the marginal distributions.

As an example of how copulas may be successfully used, consider modelling the joint distribution of a stock market index and an exchange rate. The Student's t-distribution has been found to provide a reasonable fit to the univariate distributions of daily stock market as well as of exchange rate returns. Hence, it might seem natural to model the joint distribution with a bivariate Student's t-distribution. However, the standard bivariate Student's t-distribution has the restriction that both marginal distributions must have the same tail heaviness, while the distributions of daily stock market and exchange rate returns don't. If the multivariate distribution is decomposed into marginal distributions and a copula, we obtain better models of the individual variables than would be possible if only explicit multivariate distributions were considered.

The n-dimensional Student copula has been used repeatedly for modelling multivariate financial return data. A number of papers, such as Mashal & Zeevi (2002), have shown that the fit of this copula is generally superior to that of other n-dimensional copulas for such data. However, the Student copula has only one parameter for modelling tail dependence, independent of dimension. Hence, if the tail dependence of different pairs of risk factors in a portfolio are very different, not even the copula approach will allow for the construction of an appropriate model. In paper III, we extend the theory of copulas and show how multivariate data can be modelled using a cascade of pair-copulas, acting on two variables at a time. The model construction, which is inspired by by the work of Joe (1996) and Bedford & Cooke (2001, 2002), is hierarchical in nature. The various levels correspond to the incorporation of more variables in the conditioning sets, using pair-copulas as simple building blocks. Pair-copula decomposed models also represent a very flexible way to construct higher-dimensional copulas. In paper III we propose a method to perform inference for such pair-copula decompositions and apply the methodology to a financial data set. We compare a pair-copula decomposition with Student copulas for each pair to the ordinary Student copula, and show that the latter can be rejected in favour of the pair-copula decomposition.

Part II: Financial applications

Part II of the thesis treats two areas of applications for the methodologies which have been developed in part I. Both areas are concerned with risk management. We have developed models for

- Total risk for a financial institution
- A portfolio of electricity forward products

For both problems there are methodological challenges associated with modelling the mechanisms and distributions of risk, as well as dependency structures between the financial instruments underlying the risks.

Total risk for a financial institution

Risk aggregation refers to the task of incorporating multiple types or sources of risk into a single metric (Basel Committee on Banking Supervision 2003). Most financial institutions are exposed to credit, market and operational risk. Moreover, business risk, see e.g. Saita (2004), has grown as the structure of financial institutions continues to change. For marginal evaluation of credit and market risk, most financial institutions are equipped with advanced risk assessment software. For operational risk, loss databases and measurement methodologies are currently under development. Business risk, however, has so far received less attention, probably due to the fact that there is no minimum capital linked to it. Finally, up to now, there exists no state-of-the-art approach for aggregating the marginal risk types to the total risk. Risk managers struggle with a number of important issues, including weakly founded correlation assumptions, inconsistent risk metrics and differing time horizons for the different risk types.

In paper V, we present a model that aggregates the different risk types of a financial institution to assess the total risk. The proposed model includes components for credit, market, operational and business risk. Moreover it includes a component for the ownership risk that stems from holding a life insurance company. The approach may be characterised as a base-level aggregation method. However, due to lack of appropriate data, some of the aggregation steps are done on the top level instead. The economic risk factors used in the base-level aggregation are described by a multivariate GARCH model with Student's t-distributed innovations. The loss distributions for the different risk types are determined by non-linear functions of the fluctuations in the risk factors. Hence, these marginal loss distributions are indirectly correlated through the relationship between the risk factors. The model was originally developed for DnB NOR, the largest financial institution in Norway, and one of the largest ones in the Nordic region. It has been implemented in DnB NORs system for risk management, and being adapted to the requirements in the Basel II regulations, we also believe it to be applicable in a broader context. The model in paper V is the second generation of the total risk model used by DnB NOR. The first model was developed a few years earlier, and is described in paper IV.

Joint model for portfolio of electricity forward products

The players in the deregulated power markets are faced with challenging issues of production planning, portfolio optimisation, derivatives pricing and risk management. Because of the volatile and unpredictable nature of electricity prices, these challenges are naturally approached with stochastic price models. The major market players typically have large portfolios consisting of electricity forward products. In paper VI we show how to develop a joint model for different forward products. The purpose of the model is to be able to forecast the value of the covariance matrix of the portfolio, which is needed for computing the daily Value-at-Risk (VaR). The proposed model may in principle be used for forward prices from any market. In paper VI we have fitted it to EEX forward prices and Nordpool futures.

In the proposed method, the variance of next forward price return is modelled as a function of the variance of recent returns in an ARCH-like (Engle 1982) manner, and the time to maturity. Market imperfections, such as bid-ask spreads, are taken into account. The proposed model is easily fit to historical data, and the resulting variance-covariance matrix is ensured to be positive definite. Another advantage of the approach described in Paper VI is that it can be used to compute volatility and correlations for forward products without historical observations.

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Paper I

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